Conciliating absolute and relative poverty: Income poverty measurement with two poverty lines

IARIW Conference in Washington

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- strong income growth (WB, 2018)
  $\Rightarrow$ Absolute poverty $\searrow$
- more within-country inequality (Bourguignon, 2015; Milanovic, 2016; Ravallion, 2014)
  $\Rightarrow$ Relative poverty $\nearrow$

$\Rightarrow$ Evaluate progress with measure combining absolute and relative poverty

Mainstream measures yield debatable comparisons

- Mainstream measures combining both yield debatable poverty comparisons.

- Illustration: is Colombia as poor as Bangladesh in 2015?

  Measure:
  - ◇ Threshold $= \max(z_a, z_r)$
  - ◇ Head-count ratio (HC)

	mean income ($ a month)	z_a ($ a day)	z_r ($ a day)	HC_A (%)	HC_{oR} (%)	HC (%)
Bangladesh	116	1.9	2.4	15	14	29
Colombia	442	1.9	5.5	5	24	29

Note: z_r is Societal poverty line.

- Mainstream measures do not consider that absolute poverty status is more severe than (only) relative poverty status.

Debatable comparisons are due to index used

- A poverty measure has two elements
  - ◇ poverty line(s):
    - – identification of poverty status
  - ◇ poverty **index**:
    - – "prioritization"
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- Why?

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- Why? Literature on poverty **indices** assumes that poverty line is absolute.

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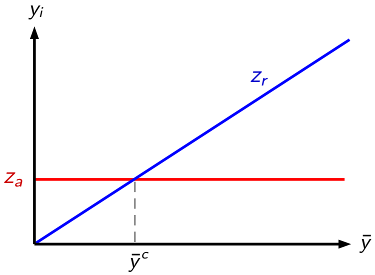
  - absolute line $z_a > 0$
  - relative line $z_r(\overline{y})$



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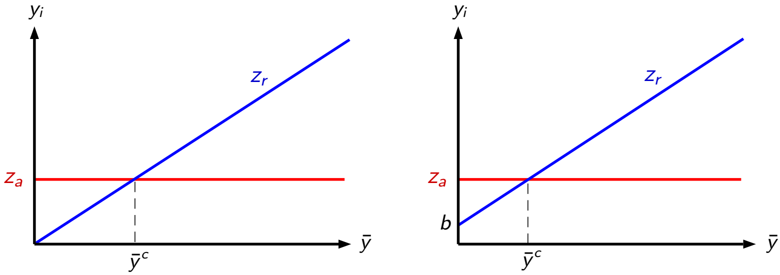

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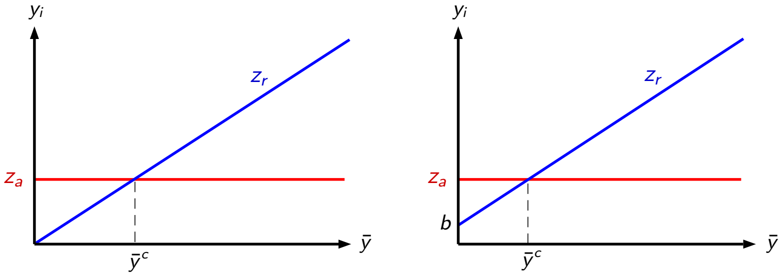

- Poverty status: i is **poor** if $y_i < \max\{z_a, z_r(\overline{y})\}$
  - ◇ **absolutely poor** if $y_i < z_a$
  - ◇ **only relatively poor** if $z_a \leq y_i < z_r(\overline{y})$.

Notation (ctn'd)

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- For fixed absolute threshold z_a and threshold function z_r, a poverty index $\mathrm{I} : Y \to \mathbb{R}_+$ represents a complete ranking on Y.
  - ◇ $\mathrm{I}(y) < \mathrm{I}(y')$ means that y' has more poverty than y.

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2. Index I is ordinally equivalent to

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(i) $p(0, \overline{y}) = 1$ and $p(y_i, \overline{y}) = 0$ if i is non-poor,
(ii) p is strictly decreasing in its first argument if i is poor,
(iii) p is continuous in both its arguments if $\overline{y} > \overline{y}^c$,

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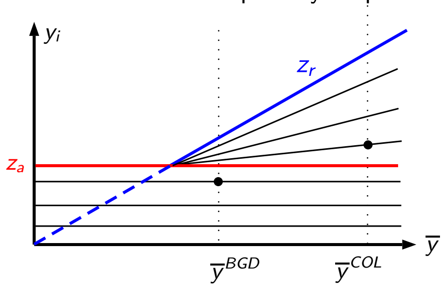
   (*iii*) p is continuous in both its arguments if $\overline{y} > \overline{y}^c$,
   (*iv*) p **is constant in its second argument if** i is absolutely poor.

Hierarchical indices consider that absolutely poor is poorer

Absolutely poor individuals are considered **poorer** than the only relatively poor.

Hierarchical iso-poverty map



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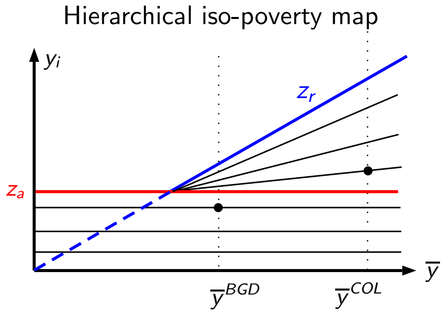

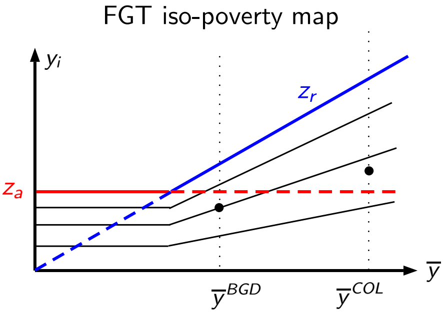

- FGT indices may consider that absolutely poor is **less poor**:

$$p_\alpha(y_i, \overline{y}) = \left(1 - \frac{y_i}{z(\overline{y})}\right)^\alpha \qquad \text{where} \qquad z(\overline{y}) = \max\{z_a, z_r(\overline{y})\}$$

New index generalizing the head-count ratio

The extended head-count ratio is defined as

$$p^{EHC}(y_i, \overline{y}) = \begin{cases} 1 & \text{if } \; y_i < z_a, \\ \frac{z_r(\overline{y}) - y_i}{z_r(\overline{y}) - z_a} & \text{if } \; z_a \leq y_i < z_r(\overline{y}), \end{cases}$$

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where

$$\omega(y) = \frac{z(\overline{y}) - \hat{y}_r}{z(\overline{y}) - z_a}$$

and $\hat{y}_r$ is mean income among the only relatively poor.

Illustration: selection of normative parameters

Objective:

- Contrast poverty comparisons of HC and EHC using data taken from PovcalNet (World Bank)

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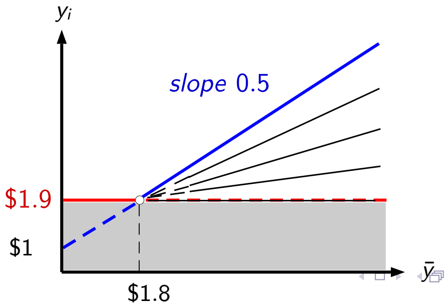
- Contrast poverty comparisons of *HC* and *EHC* using data taken from PovcalNet (World Bank)

Two poverty lines (units are $ a day):

- Absolute line $z_a = 1.9$,
  (Ferreira et al 2012)
- Societal poverty line $z_r(\overline{y}) = 1 + 0.5\overline{y}$ where $\overline{y}$ is median,
  (Jolliffe and Prydz 2017)



EHC avoids debatable comparisons because it is hierarchical

	mean ($ a month)	HC_A (%)	HC_{oR} (%)	HC (%)	$\omega(y)$ -	EHC (%)
Bangladesh	116	15	14	29	0.49	22

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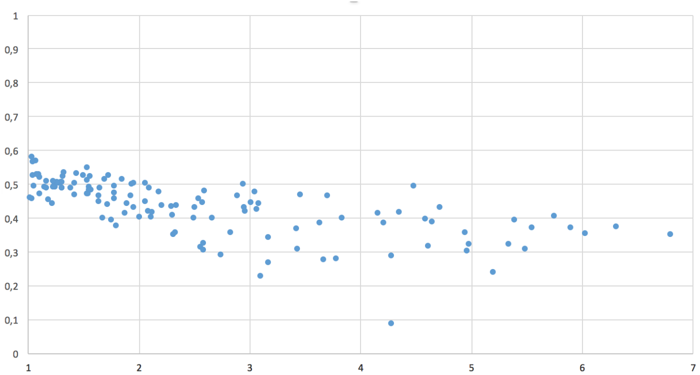
Bangladesh	116	15	14	29	0.49	22
Colombia	442	5	24	29	0.47	16

- *EHC* finds less poverty in Colombia because the “only relative” poverty status is considered less severe.

Weight $\omega(y)$ decreases with income standard

Weight $\omega(y)$ as a function of $\frac{z_r(\overline{y})}{z_a}$ in 2015.



- low-income countries: $\omega(y) \approx 0.5$
- low middle-income countries: $\omega(y) \approx 0.4$
- high middle-income countries: $\omega(y) \approx 0.3$

EHC finds more poverty reduction than HC

Over 1990-2015, *EHC* finds significantly more poverty reduction than *HC*:

	EHC $\frac{2015}{1990}$	*HC* $\frac{2015}{1990}$
Developing World	0.41	0.56

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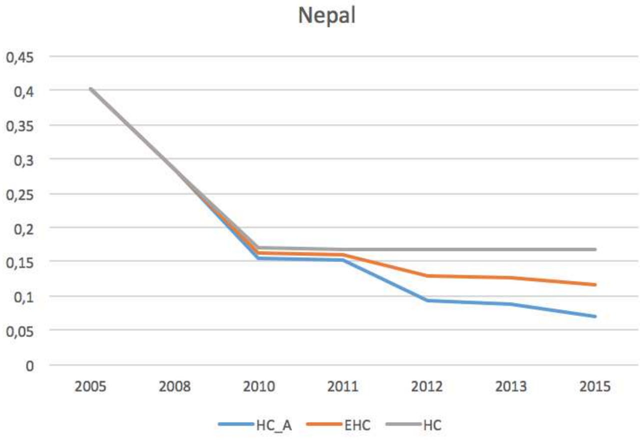
Rate of poverty reduction measured by the compound annual growth rate:

- -2.3% annually for *HC*.
- -3.5% annually for *EHC*.

EHC finds a rate at least 50% larger than that of *HC*.

EHC more reactive to growth than HC

Nepal experienced equi-proportionate growth (according to Povcalnet).



If $z_r(\overline{y}) = 0.5\overline{y}$, then equi-proportionate growth implies

- HC is constant (when $z_r(\overline{y}) \geq z_a$),
- EHC decreases as individuals escape absolute poverty.

Conclusion

- Mainstream measures yield debatable comparisons because of their index
- I show that indices based on two lines should be hierarchical
- the Extended head-count ratio is a prominent hierarchical index

$\Rightarrow$ **When using two poverty lines, HC should be replaced by EHC**.

Seven properties characterize hierarchical indices

Absolute Focus: exact income of non-poor is irrelevant when all poor are absolutely poor.

- For all $y, y' \in Y$ with $n(y) = n(y')$, if
  - $q_a(y) = q(y) = q_a(y') = q(y')$ and
  - $y_i = y'_i$ for all $i \leq q_a(y)$

  then $\mathrm{I}(y) = \mathrm{I}(y')$.

Relative Focus: exact income of non-poor is irrelevant as long as income standard is unchanged.

- For all $y, y' \in Y$ with $n(y) = n(y')$, if
  - $q(y) = q(y')$,
  - $y_i = y'_i$ for all $i \leq q(y)$,
  - $\overline{y} = \overline{y}'$,

  then $\mathrm{I}(y) = \mathrm{I}(y')$.

Hierarchical indices violate basic fairness axiom

Transfer: poverty does not increase after a Pigou-Dalton transfer

◇ For all $y, y' \in Y$ with $n(y) = n(y')$, if
 - $q(y) = q(y')$,
 - $y_j - \delta = y'_j > y'_k = y_k + \delta$ for some $j, k \leq q(y)$ and $\delta > 0$
 - $y_i = y'_i$ for all $i \neq j, k$

 then $\mathrm{I}(y) \geq \mathrm{I}(y')$.

Theorem 2: If I is a hierarchical index, then I violates Transfer.

- Trade-off between hierachical inter-personal comparisons and prioritization.
  - ◇ I argue inter-personal comparison is "deeper" than prioritization

back