2019

IARIW-World Bank

Special IARIW-World Bank Conference "New Approaches to Defining and Measuring Poverty in a Growing World" Washington, DC, November 7-8, 2019

The First Revision of the Global MPI: Empirical Insights and Robustness

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Paper Prepared for the IARIW-World Bank Conference

Washington, DC, November 7-8, 2019

The first revision of the global MPI: Empirical insights and robustness

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Abstract

The global Multidimensional Poverty Index (MPI), published since 2010, is a measure that captures acute multidimensional poverty experienced by people in the developing regions of the world. In 2018, five of the ten indicators were revised with the purpose of aligning the index to the Sustainable Development Goals (SDGs). How useful is the revised global MPI as a guide to poverty comparisons? This paper provides comprehensive analyses of the revised global MPI from three perspectives. First, this paper explores the overlap of deprivations prior to the application of the poverty cutoff on a global scale. Second, we analyse the robustness of the revised global MPI to two alternative parameters – the poverty cutoffs and weighting structures. Third, the paper systematically compares the revised 2018 and the original 2010 specifications of the global MPI. Our findings indicate that acute multidimensional poverty is consistently prevalent in Sub-Saharan Africa followed by South Asia compared to other world regions. In addition, the robustness analyses reveal that the revised index is stable across alternative weighting structures and poverty cut-offs. We also find that the aggregate results and ranking of countries have remained similar between the 2010 and 2018 specifications, suggesting the stability of the index despite the revisions.

1. Introduction

It is widely agreed in both academia and practise that poverty is multidimensional (e.g., Narayan et al., 2000; Atkinson, 2003, 2019; Bourguignon & Chakravarty, 2003; Alkire & Foster, 2011; Ferreira, 2011; Ravallion, 2011; Whelan et al., 2014; World Bank 2017, 2018). This consensus is reflected in the most visible contemporary development paradigm, the 2030 Agenda. In fact, SDG target 1.2.2 *explicitly* states that countries should "reduce […] poverty in all its forms and dimensions […][1].” Importantly, a full account of the multidimensional nature of poverty is not merely concerned with its manifold manifestations, but also their intrinsic interconnections (Atkinson, 2019).

The global Multidimensional Poverty Index (MPI) systematically implements an incomplete yet still more comprehensive notion of poverty. The global MPI developed by Alkire and Santos (2010; 2014) and the UNDP was first reported in the United Nations Development Program’s 2010 Human Development Report (UNDP 2010). The aim of the measure is to offer a global account of acute multidimensional poverty that is comparable across countries in the developing world, to the largest extent possible. Relying methodologically on the dual-cutoff counting approach pioneered by Alkire & Foster (2011), this index complements the traditional measures of monetary poverty by directly measuring the simultaneous shortfall in several dimensions of human wellbeing (see e.g. Atkinson, 2019; the Report of the A/73/78 Report of the UN Secretary General[2], and The Global Sustainable Development Report 2019[3]).

In 2018, the global MPI was subjected to the first major revision since its inception in order to take into account the progress in the availability of survey microdata, and to better align to the 2030 development agenda and related international strategies and policy actions (Alkire & Jahan, 2018). Formally, the 2018 revision consisted of adjustments in the definition of five out the ten indicators (OPHI, 2018). Indicators related to child mortality, nutrition, years of schooling, housing and asset ownership, were revisited in light of theoretical foundations, data availability and policy relevance (Alkire & Kanagaratnam, 2019). The rest of the parameters involved in the estimation of the global MPI were maintained. The global MPI still reflects equal relative importance of the component dimensions of poverty: health, education and living standards, and equal nested indicator weights within each dimension.

This paper studies consequences of this revision, hence filling a gap in the literature regarding the robustness of the revised structure. A vigorous assessment is useful because the MPI is one of the simultaneously appears in the international media[4], while also inspiring academic studies and

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[1] See *Transforming our World* report https://sustainabledevelopment.un.org/content/documents/21252030%20Agenda%20for%20Sustainable%20Development%20web.pdf
[2] https://undocs.org/A/73/298
[3] https://sustainabledevelopment.un.org/gsdr2019

[4] Wide circulating newspapers such as The Guardian, and more specialized magazines such as The Economist often make mentions to the global MPI. For instance see: https://www.theguardian.com/global-

influencing public policy strategies to tackle poverty. The theoretical and methodological underpinnings of the global MPI are often taken as benchmarks for analysis in numerous academic studies about the causes and consequences of a rich notion of poverty (see e.g. Jindra & Vaz, 2019 for governance and poverty; Ogutu & Qaim, 2019 for the impact of commercialization on poverty; Espinoza-Delgado & Klasen, 2018 for intra-household poverty disparities; Alkire et al, 2017 for a comprehensive cross-country analysis of changes over time; Pasha, 2017 for the consequences of alternative weighting scheme in MPI on country ranking; Rogan, 2016 for a gendered approach to poverty; and Alkire and Seth 2015 for analyses of over time in India), as well as country-specific poverty analyses (see e.g. Datt, 2019 for the Philippines, Suppa 2018 for Germany, Hanandita & Tampubolon, 2016 for Indonesia; Angulo, 2016 for Colombia, Trani et al., 2016 for Afghanistan). Furthermore, in recent years, measures similar to the global MPI have been developed to gauge simultaneous multiple deprivations in the specific context of children (De Neubourg et al., 2012) and an extended monetary variation that replaces health with $1.90/day income poverty and five education and living standard indicators from the global MPI (World Bank, 2018). Additionally, an increasing number of governments in developing countries draw inspiration on the global MPI to produce permanent and national poverty statistics that are tailored to their national characteristics and needs (see e.g. UNDP-OPHI, 2019)

This paper contributes to filling this knowledge gap about the effects of revising the global MPI indicators in three ways. Each of them is aimed at answering a precise question: i) What novel insights about interlinkages among poverty-related indicators in the developing world do we gain from the revised global MPI? ii) How robust is the revised structure to changes in some the fundamental parameters? iii) What are the empirical consequences of the revision for the way we understand poverty in light of the global MPI?

Providing rigorous answers to these questions entails data-intensive empirical analyses. We build upon the same data that was used to produce the results of the revised global MPI in 2018. It consists of a unique dataset that combines 105 strictly harmonized microdata surveys (see Alkire et al., 2018), each of them being nationally representative of the population in a country located in one of six developing world regions as defined by UNDP: the Arab States, East Asia & the Pacific, Europe & Central Asia, Latin America & the Caribbean, South Asia and Sub-Saharan Africa. The pooled sample from these 105 countries resulted in 8.79 million individual observations that

development/2013/sep/25/new-ways-measure-poverty ; https://www.economist.com/graphic-detail/2018/09/14/life-in-developing-countries-continues-to-improve represent around 5.7 billion people. This corresponds to nearly 77% of the global population and 91% of the population living in the developing world. Given that levels of acute multidimensional poverty are expected to be low outside the developing world, our analysis is close to having a global scale but we recognise and proactively affirm the need for MPIs in high-income settings.

To tackle the first question we perform a two-fold exploration of key novel insights offered by the revised global MPI: (a) we assess the overlap of deprivations prior to the application of the poverty cutoff, and (b) we evaluate the implications of adopting a multidimensional poverty cutoff in the way we understand poverty patterns in the developing world. Thus our paper is aligned with scholarship emphasizing the practical importance of the *joint* distribution of deprivations to understand the many facets of poverty (e.g. Duclos et al., 2006, Wolff & de-Shalit, 2007; Atkinson, 2019), as well as the literature on the importance of the definition of the multidimensional poverty cutoff for the identification of poverty sets (Dotter & Klasen, 2014, Datt, 2018, Pattanaik & Xu, 2018). For a deeper analysis, we also perform multilevel disaggregation of the overall aggregate poverty measures by world region, rural/urban areas and age-cohorts, explaining formally how this multilevel disaggregation is performed using such a large-scale dataset, which is in fact an extension of the single-level disaggregation procedure presented in Alkire & Foster (2011) and Alkire et al., (2015).

Next, the robustness of the revised version of the global MPI is assessed in terms of the stability of its aggregate poverty measures, namely the simple poverty headcount ratio, the intensity of poverty, and the simple headcount ratio adjusted by the intensity of poverty, termed the *adjusted* headcount ratio that precisely corresponds to the MPI (see Alkire & Foster, 2011; Alkire & Santos, 2010). For this analysis, we focus on the effects of changes in two key parameters of this index: the multidimensional poverty cutoff and the dimensional weighting structures. Thus our study is related to the robustness analysis of the original version of the MPI conducted by Alkire & Santos (2014), but it also relates more broadly to the robustness of aggregate poverty measures to distinct parametric choices (Santos & Villatoro, 2018; Ravallion, 2017; 2019 Angelini & Michalengili, 2012;). We extend the analysis conducted by Alkire & Santos (2014) in three ways. First, we distinguish the effects of alternative parameters on the *absolute* position of each country in a global poverty ordering and the *relative* order shifts of each possible pair of countries, termed *pairwise comparisons* in Alkire & Santos (2014) and in Alkire et al. (2015). Second, the robustness of pairwise comparisons is assessed in light of hypothesis tests instead of confidence interval overlap analyses, thus accounting for the general case of possible non-zero covariance between aggregate measure estimates. This is required for consistent comparisons when aggregate estimates draw on information from partly (or fully) overlapping population sets. Practically, we are able to account for non-zero covariances here because we are treating the combined 105 country-level information as one single dataset. Third, we highlight the robustness differentials between the simple and adjusted headcount ratios, which allows us to discuss the role of poverty intensity in the stability of the MPI.

Finally, we offer a detailed empirical comparison of the poverty patterns arising in light of the original and revised versions of the global MPI. Feeding the same data into both specifications of the index, we first analyse differences in the key aggregate poverty measures by world regions, as well as the deprivation rates suffered by the whole population and the subset of poor people. Also, we perform a country pairwise comparison analysis (with hypothesis tests) to assess the robustness of relative orderings between the two versions of the index.

The paper is structured as follows: Section 2 briefly presents the Alkire-Foster method and the global MPI data. Crucially, we introduce the notation that guides us through the rest of the paper. Section 3 delves into the state of poverty through the lens of the revised global MPI at the most aggregate level, but it also presents multilevel disaggregations by world region, rural/urban areas and age-cohorts. Section 4 analyses the robustness of the revised global MPI to changes in indicator weights and the poverty cutoff. We also present the formal methodology underlying the pairwise comparison hypothesis tests. Section 5 compares the poverty figures obtained in the light of the original and the revised versions of the global MPI. Finally, in section 6, we conclude the paper by highlighting the key take away messages.

2. The global MPI: Methods and Data

The global MPI is arguably the most well-known application of the dual cutoff counting approach to poverty developed by Alkire & Foster (2011, AF method henceforth). Whereas the innovation of the dual cutoff approach was general and methodological, the innovation of the global MPI lies, precisely, in its selection and empirical application of indicators and deprivation values or relative weights across over 100 countries. Given that the defining features of the global MPI is its indicators and weights, and given that the revision adjusted the former, it is paramount to consider how to assess the revised global MPI, as this points out exercises that could also be useful when other established measures adjust their parameters. In particular, we explore the empirical consequences of adopting alternative parametrizations and the formal mechanisms through which the revision yields empirical changes. Hence, let us make a brief formal presentation of the method, highlighting the way in which it operationalizes the two steps of poverty measurement, namely identification and aggregation (Sen, 1976). We present the *overall* global MPI aggregate measures as the primary outcome of an aggregate poverty analysis of the developing world, which can then be disaggregated down to multiple levels for more detailed analyses. However this can be easily misunderstood: we recognise that the MPI is not global - many countries are missing; the data are imperfect and constrained; and the surveys vary in size and detail. The notation assists in presenting estimations that are in fact run across all the included datasets together.

2.1 The Alkire-Foster (AF) method applied to parameters of the global MPI

Our microdata contains information about each individual in the total sample of size, N, is 8.76 million people. Let us denote as g^0 the ($N \times d$)-sized matrix containing binary deprivation indicators for all the individuals in each one considered indicators, $j = 1 \dots d$. If individual i falls short of the minimum achievement level in indicator j that is necessary for them to be considered non-deprived, then $g_{ij}^0 = 1$. Otherwise, $g_{ij}^0 = 0$. Each deprivation may have a different relative importance, which is reflected in the vector of weights $w = (w_1 \dots w_d)$ such that $w_j > 0$ and $\sum_{j=1}^{d} w_j = 1$. Each element w_j reflects the relative value or importance of each deprivation to poverty. The weighted deprivation counting vector, c sized $(N \times 1)$, is the collection of individual deprivation scores defined as $c_i = \sum_{j=1}^{d} w_j \, g_{ij}^0, \forall i$. These scores represent the number of weighted deprivations experienced by each individual in the sample.

<u>Identification:</u>

The c-vector is the underlying welfare variable in the global MPI. It covers the entire sample and it is the variable to which the multidimensional poverty cutoff, denoted as k, is applied to identify individuals as being poor. Thus drawing inspiration from Alkire & Foster (2011) and Alkire et al. (2015), we define the poverty identification function as $\rho(g_i^0, w, k) = \mathbb{I}(c_i \geq k)$, where g_i^0 is the row vector collecting all the deprivation indicators of person i. If they face k weighted deprivations or more, then $\rho(g_i^0, w, k) = 1$. Otherwise $\rho(g_i^0, w, k) = 0$. In our notation, we wish to explicitly state the set of parameters that precisely define the specification of these functions and are therefore essential to assess robustness. It is easy to see that the revision has modified the identification functions by changing the deprivation matrix g_i^0, while maintaining w and k unchanged.

Furthermore, an 'extended' identification function may be posited accounting for two k-values simultaneously may be defined as $\rho(g_i^0, w, k_1, k_2) = \mathbb{I}(k_1 > c_i \geq k_2)$. This is used in the global MPI to define people who are vulnerable, because they are deprived in 20% or more, but strictly less than 33% of weighted indicators.

Aggregation:

After identification, three empirical means applied to transformations of the identification functions yield the aggregate poverty measure MPI, and the partial indices *H* and *A* for the *entire set of countries* included in the global MPI. For notational convenience, let us collapse the exogenous elements defining the identification functions into array $\theta \equiv (g_i^0, w, k)$, such that $\rho(g_i^0, w, k) = \rho(\theta)$.

First, the simple poverty headcount ratio can be computed as $H(\theta) = \frac{1}{N}\sum_{i=1}^{N} \rho(\theta)$. This rate represents the proportion of poor people by cutoff k. Second, the rate of multidimensional poverty intensity can be computed as $A(\theta) = \frac{1}{q}\sum_{i=1}^{N}(\rho(\theta) \times c_i)$, where $q = \sum_{i=1}^{N} \rho(\theta)$ is the number of poor people by cutoff k. Thus $A(\theta)$ represents the average number of weighted deprivations *experienced by the poor*. Third, the *adjusted* poverty headcount ratio, denoted as $M_0(\theta)$, combines $H(\theta)$ and $A(\theta)$ in a multiplicative form, such that $M_0(\theta) = H(\theta) \times A(\theta) = \frac{1}{N}\sum_{i=1}^{N}(\rho(\theta) \times c_i)$. Thus this rate represents the number of weighted deprivations experienced by the poor as a proportion of the number of individuals in the whole sample. In the context of the global MPI, the adjusted headcount ratio is precisely the level of the MPI, so $M_0(\theta)$ and $MPI(\theta)$ are interchangeable notations. Note that these definitions can be applied in a straightforward manner to the generalized identification functions $\tilde{\rho}_{k_1;k_2}(X_{i.}, z, w)$ to yield aggregate measures for the population subset that have deprivation scores located *between* cutoffs k_1 and k_2.

It can be seen that the revision of the global MPI modified these aggregate rates by changing the identification functions. Thus one way of empirically evaluating the consequences of the revision is to assess the difference between these aggregate measures. This can be done for the overall values of $H(\theta)$, $A(\theta)$ and $M_0(\theta)$, but to gain richer insights it can also be done for subgroups of the entire sample.

Multilevel subgroup disaggregation of the global MPI main indices

The aggregate poverty measure $M_0(\theta)$ and subindices $H(\theta), A(\theta)$ applied to the entire population sample of included countries may be termed as the *overall* poverty aggregates. Let us recall that the overall sample is a result of pooling 105 harmonized national surveys. Each one of these surveys has a specific complex survey design (see Alkire et al., 2018), by which each household is assigned a sampling weight. In each national survey, these weights are inversely proportional to the probability of selection within the specified sampling frame (see e.g. USAID, 2012 for the specific case of DHS datasets, and Lohr (2019) for a technical discussion). Thus they expand each sample to the corresponding national population size at the moment of the survey. Hence, each national survey allows, in principle, to obtain unbiased estimators of $M_0(\theta)$, $H(\theta)$ and $A(\theta)$ for each country if the estimation procedure consisted of producing these estimates for each country separately, as well as subnational disaggregations as allowed by the survey design[5].

In this paper, however, we take the alternative route of computing the estimators of $M_0(\theta), H(\theta)$ and $A(\theta)$ for the developing world (as represented by our 105 countries), and then disaggregate, if need be, these overall measures into their country-level counterparts. In effect, this enables us to produce other disaggregations, such as developing world regions or the urban-rural population in the developing world, which are particularly useful to understand the state of poverty.

Since the national datasets survey designs are independent from each other, the sampling weights of the pooled dataset expand to the population of our representation of the developing world. Thus, effectively, the estimated values of $M_0(\theta), H(\theta)$ and $A(\theta)$ computed from the entire pooled dataset are population-weighted sums of the corresponding national estimates. A key data constraint that is currently impossible to circumvent with the existing data is related to the fact that not all the national datasets are collected in the same timespan (see Alkire et al., 2018 for more details). Thus, the 'raw' pooled dataset expands to an abstract population size that hardly has a meaningful interpretation, as it is a mixture of national population sizes at different times. So, differences between world regions or countries, for instance, could be attributable to a) different survey years or b) different levels of measured poverty or c) to different average aggregate population year reference. This creates challenges in interpreting cross-regional differences. To recover the logic of our analysis, we propose to rescale the sampling weights for each national survey so that they add up to the population of that country in one common time period. Based

[5] Note that this statement holds true without a doubt in the absence of sample drop. If sample drop occurs generating a pattern of missing values that is completely at random (MCAR, see e.g. Bhaskaran & Smeeth, 2014), the national representativity of the sample is preserved. Some preliminary analyses about this are discussed in Alkire et al., 2019.

on data availability, we thus rescaled the weights to add up to the 2016 population size as reported in UNDESA (2018). This facilitates international comparisons; aggregating using a common population year is a convention used in the global MPI reports (see e.g. OPHI, 2018). A result of this convention is that if the population date post-dates the survey, and if population has grown, and if poverty is declining, the number of poor persons will be overstated. Our results have to be interpreted keeping this in mind.

This procedure enables us to draw inspiration from Alkire et al. (2015) and disaggregate the overall measures $M_0(\theta)$, $H(\theta)$ and $A(\theta)$ obtained from the pooled dataset into a nested group structure to allow for population subgroup poverty analyses. For instance, it may be particularly informative to break down the aggregate measures into rural/urban measures within world regions. This would require a disaggregation by world region (level 1 subgroup) combined with a disaggregation by area of residence (level 2 subgroup). Similarly, it may be useful to disaggregate the overall measures into age-cohort poverty measures within each country. In this case, the level 1 subgroup would be the country and the level 2 subgroup would be the age-cohort. Let us stress that the number of levels in this nested group structure crucially depends on the sampling structure of each survey; we will limit ourselves here to present a two-level disaggregation. In Appendix A, we formally describe this multilevel disaggregation procedure, assuming known population sizes for notational simplicity, which also mirrors the sampling weight rescaling procedure that we made a case for above.

2.2 Data

As we mentioned earlier, we build upon the exact same data that was used to produce the revised global MPI results when they were first published in 2018 (OPHI, 2018). These data are based on 105 harmonized nationally representative datasets drawn from five major sources: the Demographic and Health Surveys (DHS), the Multiple Indicator Cluster Surveys (MICS), the combined DHS-MICS survey, the Pan Arab Project for Family Health (PAPFAM) surveys, and a some national surveys. Appendix 1 provides details on the region, survey, year, sample size and total indicators covered by country. The vast majority of the countries (90) had surveys that were fielded between 2011 and 2016. This suggests that some 86% of the countries in this wave of the global MPI had surveys as recent as in the last 5 years. More details of this harmonization can be found in (Alkire et al., 2018).

In 87 countries, the results were based on all 10 indicators of the global MPI. In 17 countries, the results were based on nine indicators, while Philippines was the only country that lacked two indicators. The countries lacking one indicator mainly lacked information on nutrition or child mortality, with Egypt lacking cooking fuel, Honduras lacking electricity and China not having information on housing. To account for these special cases, the elements of the indicator weighting scheme, w, is defined for each country as follows: $w_d > 0$ if indicator d exists in the considered country dataset and $w_d = 0$ otherwise. In order to preserve the normalization $\sum_{d=1}^{D} w_d = 1$ for the entire set of countries, the weight of the missing indicator was equally redistributed between all the remaining indicators *within the same dimension*. This procedure amounts to maintaining roughly equal weights across the three dimensions, while making best use of the available information. Thus it is aimed at preserving the internal theoretical rationale of the global MPI since it was conceived in 2010.

2.3 A synthesis of the revision

The revision of the global MPI consisted of adapting five of the ten component indicators. Table 1 presents a concise comparison between the original and revised indicators; more details of this revision can be found in Alkire & Kanagaratnam (2019), Vollmer & Alkire (2018) and Alkire & Jahan (2018). We limit ourselves here to state that in the revised version, the nutrition status for children under five include the union between weight-for-age (underweight) and height-for-age (stunting). The original specification was limited to only underweight. The inclusion of stunting better aligns with the SDG framework towards zero hunger (indicator 2.2.1).

In addition, for 51 countries where there is nutrition data for adults, we applied the BMI-for-age measure for teenagers 15-19 years and the BMI measure for adults 20 years and older. The original specification applied the BMI measure for all individuals 15 years and older. The BMI-for-age measure better accommodates the sporadic growth experience of teenagers than a BMI measure.

Table 1 A comparison between original and revised global MPI indicators

Dimensions of poverty	Indicator	Original global MPI Deprived if…	Revised global MPI Deprived if…
Health	**Nutrition**	Any teenagers or adults have **low BMI** or any child under 5 is **underweight**.	Any adults have **low BMI** or any teenagers have **low BMI-for-age** or any child under 5 is **underweight** or **stunted**.
	Child mortality	Any **child** has **died** in the family.	Any **child*** has **died** in the family in the **five-year** period preceding the survey.
Education	**Years of schooling**	No household member aged 10 years or older has completed **five** years of schooling.	No household member aged 10 years or older has completed **six** years of schooling.
	School attendance	Any school-aged child is not attending school up to the age at which he/she would complete class 8.	
Living Standards	Cooking fuel	The household cooks with dung, wood or charcoal.	
	Sanitation	The household's sanitation facility is not improved, or it is improved but shared with other households.	
	Drinking water	The household does not have access to improved drinking water or safe drinking water is at least a 30-minute walk from home, roundtrip.	
	Electricity	The household has no electricity.	
	Housing	The household has a dirt, sand, dung or other unspecified type of **floor**.	The household has inadequate housing: the **floor** is of natural materials or the **roof** or **walls** are of rudimentary materials.
	Assets	The household does not own more than one **radio**, **TV**, **telephone**, **bike**, **motorbike** or **refrigerator** and does not own a car or truck.	The household does not own more than one of these assets: radio, TV, telephone, **computer**, **animal cart**, bicycle, motorbike, or refrigerator, and does not own a car or truck.

***Note**: In 2019, the definition of child mortality was further revised to include age criteria. Individuals are deprived in child mortality if any child **under 18** has died in the family in the five-year period preceding the survey.

In the revised version, a child death is considered in the child mortality indicator only if it took place five years prior to the survey. This avoids capturing past mortality stocks and allows to better capture policy success in reducing it. The deprivation cut-off in years of schooling has been revised from five to six years in order to reflect the international standard duration of primary schooling. The assets indicator was expanded to include computer and animal cart and thus reflect urban and rural deprivations more adequately (Vollmer & Alkire, 2018).

3. The revised global MPI: What insights do we really gain?

Let us begin our analysis by discussing the differences of an assessment based on the simple deprivation indicators and the overlap of deprivations, both prior to identification. Subsequently, we examine some key results arising in light of the simple and adjusted headcounts, and the intensity rate based on the cutoff $k = \frac{1}{3}$, which represents the poverty line in the global MPI. We complement these results with analyses for the vulnerable ($k_1 = \frac{1}{3}$ and $k_2 = \frac{1}{5}$ in the generalized identification function presented earlier) and severe ($k = \frac{1}{2}$) aggregate measures.

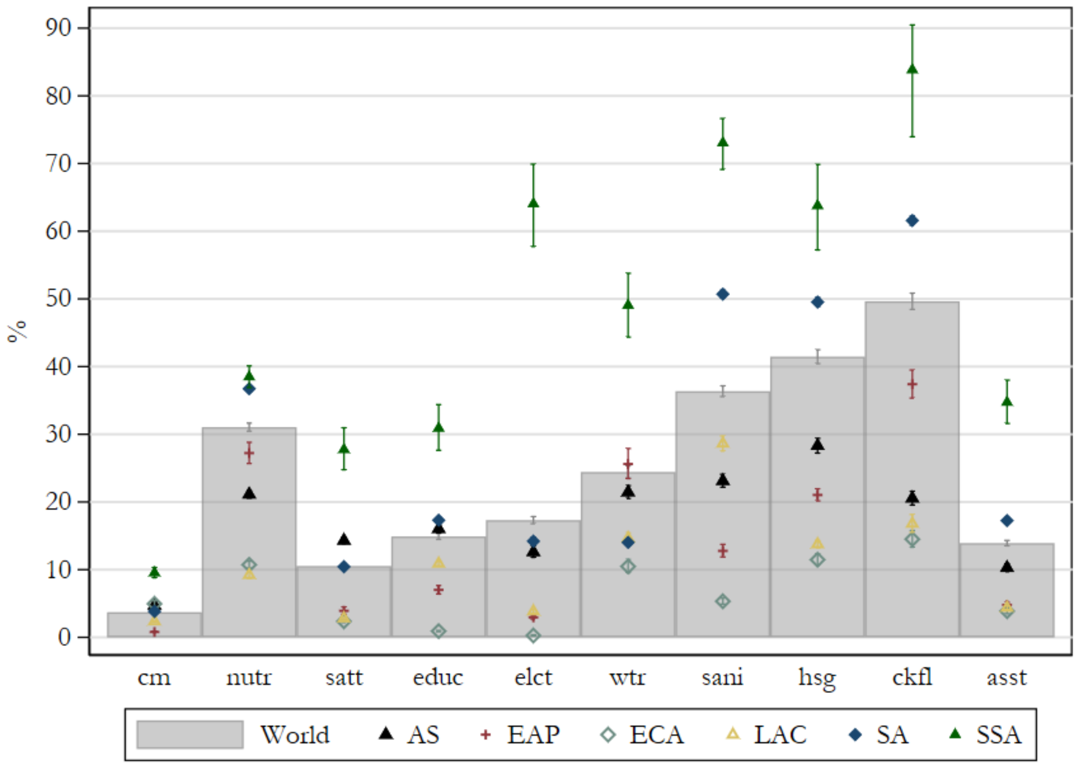
3.1 A dashboard of deprivation indicators

An analysis of isolated deprivation headcount ratios is perhaps the simplest way to start a description of poverty patterns in the developing world. This is akin to taking a dashboard approach to multidimensional poverty, which focuses on the marginal indicator distributions, one at a time (Ravallion, 2011). These ratios are here termed *uncensored headcount ratios* (see e.g. Alkire et al., 2015) and they correspond to the column-wise mean of the deprivation matrix g^0. Although partial, this analysis can be illuminating in its own. However, while analysing these headcount ratios, one has to keep in mind that it takes place prior to the identification and aggregation steps, so it does not correspond to a full-fledged poverty analysis. The focus is not on the poor population, but on the society as a whole, and the interconnections between the indicators are cast aside.

Globally, the highest overall headcount ratios correspond to *cooking fuel* (44.8%), *housing* (39.6%) and *sanitation* (37.0%) (Figure 1). Deprivations in these indicators afflict large portions of the population, regardless if and how one gauges their poverty status. In a disaggregated analysis, stark differences between world regions emerge. Deprivations in almost every indicator are unambiguously higher in Sub-Saharan Africa. Considering 95% confidence intervals, the uncensored deprivation headcount ratio in this region is over two-thirds in cooking fuel, housing, sanitation and electricity. This goes on to show the extent of geographical concentration of these deprivations.

The uncensored headcount ratios in South Asia and Sub-Saharan Africa are highest among all world regions in almost every indicator. This is a clear pattern that regularly emerges even through a purely monetary approach to poverty (World Bank, 2018; Ravallion, 2016). The only exception is *child mortality* for which we observe very low poverty headcounts even in highly populated regions such as East Asia & the Pacific. This is related to the progress made in terms under-5 mortality globally in recent years, which has been extensively discussed e.g. in You et al. (2015). This is also aligned with scholarship making a case for improvements in coverage of health determinants as a main driver of fast reductions in child (and maternal) mortality in the developing world (see e.g. Bishai et al., 2016)

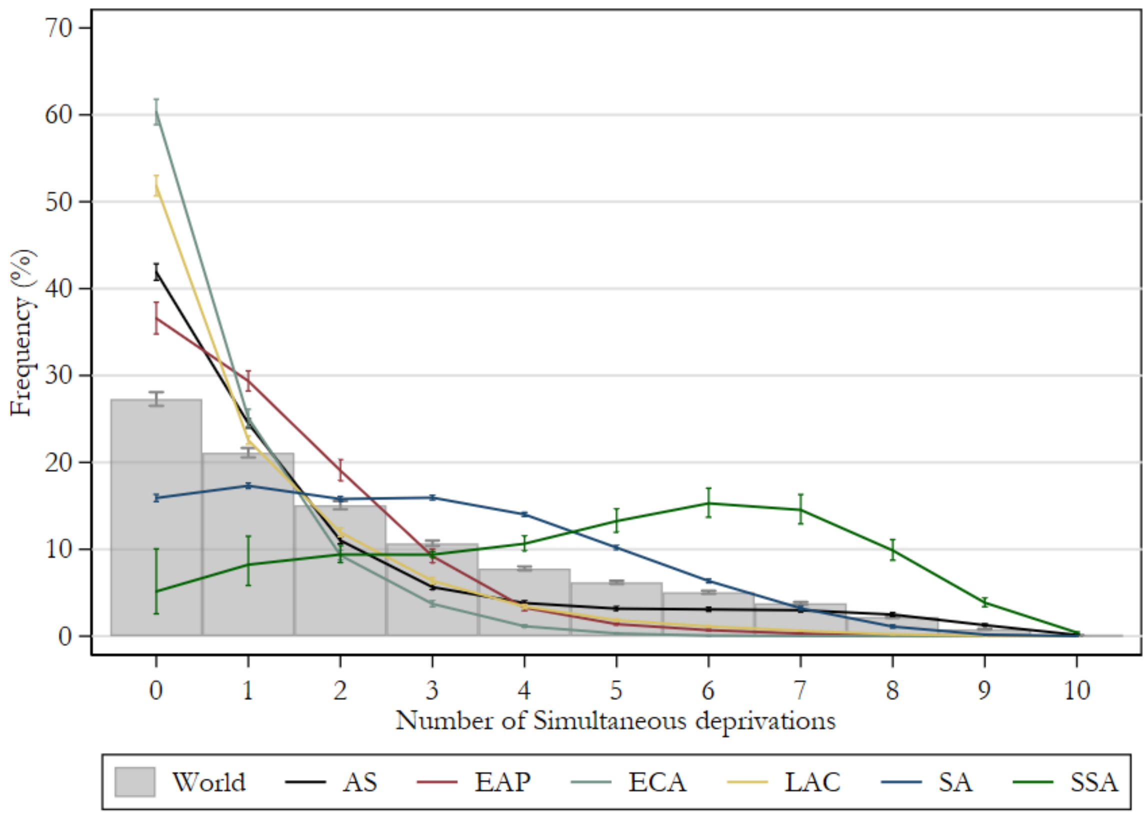
Figure 1 Uncensored headcount ratios by world region



3.2 Overlapping deprivations

The analysis of each indicator in isolation provides useful insights, but considering them as separate entities overlooks their interlinkages or natural interconnections. People who suffer one deprivation are very likely to face *other* deprivations at the same time.

Figure 2. Number of simultaneous deprivations by world region

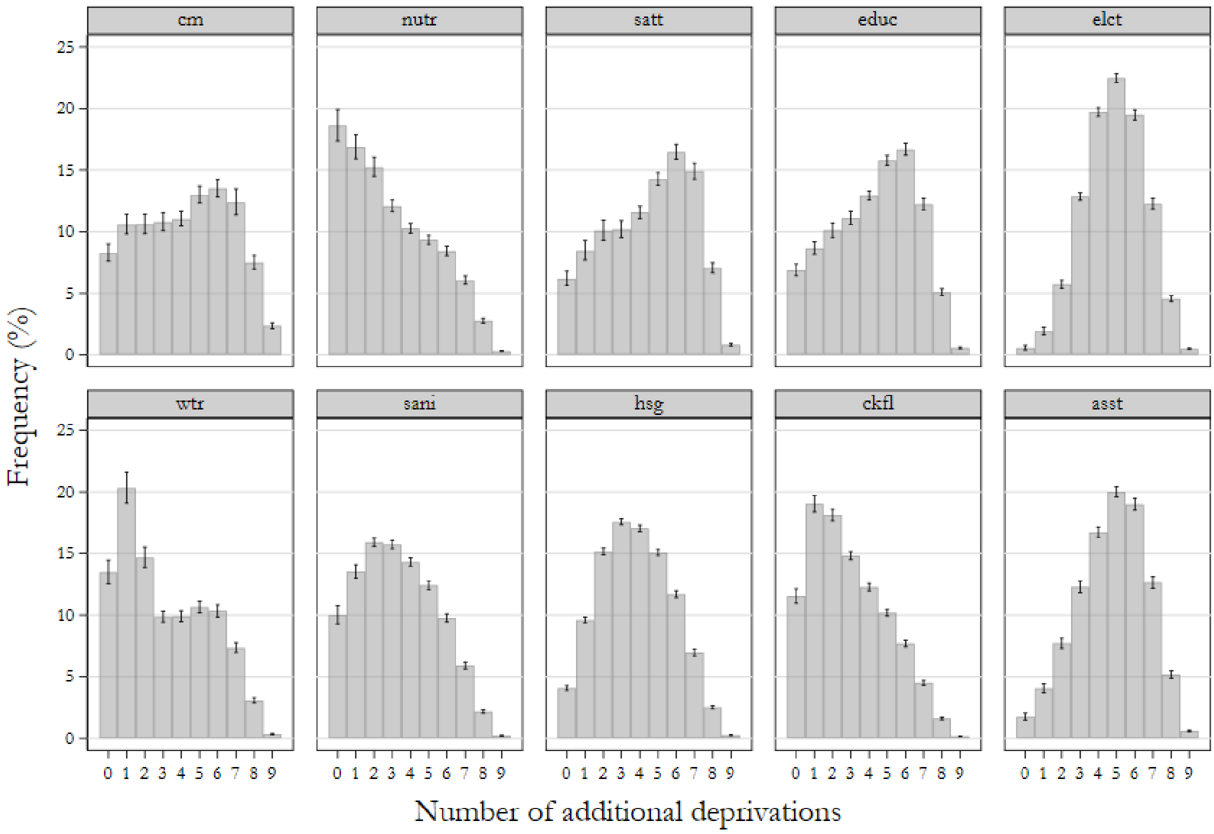


As shown in Figure 2, at a global level, around 27% of the population do not suffer any deprivation and 21% face exactly one single deprivation. The majority of the population (52%) are deprived in multiple ways; they face two or more deprivations. However, there is a high level of heterogeneity by world region around this global pattern. In South Asia, people are most likely to face one deprivation and there is a similar chance of facing two or three simlutanous deprivations. This means, for instance, that multisectoral policies with unified targeting mechanisms have more chances of being effective in the battle against these joint deprivations. In Sub-Saharan Africa, however, the most likely situation is to suffer five, six or seven simultaneous deprivations. The likelihood of living deprivation-free is the lowest in this region. This depicts much larger, more complex challenges for policymaking. More actors and institutions need to align efforts in the form of multisectoral programs, which risk to face obstacles linked to persisting institutional fragility in the region (see e.g. Deléchat et al., 2018; McKay & Thorbecke, 2019).

Although the uncensored headcount ratios provide useful information, they have the important drawback of being oblivious to the simultaneous nature of these deprivations, which is evident.

This has important consequences for policymaking. The challenges that they raise for policymaking in South Asia and South Africa may not be faced without accepting that poverty is multidimensional and that no one-proxy will do to fully grasp the livelihood of poor people. To see this, let us consider the distribution of the number of deprivations *conditional* on being deprived in each indicator. Figure 3 considers 100% of the persons who are deprived in a given indicator such as child mortality, and plots the percentage of them who are deprived in differing numbers of other indicators simultaneously. Implicitly, indicators are here equally weighted. Taking into account the confidence intervals of these conditional frequencies, facing one single deprivation alone is *never* the most likely situation (Figure 3)[6]

Figure 3. Distribution of additional deprivations by indicator



Note: Bars sum up to 100% of the deprived population in each indicator

Figure 3 is a graphical representation of the information presented in Table 2, which shows only the mean point estimates. We can see that proportion of persons who are only deprived in electricity or assets are less than one and two percent, respectively. We also see that those deprived

[6] Nutrition behaves differently with respect to the other indicators. Based on point estimates, it is the only indicator for which no additional deprivations is the most likely situation. But considering the 95% confidence intervals we find the likelihood of facing that deprivation alone or one additional deprivation to be statistically indistinguishable.

only in housing are around four percent, those deprived only in child mortality, school attendance, years of schooling and sanitation are between 5-10%. Only in three indicators out of the ten that are included in the global MPI, more than one in ten persons *only* deprived in that indicator: water, cooking fuel, and nutrition. Thus, across all ten indicators, between 81% and 99% of the population in the developing world deprived in that indicator experience one or more additional deprivations. At the bottom of Table 2, we can also see for every one of the ten indicators, the average number of additional deprivations is between 3 (nutrition and cooking fuel) and 5 (electricity and assets).

Table 2. Frequency of additional deprivations by indicator

	Frequency (%) / number of deprivations									
No. of additional deprivations	**cm**	**nutr**	**satt**	**educ**	**elct**	**wtr**	**sani**	**hsg**	**ckfl**	**asst**
0	8.28	18.60	6.19	6.88	0.56	13.48	10.01	4.08	11.54	1.75
1	10.59	16.87	8.45	8.66	1.90	20.32	13.55	9.59	19.04	4.05
2	10.61	15.25	10.08	10.11	5.73	14.68	15.92	15.17	18.13	7.72
3	10.79	12.08	10.20	11.10	12.84	9.85	15.73	17.58	14.83	12.31
4	11.02	10.27	11.56	12.92	19.71	9.91	14.32	17.04	12.28	16.73
5	12.99	9.34	14.26	15.78	22.47	10.66	12.42	15.08	10.19	20.00
6	13.51	8.43	16.48	16.67	19.49	10.34	9.77	11.70	7.69	18.99
7	12.37	6.06	14.91	12.23	12.26	7.35	5.90	6.96	4.50	12.65
8	7.50	2.78	7.06	5.08	4.55	3.07	2.16	2.52	1.62	5.19
9	2.35	0.32	0.82	0.57	0.49	0.35	0.23	0.27	0.17	0.61
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Mean no. of additional deprivations	4.2	2.9	4.4	4.2	4.9	3.1	3.3	3.8	2.9	4.7

Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

Based on this evidence, we argue that the global MPI is a useful way to account for the direct interlinkages across these deprivations. This index summarizes the multidimensional nature of poverty as measured by the manifestation of manifold deprivations, while accounting for their interlinkages.

3.3 The global MPI, its components and related measures

The overall incidence or headcount ratio of multidimensional poverty in the developing world is around 23.2%, and the average poor person experiences around 49.5% of the weighted deprivations (intensity). The value of the global MPI is 0.115. To delve deeper, let us discuss the results of a one-level disaggregation of these overall figures by world regions. They are depicted in Table 3.

Although there is considerable regional heterogeneity around these results, it is statistically unambiguous that Sub-Saharan Africa followed by South Asia have the largest proportions of their population living in poverty (57.7% and 31.3%, respectively). However, there is not a direct relationship between the incidence and the intensity of poverty. In Sub-Saharan Africa and in the Arab States we find that the average poor person experiences more than half of the weighted deprivations (54.9% and 50.8% respectively). Balancing incidence and intensity, and including 95% confidence intervals the adjusted headcount ratio depicts a clear regional poverty ordering with Sub-Saharan Africa (0.317) as the poorest region, followed by South Asia (0.143) and the Arab States (0.098).

When it comes to severe multidimensional poverty, in light of the corresponding simple headcount, Sub-Saharan Africa is undoubtedly the most affected region, with 35.3% of this population facing extreme hardships. However, one may not rule out that the incidence of severe poverty is similar in South Asia and the Arab States, as their 95% confidence intervals overlap. Around 10% of the population in these regions live in severe multidimensional poverty.

So far we have focused on people who are poor, with varying intensity, by the global MPI. We also want to stress that South Asia has the largest incidence of vulnerability to poverty in the developing world (see Table 3). It is also noticeable that a large proportion of the population are vulnerable to poverty in Sub-Saharan Africa (17.3%), which confirms the marked challenges for policymaking in this region. Not only is it home to the highest proportion of the poor population in the developing world, but those who are not poor are very close to multidimensional poverty cutoff. On average, three out of every four persons in Sub-Saharan Africa are either poor or vulnerable to multidimensional poverty.

Table 3. Poverty incidence (%) for different poverty cutoffs by world region

	H(%) Acute			**H(%) Severe**			**H(%) Vulnerable**		
	mean	**lb**	**ub**	**mean**	**lb**	**ub**	**mean**	**lb**	**ub**
World	23.24	22.57	23.90	10.66	10.26	11.07	15.56	15.15	15.98
AS	19.23	18.42	20.03	9.65	9.05	10.25	9.72	9.35	10.09
EAP	5.85	5.20	6.50	1.23	1.06	1.40	15.57	14.47	16.67

ECA	2.37	2.12	2.61	0.26	0.19	0.33	5.85	5.42	6.27
LAC	7.69	7.44	7.95	2.13	1.99	2.28	7.64	7.32	7.96
SA	31.28	30.69	31.86	11.48	10.97	11.98	18.90	18.61	19.19
SSA	57.79	51.82	63.77	35.32	31.27	39.37	17.30	16.80	17.79

Note: lb and ub denote, respectively, lower bound and upper bounds of the 95% confidence intervals

Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

Table 4. MPI and Intensity (A) by world region

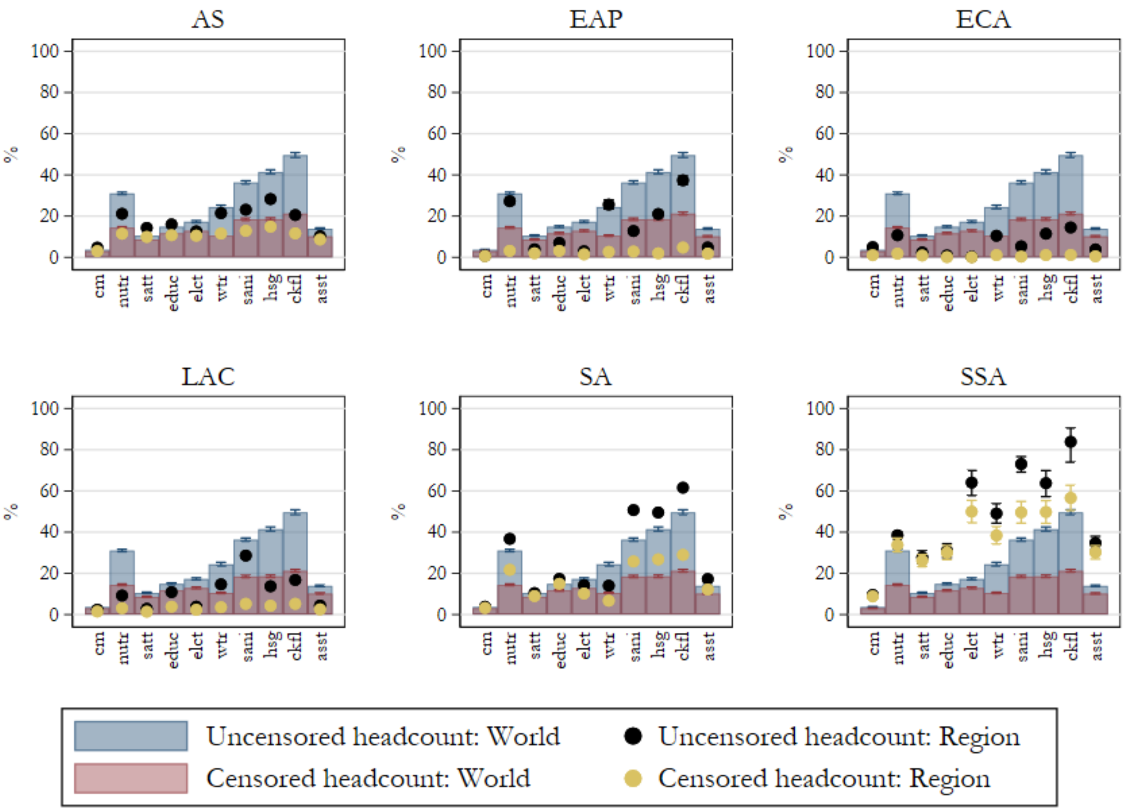
	Intensity (A, %)			**MPI**		
	mean	**lb**	**ub**	**mean**	**lb**	**ub**
World	49.50	49.27	49.73	0.115	0.111	0.119
AS	50.82	50.29	51.35	0.098	0.093	0.102
EAP	43.06	42.44	43.68	0.025	0.022	0.028
ECA	38.25	37.72	38.79	0.009	0.008	0.010
LAC	43.19	42.76	43.62	0.033	0.032	0.034
SA	45.76	45.37	46.14	0.143	0.139	0.147
SSA	54.88	54.54	55.21	0.317	0.283	0.351

Note: lb and ub denote, respectively, lower bound and upper bounds of the 95% confidence intervals

Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

After identifying the part of the population suffering multidimensional poverty across various poverty cutoff, naturally the question arises as to *how* they are poor. For this, we take a step further with respect to the analysis of uncensored headcount ratios and identify the proportion of the population who are poor *and* deprived in each indicator *censored headcount ratios*. They are denoted as $h_j(\theta), j = 1 \ldots 10$ and they can be computed as the mean of corresponding column of matrix g^0: $h_j(\theta) = \frac{1}{N}\sum_{i=1}^{N} g_{ij}^0, \forall j$. Unlike their uncensored counterparts, the censored headcounts depend on the poverty cutoff and thus they allow to lay focus on the prevalence of each deprivation only among the poor.

Figure 4 Censored and uncensored headcount ratios by world region

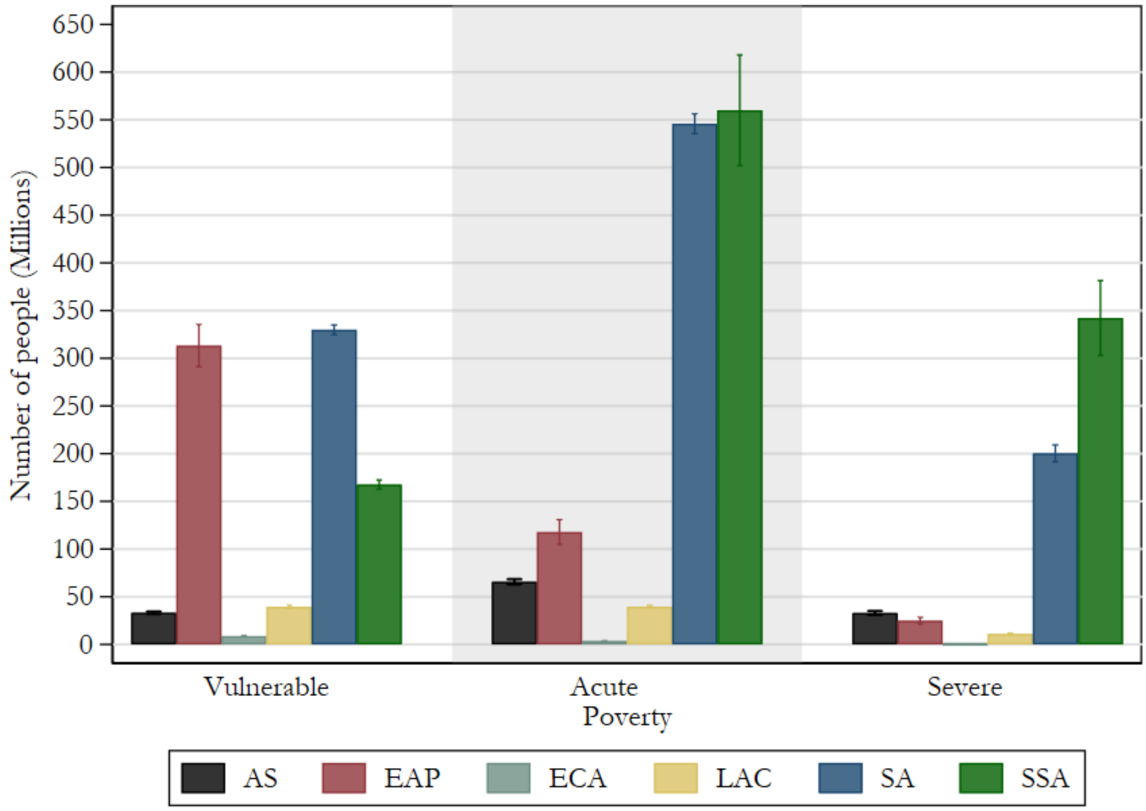


Compared to South Asia and Sub-Saharan Africa, the censored headcount ratios are very low in East Asia & the Pacific, Europe & Central Asia and Latin America & the Caribbean (see Figure 4). In contrast, the censored headcounts in Sub-Saharan Africa are highest for every single indicator, followed by those in South Asia.

There are some stark differences between the uncensored and censored headcounts in different regions. These differences denote that some deprivations are prevalent among the entire population, but are not necessarily a condition of the poor, because people deprived in those indicators are not deprived in at least one-third of the weighted indicators overall. This may be due to non-sampling measurement issues, preferences, or pervasive singleton deprivations. Empirically, the indicators which are most often censored are *nutrition, water, housing and cooking fuel* in East Asia & the Pacific; *sanitation* in Latin America & the Caribbean; and s*anitation, housing and cooking fuel* in South Asia.

So far, our assessment of the revised global MPI results has focused on proportions of the population. However, the actual number of people suffering poverty and deprivation is also important.

Figure 5 Number of poor, severely poor and vulnerable



Whereas South Asia and Sub-Saharan Africa are home to the largest number of poor people (546 and 560 million, respectively), the number of people vulnerable to poverty is highest in South Asia and East Asia & the Pacific (330 and 313 million, respectively) (Figure 5). Although on average there are more MPI-poor people in Sub-Saharan Africa than in South Asia, if we take into account the standard error of these estimates, the number of MPI-poor people in these regions is actually undistinguishable. In contrast, the number of people suffering severe multidimensional poverty is unambiguously highest in Sub-Saharan Africa (342 million), followed by South Asia (200 million).

3.4 Multilevel disaggregation of the global MPI

Taking the world regions as a level 1 subgroup, several multilevel disaggregations are feasible. We will close out this section scrutinizing two key level 2 disaggregations. The first one distinguishes urban and rural poverty within each region (see e.g. Lucci et al., 2018; Zhou & Liu, 2019) and the second disaggregates by age-cohorts (see e.g. Kim, 2019; Cribb & Emmerson, 2019). One at a time, these population sets are taken as level 2 subgroups of the each world region (level 1).

Table 5 Disaggregating H, A and MPI over area by world regions

	H (%)					
	Urban			Rural		
	mean	lb	ub	mean	lb	ub
World	8.01	7.59	8.46	35.50	34.52	36.49
AS	8.24	7.64	8.88	29.98	28.52	31.48
EAP	2.43	1.81	3.25	9.52	8.48	10.68
ECA	0.73	0.61	0.88	4.05	3.62	4.51
LAC	3.28	3.04	3.53	21.11	19.28	23.07
SA	12.01	11.43	12.61	40.50	39.79	41.22
SSA	26.44	21.55	31.98	73.20	68.96	77.05

	A (%)					
	Urban			Rural		
	mean	lb	ub	mean	lb	ub
World	44.01	43.64	44.37	50.50	50.26	50.73
AS	43.47	42.67	44.27	52.79	52.19	53.40
EAP	39.33	38.38	40.29	44.08	43.46	44.69
ECA	35.72	35.16	36.28	38.73	38.13	39.33
LAC	40.23	39.51	40.96	44.59	44.19	45.00
SA	43.12	42.62	43.62	46.13	45.71	46.55
SSA	46.83	46.33	47.33	56.30	55.97	56.64

	MPI					
	Urban			Rural		
	mean	lb	ub	mean	lb	ub
World	0.035	0.033	0.037	0.179	0.174	0.185
AS	0.036	0.033	0.039	0.158	0.150	0.167
EAP	0.010	0.007	0.013	0.042	0.037	0.047
ECA	0.003	0.002	0.003	0.016	0.014	0.018
LAC	0.013	0.012	0.014	0.094	0.086	0.103
SA	0.052	0.049	0.055	0.187	0.182	0.192
SSA	0.124	0.101	0.151	0.412	0.388	0.437

Note: lb and ub denote, respectively, lower bound and upper bounds of the 95% confidence intervals
Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

It is important to mention that the definitions of 'urban' and 'rural' are not consistent across the 105 countries that we study. Thus we take the definition of these areas directly from the datasets used to construct the MPI. The DHS surveys, for example, use national census definitions for most datasets, and these vary across countries. Unfortunately, it is not possible at this time to use a consistent definition of rurality, and this affects the interpretation of results.

Table 6. Disaggregating H, A and MPI over age groups by world regions

	H (%)											
	Age 0-9			Age 10-17			Age 18-59			Age 60+		
	mean	lb	ub	mean	ub	lb	mean	lb	ub	mean	lb	ub
World	38.14	37.20	39.08	28.32	27.60	29.05	17.73	17.17	18.30	17.38	16.76	18.02
AS	28.06	27.00	29.14	21.65	20.72	22.61	14.79	14.14	15.45	13.95	13.14	14.80
EAP	9.86	8.51	11.40	7.30	6.54	8.13	4.45	3.94	5.02	7.21	6.44	8.05
ECA	4.86	4.40	5.36	2.46	2.16	2.81	1.95	1.75	2.18	1.25	1.09	1.43
LAC	12.52	12.09	12.96	9.13	8.72	9.56	6.00	5.82	6.18	6.88	6.43	7.35
SA	44.97	44.09	45.85	31.34	30.68	32.00	26.75	26.25	27.25	28.49	27.95	29.02
SSA	67.18	62.87	71.22	58.53	53.93	62.98	50.51	43.40	57.59	55.93	47.94	63.63

	A (%)											
	Age 0-9			Age 10-17			Age 18-59			Age 60+		
	mean	lb	ub	mean	ub	lb	mean	lb	ub	mean	lb	ub
World	52.46	52.19	52.73	50.28	50.03	50.53	47.88	47.67	48.09	44.45	44.21	44.70
AS	52.76	52.16	53.37	51.05	50.47	51.63	49.32	48.82	49.81	47.64	47.07	48.21
EAP	45.27	44.36	46.19	44.17	43.44	44.91	42.60	41.96	43.24	40.70	40.02	41.38
ECA	38.93	38.30	39.56	38.14	37.47	38.81	37.99	37.46	38.52	37.28	36.56	38.02
LAC	45.01	44.44	45.58	44.17	43.73	44.61	42.60	42.15	43.06	39.53	39.22	39.85
SA	48.20	47.68	48.72	46.22	45.81	46.64	44.69	44.37	45.01	42.55	42.20	42.91
SSA	56.93	56.58	57.27	54.77	54.41	55.13	53.57	53.22	53.92	50.16	49.83	50.49

	MPI											
	Age 0-9			Age 10-17			Age 18-59			Age 60+		
	mean	lb	ub	mean	ub	lb	mean	lb	ub	mean	lb	ub
World	0.200	0.195	0.206	0.142	0.138	0.146	0.085	0.082	0.088	0.077	0.074	0.080
AS	0.148	0.142	0.155	0.111	0.105	0.116	0.073	0.069	0.077	0.066	0.062	0.071
EAP	0.045	0.038	0.052	0.032	0.029	0.036	0.019	0.017	0.021	0.029	0.026	0.033
ECA	0.019	0.017	0.021	0.009	0.008	0.011	0.007	0.007	0.008	0.005	0.004	0.005
LAC	0.056	0.054	0.059	0.040	0.038	0.042	0.026	0.025	0.026	0.027	0.025	0.029
SA	0.217	0.211	0.223	0.145	0.141	0.149	0.120	0.117	0.122	0.121	0.118	0.124
SSA	0.382	0.358	0.408	0.321	0.295	0.347	0.271	0.233	0.312	0.281	0.242	0.323

Note: lb and ub denote, respectively, lower bound and upper bounds of the 95% confidence intervals

Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

In terms of differences by age-cohort, multidimensional poverty as measured by the MPI is unambiguously higher among young children (aged 0-9), across all the world regions (Table 6). More than one in three children (38.14%) live in poor households and they face on average, 52.46% of the weighted deprivations. This situation affects children in Sub-Saharan Africa the most.

4. Robustness of the 2018 revised global MPI

As we mentioned earlier, one particular parametrization underlies all the results that we have discussed so far. When the MPI was first released in 2010, there was some scepticism about its robustness to alternative parametrizations in the academic and policy-making spheres (see Ferreira 2011 for a discussion on this matter). However, this index was found to be robust to changes in (a) the weighting scheme and (b) to the poverty cutoff in Alkire & Santos (2014). For comparison purposes, we revisit the latter study to evaluate the robustness of the revised version of the index to the same parameters.

Before presenting our results, let us briefly discuss the methodological strategy that we adopt to establish robustness of country pairwise comparisons. More formal details can be found in Appendix B. Two ways of establishing robustness consists of performing confidence interval overlap analyses and hypothesis tests on difference between estimates, both of which were implemented in Alkire & Santos (2014). Indeed, for any two poverty estimates that draw on independent samples, the absence of overlap between their corresponding confidence intervals (which can be established visually, for instance) allows to infer that they are statistically different[7] (see e.g. Alkire et al., 2015). A related, more formal approach consists of conducting hypotheses tests to assess whether the difference between estimates is zero. The key difference is that confidence interval overlap analyses do not take into account the possibility of non-zero covariance between poverty estimates. This situation may occur, for instance, when one is interested in making a *direct* comparison of two estimates (say due to a change in parameters) making use of the same sample.

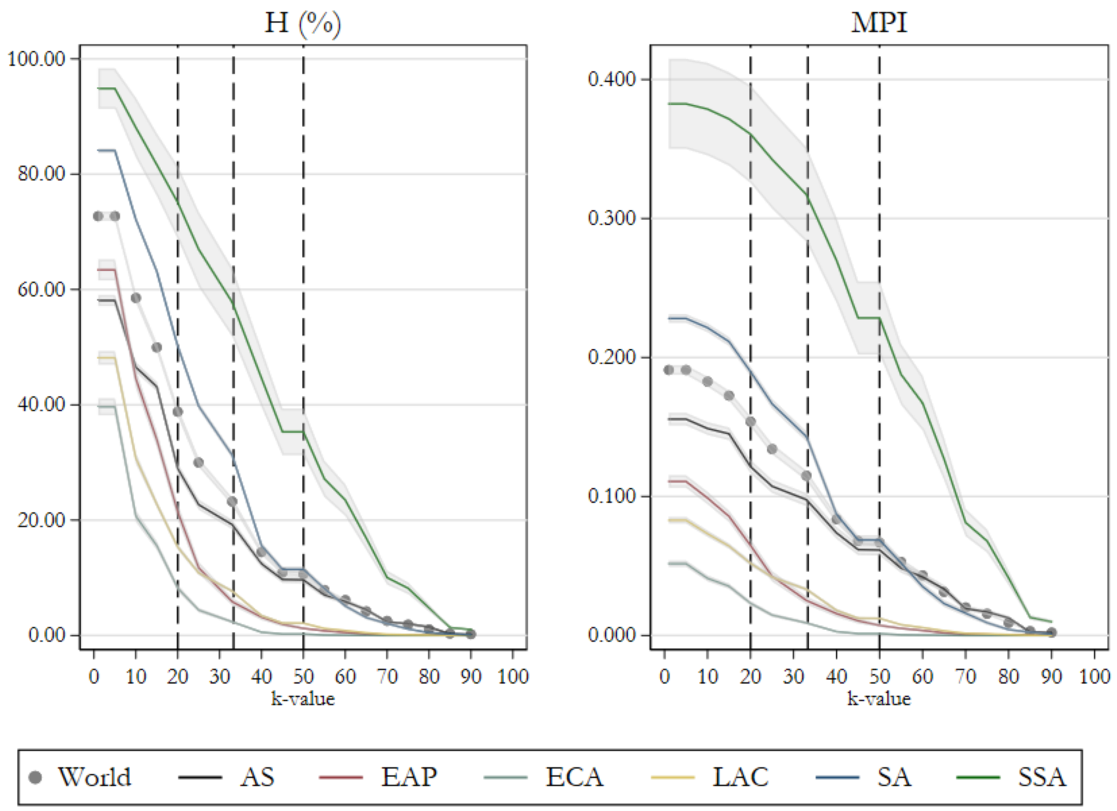
4.1 Shifting the poverty cutoff

Let us first visually describe some robustness patterns by assessing the $H(\theta)$ and $MPI(\theta)$ complementary CDFs over different poverty cutoffs k. In Figure 6, we can see that $H(\theta)$ and

[7] The converse is not true, however, as even if the estimates draw on independent samples, by definition, the difference between their individual standard errors is not equivalent to the standard error of their difference.

$MPI(\theta)$ for Sub-Saharan Africa first-order stochastically dominate the distribution of these measures with respect to k for every other world region. Conversely, the distributions for every world region, first-order stochastically dominate those for Europe & Central Asia. These are powerful results related to two statements that hold true over the entire set of possible k-values: i) the simple and adjusted headcount ratios are highest in Sub-Saharan Africa, and ii) the converse is true for Europe & Central Asia.

Figure 6. Complementary cumulative distribution functions by world region



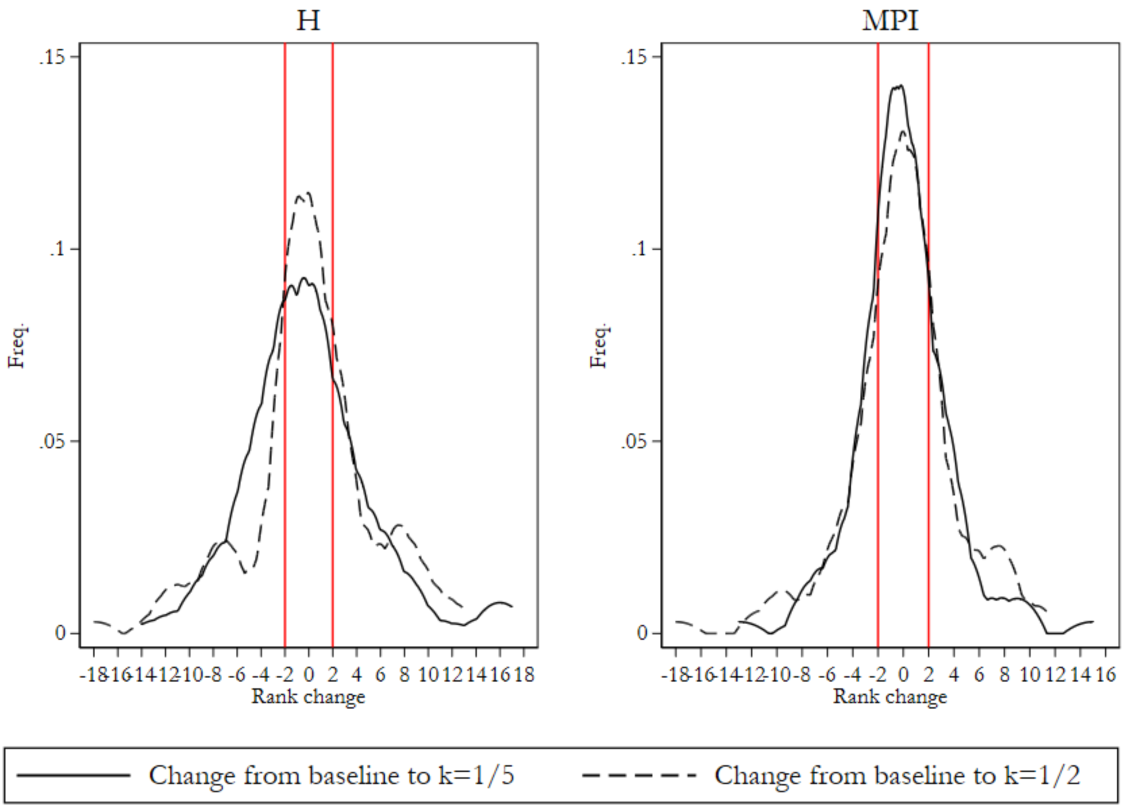
In a general way, results that hold true over the *entire range* of k are the exception, which is why the above insights are remarkable. Note, however, that both $H(\theta)$ and $MPI(\theta)$ are monotonic increasing functions of k, so different population subsets are effectively identified as multidimensionally poor by adopting distinct k-values. Each one of these subsets regroup people that experience joint deprivations to different extents and with varying intensity. Their livelihoods are different and the types of policies required to improve their situation should build upon these differences in order to be effective. Thus, we argue that if changes arise due to shifts in k, they have a meaningful interpretation and they may usefully point towards distinct poverty analyses and policy actions against different patterns and intensities of joint deprivations. Thus instead of delving deeper into a *general* robustness analysis of $H(\theta)$ and $MPI(\theta)$ distributions, it may be more informative to focus on *local* robustness within a relevant neighbourhood of k (Sen, 1999; World Bank, 2017). One useful way to establish this neighbourhood is to use the internationally comparable definitions of three population groups that regularly appear in recent global MPI reports (see e.g. OPHI, 2018), which have already guided us through this paper thus far. When the robustness of the global MPI was first presented (Alkire & Santos, 2010), the multidimensionally poor people were identified with the cutoff $k = \frac{1}{3}$, and is now the definition taken to identify people suffering acute poverty. The recent reports on the global MPI also identify the severely multidimensionally poor people with $k = \frac{1}{2}$ (which is a subset of the acutely poor population), and people that are vulnerable to multidimensional poverty are identified if $\left(\frac{1}{3} > c_i \geq \frac{1}{5}\right)$. These definitions implicitly define the range $k \in \left[\frac{1}{5}; \frac{1}{2}\right]$ as the relevant neighbourhood to assess the local robustness of $H(\theta)$ and $MPI(\theta)$ around the baseline cutoff $k = \frac{1}{3}$.

Restricting our visual analysis of Figure 6 to $k \in \left[\frac{1}{5}; \frac{1}{2}\right]$, we can also affirm that the $H(\theta)$ and $MPI(\theta)$ distributions of South Asia are the second highest in the world, followed by the ones of the Arab States. We cannot, however, establish a dominance order between East Asia & the Pacific and Latin America & the Caribbean, as their complementary CDFs cross each other. For k-values close to $\frac{1}{5}$ (i.e. vulnerability), Latin America & the Caribbean tend to be less poor by $H(\theta)$ and the $MPI(\theta)$. This means that the likelihood of being vulnerable to poverty is lower in this region. However, this relative advantage is not preserved for k-values closer to $\frac{1}{2}$ (i.e. severe poverty), meaning that the likelihood of suffering severe poverty tends to be similar in both regions.

Digging deeper, let us assess the local robustness of *absolute* country poverty orderings with respect to plausible changes of k. The poorest countries in these orderings (South Sudan according to $H\left(\bar{\theta}|k = \frac{1}{3}\right)$ and Niger according to the $MPI\left(\bar{\theta}|k = \frac{1}{3}\right)$) are thus the poorest among the 105 considered countries. Changing rank under alternative k-values is interpreted as a comparative poverty increase induced by this shift in the poverty line.

In terms of H-orderings, the absolute position of 11 and 12 countries is totally unchanged if k shifts, respectively in the extremes of the relevant k-range, $\frac{1}{5}$ and $\frac{1}{2}$,. The corresponding results for MPI-orderings depict a slightly higher stability - 23 and 13 countries. The kernel densities of all rank changes under these two alternative cutoffs is depicted in Figure 7. We can see that non-null ordering shifts take place, but they are most often equal or lower than +/- 2 ranking positions. By MPI-orderings, this is the case for 65 countries if $k = \frac{1}{5}$ and 63 if $= \frac{1}{2}$. Note that close-to-zero rank changes are more frequent for MPI-orderings compared to H-orderings. This means that the average number of weighted deprivations among the poor (A) is highly robust to changes in the poverty cutoff, endowing the MPI with a higher stability compared to H when k shifts. Thus, even if the proportion of people suffering multidimensional poverty may vary with respect to local changes in the poverty cutoff, the intensity in which these people suffer that conditions is found to be quite stable.

Figure 7 Distributions of rank changes for different poverty cutoffs



Going beyond single-country descriptions, let us now discuss the results of country pairwise comparisons. We will evaluate the extent to which the country ordering established at the baseline is preserved if the poverty cutoff shifts *across* the relevant neighbourhood $\left[\frac{1}{5};\frac{1}{2}\right]$. Taking the notation introduced earlier, we posit that a country pairwise comparison is robust across the relevant neighbourhood if the poverty order established by specification $\bar{\theta}|k=\frac{1}{3}$ (the baseline) is preserved to the array of alternative specifications $\Theta=\left\{\tilde{\theta}_1\left|k=\frac{1}{5};\ \tilde{\theta}_2\right|k=\frac{1}{2}\right\}$. The results of these tests are summarized in Table 7.

Table 7 Pairwise comparisons using alternative poverty cutoffs

Region	**Countries**	**Possible Comparisons**	**Significant comparisons at baseline**		**Same ordering: Sig. and non-sig. at baseline**		**Same ordering: only sig. at baseline**	
			Number	**%**	**Number**	$R_P^{\bar{\theta},\Theta}$	**Number**	$S_P^{\bar{\theta},\Theta}$
			$\boldsymbol{P(\theta)=MPI(\theta)}$					
World	104	5356	4957	92.6	4922	91.9	4714	95.1
AS	13	78	71	91.0	68	87.2	67	94.4
EAP	11	55	49	89.1	47	85.5	45	91.8
ECA	14	91	65	71.4	51	56.0	41	63.1
LAC	20	190	156	82.1	172	90.5	146	93.6
SA	7	21	18	85.7	17	81.0	17	94.4
SSA	37	666	595	89.3	613	92.0	569	95.6
			$\boldsymbol{P(\theta)=H(\theta)}$					
World	104	5356	4939	92.2	4155	77.6	4033	81.7
AS	13	78	69	88.5	66	84.6	65	94.2
EAP	11	55	48	87.3	39	70.9	39	81.3
ECA	14	91	66	72.5	31	34.1	21	31.8
LAC	20	190	154	81.1	127	66.8	106	68.8
SA	7	21	18	85.7	15	71.4	14	77.8
SSA	37	666	591	88.7	514	77.2	499	84.4

Note: $R_P^{\bar{\theta},\Theta}$ denotes the proportion of country pairwise poverty orderings that are similar in all three alternative k values. In this proportion, countries that have similar levels of poverty at the baseline specification are taken into account. $S_P^{\bar{\theta},\Theta}$is similar to $R_P^{\bar{\theta},\Theta}$, but omits country poverty orderings at the baseline that show undistinguishable poverty levels.
Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

In the pairwise comparisons between the entire set of countries, just under 82% of the strict country pairwise orderings by H are robust at the baseline. However, this overall figure masks stark differences between world regions. Having compact summary measures of robustness is undeniable useful, but to be clear, note two elements need to be taken into account to interpret the ratios presented in Table 7 (see Alkire & Santos, 2014). This first is that regions with a high number of countries (such as Sub-Saharan Africa) may tend to show higher robustness due to the larger number of comparisons that are possible. The second element is that regions where the differences between countries in terms of H and MPI are high, will tend to show a higher stability because the common range between poverty levels is wider. Our results have to be interpreted taking this into account.

At the extremes, 94.2% of the strict pairwise orderings in the Arab States are robust, while this ratio is only 31.8% in Europe & Central Asia, the least poor region in the developing world and a region where the levels of H and MPI are relatively less disperse. Thus, the overall low levels of inequality across country poverty levels in this region make it difficult to arrive at a stable pairwise ordering by H and the MPI.

We confirm that the country ordering by the MPI is more robust than that by H. Taking all countries into account, 95.1% of the strict pairwise comparisons by the MPI at the baseline are unchanged. Alkire & Santos (2014) found a similar rate (95.7%) in a similar robustness analysis of the 2010 version of the MPI. However, they considered $k = \frac{1}{5}$ and $k = \frac{2}{5}$ as alternative poverty cut-offs, so finding a similar robustness rate even if the upper-limit alternative cut-off is pushed to $k = \frac{1}{2}$ depicts a higher level of robustness of the revised version of this index.

The overall robust nature of the MPI is the result of similar robustness patterns across world regions. In almost all of them, more than 90% of the strict pairwise orderings at baseline are preserved. The only exception is Europe & Central Asia, but even for this region, the robustness figure for the MPI (63.1%) doubles that for H (31.8%). The poorest world regions by the MPI, namely Sub-Saharan Africa, South Asia and the Arab States have the highest proportion of strict pairwise comparisons.

4.2 Shifting the weighting structure

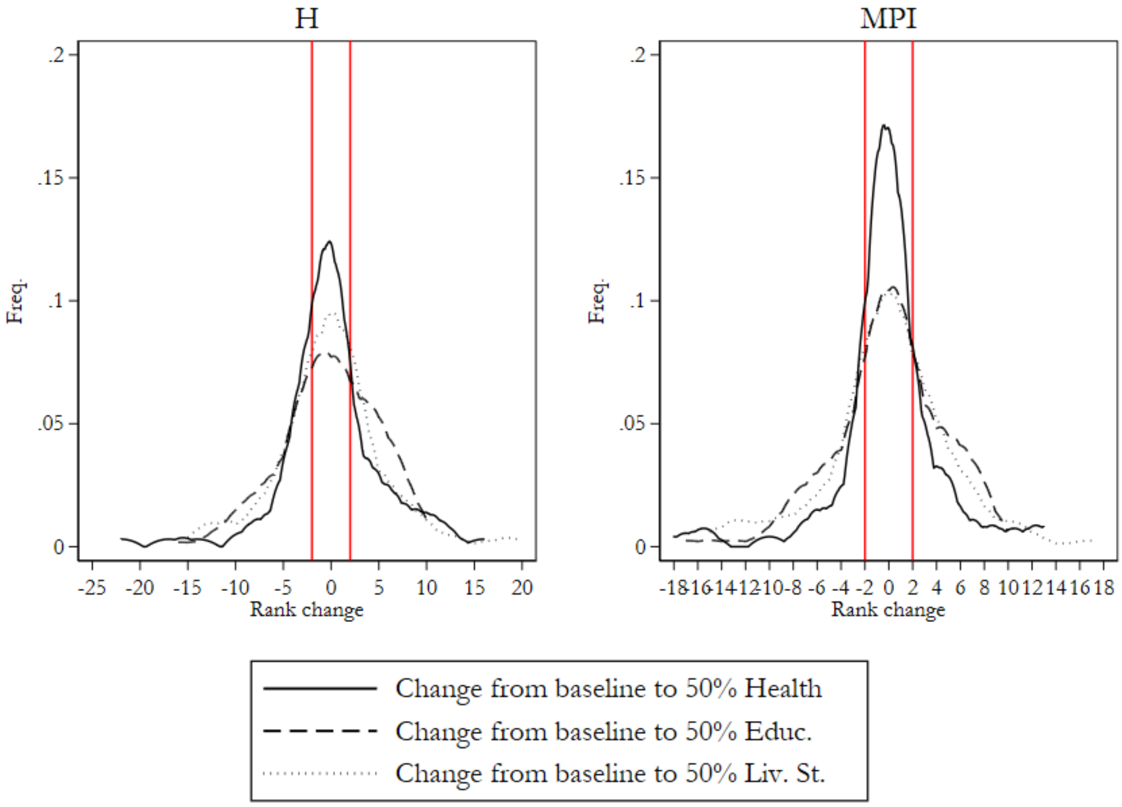
Let us now focus on a robustness analysis to changes in the indicator weighting structure, $w \in \theta$. The weights for health (he), education (ed) and living standards (ls) are denoted as w_{he}, w_{ed} and w_{ls}, respectively, so that $w = \{w_{he}, w_{ed}, w_{ls}\}$.

In a strict sense, there is an infinite combination of alternative weighting structures, so a full robustness evaluation is beyond the scope of this paper. We follow Alkire & Santos (2014) and limit ourselves to three sets of plausible weighting structures that could make sense in the practical academic and policy-making spheres, while also being easy to comprehend widely. They consist of considering, in turn, one dimension to be twice as important as the other two. Effectively, these alternative structures are computed based on different arrangements of the dimensional weight trio (25%, 25%, 50%).

Let us first conduct a robustness analysis of each country's absolute positions in the poverty orderings by H and MPI. The kernel densities of these changes are depicted in Figure . The concentration of rank changes in close-to-zero values shows that the absolute positions are unchanged to a large extent. Among the three considered alternative weighting structures, assigning a 50% weight to health generates the least number of absolute rank shifts. This is the reflection of the relatively low uncensored – and censored – deprivation headcounts of child mortality globally.

The overall changes associated to the other two alternative weighting structures depend on the aggregate poverty measure. Close-to-zero changes in the H-ranking absolute positions are more frequent if a 50% weight is assigned to living standards. Assigning this weight to education yields the least frequent close-to-zero absolute rank shifts. Even in this case, however, most of these shifts are +/- 2 positions. In terms of MPI-rankings, assigning a 50% weight to either education or living standards yields a similar frequency of close-to-zero changes.

Figure 8. Distributions of rank changes for alternative weighting structures



Let us now turn to a country pairwise comparisons analysis. Taking the notation introduced above, we assess the robustness of the baseline measure specified by $\bar{\theta}|w = \left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}$ with respect to an array of alternative specifications $\Theta = \left\{\tilde{\theta}_1 \left| w = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right\}; \tilde{\theta}_2 \right| w = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}; \tilde{\theta}_3 | w = \left\{\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right\}\right\}$.

We find that 92.2% of the strict ordering by H at the baseline are preserved across the three alternative weighting structures. In all the world regions, this rate is over 70%, with country orderings in the Arab States (97.2%) and Sub-Saharan Africa (89.6%) being the most stable. The least robust orderings are found in South Asia (72.2%). Even if these figures do not account for the number of countries or the dispersion of H in each region, they convey a useful idea about the robustness of the results discussed in previous sections.

Similar robustness patterns for all the world regions are found among the strict orderings by the MPI at the baseline, albeit to a lesser extent (Table 8). Just under 90% of all possible pairwise comparisons are preserved across all the considered alternative weighting structures. A directly comparable analysis was conducted in Alkire & Santos (2014) for the original global MPI specification, where they found a rate of 88.9%. We can thus affirm that the 2018 specification of this index is as least as stable as the original one.

Table 8 Pairwise comparisons using alternative weighting structures

Region	Countries	Possible Comparisons	Significant comparisons at baseline		Same ordering: Sig. and non-sig. at baseline		Same ordering: only sig. at baseline	
			Number	%	Number	$R_P^{\bar{\theta},\Theta}$	Number	$S_P^{\bar{\theta},\Theta}$
			$P(\boldsymbol{\theta}) = H(\boldsymbol{\theta})$					
World	104	5356	4957	92.6	4688	87.5	4569	92.2
AS	13	78	71	91.0	71	91.0	69	97.2
EAP	11	55	49	89.1	43	78.2	39	79.6
ECA	14	91	65	71.4	57	62.6	48	73.8
LAC	20	190	156	82.1	140	73.7	128	82.1
SA	7	21	18	85.7	13	61.9	13	72.2
SSA	37	666	595	89.3	549	82.4	533	89.6
			$P(\boldsymbol{\theta}) = MPI(\boldsymbol{\theta})$					
World	104	5356	5090	95.0	4579	85.5	4566	89.7
AS	13	78	73	93.6	70	89.7	70	95.9
EAP	11	55	50	90.9	39	70.9	39	78.0
ECA	14	91	69	75.8	57	62.6	53	76.8
LAC	20	190	167	87.9	132	69.5	130	77.8
SA	7	21	21	100.0	14	66.7	14	66.7
SSA	37	666	622	93.4	515	77.3	514	82.6

Note: $R_P^{\bar{\theta},\Theta}$ denotes the proportion of country pairwise poverty orderings that are similar in all three alternative k values. In this proportion, countries that have similar levels of poverty at the baseline specification are taken into account. $S_P^{\bar{\theta},\Theta}$is similar to $R_P^{\bar{\theta},\Theta}$, but omits country poverty orderings at the baseline that show undistinguishable poverty levels.
Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

5. The revised and original MPI: an empirical comparison

To empirically evaluate the consequences of the revision, we produce estimates for the original version with the exact same data used for the estimation of the revised version. In that sense, our figures do not actually reflect the original MPI values reported in 2010 (UNDP, 2010; Alkire & Santos, 2014), but rather a set of counterfactual estimations that are useful only for evaluative purposes. We compare actual (revised specification) and counterfactual (original specification) figures in three ways. First, we focus on differences between aggregate MPI figures, then we asses differences in indicator deprivation headcount ratios and finally, we perform a country pairwise comparison analysis between the 2010 and 2018 indicator specifications using the 2018 datasets.

In a nutshell, we find that the range of the overall, global proportion of people who live in multidimensional poverty (H) is very similar after the revision. With 95% confidence, the level of H level ranges between 22.6% - 23.9% in the revised specification and 23.4% - 24.7% in the original one. In that sense, the differences induced by the revision are certainly small; however given the large sample at hand (and the ensuing small standard errors for our estimates), proper hypothesis tests on the difference of H between both specifications show that the difference, although small, is statistically significant (see Table 9). Importantly however, even this strict way of assessing robustness results in a *non-statistically significant* difference for the proportion of poor people in Sub Saharan Africa, the poorest region in the world. This is also true for Europe & Central Asia if we take a 1% significance level. The similar range of poverty incidence in these regions directly implies a similarly stable nature of the number of people identified as poor in both specifications.

Turning now to the intensity of poverty, A, we find that it has significantly shifted in every region due to the revision. It ranges between 49.3% - 49.7%, in the revised specification, and between 45.3% - 45.9% in the original one. The biggest intensity shift is found in Europe & Central Asia (+15.3 percentage points), followed by Latin America & the Caribbean (+10.4 pp).

Finally, the MPI levels for the whole developing world range between 0.112-0.119 in the revised specification and 0.116-0.123 in the original one. The level of the index is around the same range after the revision, although the statistically significant shifts in A (and in H for some regions) yields statistically significant differences for the MPI as well. (see Table 9)

Table 9. MPI and its components by world region and specification

	H (%)						
	2010 specif.		**2018 specif.**				
	Mean	**SE**	**Mean**	**SE**	**Diff**	**SE**	**p value**
World	24.08	0.33	23.24	0.33	0.84	0.06	0.000
AS	17.93	0.40	19.23	0.41	-1.30	0.09	0.000
EAP	7.63	0.34	5.85	0.33	1.78	0.14	0.000
ECA	2.17	0.12	2.37	0.13	-0.19	0.08	0.020
LAC	6.84	0.14	7.69	0.16	-0.86	0.08	0.000
SA	32.43	0.29	31.28	0.30	1.15	0.10	0.000
SSA	57.91	2.97	57.73	3.04	0.18	0.13	0.169

	A (%)						
	2010 specif.		2018 specif.				
	Mean	**SE**	**Mean**	**SE**	**Diff**	**SE**	**p value**
World	45.62	0.16	49.50	0.12	-3.88	0.07	0.000
AS	44.86	0.39	50.82	0.27	-5.96	0.17	0.000
EAP	36.88	0.47	43.06	0.31	-6.18	0.41	0.000
ECA	22.91	0.78	38.25	0.27	-15.34	0.66	0.000
LAC	32.76	0.36	43.19	0.18	-10.43	0.32	0.000
SA	41.92	0.24	45.76	0.20	-3.83	0.10	0.000
SSA	52.21	0.24	54.87	0.17	-2.66	0.10	0.000
	MPI						
	2010 specif.		**2018 specif.**				
	Mean	**SE**	**Mean**	**SE**	**Diff**	**SE**	**p value**
World	0.120	0.002	0.115	0.002	0.005	0.000	0.000
AS	0.092	0.002	0.098	0.002	-0.006	0.000	0.000
EAP	0.033	0.001	0.025	0.001	0.008	0.001	0.000
ECA	0.008	0.000	0.009	0.001	-0.001	0.000	0.021
LAC	0.030	0.001	0.033	0.001	-0.003	0.000	0.000
SA	0.150	0.002	0.143	0.002	0.007	0.000	0.000
SSA	0.321	0.017	0.317	0.017	0.004	0.001	0.000

Source: Own calculations based on a pooled dataset of 105 national datasets as indicated in Appendix C

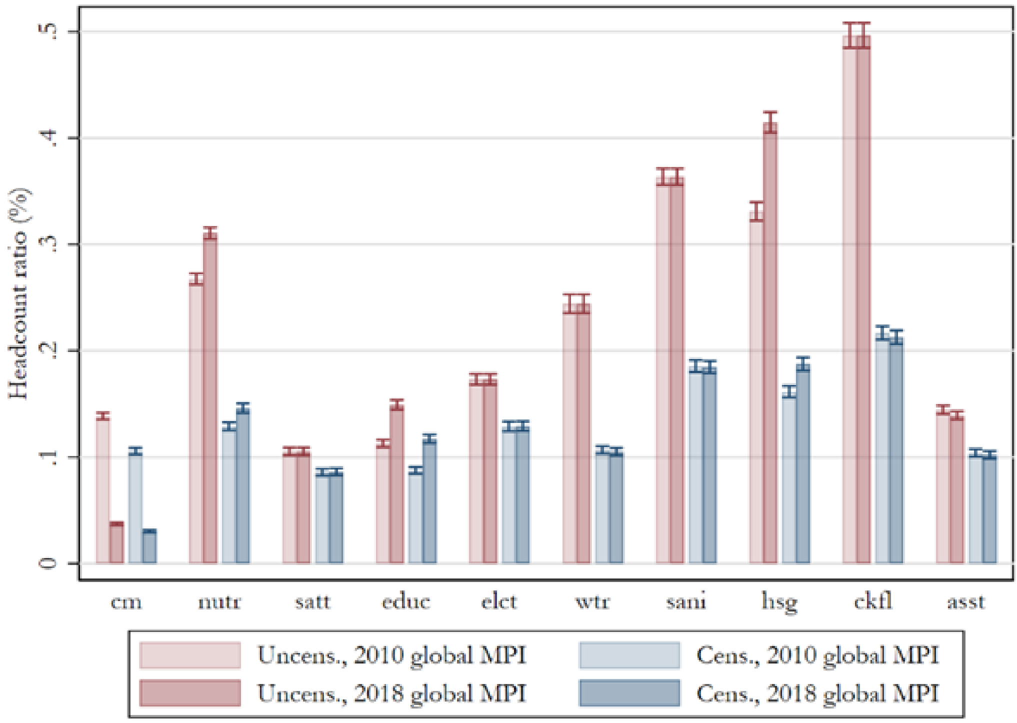
In order to gain a more in-depth insight about changes in the intensity of poverty, let us present a disaggregated analysis by indicator. Not only will we present how the revision modified the prevalence of deprivations among the poor (censored headcount ratios), but also among the entire population (uncensored headcount ratios).

The deprivation headcount ratios corresponding to four out of the five revised indicators have significantly increased in the revised specification. The only exception is the *assets* indicator, for which censored and uncensored headcounts remained unchanged, despite the inclusion of two items - computer and animal cart in the revision. This result is aligned with Vollmer & Alkire (2018) who found that these two items have relatively low difficulty and discrimination parameters in an Item-Response Theory analysis. This reflects that they are likely to be associated with the other items included in the assets indicator.

The censored and uncensored deprivations in *child mortality* are dramatically lower in the 2018 version of the MPI – by around 10 percentage points (Figure). This is because the revised indicator only considers deaths occurred during the last five years preceding the survey – as opposed to the household ever having suffered the death of a child. The information for this improved indicator was not available in 2010, but the lower headcount ratios are more accurate as well as policy-salient. This statement finds support in the official UN dMDG Report (UN, 2015), according to which the global under-five mortality rate has reduced by more than half between 1990 and 2015 (90 to 43 per 1000 children). The rate of reduction of this indicator has more than tripled in the same span. Similarly, You et al. (2015) have estimated that around 94 million children would die before they are 5 years old by 2030 if each country maintains their observed mortality rate in 2015. However, they also estimate that more than one-fourth of these could be prevented if each country manages to keep the average 2000-2015 average annual reduction pace between 2016 and 2030.

Conversely, the censored and uncensored deprivation headcounts corresponding to *nutrition, education* and *housing* are all higher in the new version of the MPI – by around 4 pp., 3pp. and 8pp., respectively. In the revision, these indicators have been assigned more demanding deprivation cut-offs, which better align with the new international standards evinced in the SDG indicators.

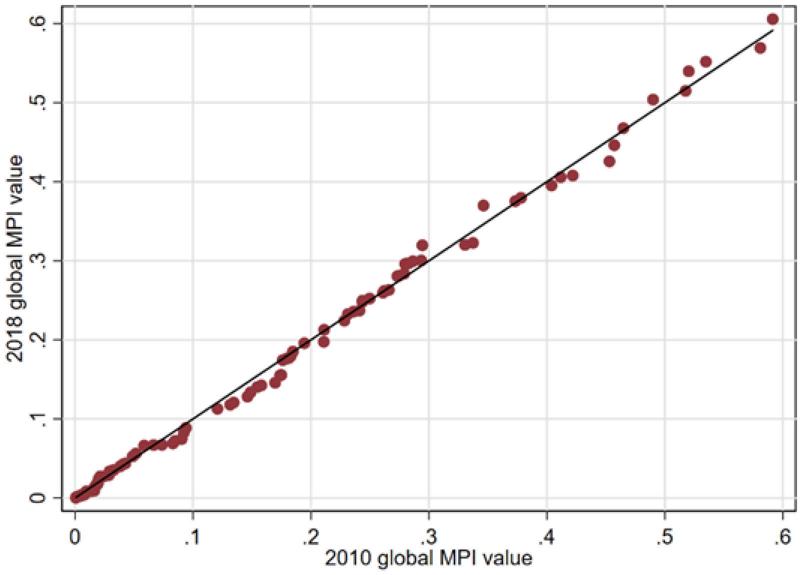
Figure 9. Censored and uncensored headcount ratios by specification



In a more detailed cross-country analysis, we find that the MPI distribution across the 105 considered countries has remained stable. As depicted in the quantile-quantile plot in

Figure , the shape of both MPI structures' distributions is similar. Their corresponding quantiles match closely and no systematic differences can be detected across the entire observed range of MPI values. Such a close distributional resemblance probably translates into a highly robust country *ordering* by the MPI (Alkire et al., 2015). To explore this, we performed a pairwise comparison analysis where the alternative specification is defined as the original definition of indicators and their deprivation cutoffs, z.

Figure 10. Quintile-quintile plot: Global distributions of MPI



Taking into account both significant and non-significant poverty orderings at the baseline (i.e. the revised specification), 93.02% of the possible country pairwise comparisons are identical in both MPI versions (4982 out of 5356). This rate can be interpreted a summary figure of the overall robustness of the MPI to the revision. To gauge the robustness of strict poverty orderings only, we can focus on 86.07% of the possible pairwise comparisons (4610 out of 5356) that are found to be strict in the 2018 MPI specification. Practically all of them (99.15%) are identical in the 2010 specification (4571 out of 4610). In our view, this is a quite powerful result showing that MPI revision manages to better identify deprivations, while maintaining country poverty orderings largely unchanged.

6. Concluding remarks

To be revised in subsequent versions

In this paper, we have presented a thorough analysis of the stability of one of the most influential development indicators today, namely the global MPI produced annually by UNDP and OPHI. Such an analysis is timely because in 2018, the index was subjected to its first major revision consisting of adapting five out its ten component indicators to better align to the 2030 Agenda and the SDGs.

We found that the index is overall robust to these changes, and that the insights that it conveys about the state of poverty in the developing world are robust to changes in its core parameters. These are the multidimensional poverty line and the weighting structure.

While focusing on analysing the new insights offered by the revision of the global MPI and assessing its robustness to key parametric changes, we have also made two methodological advances that are relevant to multidimensional poverty measurement more generally. The first consists of formal extension of a multilevel disaggregation of measures based on the dual-cutoff counting approach that are constructed using pooled surveys at the country level (105 in this case). The second one consists of establishing summary indicators of robustness to parametric changes in the form of indices that capture the extent to which country pairwise poverty orderings are preserved even when a parametric change occurs.

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8. Appendix A

Let us define g_1 exhaustive and mutually exclusive level 1 population subgroups. The generic subgroup at this level is denoted as ℓ and it has a population size denoted as $N^\ell > 0, \forall \ell = 1 \dots g_1$. We thus have $\sum_\ell N^\ell = N$. For instance, groups can be defined by world subregions, and their population add up to the global population. Let us also define g_2^ℓ exhaustive and mutually exclusive level 2 subgroups *within* ℓ, where the number of level 2 groups can vary $\forall \ell$. These groups can be urban and rural dwellers in different world regions. We will denote the generic group at this level as h^ℓ and its population size as $N_h^\ell \geq 0, \forall h = 1 \dots g_2^\ell$. Hence $\sum_{h^\ell} N_h^\ell = N^\ell$, and $\sum_\ell \sum_{h^\ell} N_h^\ell = N$. Note that we allow for the general case of different level 2 subgroups for each level 1 subgroup. This is useful to see, for instance, that subnational desegregations by country in a global analysis are in fact a particular form of a multilevel disaggregation of the global MPI (see e.g. OPHI, 2018).

Noting that $H(\theta) = \frac{1}{N}\sum_{i=1}^{N} \rho_k(X_{i.}, z, w) = \frac{1}{N}\sum_\ell \sum_{h^\ell} \sum_{i \in h^\ell} \rho_k(X_{i.}, z, w)$, this expression can be reformulated as:

$$H(\theta) = \frac{1}{N}\sum_{\ell}\sum_{h^\ell}\frac{N^\ell}{N^\ell}N_h^\ell\left[\frac{1}{N_h^\ell}\sum_{i\in h^\ell}\rho_k(X_{i.}, z, w)\right] = \sum_{\ell}\frac{N^\ell}{N}\sum_{h^\ell}\frac{N_h^\ell}{N^\ell}H_h^\ell(\theta)$$

Where $H_h^\ell(\theta)$ is the simple headcount ratio for level 2 subgroup h^ℓ. Since $\frac{N^\ell}{N}$ is the population share of subgroup ℓ in the entire sample, and $\frac{N_h^\ell}{N^\ell}$ is the population share of level 2 subgroup h^ℓ in level 1 subgroup ℓ, the above expression shows that the overall simple headcount ratio is a multilevel population-weighted sum of the simple headcount ratios.

Note that the single-level disaggregation posited in Alkire et al. (2015) is a particular case of this multilevel disaggregation procedure. It corresponds to the case where $g_2^\ell = 1, \forall \ell$, so that $N_h^\ell = N^\ell$, $i \in h^\ell$ is equivalent to $i \in \ell \; \forall h^\ell$, and $H_h^\ell(\theta) = H^\ell(\theta)$. In that case, we have $H(\theta) = \sum_\ell \frac{N^\ell}{N} \sum_{h^\ell} \frac{N_h^\ell}{N^\ell} H_h^\ell(\theta) = \sum_\ell \frac{N^\ell}{N} H^\ell(\theta)$. Among others, this shows that the overall global simple headcount ratio is the sum of country-specific simple headcount ratios, weighted by their population share in the entire developing world sample.

The disaggregation of the adjusted headcount ratio $M_0(\theta)$ follows the exact same logic:

$$M_0(\theta)=\frac{1}{N}\sum_{\ell}\sum_{h^\ell}\frac{N^\ell}{N^\ell}N_h^\ell\left[\frac{1}{N_h^\ell}\sum_{i\in h^\ell}(\rho_k(X_{i.},z,w)\times c_i)\right]=\sum_{\ell}\frac{N^\ell}{N}\sum_{h^\ell}\frac{N_h^\ell}{N^\ell}{M_0}_h^\ell(\theta)$$

The disaggregation of $A(\theta)$ also follows a similar logic, but importantly, this measure only focuses on the poor population. Thus following the notation introduced earlier, the appropriate subgroup population sizes must be denoted as q (the number of poor) instead of N (the total population). That is:

$$A(\theta)=\frac{1}{N}\sum_{\ell}\sum_{h^\ell}\frac{q^\ell}{q^\ell}q_h^\ell\left[\frac{1}{q_h^\ell}\sum_{i\in h^\ell}\rho_k(X_{i.},z,w)\right]=\sum_{\ell}\frac{q^\ell}{q}\sum_{h^\ell}\frac{q_h^\ell}{q^\ell}H_h^\ell(\theta)$$

9. Appendix B

Let us consider a set of countries $\mathbb{C}$, with $|\mathbb{C}|=m$. We denote as $P_a(\theta)$ the poverty measure of interest in country $a\in\mathbb{C}$, i.e. $P_a(\theta)=\{H_a(\theta),MPI_a(\theta)\}$. Array θ depicts a generic parametrization and we will denote as $\bar{\theta}$ the *baseline* specification. In the case of the revised global MPI, the baseline is defined as $k=\frac{1}{3}$, equal nested weights, and the indicator specifications discussed earlier. Any change in these parameters may yield a different level of the considered poverty measure, which we will denote as $P_a(\tilde{\theta})$ with $\bar{\theta}\neq\tilde{\theta}$.

For any possible *distinct* pair of countries $\{a,b\}\in\mathbb{Q}=\{\{x,y\}\in A|\ A\subset\mathbb{C}\wedge|A|=2\}$ there are three mutually exclusive possible orderings by measure P that can be represented by the following ordinal function comparing countries a and b using measure P and specifications θ: $\mathrm{O}_{ab}^P(\theta)$:

$$\mathrm{O}_{ab}^P(\theta)=\begin{cases}1 & if\quad P_a(\theta)>P_b(\theta)\\ 0 & if\quad P_a(\theta)=P_b(\theta)\\ -1 & if\quad P_a(\theta)<P_b(\theta)\end{cases}$$

Thus $\mathrm{O}_{ab}^P(\theta)=1$ indicates that a is strictly poorer than b according to measure P generated with specification θ, and the converse is true if $\mathrm{O}_{ab}^P(\theta)=-1$. $\mathrm{O}_{ab}^P(\theta)=0$ indicates that the poverty levels of a and b are indistinguishable according to specification θ. The latter statement is readily testable by means of a conventional hypothesis t-test. The appropriate test statistics can be computed as $\delta_{ab}^P(\theta)=P_a(\theta)-P_b(\theta),\forall\{a,b\}\in\mathbb{Q}$. Their corresponding analytical standard errors can be computed using the complex survey sampling structures of both countries, and we allow them to cover the general case of non-zero covariance between country estimates:

$$se\left(\delta_{ab}^{P}(\theta)\right) = \sqrt{V\left(P_a(\theta)\right) + V\left(P_b(\theta)\right) - 2Cov\left(P_a(\theta); P_b(\theta)\right)}$$

If the null hypothesis $\delta_{ab}^{P}(\bar{\theta}) = 0$ cannot be rejected at a chosen significance level, then the pairwise comparison between a and b is non-statistically significant at the baseline. It may not be stated that one country is poorer than the other. On the contrary, if the null can be rejected against the alternative hypothesis $\delta_{ab}^{P}(\bar{\theta}) \neq 0$, then the pairwise comparison between a and b is statistically significant at the baseline. One country is poorer than the other.

The pairwise comparison between a and b is deemed robust to a re-parametrization from $\bar{\theta}$ to $\tilde{\theta}$ if the ordering at the baseline ($\bar{\theta}$) is preserved under the alternative parametrization ($\tilde{\theta}$). That is $\mathrm{O}_{ab}^{P}(\bar{\theta}) = \mathrm{O}_{ab}^{P}(\tilde{\theta})$. Similarly, in a more demanding form of robustness covering more than one alternative parametrization, the pairwise comparison between a and b is deemed robust to an *array* of re-parametrizations $\Theta = \{\tilde{\theta}_1 \dots \tilde{\theta}_J\}, J < \infty$ if the ordering at the baseline ($\bar{\theta}$) is preserved under *all the alternative parametrizations* in Θ. That is $\mathrm{O}_{ab}^{P}(\bar{\theta}) = \mathrm{O}_{ab}^{P}(\tilde{\theta}_j), \forall\tilde{\theta}_j \in \Theta$.

Following these definitions, two synthetic quantitative representations of the robustness of measure P at the baseline $\bar{\theta}$ to the array of alternative array of parametrizations Θ may be defined as follows:

$$R_P^{\bar{\theta},\Theta} = \frac{\sum_{\{a,b\}\in\mathbb{Q}} \mathbb{I}\left(\mathrm{O}_{ab}^{P}(\bar{\theta}) = \mathrm{O}_{ab}^{P}(\tilde{\theta}_j), \forall\tilde{\theta}_j \in \Theta\right)}{|\mathbb{Q}|}$$

and

$$S_P^{\bar{\theta},\Theta} = \frac{\sum_{\{a,b\}\in\mathbb{Q}} \mathbb{I}\left(\mathrm{O}_{ab}^{P}(\bar{\theta}) = \mathrm{O}_{ab}^{P}(\tilde{\theta}_j), \forall\tilde{\theta}_j \in \Theta \wedge \mathrm{O}_{ab}^{P}(\bar{\theta}) \neq 0\right)}{\sum_{\{a,b\}\in\mathbb{Q}} \mathbb{I}(\mathrm{O}_{ab}^{P}(\bar{\theta}) \neq 0)}$$

Noting that $|\mathbb{Q}| = \binom{|\mathbb{C}|}{2} = \frac{1}{2}m(m-1)$, ratio $R_P^{\bar{\theta},\Theta}$represents the proportion of pairwise comparisons that are robust out of the total number of possible distinct pairwise comparisons. Ratio $S_P^{\bar{\theta},\Theta}$ represents the proportion of pairwise comparisons that are robust *and significant at baseline* out of the total number of distinct pairwise comparisons *that are significant at baseline*. Thus, ratio $S_P^{\bar{\theta},\Theta}$ lays focus on the robustness of strict poverty orderings, which allows to differentiate countries by their level of poverty, whereas ratio $R_P^{\bar{\theta},\Theta}$ measures the level of robustness for all possible orderings, including those for which making a difference between countries by their poverty levels is not possible.

Finally, note that these robustness ratios cover the particular case where there is only one alternative parametrization, i.e. $\Theta = \{\tilde{\theta}\}$ depicting a single-element set.

10. Appendix C. List of countries, dates and surveys

Country	World region	Survey	Year
Afghanistan	South Asia	DHS	2015-2016
Albania	Europe and Central Asia	DHS	2008-2009
Algeria	Arab States	MICS	2012-2013
Angola	Sub-Saharan Africa	DHS	2015-2016
Armenia	Europe and Central Asia	DHS	2015-2016
Azerbaijan	Europe and Central Asia	DHS	2006
Bangladesh	South Asia	DHS	2014
Barbados	Latin America and Caribbean	MICS	2012
Belize	Latin America and Caribbean	MICS	2015-2016
Benin	Sub-Saharan Africa	MICS	2014
Bhutan	South Asia	MICS	2010
Bolivia	Latin America and Caribbean	DHS	2008
Bosnia and Herzegovina	Europe and Central Asia	MICS	2011-2012
Brazil	Latin America and Caribbean	PNAD	2015
Burkina Faso	Sub-Saharan Africa	DHS	2010
Burundi	Sub-Saharan Africa	DHS	2016-2017
Cambodia	East Asia and the Pacific	DHS	2014
Cameroon	Sub-Saharan Africa	MICS	2014
Central African Republic	Sub-Saharan Africa	MICS	2010
Chad	Sub-Saharan Africa	DHS	2014-2015
China	East Asia and the Pacific	CFPS	2014
Colombia	Latin America and Caribbean	DHS	2015-2016
Comoros	Sub-Saharan Africa	DHS-MICS	2012
Congo	Sub-Saharan Africa	DHS	2011-2012
Congo, Democratic Republic of the	Sub-Saharan Africa	DHS	2013-2014
Côte d'Ivoire	Sub-Saharan Africa	MICS	2016
Djibouti	Arab States	MICS	2006
Dominican Republic	Latin America and Caribbean	MICS	2014
Ecuador	Latin America and Caribbean	ECV	2013-2014

Egypt	Arab States	DHS	2014
El Salvador	Latin America and Caribbean	MICS	2014
eSwatini	Sub-Saharan Africa	MICS	2014
Ethiopia	Sub-Saharan Africa	DHS	2016
Gabon	Sub-Saharan Africa	DHS	2012
Gambia	Sub-Saharan Africa	DHS	2013
Ghana	Sub-Saharan Africa	DHS	2014
Guatemala	Latin America and Caribbean	DHS	2014-2015
Guinea	Sub-Saharan Africa	MICS	2016
Guinea-Bissau	Sub-Saharan Africa	MICS	2014
Guyana	Latin America and Caribbean	MICS	2014
Haiti	Latin America and Caribbean	DHS	2012
Honduras	Latin America and Caribbean	DHS	2011-2012
India	South Asia	DHS	2015-2016
Indonesia	East Asia and the Pacific	DHS	2012
Iraq	Arab States	MICS	2011
Jamaica	Latin America and Caribbean	JSLC	2014
Jordan	Arab States	DHS	2012
Kazakhstan	Europe and Central Asia	MICS	2015
Kenya	Sub-Saharan Africa	DHS	2014
Kyrgyzstan	Europe and Central Asia	MICS	2014
Laos	East Asia and the Pacific	MICS-DHS	2011-2012
Lesotho	Sub-Saharan Africa	DHS	2014
Liberia	Sub-Saharan Africa	DHS	2013
Libya	Arab States	PAPFAM	2014
Madagascar	Sub-Saharan Africa	DHS	2008-2009
Malawi	Sub-Saharan Africa	DHS	2015-2016
Maldives	South Asia	DHS	2009
Mali	Sub-Saharan Africa	MICS	2015
Mauritania	Sub-Saharan Africa	MICS	2015
Mexico	Latin America and Caribbean	ENSANUT	2016
Moldova	Europe and Central Asia	MICS	2012
Mongolia	East Asia and the Pacific	MICS	2013
Montenegro	Europe and Central Asia	MICS	2013
Morocco	Arab States	PAPFAM	2011
Mozambique	Sub-Saharan Africa	DHS	2011
Myanmar	East Asia and the Pacific	DHS	2015-2016
Namibia	Sub-Saharan Africa	DHS	2013
Nepal	South Asia	DHS	2016
Nicaragua	Latin America and Caribbean	DHS	2011-2012
Niger	Sub-Saharan Africa	DHS	2012
Nigeria	Sub-Saharan Africa	MICS	2016-2017
Pakistan	South Asia	DHS	2012-2013
Palestine, State of	Arab States	MICS	2014
Paraguay	Latin America and Caribbean	MICS	2016
Peru	Latin America and Caribbean	DHS	2012

Philippines	East Asia and the Pacific	DHS	2013
Rwanda	Sub-Saharan Africa	DHS	2014-2015
Saint Lucia	Latin America and Caribbean	MICS	2012
Sao Tome and Principe	Sub-Saharan Africa	MICS	2014
Senegal	Sub-Saharan Africa	DHS	2016
Serbia	Europe and Central Asia	MICS	2014
Sierra Leone	Sub-Saharan Africa	DHS	2013
Somalia	Arab States	MICS	2006
South Africa	Sub-Saharan Africa	NIDS	2014-2015
South Sudan	Sub-Saharan Africa	MICS	2010
Sudan	Arab States	MICS	2014
Suriname	Latin America and Caribbean	MICS	2010
Syria	Arab States	PAPFAM	2009
Tajikistan	Europe and Central Asia	DHS	2012
Tanzania	Sub-Saharan Africa	DHS	2015-2016
TFYR of Macedonia	Europe and Central Asia	MICS	2011
Thailand	East Asia and the Pacific	MICS	2015-2016
Timor-Leste	East Asia and the Pacific	DHS	2016
Togo	Sub-Saharan Africa	DHS	2013-2014
Trinidad and Tobago	Latin America and Caribbean	MICS	2011
Tunisia	Arab States	MICS	2011-2012
Turkmenistan	Europe and Central Asia	MICS	2015-2016
Uganda	Sub-Saharan Africa	DHS	2016
Ukraine	Europe and Central Asia	MICS	2012
Uzbekistan	Europe and Central Asia	MICS	2006
Vanuatu	East Asia and the Pacific	MICS	2007
Viet Nam	East Asia and the Pacific	MICS	2014
Yemen	Arab States	DHS	2013
Zambia	Sub-Saharan Africa	DHS	2013-2014
Zimbabwe	Sub-Saharan Africa	DHS	2015