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Reducing Relative Poverty: The Role of Redistribution

Mario Negre and Espen Beer Prydz

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Abstract: When the poverty rate is measured with a strongly relative line, as is commonly the case in high income countries, the poverty rate will only change with changes in the distribution of incomes and remains insensitive to growth. This paper analyzes the relationship between relative poverty and inequality in the context of the Sustainable Development Goals (SDGs), which commits all countries to halve the proportion of people living in poverty according to national definitions by 2030. Using relative definitions of poverty, as is common among EU and OECD countries, dramatic reductions in inequality are needed to reach this target. We provide the first inequality targets and associated required redistribution for developed countries with relative poverty lines directly implied by actual commitments made under the SDGs.

*Negre (Mario.Negre@die-gdi.de) is with the German Development Institute and the World Bank Poverty and Equity Global Practice. Prydz (eprydz@worldbank.org) is with the World Bank Group, Development Economics Vice Presidency (DEC) Data Group.

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1 Introduction

Reducing poverty is at the center of the United Nations Sustainable Development Goals (SDGs). Goal number one is to "[e]nd poverty in all its forms everywhere" (United Nations, 2015:15). This first target under this goal includes ending extreme poverty, currently measured by the $1.90/day poverty line at 2011 international prices. The second target (SDG1.2) entails "reduc[ing] at least by half the proportion […] living in poverty in all its dimensions according to *national definitions*" (United Nations, 2015:15). In contrast to the Millennium Development Goals (MDGs), which focused on development progress in poor countries, the SDGs have a universal scope with rich countries also committing to make progress towards the goals, including those on poverty and inequality.

In most high income countries, national poverty is measured using a relative poverty line, set at a share of the mean or median national income. Mirroring these national approaches, the European Union's headline poverty indicator uses a poverty line set as 60% of median income, while the Organisation for Economic Cooperation and Development (OECD) uses a similar line set at 50% of the median, similar to what was proposed by Fuchs (1969).[1] Many rich countries have adopted these or similar relative definitions of poverty for monitoring national poverty, including most EU countries. The United States is a notable exception among rich countries, using a fixed value-line only adjusted for inflation, though many argue that this line should be revised or set as a relative line like in other countries of similar income level.[2] Increasingly, poverty lines defined in relative terms to the mean or median income are also relevant to low- and middle income countries, which frequently revise upwards their poverty lines as they get richer (Chen and Ravallion, 2011; Jolliffe and Prydz, 2019). A threshold of 50% of median is also used in monitoring the social inclusion in the SDG focused on inequality (target 10.2.1).

Although relative poverty is central to both national and international policy objectives and goals, little attention has been paid to the empirical relationship between relative poverty and inequality and its implications in terms of international poverty and inequality goals. Growth in itself does not reduce the poverty rate if the latter is based on a relative measure. If everyone's income grows at the same rate, poverty remains unchanged as the poverty line grows at the same rate as incomes. Only differential growth across the income distribution, and thereby changes in inequality, can change a relative poverty rate. Individuals whose income grows slower than the mean or median, will get poorer or move closer to poverty. A decrease (increase) in inequality, holding everything else constant, will lead to an decrease (increase) in poverty. Thus, in rich countries with purely relative poverty lines, reducing poverty can be directly linked to changes in inequality.

Reducing income inequality is also a central goal for many countries and in the SDGs. The 10th SDG is headlined "Reduce inequality within and among countries". Target 10.1 is to "achieve and sustain income growth of the bottom 40 percent of the population at a rate higher than the national average". Beyond the bottom 40% growing faster than the mean, this goal is not quantified. And on top of it, the target is so vague that it could arguably be met by simply achieving above-average growth in incomes of

[1] See for example OECD (2015)
[2] See for example https://www.washingtonpost.com/outlook/2019/09/16/official-us-poverty-rate-is-based-hopelessly-out-of-date-metric/

the B40 only the very last year, which would be consistent with an increase in overall inequality for the whole period up to 2030. The mechanical relationship between relative poverty and inequality, however, provides a framework to estimate what level of redistribution in terms of growth of the bottom 40 percent relative to the mean or decreases in inequality as measured by Gini are required to halve poverty by national standards in countries with relative poverty lines.

In this paper, we illustrate both theoretically and empirically, what targets for the reduction of strongly relative poverty mean in terms of required reductions in inequality. Using parametric income distributions, we first analytically show how inequality and relative poverty measures are related and assess this relationship in recent data. Second, we assess what reduction in inequality is required to reduce relative poverty in order for countries with relative poverty lines to reach the pertinent SDGs target by 2030. We find that most rich countries will need to reduce inequality to a level much lower than what even some of the most equal countries experience today. The last sections presents conclusions.

2 Modelling relative poverty and inequality

To approach the relationship between relative poverty and inequality analytically, we have explored three simple two-parameter income distribution functions, namely the lognormal, Fisk (also known as the log-logistic) and the Weibull distributions. Lognormal has been widely used to model poverty and the distribution of income (e.g. Lopez and Serven, 2006). However, the Fisk and the Weibull distributions have the advantage that they have cumulative distribution functions (CDFs) which can be solved analytically. This allows for the analytical derivation of a function establishing the relationship between the Gini ratio and the poverty headcount rate (and what reduction in Gini is required to for a given reduction in poverty).

We find that for the range of Gini and poverty rates we are assessing in this study, the Fisk distribution has a superior fit than both the Weibull and lognormal distributions. We therefore focus on the Fisk distribution for the remainder of this paper. A more thorough assessment of the fit of two-parameter distributions across distributions and income types is available in a forthcoming review (Negre and Prydz, forthcoming).

The cumulative density function (CDF) of the Fisk distribution with scale parameter α and shape parameter β, is given by:

$$F(x) = \frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}} \qquad (1)$$

Where x>0, α>0 and β>0

Key moments and quantiles of the distributions are as follows.

The median (M) is defined as

$$M = \alpha \qquad (2)$$

The mean (m) is given by

$$m = \frac{\pi\alpha}{\beta \sin\frac{\pi}{\beta}} \quad (3)$$

And the gini index is given by:

$$g = \frac{1}{\beta} \quad (4)$$

For our purpose, we use a relative poverty line (z), defined as proportion (k) of the median:

$$z = k * \alpha \quad (5)$$

The share of the distribution, or headcount ratio H, living in poverty is given by:

$$H(z) = \frac{1}{1+(k)^{-\beta}} \quad (6)$$

Substituting (4) into (6), we can express the poverty headcount ratio in terms of i g and k.

$$H(z) = \frac{1}{1+(k)^{-\frac{1}{g}}} \quad (7)$$

From here, we can isolate the final gini:

The condition is that poverty at the end of the period be a fraction (λ) so that the new gini, g1, can be expressed in terms of the initial one (g0), k and λ by substituting equation (7) in (8) to obtain through some rearrangement (9).

$$H_2(z) = \lambda H_1(z) \quad (8)$$

$$g_1 = -\frac{\ln(k)}{\ln\left[\frac{1}{\lambda}*\left(1+k^{-\frac{1}{g_0}}\right)-1\right]} \quad (9)$$

In the context of the goal of reducing poverty in half (λ = ½) , the target gini can be expressed as

$$g_1 = -\frac{\ln(k)}{\ln\left(\frac{2}{k^{\frac{1}{g_0}}}+1\right)} \quad (10)$$

$$g_1 = -\frac{\ln(k)}{ln\left(2k^{-\frac{1}{g_0}}+1\right)} \quad (11)$$

It follows that we can express the change in the poverty rate in levels with changes in the gini:

$$H'(g) = \frac{1}{g^2} * \frac{\ln(k)}{k^{-\frac{1}{g}}*\left(1+k^{-\frac{1}{g}}\right)^2} \quad (12)$$

And similarly, proportional changes in the poverty rate with g:

$$H'(g)/H(g) = \frac{1}{g^2} * \frac{\ln(k)}{k^{-\frac{1}{g}} * \left(1 + k^{-\frac{1}{g}}\right)} \quad (13)$$

We have derived a direct relationship between the relative poverty headcount rate and inequality as measured by the gini. As expected, the growth rate does not enter this relationship in the case of a perfectly relative poverty line as a proportion of the Median. However, for poverty lines that are fixed over time this is not the case. We present in this section a case of an anchored relative poverty line which is fixed in at the initial period and remains unchanged thereafter. In this case, growth does affect poverty and therefore we need to consider both growth and inequality. The reason to propose an anchored poverty line is that, based on an initially relative line, it provides a less ambitious but far more feasible goal for inequality reduction.

Initial gini is given by (g_1), while annualised growth rate (g_{ann}; δ). The relative poverty line of the initial year is given by (z=K*Median). We seek to obtain the final value of gini such that poverty will be reduced by a fraction.

If the poverty line is entirely relative, then $z_1=K*\alpha_1$ and $z_2=K*\alpha_2$. But in this case, we anchore the line to the initial value, $z_2=K*\alpha_1$ and we keep this line constant (in constant values).

In this case, the mean income does change with growth:

$$m_2 = (1 + g_{ann})^n * m_1 = \delta * m_1 \quad (14)$$

Where n is the number of years for the projected period (e.g. 2015-30: 15 years for the SDGs) and δ an income growth factor.

The expression for the mean of a Fisk distribution is:

$$m = \frac{\alpha\pi/\beta}{\sin(\frac{\pi}{\beta})} \quad (15)$$

This allows us to obtain the scale parameter of the distribution at the end of the period as a function of the initial one:

$$\alpha_2 = \delta * \frac{\beta_2}{\beta_1} * \frac{\sin\left(\pi/\beta_2\right)}{\sin\left(\pi/\beta_1\right)} * \alpha_1 \quad (16)$$

The condition is that poverty at the end of the period be a fraction of the initial one (half: λ=1/2), using the same initial poverty line (z_1).

$$H_2(z) = \lambda H_1(z) \quad (17)$$

$$\frac{1}{1+\left(\frac{z_1}{\alpha_2}\right)^{-\beta_2}} = \lambda * \frac{1}{1+\left(\frac{z_1}{\alpha_1}\right)^{-\beta_1}} \quad (18)$$

Substituting $z_2=K*\alpha_1$ and (16) into (18) we obtain the following identity:

$$1 + \left(\frac{K*\beta_1*\sin\pi/\beta_1}{\delta*\beta_2*\sin\pi/\beta_2}\right)^{-\beta_2} = \frac{1}{\lambda} * \left(1 + K^{-\beta_1}\right) \quad (19)$$

Expressed in terms of the final (g_2) and initial (g_1) Ginis this is:

$$1 + \left(\frac{K * g_2 * \sin \pi * g_1}{\delta * g_1 * \sin \pi * g_2}\right)^{\frac{-1}{g_2}} - \frac{1}{\lambda} * \left(1 + K^{\frac{-1}{g_1}}\right) = 0 \qquad (20)$$

This equation cannot be solved analytically and needs to be computed empirically for each set of initial conditions and a forecast annualized growth rate for the period. For the case of a unimodal Fisk distribution in which the shape parameter is greater than one (β>1), then the graphical representation of equation (18) shows a single solution.

3 Data: relative poverty and inequality

Having derived the analytical relationship for the Fisk distribution, we turn to assessing the relationships using empirical data for European countries and the goal of halving relative poverty when using relative poverty measures. We use the latest available Eurostat data on inequality, as measured by the Gini index, and relative poverty measures, originally calculated from the European Union Statistics on Income and Living Conditions (EU-SILC). The data is obtained from Eurostat Data explorer, and is available for 43 countries in the European Union (EU), the European Free Trade Association and five EU candidate countries. An overview of the countries and the country abbreviations used in Figures and Charts are available in Appendix 2.

Figure 1 shows the relationship between the poverty rate as measured with the relative poverty threshold of 60 percent of median. A simple bivariate assessment suggests that the gini explains 80 percent of the variation in observed relative poverty headcount rates across the EU. The figure also plots the analytical solution using the Fisk distribution for the relationship between Gini and relative poverty using a poverty line of 60 percent of median, as presented in equation (7) in the previous section, with k=0.6. It is clear that the Fisk distribution closely models the relationship, especially for countries with a Gini between 20 and 35, but with a somewhat poorer fit for the richest countries. Arguably, the superior fit at lower levels of poverty is preferable to model the required reduction in gini correctly.

Figure 1: Poverty and Inequality of 43 European Countries

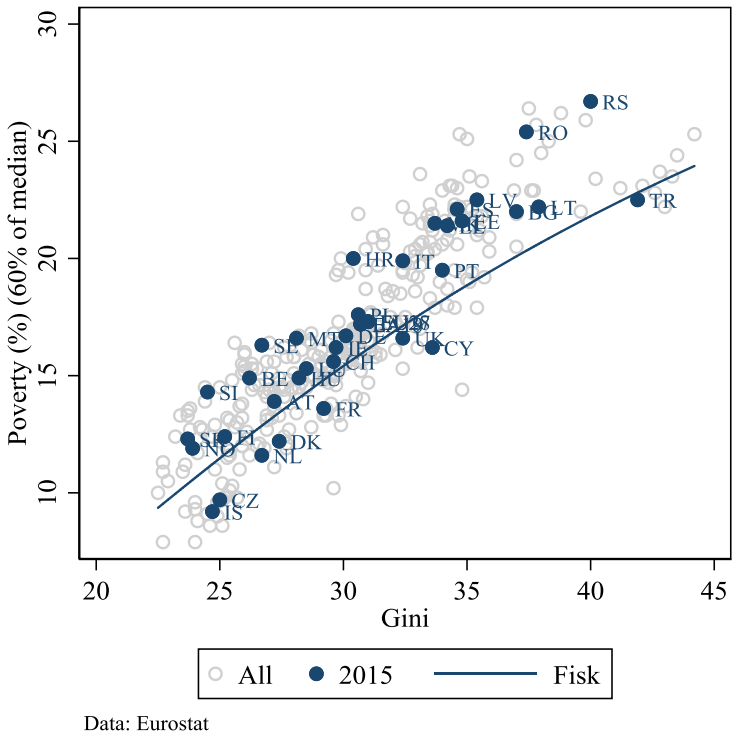


Figure 2 maps out the schedule of required gini in the final period for initial ginis.

4 Inequality reductions required to reduce relative poverty based on the Fisk distribution

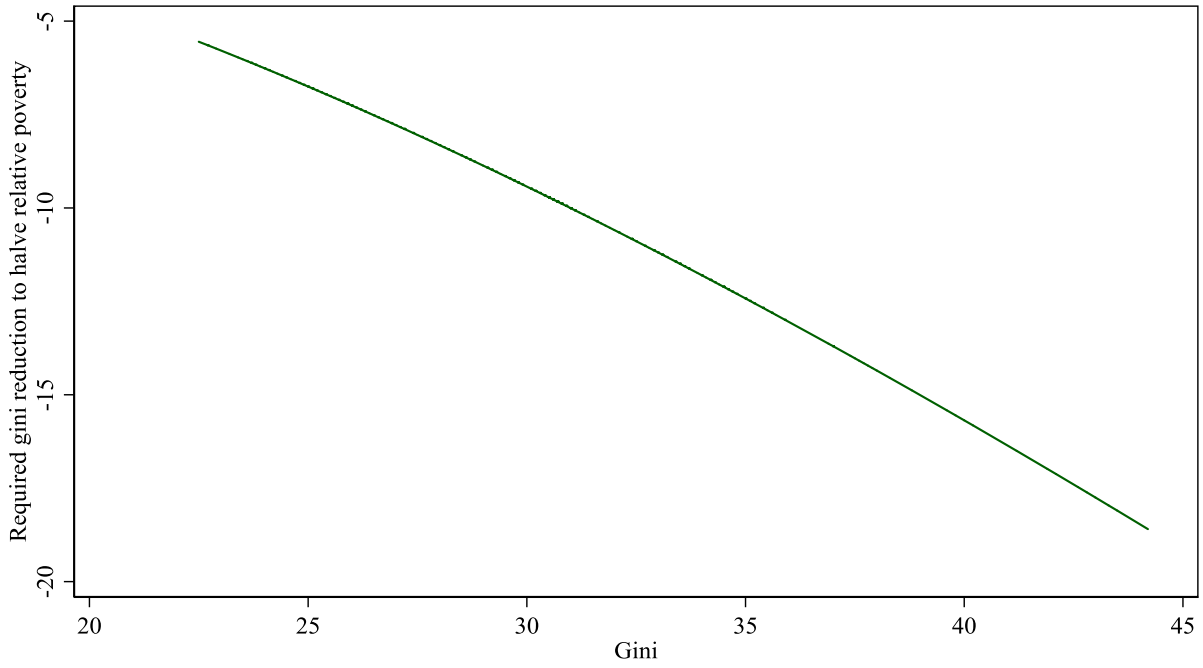
Table 1 provides results for the required reduction in Gini for various relative poverty lines from our sample. It is immediately clear for these results that the required reductions in gini are extremely large. The most unequal countries in our sample with ginis well above 40 today need to reach a level of inequality that is at the level or lower than the most equal countries today. The most equal countries need to reduce inequality to levels much lower than what has been seen in modern times. Thus according to these results, unprecedented reductions and levels of inequality will be required to halve relative poverty, as stipulated by the SDGs. Figure 2 plots the analytical relationship for the entire range of ginis. The horizontal axis shows the initial Gini while the vertical axes shows the required Gini. The two schedules show the required gini for reducing poverty by 50%.

Table 1: Initial and Required Gini to Halve Relative Poverty (Fisk, 60 percent of median)

Country Code	Initial gini (ca 2015)	Required gini	Country Code	Initial gini (ca 2015)	Required gini
AT	27.1	19.3	IT	32.2	21.5
BE	26.0	18.7	LT	37.7	23.6
BG	36.7	23.2	LU	28.2	19.8
CH	29.3	20.3	LV	35.5	22.8
CY	33.4	22.0	MK	33.8	22.1
CZ	25.2	18.3	MT	28.2	19.8
DE	29.9	20.5	NL	26.5	19.0
DK	27.3	19.3	NO	24.0	17.7
EE	34.7	22.5	PL	30.5	20.8
EL	34.2	22.3	PT	33.9	22.2
ES	34.6	22.4	RO	37.2	23.4
FI	25.1	18.3	RS	38.0	23.7
FR	29.1	20.2	SE	25.0	18.3
HR	30.6	20.8	SI	24.5	18.0
HU	28.1	19.7	SK	23.6	17.5
IE	29.7	20.4	UK	32.0	21.4
IS	23.4	17.4			

See Appendix A for country codes.

Figure 2 Fisk analytical relationship between poverty reduction and inequality change



5 Microsimulations of growth incidence

To better understand what kind of redistribution is required to achieve various degrees of relative poverty reduction, we simulate distributional changes by imposing growth incident curves that ensures various degrees of redistribution, using the methods proposed by Lakner et al (2014 and 2019). The simulations are motivated by Sustainable Development Goal Target 10.1 is to "achieve and sustain income growth of the bottom 40 percent of the population at a rate higher than the national average". The direct link between the poverty and inequality for countries using relative poverty lines, enables us to set quantifiable inequality targets in terms of required redistribution to the bottom 40 percent, which would ensure that the relative poverty targets are achieved.

Lakner et al (2019) simulate the impact on global poverty of annually changing inequality within each country. They do so by following the SDG10 target and the World Bank's shared prosperity premium (Lakner et al., 2019) based on the income or consumption of the poorest bottom 40 percent income in each country growing above average. This is done by annually redistributing from the richest 60 percent of the distribution to the poorest 40 percent using a linear growth incidence curve which monotonically decreases across percentiles so that it imposes a slightly progressive redistribution in favor of lower percentiles.

The findings presented in Lakner et al. (2019) show that the impact of reducing inequality reduction is larger than that of increasing growth for the global extreme poor count even when using the international extreme poverty line of 2011 PPP $1.90. When considering a purely relative poverty line distribution neutral growth has no impact. Using the methodology described in Lakner et al. (2019) we can estimate the necessary annual additional growth in incomes/consumption that needs to be redistributed from the top 60% to the poorest 40 percent in order to achieve the goal of reducing poverty to a half. Results are shown in figure 3.

Figure 3: Required Shared Prosperity Premium (growth of bottom 40 percent above mean)

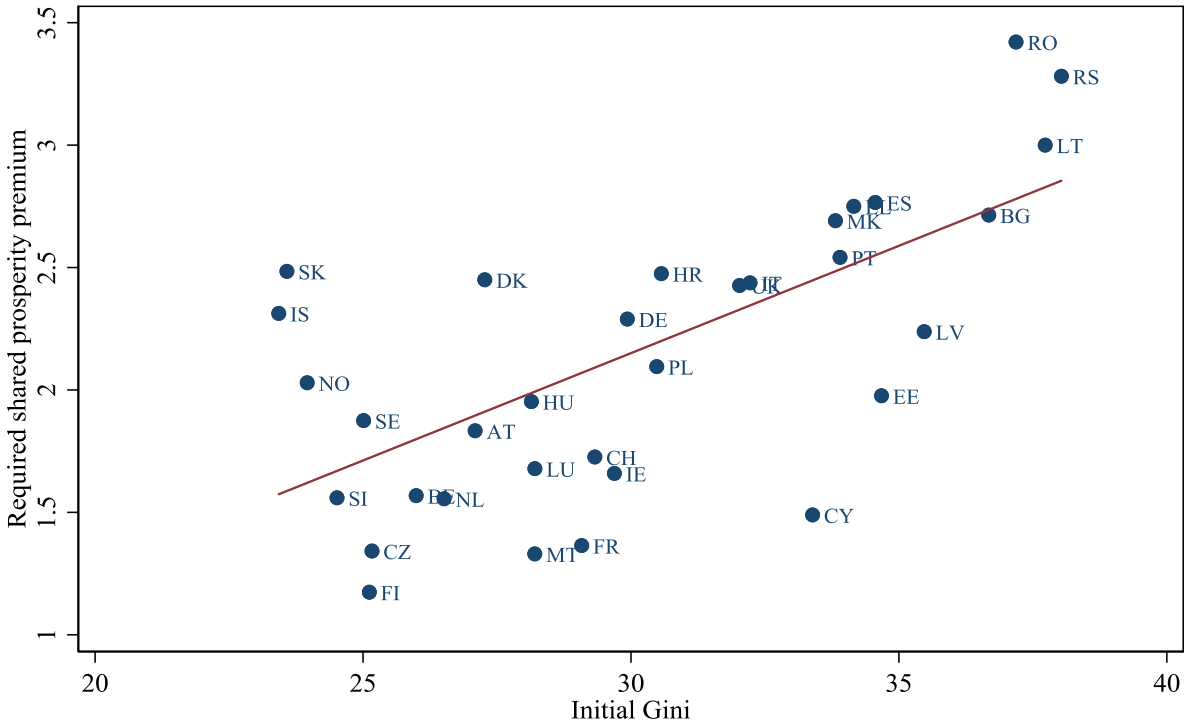


Figure 3 shows the annual shared prosperity premiums required to half poverty for each country depending on the original level of Gini. As can be seen, growth of incomes/consumption of the poorest 40 percent should in the range of between 1.2 and 3.4 percentage points above average (shared prosperity premium) if purely relative poverty is to be halved for these countries. These are large annual values, which although they are in the range of what can be empirically observed for shorter periods, such as the typical shared prosperity spells provided by the World Bank, they remain very ambitious to be sustained over a long period of time The relatively large variation shows the variation in distributions for countries with same level of gini. Table 3 shows the results for each country.

Table 3: Required redistribution (growth premium of the bottom 40 percent above mean)

Country code	Required SPP	Country code	Required SPP
AT	1.83	IT	2.44
BE	1.57	LT	3.00
BG	2.71	LU	1.68
CH	1.73	LV	2.24
CY	1.49	MK	2.69
CZ	1.34	MT	1.33
DE	2.29	NL	1.56
DK	2.45	NO	2.03

EE	1.98	PL	2.10
EL	2.75	PT	2.54
ES	2.77	RO	3.42
FI	1.17	RS	3.28
FR	1.36	SE	1.88
HR	2.48	SI	1.56
HU	1.95	SK	2.48
IE	1.66	UK	2.43
IS	2.31		

6 Conclusion

The Sustainable Development Goals set ambitious goals for halving national poverty by 2030. When national poverty measures are based on relative poverty lines, these goals are even more ambitious than for absolute poverty lines, as reaching the goal can only be attained by reductions in inequality that are very large. This is particularly relevant for countries with purely relative poverty lines, like most EU countries.

This paper has quantified the necessary changes in the level of inequality required to reduce relative poverty, either in terms of Gini reductions or above-average income growth of the bottom 40 percent, the so-called Shared Prosperity Premium, and provided both analytical and simulated results.

The required inequality reduction targets linked to poverty commitments these countries have made within the SDGs reductions in inequality which have not been seen in modern times and may thus be over ambitious. In order to overcome this and make the targets more useful, it may be necessary to set a less ambitious anchored poverty line fixed at the initial poverty line's value that provides more realistic targets.

These results not only provide defined, quantitative targets based on commitments countries have made in the 2030 Agenda but, more generally, address a key shortcoming of efforts to address high inequality, i.e. the lack of tangible targets. Unlike absolutely poverty, with very clear targets for progressive elimination, inequality reduction has remained a more difficult policy objective to quantify. The mechanical relationship between relative poverty and inequality described herein provides the opportunity to translate poverty commitments into implicit inequality targets.

7 References

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8 Appendix A – Countries in data

European Union (EU)

Belgium	(BE)	Greece	(EL)	Lithuania	(LT)	Portugal	(PT)
Bulgaria	(BG)	Spain	(ES)	Luxembourg	(LU)	Romania	(RO)
Czech Republic	(CZ)	France	(FR)	Hungary	(HU)	Slovenia	(SI)
Denmark	(DK)	Croatia	(HR)	Malta	(MT)	Slovakia	(SK)
Germany	(DE)	Italy	(IT)	Netherlands	(NL)	Finland	(FI)
Estonia	(EE)	Cyprus	(CY)	Austria	(AT)	Sweden	(SE)
Ireland	(IE)	Latvia	(LV)	Poland	(PL)	United Kingdom	(UK)

European Free Trade Association (EFTA)

Iceland	(IS)	Norway	(NO)
Liechtenstein	(LI)	Switzerland	(CH)

EU candidate countries

Montenegro	(ME)
The former Yugoslav Republic of Macedonia	(MK[1])
Albania	(AL)
Serbia	(RS)
Turkey	(TR)

9 Appendix B – Modelling Relative Poverty using Weibull and Lognormal distributions

Change in Gini based on the Weibull distribution function

Cumulative Distribution Function:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

Median:

$$M = \eta * (ln2)^{\frac{1}{\beta}}$$

Poverty line:

$$z = \alpha * M$$

Condition:

$$H(z_1) = \frac{1}{2} * H(z_0)$$

$$F(z_1) = \frac{1}{2} * F(z_0)$$

Gini:

$$g = 1 - 2^{-\frac{1}{\beta}} \qquad \leftrightarrow \qquad \beta = -\frac{ln2}{ln(1-g)}$$

1. Halving relative poverty headcount

$$\beta_1 = \frac{1}{ln\alpha} * ln\left\{1 - \frac{1}{ln2} * ln\left[1 + e^{-(ln2)*\alpha^{\beta_0}}\right]\right\}$$

Solution in terms of Gini:

$$g_1 = 1 - 2^{-\left\{\frac{(ln2)*(ln\alpha)}{\ln\left\{1-\frac{1}{\ln 2}*\ln\left[1+e^{-(ln2)*\alpha^{\left(-\frac{ln2}{ln(1-g_0)}\right)}}\right]\right\}}\right\}}$$

2. Reduction in relative poverty headcount by λ:

$$\beta_1 = \frac{1}{ln\alpha} * ln\left\{-\frac{1}{ln2} * ln\left[1 - \lambda * \left(1 - e^{-(ln2)*\alpha^{\beta_0}}\right)\right]\right\}$$

Solution in terms of Gini:

$$g_1 = 1 - 2^{-\left\{\frac{(ln2)*(ln\alpha)}{ln\left\{-\frac{1}{ln2}*ln\left[1-\lambda*\left(1-e^{-(ln2)*\alpha^{-\left(\frac{ln2}{ln(1-g_0)}\right)}}\right)\right]\right\}}\right\}}$$

3. Headcount in terms of Gini:

$$H = 1 - e^{-\left[(ln2)*\alpha^{-\left(\frac{ln2}{ln(1-g)}\right)}\right]}$$

or:

$$H = 1 - \frac{1}{e^{\left[\frac{ln2}{\alpha^{\left(\frac{ln2}{ln(1-g)}\right)}}\right]}}$$

4. Changes in Headcount from changes in Gini (first derivative)

$$H'(g) = (ln\alpha) * \left(\frac{ln2}{ln(1-g)}\right)^2 * e^{-(ln2)*\alpha^{-\left(\frac{ln2}{ln(1-g)}\right)}} * \alpha^{-\left(\frac{ln2}{\ln(1-g)}\right)}$$

Change in Gini based on the lognormal distribution function

The poverty headcount of a lognormal distribution is given by the cumulative standard normal distribution function (Φ):

$$H(z) = \Phi\left[\frac{lnz - \mu}{\sigma}\right]$$

where z is the poverty line, μ the lognormal mean and σ the lognormal standard deviation. The latter can be respectively written as follows in terms of the income distribution mean (m) and standard deviation (v):

$$\mu = \frac{m}{\sqrt{1 + \frac{v}{m^2}}}$$

$$\sigma = \sqrt{ln\left(1 + \frac{v}{m^2}\right)}$$

The median (M) and the poverty line (z) can be expressed as:

$$M = e^{\mu}$$

$$z = \alpha * M = \alpha * e^{\mu}$$

where α is again the level of the poverty line relative to the median.

Reduction in relative poverty headcount by factor λ

$$H_1(z_1) = \lambda * H_0(z_0)$$

Substituting, we obtain the following relation, independent both mean and median:

$$\Phi\left[\frac{lnz_1 - \mu_1}{\sigma_1}\right] = \lambda * \Phi\left[\frac{lnz_0 - \mu_0}{\sigma_0}\right]$$

$$\Phi\left[\frac{ln(\alpha * e^{\mu_1}) - \mu_1}{\sigma_1}\right] = \lambda * \Phi\left[\frac{ln(\alpha * e^{\mu_0}) - \mu_0}{\sigma_0}\right]$$

$$\Phi\left[\frac{ln \propto}{\sigma_1}\right] = \lambda * \Phi\left[\frac{ln\alpha}{\sigma_0}\right]$$

This equation provides the value of the required final σ for any initial one and can only be solved computationally. Since gini can be directly calculated from σ through the following relation,

$$g = 2 * \Phi\left[\frac{\sigma}{\sqrt{2}}\right] - 1$$

we can then estimate the necessary reduction in gini to reduce the poverty rate by a λ factor.