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Income Poverty has been Halved in the Developing World, even when Accounting for Relative Poverty.*

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Abstract

The first Millennium Development Goal was to halve extreme absolute poverty over the period 1990-2015. This goal has been met by a large margin, but the simultaneous increase in within-country inequality has led to an increase in relative poverty. As absolute and relative poverty evolved in opposite directions, whether or not *overall* poverty – which combines both absolute and relative poverty – has been reduced depends on the arbitrary priority assigned to absolutely poor individuals. We show that, if we assume that an individual who is absolutely poor is poorer than an individual who is only relatively poor, overall poverty in the developing world has been halved over the period, *regardless of the value chosen for the priority parameter*. This result is robust to alternative specifications of the poverty lines and to the exclusion of China or India. Alternative approaches find much less overall poverty reduction because they violate our normative assumption.

JEL: D63, I32.

Key-words: Income Poverty, Relative Poverty, Absolute Poverty, Developing World.

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1 Introduction

At the Millennium Summit in 2000, 189 countries committed to a set of ambitious social objectives known as the Millennium Development Goals. The first and maybe the most prominent of these goals was to halve extreme income poverty by 2015, taking 1990 as reference year. An individual is considered extremely poor if their income is lower than \$1.9 per day in 2011 Purchasing Power Parity (PPP) (Ferreira et al., 2016), an absolute poverty threshold capturing the minimal resources necessary to satisfy their basic *physical needs*. By 2015, this goal had been reached by a large margin, mostly due to the strong growth experienced in many developing countries (World Bank, 2018).

The absolute approach to measure global poverty is, however, not free from criticism. Its narrow focus on own income leaves relevant dimensions out. One widespread concern is that it does not take relative deprivation into account (Ravallion, 2003). Importantly, this dimension captures *social exclusion* because relatively deprived individuals experience difficulties to engage in the everyday life of their society (Townsend, 1979; Ravallion, 2008). Over 1990-2015, beside a strong growth, many developing countries experienced an increase in within-country inequality (Bourguignon, 2015; Milanovic, 2016; Anand and Segal, 2008; Ravallion, 2014). As a result of the increase in within-country inequality, the fraction of individuals who earn less than half the income standard in their society – the typical relative poverty threshold – increased over 1990-2015 in a large part of the developing world. Therefore, for a large number of developing countries there is a *disagreement* about the direction of income poverty change, depending on whether absolute or relative measures of poverty are used.

The increase in relative poverty casts some doubt on the success achieved in reducing income poverty because many policy makers are committed to helping the poor satisfy their basic physical needs and at the same time want to prevent their social exclusion. This commitment has been stated for instance by the World Bank (2015). Such policy makers must evaluate income poverty using *overall* poverty measures, *i.e.* indicators capturing both absolute and relative poverty. Importantly, overall poverty comparisons depend on a normative weight that captures the *priority* assigned to the absolutely poor. This priority measures how much more (or less) overall poverty is reduced when an additional unit of income is given to an absolutely poor individual rather than to an individual who is only relatively poor. When absolute and relative measures disagree on the direction of poverty change, the direction of *overall* poverty change typically depends on the arbitrarily chosen priority parameter. This significantly limits the usefulness of overall poverty measures.

In this paper, we show under a rather mild normative assumption that *overall* income poverty has been halved in the developing world from 1990-2015, regardless of the value chosen for the priority parameter. The normative assumption states that an individual who is absolutely poor is *poorer* than an individual who is only relatively poor, regardless of the income standard in their respective societies. This prevents debatable interpersonal comparisons. For instance, it does not allow to consider that an absolutely poor individual in a low-income country is less poor than a relatively poor individual in a middle-income country whose personal income is several times the absolute threshold. We deem this assumption mild because the idea that absolute poverty should be considered more severe than relative poverty is largely shared: it is reflected in the answers to a questionnaire experiment conducted all over the world by Corazzini et al. (2011) and has also been expressed in the poverty measurement literature (Atkinson and Bourguignon, 2001; Decerf, 2017). Our normative assumption plays a key role: it allows in some cases to obtain overall poverty comparisons that do not depend on the priority parameter even when absolute and relative measures disagree.[1]

We consider a recently developed family of indices that combine absolute and relative poverty under this normative assumption. This family is parametrized by the priority assigned to the absolutely poor. The two extreme values of this parameter attribute zero and infinite priority to the absolutely poor, respectively. Our analysis proceeds as follows. First, we characterize the conditions under which poverty judgments in this family are independent of the priority parameter. Second, we exploit these conditions in the empirical analysis in order to place a lower and an upper bound on the extent of overall poverty reduction. Using World Bank data, we show empirically that all measures in our family have declined by at least 50% when applied to the developing world from 1990-2015. This result is not entirely driven by the tremendous progress achieved by one or two populous countries such as China or India. In fact, our result holds for a third of all developing countries, when taken individually. We show that our result is robust to six different pairs of absolute and relative poverty lines. These alternative specifications reflect to a large extent the variety of proposals made in the literature (Atkinson and Bourguignon, 2001; Chen and Ravallion, 2013; Jolliffe and Prydz, 2016; World Bank, 2018). In particular, we consider mean as well as median-sensitive relative poverty lines and we consider different values for the absolute threshold and for the slope and intercept of the relative poverty line. Finally, we compare our empirical results with those obtained from standard approaches to measuring overall poverty. Alternative measures find significantly less overall poverty reduction than our lower bound estimate. In particular, all of our measures find a rate of poverty reduction that is at least 44%

[1]The following example illustrates this. Consider an income distribution for which the absolute poverty threshold is lower than the relative poverty threshold. Assume that this distribution has only one absolutely poor individual. Consider a second distribution that is obtained from the first distribution by a particular form of unequal growth: the income of all individuals increases, the income of the poor individual is lifted above the absolute threshold, but their income increases at a slower pace than the income standard. The poor individual is only relatively poor in the second distribution but their income is now further away from the relative threshold. Therefore, relative poverty is larger in the second distribution. Our normative assumption implies that overall poverty is unambiguously larger in the first distribution because the poor individual is absolutely poor in the first but not in the second distribution.

larger than the one found using the most well-known alternative measure. The reason is that alternative measures violate our normative assumption, *i.e.* they need not record progress when a growth process upgrades the status of some individuals from absolutely poor to only relatively poor. We also show that our approach is less sensitive to the definition of the relative line. This lower sensitivity, which is particularly desirable when little consensus exists on this line, is implied by our normative assumption that confers a larger role to the absolute line.

The contribution of this paper is twofold. First, we complement the literature on income poverty measurement by providing a novel assessment of the evolution of overall poverty in the developing world that avoids debatable interpersonal comparisons. We show that the reduction in absolute poverty observed in the developing world from 1990-2015 more than compensates the increase in relative poverty. Our finding confirms and strengthens positive evaluations of the success achieved against extreme income poverty (*i.e.* accomplishment of the first Millennium Development Goal). Second, we develop a new method for overall income poverty evaluation that can potentially provide judgments that do not depend on the arbitrary priority attributed to the absolutely over the relatively poor.

From a conceptual perspective, our proposal integrates the main ideas of the sizable literature on the measurement of income poverty from a world perspective,[2] but differs by satisfying our normative assumption and checking for full robustness to the priority parameter. Atkinson and Bourguignon (2001) argue that, when taking a world perspective on income poverty, both an absolute and a relative poverty line should be considered. They propose two reconciliations of the absolute and relative approaches. The first considers two measures, one absolute and one relative, and gives lexicographic priority to the absolute measure.[3] The second considers a unique measure based on an index that aggregates the income gaps with respect to both poverty lines. Our approach is in line with this second reconciliation. Yet, in contrast to our indices, the indices proposed by these authors violate our normative assumption. Ravallion and Chen (2011) stress that global relative poverty measures should satisfy a weak relativity axiom (WRA): poverty should fall when all incomes in a distribution increase in the same proportion. They show that, when the relative measure is based on the head-count ratio (the commonly used index), the relative line should be weakly relative. Several authors and institutions consider weakly relative lines (Chen and Ravallion, 2013; Jolliffe and Prydz, 2016; World Bank, 2018). They all construct global poverty measures based on the head-count ra-

[2]As stated by Atkinson and Bourguignon (2001), a world perspective on income poverty seeks to provide a framework that unifies the measurement of poverty for all countries. This is in contrast to a national perspective that evaluates each country based on its own definition of poverty (which typically differs across countries).

[3]Providing lexicographic priority to the absolute measure is much stronger than our normative assumption. In particular, our assumption allows for the income loss of an absolutely poor individual to be more than compensated by a sufficient income gain of a relatively poor individual.

tio but propose different specifications for the poverty lines. Decerf (2017) shows that global measures based on a Foster-Greer-Thorbecke (FGT) index (Foster et al., 1984) provide questionable poverty comparisons. The reason is that, when combined with a (weakly) relative line, these indices implicitly make debatable interpersonal comparisons across societies with different income standards. Unlike our measure, these indices may implicitly consider that an absolutely poor individual in a low-income country is *less poor* than a relatively poor individual in a middle-income country. In the particular case of the head-count ratio, all poor individuals are deemed *equally poor*, even when their incomes are on different sides of the absolute threshold. We consider a family of income poverty measures that (1) are based on both an absolute and a relative line (Atkinson and Bourguignon, 2001), (2) satisfy the WRA (Ravallion and Chen, 2011), (3) avoid the questionable interpersonal comparisons implicitly made by global measures based on FGT indices (Decerf, 2017) and (4) differ from each other by the value assumed for the priority parameter. Furthermore, our measures can be given a welfarist interpretation if individual welfare only depends on own income up to the absolute threshold and also depends on relative income above the absolute threshold.

The rest of the paper is organized as follows: we present the theory in section 3, the data and our main specification in section 4, the empirical analysis in section 5 and we conclude in section 6.

2 Related literature

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3 Robust overall poverty comparisons

3.1 Basic Framework

Let an income distribution $y := (y_1, \ldots, y_n)$ be a list of non-negative incomes sorted in non-decreasing order, with $n \in \mathbb{N}$. The set of such income distributions is denoted by Y. Let $\overline{y}$ denote the income standard in distribution y, *e.g.* mean or median income in y. The income standard is homogeneous of degree one. We consider two different poverty status, each identified by a specific poverty line.

The *absolute* poverty line is defined by a poverty threshold $z_a \in \mathbb{R}_{++}$, which does not depend on the income standard. An individual i is deemed absolutely poor if $y_i < z_a$. Typically, z_a is the minimal income level allowing to purchase the goods necessary to satisfy basic needs (*e.g.* food, clothes or shelter). The number of absolutely poor individuals in distribution y is denoted by $q_a(y)$.

The *relative* poverty line is defined by a threshold function $z_r : \mathbb{R}_+ \to \mathbb{R}_+$ defined as

$z_r(\overline{y}) = b + s\overline{y}$, where $s \in (0,1)$ is the slope of the relative line and $b \geq 0$ is its intercept. Strongly relative poverty lines have $b = 0$ and weakly relative lines have $b > 0$ (Ravallion and Chen, 2011). Typically, the slope takes value $s = 0.5$. An individual i is deemed relatively poor if $y_i < z_r(\overline{y})$. The relative threshold $z_r(\overline{y})$ is understood as the minimal amount necessary to engage in the everyday life of a society whose income standard is $\overline{y}$. The number of relatively poor individuals in distribution y is denoted by $q_r(y)$.

A poverty measure is a function $P : Y \to [0,1]$ that ranks all income distributions using a fixed (set of) poverty line(s). We say that P measures *absolute* (resp. *relative*) poverty if P identifies the poor using only the absolute (resp. relative) line. We say that P measures *overall* poverty if P identifies the poor using both lines. In this latter case, the number of individuals who are poor is denoted by $q(y) = max\{q_a(y), q_r(y)\}$ and the number of individuals who are only relatively poor is $q(y) - q_a(y)$. Since income distributions are sorted, if $i \leq q_a(y)$ then individual i is absolutely poor and if $q_a(y)+1 \leq i \leq q(y)$ then individual i is only relatively poor.

If for two distributions $x, y \in Y$ we have $P(x) > P(y)$, then x has more poverty than y. We say that there is a *disagreement* between two different poverty measures on two distributions when these measures draw opposite evaluations of the distributions.

Definition 1. *There is a disagreement between poverty measures P and P' over distributions $x, y \in Y$ if $P(x) > P(y)$ and $P'(x) < P'(y)$.*

3.2 Disagreement between absolute and relative measures

We present our analysis using a stylized example for which the absolute threshold is set at \$1.9 a day (*i.e.* the extreme poverty threshold of the World Bank) and the relative threshold is set at half mean income. Our example assumes a strongly relative line but it is straightforward to adapt our reasoning to the case of a weakly relative line. Consider distributions x and y shown in Table 1. Both distributions feature three individuals. Individual 1 is absolutely poor, individual 2 is only relatively poor and individual 3 is non-poor. Distribution y is obtained from x by a particular form of *unequal growth*. The income of each individual i is larger in y than in x, which yields a mean income in y (\$10) twice as large as the mean income in x (\$5). Yet, the income growth from x to y is not equi-proportional. The income of the non-poor individual 3 is more than doubled while the income of the poor individuals 1 and 2 grow at a slower pace. We show below that, when considering gap-sensitive poverty measures, there is a disagreement between absolute and relative measures over these two distributions that have different income standards.

The most popular poverty measures belong to the Foster-Greer-Thorbecke (FGT) family (Foster et al., 1984). These additive measures are computed as the average poverty

Table 1: Disagreement over the comparison of x and y

	$i=1$	$i=2$	$i=3$	z_a	z_r
Distribution x	1.6	2	11.4	1.9	2.5
Distribution y	1.8	3	25.2	1.9	5

Note: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

contribution of all individuals in a distribution. The *absolute* poverty measure A_α is defined as:

$$A_\alpha(y) := \frac{1}{n}\sum_{i=1}^{q_a(y)}(1-d_a(y_i))^\alpha \qquad \text{where} \qquad d_a(y_i) = \frac{y_i}{z_a}, \tag{1}$$

where function d_a computes the *normalized income*, *i.e.* the income divided by the poverty threshold, and the poverty aversion parameter $\alpha \geq 0$ tunes the priority given to poor individuals with smaller normalized income. This family admits the head-count ratio ($\alpha = 0$) and the poverty-gap ratio ($\alpha = 1$) as special cases.

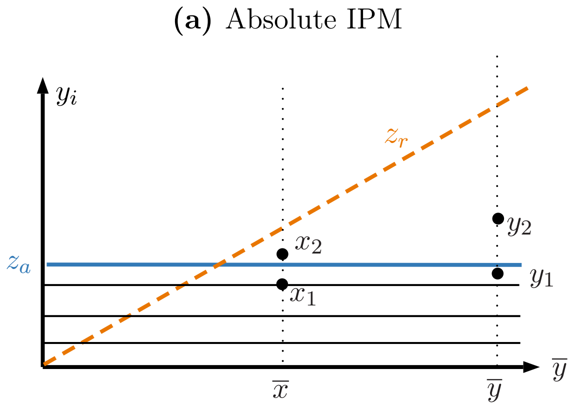
The *relative* poverty measure R_α is defined similarly. The only difference is the definition of the poverty threshold.

$$R_\alpha(y) := \frac{1}{n}\sum_{i=1}^{q_r(y)}(1-d_r(y_i,\overline{y}))^\alpha \qquad \text{where} \qquad d_r(y_i,\overline{y}) = \frac{y_i}{z_r(\overline{y})}. \tag{2}$$

Absolute and relative measures sometimes disagree on two distributions that have different income standards. This is the case for distributions x and y shown in Table 1, at least when $\alpha > 0$. To keep the exposition simple, assume $\alpha = 1$. We have $A_1(x) = 0.05 > 0.02 = A_1(y)$ but $R_1(x) = 0.19 < 0.35 = R_1(y)$. We have $A_1(x) > A_1(y)$ because the inequality $x_1 < y_1$ implies that individual 1's normalized income (with respect to the *absolute* threshold) is smaller in x than in y. We have, instead, $R_1(x) < R_1(y)$ because the incomes of individuals 1 and 2 do not grow as fast as the income standard, which implies that their normalized incomes (with respect to the *relative* threshold) are larger in x than in y. For instance, in the case of individual 1 we have $\frac{y_1}{x_1} < 2 = \frac{\overline{y}}{\overline{x}}$.

This example illustrates that the absolute and relative measures may disagree because they provide different comparisons of individual situations across distributions having different income standards. In our framework, the situation of any individual i is defined by their *bundle* $(y_i, \overline{y})$. Each additive poverty measure implicitly defines a complete ranking of individual bundles, summarized by its *iso-poverty map* (IPM) (Decerf, 2018). An iso-poverty map is a collection of *iso-poverty curves*, which are defined as the set of all individual bundles associated to a given value of poverty contribution. The IPMs implicitly defined by absolute and relative measures are graphically illustrated in Figure 1.

Figure 1: Distribution x has higher absolute poverty but lower relative poverty than y.



Note: The black lines are iso-poverty curves. These lines reveal how different bundles $(y_i, \overline{y})$ are implicitly compared across distributions with different income standards.

In the case of measures A_1 and R_1, an iso-poverty curve is the set of bundles associated to a given value of normalized income. For the absolute measure A_1, the normalized income only depends on individual income. As a result, all iso-poverty curves associated to A_1 are flat lines, as illustrated in Figure 1.a. As $(x_1, \overline{x})$ is on a lower iso-poverty curve than $(y_1, \overline{y})$, we have $A_1(x) > A_1(y)$. For the relative measure R_1, the normalized income is the individual income divided by the relative poverty threshold. As a result, all iso-poverty curves associated to R_1 are straight rays from the origin, as illustrated in Figure 1.b.[6] Figure 1.b shows that $(x_1, \overline{x})$ is on a higher iso-poverty curve than $(y_1, \overline{y})$ and that $(x_2, \overline{x})$ is on a higher iso-poverty curve than $(y_2, \overline{y})$, which implies that $R_1(x) < R_1(y)$.

Observe that the IPMs associated to (1) and (2) are the same for all values of poverty aversion such that $\alpha > 0$. Therefore, measures A_α and R_α disagree on distributions x and y for any $\alpha > 0$, which shows that the disagreement is deep.[7]

3.3 Overall poverty comparisons

When both absolute and relative poverty lines are deemed relevant for poverty identification, it is in general unclear how the *overall* poverty of distributions x and y compare. In order to partially overcome this indeterminacy, we impose the following **normative assumption**: an individual who is absolutely poor must be considered poorer than an individual who is only relatively poor, *regardless of the income standard in their respective societies*. We view this assumption as rather mild because the idea that absolute poverty should be deemed more severe than relative poverty is largely shared. It is reflected in the answers to a questionnaire experiment conducted all over the world by Corazzini et al.

[6]This is because our example assumes a strongly relative line. When the relative line is weakly relative, these iso-poverty curves are straight rays with positive intercepts.

[7]For the special case $\alpha = 0$, measures A_0 and R_0 are not gap-sensitive. All iso-poverty curves below the poverty line form a "thick" iso-poverty curve. Both A_0 and R_0 find equal poverty in x and y.

(2011) and has also been expressed in the poverty measurement literature (Atkinson and Bourguignon, 2001; Decerf, 2017). Moreover, Decerf (2018) provides an axiomatic result showing that overall poverty measures should satisfy this assumption.

Relative poverty measures violate this assumption. In Figure 1.b, $(x_1, \overline{x})$ is on a higher iso-poverty curve than $(y_2, \overline{y})$ despite the fact that individual 1 is absolutely poor in x whereas 2 is only relatively poor in y. Thus, the contribution to R_1 of individual 1 in x is smaller than the contribution to R_1 of individual 2 in y.

Importantly, commonly used overall measures also violate our normative assumption. Consider for instance the overall measures proposed by Atkinson and Bourguignon (2001), which are defined as:[8]

$$O_\alpha(y) := \frac{1}{n}\sum_{i=1}^{q(y)}(1 - d_{ar}(y_i, \overline{y}))^\alpha \qquad \text{where} \qquad d_{ar}(y_i, \overline{y}) = \frac{y_i}{\max\{z_a, z_r(\overline{y})\}} \tag{3}$$

and where $\alpha \geq 0$ is the poverty aversion parameter. The poverty line considered by these measures is the upper-contour of the two poverty lines. Therefore, the poverty line is absolute in low-income countries and relative in middle-income countries. All measures O_α are associated to the same IPM, regardless of the value taken by α.[9] This IPM is implicitly defined by function d_{ar}, which computes the normalized income. As illustrated in Figure 2.a, this IPM corresponds to the IPM of absolute measures in very low-income countries ($z_a > z_r$) and corresponds to the IPM of relative measures in higher income countries ($z_a < z_r$). As their iso-poverty curves cross the absolute threshold, these measures violate our normative assumption. This violation can be illustrated using Figure 2.a. When $\alpha > 0$, the contribution to O_α of individual 1 in x is smaller than the contribution to O_α of individual 2 in y, even if the former is absolutely poor and the latter is not. When $\alpha = 0$, their contributions are the same.

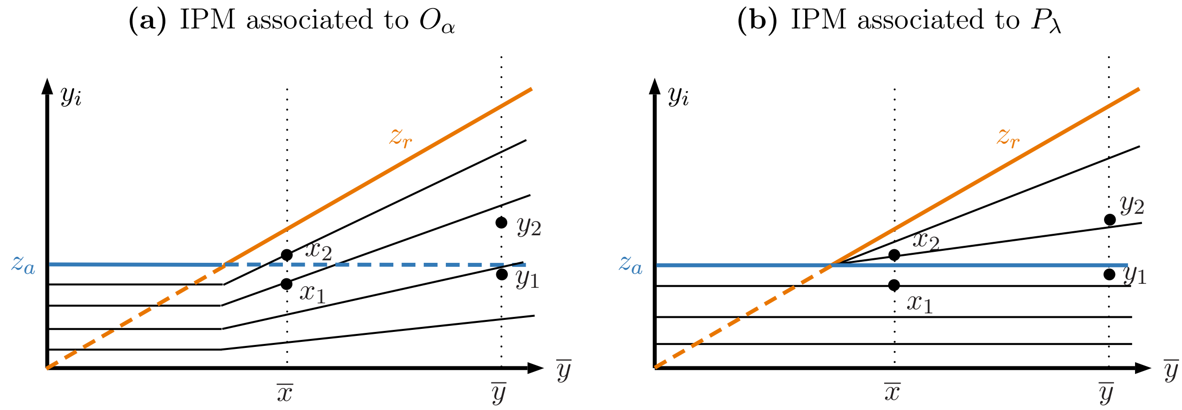
Decerf (2017) derives the following family of overall measures whose members all satisfy our normative assumption:

$$P_\lambda(y) := \frac{1}{n}\sum_{i=1}^{q(y)}\big(1 - d_\lambda(y_i, \overline{y})\big), \tag{4}$$

[8] Atkinson and Bourguignon (2001) propose a more general family of overall measures. Equation (3) corresponds to the subfamily that they consider in their empirical application. Their alternative measures also violate our normative assumption.

[9] Strictly speaking, the IPM for O_0 is different because all its iso-poverty curves form a “thick” iso-poverty curve.

Figure 2: Under our normative assumption (P_λ), distribution x has a larger overall poverty than y.



Note: The black lines are iso-poverty curves. These lines reveal how different bundles $(y_i, \overline{y})$ are implicitly compared across distributions with different income standards.

where individual i's poverty contribution is $1 - d_\lambda(y_i, \overline{y})$ and

$$d_\lambda(y_i, \overline{y}) := \begin{cases} \lambda \frac{y_i}{z_a} & \text{if } \; y_i < z_a, \\ \\ \lambda + (1-\lambda)\frac{y_i - z_a}{z_r(\overline{y}) - z_a} & \text{if } \; z_a \leq y_i < z_r(\overline{y}), \end{cases} \tag{5}$$

and where parameter $\lambda \in [0, 1]$ tunes the priority given to an individual who is absolutely poor over an individual who is only relatively poor (see below for the interpretation of this key parameter).[10] Importantly, d_λ is by definition always smaller for an absolutely poor individual than for an only relatively poor individual, regardless of the income standard in their respective societies. Thus, the former contributes more (*i.e.* is considered poorer) than the latter. As this holds for any value of λ, all members of the family satisfy our normative assumption.

All measures P_λ are associated to the same IPM (illustrated in Figure 2.b), regardless of the value taken by λ.[11] This IPM has three key features. First, all the iso-poverty curves below the absolute threshold are flat. The reason is that the poverty contribution of an absolutely poor individual, such as for measure A_α, only depends on their individual income. Importantly, this implies that no iso-poverty curve "crosses" the absolute threshold. That is, no iso-poverty curve has some of its bundles below the absolute threshold and some of its bundles above the absolute threshold. Hence, an absolutely poor individual always contributes more to P_λ than an individual who is only relatively poor.

[10]This family assumes a poverty aversion $\alpha = 1$. Therefore, all individuals who have the same poverty status (being absolutely poor or being only relatively poor) have the same priority. When $\alpha = 1$, the condition for robust overall poverty comparisons exposed in Proposition 2 is simple.

[11]Different values of parameter λ define different numerical representations of this IPM. Strictly speaking, the IPMs for P_0 and P_1 are slightly different because each of these measures has a "thick" iso-poverty curve. In the case of P_0, all the iso-poverty curves below z_a form a "thick" iso-poverty curve. In the case of P_1, all the iso-poverty curves above z_a form a "thick" iso-poverty curve.

This shows that measure P_λ satisfies our normative assumption. Second, the iso-poverty curves above the absolute threshold have a positive slope. The reason is that the poverty contribution of individuals who are (only) relatively poor, such as for measure R_α, also depends on the income standard. Third, at any bundle above the absolute threshold, the slope of the iso-poverty curve associated to P_λ is less steep than the slope of the iso-poverty curve associated to R_α. Iso-poverty curves associated to P_λ make a trade-off between the absolute and relative aspects of income, while iso-poverty curves associated to R_α only capture the relative aspect.

Observe that our measures can be given a welfarist interpretation in which the underlying utility function is expressed in equation (5). According to the utility function $d_\lambda(y_i, \overline{y})$, concerns about relative deprivation emerge only when the income standard is above some critical level and when own income is above the absolute threshold. Under this interpretation, individuals prefer to be only relatively poor in a middle-income country than absolutely poor in a low-income country. In other words, they prefer to have the possibility of satisfying their basic needs, even if having this possibility increases the cost of social participation.[12]

Parameter λ has a key normative interpretation. It tunes the priority given to an individual who is absolutely poor over an individual who is only relatively poor. Mathematically, this parameter tunes the marginal poverty contributions of these two types of poor individuals. Letting i be absolutely poor and j only relatively poor, we get from equation (5) that

$$\frac{\partial d_\lambda(y_i, \overline{y})}{\partial y_i} = \frac{\lambda}{z_a} \qquad \text{and} \qquad \frac{\partial d_\lambda(y_j, \overline{y})}{\partial y_j} = \frac{1-\lambda}{z_r(\overline{y}) - z_a}.$$

Thus, when i earns an additional ϵ of income, their contribution to poverty decreases by $\epsilon \frac{\lambda}{z_a}$, regardless of their exact income. The larger λ, the larger is the decrease in their contribution. In contrast, when j earns an additional ϵ of income, their contribution decreases by $\epsilon \frac{1-\lambda}{z_r(\overline{y})-z_a}$, regardless of their exact income. The larger λ, the smaller is the decrease in their contribution. When λ is large (close to 1), giving an additional ϵ to an absolutely poor individual reduces P_λ much more than giving it instead to an only relatively poor individual. The support of parameter λ contains all possible views on the respective priority that could be given to absolutely poor individuals. For the extreme case $\lambda = 1$, absolutely poor individuals have infinite priority because the additional ϵ is infinitely more poverty reducing when given to an absolutely poor individual. For the other extreme case $\lambda = 0$, individuals who are only relatively poor have infinite priority over absolutely poor individuals. Figure 3 graphically illustrates the impact of parameter λ on the shape of the contribution function at a fixed level of income standard. As the

[12] Ravallion and Lokshin (2010) provide empirical evidence that absolute consumption needs dominate welfare at very low levels of consumption.

graph for $\lambda = 1$ reveals, P_1 gives infinite priority to the absolutely poor because the contribution of the only relatively poor is constant in own income. Then, the graph for $\lambda = 0$ shows that P_0 gives infinite priority to the only relatively poor because the contribution of the absolutely poor is constant in own income.

Figure 3: Contribution as a function of income y_i, at a fixed income standard $\overline{y}$.

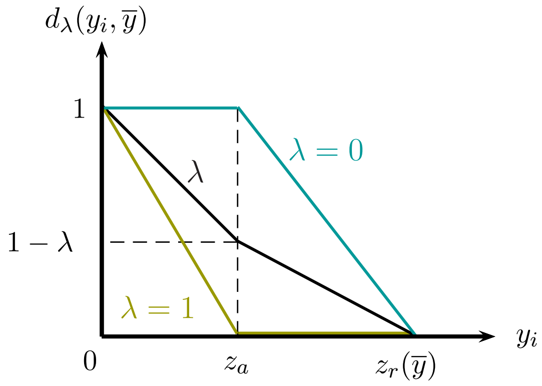


Table 2 illustrates how the value taken by parameter λ influences overall poverty comparisons. The absolute and relative poverty lines are defined as in the previous example. Distributions x' and y' have the same value of mean income and therefore share the same relative poverty threshold. Both distributions feature three individuals: individual 1 is absolutely poor, individual 2 is only relatively poor and individual 3 is non-poor. Individual 1 earns \$0.5 less in x' than in y', but individual 2 earns \$1 more in x' than in y'. Thus, distribution y' is better than x' for the absolutely poor individual but (much) worse for the individual who is only relatively poor. The absolute measure A_1 and the relative measure R_1 disagree on x' and y'. The overall poverty comparison of x' and y' depends on the priority assigned to absolutely poor individuals. When $\lambda = 0.7$, poverty as measured by P_λ is larger in x' than in y'. However, for the smaller value $\lambda = 0.3$, the comparison is reversed. Decreasing the value of parameter λ places more emphasis on individuals who are only relatively poor and less on absolutely poor individuals.

Table 2: Non-robust overall poverty comparison of x' and y'.

	$i = 1$	$i = 2$	$i = 3$	z_a	z_r	A_1	R_1	$P_{0.7}$	$P_{0.3}$
Distribution x'	1	4	25	1.9	5	0.16	0.33	0.24	0.36
Distribution y'	1.5	3	25.5	1.9	5	0.07	0.37	0.21	0.41

Note: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

The overall poverty comparison of x' and y' is not robust in the sense that the conclusion obtained depends on the value chosen for parameter λ.

Definition 2. *Distribution x has robustly more overall poverty than distribution y if we have $P_\lambda(x) \geq P_\lambda(y)$ for all $\lambda \in [0,1]$.*

The central message of this section is that it is possible to draw robust overall poverty comparisons for some pairs of distributions for which A_α and R_α disagree. For instance, we can robustly compare the overall poverty in distributions x and y given in Table 1. Consider again the hierarchical IPM in Figure 2.b, where the bundles of individuals 1 and 2 are shown for these two distributions. Both bundles $(x_1, \overline{x})$ and $(x_2, \overline{x})$ are on a lower iso-poverty curve than their respective counterparts $(y_1, \overline{y})$ and $(y_2, \overline{y})$. In this particular sense, distribution y first-order stochastically dominates distribution x (Atkinson, 1987). Thus, regardless of the value given to parameter λ, there is more overall poverty in x than in y.

Proposition 1 shows that any equi-proportionate growth robustly reduces overall poverty. In other words, any measure P_λ satisfies the WRA (Ravallion and Chen, 2011), even when the relative line considered is strongly relative ($b = 0$). Formally, distribution y is obtained from distribution x by an *equi-proportionate growth* if for some $g > 1$ we have $y_i = gx_i$ for all i. For such x and y, we have $A_\alpha(x) > A_\alpha(y)$ but $R_\alpha(x) = R_\alpha(y)$ with a strongly relative line. An equi-proportionate growth always moves the bundle of a poor individual along an iso-poverty curve associated to R_α with a strongly relative line. As explained above, the iso-poverty curves associated to R_α always "cross" the iso-poverty curves associated to P_λ from below. Therefore, any equi-proportionate growth moves the bundles of poor individuals onto higher iso-poverty curves of P_λ (except for individuals with zero income). This implies that poverty contributions are reduced, and so is P_λ.

Proposition 1. *Take any $x \in Y$ with some j for whom $z_a \leq x_j < z_r(\overline{y})$. If distribution y is obtained from distribution x by an equi-proportionate growth, then $P_\lambda(x) > P_\lambda(y)$ for all $\lambda \in [0,1]$.*

Proof. The proof is in Appendix A.1. ■

Note that it is not necessary that all bundles move onto higher iso-poverty curves in order to have a robust overall poverty comparison.[13]

The necessary and sufficient condition under which an overall poverty comparison is robust follows from Proposition 2.

Proposition 2. *For any two distributions $x, y \in Y$, either we have $\frac{P_0(x)}{P_0(y)} \leq \frac{P_\lambda(x)}{P_\lambda(y)} \leq \frac{P_1(x)}{P_1(y)}$ for all $\lambda \in [0,1]$ or we have $\frac{P_0(x)}{P_0(y)} \geq \frac{P_\lambda(x)}{P_\lambda(y)} \geq \frac{P_1(x)}{P_1(y)}$ for all $\lambda \in [0,1]$.*

[13] Poverty contributions to P_λ are linear in own income. Consider two poor individuals 1 and 2 whose incomes are on the same side of the absolute threshold. If we increase the income of individual 1 by an ϵ and decrease the income of individual 2 by more than ϵ while keeping the income standard constant, then P_λ is (weakly) increased regardless of λ. For instance, distribution $(1, 1, 4, 34)$ has robustly more overall income poverty than distribution $(0.8, 1.3, 4, 33.9)$, even if the bundle of individual 1 is on a lower iso-poverty curve under the latter distribution.

Proof. The proof is in Appendix A.2. ■

Proposition 2 directly implies that checking whether an overall poverty comparison is robust only requires computing P_λ for the two extreme values of λ.

Corollary 1. *The overall poverty in y is robustly smaller than the overall poverty in x if and only if $P_0(x) \geq P_0(y)$ and $P_1(x) \geq P_1(y)$.*

Corollary 2. *The overall poverty in y is robustly less than half the overall poverty in x if and only if $\frac{P_0(y)}{P_0(x)} \leq \frac{1}{2}$ and $\frac{P_1(y)}{P_1(x)} \leq \frac{1}{2}$.*

4 Data and parameters

4.1 Data

Our source of data is PovcalNet,[14] an online tool of the World Bank whose main goal is to replicate the Bank's poverty estimations. PovcalNet offers income or consumption data from more than 1500 household surveys across 164 countries in the world from 1981 to 2015.[15] We use data from 1990 until 2015 (the most recent year with available data). We estimate poverty for each reference year defined by the World Bank, these being designed to perform multi-country aggregations since surveys are conducted in different years across countries.[16] We take 1990 as our base year because it was the reference year used for the objective of halving global extreme poverty by 2015 (one of the United Nations' Millennium Development Goals). We restrict our sample to low and middle income countries.[17] We exclude countries with information missing for at least one year between 1990-2015. The final sample includes 117 countries, among which three have data for rural and urban areas separately. This gives a total of 120 units of analysis.

One of the main advantages of PovcalNet is that it provides poverty estimates that are internationally comparable. In order to allow for cross-country comparisons, the World Bank translates the survey data using the 2011 PPP exchange rates for household consumption from the International Comparison Program.

4.2 Poverty lines

Estimating poverty with P_λ requires selecting both an absolute line (z_a) and a relative line (z_r). We consider several pairs of poverty lines (see Section 5.2.2), but we mostly focus on our preferred pair of lines. In our main pair of lines, the absolute threshold is set at \$1.9

[14] PovcalNet: the on-line tool for poverty measurement developed by the Development Research Group of the World Bank can be found in: http://iresearch.worldbank.org/PovcalNet/povOnDemand.aspx.

[15] This figure includes high income countries that we exclude from the analysis.

[16] The reference years available between 1990 and 2015 are: 1990, 1993, 1996, 1999, 2002, 2005, 2008, 2010, 2011, 2012, 2013 and 2015.

[17] One reason to exclude high-income countries is that PovcalNet only provides systematic poverty information for these countries after 2000.

per person per day, in 2011 PPP. This has been the official extreme poverty threshold of the World Bank since 2015 and is computed from 15 national poverty lines among the poorest countries in the world (Ferreira et al., 2016). Our main relative threshold, in turn, is set at half mean income in each country. Selecting a relative line that is mean-sensitive instead of median-sensitive is a conservative assumption. This choice magnifies the relative component of our overall poverty measures because mean income is significantly larger than median income in most countries. Also, many countries saw their mean income increase faster than their median income over 1990-2015. Therefore, if the reduction in absolute poverty more than compensates the increase in relative poverty under a mean-sensitive line, it is very likely also to hold when changing the income standard to median income. Finally, a slope equal to 0.5 is standard for mean-sensitive relative lines.[18]

5 Empirical results

In this section we first show that overall poverty has been robustly halved in the developing world from 1990-2015. Second, we show that this result still holds when using alternative population weights and alternative poverty lines. Finally, we compare our results to those obtained by the alternative standard measures, both in terms of magnitude of poverty change and sensitivity to the relative line.

5.1 Evolution of overall poverty

We first analyze the evolution of poverty in a small set of developing countries (see Table 3).[19] These countries were selected for illustrative purposes. Except for Pakistan, they have all experienced a decrease in absolute poverty and an increase in relative poverty as measured by A_1 and R_1.[20] Altogether, these countries cover more than 55% of the sample population size over every year from 1990-2015. In particular, China, India, Indonesia and Pakistan are the top four most populous countries in the developing world.

Table 3 provides the evolution of mean income, inequality and poverty for each country from 1990-2015. Consider for instance the row corresponding to urban China. We observe that urban China has experienced a sharp increase both in mean income per capita and inequality as measured by the Gini index over this period (see Columns 1 to 4). The former led to a sharp decrease in absolute poverty as measured by A_1 from

[18] In section 5.2.2, we use an alternative (higher) mean-sensitive relative line (*i.e* $z_r = 0.4 + 0.5\overline{y}$), which obviously yields higher levels of poverty when combined with the same absolute line. However, given that this alternative relative line increases at a smaller rate when mean income increases, its poverty reduction estimates are *a priori* not necessarily more conservative than those of our main specification.

[19] As our data source provides separate consumption distributions for rural and urban areas for China, India and Indonesia, we analyze them separately. The relative threshold in rural (resp. urban) areas are computed using the income standard in rural (resp. urban) areas.

[20] In rural India, relative poverty measured by R_1 has remained constant over this period.

$A_1 = 0.08$ in 1990 to $A_1 \approx 0$ in 2015 (see Columns 8 and 9). In turn, the increase in inequality led to an increase in relative poverty as measured by R_1 from $R_1 = 0.02$ in 1990 to $R_1 = 0.07$ in 2015 (see Columns 10 and 11). This shows that the absolute measure disagrees with the relative measure on the evolution of poverty in urban China (as indicated in Column 12). In turn, the overall poverty measure P_1 is equal to A_1 and has thus been reduced from $P_1 = 0.08$ in 1990 to $P_1 \approx 0$ in 2015. Finally, the overall poverty measure P_0 has also been reduced from $P_0 = 0.32$ in 1990 to $P_0 = 0.09$ in 2015 (see Columns 5 and 6). As both P_1 and P_0 have decreased over the period, we can conclude from Corollary 1 that overall poverty has been robustly reduced in urban China (as indicated in the last Column). In this sense, the decrease in absolute poverty more than compensates the increase in relative poverty in urban China. Also, as both P_1 and P_0 have been at least halved over the period, we can robustly conclude from Corollary 2 that overall poverty has been (at least) halved in urban China.

Table 3: Statistics and full robustness conditions for selected countries.

	Mean (PPP$)		*Gini*			P_0		$P_1 = A_1$		R_1		Dis.	Rob.
	1990	2015	1990	2015	1990	2015	$\frac{2015}{1990}$	1990	2015	1990	2015		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Bangladesh	77	116	0.26	0.32	0.38	0.15	0.40	0.09	0.03	0.02	0.03	Yes	Yes
China													
Rural	48	225	0.31	0.33	0.79	0.09	0.11	0.30	0.00	0.02	0.04	Yes	Yes
Urban	80	418	0.26	0.36	0.32	0.09	0.28	0.08	0.00	0.02	0.07	Yes	Yes
India													
Rural	67	110	N/A	0.31	0.53	0.15	0.29	0.15	0.03	0.02	0.02	Yes	Yes
Urban	96	164	N/A	0.39	0.32	0.17	0.53	0.08	0.02	0.04	0.06	Yes	Yes
Indonesia													
Rural	55	146	0.26	0.33	0.67	0.12	0.18	0.21	0.01	0.01	0.03	Yes	Yes
Urban	84	199	0.35	0.43	0.41	0.20	0.49	0.11	0.01	0.04	0.09	Yes	Yes
Jamaica	232	354	0.41	0.45	0.16	0.17	1.03	0.01	0.00	0.09	0.11	Yes	No
Pakistan	65	142	0.33	N/A	0.57	0.10	0.17	0.19	0.01	0.04	0.02	No	Yes
Dping world	126	248	N/A	N/A	0.48	0.18	0.37	0.16	0.04	0.07	0.07	Yes	Yes

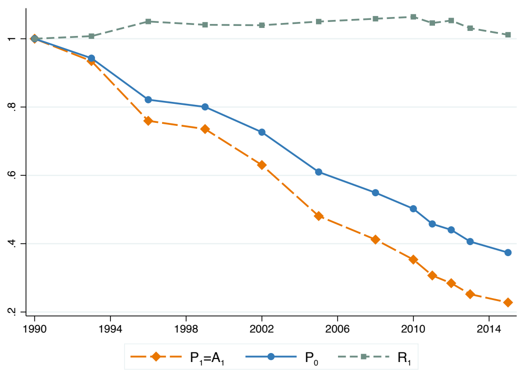
Source: PovcalNet, 1990 & 2015. Mean income per capita is expressed in PPP$ per month. A_1 and R_1 are defined as in equations (1) and (2) with $\alpha = 1$. P_0 is defined as in equation (4) with $\lambda = 0$. The column labeled "Dis." indicates whether there is a disagreement between A_1 and R_1 on the poverty change between 2015 and 1990. The last column labeled "Rob." identifies whether the poverty change according to P_λ is independent of the value of λ. For some countries, the Gini is not available for 1990 and/or 2015. We impute the Gini when there is survey data available in a window of 10 years around each reference year. The imputation concerns the following countries and reference years in the table (we indicate the survey year used to input the Gini between brackets): Bangladesh in 1990(1981), 2015(2010) & Pakistan in 1990(1981).

The evolution of poverty in urban China is not an exception as the developing world experienced both a strong growth and an increase in intra-country inequality over the period (Bourguignon, 2015; Milanovic, 2016; Anand and Segal, 2008; Ravallion, 2014). Many cases presented in Table 3, namely Bangladesh, rural China, rural and urban India and rural and urban Indonesia, experience a similar evolution: the absolute A_1

measure disagrees with the relative measure R_1 but overall poverty is robustly reduced. In most of these cases, we can robustly conclude that overall poverty has been halved. In urban India however, whether overall poverty has been halved or not depends on the priority parameter (P_0 is not halved over the period). The remaining two countries provide examples of alternative trends in poverty. In Pakistan, there was no increase in relative poverty but the strong decrease in absolute poverty has led overall poverty to be divided by a factor larger than five. In Jamaica, the decrease in absolute poverty was not large enough to offset the increase in relative poverty, leading to a slight increase in overall poverty when the priority given to absolutely poor individuals is sufficiently low (as revealed by P_0).

Figure 4 shows the evolution of poverty for the whole developing world (see statistics in Table 3). The absolute measure disagrees with the relative measure since A_1 has declined by 77% while R_1 has increased by 2%. The overall poverty measure P_1, which gives infinite priority to absolutely poor individuals, has declined by 77% as it coincides with A_1. Finally, the overall poverty measure P_0, which gives infinite priority to relatively poor individuals, has declined by 63%. Thus, there is a robust reduction in overall poverty in the developing world. Moreover, P_0 provides the lower bound for this overall poverty reduction, which is larger than 50%. This shows that overall poverty in the developing world has been halved over the period, regardless of the priority assigned to the absolutely poor.

Figure 4: Evolution of poverty in the developing world. 1990-2015.



Source: PovcalNet, 1990-2015. Evolution of poverty relative to 1990.

We look now at the evolution of overall poverty by regions of the world. The World Bank divides the developed world into six regions: (1) East Asia and Pacific, (2) Europe and Central Asia, (3) Latin America and the Caribbean, (4) Middle East and North Africa, (5) South Asia and (6) Sub-Saharan Africa. Our results on overall poverty reduction for the whole developing world are mostly driven by (populous) regions with initial large poverty. These are mainly East Asia and Pacific, South Asia and Sub-Saharan Africa, which respectively explain 54%, 23%, and 18% of global P_1 in 1990 and 49%, 27%, and 14% of global P_0 in 1990. Figures A.2a to A.2f in the Appendix show the evolution of poverty in these six regions. All regions experience a robust decline of their overall poverty over the period. Moreover, overall poverty has been robustly halved in East Asia and Pacific and in South Asia.

5.2 Robustness

In this section, we study the robustness of our results in two different ways. First, we study robustness to population weights and check whether the results are fully driven by a few major countries. Second, we study whether our results are robust to alternative definitions of the poverty lines.

5.2.1 Robustness to population weights

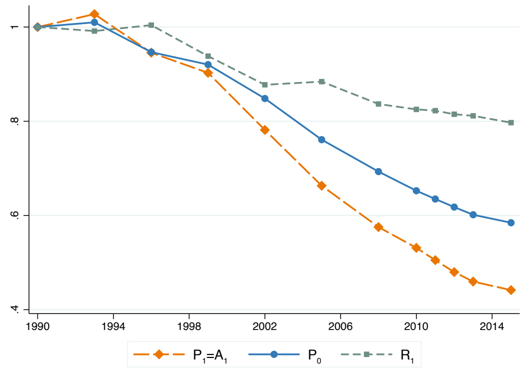
One potential concern about our analysis is whether the robust reduction in overall poverty is completely driven by the evolution of poverty in one or two large countries. In order to assess this, we perform two robustness checks. First, we exclude China and India from the sample. Second, we fully ignore population weights and compute the number of countries for which we can conclude that overall poverty has decreased (resp. has been halved) regardless of the priority parameter.

China and India represent together almost half of our sample population size (48% in 1990 and 45% in 2015). Also, they have both experienced a strong reduction in overall poverty. We first analyze whether the overall poverty reduction in the developing world also holds when we exclude these two countries. Figure 5 shows that even when these large economies are removed, overall poverty has significantly decreased. When removing China and India, absolute poverty decreases by 56% (instead of 77%) and overall poverty decreases by at least 42% (instead of 63%) (see Table A.1 in the Appendix).[21] We can almost robustly conclude that overall poverty has been halved, even when excluding both China and India. Hence, these two countries alone do not completely drive our result.

Second, we study the robustness of our results to ignoring population weights. That is, we identify the fraction of developing countries for which overall poverty has been robustly reduced and the fraction for which it has been robustly halved. To do so, we perform all within-country pairwise poverty comparisons between 1990 and 2015. For each pairwise comparison, we also identify whether there is a disagreement between A_1 and R_1. Considering all 120 units in our sample, we observe that A_1 and R_1 have evolved in opposite directions in almost 40% of the cases (see Table A.2 in the Appendix). Moreover, we observe that 78% of countries have experienced a robust overall poverty reduction and that overall poverty has been robustly halved in 30% of countries (see Table A.3 in the

[21]Table A.1 in the Appendix further shows the robust decrease in overall poverty when excluding only China or India. Figures A.1a and A.1b in the Appendix display the evolution of poverty by region excluding China and India.

Figure 5: Evolution of poverty in the developing world (excluding China and India). 1990-2015.



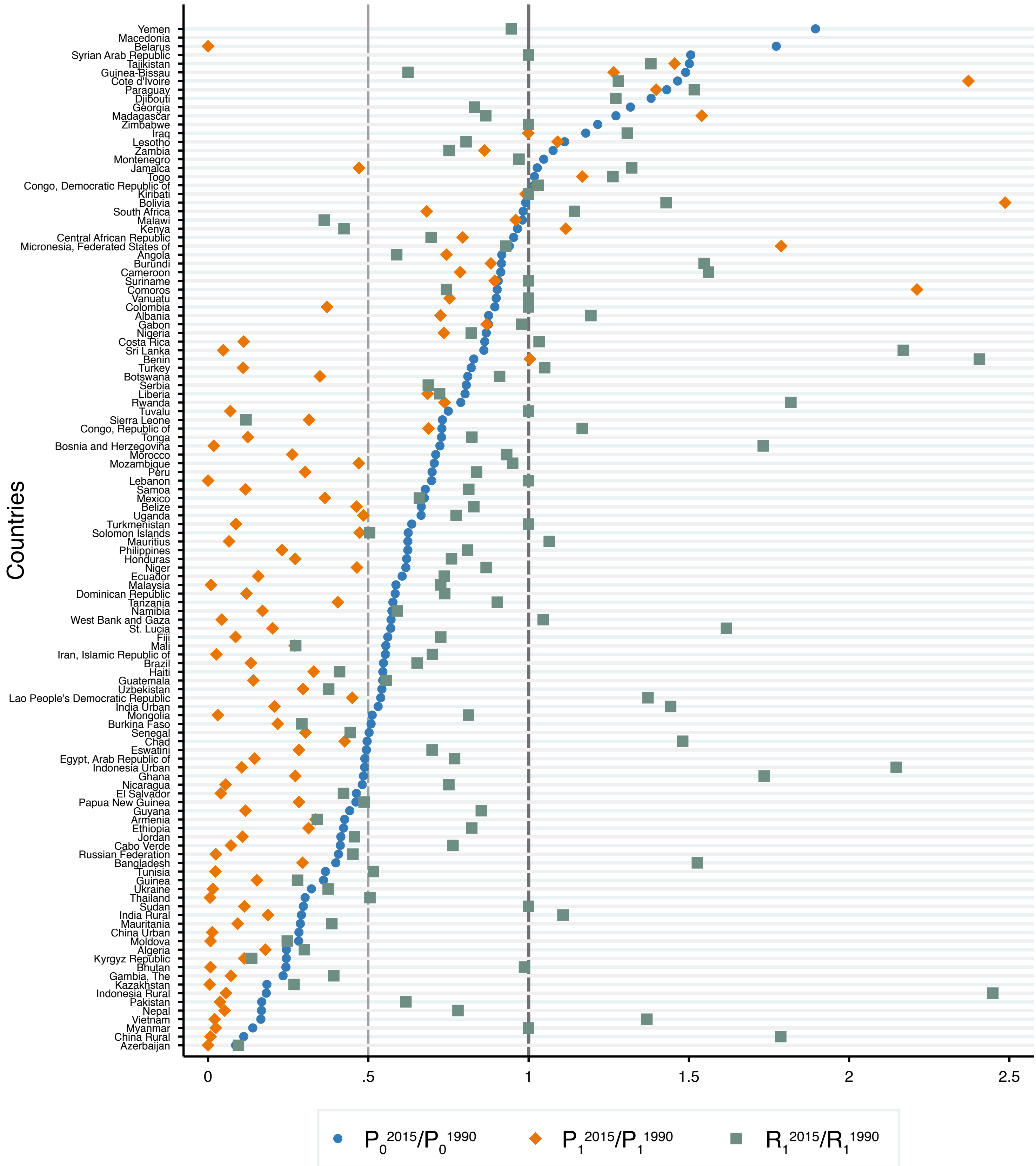
Source: PovcalNet, 1990-2015. Evolution of poverty relative to 1990.

Appendix). Figure 6 displays the ratio of P_0, P_1 and R_1 in 2015 relative to 1990 for each country in our sample.

5.2.2 Robustness to poverty lines

We show here that our results still hold for alternative pairs of poverty lines. Table 4 displays the specific combinations of absolute and relative lines that we use. The first five pairs of lines (pairs 1 to 5 in Table 4) all use different relative lines but the same absolute line. The first alternative relative line is similar to our main relative line but is based on median income instead of mean income. The second alternative relative line is also based on median income and has the same gradient as the previous one but in addition it has an intercept of $1. This line, called the societal poverty line, has been estimated by Jolliffe and Prydz (2017) from regressions of 699 national poverty thresholds against median income. The latest report from the World Bank estimates the societal poverty, which corresponds to the head-count ratio below the upper-contour of the extreme poverty line and the societal poverty line (World Bank, 2018). The third alternative relative line has an intercept of 0.4 and a relative gradient of 50% of the mean national income. This line has been estimated from regressions of national poverty thresholds by Ravallion and Chen (2017) (see their Figure 5 panel b). As some authors consider relative lines with a smaller slope parameter (see for instance Atkinson and Bourguignon, 2001), our pair 5 has a slope of 0.33 and an intercept of $1. Finally, our sixth combination of lines sets the absolute line at 3.2 PPP$ a day and uses the relative line of our main specification (pair 1). The absolute threshold of $3.2 a day corresponds to the lower-middle-income international poverty line suggested by the World Bank (see Jolliffe and Prydz, 2016).

Figure 6: Evolution of poverty by country. 2015/1990.



Source: PovcalNet, 1990-2015. For visualization purposes, ratios larger than 2.5 are not displayed. We exclude countries with at least two variables larger than 2.5. This concerns: Bulgaria, Macedonia & Romania. The following countries are displayed but have one variable larger than 2.5: Belarus, China Urban, Djibouti, Georgia, Montenegro, Serbia, Syrian Arab Republic, Yemen & Zimbabwe. Finally, note that there are several countries in the graph with a ratio $\frac{R_1^{2015}}{R_1^{1990}}$ equal to 1. For these countries, PovcalNet has survey data for only one year over the whole period. Thus, to extrapolate the distribution across years they assume equi-proportionate growth. This implies that when R_1 is defined using a strongly relative line, it does not change over time. This affects the following countries: Kiribati, Lebanon, Myanmar, Sudan, Suriname, Syrian Arab Republic, Turkmenistan, Tuvalu, Vanuatu & Zimbabwe.

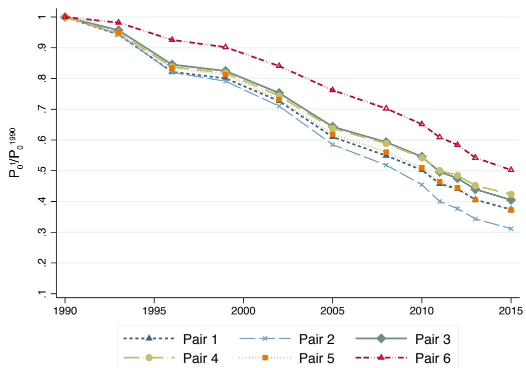
Table 4: Evolution of overall poverty in the developing world for different pairs of lines.

Pair #	z_a	z_r	Income standard $\bar{y}$	P_0 $\frac{2015}{1990}$	P_1 $\frac{2015}{1990}$
1	1.9	$0.5\bar{y}$	Mean	0.37	0.23
2	1.9	$0.5\bar{y}$	Median	0.31	0.23
3	1.9	$1+0.5\bar{y}$	Median	0.41	0.23
4	1.9	$0.4+0.5\bar{y}$	Mean	0.42	0.23
5	1.9	$1+0.33\bar{y}$	Mean	0.37	0.23
6	3.2	$0.5\bar{y}$	Mean	0.50	0.33

Source: PovcalNet, 1990 & 2015.

Figure 7 displays the evolution of overall poverty according to P_0 (relative to 1990) for all pairs of lines. For all of them, we observe a continuous decrease in overall poverty. The decline in overall poverty P_0 between 1990 and 2015 ranges from 50% to 69% (see also Table 4). Even considering the most conservative pair of lines (pair 6), overall poverty decreases by at least 50% between 1990 and 2015. This shows that our main result still holds when using alternative pairs of lines. Table A.4 in the Appendix replicates Table 3 for selected countries using the most conservative pair of lines. Results show that if we raise the absolute threshold from \$1.9 to \$3.2, the absolute measure A_1 in the developing world has decreased by 67% (which is slightly lower than the decrease of 77% obtained under \$1.9). All selected countries have experienced a substantial decrease in overall poverty under the pair of lines 6. Moreover, for all of the selected countries the decrease in overall poverty is independent of the priority parameter.

Figure 7: Evolution of poverty in the developing world by lines. 1990-2015.



Source: PovcalNet, 1990-2015. Evolution of poverty relative to 1990. Lines are defined as in Table 4.

Considering all 120 countries in our sample, we observe that 80% of them experience a robust overall poverty reduction when considering the most conservative pair of lines. Moreover, 28% of them have their overall poverty robustly halved (see Table A.5 in the Appendix). Again, this shows that our results are not fully driven by a small number of large countries.

5.3 Comparison with alternative measures

In this section, we compare the results on poverty change obtained by our family of measures with the alternative approaches most commonly used in the literature. Despite the normative appeal of our approach, its empirical relevance largely depends on the extent to which poverty change estimates differ from those obtained using standard measures. We make two types of comparisons. First, we show that alternative measures find less poverty reduction because they violate our normative assumption. Second, we show that our approach is less sensitive to the choice of the relative line.

5.3.1 Alternative approaches find less poverty reduction

The dominant practice in evaluating overall poverty is to estimate the evolution of O_0, *i.e.* the head-count ratio below the upper-contour of the absolute and relative lines. This is for instance the approach followed by Chen and Ravallion (2013); Jolliffe and Prydz (2017); Ravallion and Chen (2017). Thus, most of our comparative analysis is based on O_0.

In Table 5 we compare our estimation of overall poverty reduction with that estimated by O_0. The main takeaway is that, for all pair of lines, the poverty reduction estimated by O_0 does not lie inside our two bounds. Even our conservative estimation (associated with P_0) finds more poverty reduction than O_0.

Table 5: Evolution of overall poverty in the developing world for different pairs of lines. Alternative measures.

Pair #	z_a	z_r	Income standard $\bar{y}$	P_0 $\frac{2015}{1990}$	P_1 $\frac{2015}{1990}$	O_0 $\frac{2015}{1990}$	O_1 $\frac{2015}{1990}$	A_0 $\frac{2015}{1990}$	A_1 $\frac{2015}{1990}$	R_0 $\frac{2015}{1990}$	R_1 $\frac{2015}{1990}$
1	1.9	$0.5\bar{y}$	Mean	0.37	0.23	0.51	0.43	0.27	0.23	1.08	1.01
2	1.9	$0.5\bar{y}$	Median	0.31	0.23	0.38	0.32	0.27	0.23	1.03	0.96
3	1.9	$1+0.5\bar{y}$	Median	0.41	0.23	0.56	0.46	0.27	0.23	0.67	0.57
4	1.9	$0.4+0.5\bar{y}$	Mean	0.42	0.23	0.60	0.50	0.27	0.23	0.82	0.78
5	1.9	$1+0.33\bar{y}$	Mean	0.37	0.23	0.50	0.41	0.27	0.23	0.56	0.49
6	3.2	$0.5\bar{y}$	Mean	0.50	0.33	0.55	0.40	0.46	0.33	1.08	1.01

Source: PovcalNet, 1990 & 2015.

Importantly, this underestimation is not merely the result of O_0 being insensitive to the depth of poverty (*i.e.* the gap with respect to the poverty threshold). Indeed, we can alternatively compare our estimates with that obtained using O_1, *i.e.* the poverty-gap ratio below the upper-contour of the absolute and relative lines, a standard gap-sensitive measure. Interestingly, we observe that, except for pair 6 whose absolute threshold is much larger, O_1 also finds less poverty reduction than P_0.

The key reason that O_0 finds less poverty reduction is that it violates our normative assumption. O_0 implicitly considers that all poor individuals are equally poor, regardless of whether they are absolutely poor or only relatively poor. Growth reduces O_0 when a poor individual exits poverty, but it does not record progress when an absolutely poor individual crosses the absolute threshold and becomes only relatively poor.

In contrast, our measures do record such progress. In order to shed light on this, we contrast the mathematical expressions of O_0 and P_0, the measure associated to our lower bound for poverty reduction. O_0 computes the fraction of absolutely poor individuals plus the fraction of only relatively poor individuals:

$$O_0(y) = \frac{q_a(y)}{n} + \frac{q(y) - q_a(y)}{n}. \tag{6}$$

P_0 in turn computes the fraction of absolutely poor individuals plus the fraction of only relatively poor individuals *multiplied by an endogenous weight* $w(y) \in [0, 1]$ (see Decerf, 2018):

$$P_0(y) = \frac{q_a(y)}{n} + w(y)\frac{q(y) - q_a(y)}{n} \qquad \text{where} \qquad w(y) = \frac{z_r(y) - \hat{y}^r}{z_r(y) - z_a}, \tag{7}$$

where $\hat{y}^r$ is the average income among individuals who are only relatively poor, *i.e.*

$$\hat{y}^r = \frac{1}{q(y) - q_a(y)} \sum_{i=q_a(y)+1}^{q(y)} y_i.$$

These expressions show that O_0 and P_0 take the same value in low-income countries where no individual is only relatively poor (when $z_a > z_r$). Indeed, absolutely poor individuals all contribute one, both to O_0 and P_0. However, when these countries experience growth and the relative threshold becomes larger than the absolute threshold ($z_a < z_r$), some poor individuals exit absolute poverty and become only relatively poor. Then, P_0 takes a smaller value than O_0. The reason is that, if individuals who are only relatively poor contribute one to O_0, they contribute less than one to P_0. Therefore, P_0 records more progress than O_0 when evaluating growth.

In general, O_α tends to find less poverty reduction than P_λ because the former violates our assumption. This is easily understood when z_r is strongly relative. In that case, any equi-proportionate growth in a country with $z_a < z_r$ leaves O_α unchanged (*i.e.* WRA is violated). This behavior of O_α is debatable as such growth typically allows some part of the population to escape absolute poverty. In contrast, this growth reduces P_λ because this measure implicitly considers that being only relatively poor is a form of poverty that is less severe. The same point is more subtly made when z_r is weakly relative and the growth is not equi-proportionate, even if it remains valid. Our assumption implies less steep iso-poverty curves for P_λ than for O_α (see Figure 2). Therefore, if a given growth process moves the bundle of a poor individual onto a higher iso-poverty curve of O_α

(which implies less poverty), then it also moves their bundle onto a higher iso-poverty curve of P_λ. However, the converse is not true. A growth process that lifts the bundle of an absolutely poor individual above z_a, which automatically puts it on a higher iso-poverty curve of P_λ (which implies less poverty), could simultaneously put their bundle on a lower iso-poverty curve of O_α (which implies more poverty).

Next, we quantify the extent to which O_0 finds less poverty reduction. We show that this underestimation is substantial and that the underestimation increases as more countries have large enough income standards for relative aspects to matter, *i.e.* as $z_a < z_r$. The latter finding is not surprising given that our normative assumption only plays a role when relative poverty matters.

In order to estimate the extent to which O_0 finds a smaller decline in poverty we compute the factor by which the progress recorded by O_0 should be multiplied in order to account for the progress achieved by P_λ. More precisely, the factor computes the ratio of the compound annual growth rate of P_λ between 2015 and a given reference year t relative to the compound annual growth rate of O_0 for the same period. Formally, the factor is defined as follows:

$$F_\lambda^t = \frac{\left(\frac{P_\lambda^{2015}}{P_\lambda^t}\right)^{\frac{1}{2015-t}} - 1}{\left(\frac{O_0^{2015}}{O_0^t}\right)^{\frac{1}{2015-t}} - 1} \qquad \text{for} \quad \lambda \in [0,1] \tag{8}$$

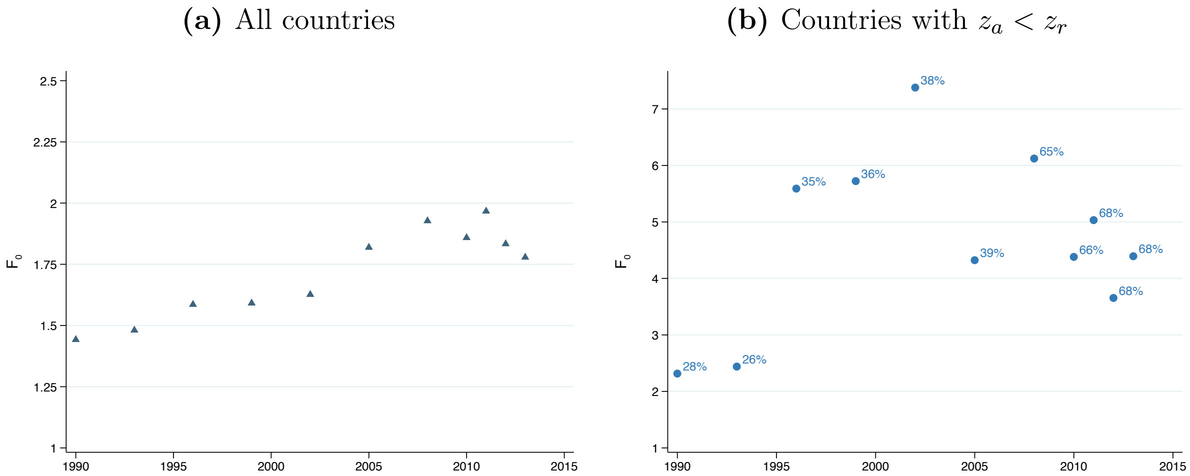
where t is a given reference year. For the sake of notation, we drop the index t and write F_λ instead of F_λ^t.

F_λ is bounded between F_1 and F_0, depending on whether we give infinite priority or zero priority to absolutely poor individuals. F_0 provides the conservative estimation given that P_0 is associated to our lower bound for poverty reduction. If we find, for instance, that F_0 is equal to 2, then we know that any index in our family will yield a rate of poverty reduction at least twice as large as O_0. Obviously, F_λ will depend on the pair of lines. For sake of simplicity, we focus on our main pair of lines (pair 1) and we provide results for alternative pairs of lines in the Appendix.

Figure 8a displays F_0 for each reference year including all countries in our sample. We observe that F_0 is always larger than 1.4 and gets closer to 2 towards the end of the period. Precisely, it lies between 1.44 and 1.97 (see also Table A.6 in the Appendix). This implies that the rate of decline in overall poverty by P_0 is at least 44% larger than by O_0. Clearly, the other extreme of Y, F_1, is even larger (see Table A.6 in the Appendix). Considering all poverty lines, we observe a large variation in F_0, which goes up to more than 2.5 for some lines and reference years (see Figure A.3a in the Appendix).

These numbers show that the underestimation of poverty is economically relevant. Moreover, they include all countries in the sample for every reference year, even those for

Figure 8: Factor F_0. 1990-2015.



Source: PovcalNet, 1990-2015. F_0 is defined as in equation (8) with $\lambda = 0$. The marker labels in panel b) indicate the share of population in the developing world that is included in the sample for each reference year.

which $z_a > z_r$. This lowers the estimates of F_0 because our normative assumption does not play a role in such countries (the IPMs of O_α and P_λ are the same when $z_a > z_r$). Indeed, the three measures O_0, P_0 and A_0 are the same for any country for which $z_a > z_r$, as revealed by equations (1), (6) and (7). Thus, these measures all register the same progress until the country grows sufficiently for relative poverty to matter. Once we have $z_a < z_r$, our assumption kicks in and P_0 registers more progress with growth than O_0. This explains why F_0 tends to increase when we increase the reference year: as time goes by, more and more countries have $z_a < z_r$.

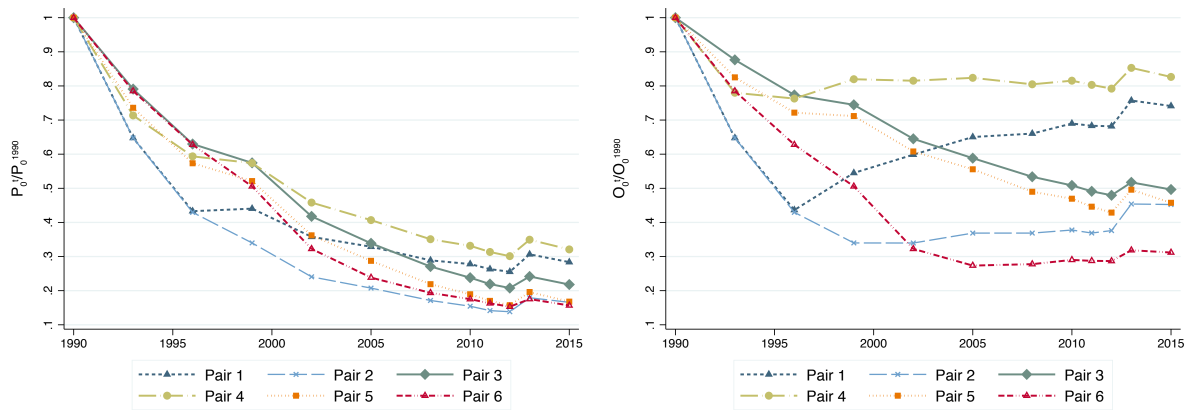
We illustrate this effect with the case of urban China. Figure 9 displays the evolution of poverty in rural China both by P_0 and O_0. We focus for now on the pair of lines 1. For the period 1990-1996, urban China has a low income standard and we have $z_a > z_r$ for pair 1.[22] Therefore, both O_0 and P_0 register the same progress over 1990-1996 (a reduction by almost 60%). After 1996, the income standard is larger and we have $z_a < z_r$, our assumption kicks in and the two measures start diverging. The unequal growth taking place in urban China after 1996 increases O_0 while it reduces P_0 (which registers progress as more and more individuals cross the absolute threshold). Hence, after 1996, the progress in poverty reduction according to P_0 is much larger than that recorded by O_0.

To account for this, we exclude from the sample those countries with $z_a > z_r$ in each reference year. Note that the sample includes different countries by reference year. We compute F_0 on this changing sample and report its evolution in Figure 8b. The marker labels indicate the share of population in the developing world that is included in the sample for each reference year. As expected, when we increase the reference year, the

[22]In 1996, $z_a = z_r$.

Figure 9: Evolution of poverty by P_0 and O_0 for urban China by lines. 1990-2015.

(a) P_0 (b) O_0



Source: PovcalNet, 1990 & 2015. Evolution of poverty relative to 1990.

share of population that is included in the sample also increases. The sample covers almost 70% of the total population by 2013. The underestimation of the rate of decline in poverty for this moving sample is striking. We find that F_0 is always larger than 2 and reaches more than 7. This conveys an important message for the evaluation of poverty reduction in the future. When most countries have $z_a < z_r$, we can expect that using O_0 will underestimate the rate of poverty reduction by at least a factor of 2.

5.3.2 Sensitivity to poverty lines

There is much less consensus on the definition of the relative line than there is on the definition of the absolute line.[23] An additional advantage of using our overall poverty measure rather than O_0 is that P_λ is less sensitive to the exact definition of the relative line. The reason for this lower sensitivity is that, when $z_a < z_r$, P_λ depends on both the absolute and the relative line, whereas O_0 only depends on the relative line.

The evolution of poverty in urban China illustrates this. Recall that, except for pair 6, all pairs of lines share the same absolute line but differ by the relative line. In Figure 9, the extent of poverty reduction seems more sensitive to the choice of lines when using O_0 than when using P_0. The decline in poverty between 1990 and 2015 as measured by O_0 varies from 17% to 69%, while the decline as measured by P_0 varies from 68% to 84%, depending on the pair of lines.

Under simplifying assumptions, we show that the poverty reduction as measured by P_0 is less elastic with respect to z_r than O_0. Assume that a country has $z_a > z_r$ in 1990 and $z_a < z_r$ in 2015 (for all pairs of lines). In 1990, O_0 and P_0 are identical regardless of the definition of the relative line (see equations (6) and (7)). Now, the relative threshold

[23] Even if the extreme line of the World Bank is widely accepted (World Bank, 2017), there is no scientific consensus on how to compute z_a (Allen, 2017).

in 2015 depends on the definition of the relative line. If the elasticity of O_0 with respect to the relative threshold z_r is larger than that of P_0, then measuring the decline in poverty with O_0 is more sensitive to the choice of the relative line. These elasticities are:

$$e_{z_r}^{O_0} = \frac{z_r}{O_0}\frac{\partial O_0}{\partial z_r} \qquad \text{and} \qquad e_{z_r}^{P_0} = \frac{z_r}{P_0}\frac{\partial P_0}{\partial z_r}$$

Of course, the values of these elasticities depend on the exact distribution under consideration. We could construct cases for which $e_{z_r}^{O_0} < e_{z_r}^{P_0}$. We show, however, that this is not likely to occur. To simplify, assume that, in 2015, the density function $f(y_i)$ characterizing the income distribution is constant on the range of incomes $[z_a, z_r + \epsilon]$, where f takes value $\overline{f} > 0$.[24] This assumption is approximately correct when z_r is not substantially larger than z_a. Under this assumption, we have:

$$O_0 = \int_0^{z_r} f(y_i)dy_i = \int_0^{z_a} f(y_i)dy_i + \overline{f}(z_r - z_a)$$

and

$$P_0 = \int_0^{z_a} f(y_i)dy_i + \frac{1}{2}\overline{f}(z_r - z_a)$$

From there, recalling that $A_0 = \int_0^{z_a} f(y_i)dy_i$, we get

$$e_{z_r}^{O_0} = \frac{z_r}{\frac{1}{\overline{f}}A_0 + (z_r - z_a)} \qquad \text{and} \qquad e_{z_r}^{P_0} = \frac{z_r}{\frac{2}{\overline{f}}A_0 + (z_r - z_a)}$$

where the factor 2 in the first term of the denominator of $e_{z_r}^{P_0}$ shows that $e_{z_r}^{O_0} > e_{z_r}^{P_0}$. In the particular case of a uniform distribution below z_a with density $\overline{f}$, we have $A_0 = \overline{f} z_a$, and thus

$$e_{z_r}^{O_0} = 1 \qquad \text{and} \qquad e_{z_r}^{P_0} = \frac{z_r}{z_r + z_a}$$

showing that, when z_r is close to z_a, we have $e_{z_r}^{P_0} \approx \frac{1}{2}$. Under the same assumptions, it is straightforward to extend this reasoning to show that $e_{z_r}^{O_0} > e_{z_r}^{P_\lambda}$ for any λ.

6 Concluding remarks

The developing world has experienced an increase in both mean income and intra-country inequality over the period 1990-2015. While this process led to a strong decrease in absolute poverty, it also increased relative poverty in many countries. By making a rather mild normative assumption, namely that an individual who is absolutely poor

[24] We consider here that an income distribution is described by a well-behaved cumulative probability distribution on the income space $\mathbb{R}_+$.

is always poorer than an individual who is only relatively poor, we show that overall income poverty, which considers both absolute and relative poverty, has declined by at least 50% in the developing world over this period. This conclusion is independent of the priority parameter and is robust to alternative definitions of the pair of poverty lines. Moreover, we find that this result holds for many developing countries individually. Alternative approaches exhibit a much lower rate of poverty decline because they violate our normative assumption and are more sensitive to the choice of the relative line. Our findings confirm and strengthen positive evaluations of the success achieved on the first Millennium Development Goal.

From a conceptual perspective, we propose a method for income poverty evaluation that accounts for the main points raised in the literature: our method combines absolute and relative poverty, satisfies the WRA and considers that absolutely poor individuals are poorer than individuals who are only relatively poor. Furthermore, our method provides, in some cases, judgments that do not depend on the arbitrary priority attributed to the absolutely over the relatively poor. This method can be readily applied in different contexts.

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A Appendix

A.1 Proof of Proposition 1

Take any distribution x with some j for whom $z_a \leq x_j < z_r(\overline{y})$ and any distribution y obtained from distribution x by an equi-proportionate growth, *i.e.* $y_i = gx_i$ for all $i \leq n$ and some $g > 1$. By equation (4), we have $P_\lambda(x) > P_\lambda(y)$ for all $\lambda \in [0,1]$ if we have

$$d_\lambda(y_i, \overline{y}) \geq d_\lambda(x_i, \overline{x}) \tag{A.9}$$

for all individuals $i \leq q(x)$, with the inequality being strict for at least one of them. Observe that, as $b \geq 0$, any individual who is non-poor in x is also non-poor after the equi-proportionate growth.

For any absolutely poor individual $i \leq q_a(x)$, function d_λ does not depend on the income standard, and thus inequality (A.9) holds as $y_i = gx_i$ and $g > 1$. Moreover, inequality (A.9) is strict if $x_i > 0$.

For any individual i who is only relatively poor (for whom $q_a(x) + 1 \leq i \leq q(x)$), we have by equation (5) that $d_\lambda(y_i, \overline{y}) > d_\lambda(x_i, \overline{x})$ if and only if

$$\frac{y_i - z_a}{z_r(\overline{y}) - z_a} > \frac{x_i - z_a}{z_r(\overline{x}) - z_a}.$$

As distribution y is obtained from distribution x by an equi-proportionate growth, we have $y_i = gx_i$ and, as the income standard is homogeneous of degree one, we have $\overline{y} = g\overline{x}$. Last inequality becomes

$$\frac{x_i}{z_a} b + s\overline{x} > x_i$$

which holds because we have $\frac{x_i}{z_a} \geq 1$ and $b + s\overline{x} > x_i$ since i is only relatively poor.

Finally, inequality (A.9) is strict for at least one poor individual because we assume that $z_a \leq x_j < z_r(\overline{y})$ for some j.

A.2 Proof of Proposition 2

Take any two distributions $x, y \in Y$.

First, we show that P_λ is *linear* in λ for any distribution $y \in Y$. That is, $P_\lambda = B + \lambda C$, where B and C do not depend on λ. P_λ adds the contributions of absolutely poor individuals P_λ^a to the contributions of only relatively poor individuals P_λ^r:

$$P_\lambda(y) = \underbrace{\frac{1}{n}\sum_{i=1}^{\mathbf{q_a(y)}} 1 - d_\lambda(y_i, \overline{y})}_{:=\mathbf{P_\lambda^a(y)}} + \underbrace{\frac{1}{n}\sum_{i=\mathbf{q_a(y)+1}}^{\mathbf{q(y)}} 1 - d_\lambda(y_i, \overline{y})}_{:=\mathbf{P_\lambda^r(y)}}. \tag{A.10}$$

Developing these two terms, we get

$$P_\lambda^a(y) = \frac{q_a(y)}{n} - \lambda\frac{q_a(y)}{n}\bar{Y}^a(y).$$

where $\bar{Y}^a(y) = \frac{\hat{y}^a}{z_a}$ and $\hat{y}^a = \sum_{i=1}^{q_a(y)} \frac{y_i}{q_a(y)}$ and

$$P_\lambda^r(y) = \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right) - \lambda\frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right)$$

where $\bar{Y}^r(y) = (\hat{y}^r - z_a)/(z_r(\overline{y}) - z_a)$ and $\hat{y}^r = \sum_{i=q_a(y)+1}^{q(y)} \frac{y_i}{q(y)-q_a(y)}$.

Together, we get:

$$P_\lambda(y) = \frac{q_a(y)}{n} + \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right) - \lambda\left[\frac{q_a(y)}{n}\bar{Y}^a(y) + \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right)\right], \tag{A.11}$$

which proves that P_λ is linear in λ.

Second, we show that $\frac{P_0(x)}{P_0(y)} \leq \frac{P_1(x)}{P_1(y)}$ implies $\frac{P_0(x)}{P_0(y)} \leq \frac{P_\lambda(x)}{P_\lambda(y)} \leq \frac{P_1(x)}{P_1(y)}$ for all $\lambda \in [0,1]$. As P_λ is linear, we can write $P_\lambda(x) = B + \lambda C$ and $P_\lambda(y) = D + \lambda E$. Inequality $\frac{P_0(x)}{P_0(y)} \leq \frac{P_1(x)}{P_1(y)}$ can be rewritten as $BE \leq CD$. Take any $\lambda \in [0,1]$, we cannot have $\frac{P_0(x)}{P_0(y)} > \frac{P_\lambda(x)}{P_\lambda(y)}$ because this inequality is equivalent to $BE > CD$. In turn, we cannot have $\frac{P_\lambda(x)}{P_\lambda(y)} > \frac{P_1(x)}{P_1(y)}$ because this inequality is also equivalent to $BE > CD$.

Finally, using the same reasoning, we also have that $\frac{P_0(x)}{P_0(y)} \geq \frac{P_1(x)}{P_1(y)}$ implies $\frac{P_0(x)}{P_0(y)} \geq \frac{P_\lambda(x)}{P_\lambda(y)} \geq \frac{P_1(x)}{P_1(y)}$ for all $\lambda \in [0,1]$, which concludes the proof.

A.3 Tables and figures

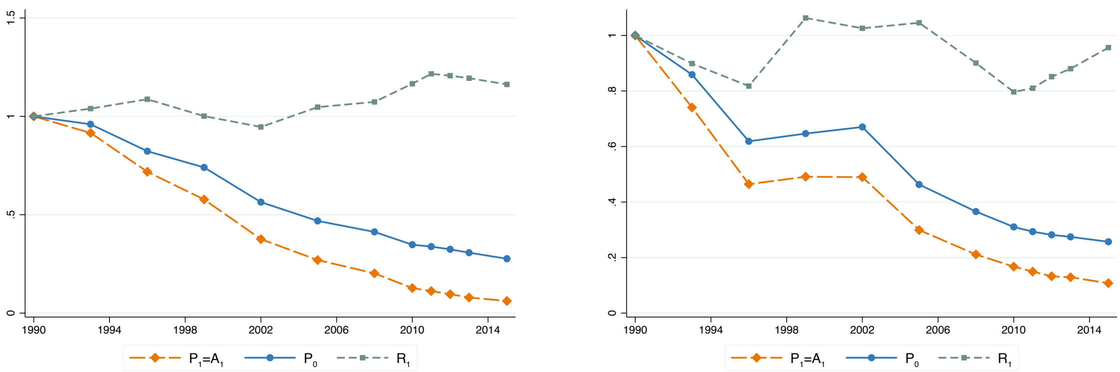
Table A.1: Statistics and full robustness conditions for the developing world excluding China and India. 1990-2015.

Countries excluded	*Mean* (PPP$)		P_0			$P_1 = A_1$		R_1		Dis.	Rob.
	1990	2015	1990	2015	$\frac{2015}{1990}$	1990	2015	1990	2015		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Both China & India	183	260	0.38	0.22	0.58	0.12	0.05	0.10	0.08	No	Yes
Only China	152	223	0.41	0.21	0.50	0.13	0.05	0.08	0.07	No	Yes
Only India	139	281	0.48	0.18	0.38	0.16	0.04	0.08	0.07	No	Yes

Source: PovcalNet, 1990 & 2015.

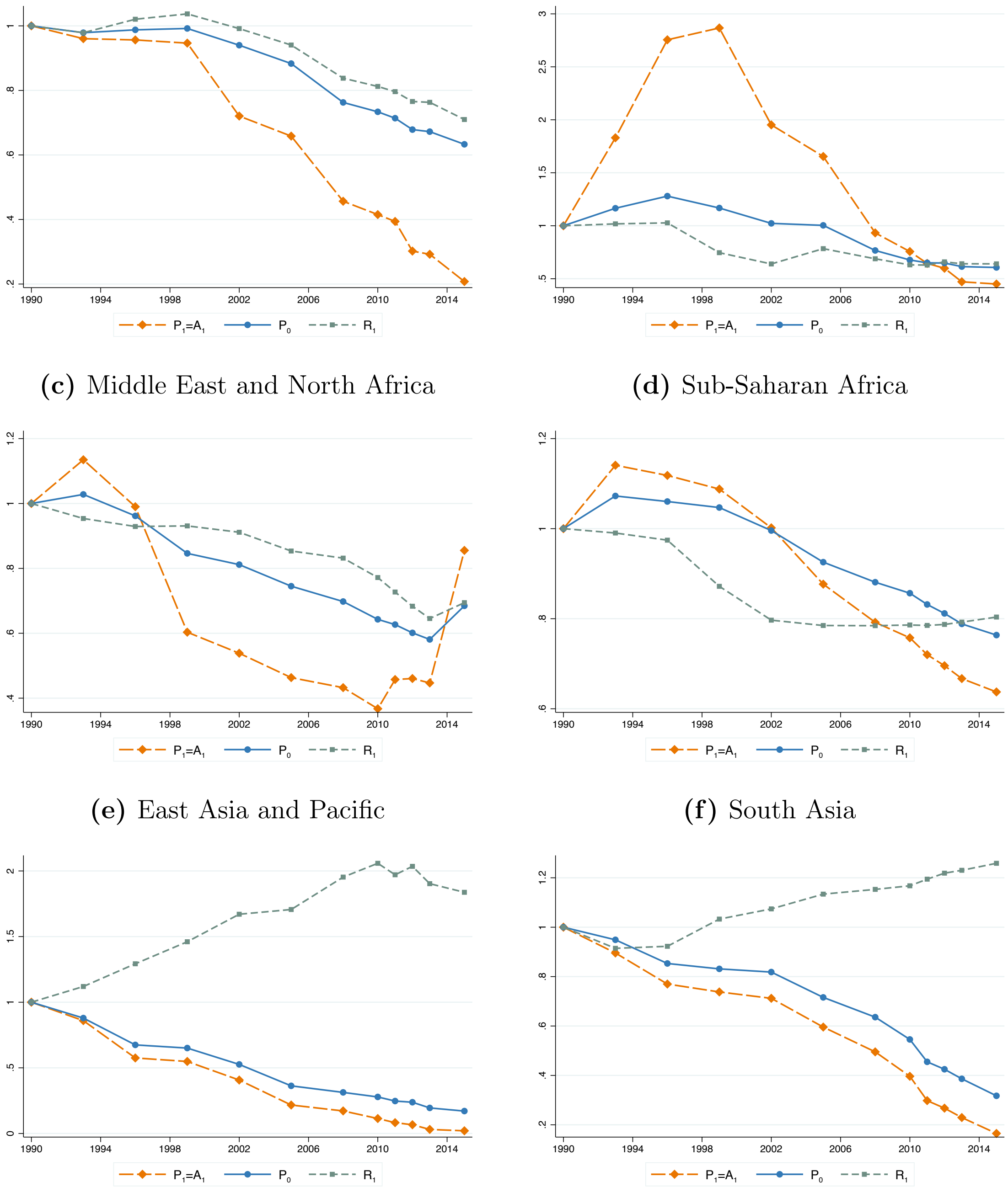
Figure A.1: Evolution of poverty for Asia excluding China and India. 1990-2015.

(a) East Asia & Pacific excluding China **(b)** South Asia excluding India



Source: PovcalNet, 1990-2015. Evolution of poverty relative to 1990.

Figure A.2: Evolution of poverty by region. 1990-2015.



(a) Latin America and the Caribbean

(b) Europe and Central Asia

(c) Middle East and North Africa

(d) Sub-Saharan Africa

(e) East Asia and Pacific

(f) South Asia

Source: PovcalNet, 1990-2015. Evolution of poverty relative to 1990.

Table A.2: Disagreement status between A_1 and R_1. 2015 vs. 1990.

Disagreement	No.	%
No	75	62
Yes	45	38
Total	120	100

Source: PovcalNet, 1990 & 2015. Table includes all countries in the sample.

Table A.3: Change in P_0 by robustness status. 2015 vs. 1990.

Evolution of P_0 by robustness status	No.	%
Partially robust		
P_0 increases	5	4
P_0 decreases (less than halved)	6	5
Fully robust		
P_0 increases	16	13
P_0 decreases (less than halved)	57	48
P_0 decreases (at least halved)	36	30
Total	120	100

Source: PovcalNet, 1990-2015. Table includes all countries in the sample.

Table A.4: Statistics and full robustness conditions for selected countries. Pair of lines 6.

	P_0			$P_1 = A_1$		R_1		Dis.	Rob.
	1990	2015	$\frac{2015}{1990}$	1990	2015	1990	2015		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bangladesh	0.80	0.54	0.67	0.31	0.16	0.02	0.03	Yes	Yes
China									
Rural	0.95	0.16	0.17	0.54	0.03	0.02	0.04	Yes	Yes
Urban	0.76	0.12	0.16	0.27	0.00	0.02	0.07	Yes	Yes
India									
Rural	0.87	0.57	0.66	0.39	0.17	0.02	0.02	Yes	Yes
Urban	0.68	0.36	0.54	0.26	0.10	0.04	0.06	Yes	Yes
Indonesia									
Rural	0.93	0.37	0.40	0.46	0.10	0.01	0.03	Yes	Yes
Urban	0.74	0.31	0.41	0.31	0.09	0.04	0.09	Yes	Yes
Jamaica	0.25	0.22	0.88	0.05	0.02	0.09	0.11	Yes	Yes
Pakistan	0.86	0.39	0.45	0.42	0.09	0.04	0.02	No	Yes
Dping world	0.70	0.35	0.50	0.33	0.11	0.07	0.07	Yes	Yes

Source: PovcalNet, 1990 & 2015. Variables are defined as in Table 3.

Table A.5: Change in P_0 by robustness status. 2015 vs. 1990. Pair of lines 6.

Evolution of P_0 by robustness status	No.	%
Partially robust		
P_0 increases	4	3
P_0 decreases (less than halved)	3	2
Fully robust		
P_0 increases	18	15
P_0 decreases (less than halved)	62	52
P_0 decreases (at least halved)	33	28
Total	120	100

Source: PovcalNet, 1990 & 2015. Table includes all countries in the sample.

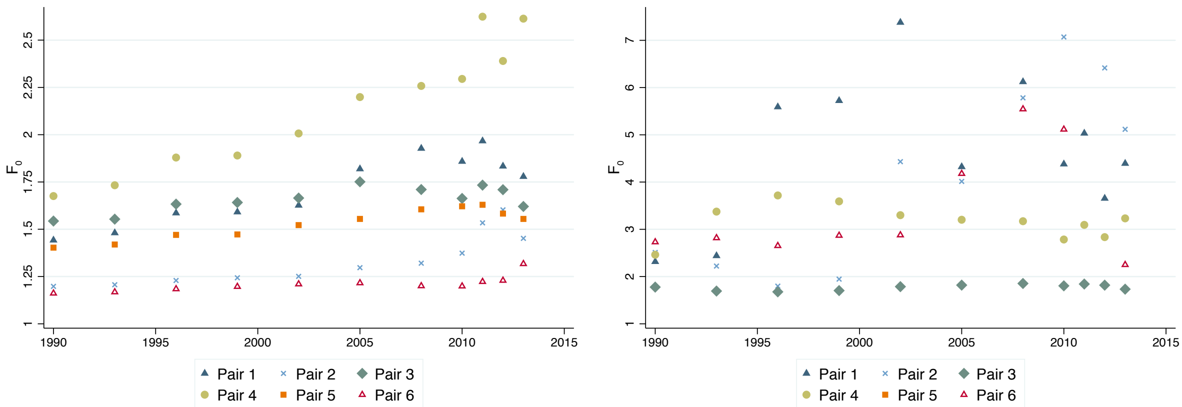
Table A.6: F_0 and F_1 by year. 1990-2013.

Reference year	F_0 All	F_0 $z_a < z_r$	F_1 All	F_1 $z_a < z_r$
1990	1.44	2.31	2.15	5.63
1993	1.48	2.44	2.23	6.47
1996	1.59	5.59	2.40	16.75
1999	1.59	5.72	2.42	17.26
2002	1.63	7.38	2.46	23.26
2005	1.82	4.32	2.74	12.71
2008	1.93	6.12	2.93	19.58
2010	1.86	4.38	2.73	13.66
2011	1.97	5.03	2.86	16.07
2012	1.83	3.65	2.45	10.62
2013	1.78	4.39	2.16	11.71

Source: PovcalNet, 1990 & 2015. F_0 and F_1 are defined as in equation (8) with $\lambda = 0$ and $\lambda = 1$ respectively.

Figure A.3: F_0 by pair of lines. 1990-2015.

(a) All countries **(b)** Countries with $z_a > z_r$



Source: PovcalNet, 1990 & 2015. F_0 is defined as in equation (8) with $\lambda = 0$.