The Role of Inequality in Poverty Measurement

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Introduction

- **Poverty** measurement pre-Sen (1976)
  - Setting poverty **line**, identifying poor
- Sen shifted focus to **aggregation** and **axioms**
- Proposed **transfer** axiom for poverty
  - Suggesting role for **inequality** in poverty measurement
  - Measure had some drawbacks
    - Unintuitive
    - Not decomposable
- Foster-Greer-Thorbecke (1984) proposed P_α (FGT) class
  - Remedied shortcomings

Introduction

Main members of class

P_0 – **Headcount** ratio (incidence)

P_1 – Poverty **gap** (depth)

P_2 – **Squared** gap (severity)

Note

P_2 satisfies **transfer** axiom

Has a place for **inequality** in poverty

Applications

World Bank PRSPs

Virtually all studies of **international poverty**

Outline

Role for **inequality**?

- Formalize via **dimensional transfer** axiom
- Expand to **M-gamma** class

Impossibility:

- Lose dimensional **breakdown**

Q: Regain using **Shapley** value?

- A: Not in general. Heirarchical weights problem; intuition.
- B: In one case, simple **formula** for Shapley breakdown; But interpretable only alongside adjusted headcount ratio; not self-standing.

Conclusion:

A. More important to end poverty than end inequality among the poor

B. Insights from inequality already present in standard MPI reports – e.g. Distribution of poor persons across 'bands' of intensity

C. Shapley is not a replacement for dimensional breakdown; can supplement

D. Nice resonance of new family: H, MPI, M-gamma2

OPI

Adjusted Headcount Ratio

Properties

Invariance Properties: **Ordinality**, Symmetry, Replication Invariance, Deprivation Focus, Poverty Focus

Dominance Properties: Weak Monotonicity, **Dimensional Monotonicity**, Weak Rearrangement, Weak Transfer

Subgroup Properties: Subgroup Consistency, Subgroup Decomposability, **Dimensional Breakdown**

Digression

Definitions of Ordinality and Dimensional Breakdown

Ordinality

Definition An *equivalent representation* rescales all variables and deprivation cutoffs.

Ordinality An equivalent representation leaves poverty unchanged.

Eg Change scale on self reported health from 1,2,3,4,5 to 2,3,5,7,9, and poverty level should be unchanged

Note

- Measure violates if relies on scale or normalized gaps
- M_0 satisfies

Dimensional Breakdown

Dimensional Breakdown after identification has taken place and the poverty status of each person has been fixed, multidimensional poverty can be expressed as a weighted sum of dimensional components.

Note

Component function for j depends only on dimension j data

M_0 satisfies, since $M_0 = \Sigma_j w_j H_j$

where H_j is the % of population both poor and deprived in j

M_0 is weighted average of censored headcount ratios H_j

Why useful?

Example

Dimensional Breakdown of MPI Cameroon

Indicator	Censored Headcount Ratio H_j	Dimensional Breakdown $w_j H_j$	Relative Contribution $w_j H_j / M_0$
Years of Schooling	16.7	2.8	11.2%
School Attendance	18.4	3.1	12.4%
Child Mortality	27.4	4.6	18.4%
Nutrition	18.3	3.1	12.3%
Electricity	37.3	2.1	8.4%
Sanitation	34.7	1.9	7.8%
Water	28.9	1.6	6.5%
Flooring	34.5	1.9	7.7%
Fuel	45.5	2.5	10.2%
Assets	23	1.3	5.2%
M_0	**24.8**	**24.8**	**100.0%**

Inequality in Poverty

Critique

- M_0 **not sensitive to distribution among the poor**

Axioms?

- Some only for **cardinal**
- Others **weak inequality**: $\leq$ and not $<$. M_0 satisfies!

Questions addressed here

- Formulate **strict** axiom?
- **Construct** practical measures satisfying this and other key properties?

New Property

Recall property in Alkire-Foster (2011)

Dimensional Monotonicity Multidimensional poverty should rise whenever a poor person becomes deprived in an additional dimension

New property

Dimensional Transfer Multidimensional poverty should fall as a result of a dimensional rearrangement among the poor

A dimensional rearrangement among the poor An association-decreasing rearrangement among the poor (in achievements) that is simultaneously an association-decreasing rearrangement in deprivations.

Note: Both start poor and stay poor.

New Property

Example with z = (13,12,3,1)

Achievements | Deprivations

$$\begin{bmatrix} 12 & \mathbf{13} & 2 & 1 \\ 10 & \mathbf{7} & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 12 & \mathbf{7} & 2 & 1 \\ 10 & \mathbf{13} & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & \mathbf{0} & 1 & 0 \\ 1 & \mathbf{1} & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \mathbf{1} & 1 & 0 \\ 1 & \mathbf{0} & 1 & 1 \end{bmatrix}$$

Dominance | No dominance | Dominance | No dominance

Dimensional Transfer implies poverty will **fall**

Question: Are there measures satisfying **DT**?

Note: Adjusted Headcount M_0

Just *violates* Dimensional Transfer

Deprivation score **unchanged**

M-Gamma Class M_0^γ

Identification: **Dual cutoff**

Aggregation:

$$M_0^\gamma = \mu(c^\gamma(k)) \text{ for } \gamma \geq 0$$

where $c_i^\gamma(k)$ is the censored deprivation score for person i raised to the γ power

Cf for union: Chakravarty and D'Ambrosio (2006), Jayaraj and Subramanian (2010), Rippin (2012)

Note: based on "normalized attainment gap"

$c_i^\gamma(\text{k}) = (\frac{d-a_i}{d})^\gamma$ for poor i

$c_i^\gamma(k) = 0$ for nonpoor i

where a_i is person i's attainment score

M-Gamma Class M_0^γ

Main measures

$\gamma = 0$ headcount ratio $M_0^0 = H$

$\gamma = 1$ adjusted headcount ratio $M_0^1 = M_0$

$\gamma = 2$ squared count measure M_0^2

Note: Multidimensional analog to P-alpha

Dimensional Transfer **satisfied** for $\gamma > 1$ ✔

But Dimensional Breakdown **violated** for $\gamma > 1$ ✖

Question: Any other measures satisfy **both**?

Impossibility

Theorem There is **no** symmetric multidimensional measure satisfying **both** Dimensional Breakdown and Dimensional Transfer

Proof

- Follows Pattanaik et al (2012)
- Idea: DT requires fall in poverty; DB requires unchanged

Conclusion

- **Easy to construct** measure satisfying Dimensional Transfer
- But at a **cost**: lose Dimensional Breakdown

Impossibility

Recall: Importance of Dimensional Breakdown

Coordination of Ministries

Coordinated dashboard of censored headcount ratios

Budget Allocation

Policies to reflect the composition of poverty

Policy Analysis

Composition of poverty across groups, space, and time

Resolution?

1. Limit Dimensional Transfer?

   Already limited ✕

2. Limit Dimensional Breakdown?

   Already limited ✕

Datt (2017) suggests Shapley methods of Shorrocks (2013)

Shapley Breakdown

- **Pros** of Shapley value approach
  - Finds **contribution** of each part to whole
  - Especially useful for **nonlinear** functions: M_0^γ for $\gamma \neq 1$
    - Where order matters
  - Takes **average marginal** contribution
    - Across all orderings or permutations
- **Cons** of Shapley value approach
  - **Tedious** to calculate
  - **Unintuitive** for policymakers
  - Problematic for **hierarchical** indicators

Shapley Breakdown

Single person examples

Pat first and then Jo on next two slides

10 indicators in **3** dimensions as in global MPI

Goal

Calculate **Shapley** contribution of **first** indicator for M_0^γ

Overall poverty is $(c_1(k))^\gamma$ (since there is only one person)

Note

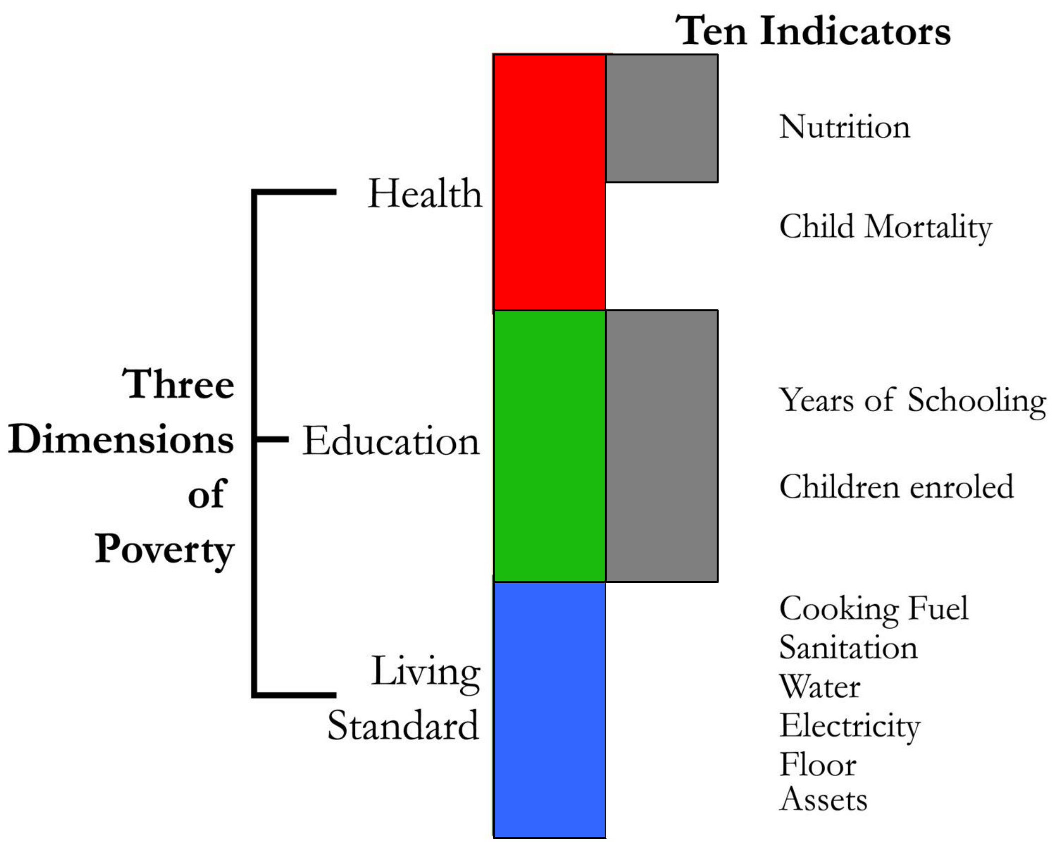
$\gamma = 1$ case easy since $M_0 = c_1(k)$

Shapley breakdown = dimensional breakdown

$\gamma \neq 1$ very **tedious**

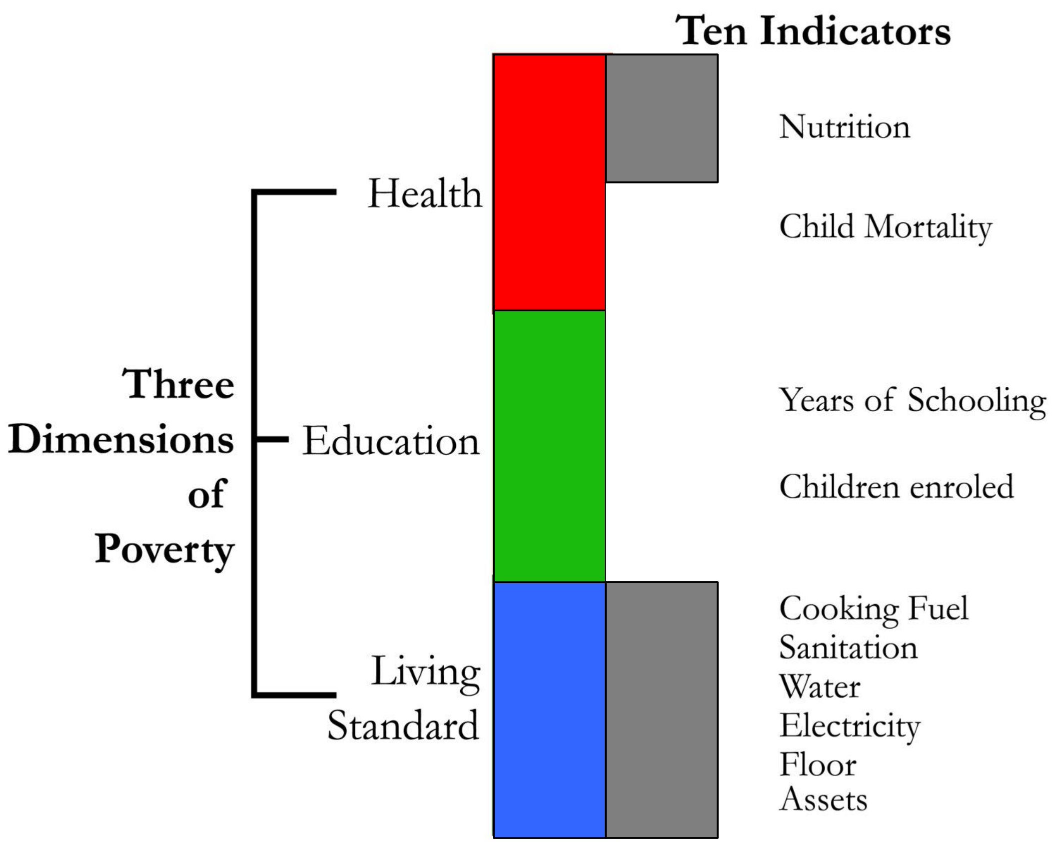
Shapley Breakdown

Example: Pat

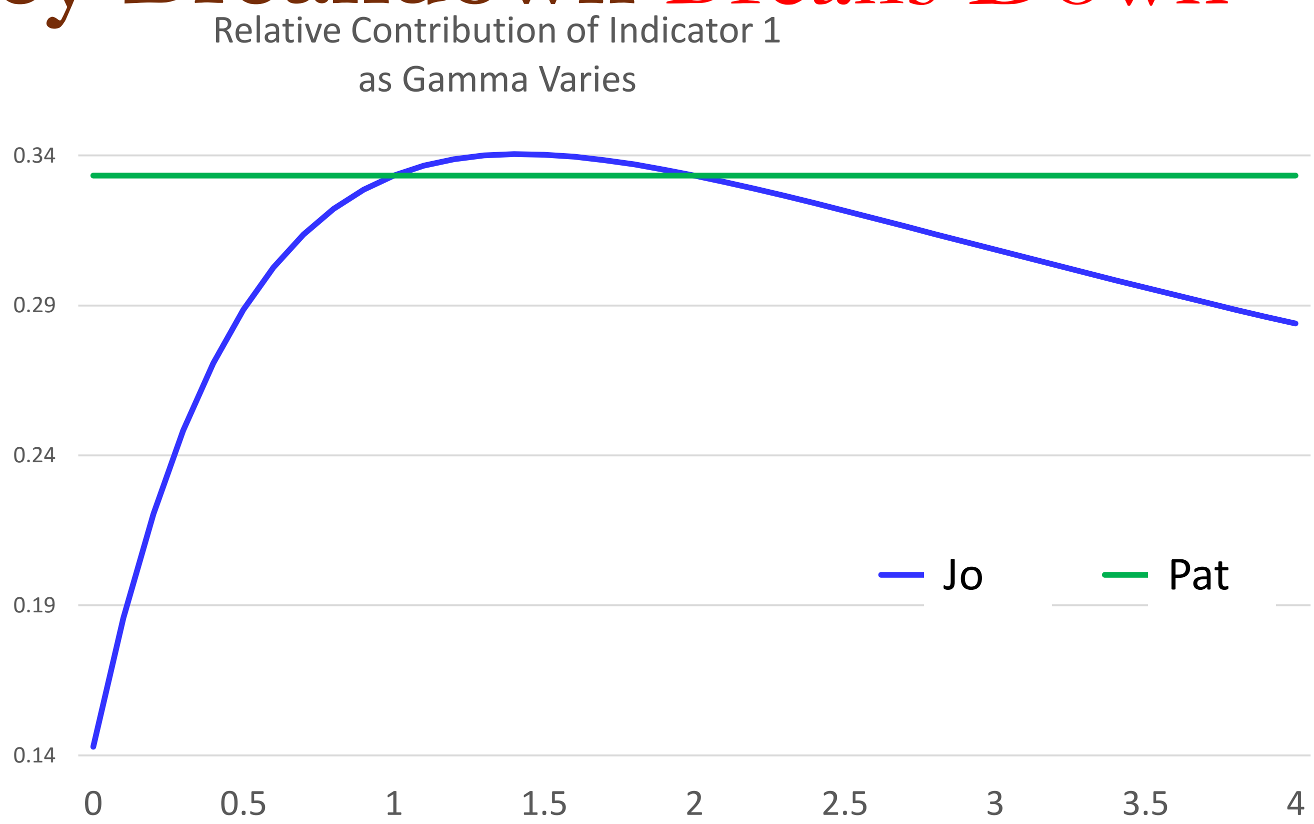


Shapley Breakdown

Example: Jo



Shapley Breakdown Breaks Down



Expect **same** relative contribution of 1 for Jo and Pat

Inconsistency here is due to **hierarchical variables**

Note: ok for $\gamma = 2$ (and for $\gamma = 1$ by Dim Breakdown)

Positive Result for Squared Count

Consider people both **poor** and **deprived** in j

Definitions

Censored headcount ratio H_j = incidence of this group in overall population

Censored intensity A_j = average intensity in this group

Censored adjusted headcount ratio $M_{0j} = H_j A_j$ = average intensity relative to entire population

Theorem The Shapley breakdown for M_0^2 yields the **elementary** formula $M_0^2 = \Sigma_j w_j M_{0j}$

Idea

Multiply each component of M_0's dimensional breakdown by A_j.

Dimensional and Shapley Breakdowns Cameroon

Table V: Breakdowns of M_0^1 and M_0^2

Indicator	Censored Headcount Ratio H_j	Dim Breakdown $w_j H_j$	Censored Intensity A_j	Censored Adjusted Headcount M_{0j}	Shapley Breakdown $w_j M_{0j}$	Relative Contribution $w_j H_j / M_0^1$	Relative Contribution $w_j M_{0j} / M_0^2$	Percentage Point Diff Δ
Schooling	16.7	2.8	66.2	11.1	1.8	11.2%	12.5%	1.3
Attendance	18.4	3.1	66.1	12.2	2.0	12.4%	13.8%	1.4
Child Mortality	27.4	4.6	57.4	15.7	2.6	18.4%	17.8%	-0.6
Nutrition	18.3	3.1	63.9	11.7	1.9	12.3%	13.3%	1.0
Electricity	37.3	2.1	56.4	21.0	1.2	8.4%	7.9%	-0.4
Sanitation	34.7	1.9	53.9	18.7	1.0	7.8%	7.1%	-0.7
Water	28.9	1.6	56.1	16.2	0.9	6.5%	6.1%	-0.4
Flooring	34.5	1.9	56.4	19.5	1.1	7.7%	7.4%	-0.4
Fuel	45.5	2.5	54	24.6	1.4	10.2%	9.3%	-0.9
Assets	23	1.3	57	13.1	0.7	5.2%	5.0%	-0.2
		M_0^1 =24.8			M_0^2 =14.7			

Note similarity of relative contributions

Conclusion

- Defined **Dimensional Transfer**
- Proved it **conflicts** with Dimensional Breakdown
- Derived **simple expression** for Shapley breakdown of squared count measure
- **Recommend** using three main M-gamma measures in tandem analogous to P-alpha measures
  - M_0 (and its dimensional breakdown) as the core measure for analysis
  - H (and A) as intuitive partial measure
  - M_0^2 (and its Shapley breakdown) to evaluate the effects of inequality among the poor