

# Assessing deprivation with an ordinal variable

## Theory and application to sanitation deprivation in Bangladesh

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# Motivation: Why Care?

- ▶ Current global development context on poverty
  - ▶ **UN's SDG Goal 1 target:** Reduce **poverty in all its dimensions** by 2030
  - ▶ **Recommendation 11 of World Bank (2017):** Publish a **portfolio of complementary indicators**
- ▶ Many dimensions are assessed by ordinal variables
  - ▶ Consist of ordered categories
  - ▶ E.g., access to basic facilities of different quality

# Motivation: An Example

## Sanitation Ladder (WHO and UNICEF)

SERVICE LEVEL	DEFINITION
<b>SAFELY MANAGED</b>	Use of improved facilities that are not shared with other households and where excreta are safely disposed of in situ or transported and treated offsite
<b>BASIC</b>	Use of improved facilities that are not shared with other households
<b>LIMITED</b>	Use of improved facilities shared between two or more households
<b>UNIMPROVED</b>	Use of pit latrines without a slab or platform, hanging latrines or bucket latrines
<b>OPEN DEFECTION</b>	Disposal of human faeces in fields, forests, bushes, open bodies of water, beaches or other open spaces, or with solid waste

# Literature

- ▶ **Cardinal variables:** [Sen \(1976\)](#); Thon (1979); Clark et al. (1981); Chakravarty (1983); [Foster et al. \(1984\)](#); Foster and Shorrocks (1988a,b); [Ravallion \(1994\)](#); Shorrocks (1995)
- ▶ **Ordinal variables**
  - ▶ **Inequality and well-being:** Allison and Foster (2004); Apouey (2007); Abul Naga and Yalcin (2008); Zheng (2011); Yalonetzky (2013); Gravel et al. (2015)
  - ▶ **Poverty (unidimensional):** Bennett and Hatzimasoura (2011); Yalonetzky (2012)

# Assessing Poverty with Ordinal Variables

- ▶ Desirable policy relevant requirements
  - ▶ Aggregation (else, too many pair-wise comparisons)
  - ▶ Additive decomposability
  - ▶ Capture **depth of deprivations**
  - ▶ Provide **precedence to poorer** during poverty alleviation
  - ▶ Have **interpretable and meaningful** values
    - ▶ E.g., bounded between 0 and 1

# Framework

- ▶  $S (\geq 2)$ : Number of ordered categories
  - ▶  $c_1 \succ_D \cdots \succ_D c_S$ :  $\succ_D$  is binary and transitive
  - ▶  $c_S$ : least poor category;  $c_1$ : poorest category
- ▶  $p_s$ : Proportion of population experiencing category  $c_s$ 
  - ▶  $p_s \geq 0$  for all  $s$  and  $\sum_{s=1}^S p_s = 1$
  - ▶  $\mathbf{p} = (p_1, \dots, p_S)$ : relative frequency distribution
  - ▶  $\mathbb{P}$ : set of all distributions over  $S$  categories
- ▶ Poverty cut-off category:  $c_k$  for  $k < S$ 
  - ▶ Poor categories:  $(p_1, \dots, p_k)$
  - ▶ Non-poor categories:  $(p_{k+1}, \dots, p_S)$

# Depth Sensitive Ordinal Poverty Measures

- ▶ The class of poverty measures we propose is

$$P(\mathbf{p}; c_k) = \sum_{s=1}^S p_s \omega_s, \text{ where:}$$

- ▶  $\omega_1 = 1$
- ▶  $\omega_{s-1} > \omega_s > 0 \forall s = 2, \dots, k$  whenever  $k \geq 2$
- ▶  $\omega_s = 0 \forall s > k$ .

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- ▶ Policy relevant features
    - 1 Additively decomposable
    - 2 Normalised between 0 and 1
    - 3 Sensitive to depth of deprivation whenever  $k \geq 2$
    - 4 For  $k = 1$ ,  $P(\mathbf{p}; c_1) = p_1$  (headcount ratio) as  $\omega_1 = 1$



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  - ▶ We provide an axiomatic characterization using four assumptions (or axioms)

# An Example: Rank-based Weights

## Sanitation Ladder

Service Level	$p$	$\omega$	Rank-based weight
Open Defecation (poorest)	$p_1$	$\omega_1 = 1$	4/4
Unimproved	$p_2$	$\omega_2 (< \omega_1)$	3/4
Limited	$p_3$	$\omega_3 (< \omega_2)$	2/4
Basic	$p_4$	$\omega_4 (< \omega_3, > 0)$	1/4
Safely Managed (non-poor)	$p_5$	$\omega_5 = 0$	0

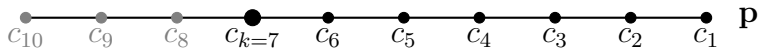
$$\blacktriangleright P(\mathbf{p}; c_k) = p_1 + \omega_2 p_2 + \omega_3 p_3 + \omega_4 p_4 = p_1 + \frac{3p_2}{4} + \frac{2p_3}{4} + \frac{p_4}{4}$$

$\blacktriangleright$  Bennett and Hatzimasoura (2011)

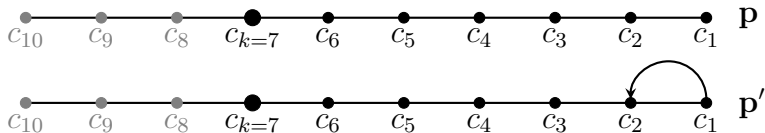
# Providing Precedence to Poorer

- ▶ How to make sure that poorer are given precedence over the less poor during poverty alleviation?
  - ▶ *Prioritarian view*: benefiting people matters more the worse off these people are (Parfit, 1997, p. 213; Esposito and Lambert, 2011)
  - ▶ *Egalitarian view*: transfer properties (Sen, 1976; Foster et al., 1984; Zheng, 1997)
  - ▶ A *prioritarian* will always find some *egalitarians* on her side (Fleurbaey, 2015)
- ▶ We operationalize the prioritarian point of view through the concept of *precedence to the poorer*

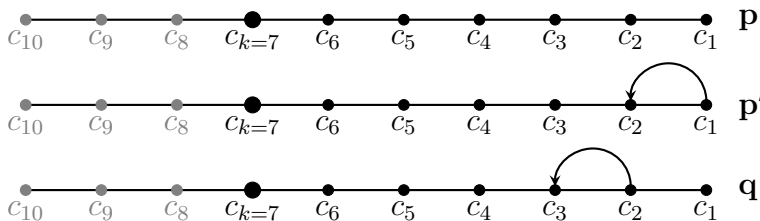
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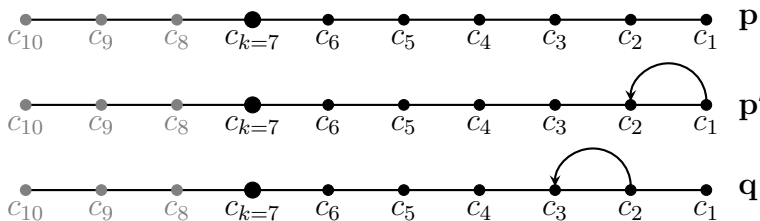
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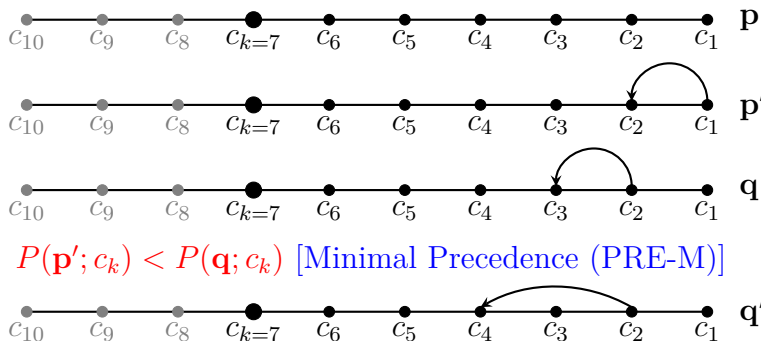


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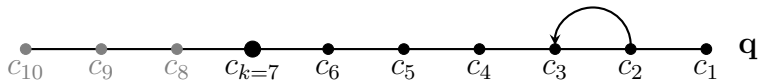
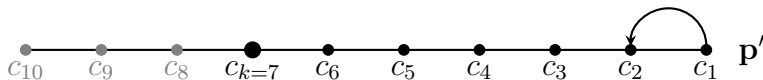
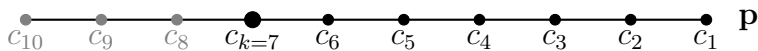
$P(p'; c_k) < P(q; c_k)$  [Minimal Precedence (PRE-M)]

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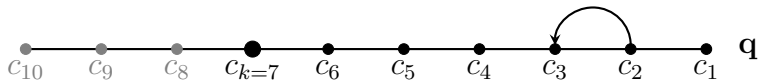
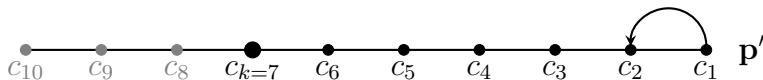
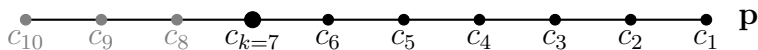


$P(\mathbf{p}'; c_k) < P(\mathbf{q}; c_k)$  [Minimal Precedence (PRE-M)]



$P(\mathbf{p}'; c_k) < P(\mathbf{q}'; c_k)$  [Order-2 Precedence (PRE-2)]

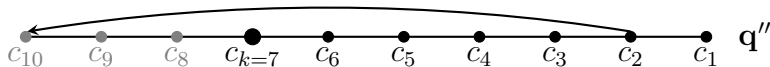
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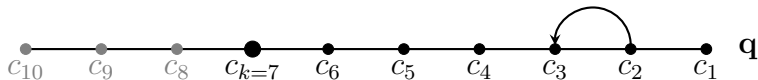
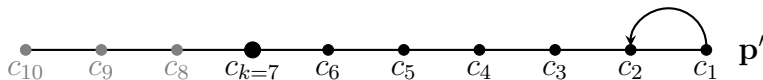
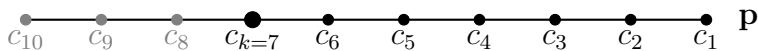
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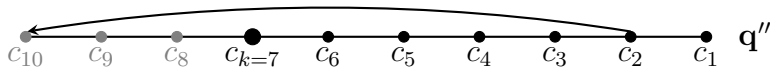
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$P(\mathbf{p}'; c_k) < P(\mathbf{q}; c_k)$  [Minimal Precedence (PRE-M)]



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$P(\mathbf{p}'; c_k) < P(\mathbf{q}''; c_k)$  [Greatest Precedence (PRE-G)]

# Minimal and Greatest Precedence to Poorer

- ▶ For any  $k \geq 2$ , our ordinal poverty measures additionally satisfy PRE-M (or PRE-1) if and only if
  - ▶  $\omega_{s-1} - \omega_s > \omega_s - \omega_{s+1}$  for all  $s = 2, \dots, k-1$  and  $\omega_{k-1} > 2\omega_k$ .
  - ▶ Analogous to cardinal Pigou-Dalton transfer
- ▶ For any  $k \geq 2$ , our poverty measures additionally satisfy PRE-G if and only if
  - ▶  $\omega_{s-1} > 2\omega_s$  for all  $s = 2, \dots, k$ .
  - ▶ Analogous to Hammond transfer (Hammond, 1976; Gravel et al., 2015)

# Providing Precedence: An Example

## Sanitation ladder

Service Level	$p$	$\omega^1$	$\omega^2$
Open Defecation (poorest)	$p_1$	1	1
Unimproved	$p_2$	$0.75^2$	0.4
Limited	$p_3$	$0.5^2$	0.15
Basic	$p_4$	$0.25^2$	0.05
Safely Managed (non-poor)	$p_5$	0	0

- ▶  $\omega^1$ : Weights are compatible with PRE-M, but not with PRE-G
- ▶  $\omega^2$ : Weights are compatible with PRE-G

# Robustness of Poverty Comparisons

- ▶ Main parameters for our class of measures are
  - ▶ The set of ordering weights  $\{\omega_1, \dots, \omega_k\}$
  - ▶ The poverty threshold category  $c_k$
- ▶ Under which circumstances are poverty comparisons robust to alternative **ordering weights** as well as alternative **poverty threshold categories**?

# Dominance Conditions for all Measures in $\mathcal{P}$

- ▶ For any  $\mathbf{p}, \mathbf{q} \in \mathbb{P}$  and for some  $k \geq 1$ ,  
 $P(\mathbf{p}, c_k) < P(\mathbf{q}, c_k) \forall P \in \mathcal{P}$  for a given  $c_k \in \mathbf{C}_{-s}$  if and only if  $\sum_{\ell=1}^s (p_\ell - q_\ell) \leq 0 \forall s \leq k$  with at least one strict inequality.
- ▶ For any  $\mathbf{p}, \mathbf{q} \in \mathbb{P}$  and for some  $k \geq 1$ ,  
 $P(\mathbf{p}, c_k) < P(\mathbf{q}, c_k)$  for any  $P \in \mathcal{P}$  and for all  $c_k \in \mathbf{C}_{-s}$  if and only if  $\sum_{\ell=1}^s (p_\ell - q_\ell) \leq 0 \forall s = 2, \dots, S-1$  and  $p_1 < q_1$ .

# Dominance Conditions for all Measures in $\mathcal{P}_\alpha$

The ordering weighting vectors for conducting dominance tests for different  $\alpha$ 's when  $S = 6$  and  $k = 5$

	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$
$\omega^1$	$(1, 0, 0, 0, 0, 0)$	$(1, 0, 0, 0, 0, 0)$	$(1, 0, 0, 0, 0, 0)$	$(1, 0, 0, 0, 0, 0)$
$\omega^2$	$(1, \frac{1}{2}, 0, 0, 0, 0)$	$(1, \frac{1}{2}, 0, 0, 0, 0)$	$(1, \frac{1}{2}, 0, 0, 0, 0)$	$(1, \frac{1}{2}, 0, 0, 0, 0)$
$\omega^3$	$(1, \frac{2}{3}, \frac{1}{3}, 0, 0, 0);$ $\bar{r} = 1$	$(1, \frac{1}{2}, \frac{1}{4}, 0, 0, 0)$	$(1, \frac{1}{2}, \frac{1}{4}, 0, 0, 0)$	$(1, \frac{1}{2}, \frac{1}{4}, 0, 0, 0)$
$\omega^4$	$(1, \frac{3}{4}, \frac{2}{4}, \frac{1}{4}, 0, 0);$ $\bar{r} = 1$	$(1, \frac{4}{7}, \frac{2}{7}, \frac{1}{7}, 0, 0);$ $\bar{r} = 1$	$(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, 0)$	$(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, 0)$
$\omega^5$	$(1, \frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}, 0);$ $\bar{r} = 2$	$(1, \frac{7}{12}, \frac{4}{12}, \frac{2}{12}, \frac{1}{12}, 0);$ $\bar{r} = 1$	$(1, \frac{8}{15}, \frac{4}{15}, \frac{2}{15}, \frac{1}{15}, 0);$ $\bar{r} = 1$	$(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, 0)$

Analogous to Foster et al. (2009), Seth and McGillivray (2018)



# Sanitation Deprivation in Bangladesh

## ► Motivation

- SDG6: [to] “achieve access to adequate ... sanitation and hygiene for all and end open defecation” by 2030
- Joint Monitoring Programme of WHO and UNICEF proposes a [service ladder approach](#) to benchmark and track progress [building on the existing datasets](#)
- We use [existing](#) Demographic Health Survey (DHS) datasets for years 2007, 2011 and 2014
  - Nationally representative household surveys
  - Samples are collected by two-stage stratified sampling design

# Comparison across Four Poverty Measures

- ▶ We use four different measures
  - ▶ Headcount ratio and three other measures with different weights
- ▶ Three different sets of weights
  - ▶  $\omega^1 = (1, 0.75, 0.5, 0.25, 0)$ : Not sensitive to precedence
  - ▶  $\omega^2 = (1, 0.75^2, 0.5^2, 0.25^2, 0)$ : Satisfies PRE-M but not PRE-G
  - ▶  $\omega^3 = (1, 0.4, 0.15, 0.05, 0)$ : Satisfies PRE-G

# Comparison across Four Poverty Measures

Region	$H$			$P_1$			$P_2$			$P_3$		
	2007	2011	2014	2007	2011	2014	2007	2011	2014	2007	2011	2014
Barisal	66.1 (2.7)	60.5 (2.1)	46.8 (3.3)	47.7 (1.9)	44.0 (1.7)	32.6 (2.9)	35.0 (1.4)	32.6 (1.4)	23.4 (2.4)	25.2 (1.1)	23.5 (1.1)	16.7 (1.8)
Chittagong	67.1 (2.9)	59.2 (2.1)	44.9 (3.1)	47.0 (2.8)	38.9 (1.7)	29.2 (2.9)	34.8 (2.7)	27.3 (1.5)	20.2 (2.7)	26.2 (2.6)	19.6 (1.2)	14.9 (2.5)
Dhaka	<b>74.8</b> (1.9)	<b>68.4</b> (1.9)	56.3 (2.5)	<b>50.1</b> (1.7)	<b>42.6</b> (1.3)	33.5 (1.8)	<b>36.6</b> (1.6)	<b>29.4</b> (1.1)	21.8 (1.6)	<b>27.8</b> (1.4)	<b>21.6</b> (1.0)	15.4 (1.3)
Khulna	69.0 (1.8)	61.4 (1.6)	50.3 (2.3)	48.9 (1.4)	41.8 (1.2)	33.0 (1.7)	35.7 (1.1)	29.4 (1.0)	22.7 (1.4)	25.9 (1.0)	20.9 (0.7)	16.1 (1.0)
Rajshahi	<b>74.0</b> (1.9)	<b>61.2</b> (2.3)	52.0 (2.3)	<b>55.2</b> (1.8)	<b>41.6</b> (1.9)	34.7 (1.8)	<b>43.0</b> (1.9)	<b>29.6</b> (1.6)	24.2 (1.5)	<b>34.1</b> (1.9)	<b>21.6</b> (1.3)	17.6 (1.3)
Sylhet	<b>69.9</b> (2.3)	<b>64.6</b> (1.9)	55.0 (1.7)	<b>50.1</b> (1.9)	<b>47.1</b> (1.6)	37.9 (1.7)	<b>36.8</b> (1.5)	<b>36.2</b> (1.4)	27.9 (1.6)	<b>26.5</b> (1.1)	<b>28.9</b> (1.4)	21.9 (1.4)
Bangladesh	71.5 (1.0)	63.4 (0.9)	52.2 (1.1)	50.4 (0.9)	42.3 (0.7)	33.5 (0.9)	37.5 (0.9)	30.1 (0.6)	23.1 (0.8)	28.5 (0.8)	22.2 (0.5)	16.8 (0.7)

Source: Authors' own computations. Standard errors are reported in parentheses.

# Dominance Tests: Overall, PRE-M and PRE-G

	2007–2011			2011–2014		
	$\mathcal{P}$	$\mathcal{P}_1$	$\mathcal{P}_G$	$\mathcal{P}$	$\mathcal{P}_1$	$\mathcal{P}_G$
Barisal	No	No	No	Yes	Yes	Yes
Chittagong	Yes**	Yes**	Yes**	No	No	No
Dhaka	Yes**	Yes**	Yes**	Yes**	Yes**	Yes**
Khulna	Yes*	Yes*	Yes*	Yes	Yes	Yes
Rajshahi	Yes***	Yes***	Yes***	Yes	Yes	Yes
Sylhet	No	No	No	Yes*	Yes*	Yes*
Bangladesh	Yes***	Yes***	Yes***	Yes*	Yes*	Yes*

Sources: Authors' own computations. A “Yes” implies reduction of the poverty levels within each region over time evaluated at the four extreme weights; a “No” implies otherwise. The levels of statistical significance for the dominance tests are: \*\*\* for 1%, \*\* for 5% and \* for 10%. We implemented a standard intersection-union test (IUT) whose alternative hypothesis is that poverty levels at the four extreme points are jointly lower in year-region “A” (e.g. Bangladesh in 2011) vis-à-vis “B” (e.g. Bangladesh in 2007). We reject the null (i.e. at least one poverty level in “A” is equal or higher than in “B”) in favour of this alternative only if every poverty level is lower in “A” than in “B” with a given significance level.

- ▶  $\mathcal{P}_1$ :  $(1, 0, 0, 0, 0)$ ,  $(1, \frac{1}{2}, 0, 0, 0)$ ,  $(1, \frac{2}{3}, \frac{1}{3}, 0, 0)$ ,  $(1, \frac{3}{4}, \frac{2}{4}, \frac{1}{4}, 0)$
- ▶  $\mathcal{P}_G$ :  $(1, 0, 0, 0, 0)$ ,  $(1, \frac{1}{2}, 0, 0, 0)$ ,  $(1, \frac{1}{2}, \frac{1}{4}, 0, 0)$ ,  $(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0)$

# Concluding Remarks & Further Research

- ▶ Potential applications
  - ▶ Monitoring progress of ordinal indicators relevant for Sustainable Development Goals
  - ▶ Evaluating pro-poorness impact of policies on ordinal indicators
  - ▶ Applications to multidimensional poverty

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- ▶ Further research
  - ▶ Develop a measure of the degree of pro-poorness of inter-temporal progress
  - ▶ A cross-country empirical study to devise a more efficient path to achieve SDG goals by 2030