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Value-Added: Why Consistency in Aggregation is Essential for Global Accounting Standards, and How It Is Achieved

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1. Introduction

Value added is the only figure not entered into, but derived from, a system of supply and use tables. In fact, one may consider its derivation as the first, if not primary aim of constructing such tables, all other detail serving to understand and explain its formation and variation. The term "value added" implies a theory of value transmission. As production of new goods is possible only in consumption of existing ones, economic value is conserved by being transferred from products consumed to newly produced goods and services. The process is called productive if "value has been added" to the value of consumed goods, in the sense that the value of new output is higher than the value of old inputs. The equation,

(1) value added = value of outputs – value of inputs,

lies thus at the heart of national accounts, and of supply and use tables, in particular, driving the economic circuit of production and income within a nation.

Defining productivity and value added, in this way, makes aggregation simple. Aggregation is performed by straight summation, and the operation obeys the associative law of algebra: You may first sum all output of enterprises, over all territories, then sum all their intermediate consumption, too, and finally deduce aggregate value added, in the last step, or you may determine all individual value added figures first, and then sum up. These statistical operations are so simple and self-understood that they are hardly ever examined if one is conscious of them, at all. Yet, they are not trivial. One of the tacit premises is that a universal, and homogeneous unit of measurement exists in the economy. The condition is fairly well realised within individual nations; between nations it is obviously violated, as a national economy is almost defined by its ability to install and manage a currency of its own. But even if existence of a general unit of measurement can be ascertained throughout the country, this does not mean that equivalence holds over time. Currencies are subject to monetary policy, and markets, and it is normal they vary with the development of an economy. Hence, comparison over time is being impaired, and this calls for a chapter 15, price and volume measures, in the SNA.

In the chapter, the SNA draws heavily on index number theory, a well established academic discipline, and older than the SNA. In fact it seems, national accountants dare not venture into the area of price measurement themselves for their respect of the experts in the field. But there is a disadvantage to this kind of outsourcing. Index number theory and national accounts are not of the same kin, their relationship may rather be described as being similar to that between

mathematics and physics. Index number theory is pure mathematics. Its proofs and theorems may be applied to any set of variables p_i and q_i joined in a scalar product. As long as they are kept as dimensionless numbers they have no specific area of application. You may give them names, such as "price" or "quantity", but the theorems hold also for any other name you choose. National accounts, in contrast, have a given empirical field which they work, and this is the set of monetary transactions constituting a capitalist economy. An essential element is the national currency, in which transactions are paid, and their value measured, at the same time, in a world where goods and services of economic value constantly change and disappear, and new products are born and grow into their markets.
The problem of inconsistency is severe in an accounting framework, but its solution is actually less complicated than it appears, it being mainly the result of insufficient research, and precocious conclusions. Consistency in aggregation had been no issue in the hundred and some years over which volume measures were calculated by means of the Laspeyres index, as it is secured there, automatically. The shift to modern chaining practice rightly abolished base year dependency, an important weakness of the Laspeyres index, and introduced inconsistency in aggregation, instead. But rather than trying to construct an index formula combining both desirable features, base year independency, and consistency in aggregation, some statistical offices exploiting their monopoly position denied that consistency was even desirable, and set out to "educate" customers and users in the "necessity" of working with inconsistent data. There are more than one switch points on this course of error. They must be identified, and the course re-set from the beginning.

The paper, therefore, starts with a re-statement of the problem, so simple that one immediately finds a solution, too. The switch point here is to employ differential algebra, instead of index number theory (section 2). With this brief mathematical background the first and non-trival step of analysis requires to look into, and to correctly describe, the empirical content of the variables figuring in the known mathematical formulas, and to investigate the grass-root process of statistical price observation, the way in which it selects some specific commodity serving as a price representative for a whole basic heading (section 3), and to clarify some of the terminology that obscures the matter here (section 4). After this conceptual groundwork the problem of aggregation, focus of the paper, is adressed, and solved by what seems to be a reasonable alternative to present practice (section 5). Having presented the theoretical option available it is still a question whether inconsistency, although undesirable, matters in practice as against other statistical errors occuring in national accounts. The question is answered with data from Denmark showing to what extremes inconsistency may grow, and even if not all industries will be equally heterogeneous as the one selected here the mere possibility would ruin credibility of the accounts (section 6). Finally, the paper ventures into an hitherto unknown area by demonstrating how an additive decomposition of growth may be developed into a full accounting picture, this time of Germany, where not only growth in volume, i.e. more production within establishments, but also its distribution through changing market prices may be monitored in a comprehensive and coherent way (section 7). It is inevitable that some traditionally cherished habits will be offended, in the course of the investigation. But it is for a common purpose, and I hope the arguments will convince, in spite.

The paper includes no review of the literature. The arguments discussed here are found in every standard text. An early attempt to formulate their critique is Reich (2001), extending thoughts of Neubauer (1974) and other German authors. For index number theory I rely on the splendid book by Bert Balk (2008). Durand (2008) has made inspiring reflections on the process of statistical price collection as it actually occurs. All versions of the SNA from 1968 onwards have also been consulted, of course. Further literature will be quoted as the argument evolves.

2. A simple point of departure

Let an aggregate of Euro values V be given by the sum of individual value figures v_i . You want to analyse the development of the values of today in respect to some time, t years earlier. So there are two aggregates to compare,

(2) $$V(t) = \sum_i v_i(t) \; [€t] ,$$

and

(3) $$V(0) = \sum_i v_i(0) \; [€0] .$$

€t represents Euros of year t, and €0 those of year 0. In the comparison you want to distinguish between a development in volumes $q_i(t)$, on the one hand, and in prices $p_i(t)$, on the other hand, within the aggregate, both indices being given on the level of the individual values v_i , but not yet defined for the aggregate. Such decomposition is achieved by writing

(4) $$V(t) = \sum_i p_i(t)\; q_i(t)\; v_i(0) \; [€t] ,$$

where $p_i(0) = q_i(0) = 1$. The aggregate V is now a sum of mathematical products which cannot be transformed into a mere sum of additive elements, by any algebraic operation. This is the impossibility theorem of index number theory standing in the way of finding "the wholy grail of the order" (Balk 2008). Two roads are open for separating price and volume on the macro level, one is logarithmation, the other is differentiation. The first road is that of traditional index number theory; it has been successfull in constructing indices for surveying the development of commodity prices, but in its application to an additive system like economic accounts it fails to produce convincing results. So I take the other road instead. Differentiation of equation 4 yields directly an additive decomposition of the muliplicative relationship:

(5) $$dV = \sum_i \left[q(t)\, dp + p(t)\, dq \right] v_i(0) \text{ [€t] .}$$

The aggregate differential movement dV has now been additively decomposed into two aggregate, weighted movements of indexes $p_i(t)$ and $q_i(t)$, in a consistent way. But statistical observation does not deliver continuous data, so you need a discrete approximation, for which there exist two well-known alternatives, either the Laspeyres form for volumes, and the Paasche form for prices, or the other way around, the ideal form being given by their average. Choosing the simple Laspeyres form for the volume index may be sufficient, for practical purposes, and the easiest one to communicate, namely,

(6) $$\Delta V^t = \sum_i (q_i^t \, \Delta p_i^t + p_i^{t-1} \Delta q_i^t)\, v_i^0 \quad \text{[€t]}$$

where $\Delta V^t = V(t) - V(t-1)$, and similarly for the other variables. The corresponding integral over the whole time period (0 , t) is then approximated by

(7) $$V_0^t = \sum_{s=1}^{t} \sum_i q_i^s \, \Delta p_i^s \, v_i^0 + \sum_{s=1}^{t} \sum_i p_i^{s-1} \, \Delta q_i^s \, v_i^0 \text{ [€t].}$$

Growth of V(t) over the time period (0, t) is now additively decomposed in two aggregates, the one describing movement of prices, movement of volumes the other, in the aggregate, in a consistent way. The decomposition is so straight-forward it requires no index number axiom for justification. (You may call it the additive variant of a Divisia-index if you want to place it there.) But still it entails more than one problems, which I will address in the following, in their logical order.

3. " Quantity" index: a misplaced concept

As said before, national accounts did not create their own method of dealing with price changes, but went to borrow from another discipline, namely price statistics, under the implicit assumption that if you get your prices right you get your volumes right, as well. Index number theory divides the economic world, (similar to its sister of economic price theory) in two classes of variables, prices *p* and quantities *q* of commodities. It is advisable to re-assure oneself about meaning, and empirical content of these two sets of variables before treating questions of formal matters such as aggregation, because it seems that there is a severe discrepancy, in concept, between index number theory, and its underlying economic theory, on the one side, and statistical practice, especially national accounts, on the other, a discrepancy which, going largely unnoticed, represents a continuous source of error.

In the theory of economic value, a commodity is defined by a specification of all its physical properties, the date at which it will be available, and the location at which it will be available. Thus a good at a certain date and the same good at a later date are different economic objects. Also a good at a certain location and the same good at another location are different economic objects. (Debreu p. 30) Actually, textbook definitions do not go to this extreme. It is accepted that one and the same commodity exists over a certain span of time, and within a certain region, but all other physical properties are assumed to be identical for every piece of a certain commodity. The value *v* of a transaction may then be defined as the mathematical product of the quantitity *q* of the commodity transferred and its price *p* expressed as Euros/physical unit of the commodity. *q*, according to equation 8,

(8) $$v \equiv p \times q.$$

In spite of some relaxation concerning space and time, the idea ruling the concept is still that of full homogeneity in physical terms, while the number of such homogeneous commodities is of no concern to the theory, only that it must be real[1] and finite (Debreu, p. 32, Balk 2006, p. 4). In essence, this reflects a bottom-up approach from a single homogeneous commodity to the overall complex of an economy, which consists, in this view, of an (arbitrarily) large number of homogeneous commodities, each with its specific price, and heterogeneity is defined as existing only between, not within commodities. In case of doubt, divide the class and make two small classes out of it. All textbooks of economics work with this understanding of physical homogeneity, be it explicitely stated, or implicitely assumed.

[1] One does not quite see the purpose of the restriction to real numbers. Actually, the mathematics developed here might be extended to complex numbers with equal utility, it seems. The restriction to homogeneity, on the other hand, is expressed so distinctly ("It is assumed that there are no new or disappearing commodities,… that comodities do not exhibit quality change, or that quality change has been accounted for by making appropriate adjustments to the prices or quantities." Balk 2008, p.4) that one wonders from where to take the courage to apply so restricted a theory to a world existing but of inhomogeneous and constantly changing products. The adjustments ought to be adressed within the theory, filling its variables with empirical content, rather than be put outside as something irrelevant.

The empirical reality of national accounts statistics, however, does not follows this road, because it is impossible to do so, in practice. There is a natural limit to the number of classes you can have in a commodity classification, it may be some hundreds in certain cases, some thousands in others. The limit is set not only by existing capacities of processing data, but also by the number of observations allowed in an actual survey. You cannot ask an enterprise for the price of every single product it sells, and even less about the corresponding quantity. And the smaller a commodity class is being defined the less likely it becomes to observe a member of it, at a certain location, at a certain time, regularly. Commodity classes defined in actual statistics are inhomogeneous, from the outset, and it is well known that even in the smallest class a lot of different commodities are being assembled. The misunderstanding comes from the fact that homogeneity does play a role in price statistics, but not in the way, theory assumes.

Homogeneity is sought, and employed, in a different way, and these two ways are easily confounded (Reich 1998). Instead of collecting all prices (and quantities) of commodities existing within a certain elementary class ("basic heading"), which is impossible, a single specific homogeneous commodity is selected from within the class. This one commodity is then assigned the task of representing of the whole class, and all commodities in the class, alone. Its price, and only it, stands for all prices in the class, and it defines the "price level" of the class as a whole. If its price rises prices of all other products in the class are deemed to rise at the same time, and by the same percentage, even if those are counted in different physical units (kg, pieces, etc.). If the price of any other commodity rises, in contrast, this is counted as an increase in quality, and thus of volume of the class. The concept of quantity has no meaning in this method of empirical observation . If the class of dairy products includes butter, as well as milk, a quantity measure of it is impossible to define. Still worse, not even the quantity of the representative product itself is observable, or in any way possible to determine, a fact that was known to, and troubled Laspeyres already (Balk 2008, p. 7), but has been forgotten. The theoretical model of supply and demand working between price and quantity of a homogeneous commodity is impossible to observe in practice, because the quantity belonging to a specific homogeneous commodity price is never known, in general. It is a deplorable matter that this elementary fact of empirical observation has not yet found its way into the theory of national accounts. Its recognition would clear up much controversy.

Actually, the SNA does not speak of “quantity” any more, employing the term “volume”, instead, for values which have been subjected to a price index adjustment. In its chapter 15 about “Price and Volume Measures” it reads: “A volume index is an average of the proportionate changes in the quantities of a specified set of goods or services between two periods of time. The quantities compared over time must be those for homogeneous items and the resulting quantity changes for different goods and services must be weighted by their economic importance, as measured by their relative values in one or other, or both, periods. For this reason volume is a more correct and apppropriate term than quantity in order to emphasize that quantities must be adjusted to reflect changes in quality” (SNA2008 para. 15.13) I cannot help but find that paragraph somewhat cryptic. First it defines volume as an average of quantities of homogeneous items, only to withdraw, and to include quality, at its end.

All these matters of empirical reality must be known, and accepted before one applies, and discusses any formal mathematical treatment of the data constructed in this manner, even if historically, - that is true, - techniques of price observation and corresponding analysis grew out of the naïve “quantity” concept. So summing up, empirical reality, rather than using equation 8, appears to be better reflected by defining a “volume” q of a commodity class by means of equation 9, namely,

$$q \equiv \frac{\sum_{i=1}^{n} v_i}{p_k}, \quad 1 \le k \le n \tag{9}$$

where p_k is the price index of some typical, homogeneous item selected from within the class for representing the heterogeneous class as a whole, and to which the corresponding quantity itself is unknown. Equation 8 takes price and quantity of a homogeneous product as given, and derives the value of a corresponding single transaction. This is the microeconomic concept and appproach. Equation 9 takes the value sum of all transactions in similar, but heterogeneous products, within a certain class as given, and derives volume from the two other data, and this is not a physical quantity, at all. Equation 9 expresses the empirically correct macroeconomic concept of variables on which are based the national accounts. The bottom-up approach being illusory in practice, you cannot but work with a top-down approach where you set up a good number of classes into which you divide all transactions expressed in monetary units, not in quantities, knowing that these classes are far from being homogeneous in their different quantitative measures, but belong together as some kind of "macro-commodity" (Durand), being distributed on the same market, perhaps.

4. "Volume index" and "real value": a new and promising distinction

In an astute analysis of "Uniqueness of the Numeraire and Consistent Valuation in Accounting for Real Values", René Durand carries the interpretation of existing statistical practice one step further. Completing equation 9 with corresponding units of measurement yields equation 10

$$(10) \qquad q = \frac{\sum_{i=1}^{n} v_i [Euros]}{p_k [Euros/kg]} = q[kg], \quad 1 \le k \le n,$$

which says that if you divide the price of the representative commodity into the value of aggregated transactions you get a measure in physical units of that particular commodity; it says what quantity of the representative commodity equals the value of the total class. You have thus come from nominal to what Durand calls "real value" of the transactions class. The interpretation corresponds to traditional price theory where an arbitrarily selected commodity ("commodity 1") is assigned the role of unit of measurement, given that money and currencies, the normal measures of economic value, are explicitely excluded from the model. Concerning these "real values" (volumes, in terminology of the SNA), Durand makes an interesting observation. As indicated in equation 10, every elementary commodity class carries its own representative, a typical item which does not exist in other classes. Hence volume of one class is measured in kg, of another class in m, of a third class in pieces of their respective standard commodites. And these different measures cannot be compared, so Durand, or even added, as they are based on different numeraires. It is wrong to aggregate them in national accounts, and this is true independent of whatever index number formula you use. These measures are non-additive as "each subaggregate is based on its own internal valuation norm (metric)" (p. 422); they are consistent only within their respective subsets, and they neglect to take into account relative prices changes across the commodity subsets (p. 420). Consistent accounting for "real values" can only be achieved, so Durand, by applying a single universal "numeraire" to all transactions registered in the accounts, such as is being used for measuring the national rate of monetary inflation, the price index of consumption expenditure (or GDP).

Durand's analysis reveals a certain lacuna in index number theory caused by its uni-laterial orientation towerds microeconomics. In this theory, prices are deduced from consumer preferences and production techniques, only. Consequently, other factors cannot be accounted for. In practice, however, prices change not only in response to changes in cosumer preferences and production possibilities, but also because the means of payment, which is the unit of measurement, itself, varies in form of inflation or, rarely, deflation. One wants to distinguish between the two possibilities, in analysing empirical price data, but if each real value is based on its own internal evaluation norm "this is as if there were as many measures of inflation in the economy (and quantity metrics) as there are commodities and subsets of commodities leaving no possibility to decompose price increases into an inflationary component and a relative price change component." (Durand 2004 p. 422)

The concept of "real value" exists in the SNA, but it is applied to a different context from that of Durand: "Many flows in the SNA, such as cash transfers, do not have price and quantity dimensions of their own and cannot, therefore, be decomposed in the same way as flows related to goods and services. While such flows cannot be measured in volume terms they can nevertheless be measured 'in real terms' by deflating their values with price indices in order to measure their real purchasing power over some selected basket of goods and services that serves as the numeraire." (SNA2008, para. 15.181) The SNA definition of "real value" coincides with that of Durand, it is a nominal value divided by a general ("unique") price index. But its field of application is different. The SNA applies real value compilation to non-commodity flows only while Durand wants to see them used for analysing product flows within supply and use tables. In essence, both authors are right. It is informative to adjust pure income flows for the change in purchasing power, as recommended in the SNA, but it also makes sense to apply the same adjustment to commodity flows, as Durand demands, if only to separate relative price change caused by preference changes and production possibilities (what may then be called the "real price change"), from the inflationary price change underlying all prices movements at equal measure. Summing up this discussion, there are three kinds of values employed in national accounts, (a) nominal values, which are the values actually established by the agents of the economy in their mutual interactions (b) imputed real values that adjust absolute prices for the change in purchasing power of the unit of measurement, leaving relative prices intact, and (c) volumes which isolate the change in relative prices as well.

Definition 10 uses an actual price for defining volume. But in statistical practice it is rather a price index that is used, so in order to agree formally with this practice, use definition 11 rather than definition 10, namely

(11) $$q = \frac{1}{p}\frac{v[€]}{v^0[€]},$$

which is equivalent to

(12) $$v = pqv^0[€],$$

where $v = \sum_i v_i$ denotes the total value of a class consisting of many commodities *i*, and likewise for v^0, the corresponding value at some base year. Equation 12 says that, for purpose of analysis, an observed nominal entry of a product transaction *v* in national accounts is decomposed into the (algebraic) product of a price index *p*, and a volume index *q*, both being applied to some base value v^0, and this value is measured in a monetary unit. There are no physical units involved in actual price collection, at all. Indexes *p* and *q* are pure ratios, and dimensionless numbers establishing an explanatory relationship between economic values *v* and v^0.

5. Chaining with consistency-in-aggregation: a mathematical impossibility?

If this has been a somewhat lengthy discussion of what the variables *p* and *q* stand for in economic statistics this may be justified as follows. You will not know what energy is, in physics, if you don't know how to measure it in actual experiments. Similarly, you will not know what volume is, or real value, if you cannot connect to the empirical reality of price observation and national accounting. So if scrupulous investigation of empirical reality leads to a threefold distinction of concepts, namely imputed "real values", and "volumes", derived from actual nominal values, it has provided a solid ground for tackling the problem of consistency in aggregation, the main topic of the paper.

Consistency in aggregation means that the order in which steps of aggregation are taken from some lower to a higher level, in national accounts, is irrelevant for the result. The law of associativity, holding for the arithmetic operation of adding, in general, holds here, too. When determining value added of an economy, for example, it makes no difference whether you first find value added for each industry, and then sum the individual balances, or whether you first add all industry output as well as all intermediate consumption, across industries, and then derive the national balance, in the last step. This sort of consistency was self-understood in national accounting, over the first hundred years of its existance, and is now discarded, silently, although it is hard to see how a general public may remain convinced that published figures are result of accounting, rather than modelling, if this feature of consistency within the accounts is missing. The “national accountants’ intuitive resistance to the non-additivity” (Durand) may be caused by this doubt.

Indeed, most national accountants do not work with non-additive figures because they like it, but because they are taught there is no other way to separate price changes from volume changes, in the accounts, more precisely, there is no other way if you want to use chain indexes, replacing old-fashioned Laspeyres indexes. The replacement is sensible as it minimises the influence of some (arbitrarily chosen) base year, on the data. But it must be paid for by loss in consistency, so the dogm.

“An aggregate is defined as the sum of its components. Additivity in a national accounts context requires this identity to be preserved for a volume series.Although desirable from an accounting viewpoint, additivity is actually a very restrictive property. Laspeyres volume indices are the only index number formulae considered here that are additive.” (SNA2008, para. 15.58). This passage, in its naivity, deserves a comment. It is true, additivity is a restrictive property, but is this undesirable? Yes, it is if you work in index number theory where you aim at measuring prices which are not additive, by their very nature. No, if you work in national accounts where “consistency in aggregation,” as it is then called is the raison d’ être of the whole system. If that consistency is ignored you transform an accounting system into a social indicator system where figures of different, heterogeneous variables are compiled side by side without that strict and rigid relationship among each other that qualifies the national accounts. Admitting logical inconsistency destroys any accounting paradigm, in the long run.

And it is possible to avoid. I challenge the dogm that inconsistency-in-aggregation is a necessary and inevitable consequence of chaining. Counterproposals have beein advanced early, but they were dismissed before serious investigation. Hillinger’s dictum that if you don’t have an additive index you must construct one is still valid, and forms the basis of the following exposition. And there is no index number theory needed to support it, just a little differential algebra, and some common sense.

Let equation 11 be the definition of volume index *q,* given a price index *p*, and two observed nominal values *v* and v^0. All these variables are functions of time . Equation 12 may then be written

(13) $$v(t) = p(t)q(t)v(0)\ [€(t)]\ .$$

The nominal value at a time t is conceived as the product of two factors, a price index and a volume index applied to the corresponding nominal value of some reference year 0. The Euros are those of year t, the year of observation. This unit of measurement is not constant over time, but varies in response to economic conditions, and monetary policy. The first adjustment required is thus the one advocated by Durand for product transactions, and by the SNA for income transactions, namely to go from nominal terms over to real terms, accounting, in this way, for variation in purchasing power of the national currency as unit of measurement.

Most countries use the index of private consumption expenditure for the purpose, which is in accordance with economic theory in that economic value is determined, there, by the coincidence of social preferences, carried by households, (utility functions) and technical options available to industries (production functions). Neither for capital formation, nor for net exports, the other components of GDP, you can derive the meaning of economic value in this coherent way. So let P(t) be the price level of the economy at time t relative to time 0. Real value $u(t)$, - *u* for utility or use value, i.e. value measured not in money, but in goods and services (of private consumption expenditure) - corresponding to nominal value $v(t)$ may then be defined by

(14) $$u(t) = \frac{1}{P(t)} v(t) = r(t)q(t)v(0)[€0]$$

where $r(t) = \frac{1}{P(t)} p(t)$ is the *r*elative, or *r*eal price index. It is the first because it eliminates the component of general inflation included in every specific commodity price index, and it is the latter because it relates price to some standard commodity basket, chosen as unique and universal unit of measurement. In this way a real price index corresponds to what is understood as price in (non-monetary) economic theory.

Applying a universal deflator to a set of nominal figures creates no problems of aggregation, it is fully consistent. It seems useful to establish, and publish, accounts of real values “in constant Euros,” besides those of nominal values “in current Euros” even if the operation requires only a simple division by means of a universal price level. It is, at any rate, a transparent.operation, easy to understand, and allows direct comparison of value figures over time, based on a constant and comparable unit ofmeasurement (Euros of year 0). In price statistics, similarly, it may be informative to supply real price indexes, besides the nominal ones, in order allow better, and direct comparison between different commodity markets.

Compilation of real values *u* in this sense is fine, and sufficient, for analysis of flows of income, and of finance. It is insufficient in respect to analysis of production, the first of the three great areas of national accounting. For a change in real value of some output may occur for two reasons, either because of a change in economic activity *q* within the industry, or because of a change in market conditions, *p* (or *r*) outside the industry. You may visualise the first as a change in the column of an input-output table and the latter as a change in some row, similar to the RAS procedure.

Indices *q* an *p* are dimensionless figures and have no empirical meaning in themselves, (in contrast to their microeconomic counterparts).They attain their concrete meaning when attached to a certain national accounts figure *v* measured in Euros of a certain year (see equation 11). What one is interested in is actually not their absolute level, which is undefined, but their level relative to some earlier year. So *q* and *p* are variables of a process (a "movement" or "growth") and not describing an economic state ("equilibrium"), in national accounts, in contrast to their use in microeconomic theory. They measure aggregate change and difference (Balk 2008), rather than some point in the commodity space. A process is described by way of differentiation, in mathematics. So from equation 14 you deduce

(15) $$du = (r\ dq + q\, dr)\ v(0) \quad [€0]$$

and together with equation 13 this yields

(16) $$dv = (p\ dq + q\ dp)\ v(0) = \left[P(r\ dq + q\ dr) + u\ dP\right] v(0)$$

As continuous functions cannot be observed directly, but only in discrete intervalls of statistical surveys, one must find an appropriate approximation. This, too, follows known tracks. Let be

(17) $$\Delta u^t = u(t) - u(t-1),$$

and similarly for all other variables. Equation 15 can then be linearly approximated by

(18) $$\Delta u^t = \left(q^t\ \Delta r + r^{t-1}\ \Delta q\right) v^0 \quad [€0].$$

The decomposition measures a change in volume in previous year real prices, similar to the traditional Laspeyres index (second term on the right hand side), while the change in real, relative price is expressed at current year volumes, the Paasche complement (first term on the right hand side). The opposite combinantion is also possible, of course, and their average would yield the best approximation, but in following the SNA, and given that yearly changes are small, the traditional decomposition may be acceptable, easy to communicate, and to understand. With it, time series of volume changes of an aggregate Q^{0t} may be constructed as follows

(19) $$Q^{0t} = \sum_i v_i^0 \sum_{s=1}^{t} r_i^{s-1} \Delta q_i^s \quad [€0],$$

which is an additive chain of Laspeyres indices. The ccoresponding time series of real, relative price changes[2] is given by

(20) $$R^{0t} = \sum_i v_i^0 \sum_{s=1}^{t} q_i^s \Delta r_i^s \quad [€0],$$

which is an additive chain of Paasche indices. Real value U^t of the aggregate in total is then given by

(21) $$U^t = Q^{0t} + R^{0t} + V^0 \quad [€0],$$

and it is connected to its original nominal value V^t by

(22) . $$V^t = P^t \times U^t \text{ [€t].}$$

The sum of all growth in real value observed since time 0 due to production (volume change Q^{0t}), and of all changes due to market forces (real price change R^{0t}) together composes the total change in real value over the time intervall (0,t). Adding to it the base year value yields total real value at time t, U^t, expressed in Euros of year 0. Multiplying by the corresponding price level P^t, finally, yields the original, actual monetary value $V(t)$ of year t. The described additive decomposition of nominal values builds on chaining of indices as required by the SNA, and it keeps consistency in aggregation fully intact, yet. The initial, naïve decomposition of equations 3 to 6 ignores variation in the unit of measurement dP embodied in any absolute price change dp.

6. Does inconsistency matter, in practice?

Having recalled that an index which combines chaining with consistency in aggregation exists, and repeating that consistency is not only desirable, but essential for viable and credible accounts, the question is still open of whether the actual inconsistency observed matters in statistical practice, taking into account that there is always a certain statistical error involved in national accounts figures, and the theoretical discrepancy may just be small enough to disappear within these known, and inevitable deviations.

[2] More precisely, and correctly: "change of value due to changes in price".

The traditional Laspeyres-Index constructing values "at constant prices" was consistent in aggregation, a natural feature which was rarely mentioned, but rather self-understood, at its time. It is only now with the introduction of chaining that consistency has been put into question. Take a case of two commodities A and B, and two consecutive periods, years 0 to1, and 1 to 2, for illustration. Equation 23 describes the situation at the base year, t = 0:

(23) $$v_A^0 = p_A^0 \, q_A^0 \ ,$$

and similarly for commodity B and the other years. For year 1, the Laspeyres-Index aggregating the two commodities is given by equation 24, namely

(24) $$v_A^0 \frac{p_A^0 \, q_A^1}{p_A^0 \, q_A^0} + v_B^0 \frac{p_B^0 \, q_B^1}{p_B^0 \, q_B^0} = p_A^0 \, q_A^1 + p_B^0 \, q_B^1 ,$$

which is consistent in aggregation. Equation 25 extends the chain into the next period,

(25) $$v_A^0 \frac{p_A^0 \, q_A^1}{p_A^0 \, q_A^0} \times \frac{p_A^1 \, q_A^2}{p_A^1 \, q_A^1} + v_B^0 \frac{p_B^0 \, q_B^1}{p_B^0 \, q_B^0} \times \frac{p_B^1 \, q_B^2}{p_B^1 \, q_B^1} ,$$

where the index chain has been attached to each product individually, before summing. The alternative aggregation is expressed by equation 26,

(26) $$\left(v_A^0 + v_B^0\right) \times \frac{p_A^0 \, q_A^1 + p_B^0 \, q_B^1}{p_A^0 \, q_A^0 + p_B^0 \, q_B^0} \times \frac{p_A^1 \, q_A^2 + p_B^1 \, q_B^2}{p_A^1 \, q_A^1 + p_B^1 \, q_B^1} \ ,$$

which clearly differs from equation 25. Chaining over one period is consistent, over more than one it is not. The consistent volume compilation follows formula 18, which yields

(27) $$\begin{aligned} Q_A^{02} + Q_B^{02} &= v_A^0 (p_A^0 \, \Delta q_A^1 + p_A^1 \, \Delta q_A^2) + v_B^0 (p_B^0 \, \Delta q_B^1 + p_B^1 \, \Delta q_B^2) \\ &= \left(v_A^0 \, p_A^0 \, \Delta q_A^1 + v_B^0 \, p_B^0 \, \Delta q_B^1\right) + \left(v_A^0 \, p_A^1 \, \Delta q_A^2 + v_B^0 \, p_B^1 \, \Delta q_B^2\right) \end{aligned}$$

where I have assumed absence of inflation so that nominal prices *p* equal real prices *r*, for purpose of simplification. In equation 27, aggregate growth is compiled for each commodity class A and B separately, and then added, in the first line, while in the second line, it goes the other way, growth of the aggregate being calculated for each year, first, and then added to a time series. The compilation is consistent.

But accepting inconsistency as undesirable, in theory, it may still be acceptable, in practice, if resulting discrepancies are small, falling within the ordinary errors of statistical observation. An empirical investigation allows the hypothesis that such hope is made in vain. Table 1 gives results of an experimental calculation applied to data of Denmark for a period of 35 years.

Table 1

Aggregative inconsistency in determining output volume of industry "Chemical, rubber, plastics and fuel" in Denmark 1970-2005

	(1)	(2)	(3)	(4)	(5)
Industry code	23t25	23	244	24x	25
Output 1970 (mill. 1970Kr.)	6,537	1,718	486	2,709	1,624
Volume index 1970 (1995 = 100)	51.6	95.6	12.5	53.7	42.7
Volume index 2005 (1995 = 100)	147.3	101.5	282.1	107.2	122.1
Volume 2005 (mill. 1970Kr.)	18,655	1,825	10,958	5,410	4,644
Sum of sub-industries (mill. 1970Kr.)	22,837				

23t25 : Industries chemical, rubber, plastic, fuels

23 : Coke, refined petroleum, nuclear fuel

244: Pharmaceuticals

24x : Chemicals excl. pharmaceuticals

25 : Rubber and plastics

Source: Reich (2010, p. 199)

Column (1) in table 1 shows aggregate figures for the industry as a whole while the adjacent columns show the break-down by its components. For year 1970, figures of output in columns (2) to (5) sum up to column (1), namely 6,537 1970Kr. (third row in table 1). Applying chained volume indices in the traditional way destroys consistency: When aggregating first and deflating the aggregated out put, thereafter, as in column (1) you get an amount of 18,655 mill. 1970Kr. for year 2005, which corresponds to a growth of 12,118 mill. 1970Kr., or 185 percent, while aggregation after separate deflation of the individual sub-industries yields an amount of 22,837 mill. 1970Kr. implying a growth of 16,300 mill.1970Kr. or 249 percent, a discrepency of 4,182 mill. 1970Kr., or 64 percent. This is just the compilation for output. When deflating intermediate inputs the discrepancy amounts to 1,886 mill. Kr . (Reich 2010, p.203). Both discrepancies fall directly on the resulting growth of value added which comes out at either 6,822 mill. 1970Kr. when deflating the aggregated industry, or at 9,076 mill. 1970Kr., in the opposite case (Reich 2010, p. 203). In relation to an original amount of 1,993 1970Kr. in year 1970, this implies an average growth rate of either 3.6 percent per year, or of 4.4 percent per over the observed period of 35 years. This is not a small error. The additive decomposition, in contrast, yields a unique growth rate of the industry's value added of 3.4 percent per year (Reich 2010, p. 208). These figures describe an extreme case, for sure, but it is for the purpose of excluding such cases with certainty that consistency in aggregation is required, in an accounting system. The inconsistency of the traditional decomposition method, existing in theory, is not irrelevant, in practice.

Summing up so far, I have shown that a consistent chain index is mathematically possible, and that it is desirable under accounting aspects; in the next, and last chapter I argue that an additive decomposition even opens up new, interesting roads towards a deeper analysis of value and growth than hitherto employed.

7. Full growth accounts

Value added is the core variable of national accounts, - its name alone implies it must be additive – and it is the most difficult one to decompose. If anywhere, it is here that the idea of a physical "quantity" looming behind the value figure appears as out of place. Being defined as a balance of payables and receivables on a monetary account value added cannot be but a pure money value. In fact, it has taken some time to have the technique of double deflation, now generally employed for determining a volume component, be accepted by the scientific community because of the evident lack of a quantity to which a price could be attached.[3] I have shown that a consistent way of determining growth in the volume of value added exists. But the analysis may not finish there; the residual, namely the movement in real, or relative prices is also worth being documented, and studied in national accounts. Conventional growth analysis, based on Cobb-Douglas or other production functions neglects this side of the growth process. But growth of an industry happens not only through an increase in production, it also occurs because of market conditions improving, in that either output is sold at higher prices, or inputs are bought at lower prices. In an accounting framework the price element of growth complementary to the growth in volume can be included as follows.
Value added of an industry is defined as the value of output minus the value of intermediate inputs. If a_i are the inputs of an industry and b_i its outputs, at constant money terms (real values) the corresponding value added u is given by

(27) $$u = \sum_i b_i - \sum_i a_i \ .$$

Growth of real value added may then be decomposed into four elements, namely,

(28) $$du = \sum_i b_i \, dq_i + \sum_i b_i dr_i - \sum_i a_i \, dq_i - \sum_i a_i \, dr_i \ .$$

[3] See Neubauer 1978 on "Irreal domestic product at constant prices", constructing drastic examples that run counter intuition.

Real growth of an industry (*du*) has thus four possible sources, either directly as an increase in volume of output ($\sum_i b_i dq_i$), or as a decrease in volume of inputs ($\sum_i a_i dq_i$), or indirectly, as an increase in real prices of sales ($\sum_i b_i dr_i$), or decrease in real prices of purchases ($\sum_i a_i dr_i$). Table 2 shows the example of the German manufacturing industry developing over the years 1991 to 2007.

Table 2

Growth of gross value added of the manufacturing industry in Germany 1991-2007 (billion Euros)

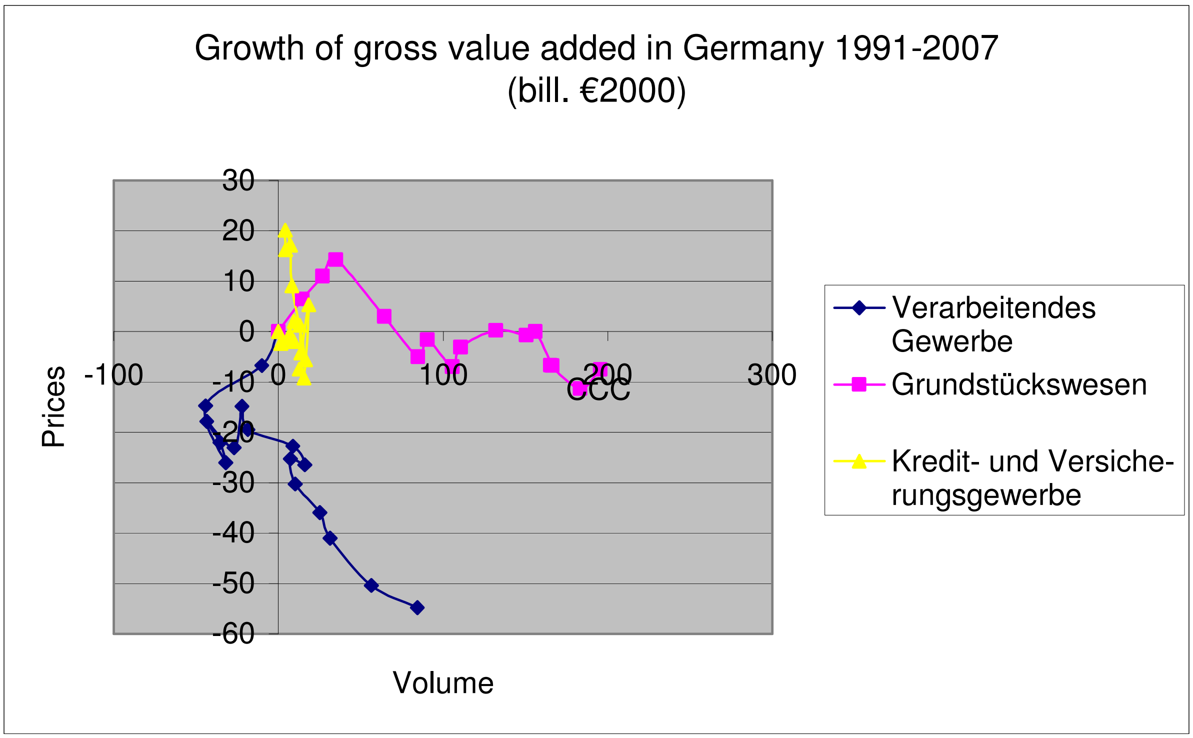
Year	Nominal values (current €)	Real values (€2000)	Total growth (€2000)	Volume (€2000)	Prices (€2000)
1991	383,45	439,91			
1992	387,04	423,03	-16,88	-10,14	-6,74
1993	361,60	381,01	-42,02	-34,05	-7,97
1994	370,93	381,75	0,75	12,07	-11,33
1995	378,47	382,35	0,59	-3,42	4,01
1996	376,57	378,53	-3,82	-8,01	4,20
1997	389,06	389,95	11,42	16,59	-5,17
1998	404,16	402,84	12,89	4,72	8,16
1999	404,74	402,00	-0,84	3,78	-4,62
2000	425,99	425,99	23,99	27,26	-3,26
2001	434,60	429,44	3,45	7,16	-3,71
2002	433,00	421,87	-7,57	-8,74	1,17
2003	436,08	419,90	-1,96	3,01	-4,97
2004	450,73	429,21	9,30	15,03	-5,73
2005	455,03	430,20	0,99	6,07	-5,08
2006	474,31	445,87	15,67	25,02	-9,35
2007	508,73	469,56	23,70	28,11	-4,41
1991-2007	---	---	29,66	84,46	-54,81

Source: Reich (2014)

Manufacturing industries generated a gross value added of 383 billion Euros in 1991, and of 508 billion Euros in 2007, 27 years later, in nominal values. However, Euros of year 2007 do not carry the same purchasing power as those of 1991, which means the unit in which to measure economic value has itself changed. Accounting for the devaluation leads to real values, uniformly measured in Euros of year 2000. So, in constant units of measurement, growth has gone from 439 to 469 billion €2000, an increase of 30 billion €2000. This advance in real value may be decomposed in growth due to more production leading to more volume of 84 billion €2000, and an accompanying loss in prices of 54 billion €2000, over the whole period. These are real, relative prices, i.e. prices where the inflationary element has been eliminated, and their movement signifies that part of the value added generated in production has been passed on either to customers through lower prices of sales, or to suppliers through higher input prices. Prices visibly act here as a mechanism of allocation of generated value added among industries, determining the terms of trade at which an industry operates within the national economy.

The yearly movement of the two components of growth can be followed up, in the table, and it may also be visualised, graphically. Graph 1 shows time series for the industries of manufacturing, of real estate and business services, and of financial intermediaries.

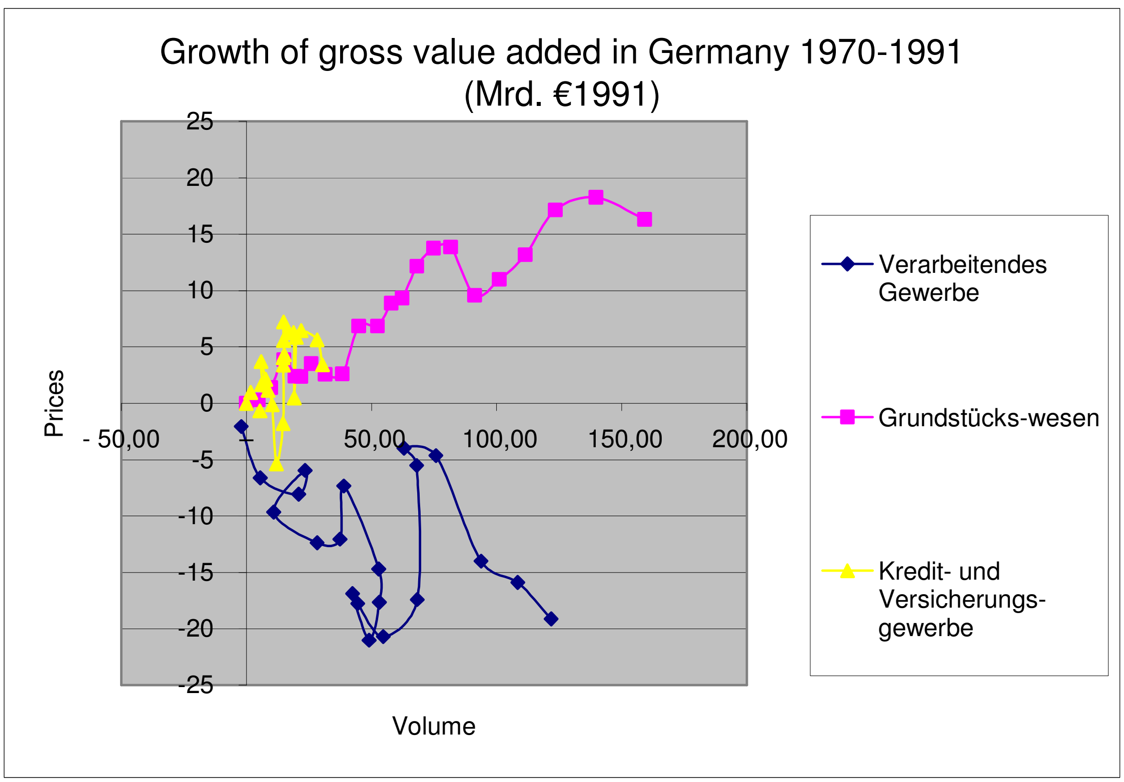
Graph 1



Source: Reich (2014)

The south-east movement of the manufacturing industry reflects the fact that at a fairly constant real value added per year, the manufacturing industry lost through deteriorating prices on its markets what it gained in volume through higher production, a situation which may be interpreted as one of increasing supply (through technical progress) at constant demand. The real estate industry, in contrast, has kept its terms of trade rather stable, - we observe a slight fall with high variation over the period, - adding 200 billion €2000 in volume to its production. Such development may be interpreted as a joint increase of supply and demand, at stable prices. A third example is provided by the financial industries, which exhibit no growth in volume, and a slight improvement in their prices, showing as a slight movement upwards with fluctuations, which may signify an increase in demand combined with a corresponding reduction in supply, at constant volume.

The existing data allow a comparison of this period of beginning gloabalisation with the growth period of 20 years earlier. Graph 2 shows the corresponding devlopment for the same industries as in graph 1. In this earlier period, the manufacturing industry , although moving rather cyclically, grows by 140 billion €1991 in volume, 20 billions of which it distributes to its trading partners through a weakening in its terms of trade. Industry real estate and business services appears again as a major growth factor, adding 160 billion €1991 to its volume, at better prices, notably. Increase in demand at constant supply may be the reason behind. For the financial industry the earlier period shows no other development than observed for the later one, high variation in prices, but almost no growth in volume, over all.



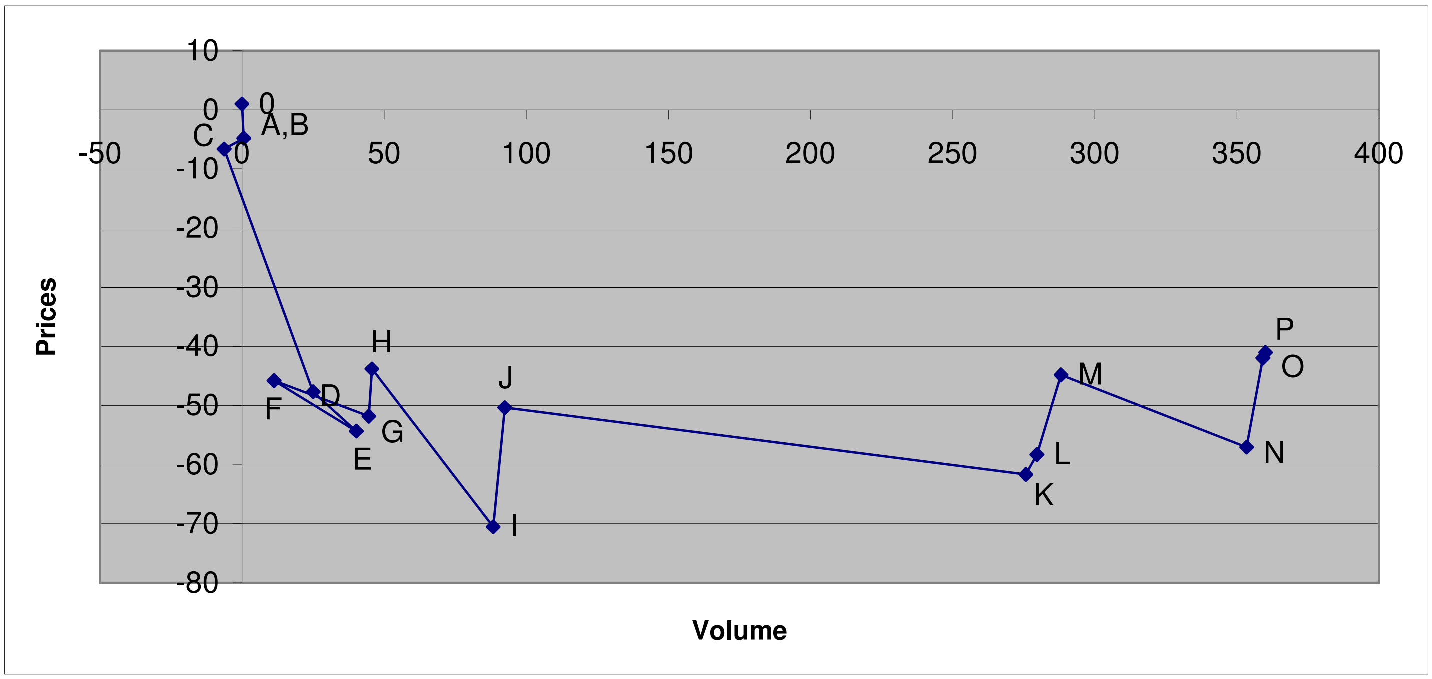
Source: Reich (2014)

One can drive the analysis even further. Graph 3 summarises the development of all German industries together over the period 1991 –2007, beginning with A: Agriculture, and ending with P: Domestic servants. One can see that of a 350 billion €2000 growth of the whole economy a major part has been contributed by industry K, real estate, and business services, as already shown in graph 1; industries I: Transport and communication, and N: Health and social services come second with a contribution of 50 billion €2000 each, while industries J, financial intermediaries, M: Education and O: Community and personal services mainly grew by improving their market position (movement upwards). The loss in terms of trade by industry D, manufacturing has already been noted in graph 1; industry I: Transport and communication shows a similar development.

Graph 3

Growth of industries' gross value added in Germany 1991 – 2007

(billion €2000)



Source: Reich (2014)

This sort of analysis is meaningful if price and volume are not treated as variables describing an economic state (of equilibrium as in microeconomics), but as measuring economic processes such as growth and inflation. In this understanding, the decision of the SNA to address price and volume measures in late chapter 15, and not earlier, is fully within its own systematic order. The SNA's first task is to construct a complete picture of the economy for a given year at actual values, i.e. in nominal terms. Neither price nor volume can be determined, or have any meaning, at that point. Only when compared to the next year these concepts make sense as they then measure the change that occurs between the two years in these two dimensions. In economic theory, on the contrary, the relationship is just the opposite. Price and quantity are assumed to be given first, and from them you derive the value that holds in a state of equilibrium, but you cannot extract from the compilation any information about how price and quantity will change within or outside of equilibrium, in the process of economic development. Microeconomic theory does not have an adequate model for macroeconomic accounts, just like in physics, speaking in the Walrasian tradition of comparing economics to physiscs, the concepts of quantum theory are beauti- und useful in themselves, but inapropriate for describing the movement of a classical mass pendulum. The scale of a phenomenon matters in the forming of fitting concepts.

8. Conclusion

Theres is a reason why consistency in aggregation is disrespected in index number theory. The theory has been designed for analysing the movement of prices on commodity markets, which still represents the main field of its application. Commodities have a natural order, which also defines a natural route of aggregation.You aggregate milk first with butter, then dairy products with meat, then food with clothes, and so on. You never aggrgegate milk with men's suits directly. Invariance with regard to the order of aggregation is not required in price analysis. This is different for national accounts. You want to compare France with Germany within the European Union. So you aggregate first over all industries in each country, and then look at the countries. But you also want to compare the European steel industry with the European construction industry aggregating, therefore, over countries first and then look at the industries. Consistency in aggregation is essential for any such kind of investigation. And it is the more important the higher the level of agggregation. Something like world accounts cannot be conceived to exist without it having this quality. The paper argues that a theoretical formula for consistency exists, and shows its applicability to data of national accounts. National accounts are, as the name says, a system of economic statistics where transactions of economic value contracted between economic units are surveyed and grouped into receivables and payables on aggregated accounts, each of these accounts closing with a meaningful, yet always monetary, balance. The arithmetic operation of adding and subtracting is the working tool of such a system; putting it aside would destroy the clue of the statistics, and transform a system of coherent accounts into a set of accidental social indicators.

When trying to establish a certain distinction in concepts, it is always useful to look back and study their genesis in people's thinking. When Laspeyres produced his first index for commodity prices of the port of Hamburg, he did so in order to investigate a macro-concept, namely the value of current money. When Fisher produced his axioms he had no particular economic object in mind, but looked for logical paradoxes. When economic theory entered the stage, money left it as unit of measurement, yielding its place to an arbitrary "numeraire" closing a Walrasian homogeneous system of mathematical equations. Aggregation in general, and national accounts in particular played no role in this research. It is only now that all these statistics are in the process of being integrated that their mutual coherence arises as an issue. True, some long favored ideas may have to be sacrificed in the process, but sacrificing coherence in aggregation for the national accounts means giving them up as an instrument of economic observation altogether.

National accounts are not based on assumptions stated at the beginning of an exercise, and then left to the reader to decide whether to accept them, or not. The assumptions on which national accounts are constructed are so evident nobody wants to question them, such as additivity of transactions. This is what distinguishes accounting from modelling. You may have different models, side by side, depending on your theoretic choice. But you can have only one system of accounts, within a firm, and within a nation. It is for this reason of uniqueness that you can say the accounts represent economic facts while models represent economic possibilities. Destroying the uniqueness of accounting by allowing inconsistency in aggregation will corrupt the mark of certainty, which alone allows a statistical office to remain neutral in economic issues, and lends it the power to withstand political pressure from wherever it comes. Similarly, for transactions crossing the national boundary, or global accounts in general, it cannot be that growth of all countries differs from growth of all industries in the world; it cannot be that value added is non-additive in a global accounting system. The dilemma is not noticed as only one way of aggregation is taken for compiling global accounting figures, at present; but a future comprehensive global accounting system cannot tolerate inconsistency, except at the price of its long run viability.

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