The Dynamics of Non-Monotonic Poverty: Theory and Application to Time Poverty in Mexico

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Abstract

The current assessment of chronic poverty relies on a notion of poverty defined by insufficiency of the means to attain a minimally decent state of wellbeing. However some indicators of functioning, e.g. measures of health status, are characterized by a non-monotonic relationship to wellbeing, *since both their scarcity and excess are associated with illfare. Likewise, one can argue that both leisure time scarcity and leisure time abundance can be symptomatic of substandard wellbeing. For the purpose of measuring illfare, when both extremes (scarcity and excess) are deemed detrimental to wellbeing, Apablaza et al. (2013) provided a family of poverty measures that satisfies a set of reasonable desirable properties. We combine their toolkit with different approaches to inter-temporal poverty measurement, in order to study the dynamics of* non-mononotic poverty. *Specifically, we propose measures of chronic and transient non-monotonic poverty following both the "permanent income" (Jalan and Ravallion, 2000, Foster and Santos, 2013) and the "spell counting" (Foster, 2009) approaches. We apply these measures to the study of leisure time poverty in Mexico.*

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JEL Classification: Non-monotonic poverty, chronic poverty, time poverty.

1 Introduction

The study of poverty experiences as they unfold across time has benefited from a mushrooming supply of concepts, measures and statistical toolkits. In addition to the analysis of poverty transitions (e.g. Cappellari and Jenkins, 2004), several proposals have been made in order to identify and understand the plight of the chronically poor vis-a-vis the transiently poor. For instance, Rodgers and Rodgers (1993), Jalan and Ravallion (2000), Aaberge and Mogstad (2007), and Foster and Santos (2013) define the chronically poor as those whose permanent income standard (e.g. a weighted average) across time falls below a poverty line, whereas Bane and Ellwood (1986), Hulme and Shepherd (2003) and Foster (2009) identify the chronically poor according to the number of spells in poverty. Following the useful distinction by Foster and Santos (2013), we consider the first to be a "permanent income" approach; while the second one can be labelled a "spell counting" approach. Likewise, there is a growing literature providing welfare evaluations of lifetime poverty experiences, where, for instance, the timing and contiguity of poverty, and non-poverty, spells is deemed to affect people's wellbeing. Prominent examples of this latter literature include Hoy and Zheng (2011), Gradin et al. (2012), Hoy et al. (2012), Bossert et al. (2012), and Dutta et al. (2013). Unlike the two aforementioned approaches, this latter inter-temporal poverty literature does not identify, or distinguish, the chronically poor from the transiently poor, and the non-poor in a dynamic sense.

Now all these concepts and methods are applied to a notion of poverty defined by insufficiency of the means to attain a minimally decent state of wellbeing, e.g. insufficient income or consumption (e.g. as is implicitly clear in the neat taxonomy of Hulme and Shepherd (2003)). These concepts have also been applied to leisure time, by regarding an insufficiency of it as time poverty [1]. However, as Apablaza et al. (2013) argue, some indicators of functioning (in Sen's terminology) are characterized by *a non-monotonic relationship to wellbeing*. For instance, measures of health status, like Body Mass Index (BMI), haemogobline, cholesterol, etc., for which both scarcity and excess are associated with illfare. Likewise, one can argue that both leisure time scarcity and leisure time abundance can be symptomatic of substandard wellbeing. For the purpose of measuring illfare, when both extremes (scarcity and excess) are deemed detrimental to wellbeing, Apablaza et al. (2013) provided a family of poverty measures that satisfies a set of reasonable desirable properties.

In this paper we combine the toolkit of Apablaza et al. (2013) with different approaches to inter-temporal poverty measurement, in order to study the dynamics of *non-mononotic poverty*. Specifically, we propose measures of chronic and transient non-monotonic poverty following both the "permanent income" (Jalan and Ravallion, 2000, Foster and Santos, 2013) and the "spell counting" (Foster, 2009) approaches.

We apply these measures in order to shed light on the patterns of leisure time poverty in the Mexican states using the ENOE panel dataset, which features a rich questionnaire on time use. We document gender differences in intertemporal non-monotonic poverty profiles. The measures of inter-temporal non-monotonic poverty are compared against

[1] For a conceptual discussion see, e.g. Bardasi and Wodon (2006).

traditional measures of poverty dynamics in order to highlight the empirical contribution of our approach.

We find that both approaches to chronic time poverty produce similar results: about 36% of the 18-65 years old population is either "shortfall" or "excess" time poor. However, when we constraint the sample to individuals in the same age cohort but who, additionally, generate earnings from a non-agriculture activity, the "permanent" income approach shows higher chronic poverty headcount rates than the "spell-counting" approach: 3.8% vs. 5.9%, respectively. Although in both cases the poverty rates significantly drop, mainly due to a decline in "excess" poverty. The larger chronic time poverty rates under the "spell counting" approach for both male and female subsamples are accompanied by a narrower gender gap. Indeed, under the "spell-counting" approach, chronic poverty for women is about 2.5 times as great as chronic poverty for men, while under the "permanent income" approach, there is a ten-fold difference favouring men.

The rest of the paper proceeds with the basic setting and notation. Then, in the next two sections we present our proposals for the measurement chronic and transient non-monotonic poverty, based on the "permanent income" and the "spell counting" approaches, respectively. The empirical illustration on time poverty in Mexico follows; and the paper concludes with some remarks.

2 Basic setting and notation

Apablaza et al. (2013) noted that some indicators measuring wellbeing attributes have a non-monotonic relationship to wellbeing. For example having a very low level of adult Body Mass Index (BMI) can be symptomatic of undernourishment; yet having high levels of BMI can also be deemed detrimental to wellbeing, to the extent that it signals obesity. Hence in those situations, traditional poverty analysis, which focuses exclusively on low values, is admittedly inappropriate. In order to make up for this analytical gap, Apablaza et al. (2013) proposed poverty indices (and poverty orderings) whose properties account for so-called "illfare" situations due both to "shortfall" and "excess" values of the wellbeing indicator. This approach appears warranted in several situations, e.g. in the case of health indicators.

Now, in the case of time poverty, the literature has rightly focused on the problem of leisure time scarcity. However, one could argue that, at least under some circumstances, an excessive amount of leisure time (among adults in working age) could be a reflection of negative situations, e.g. some inability to engage in productive work (either in the marketplace or at home) due to chronic illness or persistent unemployment conditions. Therefore it may be worth accounting for negative wellbeing outcomes due both to shortfalls and excess leisure time. In particular, gender comparisons of these time poverty profiles would be warranted (e.g. see Blackden and Wodon, 2006).

The non-monotonic poverty measurement mentioned so far is static. However, as attested by the literature, the inter-temporal dimension of poverty cannot be overstated (e.g. see Hulme and Shepherd, 2003). Hence we propose methods to study the dynamic aspects of non-monotonic poverty. The rest of this section introduces the basic notation, and the following sections discuss our proposals.

Let x_{nt} be the attribute received by individual n (out of $N \in \mathfrak{N}_+$ persons) in period t (out of $T \in \mathfrak{N}_+$ periods). Hence x_{nt} is a typical element of a matrix of incomes with N rows and T columns. We define the N-dimensional vector $\boldsymbol{x}_{.t}$ as the population's marginal income distribution at period t, and the T-dimensional vector $\boldsymbol{x}_{n.}$ as the individual's lifetime income stream. Values x_{nt} are defined on the domain $\Omega := [\omega^-, \omega^+] \subset \mathfrak{R}$. For instance, if x_{nt} denotes leisure time per day in hours, then $\Omega = [0, 24]$. Since both low and high values of this attribute are detrimental to wellbeing, we consider two deprivation lines $\{z^L, z^H\} \subset \Omega$, with $z^L < z^H$, in the poverty analysis. For the sake of simplicity, we assume without loss of generality that these thresholds do not vary from one period to the other. We then can measure individual snapshot poverty for person n at time t with the following individual poverty functions:

$$p(x_{nt}; z^L, z^H) := \psi_L(x_{nt}; z^L) g_L(x_{nt}; z^L) + \psi_H(x_{nt}; z^H) g_H(x_{nt}; z^H), \tag{1}$$

where ψ_L are ψ_H are the poverty identification functions for "shortfall" and "excess" snapshot poverty, respectively. Likewise, g_L and g_H are poverty intensity functions (i.e. measuring the severity of the individual poverty experience). More specifically:

$$\psi_L(x_{nt}; z^L) := \mathbb{I}(x_{nt} < z^L), \tag{2}$$

$$\psi_H(x_{nt}; z^H) := \mathbb{I}(x_{nt} > z^H), \tag{3}$$

where $\mathbb{I}(a) = 1$ if a in parenthesis is true, otherwise $\mathbb{I}(a) = 0$. The poverty intensity functions are characterized by: $g'_L := \frac{\partial g_L}{\partial x} \leqslant 0$ and $g'_H := \frac{\partial g_H}{\partial x} \geqslant 0$. If we want the individual poverty function to fulfill a property of monotonicity (MON) whereby values further away from the respective deprivation lines increase poverty, then we just need to set the above first partial derivatives strictly different from 0. Additionally if we want the individual poverty functions to fulfill a transfers property (TRA), whereby poverty reductions at values further away from the deprivation lines are prioritized over less reductions in less acute situations, then we need $g''_L := \frac{\partial^2 g_L}{(\partial x)^2} > 0$ and $g''_H := \frac{\partial^2 g_H}{(\partial x)^2} > 0$. Meanwhile, the ψ identification functions already guarantee that p fulfills a focus axiom (FOC) for non-monotonic poverty.[2] We also restrict ourselves to p satisfying scale invariance (SCA).

Then, in a population with N individuals, we aggregate individual poverty functions into a social index P using additively decomposable functions, also satisfying population-replication invariance (POP):

$$P(z^L, z^H) := \frac{1}{N} \sum_{n=1}^{N} p(x_n; z^L, z^H). \tag{4}$$

For example, we could use the following non-monotonic version of the Foster-Greer-Thorbecke (FGT) family proposed by Apablaza et al. (2013) for (4), that is:

$$P(z^L, z^H) := \frac{1}{N} \sum_{n=1}^{N} \left[\mathbb{I}(x_n < z^L) \left(\frac{z^L - x_n}{z^L - \omega^-} \right)^{\alpha^L} + \lambda \mathbb{I}(x_n > z^H) \left(\frac{x_n - z^H}{\omega^+ - z^L} \right)^{\alpha^H} \right], \tag{5}$$

[2] (see Apablaza et al., 2013, for more details).

where $\lambda > 0$, $\alpha^L \geqslant 1$ and $\alpha^H \geqslant 1$.

3 Chronic and transient non-monotonic poverty: a “permanent income” approach

The “permanent income” approach to the identification of the chronically poor was introduced by Rodgers and Rodgers (1993) and later popularized by Jalan and Ravallion in a series of papers (e.g. Jalan and Ravallion, 1998, 2000). Since then, it has recently been generalized by Foster and Santos (2013).[3] The original idea was to identify people as chronically poor whenever their average income (or another continuous indicator of well-being) was below a poverty line. Otherwise, people could still be considered undergoing transient poverty should their incomes be below the poverty line in certain particular periods, while their average income being still above the same line. Foster and Santos (2013) generalized the approach (so that Jalan and Ravallion, 2000, becomes a special case) by noting that using the average as a permanent income standard assumes that resources are fully transferable across periods, in a costless manner. However, they showed how that dubious assumption can be relaxed in favour of imperfect inter-temporal income substitution, by using alternative choices of generalized means as the individual's permanent income standard (e.g. the geometric or the harmonic mean). Such generalized means are of the form:

$$\mu_\beta(\boldsymbol{x}_{n.}) = \begin{cases} \left(\frac{1}{T}\sum_{t=1}^{T} x_{nt}^{\beta}\right)^{\frac{1}{\beta}} & \text{if } \beta \neq 0, \\ \prod_{t=1}^{T} x_{nt}^{\frac{1}{T}} & \text{if } \beta = 0. \end{cases} \tag{6}$$

In particular, any choice $\beta < 1$ in (6) yields permanent values standards below the mean, thereby capturing imperfect and/or costly inter-temporal transfers. The more volatile the attribute stream, the greater the shortfall of μ_β, with $\beta < 1$, from the mean. In this section we adopt this “permanent income” approach to propose some indices of chronic and transient non-monotonic poverty. We note that if we also want to account for imperfect income transfers at the high end of the distribution, we need to consider general means with $\beta > 1$ (e.g. the Euclidean mean), yielding permanent income standards above the mean. In these cases, the more volatile the income stream at the top end, the greater the excess of μ_β, with $\beta > 1$, from the mean.

Using similar notation to the previous section's, our first proposal of chronic non-monotonic *individual* poverty indices, under the permanent income approach is the following:

$$p_c(\boldsymbol{x}_{n.}; z^L, z^H) := \psi_L\left(\mu_\beta(\boldsymbol{x}_{n.}); z^L\right) g_L\left(\mu_\beta(\boldsymbol{x}_{n.}); z^L\right) + \psi_H\left(\mu_\gamma(\boldsymbol{x}_{n.}); z^H\right) g_H\left(\mu_\gamma(\boldsymbol{x}_{n.}); z^H\right), \tag{7}$$

with $\beta \leqslant 1$ and $\gamma \geqslant 1$.

The proposal in (7) follows closely the contribution by Foster and Santos (2013), but adding poverty identification and intensity functions for “excess” situations. The identification functions now compare a general mean against deprivation lines on both ends of

[3] Following Foster and Santos (2013), when we refer to “income” in the theoretical sections, we really mean any continuous variable.

the distribution. If $\mu_\beta(\boldsymbol{x}_{n.}) < z^L$ then individual n is deemed chronically poor in terms of "shortfall" poverty. Likewise if $\mu_\gamma(\boldsymbol{x}_{n.}) > z^H$ then individual n is considered chronically poor in terms of "excess" poverty. Normally, if both β and γ are fairly close to 1, and if the poverty lines are reasonably far away from each, then people will uniquely be classified as either chronically "shortfall" poor, non-poor, or chronically "excess" poor. However, otherwise, there may be situations in which not only people's classification will depend on the choice of generalized means (which is also a problem in the generalized permanent income approach), but also in which the same person could qualify as chronically poor according to both criteria. These situations are further discussed in a subsection below.

For the poverty intensity functions we impose the same signs on the derivatives as before, in order to secure fulfillment of monotonicity (and, optionally, transfers) properties, but now the derivatives are considered with respect to the general means. One good example of intensity functions is based on the Clark-Hemming-Ulph family used by Foster and Santos (2013):

$$g_L(\mu_\beta(\boldsymbol{x}_{n.})) = \begin{cases} \frac{(z^L)^\beta - \frac{1}{T}\sum_{t=1}^T x_{nt}^\beta}{(z^L)^\beta - (\omega^-)^\beta} & \text{if } \beta \in]-\infty, 0[\cup]0,1], \\ ln(z^L) - \frac{1}{T}\sum_{t=1}^T \ln(x_{nt}) & \text{if } \beta = 0, \end{cases} \tag{8}$$

$$g_H(\mu_\gamma(\boldsymbol{x}_{n.})) = \frac{\left(\frac{1}{T}\sum_{t=1}^T x_{nt}^\gamma\right) - (z^H)^\gamma}{(\omega^+)^\gamma - (z^H)^\gamma} \qquad \text{with } \gamma \geqslant 1. \tag{9}$$

If we want to ensure the fulfillment of a focus axiom in every period (whereby, basically, any income between z^L and z^H in a given period cannot compensate for incomes in the poverty subdomain $[\omega^-, z^L[\bigcup]z^H, \omega^+]$ in other periods), we need to work with intensity functions that censor any period-specific incomes in the subdomain $[z^L, z^H]$. For example, this would require restating the intensity functions in (8) and (9) the following way:

$$g_L\big(\mu_\beta(\boldsymbol{x}_{n.}); z^L\big) = \begin{cases} \frac{(z^L)^\beta - \frac{1}{T}\sum_{t=1}^T \min\{z^L, x_{nt}\}^\beta}{(z^L)^\beta - (\omega^-)^\beta} & \text{if } \beta \neq 0 \\ \ln(z^L) - \frac{1}{T}\sum_{t=1}^T \ln\big(\min\{z^L, x_{nt}\}\big) & \text{if } \beta = 0 \end{cases} \tag{10}$$

$$g_H\big(\mu_\gamma(\boldsymbol{x}_{n.}); z^H\big) = \frac{\left(\frac{1}{T}\sum_{t=1}^T \max\{z^H, x_{nt}\}^\gamma\right) - (z^H)^\gamma}{[(\omega^+)^\gamma - (z^H)^\gamma]} \quad \text{with } \gamma \geqslant 1. \tag{11}$$

Using such a strong version of the focus axiom is common in both multidimensional and intertemporal poverty measurement, but combined with the "permanent income" approach, it may entails a failure to fulfil continuity at the poverty frontier, *i.e.* marginal variations in the level of some attributes are likely to result in non-marginal variations of the estimated level of poverty.

3.1 Identification issues in the "permanent income" approach

If we use $\mu_1(\boldsymbol{x}_{n.})$, that is the arithmetic mean, as the (permanent) income standard, then we are following the identification approach proposed by Jalan and Ravallion. In our non-monotonic situation, there are no major identification challenges with this approach, since either $\mu_1 < z^L$ (shortfall chronic poverty), $z^L < \mu_1 < z^H$ (no chronic poverty), or $z^H < \mu_1$

(excess chronic poverty).

[Insert Table 1 here.]

[Insert Figure 1 here.]

However, as soon as we choose $\beta < 1 < \gamma$, potential ambiguities arise. Table 1 summarizes the six situations that could emerge and their consequences for poverty identification in the measurement of non-monotonic poverty under the permanent income approach. Figure 1 illustrates the potential issues in the two-period case.

The situations depend on the relationship between the poverty lines and the extreme values in $\boldsymbol{x}_{n.}$. Poverty identification is unequivocal in three out of the six situations. Firstly, in situation (D), whenever the range of the poverty lines is greater than the range of $\boldsymbol{x}_{n.}$, individual n is always non-chronically poor, i.e. for whichever choices of β and γ. Secondly, in situation (A), whenever the maximum value of $\boldsymbol{x}_{n.}$ is below z^L, the individual is always chronically "shortfall" poor; whereas, in situation (F), whenever the minimum value of $\boldsymbol{x}_{n.}$ is above z^H, the individual is always chronically "excess" poor.

By contrast, poverty identification is ambiguous for the other three situations. In situation (B), the individual is never "excess" poor (since $\max \boldsymbol{x}_{n.} < z^H$), but depending on the choice of β, the individual could be either "shortfall" poor or never chronically poor. For instance, given that $\min \boldsymbol{x}_{n.} < z^L$, if $\mu_1(x_{n.}) > z^L$ it is easy to show that there exists a unique value of $\beta < 1$ that switches the poverty status from "shortfall" poverty to non-poverty, or vice versa. This ambiguity problem is also pervasive in the original permanent income approach proposed by Foster and Santos (2013). Situation (E) is similar, but the ambiguity occurs at the higher end of the distribution. Since $z^L < \min \boldsymbol{x}_{n.}$, the individual is never "shortfall" chronically poor. However, because $\max \boldsymbol{x}_{n.} < z^H$, the poverty status may depend on the choice of γ. For instance, if $\mu_1(x_{n.}) < z^H$, we could find a unique value of γ that switches the poverty status between non-poverty and "excess" poverty.

Finally, the most ambiguous situation occurs in situation (C), when the range of $\boldsymbol{x}_{n.}$ is broader than the range of poverty lines. In that case, some choices of β and γ could actually render the individual with two poverty status! (i.e. both "shortfall" and "excess" poor). Regarding measurement, whether this overlap is a matter of concern depends on the choice of the functional form for the functions g_L and g_H. In the case individual poverty is assessed using functions that do not comply with the strong focus axiom, like for instance (8) and (9), overlapping "shortfall" and "excess" poverty domain are likely to result in odd behaviours of the individual poverty function. Hence, whenever those cases are spotted in the data, it would be advisable to use values of β and γ fairly close to 1 and/or to widen the gap between the two poverty lines.[4]

[4] In the two-period case, it is thus necessary to check whether the equation:

$$\left(\frac{1}{2}\left(x_{n1}^{\gamma}+x_{n2}^{\gamma}\right)\right)^{\frac{1}{\gamma}}-\left(\frac{1}{2}\left(x_{n1}^{\beta}+x_{n2}^{\beta}\right)\right)^{\frac{1}{\beta}}=z^H-z^L, \tag{12}$$

has a solution in $[z^H,\omega^+]\times[\omega^-,z^L]$. However, stronger conditions are likely to be required in order to preclude odd behaviours of the individual chronic poverty function in case (C). More specifically, a solution consists in imposing streams corresponding to situation (C) not to belong to the chronic poverty domain, *i.e.* to set $\beta \geqslant \tilde{\beta}$

3.2 Transient poverty

We can also measure transient non-monotonic poverty in the permanent income approach by adjusting both the identification and intensity functions. Since we want to capture the poverty experience of those not classified as chronically poor, the simplest method is to use the following transient identification functions:

$$\psi_L^{tra} = \left| 1 - \prod_{t=1}^{T} \mathbb{I}\big(x_{nt} > z^L\big) \right| - \psi_L\left(\mu_\beta(\boldsymbol{x}_{n.}); z^L\right), \tag{15}$$

$$\psi_H^{tra} = \left| 1 - \prod_{t=1}^{T} \mathbb{I}\big(x_{nt} < z^H\big) \right| - \psi_H\left(\mu_\gamma(\boldsymbol{x}_{n.}); z^H\right). \tag{16}$$

Finally, if we want to have a social measure of the intensity of transient poverty, we need to focus on each period's experiences, as opposed to comparisons between permanent income standards and their respective poverty lines.

3.3 Partial orderings

Some proposals in the permanent income approach transform the problem of measuring chronic poverty, from that of comparing the values in $\boldsymbol{x}_{n.}$ against a poverty line (or set thereof), to one of comparing a single income standard (e.g. a general mean like $\mu_\beta(\boldsymbol{x}_{n.})$) against a poverty line. This is the case, for instance, of Foster and Santos (2013), due to their choice of poverty identification function and their choice of poverty intensity function (the Clark-Hemming-Ulph class), both relying on general means as income standards. While this approach may be criticised for sacrificing information in the compression of the multidimensional nature of chronic poverty measurement, it bears the advantage of easy tractability for the derivation of robustness conditions based on stochastic dominance methods.

It turns out that, for the case of any additively decomposable (and population invariant) social poverty index based on our proposal for individual poverty measurement in 7, all the stochastic dominance conditions derived for the static, non-monotonic poverty case, in Apablaza et al. (2013) apply, subject to the following conditions and caveats:

- The domain of the conditions is not x_{nt} (or x_n in the static case), but rather $\mu_\beta(\boldsymbol{x}_{n.})$ and $\mu_\gamma(\boldsymbol{x}_{n.})$. Hence the conditions depend on the distributions of the general means in the populations.
- The conditions pertain to particular choices of β and γ. That is, for certain values of these parameters, chronic poverty comparisons may be robust to different choices of poverty lines and/or index, but this robustness may not necessarily hold automatically for alternative choices of β and γ.

and $\gamma \leqslant \tilde{\gamma}$ with:

$$\left(\frac{1}{2}\left(\omega^{-\tilde{\beta}} + z^{H\tilde{\beta}}\right)\right)^{\frac{1}{\tilde{\beta}}} = z^L, \tag{13}$$

$$\left(\frac{1}{2}\left(\omega^{+\tilde{\beta}} + z^{L\tilde{\gamma}}\right)\right)^{\frac{1}{\tilde{\gamma}}} = z^H. \tag{14}$$

- For the conditions to work, ambiguities in poverty identification must be ruled out.

For example, consider the dominance condition in proposition 1 of Apablaza et al. (2013). Let $F^A : \Omega \rightarrow [0,1] \subset \Re$ and $\overline{F}^A : \Omega \rightarrow [0,1] \subset \Re$ be cumulative distribution and survival functions for population A, respectively. Let also (z^{L+}, z^{H-}) be the minimum admissible gap for the poverty lines (i.e. $\omega^- \leqslant z^L \leqslant z^{L+}$ and $\omega^+ \geqslant z^H \geqslant z^{H-}$). Then we can demonstrate the following condition:[5]

Proposition 1. $P^A(z^L, z^H) \leqslant P^B(z^L, z^H)$ *for all additively decomposable* P *of the (continuous version of the) form* (4)*, and based on* p_c *of the form* (7) *with* $\beta = \beta^*$*,* $\gamma = \gamma^*$*,* $\omega^- \leqslant z^L \leqslant z^{L+}$*, and* $\omega^+ \geqslant z^H \geqslant z^{H-}$*, if and only if* $F^A(\mu_{\beta^*}) \leqslant F^B(\mu_{\beta^*}) \quad \forall \mu_{\beta^*} \in [\omega^-, z^{L+}]$ *and* $\overline{F}^A(\mu_{\gamma^*}) \leqslant \overline{F}^B(\mu_{\gamma^*}) \quad \forall \mu_{\gamma^*} \in [z^{H-}, \omega^+]$.

Proposition 1 states that chronic poverty in A will never be higher than in B, for all additively decomposable social poverty indices whose individual poverty functions rely on general means (as detailed above), for all poverty lines within a given maximal admissible set, and for a given pair of parameters for the general means, if and only if the cumulative distribution function of the μ_{β^*} in A is never above that in B, and the survival function of the μ_{γ^*} in A is also never above that in B.

As Proposition 1 supposes the right values of the parameters β and γ to be known with certainty, one can reasonably cast doubts on the ethical robustness of the results. It is then necessary to fully make use of the multidimensional nature of the data used for the measurement of poverty. For the sake of simplicity, we focus, as usually done in the related literature on the two-period case. Within this framework, we assume that the poverty domain can be split into two non-overlapping domains $\Lambda_\beta(z^L)$ and $\Lambda_\gamma(z^H)$ that are respectively associated with insufficient and excessive permanent values of the well-being attribute. Let $z^L(x_{n1}) : \Omega \rightarrow \Omega$ be the continuous non-increasing function describing the poverty frontier that separates $\Lambda_\beta(z^L)$ from the non-poverty domain, and $z^H(x_{n1}) : \Omega \rightarrow \Omega$ the continuous non-increasing counterpart for $\Lambda_\gamma(z^H)$. We then can consider broad classes of bidimensional non-monotone poverty indices of the form (4) whose individual poverty function is:

$$p_c(\boldsymbol{x}_{n.}; z^L, z^H) := \psi_L\left(\mu_\beta(\boldsymbol{x}_{n.}); z^L\right) g_L\left(x_{n1}, x_{n2}; z^L\right) + \psi_H\left(\mu_\gamma(\boldsymbol{x}_{n.}); z^H\right) g_H\left(x_{n1}, x_{n2}; z^H\right), \quad (17)$$

with:

$$g_L\big(z^L(x_{n1}), x_{n2}\big) = g_H\big(z^H(x_{n1}), x_{n2}\big) = 0 \quad \forall x_{n2} \in \Omega, \quad (18)$$

$$\frac{\partial g_L}{\partial x_{nt}} \leqslant 0 \quad \forall t \in \{1,2\}, \boldsymbol{x}_{n.} \in \Lambda_\beta\big(z^L\big), \quad (19)$$

$$\frac{\partial g_H}{\partial x_{nt}} \geqslant 0 \quad \forall t \in \{1,2\}, \boldsymbol{x}_{n.} \in \Lambda_\gamma\big(z^H\big), \quad (20)$$

$$\frac{\partial^2 g_k}{\partial x_{n1} \partial x_{n2}} \geqslant 0 \quad \forall k \in \{L,H\}, \boldsymbol{x}_{n.} \in \Lambda_\beta\big(z^L\big) \cup \Lambda_\gamma\big(z^H\big). \quad (21)$$

The first condition classically imposes continuity at the poverty frontier of each part of the poverty domain. The next two conditions expresses monotonicity in the spirit of

[5]Proof based on proof of Proposition 1 in Apablaza et al. (2013).

Apablaza et al. (2013), but within a bidimensional framework, that is poverty should not increase as individual move towards the non-poverty domain. Finally the condition on the cross second-order derivative means that poverty would increase after correlation-increasing switches (Atkinson and Bourguignon, 1982, Duclos et al., 2006), that is after transfers that leave marginal distribution unchanged but increase the correlation between the series at time 1 and 2.

Let $\overline{F}^A : \Omega^2 \to [0,1] \subset \Re$ be the survival function associated with the joint bivariate distribution for population A. More robust poverty comparisons can then be obtained using the next proposition.

Proposition 2. *$P^A(z^L, z^H) \leqslant P^B(z^L, z^H)$ for all additively decomposable P of the (continuous version of the) form* (4)*, and based on p_c of the form* (7) *with $\beta \in [\beta^*, 1]$, $\gamma \in [1, \gamma^*]$, $\omega^- \leqslant z^L \leqslant z^{L+}$, and $\omega^+ \geqslant z^H \geqslant z^{H-}$, if and only if $F^A(\boldsymbol{x}_{n.}) \leqslant F^B(\boldsymbol{x}_{n.})$ $\forall \boldsymbol{x}_{n.} \in \Lambda_{\beta^*}(z^{L+})$ and $\overline{F}^A(\boldsymbol{x}_{n.}) \leqslant \overline{F}^B(\boldsymbol{x}_{n.})$ $\forall \boldsymbol{x}_{n.} \in \Lambda_{\gamma^*}(z^{H-})$.*

Proof. See appendix (A). ■

[Insert Figure 2 here.]

Proposition 2 says that poverty is unambiguously lower for population A than for population B for all poverty sets within $\Lambda_{\beta^*}(z^{L+}) \cup \Lambda_{\gamma^*}(z^{H-})$ and for all bidimensional non-monotone poverty measures P with individual poverty indices p_c of the form (17) fulfilling conditions (18) to (21) if and only if the bidimensional distribution function is lower in A than in B for all intersection poverty frontiers in $\Lambda_{\beta^*}(z^{L+})$ and the bidimensional survival function is lower in A than in B for all union poverty frontiers in $\Lambda_{\gamma^*}(z^{H-})$. This is illustrated in Figure 2, which shows both the position of the upper poverty frontier λ^+ and some of the rectangular areas over which $F_A(x_1, x_2) - F_B(x_1, x_2)$ and $\overline{F}_A(x_1, x_2) - \overline{F}_B(x_1, x_2)$ must be computed. If $F_A(x_1, x_2)$ ($\overline{F}_A(x_1, x_2)$) is lower than $F_B(x_1, x_2)$ ($\overline{F}_B(x_1, x_2)$) for all of the rectangles that fit within $\Lambda_{\beta^*}(z^{L+})$ ($\Lambda_{\gamma^*}(z^{H-})$), then poverty is more severe for population B when compared with population A.

It is worth observing that the conditions imposed on individual poverty indices in Proposition 2 are very general and thus can be applied to measures that are not based on generalized means. For instance, the use of the generalized means $\mu_\beta(.)$ and $\mu_\gamma(.)$ entails that poverty indices are symmetric, *i.e.* poverty level is left unchanged after switching the values of the wellbeing attribute of a given individual between the different periods ($p_c(\boldsymbol{S}\boldsymbol{x}_{n.}) = p_c(\boldsymbol{x}_{n.})$ $\forall \boldsymbol{x}_{n.} \in \Omega^T$ where $\boldsymbol{S}$ is a $T \times T$ permutation matrix). Indices presented in section 3 also exhibits convex iso-poverty hypersurfaces in $\Lambda_\beta(z^L)$ and concave iso-poverty hypersurfaces in $\Lambda_\gamma(z^H)$, while such assumptions are not imposed for Proposition 2. Consequently, if we consider that chronic non-monotone poverty indices should be of the form (7) but without assuming a precise value for β and γ, it is possible to increase the ordering power of Proposition 2 by adding more structure to our class of poverty indices.[6]

[6] Dominance conditions for symmetric intertemporal poverty indices have recently been proposed in Bresson and Duclos (2012) and can easily be extended in order to fit our non-monotone framework.

4 Chronic and transient non-monotonic poverty: a "spell counting" approach

Unlike the permanent income approach in which an income standard is compared against a poverty line, the spell counting approach compares the relative proportion of periods under poverty against a cut-off value. If the individual spends a proportion of time in poverty above the threshold then he/she is deemed chronically poor. Otherwise the person is deemed either non-poor (if the person is never poor in any period) or transiently poor (if some periods are spent under poverty, albeit below the threshold). In the case of non-monotonic poverty, the approach can be extended in two different ways, depending on whether we wish to introduce a distinction between "shortfall" and "excess" forms of chronic poverty.

In the first case where identification depends on the time spent under poverty without taking into account the nature of poverty at each date, identification can be performed using the following function:

$$\phi\big(\boldsymbol{x}_{n.}; z^L, z^H, \tau\big) := \mathbb{I}\left(\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(x_{nt} < z^L \vee x_{nt} > z^H\right) \geqslant \tau\right), \tag{22}$$

where $\tau \in [0,1]$ is the cut-off function that separates the chronic poor from the rest of the population. Poverty at the individual level will then be of the form:

$$p_c(\boldsymbol{x}_{n.}; z^L, z^H) := \phi\big(\boldsymbol{x}_{n.}; z^L, z^H, \tau\big) g\big(\boldsymbol{x}_{n.}; z^L, z^H\big), \tag{23}$$

where g expresses the intensity of individual poverty. For instance, we can adapt the FGT indices proposed by Apablaza et al. (2013). If we allow the sensibility with respect to extreme forms of deprivation to differ for "shortfall" and "excess" poverty, g can take the following form:

$$g\big(\boldsymbol{x}_{n.}; z^L, z^H\big) = \frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(x_{nt} < z^L\right)\left(\frac{z^L - x_{nt}}{z^L - \omega^-}\right)^{\alpha_L} + \mathbb{I}\left(x_{nt} > z^H\right)\left(\frac{x_{nt} - z^H}{\omega^+ - z^H}\right)^{\alpha_H}, \tag{24}$$

with $\alpha_L \geqslant 1$ and $\alpha_H \geqslant 1$.

Otherwise, if we assume that chronicity is characterized by the recurrence of a specific form of deprivation, it is then necessary to make a distinction between "shortfall" and "excess" forms of chronic poverty. Let $\tau_L \in [0,1]$ and $\tau_H \in [0,1]$ be the cut-off values for "shortfall" and "excess" poverty, respectively. Then the corresponding identification functions are the following:

$$\phi_L\big(\boldsymbol{x}_{n.}; z^L, \tau_L\big) := \mathbb{I}\left(\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(x_{nt} < z^L\right) \geqslant \tau_L\right), \tag{25}$$

$$\phi_H\big(\boldsymbol{x}_{n.}; z^H, \tau_H\big) := \mathbb{I}\left(\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(x_{nt} > z^H\right) \geqslant \tau_H\right). \tag{26}$$

and the general form of individual poverty becomes:

$$p_c(\boldsymbol{x}_{n.}; z^L, z^H) := \phi(\boldsymbol{x}_{n.}; z^L, \tau_L) g(\boldsymbol{x}_{n.}; z^L, \tau) + \phi(\boldsymbol{x}_{n.}; z^H, \tau_L) g(\boldsymbol{x}_{n.}; z^H, \tau). \tag{27}$$

Concerning the aggregation step, several intensity functions can be considered. For example, we could use the formulas in (10) and (11) to measure poverty intensity in every period, in combination with the chronic-poverty identification functions in (25) and (26). Alternatively, one can decompose the function g into the following to parts:

$$g_L(\boldsymbol{x}_{n.}; z^L) = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}\left(x_{nt} < z^L\right) \left(\frac{z^L - x_{nt}}{z^L - \omega^-}\right)^{\alpha_L}, \tag{28}$$

$$g_H(\boldsymbol{x}_{n.}; z^H) = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}\left(x_{nt} > z^H\right) \left(\frac{x_{nt} - z^H}{\omega^+ - z^H}\right)^{\alpha_H}. \tag{29}$$

4.1 Identification issues in the "spell counting" approach

In the case of the "spell counting" approach, similar ambiguities in the identification of the chronically poor as the one stressed for the "permanent income" approach can also arise. The source of the problem is that the proportions of time spent in both forms of poverty are subject to different restrictions compared to the cut-off thresholds. On one hand, $\frac{1}{T}\sum_{t=1}^{T} \mathbb{I}(x_{nt} < z^L) + \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}(x_{nt} > z^H) \leqslant 1$, which means that the proportion of time spent in one form of poverty can only increase at the expense of either the proportion of time spent in the other form of poverty, or the proportion spent outside poverty. Once the proportion spent outside poverty reaches 0, then one proportion in poverty can only increase at the expense of the other one. On the other hand, the only restriction for τ_L and τ_H is that they take a real value between 0 and 1, *independently of each other*. Therefore, if the two cut-offs are simultaneously small enough (i.e. chronic poverty identification tends toward a union approach), then some people could be considered "shortfall" and "excess" poor at the same time! By contrast, with relatively high values of τ_L and τ_H, this problem is less likely to occur, given the aforementioned restriction on the proportions of time spent in both forms of poverty: they cannot be both high at the same time.

Nevertheless, observing individuals n such that $\phi_L(\boldsymbol{x}_{n.}; z^L, \tau_L) = \phi_H(\boldsymbol{x}_{n.}; z^H, \tau_H) = 1$ is only a problem if we do not make a distinction between two different forms of chronic poverty, that is "shortfall" chronic poverty and "excess" chronic poverty. If the two forms are regarded as conceptually different and can cumulate over an individual's lifetime, there is no inconsistency of having overlapping domains for the two identification functions.

However, with high value of τ_L and τ_H, we may face the opposite problem of having individuals not being considered as poor though being either "shortfall" or "excess" poor at each period. For instance, this will be the case if $\tau_L = \tau_H = 60\%$ but individuals spend 50% of their lifetime in "shortfall" poverty and the remaining half in "excess" poverty. This situation is only a problem if chronic poverty is thought as being in poverty (either "shortfall" or "excess" poverty) for a large proportions of period. If we make a distinction between chronic "shortfall" and chronic "excess" poverty,

Finally, it turns out that the results of Table 1 also hold true in the case of the spell counting approach. But since this approach does not rely on choices of parameters for general means, the only way to minimize ambiguities, in addition to choosing high values for τ_L and τ_H, is to widen the gap between the lower and upper poverty lines.

4.2 Transient poverty

As in the case of the permanent income approach, we can propose transient poverty indices in the spell counting approach, by way of adequate identification functions. As we did with (15) and eq16, we use the following identification functions:

$$\phi_L^{tra} = \left| 1 - \prod_{t=1}^{T} \mathbb{I}\big(x_{nt} > z^L\big) \right| - \phi_L\left(\boldsymbol{x}_{n.}; z^L, \tau_L\right), \tag{30}$$

$$\phi_H^{tra} = \left| 1 - \prod_{t=1}^{T} \mathbb{I}\big(x_{nt} < z^H\big) \right| - \phi_H\left(\boldsymbol{x}_{n.}; z^H, \tau_H\right). \tag{31}$$

Then the several intensity functions can be used. For instance, either the batch of (10) and (11), or an FGT set like (28) and (29) could work.

5 Empirical illustration

5.1 Data and estimation choices

We use the National Occupation and Employment Survey of Mexico (Encuesta Nacional de Ocupación y Empleo, ENOE); a nationally representative panel data set collected between 2005 and 2011. The survey provides information on time allocation to work, domestic chores, study, children or other household members (ill or elder), house building or maintenance, and community services. For the purposes of our analysis, we calculate the weekly amount of leisure for every individual included in the sample as the difference between the amount of available hours in a week (168) and the amount of time allocated to all activities: work, domestic chores, study, child care, house building and maintenance, and community services. We construct a balanced panel of individuals (5 quarters), and present results for two sub-samples: (i) individuals in the 18-to-65 age cohort (653,483), and (ii) individuals who are between 18 and 65, and currently engaged in paid work in a non-agriculture activity (141,005).We define the leisure poverty thresholds as follows: If an individual has less than 80 weekly hours of leisure (z^L), i.e. less than 11.5 hours per day, the individual is considered in "shortfall" poverty. However, if the individual has more than 140 weekly hours of leisure (z^H), i.e. more than 20 hours per day, the individual is considered in "excess" poverty.

5.2 Results

5.2.1 Static results

Table 2 presents static headcount poverty rates for both male and female subsamples, between 2005 and 2010. When considering men and women who were between 18-65 years old, we find that "shortfall" poverty rates decreased for both of them, while "excess" poverty rates increased. This trend is accompanied by an important increase in the contribution of "excess" poverty rates to time poverty, particularly for women, going from 54.8 to 94.1%—about 40 percentage points—between 2005 and 2010. When considering men and women in the previous age bracket and making earnings for work in non-agriculture activities we find similar results. However, the increase in the contribution of "excess" poverty to female time poverty between 2005 and 2010 is about 61 percentage points. These trends suggest that women earning money in non-agriculture activities are allocating more time to leisure activities. In this context further analysis regarding hourly payments and time allocated to other activities is warranted.

[Insert Table 2 here.]

5.2.2 Non-monotonic poverty transitions

Table 3 presents poverty transitions for individuals who were 18 or older. The results suggest that most individuals who were non-poor and "excess" poor during the first quarter they were observed, remained in a similar situation after five quarters (75% and 69%, respectively). However, most individuals who were "shortfall" poor during their first quarter became non-poor during the fifth quarter (69.3%). When considering the male subsample, 77.4% of "shortfall" poor during the first quarter became non-poor in the fifth quarter, and 10 % became "excess" poor (table 5). However, of those men who were not poor during the first quarter, 82.7% remained non-poor and 16% became "excess" poor in the fifth quarter (table 5). In the case of women we find a similar pattern. However, the share of women who were "shortfall" poor in the first quarter and then became "excess" poor in the fifth quarter is 20.9%—about 10 percentage points higher than for men (table 7).

If we consider the population in the 18-to-65 year-old cohort who has a paid job in a non-agriculture activity (table 4), the transitions rates among those who were "shortfall" poor and non-poor in the first quarter were qualitatively similar to those observed for the 18-to-65 sample. However, in the case of people working in non-agriculture activity, the share of "excess" poor in the first quarter who became non-poor in the fifth quarter is larger than the share who were still "excess" poor: 57.3 vs. 42%, respectively. These results are driven by the male sub-sample: of those who were "excess" poor in the first quarter, 68.8% percent became non-poor and 30.6% remained "excess" poor (table 6).

[Insert Table 3 here.]

[Insert Table 4 here.]

[Insert Table 5 here.]

[Insert Table 6 here.]

[Insert Table 7 here.]

[Insert Table 8 here.]

5.2.3 "Permanent income" results

Table 9 shows the "shortfall" and "excess" chronic time poverty headcounts according to the permanent income approach for the 18-to-65 year-old sample. About 36 % of the sample is identified as chronically poor based on this approach. This rate is driven mostly by the share of "excess" chronic poverty. The results also suggest that chronic poverty is higher among women than men: 48.8 vs. 20.4 %, respectively. In both cases, "excess" chronic poverty contributes the largest share. At the same time, the incidence of chronic time poverty is higher among individuals with no education (51.5 %), decreasing, in general, for individuals who have attained more education (e.g. it is only 21.7 % for individuals with postgraduate education). Chronic time poverty rates are substantially lower if we constraint the sample to 18-to-65 year-old individuals working in non-agriculture activity (table 10): 3.8 %. This reduction is accompanied by an increase in the contribution of "shortfall" time poverty to 25.9 %. The reduction in chronic time poverty rates is also accompanied by a reduction in the gender gap to 3.9 % points in favor of women. In terms of education level, we find a similar trend to the one observed for the 18-to-65 sample.

[Insert Table 9 here.]

[Insert Table 10 here.]

Table 11 presents the results for chronic non-monotonic time poverty when considering intensity functions, based on the permanent income approach, for the 18-to-65 year-old sample. The results indicate that women are more poor than men: 1.223 and 0.504, respectively. In line with the headcount results presented in table 9, non-educated individuals are more chronic time poor than more educated individuals. For comparison purposes, the table also shows the results based only on "shortfall" poverty. Clearly, incorporating "excess" time poverty makes a substantial difference.

Chronic non-monotonic time poverty decreases significantly when we constrain the sample to individuals in the 18-to-65 year-old age cohort who work for a remuneration in a non-agriculture activity 12. This reduction is accompanied by a smaller gender gap, although women are still more chronically time-poor than men. In terms of educational attainment, we find a similar trend to the one observed for the 18-to-65 year-old sub-sample.

[Insert Table 11 here.]

[Insert Table 12 here.]

5.2.4 "Spell counting" results

Table 13 presents the chronic time poverty headcounts under the spell-counting approach. About 36.1 % of the individuals in the 18-to-65 year-old cohort are considered chronic poor. In line with the permanent income approach, these results are driven by "excess" chronic time poverty. At the same time, chronic poverty is larger among women than men (50.2 vs. 19.5 %), and shows an inverse relation with educational attainment. While 51.1 % of individuals with no education are chronic time poor, only 19.2 % of individuals with postgraduate education fall in this category. When considering the sub-sample of 18-to-65 year-old individuals who have a paid job in a non-agriculture activity (table 14), we find a lower chronic poverty rate: 5.9 %. This reduction is accompanied by an increase in the contribution of "shortfall" time poverty, from 1.2 to 17.6 %. In addition, the gender gap fell to 7.6 percentage points in favour of women. In terms of educational attainment, the observed pattern persists: more education is related to lower chronic poverty.

[Insert Table 13 here.]

[Insert Table 14 here.]

When considering intensity functions in the spell-counting approach, we find similar trends: more chronic time poverty among women than men, as well as among less educated individuals. However, despite showing similar headcount results to the permanent income approach, when considering intensity functions the counting-spell approach provides lower values of the social measure of chronic non-monotonic poverty (Table 15). These values decrease even further when constraining the sample to individuals in the 18-to-65 year-old age cohort who work for a remuneration in a non-agriculture activity (Table 16). Table 17 provides the results for chronic non-monotonic poverty when the intensity functions are based on an FGT index, for $\alpha_L = 1, 2$. The results suggest lower values of the social measure for any sub-sample. In a similar trend, when constraining the sample to individuals in the 18-to-65 age bracket who have a paid job in a non-agriculture activity, the value of the social measure decrease even further (Table 18).

[Insert Table 15 here.]

[Insert Table 16 here.]

[Insert Table 17 here.]

[Insert Table 18 here.]

6 Concluding remarks

In this paper we explore the possibility that both leisure time scarcity and leisure time abundance can be symptomatic of substandard wellbeing, based on a *a non-monotonic relationship to wellbeing*. For this purpose, we combine the toolkit of Apablaza et al. (2013) with different approaches to inter-temporal poverty measurement, in order to study the dynamics of *non-mononotic poverty*. Specifically, we propose measures of chronic and transient non-monotonic poverty following both the "permanent income" (Jalan and Ravallion, 2000, Foster and Santos, 2013) and the "spell counting" (Foster, 2009) approaches. Then we propose measures of inter-temporal poverty (which do not distinguish between chronic and transient experiences, but are sensitive to the timing of spells) using the approach of Gradin et al. (2012). To illustrate the empirical contribution of these measures we use the ENOE panel dataset from Mexico, which features a rich questionnaire on time use.

Both approaches to chronic time poverty produce similar results: about 36% of the 18-65 years old population is either "shortfall" or "excess" time poor. However, when we constraint the sample to individuals in the same age cohort but who, additionally, generate earnings from a non-agriculture activity, the permanent income approach shows higher chronic poverty headcount rates than the spell-counting approach: 3.8% vs. 5.9%, respectively. Although in both cases the poverty rates significantly drop, mainly due to a decline in "excess" poverty. The larger chronic time poverty rates under the spell counting approach for both male and female subsamples are accompanied by a narrower gender gap. Indeed, under the spell-counting approach, chronic poverty for women is about 2.5 times as great as chronic poverty for men, while under the permanent income approach, there is a ten-fold difference favouring men.

Our empirical illustration also highlights the difference made by considering "excess" poverty on top of traditional "shortfall" time poverty analysis. When we consider "shortfall" poverty only, chronic time poverty headcounts under both permanent income and spell counting approaches, are lower. For instance, under the permanent income approach, about 1 percent of 18-65 year-old individuals in non-agricultural activities are chronically time poor in the traditional sense of "shortfall" poverty. However, this figure increases four-fold when we take into account individuals who have "excess" chronic time poverty.

For future research, it is also much worth considering combining the non-monotonic poverty framework with measures of inter-temporal poverty which do not distinguish between chronic and transient experiences, but are sensitive to the timing of spells) using recent developments in the related literature.

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A Proof of Proposition 2

The condition for $\Lambda_\beta(z^{L+})$ is the one proposed in Theorem 1 in Duclos et al. (2006). Considering the upper part of the poverty domain, please first note that the corresponding poverty measure can be expressed as:

$$\iint_{\Lambda_\gamma(z^H)} g_H(x_1,x_2)dF(x_1,x_2) = \int_{\omega^-}^{\omega^+}\int_{z^H(x_1)}^{\omega^+} g_H(x_1,x_2)f(x_1,x_2)\,dx_2dx_1, \tag{32}$$

$$= \int_{\omega^+}^{\omega^-}\int_{\omega^+}^{z^H(x_1)} g_H(x_1,x_2)f(x_1,x_2)\,dx_2dx_1, \tag{33}$$

where $f(x_1,x_2):\Omega^2\to[0,1]$ is the joint density function.

Using the same line of reasoning as Duclos et al. (2006), one then can show that:

$$\begin{aligned}\iint_{\Lambda_\gamma(z^H)} g_H(x_1,x_2)dF(x_1,x_2) = &\int_{z^H(x_1)}^{\omega^+} g_H^{(2)}(\omega^-,x_2)\overline{F}(\omega^-,x_2)\,dx_2 \\ &-\int_{\omega^-}^{\omega^+} z^{H(1)}(x_1)g_H^{(2)}(x_1,z_H(x_1))\overline{F}(x_1,z_H(x_1))\,dx_1 \\ &+\int_{\omega^-}^{\omega^+}\int_{z^H(x_1)}^{\omega^+} g_H^{(1,2)}(x_1,x_2)\overline{F}(x_1,x_2)\,dx_2dx_1,\end{aligned} \tag{34}$$

where $z^{H(1)}(x_1) := \frac{\partial z_H}{\partial x_1}$, $g_H^{(t)}(x_1,x_2) := \frac{\partial g_H}{\partial x_t}$, and $g_H^{(t,r)}(x_1,x_2) := \frac{\partial^2 g_H}{\partial x_t \partial x_r}$.

The overall poverty level can then be expressed as:

$$\begin{aligned}P = &-\int_{\omega^-}^{z^L(x_1)} g_L^{(2)}(\omega^+,x_2)F(\omega^+,x_2)\,dx_2 \\ &+\int_{\omega^-}^{\omega^+} z^{L(1)}(x_1)g_L^{(2)}(x_1,z_L(x_1))F(x_1,z_L(x_1))\,dx_1 \\ &+\int_{\omega^-}^{\omega^+}\int_{\omega^-}^{z^L(x_1)} g_L^{(1,2)}(x_1,x_2)F(x_1,x_2)\,dx_2dx_1 \\ &+\int_{z^H(x_1)}^{\omega^+} g_H^{(2)}(\omega^-,x_2)\overline{F}(\omega^-,x_2)\,dx_2 \\ &-\int_{\omega^-}^{\omega^+} z^{H(1)}(x_1)g_H^{(2)}(x_1,z_H(x_1))\overline{F}(x_1,z_H(x_1))\,dx_1 \\ &+\int_{\omega^-}^{\omega^+}\int_{z^H(x_1)}^{\omega^+} g_H^{(1,2)}(x_1,x_2)\overline{F}(x_1,x_2)\,dx_2dx_1.\end{aligned} \tag{35}$$

The necessary and sufficient conditions for Proposition 2 follow upon inspection.

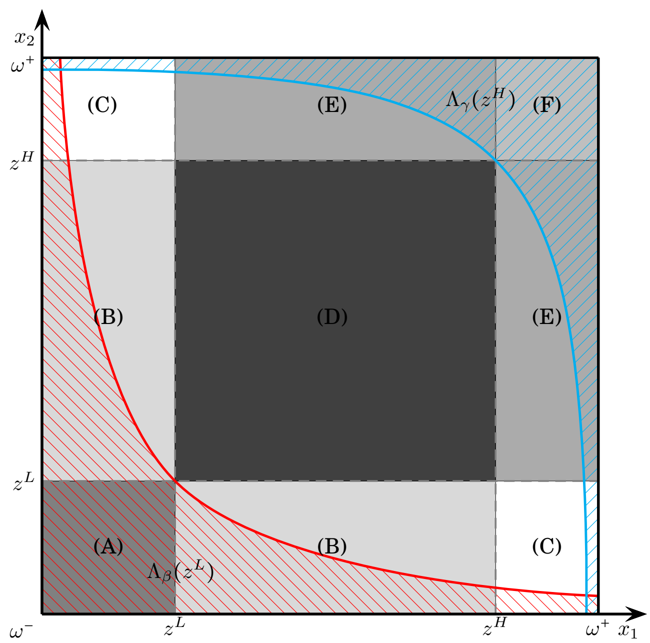

Figure 1: The identification issue in the two-period non-monotonic case with the "permanent income" approach.

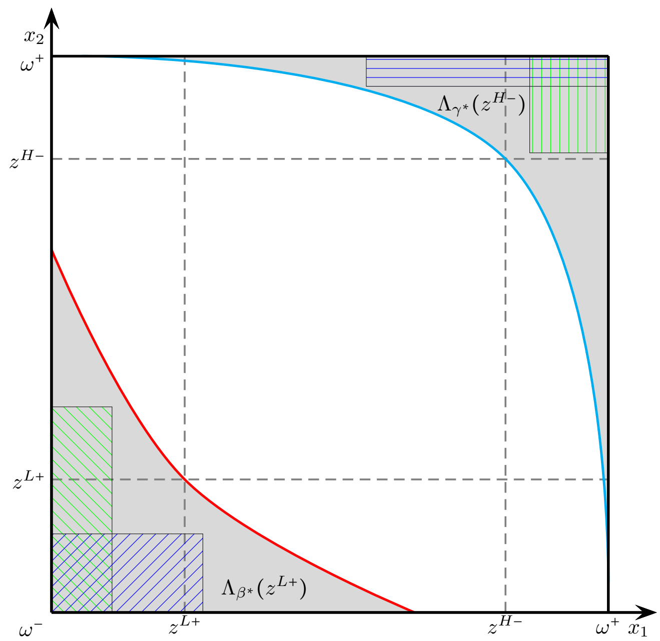

Figure 2: Bidimensional non-monotone poverty dominance.

Table 1: Chronic non-monotonic poverty identification in the permanent income approach

Situation	Shorftall poor?	Not poor?	Excess poor?
(A) $\max\{\boldsymbol{x}_{n.}\} < z^L$	Always	Never	Never
(B) $\min\{\boldsymbol{x}_{n.}\} < z^L < \max\{\boldsymbol{x}_{n.}\} < z^H$	Depends	Depends	Never
(C) $\min\{\boldsymbol{x}_{n.}\} < z^L < z^H < \max\{\boldsymbol{x}_{n.}\}$	Depends	Depends	Depends
(D) $z^L < \min\{\boldsymbol{x}_{n.}\} < \max\{\boldsymbol{x}_{n.}\} < z^H$	Never	Always	Never
(E) $z^L < \min\{\boldsymbol{x}_{n.}\} < z^H < \max\{\boldsymbol{x}_{n.}\}$	Never	Depends	Depends
(F) $z^H < \min\{\boldsymbol{x}_{n.}\}$	Never	Never	Always

Table 2: "Shortfall" and "excess" time poverty headcounts by gender, Mexico.

			18-65		18-65, Paid work Non-Agriculture	
			Mean	SD	Mean	SD
2005	Male	Shortfall	0.030	0.170	0.163	0.369
		Excess	0.043	0.202	0.028	0.166
	Female	Shortfall	0.075	0.263	0.093	0.290
		Excess	0.149	0.356	0.008	0.088
2006	Male	Shortfall	0.017	0.130	0.210	0.408
		Excess	0.024	0.154	0.045	0.208
	Female	Shortfall	0.036	0.186	0.382	0.486
		Excess	0.071	0.258	0.091	0.288
2007	Male	Shortfall	0.013	0.114	0.237	0.425
		Excess	0.019	0.137	0.051	0.221
	Female	Shortfall	0.020	0.140	0.517	0.500
		Excess	0.038	0.192	0.131	0.338
2008	Male	Shortfall	0.014	0.116	0.243	0.429
		Excess	0.02	0.141	0.056	0.23
	Female	Shortfall	0.020	0.140	0.517	0.500
		Excess	0.04	0.195	0.135	0.342
2009	Male	Shortfall	0.013	0.112	0.272	0.445
		Excess	0.02	0.139	0.065	0.246
	Female	Shortfall	0.019	0.137	0.527	0.499
		Excess	0.039	0.195	0.143	0.350
2010	Male	Shortfall	0.015	0.121	0.259	0.438
		Excess	0.025	0.155	0.061	0.239
	Female	Shortfall	0.028	0.164	0.445	0.497
		Excess	0.057	0.232	0.116	0.320

Table 3: Transition matrix of poverty status, Mexico all, 18-65.

		5th trimester		
		Shortfall	Non-poor	Excess
	Shortfall	0.131	0.693	0.176
1st Trim.	Non-poor	0.019	0.748	0.233
	Excess	0.005	0.304	0.691

Table 4: Transition matrix of poverty status, Mexico all, 18-65, paid work, non-agriculture.

		5th trimester		
		Shortfall	Non-poor	Excess
1st Trim.	Shortfall	0.179	0.789	0.033
	Non-poor	0.023	0.922	0.056
	Excess	0.007	0.573	0.42

Table 5: Transition matrix of poverty status, Mexico Men, 18-65.

		5th trimester		
		Shortfall	Non-poor	Excess
1st Trim.	Shortfall	0.126	0.774	0.100
	Non-poor	0.013	0.827	0.160
	Excess	0.005	0.450	0.544

Table 6: Transition matrix of poverty status, Mexico Men, 18-65, paid work, non-agriculture.

		5th trimester		
		Shortfall	Non-poor	Excess
	Shortfall	0.161	0.82	0.019
1st Trim.	Non-poor	0.017	0.941	0.042
	Excess	0.006	0.688	0.306

Table 7: Transition matrix of poverty status, Mexico Women, 18-65.

		5th trimester		
		Shortfall	Non-poor	Excess
1st Trim.	Shortfall	0.133	0.658	0.209
	Non-poor	0.027	0.656	0.317
	Excess	0.006	0.236	0.758

Table 8: Transition matrix of poverty status, Mexico Women, 18-65, paid work, non-agriculture.

		5th trimester		
		Shortfall	Non-poor	Excess
1st Trim.	Shortfall	0.19	0.768	0.042
	Non-poor	0.034	0.884	0.082
	Excess	0.007	0.476	0.516

Table 9: "Shortfall" and "excess" chronic time poverty headcounts according to the "permanent income" approach, Mexico, 18-65.

	Total		% Shortfall		% Excess	
	Mean	SD	Mean	SD	Mean	SD
Total	0.358	0.479	0.011	0.102	0.989	0.102
Gender						
Male	0.204	0.403	0.014	0.117	0.986	0.117
Female	0.488	0.5	0.009	0.096	0.991	0.096
Education level						
None-5	0.515	0.5	0.004	0.067	0.996	0.067
Primary	0.405	0.491	0.009	0.096	0.991	0.096
Secondary	0.318	0.466	0.016	0.124	0.984	0.124
Bachiller	0.33	0.47	0.012	0.107	0.988	0.107
Superior	0.338	0.473	0.009	0.093	0.991	0.093
Post-graduate	0.217	0.412	0.005	0.068	0.995	0.068

Note: $\beta = 0.9$ and $\gamma = 1.1$

Table 10: "Shortfall" and "excess" chronic time poverty headcounts according to the "permanent income" approach, Mexico, 18-65, paid work, non-agriculture.

	Total		% Shortfall		% Excess	
	Mean	SD	Mean	SD	Mean	SD
Total	0.038	0.191	0.259	0.438	0.741	0.438
Gender						
Male	0.024	0.153	0.257	0.437	0.743	0.437
Female	0.243	0.439	0.261	0.063	0.739	0.439
Education level						
None-5	0.077	0.267	0.134	0.341	0.866	0.341
Primary	0.05	0.218	0.225	0.418	0.775	0.418
Secondary	0.032	0.176	0.347	0.476	0.653	0.476
Bachiller	0.159	0.484	0.373	0.026	0.627	0.484
Superior	0.037	0.189	0.201	0.401	0.799	0.401
Post-graduate	0.04	0.195	0.064	0.247	0.936	0.247

Note: $\beta = 0.9$ and $\gamma = 1.1$

Table 11: Chronic non-monotonic time poverty according to the "permanent income" approach, with intensity functions (8) and (9), Mexico, 18-65.

	Non-monotonic		Only Shortfall	
	Mean	SD	Mean	SD
Total	0.894	1.271	0.0003	0.006
Gender				
Male	0.504	1.013	0.0002	0.004
Female	1.223	1.275	0.0004	0.007
Education level				
None-5	1.334	1.318	0.0002	0.005
Primary	1.018	1.254	0.0003	0.006
Secondary	0.786	1.173	0.0003	0.007
Bachiller	0.821	1.189	0.0003	0.006
Superior	0.840	1.191	0.0002	0.005
Post-graduate	0.523	1.006	0.0001	0.004

Note: $\beta = 0.9$ and $\gamma = 1.1$

Table 12: Chronic non-monotonic time poverty according to the "permanent income" approach, with intensity functions (8) and (9), Mexico, 18-65, paid work, non-agriculture.

	Non-monotonic		Only Shortfall	
	Mean	SD	Mean	SD
Total	0.064	0.374	0.0007	0.01
Gender				
Male	0.041	0.299	0.0004	0.006
Female	0.107	0.478	0.0013	0.014
Education level				
None-5	0.152	0.565	0.0008	0.011
Primary	0.089	0.44	0.0008	0.011
Secondary	0.049	0.328	0.0008	0.01
Bachiller	0.038	0.288	0.0007	0.01
Superior	0.067	0.379	0.0006	0.009
Post-graduate	0.082	0.416	0.0003	0.008

Note: $\beta = 0.9$ and $\gamma = 1.1$

Table 13: Chronic time poverty headcounts according to the "spell-counting" approach, Mexico, 18-65.

	Total		% Shortfall		% Excess	
	Mean	SD	Mean	SD	Mean	SD
Total	0.361	0.48	0.012	0.11	0.988	0.11
Gender						
Male	0.195	0.396	0.017	0.131	0.983	0.131
Female	0.502	0.5	0.01	0.102	0.99	0.102
Education level						
None-5	0.511	0.5	0.005	0.07	0.995	0.07
Primary	0.41	0.492	0.011	0.103	0.989	0.103
Secondary	0.327	0.469	0.017	0.128	0.983	0.128
Bachiller	0.331	0.471	0.015	0.12	0.985	0.12
Superior	0.338	0.473	0.011	0.103	0.989	0.103
Post-graduate	0.192	0.394	0.009	0.095	0.991	0.095

Note: $\tau_L = \tau_H = 0.6$.

Table 14: Chronic time poverty headcounts according to the "spell-counting" approach, Mexico, 18-65, paid work, non-agriculture.

	Total		% Shortfall		% Excess	
	Mean	SD	Mean	SD	Mean	SD
Total	0.059	0.235	0.176	0.381	0.824	0.381
Gender						
Male	0.032	0.175	0.216	0.412	0.784	0.412
Female	0.108	0.31	0.155	0.362	0.845	0.362
Education level						
None-5	0.11	0.312	0.086	0.281	0.914	0.281
Primary	0.073	0.26	0.162	0.369	0.838	0.369
Secondary	0.047	0.212	0.23	0.421	0.77	0.421
Bachiller	0.038	0.192	0.291	0.454	0.709	0.454
Superior	0.068	0.252	0.124	0.329	0.876	0.329
Post-graduate	0.06	0.238	0.068	0.252	0.932	0.252

Note: $\tau_L = \tau_H = 0.6$.

Table 15: Chronic non-monotonic time poverty according to the "spell-counting" approach, with intensity functions (10) and (11), 18-65, Mexico.

	Non-monotonic		Only Shortfall	
	Mean	SD	Mean	SD
Total	0.226	0.323	0.0005	0.008
Gender				
Male	0.114	0.255	0.0003	0.006
Female	0.320	0.344	0.0006	0.01
Education level				
None	0.343	0.367	0.0003	0.006
Primary	0.260	0.334	0.0005	0.009
Secondary	0.205	0.313	0.0006	0.009
Bachiller	0.210	0.317	0.0005	0.009
Superior	0.203	0.310	0.0004	0.008
Post-graduate	0.100	0.230	0.0002	0.005

Note: $\tau_L = \tau_H = 0.6$.

Table 16: Chronic non-monotonic time poverty according to the "spell-counting" approach, with intensity functions (10) and (11), paid work, non-agriculture, Mexico.

	Non-monotonic		Only Shortfall	
	Mean	SD	Mean	SD
Total	0.014	0.066	0.0011	0.013
Gender				
Male	0.007	0.045	0.0006	0.009
Female	0.027	0.091	0.002	0.018
Education level				
None	0.031	0.1	0.001	0.013
Primary	0.02	0.08	0.0013	0.014
Secondary	0.012	0.063	0.0011	0.013
Bachiller	0.009	0.053	0.0011	0.013
Superior	0.013	0.06	0.0009	0.012
Post-graduate	0.011	0.052	0.0005	0.009

Note: $\tau_L = \tau_H = 0.6$.

Table 17: Chronic non-monotonic time poverty according to the "spell-counting" approach, with intensity functions (28) and (29), 18-65, Mexico.

	Non-monotonic				Only shortfall			
	$\alpha_L = \alpha_H = 1$		$\alpha_L = \alpha_H = 2$		$\alpha_L = \alpha_H = 1$		$\alpha_L = \alpha_H = 2$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Total	0.226	0.323	0.213	0.314	0.0005	0.009	0.0005	0.009
Gender								
Male	0.115	0.255	0.104	0.241	0.0003	0.007	0.0003	0.007
Female	0.320	0.344	0.305	0.338	0.0007	0.011	0.0007	0.011
Education level								
None	0.343	0.367	0.324	0.359	0.0003	0.007	0.0003	0.007
Primary	0.260	0.355	0.245	0.326	0.0005	0.009	0.0005	0.009
Secondary	0.205	0.314	0.194	0.305	0.0007	0.01	0.0007	0.01
Bachiller	0.210	0.317	0.199	0.308	0.0005	0.009	0.0005	0.009
Superior	0.204	0.310	0.190	0.300	0.0004	0.008	0.0004	0.008
Post-graduate	0.100	0.230	0.089	0.216	0.0002	0.006	0.0002	0.006

Note: $\tau_L = \tau_H = 0.6$.

Table 18: Chronic non-monotonic time poverty according to the "spell-counting" approach, with intensity functions (28) and (29), 18-65, paid work, non-agriculture, Mexico.

	Non-monotonic				Only shortfall			
	$\alpha_L = \alpha_H = 1$		$\alpha_L = \alpha_H = 2$		$\alpha_L = \alpha_H = 1$		$\alpha_L = \alpha_H = 2$	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Total	0.014	0.067	0.007	0.041	0.0012	0.014	0.0004	0.005
Gender								
Male	0.007	0.046	0.003	0.027	0.0007	0.009	0.0002	0.003
Female	0.027	0.092	0.014	0.058	0.0022	0.019	0.0007	0.008
Education level								
None	0.031	0.1	0.018	0.064	0.0011	0.014	0.0003	0.006
Primary	0.02	0.08	0.011	0.05	0.0014	0.015	0.0004	0.006
Secondary	0.012	0.063	0.006	0.04	0.0012	0.014	0.0004	0.005
Bachiller	0.009	0.053	0.005	0.033	0.0012	0.014	0.0004	0.006
Superior	0.013	0.06	0.006	0.036	0.001	0.013	0.0003	0.005
Post-graduate	0.011	0.053	0.005	0.028	0.0005	0.01	0.0002	0.005

Note: $\tau_L = \tau_H = 0.6$.