"Pro-poorest" Poverty Reduction with Counting Measures

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Abstract

Under which conditions does poverty reduction not only reduce the average poverty score further but also decrease deprivation inequality among the poor more, thereby emphasizing improvements among the poorest of the poor? Firstly, when comparing cross-sectional datasets of the same society in different periods of time (i.e. an anonymous assessment), we derive a simple second-order dominance condition based on reverse generalized Lorenz curves*, whose fulfillment ensures that multidimensional poverty decreases along with a reduction in deprivation inequality for a broad family of inequality-sensitive poverty measures. Secondly, when we can track poverty experiences of the same individuals or households using panel datasets (i.e. a non-anonymous assessment), we adapt and extend a theorem from Benabou and Ok (2001), whose fulfillment allows us to conclude that multidimensional poverty reduction is more egalitarian in one experience vis-à-vis another one, for a broad family of poverty indices which are sensitive to deprivation inequality among the poor, and from an* ex-ante *conception of inequality of opportunity. We illustrate these methods with an application to multidimensional poverty in Peru before and after the 2008 world financial crisis.*

Keywords: Pro-poorest poverty reduction, multidimensional poverty, reverse generalized Lorenz curve.
JEL Classification: I32.

1 Introduction

The “pro-poor" nature of income or per-capita GDP growth has received much attention from both academics and policymakers for the last couple of decades. While a straightforward notion regards income growth to be “pro-poor" when the poor’s incomes rise, a more interesting notion declares growth to be “pro-poor" when the income of the poorest grows faster than the income of the less poor. Whenever income grows monotonically faster at lower initial quantiles, “pro-poor" growth reduces inequality according to a broad family of Lorenz-consistent measures. The related literature on pro-poor concepts, dominance conditions and indices is vast. (See, for instance, Deutsch and Silber (2011); for a review).

Now the “pro-poor" growth literature has traditionally worked with one continuous variable. However recently there has been an interest in connecting the “pro-poor" growth concepts with non-monetary measures of well-being, and multidimensional poverty indices in particular. For instance, Berenger and Bresson (2012) provide dominance conditions to probe the “pro-poorness" of growth when well-being is measured jointly by continuous and discrete variables. Ben Haj Kacem (2013) measures the “pro-poorness" of growth in income when the initial conditioning situation is not income itself but a non-monetary multidimensional index of poverty or well-being. Boccanfuso et al. (2009) apply the now traditional “pro-poor" growth toolkit to assess changes in the individual scores of a non-monetary poverty composite index, where the weights are determined by multiple correspondence analysis (MCA). Since they use a vast number of indicators, their scores can take several values, thereby mimicking a continuous variable.

In this paper we pose a related question in the context of *multidimensional poverty counting measures*: What are the conditions under which a poverty reduction experience is more “pro-poorest" than another one? In other words, under which conditions does poverty reduction not only reduce the average poverty score further but also decrease deprivation inequality among the poor more? In order to answer these questions we first address the most common anonymous assessment which compares cross-sectional datasets of the same society in different periods of time. In this context, Boccanfuso et al. (2009) have already shown a way in which the “pro-poor” measurement toolkit for continuous variables can be applied to the case of non-monetary deprivations if a composite index is constructed based on them, using data reduction techniques (e.g. MCA). However, in many empirical applications, the number of indicators may not be large enough, so that the number of values that the individual deprivation score can take is quite limited, for a given set of weights and deprivation lines. Hence an anonymous assessment of "pro-poor" growth linking initial-period and final-period quantiles is not really feasible. Instead we derive a simple second-order dominance condition based on *reverse generalized Lorenz curves*, which is suitable for deprivation scores with value domains of any breadth. The condition's fulfillment ensures that multidimensional poverty decreases along with a reduction in deprivation inequality for a broad family of inequality-sensitive poverty measures.

When we have a panel dataset we can also perform a non-anonymous assessment of pro-poorest poverty reduction, in which we take into account the particular poverty transitions experienced by individuals or households (depending on the unit of analysis).[1] For this purpose, we adapt and extend a theorem from Benabou and Ok (2001), who work with transition matrices. When our conditions are fulfilled then one can state that multidimensional poverty reduction is more egalitarian in one experience vis-à-vis another one, for a broad family of poverty indices which are sensitive to deprivation inequality among the poor, and from an *ex-ante* conception of inequality of opportunity. [2]

We illustrate both the non-anonymous and the anonymous conditions using a yearly panel dataset from the Peruvian National Household Surveys spanning two periods: 2002-2007 and 2007-2010 (and the respective cross-sections for the anonymous analysis). In the former period, Peru experienced a commodity boom, which translated into high GDP growth rates, from 4 % in 2003 to 8.9 % in 2007, and a steady decrease in monetary poverty headcounts, from 58.7 % in 2004 to 42.4 % in 2007. However, between 2008 and 2013, Peru's economic performance was affected by the world economic situation: GDP growth fell from 9.8 % in 2008 to 0.9 % in 2009, and then stabilizing around 7 % between 2010 and 2012. Notwithstanding this fluctuation, monetary poverty levels kept decreasing steadily, from 37.3 to 27.8 %. But how did the Peruvian population fare in terms of non-monetary multidimensional poverty? We measure non-monetary poverty with wellbeing indicators corresponding to four dimensions: household education, dwelling material infrastructure, access to services, and vulnerability related to household dependency burden.

In the anonymous assessment we compare the reverse generalized Lorenz (RGL) curves between 2002 and 2013 for the whole country, for urban and rural areas, and for each of the 25 Peruvian departments (provinces) and autonomous territories. We

[1]See Grimm (2007), for a thorough discussion of the distinction between anonymous and non-anonymous analysis of pro-poor growth in the continuous-variable context.

[2]See Fleurbaey and Peragine (2013) for an explanation of the distinction between *ex-ante* and *ex-post* inequality of opportunity.

find that the observed egalitarian reduction of our measure of poverty, between 2002 and 2013, is robust to different choices of poverty functions and poverty-identification cut-offs in 14 out of the 25 departments. However, in the other 11 departments this trend is not robust. In all these latter cases we find, instead, that the RGL curves cross once at the cut-off point where the intersection approach to poverty identification holds (i.e. when a household needs to be deprived in every dimension in order to be deemed poor). Explanations for these results are provided below.

In the anonymous assessment we rank each *poverty transition* (2002-2004, 2004-2006, 2007-2008, and 2008-2010) according to their degree of *ex-ante* “pro-poorest" poverty reduction, i.e. the extent to which they reduce expected poverty while reducing inequality among the poor at the same time, with an added interest to assess whether the ranking is affected by the particular economic conditions that characterized each observed period. We find that the mobility matrix of 2002-2004 is the pro-poorest in the sense of yielding a distribution of expected poverty experiences that second-order dominates the distributions induced by all the other matrices. Then, the matrix of 2004-2006 turns up as the second-best, since its expected distribution dominates those of 2007-2008 and 2008-2010, while being dominated by its predecessor's. However we cannot order the latter two matrices vis-a-vis each other. In summary, the pre-crisis poverty transitions induced preferable distributions of expected poverty scores from a welfare-utilitarian point of view.

The rest of the paper proceeds as follows: The next section presents our “pro-poorest” poverty-reduction conditions. First, it introduce the family of counting poverty measures for which the conditions are relevant and applicable, then it shows the condition for the anonymous case, followed by the condition for the non-anonymous case. The third section provides the empirical illustration on multidimensional poverty reduction in Peru. Finally, the paper concludes with some remarks.

2 Pro-poorest poverty reduction with counting measures

2.1 Inequality-sensitive poverty measures

Consider N individuals and D indicators of wellbeing. x_{nd} stands for the level of attainment by individual n on indicator d. If $x_{nd} < z_d$, where z_d is a deprivation line for indicator d, then we say that individual n is deprived in indicator d. In order to account for the breadth of deprivations, counting measures rely on individual deprivation scores which produce a weighted count of deprivations. If the weights are denoted by: $w_d \in [0,1] \subset \mathbb{R}_+ | \sum_{d=1}^{D} w_d = 1$, then the deprivation score for individual n is: $c_n \equiv \sum_{d=1}^{D} w_d \mathbb{I}(x_{nd} < z_d)$, where $\mathbb{I}$ is the indicator function. [3] Following Alkire and Foster (2011) we can also identify those multidimensionally poor with a flexible counting approach that compares each c_n against a multidimensional cut-off $k \in [0,1] \subset \mathbb{R}_+$, so that person n is poor if and only if: $c_n \geq k$.

Our analysis focuses on a family of social poverty counting measures that are symmetric across individuals, additively decomposable (hence also subgroup consistent), scale invariant and population-replication invariant. If $p_n : c_n \times k \to [0,1] \in \mathbb{R}_+$ is the individual poverty measure, and $P : [0,1]^N \to [0,1]$ is the social poverty measure then our family is the following:

$$P = \frac{1}{N} \sum_{n=1}^{N} p_n \tag{1}$$

Our conditions of pro-poorest poverty reduction will also be useful for a broader family of subgroup consistent measures: $Q = H(P)$ as long as $H()$ is a strictly increasing, continuous function. For the sake of subgroup consistency, the weights must be set exogenously. Additionally we want P to fulfill the following key properties:

Axiom 1. *Focus (FOC): P should not be affected by changes in the deprivation score of a non-poor person as long as for this person it is always the case that: $c_n < k$.*

Axiom 2. *Monotonicity (MON): P should increase whenever c_n increases and n is poor.*

Axiom 3. *Progressive deprivation transfer (PROG): A rank-preserving transfer of a deprivation from a poorer individual to a less poor individual, such that both are deemed poor, should decrease P.*

In relation to the latter axiom, there are different approaches to capture sensitivity to deprivation inequality in the literature, although most of the approaches are virtually equivalent. [4] Axiom PROG is critical to the assessment of "pro-poorest" poverty reduction, as it forces social poverty indices to be sensitive to the distribution of deprivation across the poor, and to prioritize the wellbeing of the most jointly deprived among them.

In order to fulfill the above key properties, we narrow down the family of social poverty indices by rendering the functional form of p_n less implicit:

[3] Taking the value of 1 if the argument in parenthesis is true, otherwise it is equal to 0.
[4] For a comparative review of these approaches see Silber and Yalonetzky (2013). A different framework is provided by Alkire and Seth (2014).

$$P = \frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n \geq k)g(c_n), \tag{2}$$

where $\mathbb{I}(c_n \geq k)$ is the Alkire-Foster poverty identification function that also secures the fulfillment of FOC; and $g : c_n \to [0,1]$, such that: $g(0) = 0$, $g(1) = 1$, $g' > 0$ and $g'' > 0$. The function g captures the intensity of poverty, which is understood as number of deprivations in the counting approach. Several examples of g have been proposed by Chakravarty and D'Ambrosio (2006).

2.2 The anonymous case

In the counting approach, there is only one vector of possible values of c_n for each particular choice of deprivation lines and weights. Moreover it is easy to show that the maximum number of possible values is given by: $\sum_{i=0}^{D}\binom{D}{i}$. In the particular, but common, case of equal weights ($w_d = \frac{1}{D}$), the number of possible values is much smaller: $D + 1$. Hence the distribution of c_n in the sample is bound to be discrete, as there will be several individuals for every value of c_n.

In this subsection we show that for an assessment of inequality-reducing poverty reduction in the anonymous case it is necessary and sufficient to compare the reverse generalized Lorenz curve of the distribution of deprivation scores at the beginning and at the end of the time period.

A reverse generalized Lorenz curve is a function $L : [0,1] \to [0,1]$ that maps from the cumulative share s of the population, *ranked from the highest to the lowest values of* c_n, to the incomplete average of c_n, i.e. the sum of all c_n in the share s divided by N:

$$L(s) = \frac{1}{N}\sum_{n=1}^{sN} c_n \tag{3}$$

Note that $L(s)$ is none other than the "adjusted headcount ratio" of Alkire and Foster (2011) for a k such that $\frac{1}{N}\sum_{n=1}^{N}\mathbb{I}(c_n \geq k) = s$. That is, $L(s)$ is the adjusted headcount ratio corresponding to the k cut-off that yields a multidimensional headcount equal to s.

Let P^A and L^A refer, respectively, to the social poverty index and the reverse generalized Lorenz curve of population A. The following theorem enables us to assert whether a poverty reduction experience has been "pro-poorest" according to any inequality-sensitive poverty index:

Theorem 1. *$P^A < P^B$ for all P in 2 satisfying FOC, MON and PROG, if and only if* $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^B(s)$.

Proof. See Appendix. □

If A stands for final period, and B stands for initial period, then whenever the condition in theorem 1 is fulfilled, any experience of poverty reduction occurs alongside decreasing inequality among the poor, as measured by indices satisfying PROG, *for every relevant value of* k.

Finally, theorem 1 can also be restricted to apply only to a subset of relevant k values, ruling out the lowest ones below a minimum k: k_{min}. In order to proceed this way, the reverse generalized Lorenz curves need to be constructed using censored deprivation scores such that: $c_n = 0$ whenever $c_n < k_{min}$. Then theorem 1 applies only to those P which rule out poverty identification approaches with $k < k_{min}$.

2.2.1 The area under the reverse generalized Lorenz curve of deprivations

It is easy to show that the area under the reverse generalized Lorenz (RGL) curve is in itself an index of counting poverty satisfying FOC, MON and PROG. The area under the RGL curve, A, is:

$$A(k) \equiv \frac{1}{N}\sum_{n=1}^{N}[N-n+1]c_n\mathbb{I}(c_n \geq k), \tag{4}$$

where the deprivation scores c_n are ranked in descending order, so that c_1 is the highest score, and c_N is the lowest. We use the area $A(0)$ below in order to complement our robustness analysis with some quantitative poverty reduction comparisons.

2.3 The non-anonymous case

In the non-anonymous case we can track the experience of each individual across periods with a panel dataset. More precisely we can construct a transition matrix with the social probabilities of attaining a particular deprivation score in the final year of the period, conditional on having had a specific deprivation score in the initial year. Then we can compute the expected value of the deprivation score conditional on a given value of the deprivation score in the initial year, by adding the products of the conditional probabilities of attaining each score in the final year times the score itself.

Thus we have as many conditional expected values of the score as the values for the score. Then we can provide social evaluations of the distribution of the conditional expected values of the scores. For instance, we may require properties similar to MON and PROG in the social evaluation of expected values. Hence, inspired by

Benabou and Ok (2001), we can implement an ex-ante non-anonymous assessment of "pro-poorest" poverty reduction. In this assessment, we compare different transition matrices and rank them in terms of their capacity to reduce poverty, prioritizing reductions in the expected deprivation score of those who start the poorest. If this assessment is applied to samples of parents and their adult offspring (so that the initial period corresponds to the former, and the final period to the latter), or at least to relatively long periods (e.g. several years), then it can also become an analysis of ex-ante inequality of opportunity (i.e. as long as we normatively posit that poverty prospects should not depend on past poverty experiences over which there is little or no control).

Let c_n^t be the score of individual n in period t. The probability of attaining a particular score in period t conditional on a specific score attained in period $t-1$ is defined as: $m_{i|j} = \Pr[c_n^t = i | c_n^{t-1} = j]$. The array of all these probabilities (i.e. from $m_{0|0}$ to $m_{1|1}$) constitutes a transition matrix M. If the number of values for the deprivation score (given a choice of weights and deprivation lines) is v then the transition matrix is a v-dimension square matrix. For any initial score value in period $t-1$ the conditional expected score in period t is:

$$E[c_n^t | j] = \sum_{i=0}^{1} i \times m_{i|j}, \tag{5}$$

where the sum in 5 has v elements. Consider also a v-dimensional vector Π containing the probability distribution of scores in period $t-1$: $\Pi := (\pi(0), ...\pi(1))$.[5] Now using superscripts to denote populations where appropriate (so, for instance, $E^A[c_n^t | j] = \sum_{i=0}^{1} i \times m_{i|j}^A$ and $m_{i|j}^A$ is an element of M^A), we propose the following theorem:

Theorem 2. $\sum_{j=0}^{1} \pi(j) g(E^A[c_n^t | j]) < \sum_{j=0}^{1} \pi(j) g(E^B[c_n^t | j])$ *for all convex, strictly increasing, continuous functions* g, *if and only if* $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^B(s)$.[6]

Proof. See Appendix. □

When theorem 2 holds, M^A induces a stronger reduction in poverty than M^B, in terms of prioritizing the expected deprivation scores of those who start with higher scores in $t-1$.

Note that theorem 2 assumes a union approach to poverty identification. However, the theorem can be restricted in order to apply it to less lenient poverty identification

[5] $\sum_{i=0}^{1} \pi(i) = 1$.

[6] Note that, unlike the non-anonymous case, the RGL curves of theorem 2 are based on the distributions of expected deprivation scores, which have v elements.

criteria. The route to follow is as in the previous section, i.e. to censor all scores whose value is below a chosen k_{min}. Then the rest of the analysis proceeds as established above, noting that some cells in the transition matrices will be merged.

Finally, if the transition matrices are *monotone*, so that: $E[c_n^t|1] \geq ... \geq E[c_n^t|0]$, we reach a theorem slightly resembling theorem 2 by Benabou and Ok (2001):

Theorem 3. *If* $1 \geq \frac{E^A[c_n^t|1]}{E^B[c_n^t|1]} \geq ... \geq \frac{E^A[c_n^t|0]}{E^B[c_n^t|0]}$ *with at least one inequality being strict, then* $L^A(s) \leq L^B(s) \quad \forall s \in [0,1] \quad \wedge \exists s|L^A(s) < L^B(s)$.

Proof. See Appendix. □

Theorem 3 provides a useful method to check whether the condition in theorem 2 holds. Instead of computing the reverse generalized Lorenz curves, one only needs to compute the ratios of conditional expected scores between two populations (of two societies or two different years) and check whether the inequality between the ratios holds as expressed in 3. However, this convenient condition only applies whenever $E[c_n^t|1] \geq ... \geq E[c_n^t|0]$, which is the case with monotone matrices (albeit not exclusively).

3 Empirical illustration: Multidimensional poverty in Peru

3.1 Background and data

As mentioned, Peru experienced a commodity boom between 2003 and 2007, which translated into high GDP growth rates, from 4 % in 2003 to 8.9 % in 2007, and a steady decrease in monetary poverty headcounts, from 58.7 % in 2004 to 42.4 % in 2007. However, between 2008 and 2013, Peru's economic performance was affected by the world economic situation: GDP growth fell from 9.8 % in 2008 to 0.9 % in 2009, and then stabilizing around 7 % between 2010 and 2012. Notwithstanding this fluctuation, monetary poverty levels kept decreasing steadily, from 37.3 to 27.8 %. How did the Peruvian population fare in terms of non-monetary multidimensional poverty?

For our two analyses we use Peruvian National Household Surveys (ENAHO). For the anonymous assessment, we have all the cross-sections between 2002 and 2013, which deliver more than 258,000 household-year observations. For the non-anonymous assessment, we also exploit ENAHO's two recent household panel surveys, spanning 2002-2006 and 2007-2010, each providing 1,570 and 2,260 households, respectively.

Our multidimensional poverty measure relies on four dimensions, and on the household as the unit of analysis. Firstly, household education, comprising two indicators: (1) school delay, which is equal to one if there is a household member in school age who is delayed by at least one year, and (2) incomplete adult primary, which is equal to one if the household head or his/her partner has not completed primary education. The household is considered deprived in education if any of these indicators takes the value of one.

The second dimension considers two indicators on infrastructure dwelling conditions: (i) overcrowding, which takes the value of one if the ratio of the number of household members to the number of rooms in the house is larger than three; and (ii) inadequate construction materials, which takes the value of one if the walls are made of straw or other (almost certainly inferior) material, if the walls are made of stone and mud or wood combined with soil floor, or if the house was constructed at an improvised location inadequate for human inhabitation. The household is deprived in living conditions if any of the above indicators take the value of one.

The third dimension is access to services. The household is deemed deprived in this dimension if any of the following indicators takes the value of one: (i) lack of electricity for lighting, (ii) lack of access to piped water, (iii) lack of access to sewage or septic tank, and (iv) lack of access to a telephone landline. The fourth dimension is household vulnerability to dependency burdens. The household is deprived or vulnerable if household members who are younger than 14 or older than 64 are three times or more as numerous as those members who are between 14 and 64 years old (i.e. in working age).

We weigh each dimension equally. Therefore the household score can take only any of the following five values: $(0, 0.25, 0.5, 0.75, 1)$.

3.2 Results

3.2.1 Anonymous case

Figure 1 shows the reversed generalized Lorenz (RGL) curves for Peru in 2002 and 2013, based on the measurement choices of the previous subsection. The curve for 2013 is never above that of 2002. Hence theorem 1 is fulfilled and we can conclude that, given a particular choice of deprivation lines and dimensional weights, multidimensional poverty in Peru decreased, *along with a reduction in deprivation inequal-*

ity among the poor, between 2002 and 2013, for a broad family of inequality-sensitive poverty indices (at least those in 2) and for any relevant choice of the poverty cut-off k.

The coordinates for the four kink points[7] in the figure represent combinations of the multidimensional headcount (horizontal axis) and the adjusted headcount ratio (vertical axis, i.e. $L(s)$ itself) for the same relevant value of k (1, 0.75, 0.5, 0.25, in our case). For 2002 these values are: (0.004,0.004), (0.158,0.119), (0.471,0.276), (0.741,0.343). Meanwhile for 2013 they are: (0.003,0.003), (0.078,0.059), (0.266,0.153), (0.581,0.232). Clearly, 2013 dominates 2002.

[Figure 1 about here.]

Figure 2 shows the experiences of urban and rural areas between 2002 and 2013. Again, clearly, both regions experienced poverty reduction accompanied by lower inequality among the poor for any inequality-sensitive index and choice of k. However, the reduction in urban areas was stronger even though the poverty situation in the cities was already less serious in 2002. This is apparent if we compute $A(0)$ between the 2002 and 2013 RGL curves (i.e. the poverty reduction areas) and divide them by the initial areas of the 2002 curves, for both urban and rural regions. In the urban case, there was a 35% reduction in the RGL curve area $A(0)$, (i.e. under the union identification approach, deeming any person poor as long as they are deprived in at least one dimension). By contrast, the area of the RGL curve only decreased 16.6% (albeit from a RGL curve further away from the horizontal axis).

[Figure 2 about here.]

Figure 3 shows the RGL curves for the five rainforest Peruvian departments (the dotted lines are for 2002, and the dashed lines for 2013). In all cases our non-anonymous poverty reduction condition is fulfilled. The RGL curve area reductions (i.e. percentage reductions in the adjusted headcount ratios under the union approach) were: Amazonas (18.4%), Madre de Dios (31.4%), Loreto (17.4%), San Martin (26.7%), and Ucayali (16.0%). Compared to other regions (see below), some rainforest departments like Amazonas, Loreto, and Ucayali experienced some of the lowest RGL curve area percentage reductions, while also departing from relatively higher RGL curves in 2002 (i.e. signalling also higher adjusted headcount ratios for every relevant value of k).

[7] One of them is barely visible as it stands very close to the origin.

[Figure 3 about here.]

Figure 4 shows the RGL curves for the four southernmost Peruvian departments, comprising both coastal and highland regions. In the case of Arequipa the two curves cross once very close to the origin. Then the 2013 curve appears always below the 2002 curve. This means that the assessment of inequality and poverty reduction depends on measurement choices. For example, with the intersection approach ($k = 1$), poverty actually increased in Arequipa during the period, whereas for less stringent identification approaches ($k < 1$) the conclusion depends on the choice of both k and individual poverty functions (the department still had a reduction in the RGL curve area of 21.2%). By contrast, Moquegua, enjoying the benefits of a thriving mining industry spread across a small population, saw a robust reduction of poverty accompanied by a decrease in inequality among the poor. Moreover its RGL curve area experienced one of the largest drops, 38.1%. Puno's case is similar to Arequipa's: the curves cross once at the highest levels of k rendering the poverty assessment inconclusive. Likewise, the most severe forms of poverty (with $k = 1$) in Puno increased between 2002 and 2013. Unlike Arequipa though, its reduction in the RGL curve area was much more modest (12.9%). Puno is landlocked, of mostly highland. Finally, Tacna's situation mimicks Puno's and Arequipa's: curve-crossing at the highest levels of k (and with an $A(0)$ reduction of 17.7%). However, unlike Puno and Arequipa, Tacna's curves are closer to the origin in both years, reflecting lower (robust) poverty levels in both years. Tacna benefits both from its mining industry and its active border with Chile.

[Figure 4 about here.]

Figure 5 shows the RGL curves for five south-central Peruvian departments, also comprising both coastal and highland regions. In the cases of landlocked Cusco and mainly coastal Ica their respective pairs of curves cross at the highest levels of k so that poverty reduction is not robust. Again, with an intersection approach, both departments actually saw increases in the most severe forms of poverty. Notwithstanding that, their curve areas reduced by 22.8% and 36.4% (one of the largest), respectively. By contrast, the other three departments did experience fully robust poverty reduction with lower deprivation inequality among the poor. Their curve area reductions were: Apurimac (29.4%), Ayacucho (26.5%), and Huancavelica (23.8%).

[Figure 5 about here.]

Figure 6 shows the RGL curves for five central Peruvian departments (including coastal and highland regions) and the coastal, city-sized, autonomous Callao province. All cases show a robust poverty reduction accompanied by inequality reduction, with the exceptions of the landlocked Junin and Huanuco. For the two latter departments, there is, again, a curve crossing at the highest levels of k, just as in the previous situations of curve-crossing encountered so far. However, all departments exhibit curve area ($A(0)$) reductions: Ancash (37%, among the largest), Callao (30.4%, where the main seaport is), Lima (26.4%, where the capital city is), Junin (30.7%), Huanuco (21.3%), Pasco (19.2%). The latter two reductions are relatively low compared to other robust experiences, and depart from relatively high RGL curves.

[Figure 6 about here.]

Figure 7 shows the RGL curves for the five northern Peruvian Departments along the coast and the highlands. Except for Cajamarca, all departments feature curve-crossing at the highest levels of k, just as in the previous cases of crossing above. Hence the conclusion of poverty reduction with lower inequality between 2002 and 2013 is not robust to all choices of k or functional forms. For instance, with an intersection approach, this extreme form of poverty actually increased throughout the region (except for Cajamarca). Despite those crossings close to the origin, all departments experienced curve area reductions: Cajamarca (15.4%), La Libertad (20.5%), Lambayeque (35.8%, one of the largest), Piura (30.5%), Tumbes (21.0%).

[Figure 7 about here.]

3.2.2 Non-anonymous case

Tables 1 through 4 show the transition matrices for deprivation scores. In each matrix the row π shows the initial distribution of scores, and the row $E[c_n^t|j]$ shows the expected deprivation score conditional on a score value of j in the initial year. Overall, all matrices are monotone, therefore the expected deprivation scores increase with the value of the initial, conditioning score. The matrices also exhibit relatively high levels of path-dependence (likelihood of replicating initial conditions in after the transition) as measured by Shorrock's trace index (where 0 means complete immobility and 1 means equality of conditional distributions). The respective values in chronological order are: 0.41, 0.41, 0.33, 0.37. For instance, the probability of being non-deprived in any dimension conditional on having that initial status remains fairly stable, across the matrices, between 82% (2) and 88% (3). Whereas the probability of being deprived in every dimension conditional on having that initial status fluctuates between 42% (2) and 70% (4).

Table 5 provides the main finding in the non-anonymous assessment. It features the vertical coordinates of the RGL curves of expected deprivation scores (the five horizontal coordinates correspond to the number of expected deprivation scores and are common to the four transition matrices). The ensuing ordering, related to theorem 2, states that if the initial score distributions were identical, then the ex-ante expected social poverty induced by matrix 1 would be lower than the levels produced by all the other matrices, for any social poverty function that increases both with higher conditional expected deprivation scores and with higher inequality between them. Likewise, matrix 2 induces lower ex-ante expected poverty than matrices 3 and 4. Finally, a similar robust ordering cannot be established between matrices 3 and 4 since their two respective RGL curves cross. In summary, the conditional distributions of expected deprivation scores produced by the pre-crisis transition matrices second-order dominate the distributions yielded by the crisis/post-crisis matrices.

[Table 1 about here.]

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

4 Concluding remarks

This paper proposed two methods to ascertain whether pro-poorest poverty reduction is robust to different choices of functional forms and identification cut-offs, in the context of counting poverty measures for discrete, or discretized, variables. The first method, based on reverse generalized Lorenz (RGL) curves, is pertinent for the so-called anonymous approach to poverty dynamics, in which we compare two cross-sectional samples of the same population in different periods of time, i.e. without tracking the outcomes of the same individuals. When the anonymous conditions based on the RGL curves are fulfilled, we can conclude that poverty decreased or increased consistently, in the sense that the result is robust to different choices of inequality-sensitive social poverty indices and counting identification approaches.

However these conditions only hold for specific choices of deprivation lines and dimensional weights. With alternative selections of the latter two parameter sets, the conditions would need to be tested again. Conditions that ensure robustness for broader sets of parameter and functional choices exist (e.g. see Yalonetzky, 2014), but they are more difficult to implement and have a limited scope of application (e.g. they only work for extreme identification approaches whenever three or more variables are considered).

Measuring poverty with indicators capturing dimensions of education, dwelling infrastructure, service access and dependency burdens, the empirical illustration of this condition's theorem 1 showed that 14 out of 25 Peruvian departments experienced robust poverty reduction between 2002 and 2013, accompanied by lower deprivation inequality among the poor. By contrast, 11 departments showed one curve-crossing, always at the highest levels of the identification threshold, k. There is a literally good reason behind these crossings: very few people are found to be deprived in every singe dimension *jointly*. Therefore, even when region A has a higher RGL curve than B, for any value $k < 1$, it can happen that, due to random sample variation, A has fewer people deprived across all dimensions (i.e. poor according to $k = 1$), which induces curve crossing close to the origin.

The second method, based on a combination of transition matrices with RGL curves, is relevant for the non-anonymous approach, which is basically an intra-generational mobility assessment, and compares the outcomes of the same people across different periods of time. When non-anonymous conditions are fulfilled, we can conclude that, *ex-ante*, the distribution of expected deprivation scores (conditioned on different initial deprivation scores) of A second-order dominates that of B (meaning, inter alia, that the distribution in A features both lower average expected deprivation scores, and less dispersion than B's in Lorenz-consistency terms), if the two distributions of initial deprivation scores are identical. Again, this conditions hold only for specific choices of deprivation lines and dimensional weights. With alternative selections, the conditions must be tested again.

Our empirical illustration of the non-anonymous condition, using the Peruvian panel datasets, showed that the mobility matrix of deprivation scores corresponding to the 2002-2004 periods induced a preferable distribution of expected deprivation scores vis-a-vis all the other matrices, i.e. those of 2004-2006, 2007-2008, and 2008-2010. The second-best matrix was that of period 2004-2006, which was also preferable to the two crisis/post-crisis matrices (but not to its predecessor, 2002-2004). By contrast it was not possible to rank 2007-2008 and 2008-2010 in terms of expected egalitarian poverty reduction. Interestingly, this quasi-ordering held despite the fact that the respective distributions of initial deprivation scores (i.e., respectively, for 2002, 2004, 2007, and 2008) were gradually experiencing probability mass moving toward lower deprivation score values. This means that even though the distribution of poverty scores has improved over the years in terms of higher proportions of lower scores, each subsequent transition has been less conducive to egalitarian poverty reduction as the previous ones (with the exception of the inconclusive comparison between 2007-2008 and 2008-2010). A diminishing marginal return, so to speak.

References

Alkire, S. and J. Foster (2011). Counting and multidimensional poverty measurement. *Journal of Public Economics 95*(7-8), 476–87.

Alkire, S. and S. Seth (2014). Measuring and decomposing inequality among the multidimensionally poor using ordinal data: A counting approach. OPHI Working Paper 68.

Ben Haj Kacem, R. (2013). Monetary versus non-monetary pro-poor growth: Evidence from rural ethiopia between 2004 and 2009. *Economics: The Open-Access, Open-Assessment E-Journal, 7*(26).

Benabou, R. and E. Ok (2001). Mobility as progressivity: ranking income processes according to equality of opportunity. Working Paper 8431 NBER.

Berenger, V. and F. Bresson (2012). On the "pro-poorness" of growth in a multidimensional context. *Review of Income and Wealth 58*(3), 457–80.

Boccanfuso, D., J. Bosko Ki, and C. Menard (2009). Pro-poor growth measurements in a multidimensional model: A comparative approach. GREDI Working Paper 09-22.

Chakravarty, S. and C. D'Ambrosio (2006). The measurement of social exclusion. *Review of Income and wealth 52*(3), 377–98.

Deutsch, J. and J. Silber (2011). On various ways of measuring pro-poor growth. *Economics: The Open-Access, Open-Assessment E-Journal 5*(13).

Fleurbaey, M. and V. Peragine (2013). Ex ante versus ex post equality of opportunity. *Economica 80*(317), 118–30.

Grimm, M. (2007). Removing the anonymity axiom in assessing pro-poor growth. *Journal of Economic Inequality 5*, 179–97.

Marshall, A., I. Olkin, and B. Arnold (2011). *[I]nequalities: [T]heory of [M]ajorization and its applications* (Second ed.). Springer Series in Statistics. Springer.

Silber, J. and G. Yalonetzky (2013). *Poverty and Social Exclusion. New methods of analysis*, Chapter 2, pp. 9–37. Routledge Advances in Social Economics. Routledge.

Yalonetzky, G. (2014). Conditions for the most robust multidimensional poverty comparisons using counting measures and ordinal variables. *Social Choice and Welfare*.

5 Appendix

5.1 Proof of theorem 1

First we prove the necessity of $L^A(s) \leq L^B(s) \quad \forall s \in [0,1] \quad \wedge \exists s | L^A(s) < L^B(s)$ for $P^A < P^B$, for all P in 2 satisfying FOC, MON and PROG, to hold. Then we prove that the condition on the reversed generalized Lorenz curve is sufficient to fulfill the condition on the indices.

NECESSITY:

We need to show that whenever the reversed generalized Lorenz curves cross, we can always find two poverty indices, P_1 and P_2 satisfying FOC, MON, and PROG, such that: $P_1^A < P_1^B$ and $P_1^A > P_1^B$. Consider a case with $D = 4$ and equal weights. For each possible value of c_n (i.e.: 0,0.25,0.5,0.75,1), the set of poverty headcounts (i.e. $H(k) \equiv \frac{1}{N}\sum_{n=1}^{N} \mathbb{I}(c_n \geq k)$) for country A is: (0.7,0.6,0.5,0.4,0.3), and for country B it is: (0.8,0.7,0.6,0.4,0.2). Then it is easy to show that L^A and L^B cross. Moreover, if we choose $p_n = \mathbb{I}(c_n \geq k)[c_n]^2$ (which fulfills FOC, MON and PROG) with a union identification approach (e.g. $k = 0$), it turns out that $P^A > P^B$. By contrast, if we choose the following member of the family proposed by Silber and Yalonetzky (2013), which fulfills FOC, MON and PROG: $P = \frac{1}{D-kD+1}\sum_{d=kD}^{D}[H(\frac{d}{D})]^{0.5}$, with a union approach, then it turns out that $P^A < P^B$. Therefore, we conclude that the absence of curve-crossing is necessary for the robustness of orderings based on poverty indices satisfying FOC, MON and PROG.

SUFFICIENCY:

Now we need to show that, whenever $L^A(s) \leq L^B(s) \quad \forall s \in [0,1] \quad \wedge \exists s | L^A(s) < L^B(s)$, we can obtain distribution A from B through a series of rank-preserving progressive transfers of deprivations between poor people and/or reductions in some deprivation scores (to which poverty indices fulfilling MON should react). For this purpose we need to use Muirhead's theorem (Marshall et al., 2011, p. 7-8). Let X and Y be the distributions of two non-negative, real-valued variables x and z, respectively. The population size of both distributions is N. The theorem says that: $\sum_{i=1}^{m} x_i \geq \sum_{i=1}^{m} y_i \;\; \forall m \in [1, N] \subset \mathbb{N}_{++}$ and $\sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i$, with both x and y added from the highest to the lowest value, if and only if Y can be obtained from X through a series of rank-preserving progressive transfers.

Now if $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^B(s)$, then either: $L^A(1) = L^B(1)$ or $L^A(1) < L^B(1)$. If $L^A(1) = L^B(1)$ then we can apply Muirhead's theorem, realizing that the reversed generalized Lorenz curves accumulate c_n in descending order. Given the relationship between L^A and L^B, the theorem allows us to conclude that the distribution of deprivation scores in A can be obtained from that in B through sequences of rank-preserving progressive transfers, i.e. transfers in which some deprivation, whose weight is w_d is transferred from c_i to c_j, such that: $c_i - w_d \geq c_j + w_d$. Therefore, by definition, it would have to be the case that $P(A) < P(B)$ for any P satisfying PROG.

However, if $L^A(1) < L^B(1)$, then before being able to apply Muirhead's theorem, we need to derive a distribution C from B such that $L^A(s) \leq L^C(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^C(s)$,and $L^A(1) = L^C(1)$. The natural starting point is to subtract the least-weighted deprivation from some among those individuals in B for whom $c_n = \min w_1, ...w_D$, thereby leaving them now belonging to the share of the population who is non-poor even by the union approach (i.e. $c_n = 0$). If rendering all individuals with the least-weighted deprivation without any deprivation at all is insufficient to yield $L^A(1) = L^C(1)$, then we keep subtracting deprivations from individuals with values immediately above $c_n = \min w_1, ...w_D$ (starting with the least-weighted deprivations), and so forth, until finally $L^A(1) = L^C(1)$. Then, once we have $L^A(s) \leq L^C(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^C(s)$,and $L^A(1) = L^C(1)$, we apply Muirhead's theorem to conclude that A can be obtained from C through a series of rank-preserving, progressive transfers of deprivations.

Finally, we have proven that if $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s | L^A(s) < L^B(s)$, and $L^A(1) < L^B(1)$, we can always derive A from B through a sequence of deprivation subtractions/reductions and rank-preserving progressive transfers (more precisely deriving C from B using deprivation subtractions, and then A from C using progressive transfers). Therefore, any P satisfying MOC and PROG must consistently conclude that $P^A < P^B$.

5.2 Proof of theorem 2

For this proof we prove, first, that $\sum_{j=0}^{1}\pi(j)g(E^A[c_n^t|j]) < \sum_{j=0}^{1}\pi(j)g(E^B[c_n^t|j])$ for all convex, strictly increasing, continuous functions g implies $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \; \exists s|L^A(s) < L^B(s)$. Then we prove that the reverse is also true.

FIRST PART: $\sum_{j=0}^{1}\pi(j)[g(E^A[c_n^t|j]) - g(E^B[c_n^t|j])] < 0$ for all convex, strictly increasing, continuous functions g implies: $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s|L^A(s) < L^B(s)$.

Note first that $\sum_{j=0}^{1}\pi(j)[g(E^A[c_n^t|j]) - g(E^B[c_n^t|j])] < 0$ if and only if $g(E^A[c_n^t|j]) - g(E^B[c_n^t|j]) \leq 0 \; \forall j \;\; \wedge \exists i|g(E^A[c_n^t|i]) - g(E^B[c_n^t|i]) < 0$. The "only if" part is true because, otherwise, if even one of the gaps (e.g. $g(E^A[c_n^t|l]) - g(E^B[c_n^t|l])$) were positive, then one could always fine a suitable vector Π such that $\sum_{j=0}^{1}\pi(j)[g(E^A[c_n^t|j]) - g(E^B[c_n^t|j])] > 0$.

Then, since g is continuous and strictly increasing, it must be the case that: $g(E^A[c_n^t|j]) - g(E^B[c_n^t|j]) < 0 \; \forall j \;\; \wedge \exists i|g(E^A[c_n^t|i]) - g(E^B[c_n^t|i]) < 0$ implies that $E^A[c_n^t|j] \leq E^B[c_n^t|j] \; \forall j \;\; \wedge \; \exists i|E^A[c_n^t|i] < E^B[c_n^t|i]$. Now, both reversed generalized Lorenz curves based on the conditional expected scores will be constructed each with v elements cumulating them in decreasing order of value. Since the cumulative sum of the largest l elements of A (l being a natural number running from 1 to v) is never higher than the respective cumulative sum for B, then it must be the case that: $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s|L^A(s) < L^B(s)$. Finally, since the set of continuous, strictly increasing functions g includes a subset of convex functions, then the first part of the statement in the theorem holds for convex functions, given that it really applies to all continuous, strictly increasing functions g.

SECOND PART: $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \exists s|L^A(s) < L^B(s)$ implies: $\sum_{j=0}^{1}\pi(j)g(E^A[c_n^t|j]) < \sum_{j=0}^{1}\pi(j)g(E^B[c_n^t|j])$ for all convex, strictly increasing, continuous functions g.

Here we follow a proof similar to that for theorem 1. Firstly, we note that $L^A(s) \leq L^B(s) \;\; \forall s \in [0,1] \;\; \wedge \; \exists s|L^A(s) < L^B(s)$ implies that either $L^A(1) \leq L^B(1)$ or $L^A(1) = L^B(1)$. If $L^A(1) = L^B(1)$ then we directly apply Muirhead's theorem to conclude that A can be obtained from B through a series of rank-preserving, progressive transfers of expected conditional scores. Now note that every time one such transfer takes place, say between individuals i and j such that $E^B[c_n^t|i] > E^B[c_n^t|j]$, there is a change in the expected welfare function proportional to $-g'(E^B[c_n^t|i]) + g'(E^B[c_n^t|j])$, which must be negative because $g'' > 0$. Therefore the second part holds.

However if $L^A(1) \leq L^B(1)$, we first need to derive a distribution C from B such that: $L^A(s) \leq L^C(s) \;\; \forall s \in [0,1] \;\; \wedge \; \exists s|L^A(s) < L^C(s)$ and $L^A(1) \leq L^C(1)$, so that then we can apply Muirhead's theorem again. As in the previous proof, the natural procedure is to subtract value from the expected deprivation scores starting from those with the lowest values. Once distribution C is derived, then we can apply Muirhead's theorem to conclude that distribution A can be derived from C through a sequence of rank-preserving, progressive transfers. Finally, based on the transitive argument enabling us to derive A from C from B, we can state that if $L^A(s) \leq L^B(s) \ \ \forall s \in [0,1] \ \ \wedge \exists s | L^A(s) < L^B(s)$ and $L^A(1) \leq L^B(1)$, we can derived A from B through a sequence of reductions in expected deprivation scores and rank-preserving, progressive transfers among them. Finally, it becomes clear to note that both types of operations induce reductions in the expected welfare function, such that: $\sum_{j=0}^{1} \pi(j) g(E^A[c_n^t|j]) < \sum_{j=0}^{1} \pi(j) g(E^B[c_n^t|j])$ for all convex, strictly increasing, continuous functions g.

5.3 Proof of theorem 3

First note that, in the context of the v expected scores: $L^A(l) - L^B(l) = (E^A[c_n^t|1] - E^B[c_n^t|1]) + \ldots + (E^A[c_n^t|l] - E^B[c_n^t|l])$. If, as stated by the first condition in the theorem, $1 \geq \frac{E^A[c_n^t|1]}{E^B[c_n^t|1]}$, then the first element of $L^A(l) - L^B(l)$ will be non-positive for any value of l.

Then for any given relationship $\frac{E^A[c_n^t|1]}{E^B[c_n^t|1]} \geq \frac{E^A[c_n^t|r]}{E^B[c_n^t|r]}$, where r is any of the admissible values of c_n in the initial period (except 1), let $\lambda^A(r) \equiv \frac{E^A[c_n^t|1]}{E^A[c_n^t|r]}$ (same for B, and note that $\lambda > 1$ since by assumption $E[c_n^t|1] \geq \ldots \geq E[c_n^t|0]$). Realizing that $E^A[c_n^t|1] - E^B[c_n^t|1] \leq 0$ and that $\lambda^A(r) \geq \lambda^B(r)$ we can deduce that: $E^A[c_n^t|r] - E^B[c_n^t|r] = \frac{E^A[c_n^t|1]}{\lambda^A(r)} - \frac{E^B[c_n^t|1]}{\lambda^B(r)} \leq 0$. Proceeding the same way with every conditioning value of c_n in the initial period (except 1), we conclude that $L^A(l) - L^B(l)$ must be non-positive for every relevant value l that c_n can take. Finally, it is straightforward to show that only one strict inequality in $1 \geq \frac{E^A[c_n^t|1]}{E^B[c_n^t|1]} \geq \ldots \geq \frac{E^A[c_n^t|0]}{E^B[c_n^t|0]}$, suffices to render $L^A(s) \leq L^B(s) \ \ \forall s \in [0,1] \ \ \wedge \exists s | L^A(s) < L^B(s)$.

Figure 1: Reversed generalized Lorenz curves of deprivation counts. Peru, 2002-2013.

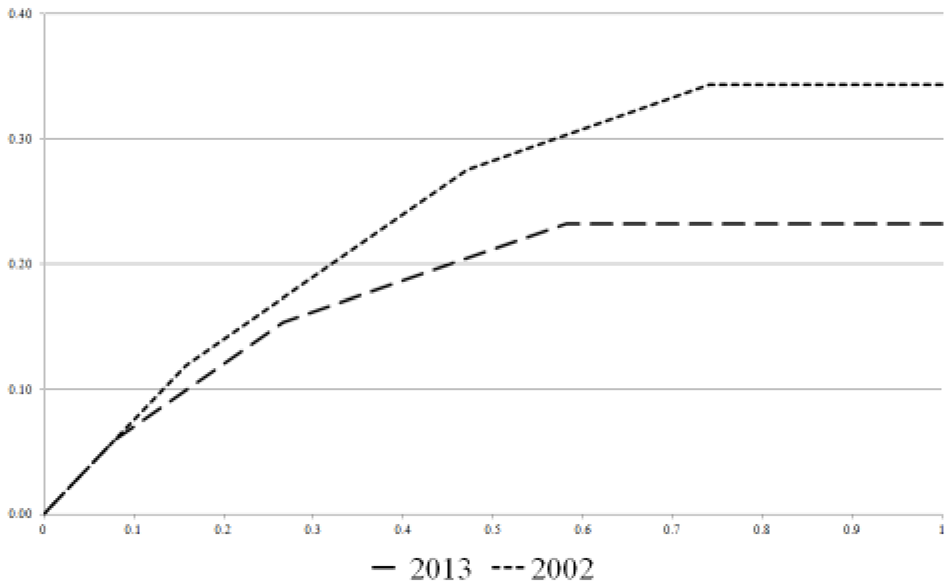
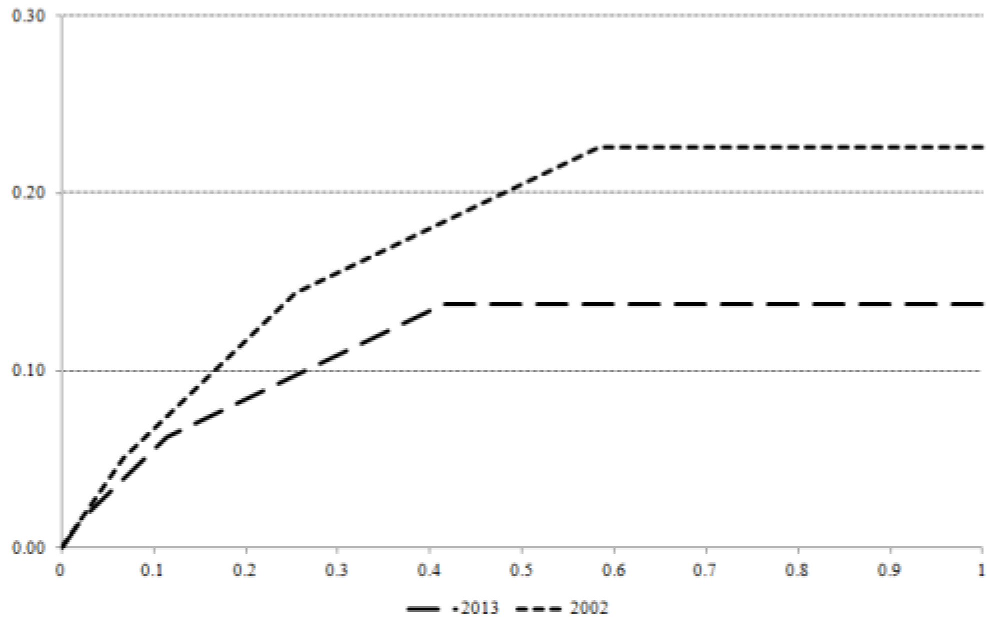


Figure 2: Reversed generalized Lorenz curves of deprivation counts. Urban and rural Peru, 2002-2013.



a) Urban (above)
b) Rural (below)

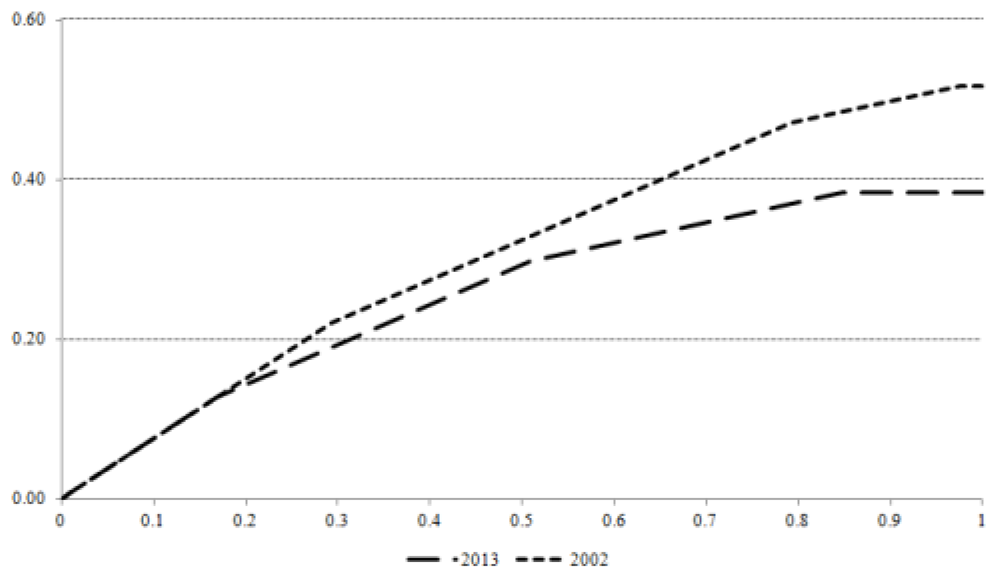


Figure 3: Reversed generalized Lorenz curves of deprivation counts. Peruvian rainforest provinces, 2002-2013.

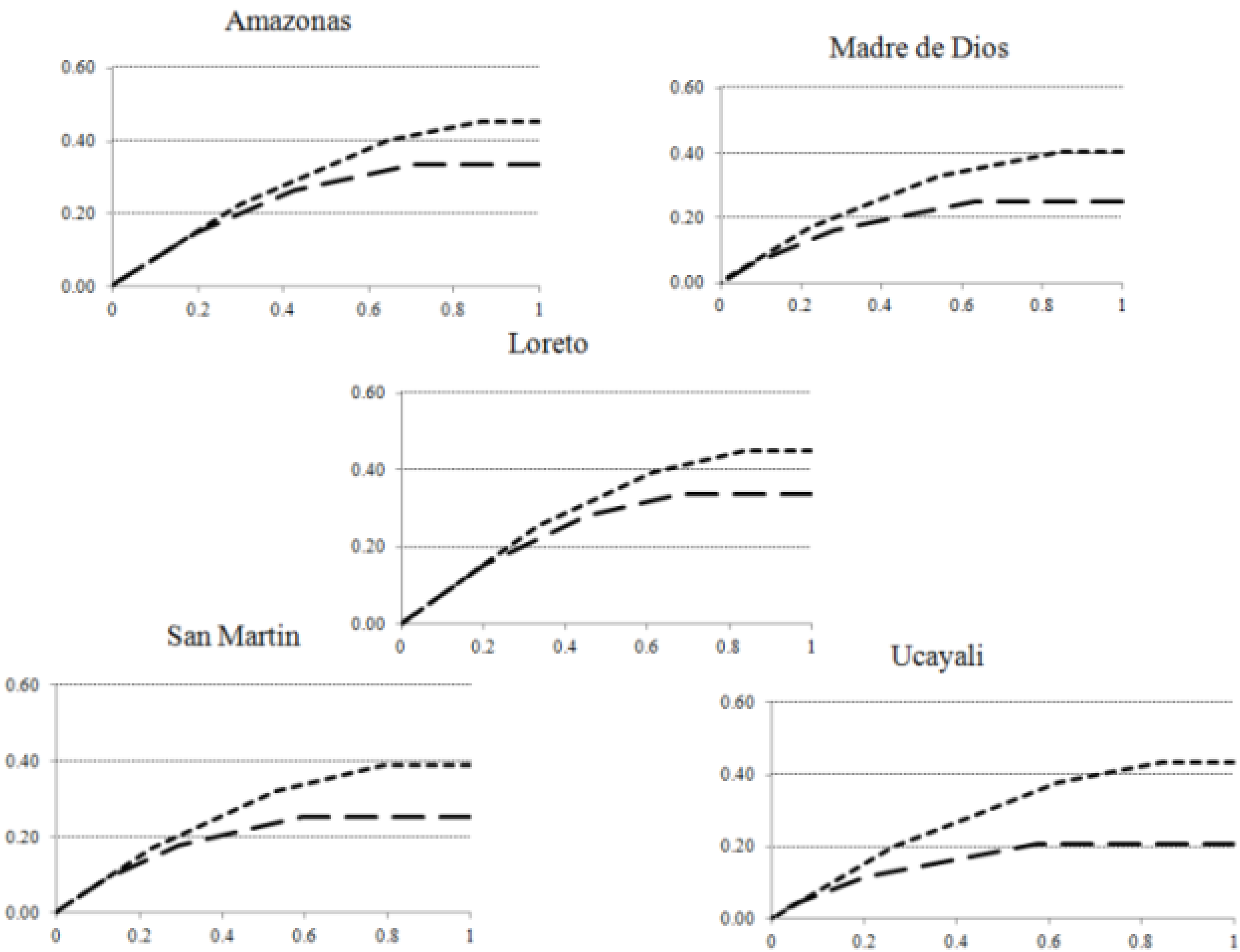


Figure 4: Reversed generalized Lorenz curves of deprivation counts. Peruvian southern coastal and highland provinces, 2002-2013.

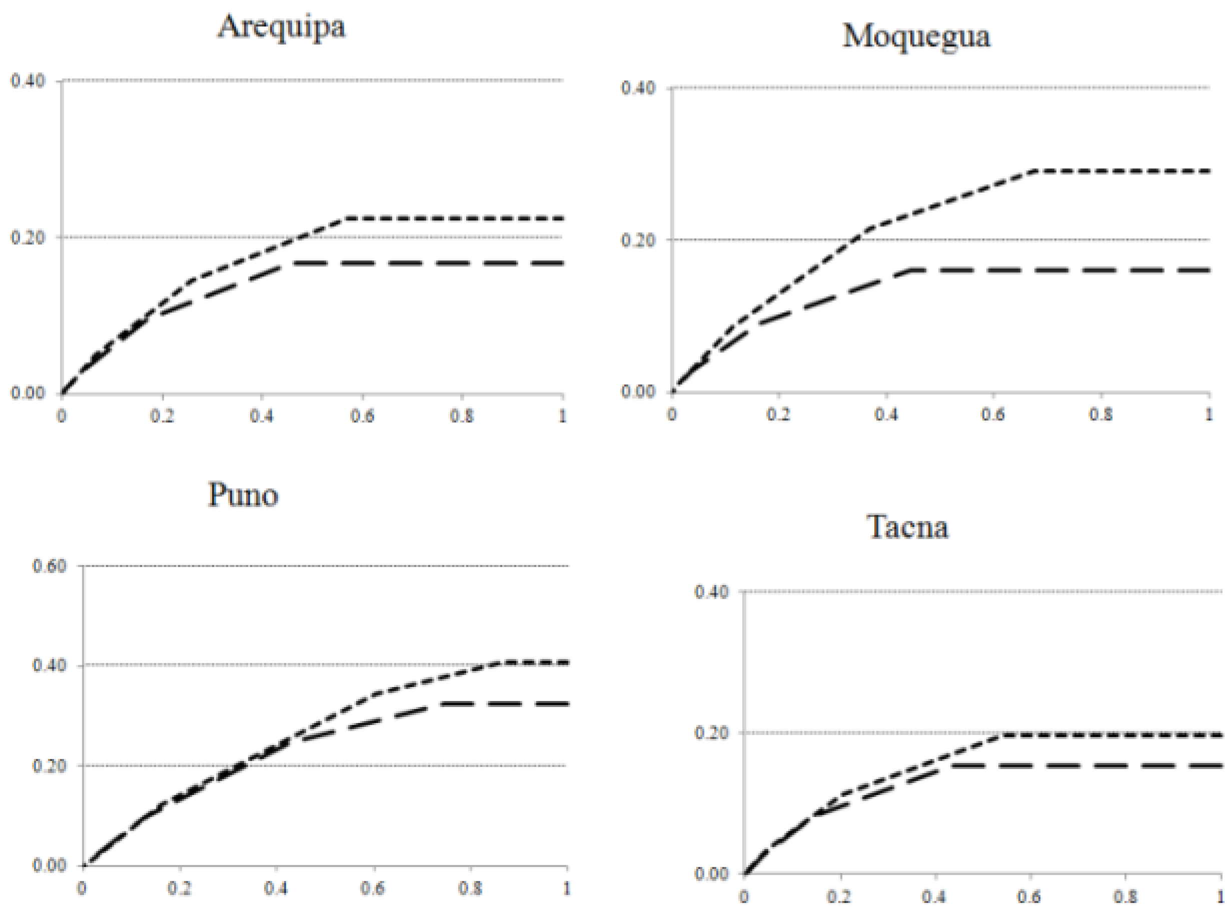


Figure 5: Reversed generalized Lorenz curves of deprivation counts. Peruvian south-central coastal and highland provinces, 2002-2013.

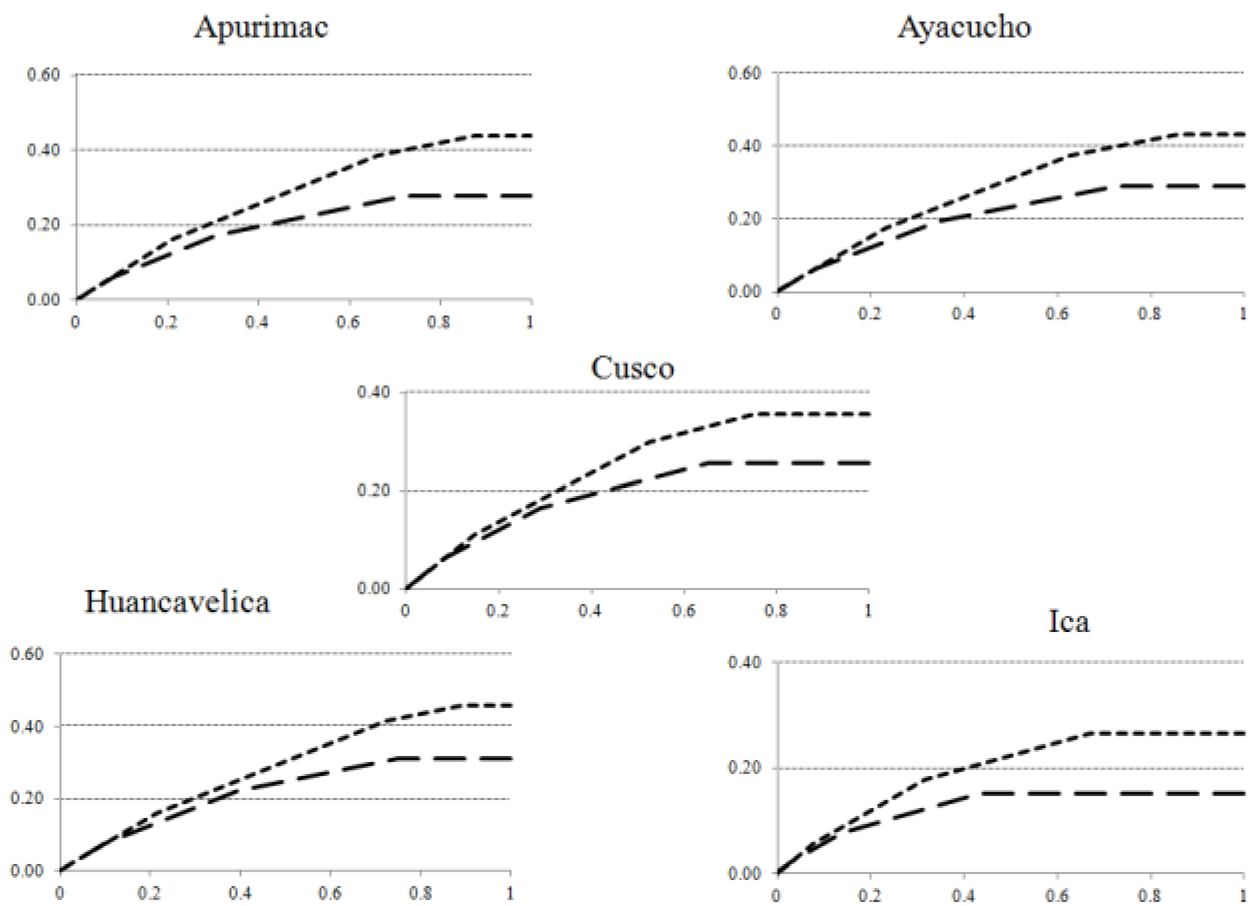


Figure 6: Reversed generalized Lorenz curves of deprivation counts. Peruvian central coastal and highland provinces, 2002-2013.

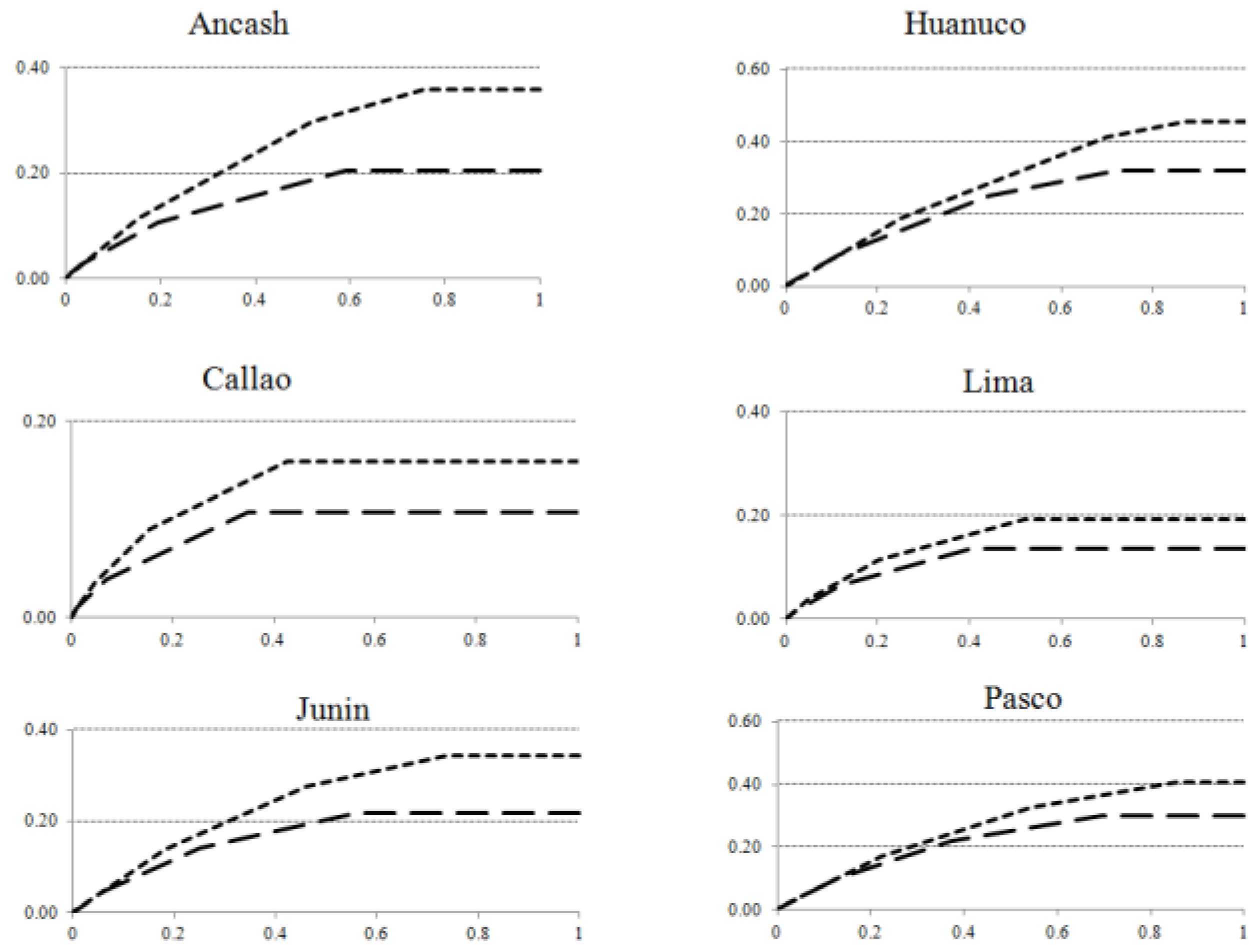


Figure 7: Reversed generalized Lorenz curves of deprivation counts. Peruvian northern coastal and highland provinces, 2002-2013.

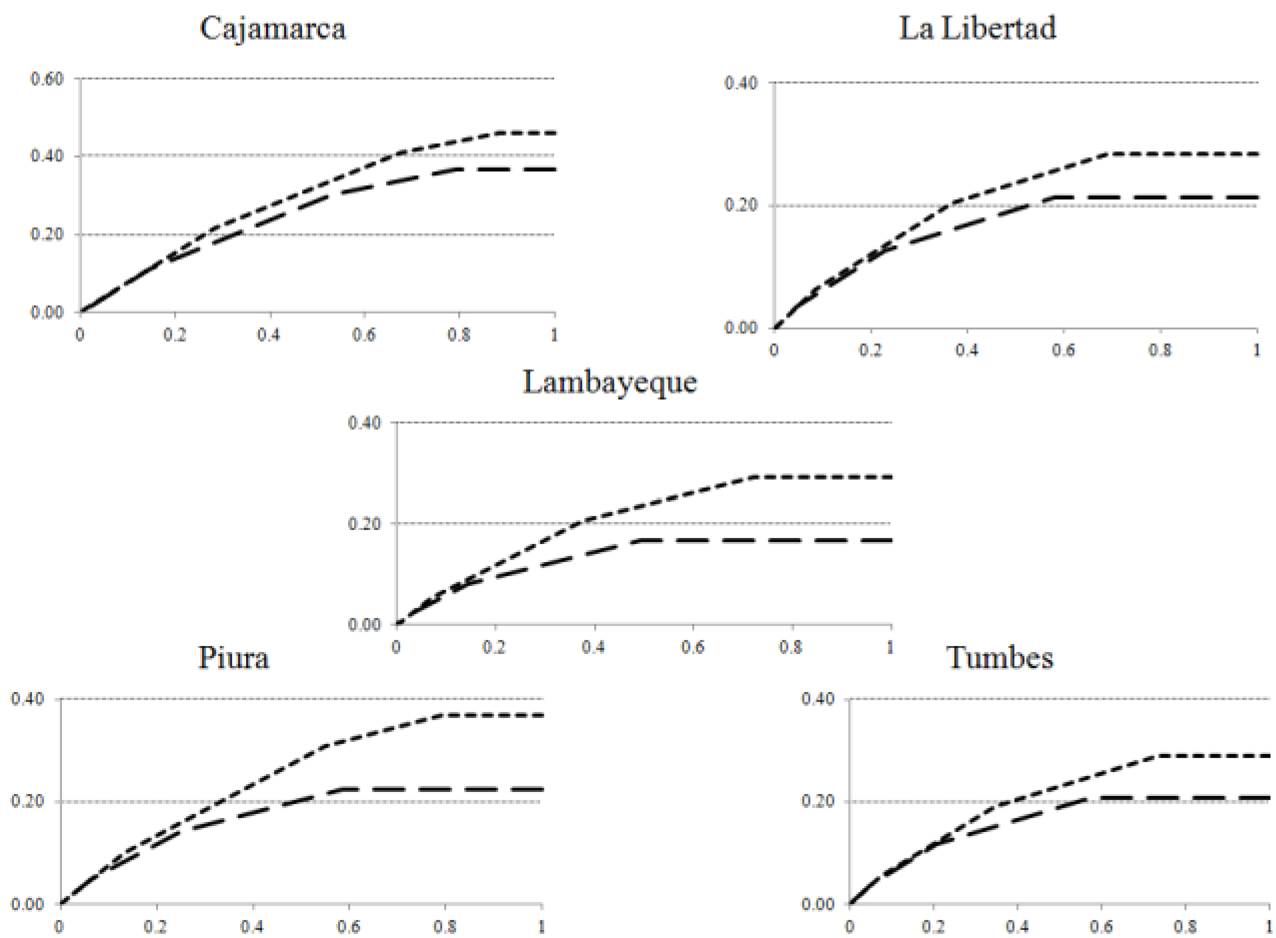


Table 1: Transition matrix of deprivation scores, Peru, 2002-2004

		2002				
		0	0.25	0.5	0.75	1
	0	0.87	0.21	0.02	0.0	0.0
	0.25	0.11	0.65	0.20	0.04	0.0
2004	0.5	0.02	0.14	0.67	0.33	0.09
	0.75	0.0	0.0	0.11	0.61	0.36
	1	0.0	0.0	0.0	0.02	0.55
	π	0.18	0.28	0.36	0.17	0.01
	$E[c_n^t \mid j]$	0.039	0.235	0.467	0.653	0.864

Table 2: Transition matrix of deprivation scores, Peru, 2004-2006

		2004				
		0	0.25	0.5	0.75	1
	0	0.82	0.19	0.02	0.0	0.0
	0.25	0.15	0.68	0.19	0.03	0.0
2006	0.5	0.03	0.12	0.69	0.21	0.0
	0.75	0.0	0.01	0.11	0.74	0.58
	1	0.0	0.0	0.0	0.02	0.42
	π	0.23	0.28	0.34	0.15	0.01
	$E[c_n^t \mid j]$	0.051	0.239	0.473	0.686	0.854

Table 3: Transition matrix of deprivation scores, Peru, 2007-2008

		2007				
		0	0.25	0.5	0.75	1
	0	0.88	0.14	0.01	0.0	0.0
	0.25	0.10	0.71	0.15	0.01	0.0
2008	0.5	0.02	0.14	0.74	0.24	0.0
	0.75	0.0	0.0	0.10	0.74	0.40
	1	0.0	0.0	0.01	0.01	0.60
	π	0.27	0.27	0.29	0.15	0.01
	$E[c_n^t \mid j]$	0.034	0.251	0.489	0.686	0.900

Table 4: Transition matrix of deprivation scores, Peru, 2008-2010

		2008				
		0	0.25	0.5	0.75	1
	0	0.86	0.17	0.03	0.0	0.0
	0.25	0.13	0.68	0.24	0.05	0.0
2010	0.5	0.01	0.14	0.64	0.26	0.05
	0.75	0.0	0.01	0.10	0.67	0.25
	1	0.0	0.0	0.0	0.02	0.70
	π	0.283	0.267	0.293	0.148	0.01
	$E[c_n^t \vert j]$	0.039	0.250	0.451	0.666	0.913

Table 5: Generalized Lorenz curves of expected deprivation scores. Vertical coordinates.

$E[c_n^t \| j]$	1	2	3	4	5
$E[c_n^{2004} \| 2002]$	0.836	1.516	1.983	2.218	2.257
$E[c_n^{2006} \| 2004]$	0.854	1.540	2.013	2.252	2.304
$E[c_n^{2008} \| 2007]$	0.900	1.586	2.074	2.325	2.359
$E[c_n^{2010} \| 2008]$	0.913	1.578	2.029	2.279	2.318