



## **Duration and Multidimensionality in Poverty Measurement**

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**Duration and Multidimensionality  
in Poverty Measurement**

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## **ABSTRACT**

We propose a dynamic multidimensional measure of poverty that is sensitive to switches in deprivations across individuals. We demonstrate that this property is a natural extension of the transfer axiom; despite this it is essentially ignored in numerous poverty measures, including static multidimensional measures, dynamic unidimensional measures, and generalisations of both – dynamic multidimensional measures. Our measure is decomposable both by dimensions and periods of time; it separately identifies deprivations that are concentrated within specific dimensions versus those that are concentrated within specific periods. We apply the measure to data from China and compare the estimates with existing measures.

**Keywords:** *Multidimensional Poverty; Poverty Duration; Transfer Axiom; Subgroup Decomposability.*

**JEL classification:** *I31, I32*

## 1. Introduction

Traditional income-based measures of poverty have been extended along two major directions: the broadening of the measures to incorporate a wider set of dimensions that together give a more accurate representation of welfare; and the lengthening of the measures to incorporate information that spans over several periods of observations.

Extensions along the first direction, largely influenced by the writings of Sen (1985), move away from unidimensional measures and into a multidimensional approach based on the individual's lack of access to a wide set of dimensions that include both market and non-market goods. Sen (1976)'s pioneering contribution provided the basis for the recent axiomatic approach to the multidimensional measurement of poverty – examples include Tsui (2002), Atkinson (2003), Bourguignon and Chakravarty (2003), and Alkire and Foster (2011a) [henceforth AF].<sup>1</sup>

Extensions along the second direction consider a dynamic approach where repeated observations (or 'spells') of poverty are treated differently to cases where poverty is temporary. In this strand of the literature, the issue of the duration of poverty endured by an individual or household is considered important in the measurement and analysis of poverty. There is now increasing realisation that long, uninterrupted spells of poverty may lead to social exclusion from which recovery may be very difficult; see, for example, Walker (1995). Examples of extensions based on such a view include Foster (2009), Calvo and Dercon (2009), Hojman and Kast (2009), Duclos et.al. (2010), Hoy and Zheng (2011), Bossert et al (2012), and Gradin et al (2012).

Despite the usefulness provided by extensions along both directions, the literature has largely considered both extensions independent of each other, retaining either the unidimensional or static property of traditional measures.<sup>2</sup> This paper aims to provide a generalised framework for measuring poverty that jointly incorporates both aspects – multidimensionality and the duration of deprivation – with a particular emphasis on the

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<sup>1</sup> See, also, Chakravarty and D'Ambrosio (2006), Bossert et al (2007), Jayaraj and Subramanian (2010) and Permanyer (2014) for closely related work on the measurement of multidimensional deprivation. In contrast to measures such as the Human Development Index and Human Poverty Index, these measures aggregate first over dimensions for each individual prior to aggregating over individuals – Dutta et al (2003) highlight the advantages of doing so.

<sup>2</sup> Exceptions include Nicholas and Ray (2012), Bossert, Ceriani, Chakravarty and D'Ambrosio (2012) and Alkire, Apablaza, Chakravarty and Yalonetzky (2013). See Merz and Rathzen (2014) for a recent attempt to introduce the time element in multidimensional poverty measurement by proposing a measure that “quantifies the shortest path to escape multidimensional poverty”.

information gained by considering the joint distribution of deprivations over both dimensions and time. The highlight of such a measure is its ability to take advantage of panel data when making poverty comparisons between different countries or subgroups of a population. With the increasing availability of panel data in both developing and developed countries, the proposed measure can be usefully applied in a variety of contexts as our illustrative application on panel data from China shows.

We first highlight a limitation associated with the popular AF multidimensional measure. Consider the case of two individuals, with two possible dimensions in which to be deprived. Assume for the sake of illustration that only the count of deprivations matter. In situation *A*, the first individual is deprived in both dimensions, while the second individual is not deprived at all. In situation *B*, both the first and second individual have one deprivation each. While the transfer axiom<sup>3</sup> implies that poverty should be higher in situation *A*, the AF measure does not differentiate between situation *A* and *B*; that is, it is insensitive to a switch of deprivations across individuals. Switches in deprivations are particularly relevant in multidimensional measures, where typically only qualitative information is available and only the count of deprivations are imputed. Insensitivity to switches is also inconsistent with the idea that at any given time, “the consequences for quality of life of having multiple disadvantages [across different domains] far exceed the sum of their individual effects” (Stiglitz et al, 2009). While the measures in Bourguignon and Chakravarty (2003), Chakravarty and D’Ambrosio (2006), Jayaraj and Subramaniam (2010) and Datt (2013) extend the model in AF to allow for this, they do so at the cost of a desirable property: *dimensional decomposition*; that is, the ability to decompose the poverty measure according to the percentage contribution of each dimension.

Similarly, the dynamic measure introduced in Foster (2009) cannot differentiate between situation *A'*, where the first individual is deprived in both periods, while the second individual is not deprived at all, from situation *B'*, where both the first and second individual have one deprivation each. This is inconsistent with the idea that recurring deprivations incur an increasing cost on the individual (see, for example, Sengupta, 2009,

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<sup>3</sup> “Any transference of income from a relatively rich man to a relatively poor man of similar temperament, since it enables more intense wants to be satisfied at the expense of less intense wants, must increase the aggregate sum of satisfaction” (Pigou, 1932, pg 89). If we take ‘income’ to cover the broader notion of ‘achievements’ from the multidimensional literature, then Pigou’s notion of ‘transfer’ implies that in situation *A*, taking one deprivation from the first individual and giving it to the second (thus creating situation *B*) should increase the “aggregate sum of satisfaction”. Further insight into the transfer axiom can be found in Pigou (1912) Dalton (1920), and a poverty-measure characterisation in Sen (1976).

and Hoy and Zheng, 2011). Analogous to the multidimensional case, while Gradin et al (2012) extend Foster (2009) to account for this, they do so at the cost of *dynamic decomposition*; that is, the ability to decompose the poverty measure according to the percentage contribution of each period of time.

We propose extensions to AF's (static) multidimensional measure and Foster (2009)'s dynamic (unidimensional) measure respectively where each measure is able to account for the type of violations just described; in addition, they are able to do so whilst satisfying *dimensional* and *dynamic decomposition* respectively. We then generalise the two extensions into a dynamic multidimensional index that satisfies the properties associated with the two extensions.

Our proposed measure therefore differs from existing subgroup decomposable dynamic multidimensional measures (Nicholas and Ray (2012) [NR], Bossert, Ceriani, Chakravarty and D'Ambrosio (2012) [BCCD] and Alkire, Apablaza, Chakravarty and Yalonetzky (2013) [AACY]) in the following ways:

- 1) It is sensitive to switches in deprivations across individuals; more specifically, it satisfies *dimensional* and *dynamic rearrangement* – two properties that are implied by the transfer axiom
- 2) It satisfies (1) while retaining *dimensional* and *dynamic decomposability*
- 3) It is able to differentiate between individuals for whom deprivations are concentrated over multiple dimensions for specific periods of time, versus individuals for whom deprivations are repeated over multiple periods of time for specific dimensions

Point (3) is important in allowing the analyst a choice over how to weight deprivation in multiple dimensions relative to deprivation in repeated time periods – a luxury available only with the advent of panel data. There may be a case to believe, for example, that an additional period of deprivation deserves more policy attention than an additional dimension of deprivation, or vice versa. The differentiation of these two aspects also deals with an issue associated with unbalanced panels; for example, due to data availability, one may have data with 12 dimensions of deprivation but with only 4 periods of observations for each dimension. Using the models adopted in NR, BCCD or AACY will result in implicitly assuming that being deprived in all 12 dimensions for 1 period is equivalent to being deprived for all 4 periods in 3 dimensions.

We apply the proposed dynamic multidimensional poverty measure to China. The absence of information on a panel of households in developing countries for a sufficiently long time period containing information on a reasonably wide set of dimensions has made applications of dynamic multidimensional poverty measures to developing countries quite limited.<sup>4</sup> Such panel data sets are rare even in the context of developed countries and, until recently, almost non-existent in the case of developing countries.<sup>5</sup> For example, Mishra and Ray (2012) have recently compared multidimensional deprivation in the static framework between China and India, but their study was not on panel data and, consequently, was unable to incorporate any of the dynamic elements of the present study.

The rest of the paper is organised as follows. The proposed dynamic multidimensional poverty measure is introduced in Section 2. The data sets are described in Section 3 along with some summary features of the data that are relevant for this study. The empirical results are presented in Section 4 while Section 5 concludes the paper.

## 2. Analytical Framework

### 2.1. Notation

Assume we observe, for  $N$  individuals in a population of interest,  $J$  different dimensions of deprivation and  $T$  equally-spaced periods of time.  $x_{njt}$  is individual  $n$ 's *achievement* in dimension  $j \in \{1, 2, \dots, J\}$  at time  $t \in \{1, 2, \dots, T\}$ . Each individual  $n$  can be said to have an *individual achievement profile*

$\mathbf{A}_n = \begin{pmatrix} x_{n11} & \dots & x_{n1T} \\ \cdot & \dots & \cdot \\ x_{nJ1} & \dots & x_{nJT} \end{pmatrix}$ . The *population achievement profile* is a vector  $\boldsymbol{\rho} = (\mathbf{A}_1, \dots, \mathbf{A}_N)$ . Define the

identification vector  $\mathbf{v} = (C_1, \dots, C_N)$  where  $C_n$  takes the value 1 if the individual is considered *poor*, and 0 otherwise. An individual is considered poor if he has at least  $z$  count(s) of deprivations; this can be based on a

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<sup>4</sup> As BCCD report, the problem of missing information at the household level on material deprivation in several individual dimensions is present in the data sets of developed countries as well. This forces one to adopt either the unsatisfactory practice of treating missing information as the household having access to the dimensions concerned as done in BCCD, or simply not including such households as done in the present study.

<sup>5</sup> There is now increasing availability of such panel data in developing countries. Besides the CHNS data set from China that has been used here, there is the IFLS data set from Indonesia used in AF, and household surveys conducted by the Research Centre on Rural Economy (RCRE) in Beijing used in Duclos, et al (2010).

minimum number of periods, or dimensions, or a combination of both. We consider a poverty index of the form  $g(\boldsymbol{\rho}, \boldsymbol{\nu})$  that ranks individuals and aggregates their rankings into a single non-negative real number.

Typically,  $\boldsymbol{\rho}$  is transformed into the *population deprivation profile*  $\boldsymbol{\delta} = (\mathbf{D}_1, \dots, \mathbf{D}_N)$  where  $\mathbf{D}_n$  is the *individual deprivation profile*: a matrix where each element of the individual achievement profile is transformed

into *deprivation inputs*  $d_{njt}^\alpha = \begin{cases} \left(1 - \frac{x_{njt}}{F_j}\right)^\alpha & \text{if } x_{njt} < F_j \quad \forall j, t \\ 0 & \text{otherwise} \end{cases}$  and where  $\alpha \geq 0$  is a sensitivity parameter

along the lines of the poverty measure in Foster, Greer, and Thorbecke (1984) [FGT]. When observed achievement levels are discrete or ordinal in at least one dimension, it is common to restrict  $\alpha = 0$  such that  $d_{njt}^\alpha \in \{0, 1\}$ .

We say that an individual  $n$  is *deprived* in dimension  $j$  at time  $t$  when  $x_{njt} < F_j$ , where  $F_j$  is a cut-off point that determines whether or not an individual is considered deprived in a particular dimension at a particular time. For example, in the dimension ‘health’,  $x$  may be the individual’s Body Mass Index, in which case  $F_{health}$  would be some threshold below which the individual would be considered underweight and therefore deprived in the health dimension. For brevity we assume these cut-offs do not vary across time, though it is not difficult to allow them to do so should the need arise.

These class of poverty measures are also known as the ‘dual-cutoff’ method due to  $F_j$ , the deprivation cut-off, and  $z$ , the poverty cutoff. The union method of identification sets  $z = 1$  while the intersection method sets  $z = (J * T)$ . Clearly the choice of who to consider poor will affect the final measure of poverty. However, the contribution of our proposed measure is the expansion of ways in which to think about the depth of poverty among the poor, rather than whom to consider poor.<sup>6</sup> We therefore restrict our attention to the union method of identification but define our axioms to be satisfied regardless of the choice of  $z$ .

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<sup>6</sup> Interested readers may refer to AACY where they consider the definition of the poverty cut-off in two stages: firstly, an individual is poor in a particular period if they are deprived in  $\nu$  dimensions, and an individual is chronically poor if they are poor for  $\tau$  periods (as introduced in Foster, 2009). Poverty is then calculated over the set of chronically poor. AACY also generalise the cut-offs to allow different weights across dimensions. Permanyer (2014) relaxes the ‘Strong Focus’ axiom maintained in the multidimensional poverty measurement literature that states that poverty measures should be unaffected by changes occurring for ‘poor households’ in dimensions where they do not suffer any deprivation. An extension of his measure to the dynamic multidimensional case is interesting but not considered here.



## 2.2. Static multidimensional measures of poverty and sensitivity to the distribution of deprivations

Let  $\mathbf{A}_{n|t}$  be the column vector of any individual achievement profile  $\mathbf{A}_n$ . The AF measure produces a separate multidimensional poverty score for each period of time using the static population achievement profile  $\boldsymbol{\rho}_t = (\mathbf{A}_{1|t}, \dots, \mathbf{A}_{N|t})$ . One limitation of AF is the following: given any distribution of achievements across dimensions and individuals in  $\boldsymbol{\rho}_t$ , a switch in the achievements in a particular dimension across poor individuals has no effect on the poverty measure, even when the switch effectively results in an increase in the inequality of the distribution of achievements across individuals. Consider the following 2 person, 2 dimension example of two different population *achievement* profiles where the non-zero entries are strictly positive.

$$\boldsymbol{\rho}_t = \left\{ \begin{pmatrix} x_{11t} \\ x_{12t} \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} \quad \boldsymbol{\rho}_t' = \left\{ \begin{pmatrix} 0 \\ x_{12t} \end{pmatrix} \quad \begin{pmatrix} x_{21t} \\ 0 \end{pmatrix} \right\}$$

$\boldsymbol{\rho}_t'$  can be constructed from  $\boldsymbol{\rho}_t$  by switching the achievement levels in dimension 1 across both individuals where  $x_{21t} = x_{11t}$ .

Intuitively, the distribution of achievements in  $\boldsymbol{\rho}_t$  is more unequal than in  $\boldsymbol{\rho}_t'$  because in  $\boldsymbol{\rho}_t$  individual 2 has no achievements whatsoever. AF allots the same poverty score to each profile. This can be seen as a violation of a general notion of the transfer axiom: given any fixed level of total deprivation across the population, poverty increases with a greater concentration of deprivation within the same individual.<sup>7</sup> The AF measure satisfies a very weak notion of this, requiring only that poverty *not increase* as we move from  $\boldsymbol{\rho}_t$  to  $\boldsymbol{\rho}_t'$  (as per the 'weak rearrangement' axiom) but not that it actually decreases.

This problem arises because the parameter  $\alpha \geq 0$  is raised over each normalised achievement gap  $\left(1 - \frac{x_{njt}}{F_j}\right)$  separately, rather than over the sum of these gaps for each individual. This is appropriate in the static unidimensional case in FGT since the normalised achievement gap is the only element that determines whether an individual is poor: a switch in achievements would have no effect on the distribution of achievements across the population, given the anonymity axiom. However, in the multidimensional case, poverty is determined by the normalised achievement gaps across multiple dimensions such that a switch of achievements across

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<sup>7</sup> While it has typically been formulated over the unidimensional case, at a general level the transfer axiom captures the notion of inequality-sensitivity; this is apparent in its position as a core property of inequality measures (the 'Pigou-Dalton transfer principle').

individuals for a specific dimension may change the distribution of achievements across individuals. While AF do not directly recognise this limitation, they suggest, in their conclusion a modification which coincidentally allows the limitation to be overcome: for each individual,  $\sum_j d_{nj}^\alpha$  can be raised by some parameter  $\gamma > 1$  to allow for deprivations to be 'complements' in the poverty index, effectively ranking  $\rho_t$  as a poorer population relative to  $\rho_t'$ . This is done in applications such as Bourguignon and Chakravarty (2003), Chakravarty and D'Ambrosio (2006), Jayaraj and Subramaniam (2010) and Datt (2013). However, doing so removes one of the desirable properties of AF: *dimensional decomposability*, which is the ability to identify the percentage contribution of each dimension to the overall poverty score.

We recommend a simple extension to AF that is sensitive to the type of inequality just described while retaining dimensional decomposability. Let us first define the AF measure in our notation as:

$$M_\alpha = \left[ \sum_n \left( \left( \frac{\sum_j d_{nj}^\alpha}{J} \right) * C_n \right) \right] / N \quad (1a)$$

where  $\alpha \geq 0$ ;  $C_n = \begin{cases} 1 & \text{if } \sum_t \sum_j d_{njt}^{\alpha=0} \geq z \\ 0 & \text{otherwise} \end{cases}$

Now consider our modified-AF measure:

$$\Omega_{\alpha|t} = \left[ \sum_n \left( \left( \frac{\sum_j d_{nj}^\alpha S_n^\alpha}{J} \right) * C_n \right) \right] / N \quad (1b)$$

where  $S_n^\alpha = \left( \frac{\sum_k d_{nkt}^\alpha}{J} \right)$  ;  $k \in \{1, 2, \dots, J\}$

Notice that because the measure remains the sum over the contribution of each dimension, it can be precisely decomposed and retain dimension decomposability. However, the term  $S_n^\alpha$  weights each deprivation input according to the normalised sum of all deprivation inputs. Consider the following example, where the previous static population achievement profiles have been converted to static population *deprivation* profiles.

$$\delta_t = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} \quad \delta_t' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

Which gives us:

$$\Omega_{\alpha|t}(\delta_t) = 0 * \left( \frac{0}{2} \right) + 0 * \left( \frac{0}{2} \right) + 1 * \left( \frac{2}{2} \right) + 1 * \left( \frac{2}{2} \right) = 2 / (J * N)$$

$$\Omega_{\alpha|t}(\delta_t') = 1 * \left(\frac{1}{2}\right) + 0 * \left(\frac{1}{2}\right) + 0 * \left(\frac{1}{2}\right) + 1 * \left(\frac{1}{2}\right) = 1/(J * N)$$

$\Omega_{\alpha|t}$  therefore satisfies our requirement that  $\delta_t$  be considered a poorer population relative to  $\delta_t'$ . In the example,  $\delta_t$  ends up two times poorer relative to  $\delta_t'$ . While some discounting of  $S_n^\alpha$  may be appropriate to narrow this gap, the choice of how much to narrow this gap by would be no different to the choice of the parameter  $\gamma$  recommended in AF.

We now state the inequality-sensitivity property formally:

**Definition (progressive dimensional rearrangement)**

We say that  $\rho'$  is obtained from  $\rho$  by a dimensional rearrangement among the poor if  $J > 1$  and there are two persons  $n$  and  $n'$  who are poor in  $\rho$  such that for one  $\rho_t$ , the following are true:

- 1) for one  $j$ , the achievement levels between  $n$  and  $n'$  are switched across  $\rho_t'$  and  $\rho_t$
- 2) the two achievement levels are non-identical

We say that  $\rho'$  is obtained from  $\rho$  by a progressive dimensional rearrangement if  $\rho'$  is obtained from  $\rho$  by a dimensional rearrangement among the poor, and in addition:

- 3) each element of  $A_{n|t}$  is *strictly* greater than the corresponding element in  $A_{n'|t}$  for  $\rho$  and
- 4) every element of  $A_n$  not belonging to  $A_{n|t}$  is *weakly* greater than the corresponding element in  $A_{n'}$  not belonging to  $A_{n'|t}$  for  $\rho$

The fourth requirement is redundant in the AF case but crucial when extended to the dynamic multidimensional case. Essentially, the progressive dimensional rearrangement requires a switch where one individual's deprivation profile for all dimensions in a specific period of time strictly dominates the other prior to the switch but where all remaining elements of the deprivation profile only weakly dominate.

**Axiom 1 (dimensional rearrangement)**

$g(\rho, \nu) > g(\rho', \nu)$  if  $\rho'$  is obtained from  $\rho$  by a progressive dimensional rearrangement.

This can be seen as a 'strict' version of AF's 'weak rearrangement' axiom.

As we will show later,  $\Omega_{\alpha|t}$  satisfies *dimensional rearrangement* and all the desirable properties of AF. In many applications of counting-based multidimensional poverty measures,  $d_{njt}^{\alpha} \in \{0,1\}$ ; in such a case *dimensional rearrangement* has the simple interpretation as increasing sensitivity to an increased count of dimensions of deprivation within the same individual.

### 2.3. Dynamic unidimensional measures of poverty and sensitivity to the distribution of deprivations

The dynamic unidimensional measures suggested by Foster (2009) and Bossert, Chakravarty and D'Ambrosio (2012) also suffer from a similar problem as that found in AF, though the problem now relates to achievements across time, rather than across dimensions.

Let  $\mathbf{A}_{nj}$  be the row vector of any individual achievement profile  $\mathbf{A}_n$ . The Foster (2009) measure produces a single poverty score over one dimension using the unidimensional population achievement profile  $\boldsymbol{\rho}_j = (\mathbf{A}_{1j}, \dots, \mathbf{A}_{Nj})$ . Given any distribution of achievements across time and individuals in  $\boldsymbol{\rho}_j$ , a switch of the achievements in a particular period of time across poor individuals may have no effect on the poverty score.<sup>8</sup> Like before, this is inconsistent with the general spirit of the transfer axiom since there are cases where it ignores the concentration of poverty within specific individuals, where concentration now refers to deprivations repeated over time.

This arises because it is the normalised achievement gap rather than the sum of the gaps across each period of time that is raised by the power  $\alpha \geq 0$ . Gradin et al (2012) do define an  $\alpha \geq 0$  over the sum of gaps that allows such sensitivity to be taken into account, but like the static multidimensional case, this violates the notion of *dynamic decomposability*; that is, the identification of the percentage contribution of each period of time to the overall poverty score.

We can define a modification of the basic Foster (2009) model that gives us sensitivity to inequality while retaining decomposability of poverty over time.

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<sup>8</sup> We say 'may' here because the incorporation of sensitivity towards persistence (deprivations occurring in consecutive periods) accounts for this to some extent, though not completely.

$$\Omega_{\alpha|j} = \left[ \sum_n \left( \left( \frac{\sum_t d_{nt}^\alpha S_n^\alpha}{T} \right) * C_n \right) \right] / N \quad (1c)$$

where  $S_n^\alpha = \left( \frac{\sum_u d_{nju}^\alpha}{T} \right); u \in \{1, 2, \dots, T\}$

As we will show later,  $\Omega_{\alpha|j}$  satisfies *dynamic rearrangement* and *dynamic decomposability*.

**Definition (progressive dynamic rearrangement)**

We say that  $\rho'$  is obtained from  $\rho$  by a dynamic rearrangement among the poor if  $T > 1$  and there are two persons  $n$  and  $n'$  who are poor in  $\rho$  such that for one  $\rho_j$ , the following are true:

- 1) for one  $t$ , the achievement levels between  $n$  and  $n'$  are switched across  $\rho_j'$  and  $\rho_j$
- 2) the two achievement levels are non-identical

We say that  $\rho'$  is obtained from  $\rho$  by a progressive dynamic rearrangement if  $\rho'$  is obtained from  $\rho$  by a dynamic rearrangement among the poor, and in addition:

- 3) each element of  $A_{n|j}$  is *strictly* greater than the corresponding element in  $A_{n'|j}$  for  $\rho$  and
- 4) every element of  $A_n$  not belonging to  $A_{n|j}$  is *weakly* greater than the corresponding element in  $A_{n'}$  not belonging to  $A_{n'|j}$  for  $\rho$

**Axiom 2 (dynamic rearrangement)**

$g(\rho, \mathbf{v}) > g(\rho', \mathbf{v})$  if  $\rho'$  is obtained from  $\rho$  by a progressive dynamic rearrangement.

If  $d_{njt}^\alpha \in \{0, 1\}$ , *dynamic rearrangement* has the simple interpretation as increasing sensitivity to an increased count of periods of deprivation within the same individual.

**2.4. Dynamic multidimensional measures of poverty and sensitivity to the distribution of deprivations**

We have shown that the static multidimensional model of AF and the dynamic unidimensional model of Foster (2009) do not satisfy variants of the transfer axiom that extend over counts of dimensions in the former, and counts of time in the latter. For each, we have suggested an alternative measure that does; in addition, these

measures satisfy desirable decomposability properties. Our key objective however, is to combine the properties of both  $\Omega_{\alpha|j}$  and  $\Omega_{\alpha|t}$  into a single dynamic multidimensional poverty index that contains both measures as special cases.

In existing dynamic multidimensional measures such as NR, BCCD and AACY, an individual's deprivation score is the double sum of deprivation inputs (summed over time and over dimensions). Axiom 1 and 2 can therefore be conveniently satisfied by raising the entire double sum to the power of some parameter  $\delta > 1$ . This is done in NR. However, once again, this violates the properties of *dimensional decomposability* and *dynamic decomposability*.

Consider instead:

$$\Omega_{\alpha\beta} = \left[ \sum_n \left( \left( \frac{\sum_t \sum_j d_{njt}^\alpha S_{njt}^{\alpha\beta}}{J*T} \right) * C_n \right) \right] / N \quad (2a)$$

where  $\alpha \geq 0$

Like NR and BCCD, the measure continues to be a double-sum across time and dimensions, therefore preserving decomposition across time and dimensions. However, each deprivation input is now weighted by  $S_{njt}^{\alpha\beta}$ , which we now define.

$$S_{njt}^{\alpha\beta} = \beta \left( \frac{\sum_k d_{nkt}^\alpha}{J} \right) + (1 - \beta) \left( \frac{\sum_u d_{nju}^\alpha}{T} \right) \quad (2b)$$

$$1 \geq \beta \geq 0$$

$$1 \geq S_{njt}^{\alpha\beta} \geq 0$$

$S_{njt}^{\alpha\beta}$  endogenously weights each deprivation input according to deprivations in all the dimensions associated with that period [first term of (2b)] and according to deprivations in all the periods associated with that dimension [second term of (2b)]. This would therefore give increasingly higher weight to individuals whose

deprivations are located along the same period/dimension relative to those whose deprivations are unrelated by time or dimensions.<sup>9</sup>

The exogenously chosen parameter  $\beta$  allows the analyst to assign additional weight to either deprivation across multiple dimensions, or deprivation across multiple periods (within the same individual). By setting  $1 > \beta > 0$  both *dimensional* and *dynamic rearrangement* are satisfied. When  $\beta = 0$  only *dynamic rearrangement* is satisfied and when  $\beta = 1$  only *dimensional rearrangement* is satisfied. However, regardless of the choice of  $\beta$ ,  $\Omega_{\alpha\beta}$  always satisfies *dimensional* and *dynamic decomposability*.

We now illustrate the poverty score generated by  $\Omega_{\alpha\beta}$  through the use of a numerical example. Consider the following three population deprivation profiles where we have 2 individuals, 2 dimensions, and 2 periods of time:

$$\boldsymbol{\delta} = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\} \quad \boldsymbol{\delta}' = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right\} \quad \boldsymbol{\delta}'' = \left\{ \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \right\}$$

As per the intuition in the previous examples, we argue that population  $\boldsymbol{\delta}$  is relatively poorer to  $\boldsymbol{\delta}'$  and  $\boldsymbol{\delta}''$  because while total deprivation is the same, deprivation is more concentrated in the former.

$\boldsymbol{\delta}'$  is constructed from  $\boldsymbol{\delta}$  by a progressive dynamic rearrangement in dimension 1, while  $\boldsymbol{\delta}''$  is constructed from  $\boldsymbol{\delta}$  by a progressive dimensional rearrangement in period 1.

Let us set  $\beta = 0.5$  to give equal weighting to deprivations in multiple dimensions and multiple periods; consider the scores that  $\Omega_{\alpha\beta}$  assigns to the three profiles:

$$\Omega_{\alpha\beta}(\boldsymbol{\delta}) = \left[ 0 * 0.5 \left( \frac{0}{2} + \frac{0}{2} \right) + 0 * 0.5 \left( \frac{1}{2} + \frac{0}{2} \right) + 0 * 0.5 \left( \frac{0}{2} + \frac{1}{2} \right) + 1 * 0.5 \left( \frac{1}{2} + \frac{1}{2} \right) \right]$$

$$+ (\text{the term for the second individual is the maximum value of 4}) = 4.5 / (J * N)$$

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<sup>9</sup> Note that  $\Omega_{\alpha\beta}$  does not give increasing weight to deprivations that are unrelated by time or by dimensions (i.e. deprivations lying diagonally across each other in the deprivation matrix). Analysts interested in doing so have to use the measure proposed in NR. However doing so sacrifices *dimensional* and *dynamic decomposition*, as well as the ability to adjust the weights given to deprivations repeated in the same dimensions over time, relative to deprivations across multiple dimensions in the same period. In addition, it is difficult to argue, for example, that in the case of three dimensions and three periods of time, an individual who is deprived in  $d_{n11}^\alpha, d_{n22}^\alpha, d_{n33}^\alpha$  is equally poor to an individual deprived in  $d_{n11}^\alpha, d_{n12}^\alpha, d_{n13}^\alpha$  since in the case of the latter, there is a case to argue that deprivation is 'concentrated'.

$$\begin{aligned} \Omega_{\alpha\beta}(\delta') &= \left[ 0 * 0.5 \left( \frac{0}{2} + \frac{1}{2} \right) + 1 * 0.5 \left( \frac{2}{2} + \frac{1}{2} \right) + 0 * 0.5 \left( \frac{0}{2} + \frac{1}{2} \right) + 1 * 0.5 \left( \frac{2}{2} + \frac{1}{2} \right) \right] \\ &+ \left[ 1 * 0.5 \left( \frac{2}{2} + \frac{1}{2} \right) + 0 * 0.5 \left( \frac{1}{2} + \frac{1}{2} \right) + 1 * 0.5 \left( \frac{2}{2} + \frac{2}{2} \right) + 1 * 0.5 \left( \frac{1}{2} + \frac{2}{2} \right) \right] = 4/(J * N) \end{aligned}$$

At  $\beta = 0.5$ ,  $\delta'$  and  $\delta''$  will yield the same scores since their elements consists of transposes of each other.

Since  $\delta''$  can be constructed from  $\delta$  by a progressive dimensional rearrangement of the first period,  $\Omega_{\alpha\beta}(\delta) > \Omega_{\alpha\beta}(\delta'')$  is consistent with Axiom 1. Similarly, since  $\delta'$  can be constructed from  $\delta$  by a progressive dynamic rearrangement of the first dimension,  $\Omega_{\alpha t}(\delta) > \Omega_{\alpha t}(\delta')$  is consistent with Axiom 2.

It is not clear if  $\delta'$  should be weighted equally to  $\delta''$  in all applications: should a poverty measure be more sensitive to deprivations repeated over time (in a given dimension), or to deprivations across multiple dimensions (at a given period of time)? Anyone wishing to compare poverty levels across two populations using panel data will, however, have to make this choice. Existing measures (NR, BCCD and AACY) essentially assume that both aspects are equally important. One may argue, for example, that a poverty measure should be relatively more sensitive to concentrations of deprivation across time rather than across dimensions. For example, individual 1 in  $\delta''$  should be considered poorer than individual 1 in  $\delta$  because single-period deprivations could be the result of short-term risky decision, or a temporary stroke of bad luck that, while debilitating in that period, requires less policy attention relative to individuals whose deprivations are repeated over time.<sup>10</sup> The measure  $\Omega_{\alpha\beta}$  allows us flexibility in deciding how to trade off deprivations repeated over time, versus those spread across multiple dimensions: increasing  $\beta$  increases the weight the measure allots to the latter.

Consider an extreme example where we allow only *dynamic rearrangement* by setting  $\beta = 0$ . The following are the new scores for the three profiles:

$$\Omega_{\alpha\beta'}(\delta) = \left[ 0 * \frac{0}{2} + 0 * \frac{0}{2} + 0 * \frac{1}{2} + 1 * \frac{1}{2} \right] + [4] = 4.5/(J * N)$$

---

<sup>10</sup> Clearly one other way to capture this chronicity is to extend the dual-cutoff method in Foster (2009) to the multidimensional case, as is done in AACY. However, the disadvantage in using additional cut-offs to capture this is that they may lead to violations of some notion of the transfer axiom; notably, a regressive transfer could reduce poverty by making a poor person non-poor (see Datt, 2013 for the case of AF; Permanyer, 2014 considers a measure that allows achievements above the threshold to contribute to the measure).



$$\Omega_{\alpha\beta'}(\boldsymbol{\delta}') = \left[ 0 * \frac{1}{2} + 1 * \frac{1}{2} + 0 * \frac{1}{2} + 1 * \frac{1}{2} \right] + \left[ 1 * \frac{1}{2} + 0 * \frac{1}{2} + 1 * \frac{2}{2} + 1 * \frac{2}{2} \right] = 3.5/(J * N)$$

$$\Omega_{\alpha\beta'}(\boldsymbol{\delta}'') = \left[ 0 * \frac{0}{2} + 0 * \frac{0}{2} + 1 * \frac{2}{2} + 1 * \frac{2}{2} \right] + \left[ 1 * \frac{2}{2} + 1 * \frac{2}{2} + 0 * \frac{1}{2} + 1 * \frac{1}{2} \right] = 4.5/(J * N)$$

We see that  $\Omega_{\alpha\beta'}(\boldsymbol{\delta}) = \Omega_{\alpha\beta'}(\boldsymbol{\delta}'') > \Omega_{\alpha\beta'}(\boldsymbol{\delta}')$ ; that is, dimensional rearrangement has no effect on the poverty score while dynamic rearrangement does. Since at  $\beta = 0$  increasing weight is only given to deprivations repeated over time, individual 1's increase in deprivation is exactly offset by individual 2's decreases in deprivation as we move from  $\boldsymbol{\delta}$  to  $\boldsymbol{\delta}''$ . That is, the measure now ignores the fact that the deprivation being switched ( $d_{n21}^\alpha$ ) shares the same time period with another dimension of deprivation for individual 2, but not for individual 1 (i.e.  $d_{111}^\alpha = 0$  but  $d_{211}^\alpha = 1$ ).

We explore the properties of  $\beta$  in greater detail in section 2.6.

**[Figure 1 around here]**

Consistent with the class of subgroup decomposable measures,  $\Omega_{\alpha\beta}$  has a convenient interpretation as the average poverty score for individuals in the population of interest. Figure 1 presents a summary of the subgroup decomposable measures that  $\Omega_{\alpha\beta}$  can be said to generalise, with the arrow pointing to measures of greater generality.

## 2.5. Other properties

### **Axiom 3 (dimensional decomposability):**

$g(\boldsymbol{\rho}, \mathbf{v}) = \omega_1 g(\boldsymbol{\rho}_1, \mathbf{v}) + \dots + \omega_J g(\boldsymbol{\rho}_J, \mathbf{v})$ , where  $\sum_j \omega_j = 1$  and  $\omega_j \in (0,1)$  are exogenously imposed dimensional weights, and  $g(\boldsymbol{\rho}_j, \mathbf{v})$  is the poverty index calculated using only the row vector  $\mathbf{A}_{nj}$  of each individual's achievement profile.

*Dimensional decomposability* allows the analyst to calculate the percentage contribution of each dimension towards the overall poverty score.

### **Axiom 4 (dynamic decomposability):**

$g(\boldsymbol{\rho}, \boldsymbol{v}) = \omega_1 g(\boldsymbol{\rho}_1, \boldsymbol{v}) + \dots + \omega_T g(\boldsymbol{\rho}_T, \boldsymbol{v})$ , where  $\sum_t \omega_t = 1$  and  $\omega_t \in (0,1)$  are exogenously imposed dynamic weights, and  $g(\boldsymbol{\rho}_t, \boldsymbol{v})$  is the deprivation index calculated using only the column vector  $\mathbf{A}_{n|t}$  of each individual's achievement profile.

*Dynamic decomposability* allows the analyst to calculate the percentage contribution of each period towards the overall poverty score. While NR, BCCD and AACY satisfy *dimensional* and *dynamic decomposability*, our measure has the advantage of allowing the decompositions to be sensitive to information outside the period or dimension being decomposed since each deprivation input is transformed to be a function of a row and column of the deprivation profile.

Because  $\Omega_{\alpha\beta}$  remains the average of the sum of individual poverty scores, it retains the subgroup decomposability property common to the class of measures originating in FGT.

Define  $N_s$  as the number of individuals in a population subgroup  $s \in \{1 \dots S\}$  where  $S$  is the total number of subgroups and  $\sum_s N_s = N$ ; define  $g(N_s, \boldsymbol{\rho}, \boldsymbol{v})$  as the poverty index calculated over subgroup  $s$ .

**Axiom 5 (subgroup decomposability):**

$$g(N, \boldsymbol{\rho}, \boldsymbol{v}) = \sum_s \frac{N_s}{N} g(N_s, \boldsymbol{\rho}, \boldsymbol{v}).$$

The axiom allows deprivation to be decomposed into a subgroup's percentage contribution towards the overall poverty score. For our measure, define  $g(N_s, \boldsymbol{\rho}, \boldsymbol{v})$  as  $\Omega_{\alpha\beta}^s$ . The subgroup's proportion of contribution can be calculated by:

$$\Omega \left( \frac{N_s}{N} \right)_{\alpha\beta}^s = \frac{N_s}{N} * \frac{\Omega_{\alpha\beta}^s}{\Omega_{\alpha\beta}} \quad (3)$$

This means that  $\Omega_{\alpha\beta}$  can effectively produce two subgroup-specific measures of poverty:  $\Omega_{\alpha\beta}^s$  allows the ranking of subgroups according to the highest average poverty score per person while  $\Omega \left( \frac{N_s}{N} \right)_{\alpha\beta}^s$  gives the percentage contribution of each particular subgroup to the overall poverty score. The two measures need not give the same ranking for subgroups; for example consider a 'subgroup' consisting of only one individual, who is deprived in every dimension for every period. While his  $\Omega_{\alpha\beta}^s$  will be 1 (i.e. the highest possible poverty score), his contribution to overall poverty in the population may be extremely small assuming there are

numerous others in the population who are deprived, even if their deprivations are nowhere near the extent of this one individual.

$\Omega_{\alpha\beta}$  also satisfies the following properties, which are based on AF but extended to the dynamic multidimensional framework.

**Axiom 6 (replication invariance):**

Define a single replication of  $N$  as  $A = (N, N)$ ; of  $\rho$  as  $B = (\rho, \rho)$ ; of  $\mathbf{v}$  as  $E = (\mathbf{v}, \mathbf{v})$ .

Replication invariance implies  $g(N, \rho, \mathbf{v}) = g(A, B, E)$  for any number of replications of  $N$ ,  $\rho$  and  $\mathbf{v}$  so long as the number of replications are the same for  $N$ ,  $\rho$  and  $\mathbf{v}$ .

This ensures that the size of the population in itself has no direct bearing on the poverty score.

**Axiom 7 (symmetry/anonymity):**

$g(\rho, \mathbf{v}) = g(\rho', \mathbf{v})$  where  $\rho'$  is any permutation of the vector  $\rho$ .

Therefore, all individuals are identical except for their deprivation profiles.

**Axiom 8 (normalisation and nontriviality):**

$g(\rho, \mathbf{v})$  achieves at least two distinct values: a minimum value of 0 and a maximum value of 1.

**Axiom 9 (poverty focus):**

$g(\rho, \mathbf{v}) = g(\rho', \mathbf{v})$  if  $\rho'$  is obtained from  $\rho$  by having a non-poor individual experience an achievement increase in any deprivation.

If an individual is not considered poor, then improvements in that individual's achievements are irrelevant to the measure, analogous to some variant of a Rawlsian social welfare function.

**Axiom 10 (deprivation focus):**

$g(\rho, \mathbf{v}) = g(\rho', \mathbf{v})$  if  $\rho'$  is obtained from  $\rho$  by an increase in the achievement in a dimension/period where an individual is not considered deprived.

That is, if  $x_{njt} \geq F_j$ , then improvements in the achievement associated with that deprivation input is irrelevant to the measure.

Define a deprivation decrement as a decrease in any one of an individual's deprivation inputs.

**Axiom 11 (deprivation monotonicity):**

$g(\boldsymbol{\rho}, \boldsymbol{\nu}) > g(\boldsymbol{\rho}', \boldsymbol{\nu})$  if  $\boldsymbol{\rho}'$  is obtained from  $\boldsymbol{\rho}$  by a deprivation decrement among the poor.

**Proposition 1:**  $\Omega_{\alpha\beta}$  satisfies the following **core properties**: *dimensional decomposability, dynamic decomposability, subgroup decomposability, replication invariance, symmetry, nontriviality, normalisation, poverty focus, deprivation focus* and *deprivation monotonicity*. In addition, when achievements are restricted to only two levels (e.g. zero and non-zero), it satisfies *dimensional rearrangement* when  $\beta > 0$  and *dynamic rearrangement* when  $\beta < 1$ . When achievements can take on more than two levels, it satisfies *dimensional rearrangement* when  $\beta > 0, \alpha > 0$  and *dynamic rearrangement* when  $\beta < 1, \alpha > 0$ .

**Proof:** *dynamic decomposability* and *dimensional decomposability* are straightforward implications of  $\Omega_{\alpha\beta}$  being a double sum of (the transformation of) individual deprivations over time and over dimensions. Since individual scores are simply summed and averaged over individuals,  $\Omega_{\alpha\beta}$  always satisfies *subgroup decomposability, replication invariance, symmetry, normalisation and nontriviality*. It also satisfies *poverty focus* based on the cutoff  $z$  and *deprivation focus* based on the cutoff  $F_j$ . *Deprivation monotonicity, dimensional rearrangement* and *dynamic rearrangement* are proved in Appendix A.

It is not uncommon in applications of multidimensional poverty measures that achievements have only two possible values (e.g. whether a person has access to water or otherwise). In such cases  $\alpha$  becomes irrelevant – *dimensional* and *dynamic rearrangement* are always satisfied since a rearrangement of achievements as defined in section 2.2 and 2.3 will always imply a switch between a deprivation input of zero and one.

Further extensions to the measure, including the assignment of different weights to different dimensions, or the incorporation of the concept of 'persistence' (Gradin et al, 2012; Bossert et al 2012) can be found in Appendix B.

## 2.6. Trading-off dimensions versus duration

In section 2.4 we discussed how  $\beta$  could be used to weight deprivations concentrated across dimensions more heavily than deprivations concentrated across time, or vice versa. In  $\Omega_{\alpha\beta}$  the increase in the poverty score caused by an increase in a deprivation input  $d_{njt}^\alpha$  is an increasing function of deprivations lying on the same column and row as  $d_{njt}^\alpha$ . Changing  $\beta$  changes the increase in any weighting term,  $S_{njt}^{\alpha\beta}$  caused by an increase in  $d_{nkt}^\alpha$  relative to an increase in  $S_{njt}^{\alpha\beta}$  caused by an increase in  $d_{nju}^\alpha$ .

To further illustrate the usefulness of  $\beta$ , assume  $d_{njt}^\alpha \in \{0,1\}$  and therefore that an increase in  $d_{njt}^\alpha$  only occurs when there is an increase in the count of deprivation. Let  $\frac{\Delta S_{nkt}^{\alpha\beta}}{\Delta d_{nkt}^\alpha}$  where  $k \neq j$  be the change in  $S_{nkt}^{\alpha\beta}$  caused by an increase in a count of deprivation belonging to  $t$  but not to  $j$ . Similarly let  $\frac{\Delta S_{nju}^{\alpha\beta}}{\Delta d_{nju}^\alpha}$  where  $u \neq t$  be the change in  $S_{nju}^{\alpha\beta}$  caused by an increase in a count of deprivation belonging to  $j$  but not to  $t$ . We can define the ratio of these two as  $\Psi \equiv \frac{\Delta S_{nkt}^{\alpha\beta}}{\Delta d_{nkt}^\alpha} / \frac{\Delta S_{nju}^{\alpha\beta}}{\Delta d_{nju}^\alpha}$ , which can be interpreted as  $S_{nkt}^{\alpha\beta}$ 's marginal rate of substitution between an additional dimension versus an additional period of time; equally, it can be interpreted as the ratio of the sensitivity of any weight  $S_{nkt}^{\alpha\beta}$  to additional dimensions relative to additional periods.  $\Psi = 2$  for example, means an additional dimension of deprivation along time  $t$  increases  $S_{nkt}^{\alpha\beta}$  by a factor twice that of the contribution from an additional period of deprivation along dimension  $j$ . In the example in section 2.4, this would lead to  $\delta$  being classified as poorer than  $\delta'$ .  $\Psi$  can be 'chosen' by setting an appropriate value for  $\beta$ .

**Proposition 2:** Given  $J$  and  $T$ , for any choice of  $\Psi$ ,  $\beta$  must be set equal to  $\frac{\Psi J}{T + \Psi J}$

**Proof:** Appendix C

From Proposition 2 we can see that when  $\Psi = 1$  (i.e. equal weighting is desirable),  $\beta$  should be set equal to 0.5 only when  $T = J$ .

Recall that the main point of dynamic multidimensional measures is to compare the deprivation of different populations, or the subgroups within a given population. Therefore even when the precise choice of  $\Psi$  (and

hence,  $\beta$ ) is unclear to the analyst, it may be useful to consider whether the *group rankings* from any of these comparisons change with the value of  $\beta$ . We can term groups whose rankings do not change with  $\beta$  as ' *$\beta$  invariant*' and avoid the need to worry about the choice of  $\beta$  (at least, for ranking purposes).

## 2.7. Decomposing the measure according to overall multidimensional versus durational contributions

While dynamic multidimensional poverty measures are able to rank different populations or subgroups of populations, a policy-maker may be interested in knowing whether a population or subgroup's poverty score reflects deprivations concentrated across particular periods of time, deprivations concentrated across specific dimensions, or perhaps, neither. This will inform the policy-maker as to whether poverty is driven by brief or periodic, but deeply debilitating deprivation, versus whether it is instead driven by recurring deprivation in specific dimensions. The policy prescriptions for both may be quite different: in the case of the former, programs that insure against shocks may be optimal; in the case of the latter, policies directed at the specific dimension may be superior.

This information cannot be gleaned by looking at the dimensional and dynamic decompositions; this is because  $\Omega_{\alpha\beta}$  cannot be decomposed by dimensions and periods simultaneously. We instead propose a simple summary statistic,  $Bp$ , that gives us such information. In addition,  $Bp$  allows us to infer the sensitivity of the various subgroup-specific poverty measures,  $\Omega_{\alpha\beta}^s$ , to changes in  $\beta$ .

$$Bp = \frac{\sum_n^N \left( \sum_t^T \sum_j^J \left[ \left( \frac{\sum_j^J d_{njt}^\alpha}{J} \right) d_{njt}^\alpha \right] \right)}{\sum_n^N \left( \sum_t^T \sum_j^J \left[ \left[ \left( \frac{\sum_j^J d_{njt}^\alpha}{J} \right) + \left( \frac{\sum_t^T d_{njt}^\alpha}{T} \right) \right] d_{njt}^\alpha \right] \right)} = \frac{\Omega_{\alpha,\beta=1}}{2 * \Omega_{\alpha,\beta=0.5}} \quad (4)$$

where  $Bp$  is the proportion of overall poverty attributable to a concentration of multiple dimensions of deprivation in particular periods (i.e., 'breadth' of deprivation) and  $Lp$  is the proportion of overall poverty attributable to a concentration of multiple periods of deprivation in particular dimensions (i.e., 'length' of deprivation). Since  $Bp + Lp = 1$ , for convenience we focus solely on  $Bp$ .  $Bp$  can be calculated for the population as a whole, or for each subgroups separately by replacing  $N$  with  $N_s$ .

A  $Bp$  above 0.5 indicates that deprivation is more likely to be concentrated within specific periods of time in the form of multiple dimensions. A  $Bp$  below 0.5 indicates that deprivation is more likely to be concentrated within specific dimensions in the form of multiple periods.<sup>11</sup>

When calculated for each subgroup, relative differences in  $Bp$  across subgroups give us an indicator of the relative sensitivity of the subgroup scores  $\Omega_{\alpha\beta}^s$  to changes in  $\beta$ . For example, consider two subgroups  $s \in \{1, 2\}$ . Assume  $Bp_{s=1} > Bp_{s=2}$ . This means subgroup 1 has a more deprivations spread over multiple dimensions relative to subgroup 2. Increasing  $\beta$  would therefore increase subgroup 1's poverty score relative to subgroup 2 while decreasing  $\beta$  does the opposite.

### 3. Data set and summary features

We apply the proposed dynamic measure of multidimensional poverty to a panel data set from China from 1993-2009. The empirical evidence is of particular interest since they cover periods that include the Asian Financial Crisis and the Global Financial Crisis.

The Chinese data came from the China Health and Nutrition Survey (CHNS). This is an ongoing international project between the Carolina Population Center at the University of North Carolina at Chapel Hill and the National Institute of Nutrition and Food Safety at the Chinese Centre for Disease Control and Prevention. This project was designed to examine the effects of health, nutrition and family planning policies and programs implemented by the national and local governments and to see how the social and economic transformation is affecting the health and nutritional status of the population. A detailed description of the CHNS data base has been presented in Popkin, Du, Zhai and Zhang (2010). The surveys took place over a three day period using a multi-stage, random cluster process to draw a sample of over 4000 households in nine provinces that vary

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<sup>11</sup> Note for any individual, the breadth and length aspects of deprivation can only differ when the count of deprivations,  $\sum_j \sum_t d_{njt}^\alpha$ , is greater than zero and less than  $(J * T)$  – i.e., the theoretical maximum number of deprivations. When the count of deprivations is  $(J * T)$ , the deprivation profile is ‘full’ and therefore both breadth and length aspects are at their respective maxima. For values of  $\sum_j \sum_t d_{njt}^\alpha$  that lie between zero and  $(J * T)$ , the differentiation of breadth and length becomes meaningful and  $Bp$  can take on different values depending on the deprivation profile. In applications, the average deprivation count  $\sum_j \sum_t d_{njt}^\alpha / N$  can be used as an indicator of the usefulness of distinguishing length from breadth.

substantially in geography, economic development, public resources and health indicators. We converted household level information to the individual level by assuming that the household's access to a facility such as drinking water or electricity is the same for all individuals in that household. In a departure from previous applications on CHNS data, we supplemented the individual and household level information with the community level information available in the CHNS community identifiers. The community questionnaire (filled out for each of the primary sampling units) collected information from a knowledgeable respondent on community infrastructure (water, transport, electricity, communications, and so on), services (family planning, health facilities, retail outlets), population, prevailing wages, and related variables. Only individuals aged 18 years and above in the first year of the panel were included in construction of the balanced panel.

The dimensions at the household, individual and community levels considered in this study are described in Appendix D1. Appendix D2 describes the two different samples used for China. Appendix D2 also provides the deprivation cut offs used in the quantitative dimensions (years of education, BMI, BP). While the CHNS data set is longitudinal, there is a trade-off between the length of the time interval and the number of dimensions on which information is available for the panel of individuals. The first sample considers different combinations of time and dimensions through the use of exclusively 'qualitative' dimensions where an individual is either deprived or not deprived in a dimension – there is no information on gradations of deprivation.<sup>12</sup> In such a case,  $\alpha$  has no effect on the measure – *dimensional* and *dynamic rearrangement* are always satisfied. In contrast, sample two contains only dimensions that are quantitative, meaning there is information on the size of the achievement gap. This means that changes in  $\alpha$  will have an effect on the measure.

For the analysis, we focus initially on Sample 1 given the breadth of dimensions it covers: Appendix D3 presents the summary statistics, year and dimension-wise, of the deprivation rates in China for the panel of individuals in sample one. While some dimensions such as electricity, drink water, radio/TV recorded large improvements over the period, the opposite is true for other dimensions; for example, the proportion of individuals with abnormal blood pressure and the proportion of individuals without access to a vehicle increased over the period.

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<sup>12</sup> Note that though in case of some of these dimensions, for example, BMI, blood pressure and years of schooling, the quantitative information is available, we converted them into qualitative dimensions for consistency with the others, such as access to toilet, fuel, etc.



## 4. Results

This section provides an empirical illustration of the use and interpretation of the proposed measure,  $\Omega_{\alpha\beta}$ , and its associated properties. As the main use of this class of measures is the comparison of poverty across different population subgroups, in Section 4.1 we present the results according to this decomposition. In Section 4.2 we consider how sensitive the results are to changes in  $\beta$ . We present results based on dimensional and dynamic decomposition in Section 4.3. Such decompositions allow direct comparisons with Foster (2009) and AF. In all our calculations we assume  $z = 1$  (the union approach).

The results presented in Tables 1-4 contain three models each. Model 1 is the base case, where  $S_{njt}^{\alpha\beta}$  is restricted to be equal to one. This leads to special cases of existing dynamic multidimensional measures (NR, BCCD and AACY), and in the case of dimensional and dynamic decomposition, also leads to particular specifications of Foster (2009) and AF respectively.<sup>13</sup> Model 2 allows  $S_{njt}^{\alpha\beta}$  to endogenously vary as defined in equation (2b), therefore satisfying *dimensional* and *dynamic rearrangement* properties. However we also set  $\beta = 0.5$  in Model 2. Since  $J > T$  in Sample 1, this implicitly amounts to allotting more weight to repeated periods of deprivation relative to additional dimensions of deprivation. In Sample 2,  $J < T$  and the reverse is true. To account for this, in Model 3 we set  $\beta$  according to Proposition 1 by choosing  $\chi = 1$  (that is, the weights are equally sensitive to additional periods and dimensions).

### 4.1. Subgroup comparisons: Existing dynamic multidimensional measures and $\Omega_{\alpha\beta}$

Table 1 presents the subgroup comparisons for Sample 1.

[Table 1 around here]

We define subgroups according to three different groupings: 1) male/female; 2) province; 3) rural/urban. The reported calculations present four numbers: the first is the proportion contributed by each population subgroup to the overall score (equation 3); the second number is the rank of each subgroup according to this proportion

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<sup>13</sup> In the case of Foster (2009), this is equivalent to a specification where  $\tau = 1/T$ ; in the case of AF, this is equivalent to a specification where  $\alpha = 0$  for Sample 1 and  $\alpha = 1$  for Sample 2.

(from the most to least poor); the third is the poverty score calculated for each subgroup,  $(\Omega_{\alpha\beta}^S)$ ; and lastly, the rank of each subgroup (from the most to least poor) according to  $\Omega_{\alpha\beta}^S$ . Note that while the proportion calculation is sensitive to the size of the subgroup, the actual score  $\Omega_{\alpha\beta}^S$  is not, by virtue of it being an average across individuals (as per the *replication invariance* axiom). However, the actual score  $\Omega_{\alpha\beta}^S$  suffers from not being comparable across models because its scale changes with the choice parameters. For example in Model 1 we see that the Henan province contributes the least proportion to overall poverty, even though it has the 3<sup>rd</sup> highest deprivation score, which is attributable to Henan's relatively smaller population. These features are present in the class of subgroup-decomposable measures consistent with FGT.

A comparison of proportion contributions between Model 1 and 2 of Table 1 reveal relatively minor changes in terms of the rural/urban and male/female comparison. However, while the rankings of the provinces according to  $\Omega_{\alpha\beta}^S$  do not change across the two models, the rankings according to proportion contributions do, notably, for the Henan, Hunan and Guangxi provinces. The increased relative poverty level associated with Henan in Model 2 suggests that deprivation in Henan is more likely to be concentrated, whether in particular periods or particular dimensions. These changes highlight that  $\Omega_{\alpha\beta}$  can lead to rankings that are different from existing dynamic multidimensional measures due to the satisfaction of *dimensional* and *dynamic rearrangement* properties.

**[Table 2 around here]**

Table 2 presents the results from Sample 2, which contains quantitative information on deprivations, and in which we set  $\alpha = 1$ . As in Table 1, we see several provinces continue to switch ranks in terms of proportion contribution as we move from Model 1 to Model 2. Furthermore the Jiangsu and Hubei provinces switch ranks in terms of  $\Omega_{\alpha\beta}^S$  as well. Table 2 also suggests that the use of quantitative dimensions increases the differences in proportion contributions between males and females considerably: the proportion of deprivation attributable to females increases by 8% (to 75%) in Model 2.

#### 4.2. Subgroup comparisons: $\Omega_{\alpha\beta}$ and $\beta$

In all the tables, Models 2 and 3 differ by the choice of  $\beta$ . Where the former sets  $\beta = 0.5$ , the latter sets it by choosing  $x = 1$  and through the use of Proposition 1. For Sample 1 (Table 1), this results in  $\beta$  increasing to 0.722. As highlighted in Section 2.7, the sensitivity of the poverty score to changes in  $\beta$  can be gauged by looking at the proportion of deprivation concentrated in specific periods, but across multiple dimensions. This is summarised by the statistic  $Bp$ . Looking at  $Bp$  for the sample as a whole, the first thing we notice is that it is less than 0.5. This value of  $Bp$  can be interpreted thus: approximately 36% of overall poverty in the sample can be attributed to multiple dimensions within specific periods and the remaining 64% can be attributed to repeated periods within specific dimensions. The overall poverty score,  $\Omega_{\alpha\beta}$ , should therefore decrease as  $\beta$  increases (as it does in Table 1 from 0.2155 to 0.1893).

Along the same lines, the sensitivity of the subgroup measure,  $\Omega_{\alpha\beta}^S$ , to changes in  $\beta$  can be ascertained by looking at the differences between the  $Bp$  calculated for each subgroup. Looking at the provinces in Table 1,  $Bp$  lies within the range of 0.34-0.38, therefore suggesting that  $\Omega_{\alpha\beta}^S$  is unlikely to vary wildly with  $\beta$ . However, for subgroups whose poverty scores are close, sensitivity to  $\beta$ , regardless of how low, may have an impact on poverty rankings. Consider, for example, the Shandong and Henan provinces, who, in Model 2, have scores of 0.2541 and 0.2514 respectively. Because Henan's  $Bp$  score is higher than Shandong, we can expect Henan's poverty to increase relative to Shandong as  $\beta$  is increased. In Model 3 the corresponding scores are 0.2215 and 0.2202. While their ranks have not switched, the gaps have narrowed: further increases in  $\beta$  may switch their rankings. Figure 2 depicts the poverty score of each province as a ratio of the sum of the scores of all the provinces according to all possible settings of  $\beta$  – we see that at values of  $\beta$  close to 1, the Henan province is considered to be in deeper poverty than Shandong.

**[Figure 2 around here]**

In Table 2, we have the quantitative Sample 2 where we set  $\beta = 0.375$  in Model 3. Therefore while in Table 1, the movement from Model 2 to 3 involved an increase in  $\beta$ , in Table 2 it instead involves a decrease to  $\beta = 0.375$ . This is because in the quantitative sample we have three dimensions but six periods. Similar to Table 1, we do not see any change in rankings, though the closeness of the poverty scores of Jiangsu and Shandong

suggest that further *decreases* in  $\beta$  may switch their rankings.<sup>14</sup> One interesting thing to note is that in both Tables 1 and 2, the *Bp* score of Guizhou – consistently the poorest district in both measures and in all models across both tables – is markedly higher than the other provinces. This suggests that a relatively wider breadth of deprivation, i.e. deprivation in multiple dimensions for certain periods, may be a key driver of Guizhou’s consistent higher poverty score.

### 4.3: Dynamic and Dimensional Decomposition

$\Omega_{\alpha\beta}$  is decomposable according to each period and dimension. Model 1 in Tables 3 and 4 show us the decompositions according to periods and dimensions when  $S_{njt}^{\alpha\beta}$  is restricted to be equal to 1. When decomposed according to each period and when  $S_{njt}^{\alpha\beta}$  is restricted to be equal to 1, the measure can be interpreted as a special case of AF with one score calculated for each period; the percentage contribution of each period is therefore simply the ratio of the score for the specific period of time as a ratio to the sum of scores for all periods. Similarly, when decomposed according to each dimension and when  $S_{njt}^{\alpha\beta}$  is restricted to be equal to 1, the measure can be interpreted as a special case of Foster (2009) with one score calculated for each dimension.<sup>15</sup> Models 2 and 3 allow  $S_{njt}^{\alpha\beta}$  to vary as per equation (2b); this means, dynamic decompositions are not only a function of deprivations belonging to the specific time period, and dimensional decompositions are not only a function of deprivations belonging to the specific dimension.

[Table 3 around here]

Table 3 depicts the dynamic and dimensional decompositions for Sample 1 in terms of the proportion contribution to overall poverty. In terms of dynamic decomposition, we see that 1993 remains the most deprived period in all three models and that deprivation has been falling over time. The qualitative conclusions are the same for all 3 models, with very minor changes in the percentage contributions. In terms of dimensional

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<sup>14</sup> We require a decrease in this case because the *Bp* for Shandong is higher than Jiangsu’s, therefore decreasing  $\beta$  decreases Shandong’s poverty relative to Jiangsu.

<sup>15</sup> Since Foster’s application is unidimensional and focussed on the traditional income measure, such a characterisation is not immediately obvious.

decomposition, we see that Blood Pressure and No Access to Toilets switch ranks as we move from Model 1 to Model 2. The increase in  $\beta$  as we move from Model 2 to Model 3 has little effect on the results. No Access to Fuel consistently remains the dimension that contributes the most to poverty, while No Access to Electricity, the least.

**[Table 4 around here]**

Table 4 depicts the dynamic and dimensional decompositions for Sample 2. While the rankings of the decompositions across time and dimensions do not change across the models, two features are worth noting. First, while the year 2006 remains fifth ranked in terms of poverty in Table 3, it is second ranked in Table 4. This highlights that the choice of dimensions clearly has an effect on our conclusions; the unrepresentative dimensions for the quantitative sample should not be taken as indicative of overall poverty in the sample. Secondly, the proportion contribution of the blood pressure dimension (Table 4) falls by 19% as we move from Model 1 to Model 2. This is primarily attributable to the increase in the proportion contribution of the education dimension. We had chosen the education variable because of its correlation over time; i.e. an adult individual who does not have a primary education today is not likely to obtain one over time. Because Models 2 and 3 are sensitive to the repeated deprivation in this dimension, the proportion contribution has increased dramatically. Clearly this leads to the question of whether education is an appropriate dimension to include in measures that are interested in the duration of deprivation, or whether it should be, at least, discounted in the overall calculations (Appendix B details how this can be done). We have included it here to highlight one aspect that can be captured by our measure: sensitivity to repeated deprivations over time within the same dimension. The flexibility of our measure allows the analyst to set  $\beta = 1$  when sensitivity to repeated periods (*dynamic rearrangement*) may be deemed unnecessary.<sup>16</sup>

Notice that the measure  $\Omega_{\alpha\beta}$  cannot be decomposed into its dimensional and time components simultaneously. This means that while *within* any set of decompositions, we can identify which component contributes more to poverty (e.g. that 1993 contributed more than 2004, or that BMI contributed more than blood pressure) we do

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<sup>16</sup> Indeed, it is also possible to think of other ways to *decrease* the weight allotted to repeated time periods; for example, the term  $\left(\frac{\sum_t^T \alpha_{nju}^\alpha}{T}\right)$  in the weighting function  $S_{njt}^{\alpha\beta}$  can be raised to a power of less than one but greater than zero.

not know which *set* contributes relatively more to poverty. For that, the  $Bp$  statistic serves as a convenient indicator. In Sample 1, for example, we know that  $Bp = 0.36$  (Table 1). We therefore know that deprivation is more likely to be concentrated in particular dimensions, which in turn suggests that attention should be paid to dimensional, rather than dynamic decompositions.

## 5. Discussion and Conclusion

There has been a large and rapidly proliferating literature on the multidimensional measurement of poverty and its application to household survey data. The increasing availability of micro datasets containing a wealth of household and individual information has helped in the development of increasingly sophisticated measures that take into account the richness of available information.

We have proposed a dynamic multidimensional measure of poverty that unites two strands of a literature that finds its origins in Foster, Greer and Thorbecke (1984). Our measure allows population subgroups to be compared when information regarding both multiple dimensions of deprivation and multiple periods of time are available. We lay out two unique properties of our measure that are natural extensions of the transfer axiom, both of which are not satisfied by the Alkire and Foster (2011a) and Foster (2009) measure. We argue that the separate identification and weighting of deprivation that occurs as multiple dimensions in the same period, versus repeated periods in the same dimension is useful to the analyst for two reasons: 1) due to the general idea that a concentration of deprivations within the same individual (whether within specific dimensions or within specific periods of time) should be given more attention than the simple count of deprivations; 2) there may be cases where a concentration of deprivation within specific periods is more important than deprivation within specific dimensions and vice versa, in which case the ability to change the relative weights of the two components becomes important.

The empirical application has highlighted that our measure allocates subgroup poverty scores, as well as subgroup, dimensional, and dynamic shares that may differ from other models in the literature. Another point illustrated by our application has been the importance of the 'length' relative to the 'breadth' aspect of poverty in China, suggesting that for several dimensions, deprivation remains chronic. Despite this, the province of

Guizhou, which consistently appears as the province with the deepest extent of poverty, has a relatively higher breadth aspect. Overall therefore, while chronic deprivation within specific dimensions best explains overall poverty, deprivation over multiple dimensions within specific periods explains Guizhou's *relatively* higher poverty score.

The generality and flexibility of our measure comes at a cost since any researcher or policy-maker will have to make a choice with regards to the additional parameter  $\beta$ . This additional choice may then encourage interested users who have panel data in defaulting to the use of Alkire and Foster (2011a) when comparing poverty of a fixed population across time, and using Foster (2009) when comparing poverty of a fixed population across dimensions. That is, our extension may be deemed unnecessary if comparison across subgroups is not required. However, we have also highlighted that even if one was only interested in comparing across dimensions, or comparing across periods, our measure, when decomposed to the component of interest, may still lead to different conclusions than that generated by existing models given its sensitivity to the overall distribution of deprivations within individual deprivation profiles.

For multidimensional poverty measures in general, the choice of dimensions of deprivation, dimensional cut-offs, poverty cut-offs, as well as the appropriate weights to attach to them remains a fertile research agenda beyond the scope of this paper [for a brief discussion, see Alkire and Foster (2011b)]. When faced with a large degree of undecidedness with regards to parameter choices, we would advocate calculating the results for various combinations of the parameters. Indeed, understanding how and why poverty measurement changes with the various parameters is arguably more informative than poverty conveyed by a single number.

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**Table 1:** Multidimensional Deprivation and its Subgroup Decomposition for CHNS Sample One

		MODEL 1		MODEL 2		MODEL 3		$Bp$
$\alpha$		irrelevant		irrelevant		irrelevant		.
$\beta$		$S_{njt}^{\alpha\beta} = 1$		0.5		0.7222		.
$\Omega_{\alpha\beta}$		0.3597		0.2155		0.1893		0.3632
	<b>No. of observations</b>	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^s, \text{Rank})$	$(\Omega_{\alpha\beta}^s, \text{Rank})$	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^s, \text{Rank})$	$(\Omega_{\alpha\beta}^s, \text{Rank})$	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^s, \text{Rank})$	$(\Omega_{\alpha\beta}^s, \text{Rank})$	
<i>Male</i>	839	(0.4381, 2)	( 0.3520, 2)	(0.4386, 2)	(0.2092, 2)	(0.4329, 2)	(0.1831, 2)	0.3595
<i>Female</i>	1035	(0.5619, 1)	( 0.3659, 1)	(0.5654, 1)	( 0.2207, 1)	(0.5671, 1)	(0.1944, 1)	0.3661
<i>Jiangsu</i>	303	(0.1539, 3)	( 0.3425, 5)	(0.1492, 3)	(0.1988, 5)	(0.1490, 3)	( 0.1745, 5)	0.3627
<i>Shandong</i>	223	(0.1327, 4)	( 0.4010, 2)	(0.1403, 4)	(0.2541, 2)	(0.1392, 4)	(0.2215, 2)	0.3555
<i>Henan</i>	179	(0.1063, 7)	( 0.4003, 3)	(0.1114, 5)	(0.2514, 3)	(0.1111, 5)	(0.2202, 3)	0.3605
<i>Hubei</i>	299	(0.1621, 2)	( 0.3655, 4)	(0.1633, 2)	(0.2205, 4)	(0.1633, 2)	(0.1938, 4)	0.3638
<i>Hunan</i>	248	(0.1176, 5)	( 0.3195, 6)	(0.1095, 6)	( 0.1783, 6)	(0.1094, 6)	( 0.1564, 6)	0.3625
<i>Guangxi</i>	277	(0.1068, 6)	( 0.2600, 7)	(0.0923, 7)	( 0.1346, 7)	(0.0903, 7)	(0.1157, 7)	0.3418
<i>Guizhou</i>	345	(0.2205, 1)	( 0.4308, 1)	(0.2341, 1)	( 0.2741, 1)	(0.2376, 1)	( 0.2443, 1)	0.3779
<i>Rural</i>	1399	(0.8119, 1)	( 0.3912, 1)	(0.8316, 1)	( 0.2401, 1)	(0.8360, 1)	(0.2120, 1)	0.3684
<i>Urban</i>	475	(0.1881, 2)	(0.2669, 2)	(0.1684, 2)	( 0.1432, 2)	(0.1640, 2)	(0.1225, 2)	0.5091

**Table 2:** Multidimensional Deprivation and its Subgroup Decomposition for CHNS Sample Two

		MODEL 1		MODEL 2		MODEL 3		$Bp$
$\alpha$		1		1		1		.
$\beta$		$S_{njt}^{\alpha\beta} = 1$		0.5		0.375		.
$\Omega_{\alpha\beta}$		0.1396		0.0572		0.0623		0.3222
	No. of observations	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^S, \text{Rank})$	$(\Omega_{\alpha\beta}^S, \text{Rank})$	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^S, \text{Rank})$	$(\Omega_{\alpha\beta}^S, \text{Rank})$	$(\Omega \left(\frac{N_S}{N}\right)_{\alpha\beta}^S, \text{Rank})$	$(\Omega_{\alpha\beta}^S, \text{Rank})$	
<i>Male</i>	1159	(0.3713, 2)	(0.098, 2)	(0.2812, 2)	(0.0305, 2)	(0.2792, 2)	(0.0329, 2)	0.3380
<i>Female</i>	1299	(0.6687, 1)	(0.177, 1)	(0.7490, 1)	(0.0811, 1)	(0.7509, 1)	(0.0889, 1)	0.3168
<i>Heilongjiang</i>	368	(0.0973, 7)	(0.0907, 8)	(0.0680, 8)	(0.0259, 8)	(0.0679, 8)	(0.0283, 8)	0.3237
<i>Jiangsu</i>	344	(0.1421, 2)	(0.1418, 5)	(0.1568, 2)	(0.0641, 4)	(0.1576, 2)	(0.0702, 4)	0.3111
<i>Shandong</i>	303	(0.1314, 3)	(0.1488, 3)	(0.1408, 3)	(0.0654, 3)	(0.1412, 3)	(0.0714, 3)	0.3165
<i>Henan</i>	197	(0.0913, 8)	(0.1590, 2)	(0.0991, 5)	(0.0708, 2)	(0.0993, 5)	(0.0772, 2)	0.3190
<i>Hubei</i>	301	(0.1255, 4)	(0.1430, 4)	(0.1256, 4)	(0.0588, 5)	(0.1257, 4)	(0.0639, 5)	0.3254
<i>Hunan</i>	296	(0.0975, 6)	(0.1130, 7)	(0.0798, 6)	(0.0379, 7)	(0.0795, 6)	(0.0411, 7)	0.3301
<i>Guangxi</i>	285	(0.0980, 5)	(0.1179, 6)	(0.0792, 7)	(0.0391, 6)	(0.0788, 7)	(0.0424, 6)	0.3331
<i>Guizhou</i>	364	(0.2168, 1)	(0.2043, 1)	(0.2963, 1)	(0.1971, 1)	(0.2499, 1)	(0.1051, 1)	0.4826
<i>Rural</i>	1698	(0.7262, 1)	(0.1467, 1)	(0.7359, 1)	(0.0610, 1)	(0.7350, 1)	(0.0458, 2)	0.4806
<i>Urban</i>	760	(0.2738, 2)	(0.1236, 2)	(0.2641, 2)	(0.0489, 2)	(0.2649, 2)	(0.0533, 1)	0.4609

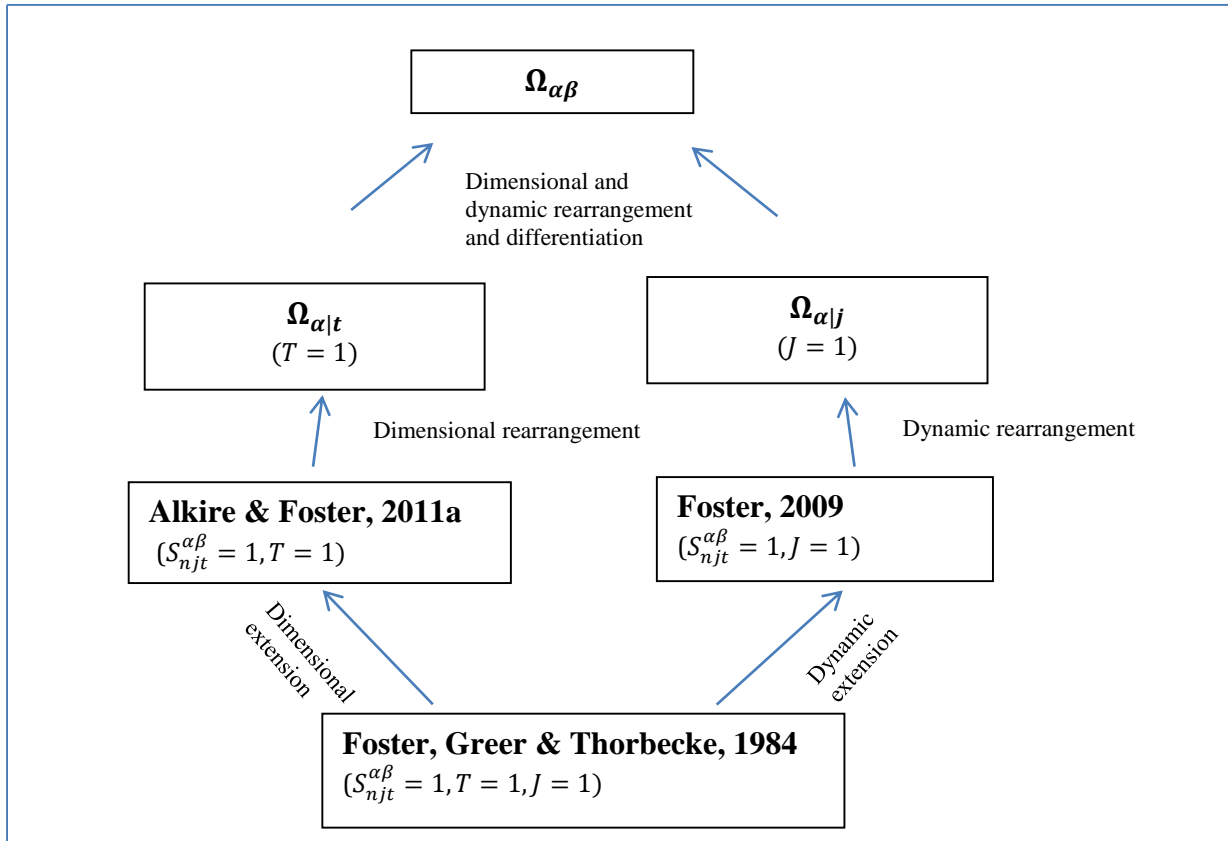
**Table 3:** Dynamic and Dimensional Decomposition (CHNS Sample One)

	<b>Model 1</b>		<b>Model 2</b>		<b>Model 3</b>	
$\alpha$	irrelevant		irrelevant		irrelevant	
$\beta$	$S_{njt}^{\alpha\beta} = 1$		0.5		0.7222	
$\Omega_{\alpha\beta}$	0.3597		0.2155		0.1893	
<b>Dynamic Decomposition</b>						
Period	Contribution	Rank	Contribution	Rank	Contribution	Rank
1993	0.2176	1	0.2141	1	0.2182	1
1997	0.2025	2	0.2046	2	0.2052	2
2000	0.1967	3	0.2011	3	0.2004	3
2004	0.1917	4	0.1921	4	0.1896	4
2006	0.1915	5	0.1880	5	0.1866	5
<b>Dimensional Decomposition</b>						
	Contribution	Rank	Contribution	Rank	Contribution	Rank
<i>No access to toilet</i>	0.1240	3	0.1352	2	0.1325	2
<i>No access to fuel</i>	0.1568	1	0.1699	1	0.1627	1
<i>No access to electricity</i>	0.0017	13	0.0013	13	0.0016	13
<i>No access to drink water</i>	0.1165	4	0.1274	4	0.1245	4
<i>No access to at least one vehicle type</i>	0.0549	9	0.0525	9	0.0533	9
<i>No access to radio/TV</i>	0.0595	8	0.0547	8	0.0595	8
<i>BMI not in normal range</i>	0.0212	12	0.0181	12	0.0192	12
<i>Illness in the last four weeks</i>	0.0375	11	0.0250	11	0.0290	11
<i>Blood pressure not normal</i>	0.1339	2	0.1289	3	0.1249	3
<i>Individual below primary education</i>	0.0826	6	0.0940	5	0.0910	5
<i>No access to road in community</i>	0.0939	5	0.0867	6	0.0903	6
<i>No access to bus station in community</i>	0.0776	7	0.0753	7	0.0776	7
<i>No access to school in community</i>	0.0399	10	0.0313	10	0.0338	10

**Table 4:** Dynamic and Dimensional Decomposition (CHNS Sample Two)

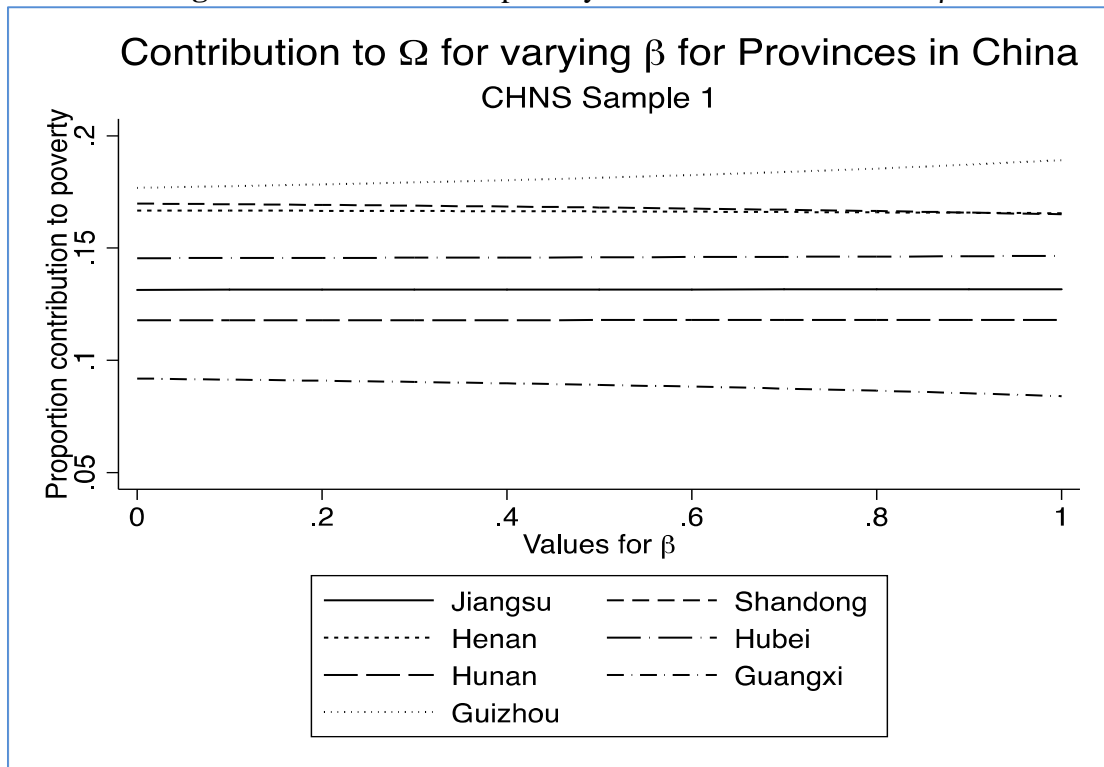
	Model 1		Model 2		Model 3	
$\alpha$	1		1		1	
$\beta$	$S_{njt}^{\alpha\beta} = 1$		0.5		0.375	
$\Omega_{\alpha\beta}$	0.1396		0.0572		0.0623	
<b>Dynamic Decomposition</b>						
Period	Contribution	Rank	Contribution	Rank	Contribution	Rank
1997	0.2160	1	0.2135	1	0.2126	1
2000	0.2035	3	0.2030	3	0.2031	3
2004	0.1784	5	0.1785	5	0.1802	5
2006	0.2088	2	0.2083	2	0.2071	2
2009	0.1933	4	0.1967	4	0.1969	4
<b>Dimensional Decomposition</b>						
	Contribution	Rank	Contribution	Rank	Contribution	Rank
<i>BMI not in normal range</i>	0.0071	3	0.0024	3	0.0019	3
<i>Blood pressure not normal</i>	0.3329	2	0.1351	2	0.1262	2
<i>Individual below primary education</i>	0.6600	1	0.8625	1	0.8719	1

**Figure 1:** Generalisation-tree of the class of subgroup decomposable poverty measures <sup>a</sup>



<sup>a</sup>: Note that the strands of literature referred to in the figure do not necessarily represent the origin of the extensions; for example, multidimensional extensions had appeared as early as Bourguignon and Chakravarty (2003). We simply refer to measures that are special cases of  $\Omega_{\alpha\beta}$ .

**Figure 2:** Contribution to poverty across different values of  $\beta$



## APPENDIX A: Proofs

**(1) Proposition:**  $\Omega_{\alpha\beta}$  satisfies *deprivation monotonicity*.

$$\text{Define } I_{njt}^{\alpha\beta} = S_{njt}^{\alpha\beta} d_{njt}^{\alpha} = \left[ \beta \left( \frac{\sum_h^J d_{nht}^{\alpha}}{J} \right) d_{njt}^{\alpha} + (1 - \beta) \left( \frac{\sum_s^T d_{njs}^{\alpha}}{T} \right) d_{njt}^{\alpha} \right]$$

*Deprivation monotonicity* requires that  $\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{nku}^{\alpha}} > 0$ ; that is, an increase in a deprivation input increases individual poverty.

$$\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{nku}^{\alpha}} = \frac{\partial \Omega_{\alpha\beta n}}{\partial (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})} \frac{\partial (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})}{\partial d_{nku}^{\alpha}} \quad (5)$$

First multiplicative term on the RHS of equation (4):  $\frac{\partial \Omega_{\alpha\beta n}}{\partial (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})} = \frac{C_n}{J^* T} > 0$  for  $C_n = 1$ .

Second multiplicative term on the RHS of equation (4):  $\frac{\partial (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})}{\partial d_{nku}^{\alpha}} = \sum_t^T \sum_j^J \frac{\partial I_{njt}^{\alpha\beta}}{\partial d_{nku}^{\alpha}}$

A change in  $d_{nku}^{\alpha}$  changes all  $I_{njt}^{\alpha\beta}$  sharing the same period and dimension as it. We therefore consider its effects on three possible cases:

**Case 1 ( $k = j, u = t$ )**

$$\frac{\partial I_{nku}^{\alpha\beta}}{\partial d_{nku}^{\alpha}} = \beta \left( \frac{\sum_h^J d_{nhu}^{\alpha} + d_{nku}^{\alpha}}{J} \right) + (1 - \beta) \left( \frac{\sum_s^T d_{nks}^{\alpha} + d_{nku}^{\alpha}}{T} \right) \quad (6a)$$

**Case 2 ( $k = j, u \neq t$ )**

$$\frac{\partial I_{nkt}^{\alpha\beta}}{\partial d_{nku}^{\alpha}} = (1 - \beta) \left( \frac{d_{nkt}^{\alpha}}{T} \right) \quad (6b)$$

**Case 3 ( $k \neq j, u = t$ )**

$$\frac{\partial I_{nju}^{\alpha\beta}}{\partial d_{nku}^{\alpha}} = \beta \left( \frac{d_{nju}^{\alpha}}{J} \right) \quad (6c)$$

From equations (6a), (6b) and (6c) and the restriction that  $0 \leq \beta \leq 1$ , we know that  $\frac{\partial (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})}{\partial d_{nku}^{\alpha}} > 0$ , which in turn implies that  $\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{nku}^{\alpha}} > 0$ .

**(2) Proposition:**  $\Omega_{\alpha\beta}$  satisfies *dynamic rearrangement* when  $\beta < 1$  and  $\alpha > 0$  (achievements have more than two possible values).

**Proof:**

Consider the dimension  $k$  at period  $u$  and two individuals  $n$  and  $n'$  with achievements in dimension  $k$  and period  $u$  represented by  $x_{nku}$  and  $x_{n'ku}$  respectively.

Condition 1:  $x_{nku} > x_{n'ku} \forall u$

Condition 2:  $x_{ngu} \geq x_{n'gu} \forall g \neq k$  and  $\forall u$

A switch in achievements across the two individuals would therefore increase the achievement level of individual  $n'$  by some amount  $x_{nku} - x_{n'ku}$  while reducing the achievement level of individual  $n$  by the exact same amount.

*Dynamic rearrangement* requires that the poverty measure register a decrease in poverty from such a switch. Condition 1 and 2 jointly ensure that  $n'$  is unambiguously poorer prior to the switch.

Since the poverty measure  $\Omega_{\alpha\beta}$  is simply the sum of individual poverty scores, it is sufficient to show that  $\left| \frac{\partial \Omega_{\alpha\beta n}}{\partial x_{nku}} \right| < \left| \frac{\partial \Omega_{\alpha\beta n'}}{\partial x_{n'ku}} \right|$  under both conditions, where  $\Omega_{\alpha\beta n}$  is the individual poverty score.

For the poverty score to change with a change in achievements,  $\frac{\partial d_{nku}^\alpha}{\partial x_{nku}} > 0$  is required. This in turn requires  $\alpha > 0$ . For the rest of the proof we assume this. With this assumption it is sufficient to show  $\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha} < \frac{\partial \Omega_{\alpha\beta n'}}{\partial d_{n'ku}^\alpha}$ .

Under Condition 1 it is then sufficient to show that  $\frac{\partial^2 \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} > 0$  where  $r \neq u$ . Since individual  $n$  has less periods deprivations in dimension  $k$  relative to  $n'$ , this would confirm that the increase in deprivation to  $n$  is more than offset by the decrease in deprivation to  $n'$  from the switch.

From *deprivation monotonicity* we know that  $\frac{\partial \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha} > 0$ . Differentiating equation (6a), (6b) and (6c) by  $d_{nkr}^\alpha$  we get the following:

**Case 1 ( $k = j, u = t$ )**

$$\frac{\partial^2 I_{nku}^{\alpha\beta}}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} = (1 - \beta) \frac{1}{T} \quad (7a)$$

**Case 2 ( $k = j, u \neq t$ )**

$$\frac{\partial^2 I_{njt}^{\alpha\beta}}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} = 0 \quad (7b)$$

**Case 3 ( $k \neq j, u = t$ )**

$$\frac{\partial^2 I_{njt}^{\alpha\beta}}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} = 0 \quad (7c)$$

From equations (7a), (7b) and (7c) we know that  $\frac{\partial^2 (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} > 0$  for  $\beta < 1$ , which in turn implies that

$$\frac{\partial^2 \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} > 0 \text{ for } \beta < 1.$$

**(3) Proposition:**  $\Omega_{\alpha\beta}$  satisfies *dimensional rearrangement* when  $\beta > 0$  and  $\alpha > 0$  (achievements have more than two possible values).

Consider the dimension  $k$  at period  $u$  and two individuals  $n$  and  $n'$  with achievements in dimension  $k$  and period  $u$  represented by  $x_{nku}$  and  $x_{n'ku}$  respectively.

Condition 1:  $x_{nku} > x_{n'ku} \forall k$

Condition 2:  $x_{nkv} \geq x_{n'kv} \forall v \neq t$  and  $\forall k$

Following the proof for *dynamic rearrangement*, under Condition 1 it is sufficient to show that  $\frac{\partial^2 \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha \partial d_{n'fu}^\alpha} >$

0 where  $f \neq k$ . Since  $\frac{\partial^2 (\sum_t^T \sum_j^J I_{njt}^{\alpha\beta})}{\partial d_{nku}^\alpha \partial d_{nkr}^\alpha} = \beta \frac{1}{J}$  we know that  $\frac{\partial^2 \Omega_{\alpha\beta n}}{\partial d_{nku}^\alpha \partial d_{n'fu}^\alpha} > 0$  for  $B > 0$ .



## APPENDIX B: Weights and Extensions

$\Omega_{\alpha\beta}$  can easily be extended to ‘persistence’ (Gradin et al, 2012; Bossert et al 2012) and ‘loss-aversion’ (Hojman and Kast, 2009). Given our focus on the interaction between ‘length’ and ‘breadth’ of deprivations, we avoided the use of such extensions in our empirical applications as they would make it harder to identify the effect of length versus breadth given the additional trade-offs introduced with such extensions. However, we briefly describe how these can be incorporated into our measure – the interested reader is advised to refer to the aforementioned papers directly for more details regarding their properties.

Define  $I_{njt}^{\alpha\beta} = S_{njt}^{\alpha\beta} d_{njt}^{\alpha}$ .

$$\Omega_{\alpha\beta}^{extend} = \left[ \sum_n \left( \left( \frac{\sum_t \sum_j I_{njt}^{\alpha\beta} * w_{njt}}{J * T} \right) * C_n \right) \right] / N \quad (8a)$$

$w_{njt}$  is a generic weight associated with each  $I_{njt}^{\alpha\beta}$  (the transformed deprivation input). These weights can be defined in a variety of ways, though they clearly change the interpretation of the measure. ‘Persistence’, which gives increasing weight to deprivations that occur in consecutive periods, for example, can be incorporated by defining  $w_{njt} = (p_{njt} / T)$  where  $p_{njt}$  is the length of the deprivation *spell* associated with a particular  $I_{njt}^{\alpha\beta}$ . Another way of defining persistence could be  $w_{nt} = (p_{nt} / T)$  where the weights are increasing in the number of consecutive periods that an individual experiences breath of deprivation above a predetermined cutoff (BCCD).

Persistence may not always be relevant to every dimension of deprivation. One can, for example, imagine that being unemployed for three consecutive periods and then being employed for three consecutive periods is superior to alternating in and out of employment for six periods since one incurs an ‘adjustment cost’ when changing states. This can be captured with the concept of loss-aversion (Kahneman and Tversky, 1979; applied to poverty indices in Hojman and Kast, 2009). Consider, for example:

$$w_{njt} = \begin{cases} \left[ \frac{2}{4} + \left( \frac{\sum_j I_{nj(t+1)}^{\alpha\beta}}{J} - \frac{\sum_j I_{njt}^{\alpha\beta}}{J} \right) \left( \frac{2}{4} \right) \right] & \text{when } \sum_j I_{nj(t+1)}^{\alpha\beta} > \sum_j I_{njt}^{\alpha\beta} \\ \left[ \frac{2}{4} + \left( \frac{\sum_j I_{nj(t+1)}^{\alpha\beta}}{J} - \frac{\sum_j I_{njt}^{\alpha\beta}}{J} \right) \left( \frac{1}{4} \right) \right] & \text{otherwise} \end{cases} \quad (8b)$$

This weighting scheme has two features: 1) it allows a particular period to be weighted heavier if it is preceded by more dimensions of deprivation and to be weighted by less if it is preceded by less dimensions of deprivation. 2) it captures the concept of loss-aversion by weighting increases in the breadth of deprivation in the subsequent period more heavily than an equivalent decrease in the breadth of deprivation in the subsequent period (in absolute terms).

Another potential use of the general weights is to assign different importance to different dimensions. As suggested by Atkinson (2003), it is not unreasonable in most applications to start with the case where  $w_j = \frac{1}{J}$ ; that is, where each dimension is weighted equally. With additional information, however, a researcher or policy-maker may assign different weights (so long as  $\sum_j w_j = 1$ ). Reasons behind changing the weights include ‘double counting’ in the sense that some dimensions may essentially be capturing the same aspect of deprivation, which justifies the discounting of the importance of the associated dimensions; another reason is that individuals may actually put very low importance on certain dimensions; e.g. in Bossert et al (2013) dimensions are weighted based on the views of society regarding the importance of those dimensions – ‘consensus weighting’.

## APPENDIX C: Choosing $\beta$

**Proposition:** Given  $J$  and  $T$ , for any choice of  $\Psi$ ,  $\beta$  must be set equal to  $\frac{\Psi J}{T + \Psi J}$

**Proof:**

We present the proof for the continuous case though the logic extends to the case of discrete levels of achievements.

Define  $\Psi = \frac{\partial S_{njt}^{\alpha\beta}}{\partial d_{nht}^{\alpha}} / \frac{\partial S_{njt}^{\alpha\beta}}{\partial d_{njs}^{\alpha}}$  where  $h \neq j$  and  $s \neq t$

$$S_{njt}^{\alpha\beta} = \beta \left( \frac{\sum_k d_{nkt}^{\alpha}}{J} \right) + (1 - \beta) \left( \frac{\sum_u d_{nju}^{\alpha}}{T} \right)$$

Therefore,  $\frac{\partial S_{njt}^{\alpha\beta}}{\partial d_{nht}^{\alpha}} = \frac{\beta}{J}$  and  $\frac{\partial S_{njt}^{\alpha\beta}}{\partial d_{njs}^{\alpha}} = \frac{(1-\beta)}{T}$

$$\Psi = \frac{\beta}{J} / \frac{(1-\beta)}{T}$$

$$\beta = \frac{\Psi J}{T + \Psi J}$$

**APPENDIX D1: Description of Dimensions used for analysis**

Type	Variable	Description
<b>Household</b>	Access to Toilet (D1)	Individual's household has access to improved toilet facility as per UN norms. UN defines improved facility as having own flush toilet, own pit toilet, traditional pit toilet, ventilated improved pit latrine, pit-latrine with slab, flush toilet, and composting toilet
	Access to fuel (D2)	Fuel used for cooking by individual's household is kerosene, electricity, LPG or biogas
	Access to electricity (D3)	Individual's household has access to electricity
	Access to drink water (D4)	Individual's household has access to improved drinking water source. UN defines improved drinking water source as piped water into dwelling, plot or yard, public tap/standpipe, tube well, borehole, protected dug well, protected spring and rainwater.
	Access to atleast one vehicle type (D5)	Individual's household owns at least one of the following mode of transport: a bicycle, motorcycle or car.
	Access to radio/TV (D6)	Individual's household owns at least a radio, b/w Television or a colour Television
<b>Individual</b>	BMI<18.5 or BMI>30 (D7)	If BMI of the individual is not normal (i.e., either greater than 30 or less than 18.5.)
	Illness in the last four weeks (D8)	whether individual suffered illness in the last four weeks.
	Blood pressure not normal (D9)	If the blood pressure is normal (i.e., 90= $\leq$ systolic<120 and 60= $\leq$ diastolic<80)
	Individual atleast primary educated (D10)	Individual is educated up to primary (year 6).
<b>Community</b>	Access to road (D11)	If the village/neighbourhood has a stone/gravel or paved roads.
	Access to bus/train station (D12)	If there is a bus stop in the village/neighbourhood.
	Access to school (D13)	If there is a primary/middle/upper middle school in the village/neighbourhood.

**APPENDIX D2: Description of balanced samples used for analysis**

<b><u>China Analysis: Sample Description</u></b>		
<b>Sample 1</b>	Balanced panel	Sample of 1874 individuals over five years (1993-1997-2000-2004-2006) covering 7 provinces.
	Dimensions	D1-D2-D3-D4-D5-D6-D7-D8-D9-D10-D11-D12-D13
	Description	This sample comprises of household, individual and community dimensions. All dimensions are qualitative.
<b>Sample 2</b>	Balanced panel	Sample of 2458 individuals across five years (1997-2000-2004-2006-2009)
	Dimensions	<ol style="list-style-type: none"> <li>1. Years of education of individual (cut off is primary education i.e., 6 years)</li> <li>2. Individual's Body Mass Index (cut off is 18.5)</li> <li>3. Individual's Blood Pressure (systolic/diastolic).i.e., more than 120/80 then high BP.</li> </ol>
	Description	All dimensions in this sample are quantitative

**Appendix D3: Dimension wide poverty rates for China (Sample 1, Qualitative – balanced panel of 6 years and 13 dimensions)**

CHNS wave	No Access to Toilet	No Access to fuel	No access to electricity	No access to drink water	No access to atleast one vehicle type	No access to radio/TV	BMI not in normal range	Illness in the last four weeks	Blood pressure not normal	Individual below primary educated	No access to road in community	No access to bus/train station in community	No access to school in community	Income less than 0.5*Median <sup>b</sup>
1991	0.6665	0.9189	0.0523	0.6526	0.1827	0.4823	0.1185	0.2391	0.4726	0.4454	0.8538	0.4885	0.2560	0.2344
1993	0.6573	0.8769	0.0164	0.6660	0.1637	0.4330	0.0944	0.1596	0.5239	0.4228	0.8964	0.4495	0.1837	0.2344
1997	0.6147	0.7414	0.0036	0.5557	0.2376	0.3740	0.0867	0.1585	0.6039	0.3982	0.7542	0.3217	0.1611	0.2488
2000	0.5952	0.6952	0.0092	0.5485	0.2550	0.2950	0.0841	0.1827	0.6485	0.3833	0.8481	0.3638	0.1432	0.2904
2004	0.5428	0.7086	0.0056	0.5141	0.2996	0.1652	0.1067	0.3479	0.6963	0.3366	0.9389	0.3628	0.2365	0.2776
2006	0.4772	0.6321	0.0056	0.4962	0.3258	0.1113	0.1042	0.2986	0.7045	0.4310	0.9656	0.4228	0.2817	0.3074

<sup>b</sup>:income used as a benchmark and not an actual dimension