Polarization of Time and Income – A Multidimensional Analysis for Germany

Joachim Merz (Leuphana University Lüneburg, Germany)

Bettina Scherg (Leuphana University Lüneburg, Germany)

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Joachim Merz und Bettina Scherg*

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*Univ.-Prof. Dr. Joachim Merz, Dipl.-Vw. Bettina Scherg, LEUPHANA University Lüneburg, Department of Economics, Research Institute on Professions (Forschungsinstitut Freie Berufe (FFB)), Chair of Statistics and Professions, Scharnhorststr. 1, 21332 Lüneburg, Tel.: +49 4131 / 677- 2051, Fax: +49 4131 / 677- 2059, Email: merz@uni.leuphana.de, scherg@uni.leuphan.de, URL: www.leuphana.de/ffb

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Abstract

A growing polarization of society accompanied by an erosion of the middle class is receiving increasing attention in recent German economic and social policy discussion. Our study contributes to this discussion in two ways: First, on a theoretical level we propose extended multidimensional polarization indices based on a CES-type well-being function and present a new measure to multidimensional polarization, the mean minimum polarization gap 2DGAP. This polarization intensity measure provides transparency with regard to each single attribute, which is important for targeted policies, while at the same time respecting their interdependent relations. Second, in an empirical application time is incorporated, in addition to the traditional income measure, as a fundamental resource for any activity. In particular, genuine personal leisure time will account for social participation in the sense of social inclusion/exclusion and Amartya Sen's capability approach.

Instead of arbitrarily choosing the attribute parameters in the CES well-being function, the interdependent relations of time and income are evaluated by the German population. With the German Socio-Economic Panel (GSOEP) and detailed time use diary data from the German Time Use Survey (GTUS) 1991/92 and 2001/02, we quantify available and extended multidimensional polarization measures as well as our new approach to measuring the polarization of the working poor and affluent in Germany.

There are three prominent empirical results: Genuine personal leisure time in addition to income is an important and significant polarization attribute. Compensation is of economic and statistical significance. The new minimum 2DGAP approach reveals that multi-dimensional polarization increased in the 1990s in Germany.

JEL: I32, D31, J22

Keywords: *Multidimensional polarization, intensity of time and income poverty and affluence, interdependent multidimensional time and income poverty and affluence, minimum multidimensional polarization gap (2DGAP), extended economic well-being, satisfaction/happiness, working poor and affluernt, CES well-being function, German Socio-Economic Panel, German Time Use Surveys 1991/92 and 2001/02*

1 Introduction

A growing polarization in society accompanied by an erosion of the middle class is receiving greater attention in recent economic and social policy discussion in Germany. This drifting apart has many far reaching consequences for the economy and for the quality of life in general and requires engagement on many levels. If this complex topic is examined from an economic perspective, the question is about a growing income gap, which means that "the rich get richer and the poor get poorer" (Grabka and Frick 2008). Though the case is important and there is a large literature about inequality and in particular income inequality with a focus on the poor, there are only a few theoretical and empirical studies which explicitly investigate both poles of income distribution (but see the approaches discussed below). Even less empirical evidence and fewer theoretical approaches about polarization can be found when not only income but a multidimensional approach to economic well-being is considered, though multidimensional approaches have proven to be important for extended poverty studies (Alkire and Foster 2011, Chakravarty 2009, Chakravarty and Silber 2008, Bourguignon and Chakravarty 2003, Atkinson 2003).

Our study contributes to the polarization discussion with respect to interdependent multidimensional (IMD) theoretical measurement and its empirical application in two ways:

First, we propose a methodology using extended multidimensional polarization indices based on a CES-type well-being function, and we present new well-being measures of multi-dimensional polarization, and in particular the new mean minimum polarization gap 2DGAP, a shortest way to escape poverty or to leave affluence. This unique polarization intensity measure provides transparency with regard to each single attribute *and* ensures at the same time their interdependent relations.

Second, since polarization has both an economic and social dimension, in the empirical application we respect both and argue that – in addition to the traditional income measure – time as a fundamental resource for any activity should be incorporated into multidimensional analysis. Genuine personal leisure time in particular will account for the social participation aspect in the sense of social inclusion/exclusion and Amartya Sen's capability approach (e.g. Sen 1999, 1985). The interdependent relations of time and income via the polarization attribute parameters in the CES well-being function will be estimated and evaluated by the German population instead of arbitrarily assigning them values (as in Bourguignon and Chakravarty 2003 or Lugo and Maasoumi 2009).

This empirical application is based on the German Socio-Economic Panel (GSOEP) with additional detailed time use diary data from two available German Time Use Surveys (GTUS) 1991/92 and 2001/02. We quantify available and extended polarization measures as well as our new approaches to polarization development over that decade in Germany according to time and income. In addition to one-value multidimensional polarization indices quantified and discussed in our study, we argue that our new transparent multidimensional polarization 2DGAP components has empirical significance for Germany and is important for targeted policies aiming to overcome polarization.

Current approaches to polarization measurement can be divided into two strands, regardless of whether they are unidimensional or multidimensional. One strand, based on Foster and Wolfson 2010, considers bipolarity with two income groups, one above and one below the income median. This family of measures includes the unidimensional indices of Foster and Wolfson 2010 and Wang and Tsui 2000 as well as the multidimensional extension of Scheicher 2010. The second strand, which is based on the work of Esteban and Ray 1994, relies on a separation of the income distribution into several groups and defines polarization

by inequality within the groups and the distance between the groups. This family of polarization measurements includes the unidimensional measure by Esteban and Ray 1994, Esteban et al. 2007 and Duclos et al. 2005, as well as the multidimensional approach of Gigliarano and Mosler 2009. In the following we only briefly characterize these multidimensional approaches; the respective formulas can be found in the Appendix.

Gigliarano and Mosler 2009 construct a class of multidimensional polarization measures by decomposing different inequality measures and measuring the relative sizes of various groups. Polarization is then described as inequality within groups and as inequality between groups. This measure is a multidimensional extension of the earlier group approach of Esteban and Ray 1994. Scheicher's 2010 multidimensional polarization index is based on individual middle class attribute distances to middle class thresholds. For each individual the single attribute distances are summed up so as to provide individual mean distances of the poor and of the affluent, which are then aggregated. An advantage is certainly the incorporation of multiple attributes and this was an inspiration for our following proposal. However, since the attributes could have different dimensions (euros for income and minutes for time, say), it is difficult to provide a convincing and comprehensive argument in favor just of adding up. While the approach of Gigliarano and Mosler allows a substitution of attributes, the distance measure by Scheicher does not include a substitution parameter.[1]

Though the multidimensional approach is obviously the virtue of these approaches, because of the differences in the attribute dimensionalities, it is necessary to find a combined "umbrella" which aggregates the single polarization attributes and respects their interdependence. The contribution of this paper will be to fill that gap.

This study is the multidimensional polarization extension of our minimum 2DGAP approach and application recently proposed for multidimensional poverty studies (Merz and Rathjen 2014b).

The remainder of this paper is organized in a theoretical and an application part as follows: Section 2 is about the identification and aggregation of multidimensional polarization (2.1), and the specification of the multidimensional CES well-being function (2.2). Section 3 proposes our new well-being measures of interdependent multidimensional polarization. Section 4 introduces the minimum multidimensional polarization gap (2DGAP) with transparent attribute contributions in the compensation context.

The entire application part consists of different sections which build on each other. Section 5 justifies time and income as the two most important MDP attributes, describes their expected interdependence and discusses the polarization threshold lines concepts. Section 6 describes the data base and the empirical polarization threshold lines. Section 7 provides polarization results based on measures reported in the literature and in particular on the results of our new (CES) well-being gap and minimum 2DGAP interdependent multidimensional polarization indices. Section 8 is about the importance of polarization for various socio-demographic groups. Section 9 provides a conclusion.

There are three prominent empirical results for Germany: First, genuine personal leisure time, in addition to income, appears to be an important and significant polarization attribute. Second, compensation between these two attributes, evaluated by the German population, is found to be of economic and statistical significance. Third, as shown by the new minimum 2DGAP approach, multidimensional polarization significantly increased in Germany over the decade 1991/92-2001/02.

[1] For a more in-depth discussion of uni- and multidimensional polarization indices see Merz and Scherg 2013.

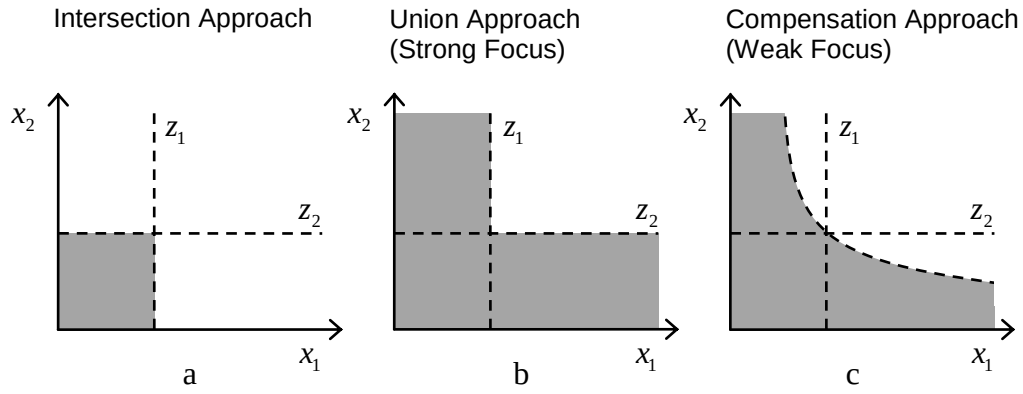
2 Multidimensional Polarization: Identification. Aggregation and Multidimensional CES Well-Being Function

2.1 Multidimensional Polarization: Identification and Aggregation

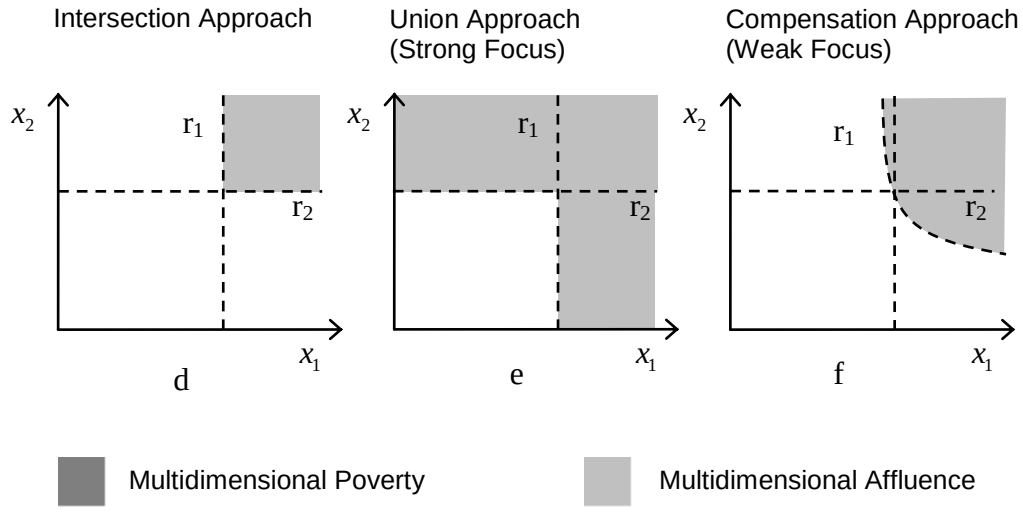
To define multidimensional polarization, we mainly build on the multidimensional poverty discussion (Kakwani and Silber 2008, Deutsch and Silber 2005). In *multidimensional poverty identification* (Bourguignon and Chakravarty 2003), there are two approaches. In the so-called *union approach (strong focus axiom)* a person is judged to be multidimensional poor as soon she or he is deprived in at least one dimension (see Figure 1b for the two-dimension case, the shaded area with the corresponding poverty thresholds z_1 and z_2). The *intersection approach*, by contrast, judges an individual to be multidimensional poor when she is deprived in all dimensions (Figure 1a). Intermediate concepts are conceivable as well.

Figure 1: Identification of Multidimensional Polarization

Multidimensional Poverty



Multidimensional Affluence



Note: x_1 and x_2 are the quantities of the first and second dimension, z_1 and z_2 are the corresponding poverty and r_1 and r_2 the corresponding affluence dimension thresholds.

Source: Own figure.

The selection of the union, intersection or intermediate approach ultimately depends on the relationship between poverty dimensions. Given a substitutive situation the intersection approach is preferable. If a complementary relation is given, then the union approach would be appropriate.

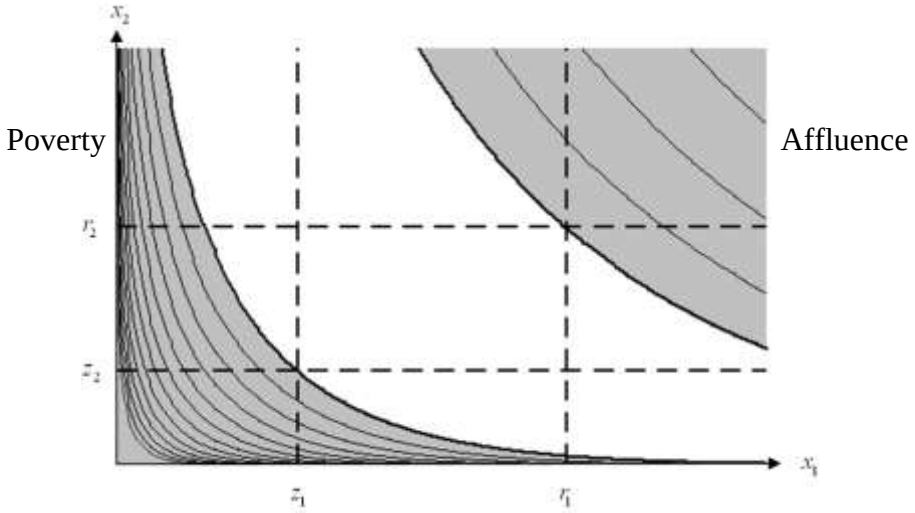
Since the union and intersection approaches seem to be too rigid as identification strategies for most cases, an intermediate approach would be most appropriate (Lugo and Maasoumi 2009). It allows compensation (weak focus axiom) for all ranges in one dimension given poverty in the other (Figure 1c). In the *compensation approach* thus, besides being poor in both dimensions (intersection), a person is multidimensional poor when she cannot compensate poverty in one dimension by the other non-poverty dimension.

The empirical question is whether and to which extent a poverty gap in one dimension might be compensated for by higher quantities in the other one. If a gap in one dimension can be compensated by another's dimension quantity above the dimension threshold, then a person is off poverty (Figure 1c, unshaded area). Thus, the multidimensional poverty line in the two dimensional case runs through the intersection of the dimension thresholds $z = (z_1, z_2)$ dividing the poor (shaded areas in Figure 1a,b,c) and the non-poor (unshaded areas in Figure 1a,b,c).

We extend multidimensional poverty identification to *multidimensional polarization* in a similar though not identical way to poverty, and then we identify the other pole of the distribution, the affluent, by mirroring the poverty areas along the single affluence thresholds $r = (r_1, r_2)$ (Figure 1d,e,f). The kind of relationships between dimensions does not change with respect to the intermediate, union and compensation ranges. Figure 1 also allows the identification of unidimensional polarization with regard to the x_1 dimension (poor to the left of z_1, affluent to the right of r_1) and to the x_2 dimension (poor below z_2, affluent above r_2).

In the following we focus on the compensation approach (weak focus axiom), where the grade of compensation will be evaluated empirically by the German population. For the compensation approach, Figure 2 illustrates the poverty *and* the affluence situation (two-dimensional case). Again, the shaded areas in Figure 2 describe the poor under the poverty line and the affluent above the affluence line. Take for example the affluence compensation above the income threshold (right of r_1 in

Figure 2: Multidimensional Isopolarization Contours – Compensation Approach (Weak Focus Axiom) in the Two-Dimension Case



Source: Own figure

Figure 2). The time deficit (being below the time affluence line r_2) is considered to be compensated by a high income above the multidimensional affluence line to be assigned IMD affluent. The time deficit, however, is considered to be not compensated by high income below the multidimensional affluence line; there is no more multidimensional affluence.

The poverty and affluence lines (in the two-dimension case) can be interpreted as *multidimensional isopolarization contours*, which are isoquants of an underlying well-being function, a function which comprises all polarization attributes and evaluates their interdependent relation. This well-being function is specified in the next section.

To measure then multidimensional polarization for a population, the possible interdependence of attributes has to be specified by the aggregation across the dimensions for each individual well-being index as well as the aggregation across individuals obtaining a poverty measure; see the development for our approach in Section 3.

2.2 Multidimensional Polarization: Multidimensional CES Well-Being Function

Following the compensation approach, this section specifies a particular multidimensional well-being function, a Constant Elasticity of Substitution (CES) function, whose multidimensional isopolarization contours, as described above, identifies the individuals in the polarization poles, both the poor and the affluent. The CES well-being function accounts for the interdependence of the polarization attributes and will be the key element in our new multidimensional polarization indices (described in the next section) and the foundation for our new minimum 2DGAP polarization approach.

Our CES-type well-being function with its individual well-being indicator V_i (weak focus axiom) is based on the multidimensional poverty approach by Merz and Rathjen 2014a and evaluates the interdependence of dimensions by

$$(1) \qquad V_i = \gamma \left[w_1 \left(x_{1i} \right)^{-\rho} + w_2 \left(x_{2i} \right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}}$$

with the substitution elasticity $\sigma = 1/(1+\rho)$ measuring the curvature of the isoquants, ρ as a substitution parameter of the isopolarization contours with $\rho \neq 0$, γ as a constant, υ as returns to scale, x_{1i} and x_{2i} as the polarization attribute quantities, and the coefficients w_1 and $w_2 = 1 - w_1$ as distribution and weighting parameters describing the skewness of the isopolarization contours.

The degree of substitution between genuine personal leisure time and income is measured by the Hicks' elasticity of substitution as the relative change in the proportion of the two attributes dependent on the relative change of the corresponding marginal rate of substitution. With the CES function, the intersections of all isoquants with a ray from the origin have the same marginal rate of substitution. Substitution/compensation, however, is different between different rays from the origin, which allows different degrees of substitution with different time and income ratios.

Compared to other specifications the CES function has the virtue that the elasticity of substitution/compensation can be estimated empirically and is not restricted to a certain value, like a value of one as with the Cobb-Douglas function (a special case of the more general CES function). Since in our empirical application we estimate the CES by a log-Taylor

approximation, the results can even be interpreted as being a more flexible specification like a translog one.[2]

Following the CES well-being compensation approach to quantifying the interdependence of the polarization attributes, the aggregated *multidimensional poverty line* and the aggregated *multidimensional affluence line* will be defined at their respective thresholds by

$$(2) \qquad V_z = \gamma\left[w_1 \left(z_1\right)^{-\rho} + w_2 \left(z_2\right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}} \quad and \quad V_r = \gamma\left[w_1 \left(r_1\right)^{-\rho} + w_2 \left(r_2\right)^{-\rho} \right]^{\frac{\upsilon}{-\rho}}$$

resulting in the two isopolarization contours, the isopoverty and the isoaffluence contours, which cross the poverty threshold intersection at $z = (z_1, z_2)$, or respectively the affluence threshold intersection at $r = (r_1, r_2)$. All individuals with their calculated multidimensional well-being $V_x = V(x_{i1}, x_{i2})$ below the isopoverty line are assigned to be poor, above the isoaffluence line to be affluent, and together assigned to be polarized (see again Figure 2).

3 Multidimensional Polarization: New Indices Based on a Multidimensional (CES) Well-Being Function

We propose a new straightforward measurement approach to multidimensional polarization which respects the interdependence of the attributes based on the above compensation perspective by a multidimensional well-being function like our CES function. With regard to the aggregation of individual well-beings to a multidimensional polarization index for a population, we follow the so-called "shortfall of well-being" aggregate approach used also by Lugo and Maasoumi (2008) for the multidimensional *poverty* aggregation of well-being levels. In our empirical application in particular we follow this well-being level approach rather than the "well-being of shortfalls" component approach based on the relative differences between individual attributes and their thresholds.

Multidimensional Well-Being Polarization (Median)

The first multidimensional polarization measure is the multidimensional well-being extension of the unidimensional Wang and Tsui 2000 polarization measure which considers polarization poles below and above the respective medians. Our *multidimensional well-being polarization index (median)* (weak focus axiom) measures the polarization intensity as a mean relative well-being gap with respect to the median, and is defined (for the two dimensional case) by

$$(3) \qquad P_{median} = \frac{1}{n} \sum_{i=1}^{n} \left[\left| \frac{V(x_{i1}, x_{i2}) - V(m_1, m_2)}{V(m_1, m_2)} \right| \right]^{\alpha}$$

where $V(.)$ is a (CES) well-being function as in Equation 1, m_i is the median value of the polarization attributes, and n is the population size. The greater the distance from the median well-being to the individual well-being, the greater is this index. In contrast to Wang and Tsui 2000, who relate α to the interval [0.1], we follow the Foster-Greer-Thorbecke (FGT) 1984 idea of α describing here a polarization aversion index, with α = 1 as the relative well-being distance to the median, and α = 2 (or α ≥ 1) for a greater polarization sensitivity with greater weights for larger gaps.

[2] For a further discussion, reasoning and justification of the CES well-being function with multidimensional poverty application, the reader is referred to Merz and Rathjen 2014a.

Multidimensional Well-Being Polarization (Poverty and Affluence Threshold Lines)

Whereas the last index comprises all gaps below and above the well-being median, the next well-being measure considers the individual gaps with respect to distinct poverty and affluence threshold lines.[3]

Our *multidimensional well-being polarization index (poverty and affluence threshold lines)* P_{all} is the sum of a mean relative poverty and a mean relative affluence well-being gap under the weak focus axiom as defined by

$$(4) \qquad P_{all} = \frac{1}{n}\sum_{i=1}^{n}\left[max\left(\frac{V(z_1,z_2)-V(x_{1i},x_{2i})}{V(z_1,z_2)},0 \right)\right]^{\alpha} + \frac{1}{n}\sum_{i=1}^{n}\left[max\left(\frac{V(x_{i1},x_{i2})-V(r_1,r_2)}{V(x_{i1},x_{i2})},0 \right)\right]^{\beta}$$

The exponents α and β serve as polarization aversion coefficients, with $\alpha = 0$ and $\beta = 0$ delivering the multidimensional polarization headcount ratios. With $\alpha = 1$ and $\beta = 1$ an average relative polarization gap in well-being units is measured, and with $\alpha > 1$ and $\beta > 1$ a higher aversion against strong polarization (which may be different for the poor and the affluent) is reflected. The proposed polarization index (4) has its origins in a multidimensional Foster-Greer-Thorbecke 1984 poverty index under the weak focus axiom according to well-being units (Lugo and Maasoumi 2009, Merz and Rathjen 2014a,b).

The construction principle of this index – which transfers gap measures from poverty analysis to the analysis of the affluent – reveals a general problem of measuring any gap for the affluent. Whereas a poverty gap is restricted to the maximum interval *z*, the affluence gap would be unbounded. If the affluence part were specified in the same manner as the poverty part, then a reference to the affluence threshold $V(r_1,r_2)$ might deliver values greater than one (further implications are discussed e.g. in Peichl et al. 2010). Thus the second part of our multidimensional polarization index refers to the individual situation $V(x_{i1},x_{i2})$ ensuring affluence percentage ratios in the interval [0,1].

Though both parts have a different reference, for both parts a larger index characterizes an increasing polarization as increasing mean distances within the multidimensional poles.

Since in both parts of the multidimensional polarization index (Equation 4), the average is related to the whole population (n), relatively small values have to be expected in an empirical application. A more intuitively appropriate average would be related to only the poor or affluent population numbers. The *multidimensional well-being polarization index (poverty and affluence threshold lines)* P_{poles} then is defined by

$$(5) \qquad P_{poles} = \frac{1}{n_{poor}}\sum_{i\in poor}^{n_{poor}}\left[\frac{V(z_1,z_2)-V(x_{i1},x_{12})}{V(z_1,z_2)}\right]^{\alpha} + \frac{1}{n_{rich}}\sum_{i\in rich}^{n_{rich}}\left[\frac{V(x_{i1},x_{i2})-V(r_1,r_2)}{V(x_{i1},x_{i2})}\right]^{\beta}$$

with exponents α and β greater or equal one. Our polarization index (Equation 5) respects well-being units, whereas the unidimensional Scheicher 2010 polarization index refers to income units.

[3] It is an extension of the Scheicher 2010 index, which for each individual gap only sums up attribute values without any compensation possibilities.

With regard not to relative but absolute deviations, the respective *multidimensional well-being polarization index (poverty and affluence contour)* in *absolute well-being deviations* then is

$$(6) \qquad P_{all,abs} = \frac{1}{n}\sum_{i=1}^{n}\left[max\left(V(z_1,z_2) - V(x_{1i},x_{2i}),0\right)\right]^{\alpha} + \frac{1}{n}\sum_{i=1}^{n}\left[max\left(V(x_{i1},x_{i2}) - V(r_1,r_2),0\right)\right]^{\beta}$$

and

$$(7) \qquad P_{poles,abs} = \frac{1}{n_{poor}}\sum_{i\in poor}^{n_{poor}}\left[V(z_1,z_2) - V(x_{i1},x_{12})\right]^{\alpha} + \frac{1}{n_{rich}}\sum_{i\in rich}^{n_{rich}}\left[V(x_{i1},x_{i2}) - V(r_1,r_2)\right]^{\beta}$$

with an analogous interpretation as the relative indices (4) and (5).

Multidimensional Well-Being Polarization Asymmetry

The building blocks of our multidimensional well-being polarization index allows a characterization of the polarization asymmetry between the poverty part and the affluence part. This asymmetry might be described by a *multidimensional well-being polarization gap ratio* between the poverty and affluence gaps for the different relative mean concepts by

(8)

$$P_{all,ratio} = \left\{\frac{1}{n}\sum_{i=1}^{n}\left[max\left(\frac{V(z_1,z_2) - V(x_{1i},x_{2i})}{V(z_1,z_2)},0\right)\right]^{\alpha}\right\} \Big/ \left\{\frac{1}{n}\sum_{i=1}^{n}\left[max\left(\frac{V(x_{i1},x_{i2}) - V(r_1,r_2)}{V(x_{i1},x_{i2})},0\right)\right]^{\beta}\right\}$$

and

$$(9) \qquad P_{poles,ratio} = \left\{\frac{1}{n_{poor}}\sum_{i\in poor}^{n_{poor}}\left[\frac{V(z_1,z_2) - V(x_{i1},x_{12})}{V(z_1,z_2)}\right]^{\alpha}\right\} \Big/ \left\{\frac{1}{n_{rich}}\sum_{i\in rich}^{n_{rich}}\left[\frac{V(x_{i1},x_{i2}) - V(r_1,r_2)}{V(x_{i1},x_{i2})}\right]^{\beta}\right\}$$

and in a similar way for the absolute gaps of Equations 6 and 7 as

(10)

$$P_{all,ratio,abs} = \left\{\frac{1}{n}\sum_{i=1}^{n}\left[max\left(V(z_1,z_2) - V(x_{1i},x_{2i}),0\right)\right]^{\alpha}\right\} \Big/ \left\{\frac{1}{n}\sum_{i=1}^{n}\left[max\left(V(x_{i1},x_{i2}) - V(r_1,r_2),0\right)\right]^{\beta}\right\}$$

and

$$(11) \qquad P_{poles,ratio,abs} = \left\{\frac{1}{n_{poor}}\sum_{i\in poor}^{n_{poor}}\left[V(z_1,z_2) - V(x_{i1},x_{12})\right]^{\alpha}\right\} \Big/ \left\{\frac{1}{n_{rich}}\sum_{i\in rich}^{n_{rich}}\left[V(x_{i1},x_{i2}) - V(r_1,r_2)\right]^{\beta}\right\}$$

Larger *multidimensional well-being polarization gap ratios* than a value of one describe a greater asymmetry between the pole gaps.

4 Minimum Multidimensional Polarization Gap (2DGAP)

Multidimensional polarization by the compensation approach, as so far discussed, is captured by a multidimensional well-being function which is at the heart of our new polarization indices described above. The virtue of measuring multidimensional well-being and any well-being gap by a CES-type or other type well-being function is that it respects and quantifies the interdependence of multiple well-being attributes by a one-value well-being index. However, such an aggregation of dimensions into a single well-being value is questionable if it is still

measuring "multidimensional" poverty, affluence or polarization, since the single attributes are no longer transparent. Transparency for the single attributes in the multidimensional approach is however desirable in a manner which allows a targeted policy aimed at the specific attributes of poverty, affluence or polarization.

The main motivation for developing the multidimensional polarization 2DGAP is thus to "disentangle" the single attributes of a well-being gap to obtain a unique multidimensional intensity measure with transparent single attributes. This approach extends the minimum poverty 2DGAP concept proposed in Merz and Rathjen 2014b to the multidimensional polarization case.

In the polarization case, both poles of a distribution with their respective poverty threshold and affluence threshold contours are of interest. Figure 3 (top) shows the CES well-being graph and describes the two-dimensional poverty and affluence case: $V_z = V(z_1, z_2)$ is the well-being contour at the poverty threshold $z = (z_1, z_2)$, $V_i = V(x_{1i}, x_{2i})$ is the individual well-being contour at $x_i = (x_{i1}, x_{i2})$. The difference $V_z - V_i$ is the multidimensional poverty well-being gap. In an analogous way $V_i^r - V_r$ defines the multidimensional affluence well-being gap for a rich person with $V_i^r = V(x_{i1}^r, x_{i2}^r)$ and $V_r = V(r_1, r_2)$ as the affluence threshold.

The mapping of the multidimensional well-being on its (two) single dimensional input space now allows for another attractive integrated approach for describing multidimensional polarization intensity while disentangling the single attribute contributions. It consists of a unique distance between an individual situation and the poverty or, respectively, the affluence threshold.

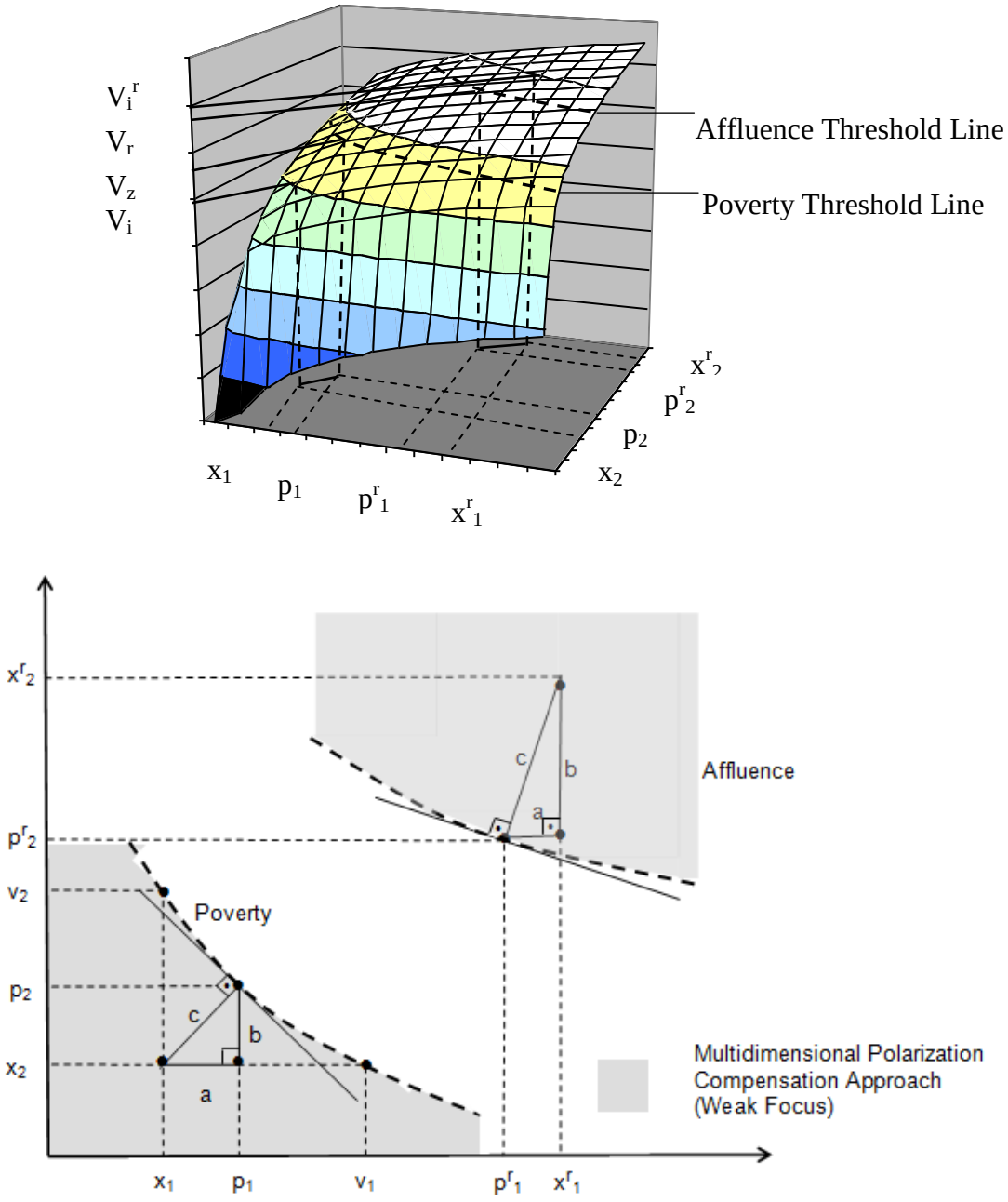
As an illustration, consider the two-dimensional case from the compensation approach and its attribute space as in Figure 3 (at the bottom) and regard first the poverty situation at $x = (x_1, x_2)$ for an individual. With respect to both dimensions there is a fan of distances from that point $x = (x_1, x_2)$ to the IMD isopoverty threshold. Indeed, each distance yields the same well-being difference $V_z - V_i$. However, each distance requires that different single attribute input mixtures need to be overcome in order to escape multidimensional poverty.

The *shortest* path between $x = (x_1, x_2)$ and the corresponding point $p = (p_1, p_2)$ at the isopoverty threshold contour is shown in Figure 3. It requires a minimum input mixture in the sense of an optimized compensation intensity, a minimum combined input "length", in order to escape multidimensional poverty. A natural measure for that length is the Euclidian distance of the single attributes $c = \left[a^2 + b^2 \right]^{0.5}$ with their components a and b as the single poverty attribute gap intensities.

The same approach will be used for a shortest path from a multidimensional affluent individual situation to the isoaffluence line. It is the minimum combined input length necessary to lose multidimensional affluence.

Following the proposed minimum 2DGAP for the poverty case in Merz and Rathjen 2014b, we call this shortest distance *c* the *minimum multidimensional poverty/affluence 2DGAP* (for two polarization dimensions).

Figure 3: Multidimensional Polarization: Well-Being Gap and Minimum 2DGAP



Source: own illustration

The solution for the shortest (or closest) length is characterized by the orthogonal path from a poor individual $x=(x_1,x_2)$ to the tangent at $p=(p_1,p_2)$ for the poverty case of the CES-type isothreshold line by

$$c = \square c \square = \left[a^2+b^2\right]^{0,5} = \left[(p_1-x_1)^2+(p_2-x_2)^2\right]^{0,5} = \left[(p_1-x_1)^2+(f(p_1|V_z)-x_2)^2\right]^{0,5} = \min! \quad (12)$$

where $f(p_1|V_z)$ is the isothreshold contour with regard to the ordinate values x_2 (say, time) of the CES multidimensional well-being function

$$(13) \qquad f(p_1|V_z) = \left(\left(\left(\frac{V_z}{\gamma} \right)^{-\frac{\rho}{\upsilon}} - w_1 p_1^{-\rho} \right) / w_2 \right)^{-\frac{1}{\rho}} .$$

The solution p_1 of the minimizing problem then allows calculation of c by Equation 12 for a given $x = (x_1, x_2)$. Since the proposed CES well-being function is well behaved, there is always a unique solution for the minimum 2DGAP (distance c).

Because of the quadratic distances in Equation 12, the solution procedure for the affluence situation is similar to the poverty situation. The solution p_1, respectively $p^r = (p_1^r, p_2^r)$ in the affluent case, of the nonlinear Equation 12 (see also Figure 3) then allows calculating c again by Equation 12 for a single poor $x = (x_1, x_2)$ or a single affluent $x_i^r = (x_{i1}^r, x_{i2}^r)$ individual.

Single Poverty/Affluence Attribute Gaps

Once $p = (p_1, p_2 = f(p_1|V_z))$ is found, the single poverty and affluence *attribute gaps* can be calculated by

$$(14) \qquad a = p_1 - x_1 \quad und \quad b = f(p_1) - x_2$$

Relative Minimum 2DGAP

For the *poverty case* the 2DGAP might be defined relative to the maximum 2DGAP distance $c_{\max}$, which is the distance from the origin $(0,0)$ to the respective orthogonal slope of the IMD isopoverty threshold:

$$(15) \qquad c_{rel} = c / c_{\max} \quad \text{where} \quad c_{\max} = \square c_{\max} \square = \left[(p_1)^2 + (f(p_1, V_z))^2 \right]^{0,5} = \min!$$

with its corresponding *relative single poverty attribute gap intensities*

$$(16) \qquad a_{rel} = \left[a / p_1 \,|\, a_{\max} \right] \quad and \quad b_{rel} = \left[b / f(p_1, V_z) \,|\, b_{\max} \right]$$

However, for the *affluence case*, as already discussed, there is no comparable genuine maximum distance since any affluence well-being or 2DGAP gap faces the problem of an open top interval.

A possible relative minimum affluence 2DGAP relating to the isoaffluence line precludes direct comparisons to poverty pole measures. Relating it to the overall median for both pole minimum 2DGAPs would have a comparable reference but would not deliver transparent single attribute components.

Aggregation and Mean Minimum Polarization 2DGAP

To retain the polarization single attribute contributions in the multidimensional approach, we propose a straightforward aggregation by the sum of the respective minimum 2DGAP pole means, the *mean minimum polarization 2DGAP*:

$$(17) \qquad C = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} c_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} c_i$$

with its single aggregated components[4]

$$(18) \qquad A = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} a_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} a_i, \qquad B = \frac{1}{n_{poor}} \sum_{i \in poor}^{n} b_i + \frac{1}{n_{rich}} \sum_{i \in rich}^{n} b_i$$

The Benefit: Transparency of Single Attributes of Multidimensional Polarization

The minimum 2DGAP distance c itself measures the shortest multidimensional gap as the shortest length of all dimensional gap intensities in combined attribute units but without direct interpretation in terms of, say, money or time units. However, and this is the benefit of our proposed approach, both sides (attributes) of the right-angled triangle (a and b) are measurable and interpretable in the single dimension, say, income in money units (euros) and time in time units (minutes). This information and transparency then allows targeted single-dimension anti-polarization policies while respecting its multidimensional interdependence. We briefly discuss such polarization policies in our concluding section.

The n-Dimensional Case

The minimum multidimensional polarization gap (2DGAP) can be extended to the n-dimensional case, called minimum NDGAP, by a multivariate minimum search, where the slopes of the NDGAP linear distance are subject to the orthogonality of the n-dimensional tangents to their respective isothreshold contours.

Input Distance Function Approach and Minimum 2DGAP

Another input-related gap measurement is the input distance function (IDF) approach (Malmquist 1953, Deutsch and Silber 2005, Anderson et al. 2008). It is a measure of efficiency where all input factors would have to be changed proportionally by a certain value to achieve a total (technical and allocative) economically efficient output.[5] This measurement requires that the slope (marginal rate of substitution) of the output level isoquants is identical at any crossing point with a given input vector at a given fixed allocative input price ratio. Then the IDF is an appropriate scaling ratio along the input vector, an arrow from the input space origin with a fixed input factor ratio, to efficiently achieve a certain output level.[6]

With respect to multidimensional poverty (Deutsch and Silber 2005), the IDF approach can be interpreted and adapted as a standard of living gap measure in the input (resource) space. The farther the resource vector is below from a well-being (utility) frontier, the lower is the standard of living. In particular, the IDF approach describes the proportional increase of the multidimensional poverty attributes (time and income, say) to reach the efficient allocation at

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[4] The aggregation of the single poverty attributes a and b and of the 2DGAP c over all individuals might not result in the joint aggregate condition $c = (a^2 + b^2)^{0.5}$. With two degrees of freedom one remaining component (a, b or c) is computable from the other aggregates. In our application, alternative computations of the respective remaining components have shown close accordance with the orthogonal condition.

[5] The distance value is one if the input factors result exactly in a certain output level (total economically efficient situation). Depending on the definition of the input arrow lengths relation, a distance value different to one requires more or less input factor quantities (in a fixed relation) to be efficient.

[6] Imagine an individual two-dimensional input situation $(x = (x_1, x_2))$ below a multidimensional well-being threshold with its respective individual optimal allocation at x and regard the input space of a linear homogeneous output function (e.g. Cobb Douglas well-being function). Then a distance function value of 1.5 (times the length of x) requires 50% more of both inputs x_1 and x_2 to achieve the poverty threshold isoquant in an efficient way.

the multidimensional poverty isoquant. Thereby, and this is the important difference to our minimum 2DGAP intensity, the individual input price ratio (individually evaluated time and income) from a budget constraint with regard to the optimal allocation at the individual situation remains constant also at the crossing point of the input vector and the poverty isoquant.

Take as an example the wage rate as the price for income (working hours). In case of a higher wage offer for a working poor, a constant price ratio in the IDF approach would require foregoing a better paid working situation to escape poverty.

A poverty gap measured by our minimum 2DGAP approach, however, provides a *minimum* gap from the individual situation to the multidimensional threshold isoquant while respecting the attribute compensation. The minimum 2DGAP thereby allows a changing input factor ratio and a changing individual price ratio on the path to the multidimensional poverty threshold isoquant. In contrast to the IDF approach, this would allow an individual, for example, to take advantage of better paid jobs to escape poverty.

In brief, whereas with the IDF gap the individual situation (slope of the individual budget line of the optimal allocation ratio of time and income) remains constant, the 2DGAP allows a changing individual situation and allocation to achieve the poverty respectively affluence frontier led by the minimum compensation as evaluated by, in our study, the German population.

5 Multidimensional Time and Income Polarization in Germany – Justification of Attributes and Polarization Threshold Line Concept

The empirical application part of our study is about multidimensional polarization in Germany. Some empirical polarization findings for Germany are already available (see Goebel et al. 2010, Grabka and Frick 2008); they are however primarily based on unidimensional income polarization. Merz 2006 and Merz and Zwick 2005 analyse unidimensional income polarization of the self-employed as free-lancers or entrepreneurs in particular. Scheicher 2009 provides empirical results based on his multidimensional approach with working hours and income as polarization attributes. Education and income are the polarization attributes of the Gigliarano and Mosler 2009 multidimensional analysis according to their group-specific approach.

Though the available German results show, roughly speaking, some increasing polarization, mainly in the decade 2001-09, the empirical results with our new polarization measures, polarization attributes, and type and periods of data are obviously new.

In the empirical part of this study, Section 5 justifies time and income as multidimensional polarization attributes and their expected interdependence, and discusses the polarization threshold lines concept. Section 6 describes the data base and the empirical polarization threshold lines. Section 7 provides polarization results based on other measures found in the literature as well as the results of our new (CES) well-being gap and minimum 2DGAP interdependent multidimensional polarization indices. Section 8 is about the degree of polarization for various socio-demographic groups.

A general remark to the empirical part: since our study is an extension of our multidimensional time and income poverty study (Merz and Rathjen 2014a, b), further in-depth empirical justifications and information is available there concerning the variables under investigation, time and income, and all further empirical definitions regarding poverty. These will not be elaborated on in this study.

5.1 Time and Income as Multidimensional Polarization Attributes

To understand poverty in a broader sense, empirical multidimensional poverty studies incorporate various poverty attributes. An example is the European Union Laeken social inclusion/exclusion indicator set with educational disadvantages, health inequalities, unemployment and worklessness as poverty dimensions (Atkinson 2003). Whereas these and a number of other attributes are broadly accepted and discussed in the context of economic and social policies combatting poverty, a simple mirror image of affluence is obviously misleading in many aspects.

Why use income as a polarization dimension?

Income is the traditional and most-widely accepted poverty attribute and typically the focus of much policy-making. The affluent are commonly defined as controlling a large amount of material resources, in particular income and wealth. Thus, income is a natural candidate as a polarization dimension for both poles.

Why use time as a polarization dimension?

We argue that, in addition to income as a fundamental material resource, time, though immaterial, is a similar fundamental resource and should be incorporated as a second attribute to better understand societal polarization. Time is important for individual well-being simply by allowing or prohibiting desired everyday activities for the poor and the affluent alike. The importance of the time dimension for poverty studies is stressed meanwhile by other studies (see the discussion in Merz and Rathjen 2014a, Goodin et al. 2008, Burchardt 2008, Harvey and Mukhopadhyay 2007, Bittman 1999, or Vickery 1977).

Time as an enabling factor is also a condition for social participation, or social inclusion/exclusion, which is an important aspect in the extended poverty discussion (Sen 1999, 1995). We are also convinced that social participation has a similar importance for the affluent if they are to lead an integrated social life.

Instead of a broad leisure time concept we propose g*enuine personal leisure time* as being essential to the multidimensional approach. Time poverty occurs when genuine personal leisure time – which is defined as the time left after all paid and unpaid, market and non-market obligations have been met – is below a given threshold level and does not allow or limits social participation in society (see Merz and Rathjen 2014a for further discussion). Correspondingly, time affluence occurs when genuine personal leisure time is above a given threshold level.

Why interdependent time and income polarization?

Time availability restricts both market and non-market activities. Thus the more time is spent for income-related activities; the less is available for leisure, and vice versa. This trade-off is well-known and is central to optimal allocation theory in microeconomics and Becker's 1965 household production function approach. The microeconomic approaches should illustrate the general competing time-income interdependence, however, the individual time-income optimal allocation admittedly is not in the focus of our analysis.

As discussed above, the trade-off will be quantified in our study by a CES well-being function with time and income as weighted input factors. Instead of arbitrarily chosen trade-off weights and situations with different compensation degrees, we let the German population (through survey data) identify the degree of interdependence and substitution between income and genuine personal leisure time.

5.2 Income, Time, and Multidimensional Poverty and Affluence Threshold Line Concepts

Single poverty threshold lines z_i and single affluence lines r_i $(i = 1, 2)$ identify the poor and the affluent respectively, the respective multidimensional well-being thresholds and finally the set of individuals in our polarization analysis. Yet, empirical analysis requires concrete values.

Income: Poverty and Affluence Threshold Line Concept

Income poverty studies commonly use monthly household net equivalence income with equivalence scales like the OECD scale[7]. Conventional income-based poverty studies in the European Union identify a person as income poor if her net equivalence income is below 60% of the median income of all households (Bundesregierung 2005, XV). As a result, the 60% median line of the monthly household net equivalence income is adopted in this study as the *income poverty line*. For the sake of comparison, all subsequent income information for 1991/92 is adjusted to 2001/02 price levels.

Whereas there is common agreement about the income poverty line, there is a longstanding and still open discussion about a respective affluence line. The German government explicitly focused for the first time on affluence in addition to poverty in their first "Poverty and Affluence Report" (Bundesregierung 2002), which was followed by three further government reports (Bundesregierung 2004, 2011, 2013). During that period, top incomes gained increasing attention not only in Germany (Atkinson and Piketty 2007, Dell 2007 with German income tax microdata from 1891-1998, Merz, Hirschel and Zwick 2007 with German income tax microdata from 1992-2003). Several affluence lines were proposed in this literature, including an affluence line as a multiple of an income fraction, such as 200% or 150% of mean median income, or as a top income percentile.

As a pragmatic approach, we are choosing 150% as the cut-off for the median monthly household net equivalence income affluence threshold line (this figure is supported, for example, by the polarization studies of Goebel et al. 2010 or Grabka and Frick 2008 from the German Economic Institute).

Time: Poverty and Affluence Threshold Line Concept

Compared to income, the discussion about time poverty or even time affluence is still in its infancy. Bittman 1999 mentions a 50% time poverty line. To be consistent to our income poverty and affluence line, we chose 60% of the median genuine personal leisure time as defining an individual which is time poor, and 150% of the median as the time affluence threshold line. Admittedly, such threshold lines are certainly debatable.

6 Multidimensional Time and Income Polarization in Germany – Data and Empirical Threshold Lines

6.1 Data: GSOEP 2002 and GTUS 1991/92 and 2001/02

The German Socio-Economic Panel (GSOEP)

[7] With a weight 1 for a household head, a weight of 0.5 for additional household members aged 15 years or older, and a weight of 0.3 for all others.

The German Socio-Economic Panel (GSOEP) provides representative individual longitudinal data for all persons older than 16 years living in German households. The representative panel study started in 1984 and provides annual subjective as well as objective information about individual living conditions in Germany (for details, see Wagner, Frick and Schupp 2007). In particular, the GSOEP asks about satisfaction with regard to different topics, such as income, as well as general questions about life satisfaction. The 11-point scale regarding general satisfaction information is used for our CES well-being estimation and refers to the recent happiness/satisfaction literature (Clark et al. 2008, Frey and Stutzer 2005).

Since appropriate well-being data are only available in the German Socio-Economic Panel, we use the GSOEP 2002 for the CES well-being estimation. Although in principle we could use the GSOEP for further analyses, we prefer to use time use diary data from the German Time Use Surveys (GTUS) from 1991/92 and 2001/02 (with no appropriate well-being information) since the time use diaries provide more in-depth information.

The German Time Use Surveys (GTUS) 1991/92 and 2001/02

The German Federal Statistical Office conducted two large representative time use surveys, the German Time Use Surveys 1991/92 and 2001/02 (Ehling, Holz and Kahle 2001, Ehling 2003). In these surveys all respondents in a household older than 11 years noted their everyday routines in diaries in their own words for two working days and a Saturday or Sunday. Personal and household questionnaires also provided socio-economic background information. The final available data for our analysis comprised 6,774 households with 15,366 persons and 30,732 diaries for 1991/92, and 5,144 households with 11,908 persons and 35,685 diaries for 2001/02.

6.2 Empirical Time, Income and Well-Being Multidimensional Poverty and Affluence Threshold Lines, Germany 1991/92 and 2001/02

The time and income single poverty and affluence threshold lines are summarized in Table 1.[8] All income data are adjusted for price inflation.

The *median income,* and as a result the income poverty and affluence thresholds, increased by 19.2% to 1322.59 euros in the ten-year period between 1991/92 and 2001/02.

The *median time,* and again the time poverty and affluence thresholds, increased somewhat less by 17.0% to 310 minutes per day within that ten-year period.

Concerning the empirical well-being and isopolarization thresholds an estimation of the CES well-being function is necessary. With the GSOEP-reported general life satisfaction data on an 11-point scale[9], an estimation of individual well-being requires rather a type of ordered response modelling. The Kmenta 1967 Taylor series approach, however, allows a simple OLS estimation of the log-transformed non-linear CES well-being function (see Merz and Rathjen 2014a) and results in the CES well-being function

$$(19) \qquad V = f(I,L) = 3.550 \cdot \left(0.519 \cdot I^{0.297} + 0.481 \cdot L^{0.297}\right)^{\frac{0.108}{0.297}}$$

[8] As mentioned, income is defined as monthly net equivalence income. Time is personal genuine leisure time, which is detailed in the individual time use diaries and includes one of the main categories "Contact, Conversations, Sociality" or "Media Use, Free-time Activities" in GTUS 1991/92 and the categories "Social Life and Entertainment", "Participation in Athletic Activities e.g. Outdoor Activities", "Hobbies and Games" and "Mass Media" in GTUS 2001/02.

[9] GSOEP 2002 question 11 in the personal questionnaire.

with $I = x_{i1}$ for income and $L = x_{i2}$ for genuine personal leisure time. Significantly estimated coefficients together with the fulfilment of further consistency rules quantify the relevance of substitution/compensation between time and income. The population-based evaluation[10] of the substitution/compensation between genuine time and income yields a substitution elasticity of $\sigma = 1.422$, which shows an easier substitution than in the Cobb-Douglas type ($\sigma = 1$) situation[11]. For comparison reasons, the 1991/92 well-being function is specified by the same estimated parameters as in 2001/02.

Table 1: Income, Time and Well-Being Multidimensional Poverty and Affluence Lines, Germany 1991/92 and 2001/02

	1991/92	2001/02
Median Net Equivalence Income (in euros per month and 2002 prices)	1109.64	1322.58
Median Personal Leisure Time (in minutes per day)	265	310
Income Poverty Line (=60% median net equivalence income)	665.78	793.55
Time Poverty Line (=60% median personal leisure time)	159	186
Well-Being Poor $V^{poor} = f(I^{poor}, L^{poor})$	6.704	6.827
Income Affluence Line (=150% of median)	1664.46	1983.97
Time Affluence Line (=150% of median)	397.50	465.00
Well-Being Rich $V^{rich} = f(I^{rich}, L^{rich})$	7.402	7.538

Source: own calculations with data from GTUS 1991/92 and 2001/02, The time and income poverty lines and affluence lines by GTUS data are calculated for the total population for the median income, and the median genuine personal leisure time for the available population older than 11 years.

The evaluated *well-being poverty threshold line* at the intersection of the single time and income thresholds in Equation 19 has a well-being level of 6.704 in 1991/92 and 6.827 in 2001/02. By contrast, the evaluated *well-being affluence threshold line* at the intersection of the single time and income thresholds yields a well-being level of 7.402 in 1991/92 and 7.538 in 2001/02.

Thus, the CES results suggest a slight increase in overall well-being over the ten-year period. The estimated input coefficients, the weight w for income and $(1-w)$ for personal leisure, indicate a relative importance of income. However, the evaluated time contribution is not that far away from a balanced 50% situation, and reflects the importance of time.

[10] Our CES well-being function estimates are based on the working population because the economically active population actually experiences both work and leisure and therefore its judgment of the trade-off between the two dimensions is probably more appropriate.

[11] Perfect substitution: ($\rho = -1, \sigma = \infty$), Cobb-Douglas case with ($\rho = 0,\ \sigma = 1$), no substitution at all (complementary input factors, $\rho = \infty, \sigma = 0$).

7 Multidimensional Time and Income Polarization in Germany – Headcount Ratios, Well-Being Gap and Minimum 2DGAP Results

7.1 Overall Multidimensional Polarization Results: Headcount Ratios and Well-Being Gaps

Our analysis concentrates on the economically active population. With regard to the working poor, we shed light on a situation where despite governmental efforts poverty still exists. Similarly, we regard the economically active affluent without taking any further wealth situation into account. Thus, we will focus for both distributional poles on those who are economically active with more than five daily working hours.[12]

We identify the poor and the affluent by their headcount ratios subdivided in six multidimensional regimes for both available years (Figure 4). The regimes identify unidimensional time and income polarization as well as interdependent multidimensional time and income polarization with their compensation regimes. Table 2 provides unidimensional and multidimensional polarization well-being results of our respective indices as well as of indices from the literature. With a focus on our new well-being indices, we only briefly summarize some results of the literature indices. Together with their formulas, they are discussed in more detail in Merz and Scherg 2013.

Unidimensional Polarization Results

Table 2 and Figure 4 summarize the unidimensional income and time results. The combined *time* polarization situation is less selective than the *income* polarization situation; the still divergent poverty and affluence developments only result in a significantly increased median specific polarization (Wang and Tsui). All other polarization measures (Foster and Wolfson; Esteban, Gradin and Ray; and Scheicher) are insignificant with respect to time. However with respect to income, all these measures (except Scheicher) describe a significant income polarization decrease.

Multidimensional Well-Being Results

The discussed *compensation approach (weak focus axiom)* allows substitution between time and income and respects the interdependence of the polarization dimensions. New results of our *well-being multidimensional polarization measures* and components over that decade in Germany are presented (IMD results in Figure 4 and Table 2). We regard P_{median} (Equation 3) which is related to the median P_{poles} (Equation 4), which in turn is related to the poverty and affluence lines, and $P_{poles,ratio}$ (Equation 9) which measures the asymmetry of both pole gaps. We divide the discussion with respect to the poverty and the affluence poles, and then discuss them combined.

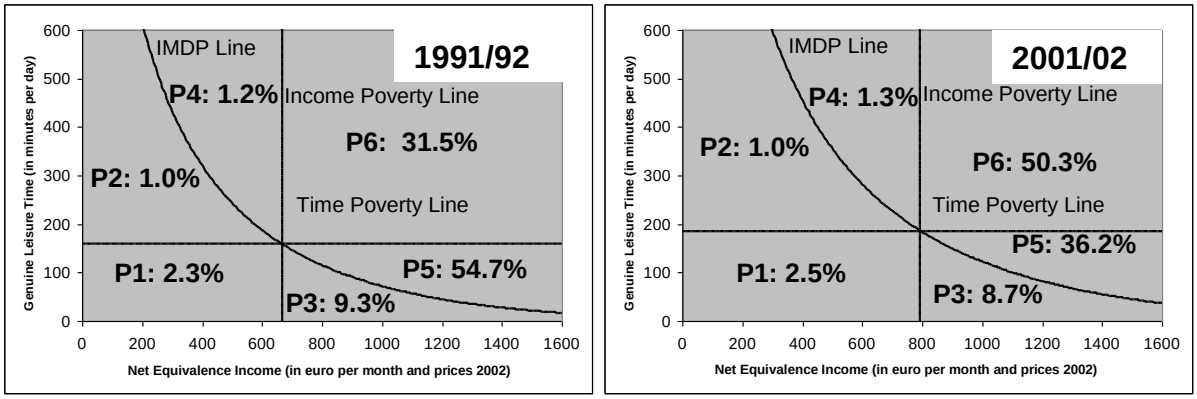
Interdependent Multidimensional Poverty: The **headcount ratios** of the multidimensional poor (Figure 4 top, regimes P1, P2, P3) declined slightly from 12.6% in 1991/92 to 12.2% in 2001/02. Though the change is not significant, the absolute level of the working poor in both years is still remarkable. Regime P3 is of particular importance as even an income above the poverty threshold is assigned not to compensate time poverty for 9.3% and 8.7%, respectively, of the economically active population. Regime P3 is also the prominent poverty regime in the multidimensional perspective. A detailed discussion of time and income

[12] A graphical inspection of the time and income situations is found in our discussion paper Merz and Scherg 2013.

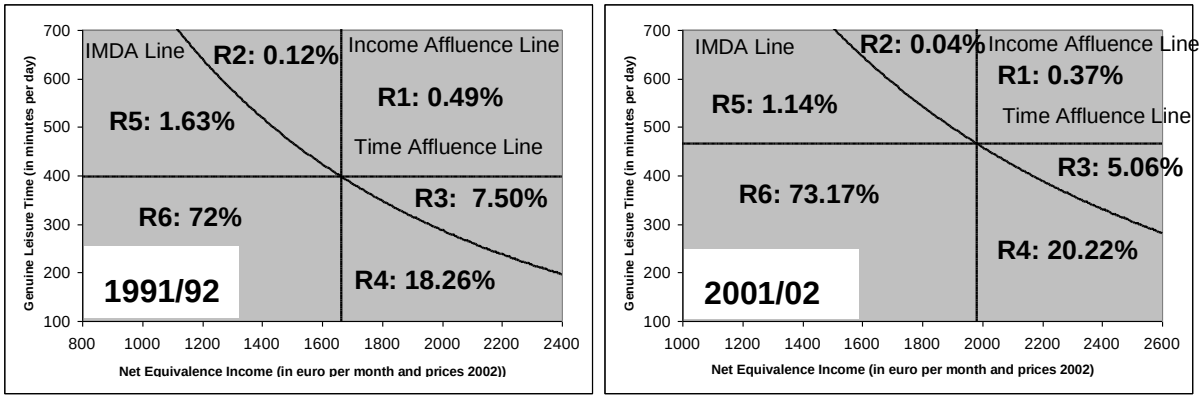
multidimensional *poverty* development (compensation approach, weak focus axiom) with the same data is provided by Merz and Rathjen 2014a.

Figure 4: Multidimensional Polarization Headcount Ratios in Different Poverty and Affluence Regimes for Germany 1991/92 and 2001/02

Poverty



Affluence



IMDP (IMDA) line is the multidimensional time and income isopoverty/isoaffluence threshold based on the GTUS 2002 CES estimates.

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population

***Interdependent Multidimensional Affluence*:** there is a significant reduction in multidimensional affluence by **headcount ratios** from 1991/92 to 2001/02 from 8.1% to 5.5% (Table 2 and Figure 4 bottom, regimes R1, R2, R3). The prominent regime in both years is regime R4. These individuals (18.26% respectively 20.22%) are assigned not to be multidimensional affluent though income rich but time poor; their time deficit is assigned not to be compensated by high income. In contrast, regime R3 individuals (only 7.50% respectively 5.06%) are assigned to be compensated their time deficit by their high income; they are multidimensional affluent.

Furthermore, whereas this group and regime decreases from 7.5% to 5.06% in 2001/02, regime R4 developed in the opposite direction, from 18.26% to 20.22%. Thus the headcount ratio of the income but not time affluent increased in Germany over that decade. Notably, there is only less than a half percent of the economically active population which is affluent in both attributes and both years (regime R1).

Interdependent Multidimensional Polarization: Now regard the combined poverty and affluence situation. The IMD polarization **headcount ratio** decreases significantly from a remarkable 20.7% in 1991/92 to 17.6% in 2001/02. Of specific interest and empirical

Table 2: Interdependent Multidimensional and Unidimensional Time and Income Polarization - Headcount Ratios and Well-Being Measures: 1991/92 and 2001/02, Germany

			1991/92			2001/02					
			Index	95% Confidence Interval		Index	95% Confidence Interval		Ratio 1991/92 =100	Diff. Test[1] p-values	
Headcount Ratio (%)	Unidimensional										
	Income	poor	4.19	3.79	4.58	4.82	4.35	5.28	115	0.043	*
		rich	26.25	25.39	27.12	25.65	24.71	26.6	98	0.361	
	Time	poor	43.06	42.09	44.03	47.34	46.26	48.43	110	0.000	***
		rich	2.24	1.95	2.53	1.55	1.28	1.82	69	0.000	***
	Multidimensional										
	IMD[2]	poor	12.55	11.9	13.2	12.16	11.45	12.87	97	0.425	
		rich	8.11	7.57	8.65	5.47	4.97	5.96	67	0.000	***
		poor & rich	20.66	19.81	21.50	17.63	16.85	18.41	85	0.000	***
Polarization	Unidimensional										
	Income										
	Foster & Wolfson		0.0996	0.0969	0.1023	0.0908	0.0878	0.9386	91	0.000	***
	Esteban, Gradin & Ray		0.0506	0.0498	0.0516	0.0458	0.0445	0.047	91	0.000	***
	Gap Wang & Tsui		0.4356	0.4257	0.4455	0.3894	0.3801	0.3988	89	0.000	***
	Gap Scheicher		0.4104	0.4017	0.419	0.4034	0.3941	0.4127	98	0.286	
		poor	0.1905	0.1749	0.2061	0.2215	0.2063	0.2366	116	0.005	**
		affluent	0.2199	0.2134	0.2263	0.1820	0.1745	0.1895	83	0.000	***
	Time										
	Foster & Wolfson		0.1239	0.1201	0.1276	0.1214	0.1172	0.1255	98	0.379	
	Esteban, Gradin & Ray		0.0608	0.0596	0.0619	0.0600	0.0586	0.0614	99	0.357	
	Gap Wang & Tsui		0.4074	0.4022	0.4125	0.4205	0.4149	0.426	103	0.000	***
	Gap Scheicher		0.5115	0.5033	0.5197	0.5073	0.4987	0.5158	99	0.484	
		poor	0.4037	0.3956	0.4118	0.3899	0.3818	0.3980	97	0.018	*
		rich	0.1078	0.0964	0.1192	0.1174	0.0989	0.1359	109	0.388	
	Multidimensional										
	Scheicher (Gap)		223.64	214.91	232.38	224.02	214.83	233.22	100	0.953	
	Multidimensional										
	IMD										
	P_{median} (%)		3.28	3.23	3.34	3.09	3.03	3.15	94	0.000	***
	P_{poles} (%)		5.20	5.02	5.39	4.87	4.65	5.08	94	0.020	*
		poor	3.49	3.28	0.37	3.35	0.31	3.59	96	0.394	
		rich	1.72	1.61	1.83	1.52	0.14	1.64	88	0.018	*
	$P_{poles, ratio}$		2.03			2.2					

[1] Two sample difference in means test with variance inhomogeneity and unequal variances; *** = significant on the 0.1% level; ** = significant on the 1% level; * = significant on the 5% level.
[2] IMD: Interdependent Multidimensional (IMD) compensation approach ; Poverty: CES well-being at 60% of income respective time median (CES well-being (1991/92) = 6.704, CES well-being (2001/02 = 6.827)
IMD: Interdependent Multidimensional (IMD) compensation approach; Affluence: CES well-being at 150% of income respective time median (CES well-being (1991/92) = 7.402, CES well-being (2001/02 = 7.538)

Source: GTUS 1991/92 and 2001/02, own calculations, weighted data

importance are regimes with time deficits (Table 3): for poverty, where the time deficit is assigned not to be compensated even by above poverty threshold income (regime P3); for affluence, where the time deficit is assigned to be compensated by above affluence threshold income (regime R3). These regimes show the highest IMD polarization headcount ratios for both years and emphasize the importance of genuine personal leisure time for the German population.

IMD polarization well-being intensity decreases significantly over that decade if the common polarization threshold is the median (P_{median}). However, if the isopoverty and isoaffluence thresholds are farther from the median, then the multidimensional polarization decrease is only of minor significance (P_{poles}). This indicates particular differences between the lower and upper middle classes relatively close to the median.

Thus, based on the compensation evaluation of the German population with a CES well-being function, time and income polarization only declined in the 1990s when both distributional poles were measured with respect to the median time and income values.

If differences to median pole thresholds are regarded, which pole is driving the polarization measure? An answer is given by our *multidimensional well-being polarization gap ratio* $P_{poles,ratio}$ (Equation 9), which measures the **pole asymmetry**. The mean well-being poverty gap is more than twice as large as the mean well-being affluence gap, and, the pole asymmetry grew. The dominance of the poverty well-being gap might be explained as follows. First, less than 1% is considered affluent in both dimensions (possibly due to underreporting) and are expected to be not far away from the IMD affluence line, which results in small well-being differences. Second, and a related reason, instead of being affluent in both dimensions it is expected instead that high income together with high working hours restricts genuine personal leisure time. Relatively high regime R4 headcount ratios (Figure 4) support this argument.

To summarize the overall situation: the *unidimensional* consideration of income and time suggests a decrease in income polarization and a slight increase in time polarization in Germany over the decade. Concerning the single poles, time as well income poverty increases whereas income affluence decreases. Therefore, there is no definite evidence of an increasing bipolarity of income and time distribution if the unidimensional measures are regarded separately,.

The proposed *interdependent multidimensional* polarization well-being approach however provides such a combined analysis. The results up to now show, only with respect to the median but not to the specific poverty and affluence lines, a significant overall decrease in interdependent multidimensional time and income polarization.

7.2 Multidimensional Polarization by the Minimum Multidimensional Polarization Gap (2DGAP)

The multidimensional polarization gap results so far discussed embrace the interdependence of time and income under the CES well-being shield. However, and in particular for targeted policy analysis, one might argue that the single polarization attributes suffer from a lack of transparency when only a one-value indicator is given. Our proposed multidimensional polarization 2DGAP measure, developed in Section 4 above, in fact provides such a single time and income polarization contribution while respecting the assigned attribute compensations.

Table 3 shows polarization results as *mean minimum multidimensional polarization gaps* (2DGAP) (Equation 17) with the disentangled income and genuine personal leisure time components (Equation 18) for 1991/92 and 2001/02 overall and divided for the respective poverty and affluence regimes. In addition, Table 3 presents regime specific mean well-being gaps as the underlying one-value well-being indicator, and further regime specific polarization headcount ratios.

2DGAP Overall (*C*) (see the last line of Table 3): the *mean multidimensional polarization 2DGAP* (*C*, Equation 17) increased significantly by 18% starting at 141.30 in 1991/92. Thus, respecting the compensation between genuine personal leisure time and income as evaluated by the German population, polarization increased in the 1990s, a remarkable result.

Of particular interest are its mean components for income and time (*A* and *B* of Equation 18) under compensation:

2DGAP Income component (*A*): The summarized mean minimum *income 2DGAP* component increases from €29.27 to €38.58 (2DGAP a). Though the mean income gap is small, the relative monetary increase of 32% is remarkable and highly significant. There is an asymmetry with a stronger poverty than affluence intensity: the poverty income component (€17.72) is greater than the affluence income component (€11.55). This might be an indication for an antipoverty policy instead of a policy to tax higher income in an effort to successfully decrease polarization.

2DGAP Time component (*B*): The summarized mean minimum *time 2DGAP* component of the poverty and affluence gaps (2DGAP b) increased significantly from 136 minutes per day to 150 minutes of genuine personal leisure time by 18%. The asymmetry between the poor and the affluent mean minimum time gap component is remarkable. The affluence time gap in 1991/92 is twice as large as the poor time gap in 1991/92. It is reduced to a 1.6 multiple in 2001/02. A steeper mean minimum 2DGAP c for the affluent with a higher time/income relation indicates the importance of the compensated time deficits (regime R3, Figure 4). By some contrast, the less steep mean minimum 2DGAP *c* for the poor with a lower time/income relation indicates the importance of the time situation below the income threshold.

Overall, though genuine personal leisure time is an important contribution to well-being polarization, the significant overall 2DGAP polarization growth is mainly due to the significant growth of the income 2DGAP component.

2DGAP polarization regimes: The minimum polarization gaps (2DGAP) measure the poverty and affluence intensities. The strongest polarization intensity is given for the intersection of time and income poverty together and for each respective pole separately (Table 3, regimes P1, R1 and IMD poor and IMD rich). Those individuals face poverty or affluence beyond any compensation. This holds for the combined 2DGAP *C* as well as for the single income (2DGAP *A*) and time (2DGAP *B*) components.

The strongest gap growth rate is seen in the R2 regime for the affluent, based however on a relatively low headcount ratio. Individuals there are multidimensional affluent though income is below the affluence income threshold; less income there is assigned to be compensated by time affluence. Further results concerning the single income and time 2DGAP components can be found in Table 3.

The 2DGAP results, different for various regimes, show different combinations of different time/income compensation and stress the differences to escape poverty or to leave affluence by the shortest way.

Table 3: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Gap (2DGAP) of Interdependent Multidimensional Time and Income, Regime Specific Headcount Ratios and Mean Well-Being Gaps 1991/92 and 2001/02, Germany*

		Headcount Ratio				**Well-Being Gap**				**2DGAP: Mean Minimum 2DGAP c**				**2DGAP: Mean Minimum Income 2DGAP a (in €)**				**2DGAP: Mean Minimum Time 2DGAP b (in minutes per day)**			
	Year	1991	2001	Index 1991 =100	Diff test[4]	1991	2001	Index 1991 =100.	Diff test	1991	2001	Index 1991 =100	Diff test	1991	2001	Index 1991 =100	Diff test	1991	2001	Index 1991 =100	Diff . test
Poor	P1[1]	2.28	2.46	107		0.2593	0.3435	131	***	106.48	152.21	143	***	50.52	72.09	143	***	92.85	133.11	144	***
	P2[2]	1.04	1.02	98		0.0932	0.1080	18		56.32	74.75	133	*	35.51	46.67	131		43.19	57.82	134	*
	P3[3]	9.27	8.69	94		0.2186	0.1864	86	**	34.54	44.10	128	***	7.64	10.71	140	***	33.58	42.66	127	***
	IMD Poor	12.9	12.17	97		0.2160	0.2116	95		49.38	68.50	139	***	17.72	26.11	147	***	45.11	62.20	138	***
Rich	R1	0.49	0.37	76		0.1831	0.1639	88		188.66	204.65	108		36.26	40.30	111		183.98	199.59	108	
	R2	0.12	0.04	33	*	0.0291	0.0557	21		39.79	95.74	241	*	16.30	46.04	282	*	36.27	83.92	231	*
	R3	7.50	5.06	67	***	0.1296	0.1146	85	*	86.42	90.96	105		9.85	9.76	99		85.63	90.30	105	
	IMD Rich	8.11	5.47	67	***	0.1314	0.11752	92	*	91.92	98.73	107		11.55	12.09	107		90.87	97.69	108	
Rich & Poor	P1+R1	2.77	2.83	102		0.4424	0.5075	114	**	295.14	356.86	121	***	86.78	112.39	130	***	276.56	332.7	120	***
	P2+R2	1.16	1.06	91		0.1223	0.1638	141	**	96.11	170.49	177	***	51.81	92.71	179	***	79.46	141.74	178	***
	P3+R3	16.77	13.75	82	***	0.3483	0.3010	86	***	120.96	135.06	117	***	17.49	20.47	117	***	119.21	132.96	112	***
	IMD P + IMD R	**20.70**	**17.64**	**85**	***	**0.3473**	**0.3291**	**94**	*	**141.30**	**167.23**	**118**	***	**29.27**	**38.56**	**132**	***	**135.98**	**159.89**	**118**	***

* Poor describes multidimensional poverty, Rich multidimensional affluence and Rich & Poor multidimensional polarization in respective regimes; Headcount Ratio as P_{all} of Equation 4 with $\alpha=\beta=0$; Mean well-being gap as P_{poles} of Equation 5 with $\alpha=\beta=1$

[1] P1/R1: regime of income *and* time poor/rich individuals. [2] P2: regime of income poor individuals but not time poor. [3] P3: regime of time poor individuals but not income poor; R2: regime of time rich individuals but not income rich; R3: regime of income rich individuals but not time rich. [4] Two-sample difference in means test with variance inhomogeneity and unequal variances; *** = significant at the 1% level; ** = significant at the 5% level; * = significant at the 10% level. [5]Poverty: CES well-being at 60% of income respective time median (CES well-being in 1991/92 = 6.704, CES well-being in 2001/02 = 6.827); IMD: interdependent multidimensional compensation approach; Affluence: CES well-being at 150% of income respective time median (CES well-being in 1991/92 = 7.402, CES well-being in 2001/02 = 7.538).

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population

Polarization centers: The mean minimum multidimensional polarization gaps (2DGAP) can also describe polarization centers. In particular, a polarization center is the starting point (x_{1c}, x_{2c}) of the mean minimum 2DGAP (Table 4). [13]

Table 4: Multidimensional Polarization Centers 1991/92 and 2001/02, Germany

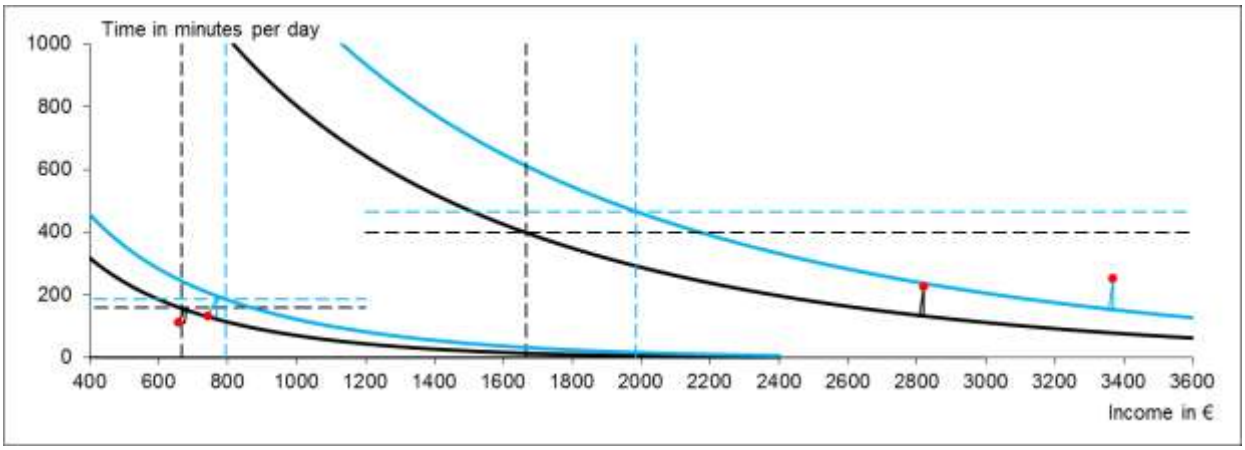
	1991/92		2001/02	
2DGAP components	poor	rich	poor	rich
Income (x_{1c}) (€)	656.72	2818.95	735.98	3344.46
Time (x_{2c}) (minutes)	109	224	131	56

Source: own calculations with GTUS 1991/92 and 2001/02, economically active population

At the polarization center the relation B/A of the 2DGAP poverty intensity components illustrates the compensation there. For 2001/02 poverty, the slope at the IMD isopoverty line where c is crossing (= marginal rate of substitution) is $c'_{\perp} = -A/B = -0.42$. Thus the assigned amount of time to compensate one euro income locally is about 0.42 minutes; i.e. it is less than a 1:1 compensation; 0.42 minutes are enough to compensate one euro, which highlights the particular importance of time. For 2001/02 affluence, the comparable slope is 0.12; i.e. 0.12 minutes are enough to compensate one euro, which highlights the even stronger time importance for the affluent.

Figure 5 illustrates the positions of the mean minimum multidimensional polarization gaps (2DGAP) and their polarization centers for 1991/92 (black) and 2001/02 (blue). Three results illustrate the numerical findings: first, the mean gaps are relatively small, thus the poverty and affluence positions are relatively near their respective thresholds. Second, there is a particular move of the mean affluence gap to higher income over the decade. Third, relatively steep rising mean gaps pinpoint the importance of the time component.

Figure 5: Multidimensional Polarization: Mean Minimum Multidimensional Polarization Pole Gaps (2DGAP c) and Polarization Centers 1991/92 and 2001/02 (blue)

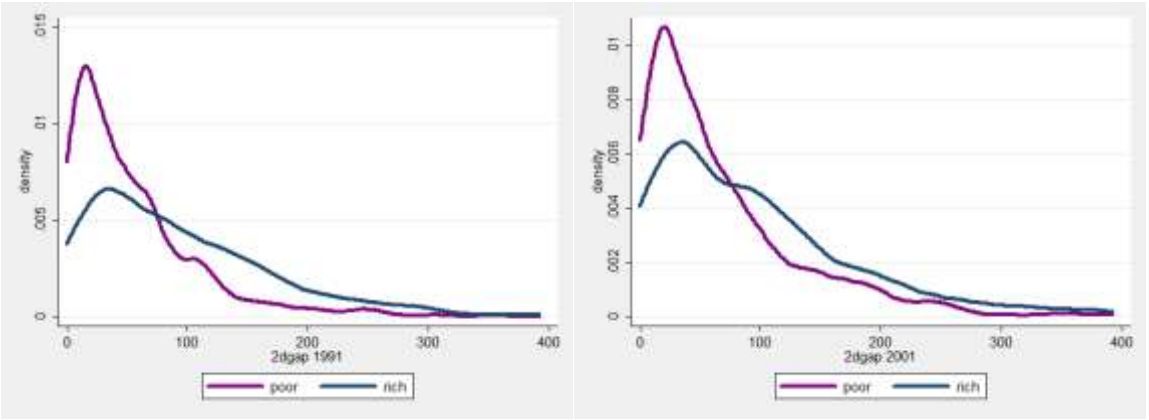


Source: Own calculations with GTUS 1991/92 and 2001/02, economically active population

The Kernel densities of the poverty and affluence gap distributions for 1991/92 and 2001/02 shown in Figure 6 illustrate the different pole distributions of the minimum 2DGAP c measure. The affluent pole distributions are more right-skewed than the poor pole distributions and characterize situations farther away from their polarization threshold.

[13] It is calculated by an iterative process via 2DGAP mean components and the orthogonal requirement.

Figure 6: Kernel Densities of Minimum Multidimensional Polarization Pole Gaps (2DGAP c) 1991/92 and 2001/02, Germany



Source: Own calculations with GTUS 1991/92 and 2001/02, economically active population

Multidimensional Polarization Well-Being and Minimum 2DGAP Results Compared

A last remark is about differences between well-being and minimum 2DGAP results. The mean minimum polarization gaps (2DGAP) indicate a significant increase in multi-dimensional time and income polarization, whereas the well-being polarization gaps indicate some decreasing polarization, which is however significant only for the median reference and of minor importance for the pole threshold specific measure (P_{poles}).

One explanation is the following: Any well-being difference is described by two respective contours as isoquants in the 2D attribute space. There, a fan of multitude gaps describes the differences from one individual situation to the isopoverty or the isoaffluence line. Yet, there is only a one-value well-being gap, which covers the whole area between the individual isoquant and the respective isothreshold. However, the indefinite number of possible distances (gaps) between a single situation and the isopoverty or the isoaffluence line results in a fuzzy overall well-being situation. The minimum 2DGAP approach, by contrast, builds on a unique well-defined multidimensional shortest distance from the individual situation to the respective threshold with interpretable polarization components. So, the "fuzziness" of the well-being gap might be the reason for different well-being gap and minimum 2DGAP results.

To summarize the 2DGAP multidimensional polarization results: With highly significant results we face a strong case for an increased interdependent multidimensional time and income polarization intensity for Germany between 1991/92 and 2001/02.

8 Multidimensional Time and Income Polarization in Germany – Results for Socio-Demographic Groups

It is to be expected that different individual resources and limitations will result in a different polarization situation for different socio-economic and socio-demographic groups. Finally, for those groups which experience attention in the public discussion, Table 5a presents polarization headcount ratios, mean IMDP well-being gaps (compensation approach, weak focus) and mean minimum multidimensional 2DGAPs with their income and time components respecting compensation. The 2001/02 information also provides indices which describe the development since 1991/92 in Germany (detailed results are given in the Appendix, Tables 5b,c including information for the single poles in our downloadable discussion paper Merz and Scherg 2013).

There are many interesting individual results given the compensation evaluation by the German society. To be brief, we will focus on selected results, in particular with regard to our new multidimensional 2DGAP polarization measure and its components, which describe the polarization intensity concerning income (in euros) and genuine personal leisure time (in minutes).

Gender: Females are affected more often than males by poverty or affluence referring to unidimensional income and time polarization. In contrast males face a deeper multidimensional polarization gap (2DGAP c) overall *and* with respect to income and time. And, the 2DGAP polarization increased the most (2DGAP a,b,c) between 1991/92 to 2001/02 for males.

Age: Individuals who are over 65 years old and who are still working more than 5 daily hours are the group with the highest unidimensional polarization headcount ratios in the poles as well as under the IMD polarization regimes. The older the individuals are, the deeper are the polarization gaps (2DGAP c), too. The importance of age for both distributional poles is remarkable and underlines a particular erosion of the middle class for the elderly.

Education: Individuals with an A level (*Abitur*) – in contrast to all other educational levels – show the most intense polarization. The higher the education level, the higher is the headcount ratio of the affluent (Merz and Scherg 2013, Appendix Table 4b). Secondary schooling is connected with the fastest polarization growth, which in contrast to A-level education is driven by a prominent poverty pole importance.

Occupation: The self-employed are remarkably more often affected by income (52.88%), time (59.44%) and IMD polarization (33.95%) than any other occupational group. Subdividing the self-employed into the liberal professions (architects, lawyers, physicians etc.) and entrepreneurs reveals that the high percentages should be traced back to the high percentages of entrepreneurs with regard to time and IMD polarization, however not for the reverse income polarization. Furthermore, polarization intensity measured by multidimensional gaps show the highest spread for the self-employed (2DGAP c) and in particular for genuine personal leisure time (2DGAP b). Finally, multidimensional polarization grew fastest for the self-employed, followed by blue-collar workers.

For the self-employed, one notable result is that since two-thirds of the self-employed in the two poles under IMD polarization are found at the poverty pole, the deprived situation is of particular importance for the self-employed despite the relatively dominant affluence gap contribution (see Merz and Scherg 2013, Appendix Tables 5b,c). This underpins and is in line with the self-employed multidimensional time and income *poverty* results of Merz and Rathjen 2011.

Working Hours: The highest polarization headcount ratios and the largest IMD polarization intensity (2DGAP c) are found for those with the most weekly working hours.

Household/Family Structure: Whereas the IMD polarization headcount ratio for couples with two or more children is the highest among the family groups, the polarization is strongest for single parents with children (2DGAP c). As to be expected, single parents with children also show the relatively highest time gap. Further analysis show that this is mainly due to the poverty pole. They faced a strong polarization increase by 33% (2DGAP c) over that decade. The increase is even stronger for single parents with more children (51%) and pinpoints growing difficulties for single parents.

Region: Though unidimensional income and time poverty headcount ratios are higher in the old German states, this is not the case for the multidimensional situation. The relative number of individuals in the distributional poles is higher in the new German states (23.65% vs.

Table 5a: Multidimensional Polarization in Socio-Economic Groups of Interdependent Multidimensional Time and Income Polarization 2001/02, Germany

	Polarization Headcount Ratio						Well-Being Gap		Multidimensional Polarization Minimum 2DGAP					
	Income	Index 1991 =100	Time	Index 1991 =100	IMD*	Index 1991 =100	Mean	Index 1991 =100	Mean c	Index 1991 =100	Mean a Income (€)	Index 1991 =100	Mean b Time (min.)	Index 1991 =100
Gender														
Male	29.84	99	47.29	115	17.76	92	0.3525	103	182.97	134	40.51	152	175.34	133
Female	31.55	99	51.65	98	17.40	76	0.2845	80	135.34	92	33.88	105	128.46	91
Age														
12-17	17.05	76	40.57	103	17.90	135	0.2846	114	158.86	89	69.38	105	140.47	86
18-24	20.32	79	38.45	95	14.65	81	0.2776	84	158.23	111	45.83	126	148.90	111
25-44	28.79	101	50.92	109	17.97	84	0.3171	95	157.61	117	37.47	136	150.20	116
45-65	36.16	97	49.75	109	18.16	86	0.3520	95	174.20	120	36.08	130	167.79	120
>65	54.33	139	59.91	161	20.65	84	0.2662	45	189.54	110	55.76	283	175.89	103
Education														
A-Level	45.86	107	52.52	114	19.07	76	0.3579	97	172.75	113	36.81	113	166.35	115
Vocational Dipl.	33.50	71	48.46	104	18.46	70	0.2919	74	163.61	115	37.92	134	156.19	114
Second. School II	24.92	93	49.54	107	18.33	85	0.3212	96	167.12	117	42.47	130	158.45	116
Second. School I	22.90	87	45.16	103	15.08	91	0.3217	100	155.57	127	32.79	135	149.65	126
No certificate	28.69	219	48.84	158	17.64	145	0.2595	54	131.82	63	32.41	95	126.62	63
Occupation														
Self-employed	52.88	100	59.44	114	33.95	96	0.4563	95	240.22	129	59.59	168	227.72	128
Liberal. Prof.	59.84	-	49.64	-	28.29	-	0.4278	-	230.50	-	50.67	-	220.54	-
Entrepreneur	48.16	-	66.11	-	37.79	-	0.4510	-	231.47	-	62.20	-	218.11	-
Civil Servant	53.22	97	47.83	124	18.97	82	0.3220	80	135.19	104	16.37	84	133.84	104
White-Collar	33.88	102	50.14	103	14.34	69	0.2886	88	130.23	103	24.43	72	126.69	104
Blue-Collar	15.41	78	46.35	110	14.95	85	0.2561	88	155.62	127	37.06	124	149.97	128
Working Hours														
<20	21.70	75	47.74	116	22.24	105	0.2597	88	170.52	112	44.65	94	162.31	116
21-38	24.74	71	44.03	107	13.62	78	0.2745	82	151.23	112	31.58	115	145.84	112
39-40	27.98	115	47.70	103	14.23	71	0.3001	102	151.38	130	31.14	118	146.37	131
41-44	32.57	149	50.91	113	16.84	90	0.3508	78	187.05	100	36.04	106	182.01	100
…>45	46.75	106	60.99	115	27.9	92	0.4024	91	187.40	115	45.74	163	177.82	113
HH-Size														
Single-HH	30.13	132	50.40	122	15.88	81	0.3067	88	178.73	109	36.52	77	172.49	123
Couple 0 Kids	56.07	110	46.82	108	18.17	69	0.3294	90	150.37	106	26.76	112	145.84	106
Couple 1 Kid	23.95	136	47.47	102	14.04	95	0.2780	94	127.73	98	31.69	106	122.67	98
Couple 2 Kids	14.61	97	49.36	97	16.34	78	0.2833	85	146.62	97	42.85	148	138.69	94
Couple >2 Kids	20.94	114	57.65	100	31.68	118	0.2973	96	191.76	147	84.28	294	170.43	135
Single par. 1 Kid	31.18	121	40.46	78	22.23	105	0.3337	90	232.64	133	65.03	145	218.50	132
Single par. >1Kid	22.96	81	51.77	82	17.43	40	0.2491	119	146.09	151	47.44	114	135.88	158
Other structure	25.96	73	50.99	120	17.46	96	0.3693	102	160.14	129	52.63	212	148.68	124
Region														
West Germany	32.98	90	46.43	110	16.26	87	0.3193	85	161.59	121	36.16	141	154.57	120
East-Germany	19.38	114	59.77	114	23.65	93	0.3536	113	191.00	126	46.06	128	182.51	126

* IMD: Interdependent Multidimensional polarization compensation approach Source: own calculations with GTUS 1991/92 and 2001/02, active population

Source: Own calculations with GTUS 1991/92 and 2001/02, economically active population

16.26% in the old German states) and the polarization intensity overall and with regard to income and time is greater in the new than in the old German states.

To summarize: As expected, different socio-demographic groups show different uni- and multidimensional polarization and different polarization growth depending on gender, age, education, family structure and geographic region. Notably, multidimensional polarization of time and income of the self-employed as well single parents deserves specific attention.

Our quantification of multidimensional time and income polarization for various socio-demographic groups Germany is important to detect groups of specific concern. It is expected that many further factors will be needed to explain and to formulate targeted policies. This discussion has to be postponed until further research has been done.

9 Concluding Remarks

This study contributes to the conceptual refinement of multidimensional polarization by using new methodological approaches which provide new empirical results. In particular, we propose a CES well-being function to capture interdependence/compensation/substitution between polarization attributes. This is the basis for new well-being polarization measures and for the new minimum multidimensional polarization (2DGAP) approach. In particular, the 2DGAP approach disentangles the single polarization attributes *and* ensures at the same time that compensation between the polarization attributes is accounted for. This is important for the development of targeted economic and social policies.

The empirical application uses genuine personal leisure time in addition to income as polarization attributes in order to supplement the traditional use of income in polarization indices with social participation aspects. This is done by using the German time use diary data for the period 1991/92 to 2001/02. Beyond unidimensional and multidimensional polarization results of measures found in the literature, we develop new findings about polarization while accounting for compensation, which is not given an arbitrary value but instead one which is estimated and evaluated by the German population.

There are three prominent empirical results for Germany: First, genuine personal leisure time, in addition to income, appears to be an important and significant polarization attribute. Second, compensation between these two attributes, evaluated by the German population, is found to be of economic and statistical significance. Third, as shown by the new minimum 2DGAP approach, multidimensional polarization significantly increased in Germany over the decade 1991/92-2001/02.

In particular, the **IMD polarization headcount ratio** decreases significantly from a remarkable 20.7% in 1991/92 to 17.6% in 2001/02. Of specific interest and empirical importance are regimes with time deficits (Table 3): for poverty, where the time deficit is assigned not to be compensated even by above poverty threshold income (regime P3); for affluence, where the time deficit is assigned to be compensated by above affluence threshold income (regime R3). These regimes show the highest IMD polarization headcount ratios for both years and emphasize the importance of genuine personal leisure time for the German population.

IMD polarization well-being intensity decreases significantly over that decade if the common polarization threshold is the median. However, if the isopoverty and isoaffluence thresholds are farther from the median, then the multidimensional polarization well-being intensity decrease is only of minor significance. This indicates particular differences between the lower and upper middle classes relatively close to the median. The **new minimum 2DGAP polarization** measure however shows a significant increase of multidimensional

time and income polarization intensity by 18%. The disentangled components increased significantly for income (32%) and less distinct but also significantly for genuine personal leisure time (18%). With regard to poalarization regimes, the strongest multidimensional polarization 2DGAP intensity is given for the intersection of time and income for the poverty as well as for the affluence pole.

The starting point of the overall mean minimum 2DGAP intensity can be interpreted as a **polarization center**. For the poverty pole this center is in the intersection of time as well income poverty. For the affluence pole this center is in the regime where the time deficit is assigned to be not compensated by above affluence threshold income. The polarization centers thus show time deficits in both poles and highlights the strong importance of genuine personal leisure time for the multidimensional polarization in Germany that time period.

We argue that some differences in well-being and minimum 2DGAP intensity results are due to the "fuzziness" of the well-being gap. In summary, with highly significant minimum 2DGAP results we face a strong case for an increased interdependent multidimensional time and income polarization intensity for Germany between 1991/92 and 2001/02.

Further results for socio-demographic groups show greater polarization for the self-employed and increasingly for single parents than for differences based on gender, age or geographic region.

These and all other findings in this study stress the relevance of genuine personal leisure time with its social participation aspect as an important polarization dimension both for the poor and for the affluent. Economic and social policy will probably deal differently with the poverty and affluence pole when a reduction in polarization is sought. However, more detailed information about both the poor and the affluent is needed to understand polarization trends; and this is what our contribution provides to a certain extent.

Obviously, data both contributes to and restricts the explanatory power of any analysis. In our case it is questionable whether available survey data would describe in sufficient detail the situation of the affluent and in particular the high income affluent. Compulsory income tax data, which arguably provides the most meaningful information about high income individuals, showed an increased unidimensional income polarization in Germany during the same period of analysis (Merz 2006). This is an indication that the increase of the multidimensional time and income polarization gap described in our study would probably have been even greater if more detailed data had been available.

Appendix: Polarization Measures in the Literature

Unidimensional Polarization Measures

Foster and Wolfson (mimeo 1992, released 2010)

(A1) Income spread: $S(q) = \left| \frac{F^{-1}(q) - m}{m} \right|$

(A1b) Bipolarity: $B(p) = \left| \int_{q}^{0.5} \frac{F^{-1}(p) - m}{m} \right| dp$

(A1c) $P^{FW} = 2 \left| \int_{q}^{0.5} \frac{F^{-1}(p) - m}{m} \right| dp$

where F is the cumulative distribution function of income, q is the population fraction, and m is the median income. This polarization index is closely related to the Gini coefficient

(A1d) $P^{BW} = (G^{W} - G^{B}) \frac{L}{n}$

Wang and Tsui 2000

(A2) $P^{WT} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - m}{m} \right|^{\alpha}$

where $\alpha \in [0,1]$, m is the median income, y_i is the income of individual i, and n is the number of observations.

Scheicher 2010

(A3) $P^{S}_{univ} = \frac{1}{n_{poor}} \sum_{i \in poor} \left| \frac{z - y_i}{z} \right|^{\alpha} + \frac{1}{n_{rich}} \sum_{i \in rich} \left| \frac{y_i - r}{y_i} \right|^{\beta}$

where z is the poverty line, r is the affluence line, m is the median, and y_i describes individual income.

Polarization with separate groups by Esteban and Ray 1994

(A4) $P^{ER} = K \sum_{i=1}^{g} \sum_{j=1}^{g} \pi_i^{1+\alpha} \pi_j \left| \mu_i - \mu_j \right|$

where π is the population fraction of group i, and μ is the mean income of group i. Thus if all individuals have the same income, the index will be zero so that the minimum of the index is achieved at minimal inequality. The maximum of the measure is achieved when half of the

population has the minimum income and half of the population has the maximum income. Then the measure will be one normalized by a constant *K*.

Esteban, Gradín and Ray 2007

$$\text{(A5)} \quad P^{ERG} = \sum_{i=1}^{g}\sum_{j=1}^{g} \pi_i^{1+\alpha}\pi_j \left|\mu_i - \mu_j\right| - \beta\varepsilon(G - G^{grouped})$$

β is a weight for measurement error. An advantage of this extended polarization measure is that it is not necessary to classify the groups on the basis of arbitrary income constraints. Only the number of groups has to be chosen, and the group assignment is instead done by a classification algorithm.

Duclos, Esteban and Ray 2004

$$\text{(A6)} \quad P_\alpha^{DER}(F) = \int_y f(y)^\alpha a(y) dF(y) \text{ with } \quad \int_y$$

where F is the distribution function, f the associated density, μ is the income mean, $\alpha \in [0,1]$.

Multidimensional Polarization Measures

Gigliarano and Mosler 2009

$$\text{(A7a)} \quad P_1^{GM} = \phi\left(\frac{B(X)}{W(X)+c}\right)\cdot S(X)$$

$$\text{(A7b)} \quad P_2^{GM} = \psi\left(B(X) - W(X)\right)\cdot S(X)$$

$$\text{(A7c)} \quad P_3^{GM} = \tau\left(\frac{B(X)}{B(X)+W(X)+c}\right)\cdot S(X)$$

where *B(X)* is the inequality between the groups, and *W(X)* is the inequality within the groups. φ, ψ, τ are strictly increasing functions of a specific inequality measure. *S(X)* is the relative group size, which is measured by an inverse concentration measure like the Herfindahl index. *X* is a matrix which contains all individuals in the columns and their multiple attributes in the rows. Polarization is measured either via 7a, 7b or 7c. An increasing inequality or an increasing group size raises polarization, an increasing inequality within the groups lowers polarization.

Scheicher 2010

$$\text{(A8a)} \quad d\left(y_{ij}, \left[z_j, r_j\right]\right) = \begin{cases} \min\left\{\left|y_{ij} - z_j\right|, \left|y_{ij} - r_j\right|\right\} \text{ if } y_{ij} \notin \left[z_j, r_j\right] \\ 0 \text{ if } y_{ij} \in \left[z_j, r_j\right] \end{cases}$$

where z_j is the poverty line, r_j is the affluence line of attribute *j*, y_{ij} stands for the value of the j^{th} attribute of individual *i*. For aggregation, the single attribute specific distances are summed up over all attributes:

(A8b) $d\left(y_i,\left[z,r\right]\right)=\sum_j d(y_{ij},\left[z_j,r_j\right])$

Finally, the mean of all aggregated distances of the poor and the rich individuals given the respective middle class thresholds builds the Scheicher 2010 multidimensional polarization index:

(A8c) $P_{mult}^{S}=\frac{1}{n}\sum_i d(y_i,\left[z,r\right])$

References

Alkire, S. and J. Foster (2011), *Counting and multidimensional poverty measurement*, in: Journal of Public Economics, 95/7-8, 476-487.

Anderson, G. Crawford, I. and A. Leicester (2008), *Efficiency Analysis and the Lower Convex Hull Approach*, in: Kakwani, N. and J. Silber (Eds,), Quantitative Approaches to Multidimensional Poverty Measurement, Palgrave, Macmillan, New York, 176-191.

Atkinson, A.B. (2003), *Multidimensional Depriviation: Contrasting Social Welfare and Counting Approaches*, in: Journal of Economic Inequality, 1, 51-65.

Atkinson, A.B. and T. Piketty (eds.), 2007, *Top Incomes Over the 20th Century A Contrast Between Continental European and English Speaking Countries*, Oxford University Press, Oxford.

Becker, G.S. (1965), *A Theory of the Allocation of Time*, in: Economic Journal 75, 493-517.

Bittman, M. (1999), *Social Participation and Family Welfare: The Money and Time Cost of Leisure*, SPRC Discussion Paper No. 95, Sydney.

Bourguignon, F. and S.R. Chakravarty (2003), *The Measurement of Multidimensional Poverty*, in: Journal of Economic Inequality 1, 1, 25-49.

Bundesregierung (2013, 2011, 2004, 2002), Lebenslagen in Deutschland, Der erste (zweite, dritte, vierte) Armuts- und Reichtumsbericht der Bundesregierung, Bundesministerium für Gesundheit und Soziale Sicherung, Berlin.

Burchardt, Tania (2008), *Time and Income Poverty*, Centre for Analysis of Social Exclusion, London School of Economics, London (UK).

Chakravarty, S. R. (2009): *Inequality, Polarization and Poverty. Advances in Distributional Analysis.* New York, NY: Springer-Verlag New York.

Chakravarty, S.R. and Silber, J. (2008), *Measuring Multidimensional Poverty: The Axiomatic Approach*, in: Kakwani, N. and J. Silber (Eds.), Quantitative Approaches to Multidimensional Poverty Measurement, Palgrave Macmillan, New York, 192-209.

Clark, A.E., Frijters, P. and M. Shields (2008), *Relative Income, Happiness and Utility: An Explanation for the Easterlin Paradox and Other Puzzles,* in: Journal of Economic Literature, 46, 95-144.

Dell, F., (2007), *Top Incomes in Germany Throughout the Twentieth Century: 1891-1998*, in: Atkinson and Piketty (2007), 365-425.,

Deutsch, J. and J. Silber (2005), *Measuring Multidimensional Poverty: An Empirical Comparison of Various Approaches*, in: The Review of Income and Wealth, 51, 1, 145-174.

Duclos, J.-Y., Esteban, J.-M. and D. Ray (2004), *Polarization: Concepts, Measurement, Estimation*. in: Econometrica 72 (6), 1737–1772.

Ehling, M. (2003), *Erhebungen zur Zeitverwendung 1991-92 und 2001-02. Skizzierung des methodischen Vorgehens*, in: Wirtschaft und Statistik - Sonderausgabe anlässlich des 54.

Weltkongresses des Internationalen Statistischen Instituts vom 13. bis 20. August 2003 in Berlin, 85-87.

Ehling, M., Holz, E. and I. Kahle (2001), *Erhebungsdesign der Zeitbudgeterhebung 2001/2002,* in: Wirtschaft und Statistik, 427–436.

Esteban, J.-M. and D. Ray (1994), *On the Measurement of Polarization*, in: Econometrica, 62 (4), 819–851.

Esteban, J.-M., Gradín, C. and D. Ray (2007), *An Extension of a Measure of Polarization, with an Application to the Income Distribution of Five OECD Countries*, in: Journal of Economic Inequality 5 (1), 1–19.

Foster, J. and M.C. Wolfson (2010), *Polarization and the Decline of the Middle Class*. Canada and the U.S, in: Journal of Economic Inequality 8 (2), 247–273.

Foster, J., Greer, J. and E. Thorbecke (1984), *A Class of Decomposable Poverty Measures,* in: Econometrica, Vol. 52, No. 3, 761-766.

Frey, B.S. and A. Stutzer (2005), *Happiness Research: State and Prospects,* in: Review of Social Economy, Vol. LXII, No. 2, 207-228.

Gigliarano, C. and K. Mosler (2009), *Constructing Indices of Multivariate Polarization*, in: Journal of Economic Inequality, 7 (4), 435–460.

Goebel, J., Gorning, M. und H.Häußermann (2010), *Polarisierung der Einkommen: Die Mittelschicht verliert*, Wochenbericht des DIW Berlin, Nr. 24/2010, Berlin.

Goodin, R., Rice, J., Parpo, A. and Eriksson, L. (2008), *Discretionary Time: A New Measure of Freedom*, Cambridge University Press, Cambridge (UK).

Grabka, M. und J.R Frick (2008), *Schrumpfende Mittelschicht – Anzeichen einer dauerhaften Polarisierung der verfügbaren Einkommen*, Wochenbericht des DIW Berlin, Nr. 10/2008, Berlin.

Harvey, A.S. and A.K. Mukhopadhyay (2007), *When Twenty-four Hours is Not Enough: Time Poverty of Working Parents*, in: Social Indicators Research, 82, 1, 57-77.

Kakwani, N. and J. Silber (2008): *Quantitative Approaches to Multidimensional Poverty Measurement*, Houndmills, Basingstoke, Hampshire: Palgrave Macmillan.

Kmenta, J. (1967), *On Estimation of the CES Production Function,* in: International Economic Review, Vol. 8, No. 2.

Lugo, M.A. and E. Maasoumi (2009), MultidimensionalPoverty Measures from an Information Theory Perspective, OPHI working paper No. 10, Oxford Poverty & Human Development Initiative, Oxford (UK).

Maasoumi, E. and M.A. Lugo (2008), *The Information Basis of Multivariate Poverty Assessments*, in: Kakwani, N. and J. Silber (Eds,), Quantitative Approaches to Multidimensional Poverty Measurement, Palgrave, Macmillan, New York, 1-29.

Malmquist, S. (1953), *Index Numbers and Indifference Surfaces*, in: Trabajos Estatistica, 2, 209-242.

Merz, J. (2006), *Polarisierung der Einkommen von Selbständigen? – Zur Dynamik der Einkommensverteilung von Freiberuflern und Unternehmern*, in: Merz, J. (Ed.), Fortschritte der Mittelstands Forschung, CREPS-Schriftenreihe Vol. 3, Lit Verlag, Münster, 395-415.

Merz, J. and B. Scherg (2013), Polarization of Time and Income – A Multidimensional Approach with Well-Being Gap and Minimum 2DGAP: German Evidence, Forschungsinstitut Freie Berufe (FFB), Leuphana Universität Lüneburg, FFB-Discussion Paper No.95, Lüneburg (download via www.leuphana.de/ffb).

Merz, J. and M. Zwick (2005), *Hohe Einkommen: Eine Verteilungsanalyse für Freie Berufe, Unternehmer und abhängig Beschäftigte mit Mikrodaten der Einkommensteuerstatistik*, in: Journal of Applied Social Science Studies, Jg. 125/2, 313-328.

Merz, J. and T. Rathjen (2011), Sind Selbständige zeit- und einkommensarm? – Zur Dynamik interdependenter multidimensionaler Armut mit den deutschen Zeitbudgeterhebungen, in: Bekmeier-Feuerhahn, S., Martin, A., Merz, J. and U. Weisenfeld (Eds.), Die Dynamik tiefgreifenden Wandels in Gesellschaft, Wirtschaft und Unternehmen, LIT-Verlag, 219-239.

Merz, J. and T. Rathjen (2014a), *Time and Income Poverty – An Interdependent Multidimensional Poverty Approach with German Time Use Diary Data*, forthcoming in: The Review of Income and Wealth .

Merz, J. and T. Rathjen (2014b), *Multidimensional Time and Income Poverty: Well-Being Gap and Minimum 2DGAP Poverty Intensity – German Evidence,* forthcoming in: Journal of Economic Inequality.

Merz, J., Hirschel, D. and M. Zwick (2005), *Struktur und Verteilung hoher Einkommen - Mikroanalysen auf der Basis der Einkommensteuerstatistik*, Gutachten zum zweiten Armuts- und Reichtumsbericht der Bundesregierung, Lebenslagen in Deutschland, Bundesministerium für Gesundheit und Soziale Sicherung, Berlin.

Peichl, A. Schaefer, T. and C. Scheicher (2010), *Measuring Richness and Poverty – A Micro Data Application to Europe and Germany*, in: The Review of Income and Wealth, 56 (3), 597–619.

Ritter, C. (1896), *Platos Gesetze*, Neudruck der Ausgabe Leipzig 1896, Scientia Verlag Aalen.

Scheicher, C. (2009), Armut, Reichtum, Umverteilung: Begriff und statistische Messung, Eul Verlag, Lohmar.

Scheicher, C. (2010), *Measuring Polarization via Poverty and Affluence*, Köln Discussion Papers in Statistics and Econometrics, Köln.

Schmidt, A. (2004), *Statistische Messung von Einkommenspolarisierung*, Eul Verlag, Lohmar.

Sen, A.K. (1985), *Commodities and Capabilities*, North-Holland, Amsterdam.

Sen, A.K. (1999), *Development as Freedom*, Knopf Publishers, New York.

Sen, A.K. (2008), *The Economics of Happiness and Capability*, in: Bruni, L., Comim, F. and M. Pugno, Capabilities & Happiness, Oxford University Press, Oxford (UK), 16-27.

Vickery, C. (1977), *The Time-Poor: A New Look at Poverty*, in: Journal of Human Resources, 12, 1, 27-48.

Wagner, G.G., Frick. J.R. and J. Schupp (2007), *The German Socio-Economic Panel Study (GSOEP): Scope, Evolution and Enhancements*, in: Journal of Applied Social Science Studies, 127, 1, 139-179.

Wang, Y.-Q. and K.-Y. Tsui (2000), *Polarization Orderings and New Classes of Polarization Indices*, in: Journal of Public Economic Theory, 2 (3), 349–363.