On the Valuation of Leisure, Labour Supply and Household Production

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Abstract

The paper provides a generalization of Becker's theory of the allocation of time. The paper assumes that household time plays three roles: as leisure, household work and household labour supply, with separate utility valuations for each use of time. The paper also considers the case where the household does not provide external market labour, a case not considered by Becker and Pollak and Wachter. The present paper elaborates on the earlier analysis of Schreyer and Diewert in that various corner solutions to the household's time allocation problem are considered in more detail. The paper also consider the econometric problems that these corner solutions create. Finally, as in Schreyer and Diewert, the paper relates the analysis to the difficult problems associated with the valuation of household work at home, an issue that national income accountants have attempted to address over the years.

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1. Introduction

Becker (1965) introduced the household's allocation of time as an additional constraint into the traditional household utility maximization problem. However, Pollak and Wachter (1975; 266) recognized some limitations of his analysis: namely, that Becker neglected the role of household work at home in his model and did not model the direct disutility of work at home. In addition, Becker assumed that the household could provide market labour supply and this enabled Becker to consolidate the budget constraint and the time constraint into a single constraint and enabled him to value household time in an unambiguous way. Schreyer and Diewert (2013) generalized these models of the household allocation of time by allowing time to play three roles: as leisure, household work and household labour supply, with separate utility valuations for each use of time. Schreyer and Diewert also considered the case where the household did not provide external market labour, a case not considered by Becker and Pollak and Wachter. The present paper elaborates on the analysis of Schreyer and Diewert in that we now consider various corner solutions to the household's time allocation problem in more detail. We also consider the econometric problems that these corner solutions create. Finally, as in Schreyer and Diewert (2013), we will relate our analysis to the difficult problems associated with the valuation of household work at home, an issue that national income accountants have attempted to address over the years.

To conclude this introduction, we will elaborate on the above points in a bit more detail. In Becker's model of consumer behavior, a household purchases q_n units of *market commodity n* and combines it with a household input of time, t_n, to produce $z_n = f_n(q_n, t_n)$ units of a *finally demanded commodity* z_n for n = 1,2,…,N say, where f_n is *the household production function* for the nth finally demanded commodity[2]. Some examples of such finally demanded "produced" commodities are:

- Eating a meal; the inputs are the prepared meal and time spent eating and the output is a consumed meal.
- Reading a book; the inputs are computer services or a physical book and time and the output is a book which has been read.
- Cleaning a house; the inputs are cleaning utensils, soapy water, polish and time and the output is a clean house.
- Gardening services; the inputs are tools used in the yard, fuel (if power tools are used) and time and the output is a beautiful yard.
- Making a meal; the inputs are the ingredients used, the use of utensils and possibly a stove and time required to make the meal and the output is the prepared meal.

We will modify Becker's framework in two ways:

[2] For additional work on the allocation of time and household production, see Pollak and Wachter (1975) (1977), Barnett (1977), Landefeld and McCulla (2000), Diewert (2001), Abraham and Mackie (2005), Fraumeni (2008), Hill (2009), Landefeld, Fraumeni and Vojtech (2009), Schreyer and Ranuzzi de Bianchi (2009) and Schreyer and Diewert (2013).

- We will decompose the finally demanded produced commodities into two classes: one class of *finally demanded services* where the final service cannot be purchased in the marketplace (such as eating a meal or reading a book) and another class of *household production function services* where the service could be purchased in the marketplace (such as cleaning, gardening and cooking services) or it could be produced internally by the household. The time spent on the second class of activities can be classified as household work time.
- Becker assumed that the opportunity cost of household time was the (after tax) market wage rate that household members could earn. But what is the appropriate price of household time for a retired household? We will address this question.

In our model of household behavior, a household member can divide its time among *three uses*: time spent on producing finally demanded services t_F, time spent on household production or work t_H and for nonretired members, time spent on market employment or labour supply, t_L.

Our main focus will be to cast some light on a problem that has troubled national income accountants for some time: *how should household leisure and work time be valued*: at the household's opportunity cost market wage rate or at the wage rate at which household services could be purchased? We will show below that it is not always possible to give an unambiguous answer to this question.

2. The Household's Utility Maximization Problem as a Concave Programming Problem

For the sake of simplicity, we will consider the utility maximization problem of a single person household that has preferences defined over four commodities: Q_F is the quantity of *finally demanded leisure type services* that the household consumes (these services cannot be purchased in the marketplace), Q_H is the quantity of *household produced services* that could be produced by using market goods q_H and household time t_H or externally purchased time inputs q_S, t_H is *household working time* and t_L is the quantity of *time spent working in the external marketplace*. The household has preferences over these four commodities that can be represented by the utility function $U(Q_F,Q_H,t_H,t_L)$ where the utility function U is defined over the nonnegative orthant and is concave,[3] continuous, differentiable,[4] increasing in Q_F and Q_H and nonincreasing in t_H and t_L.[5] Household leisure services Q_F are produced by the household subutility function F that uses inputs of purchased goods q_F and household leisure time t_F. Household production services Q_H are produced by the production function H that uses purchased goods q_H and $t_H + q_S$ units of time

[3] The concavity assumption is stronger than the usual quasiconcavity assumption but the results of Afriat (1967; 75) and Diewert (1973; 423) show that from an empirical point of view, it is not restrictive to assume concavity of the utility function.

[4] We require only the existence of first order partial derivatives. We assume that $\partial U(Q_F,Q_H,t_H,t_L)/\partial Q_F \equiv U_1(Q_F,Q_H,t_H,t_L) > 0$, $\partial U(Q_F,Q_H,t_H,t_L)/\partial Q_H \equiv U_2(Q_F,Q_H,t_H,t_L) > 0$, $\partial U(Q_F,Q_H,t_H,t_L)/\partial t_H \equiv U_3(Q_F,Q_H,t_H,t_L) \leq 0$ and $\partial U(Q_F,Q_H,t_H,t_L)/\partial t_L \equiv U_4(Q_F,Q_H,t_H,t_L) \leq 0$ for all $(Q_F,Q_H,t_H,t_L) \geq 0_4$. Thus we assume that the marginal utility of an additional unit of Q_F and Q_H is positive and the marginal utility of an additional hour of household work t_H and of market labour supply t_L is nonnegative so if these derivatives are negative, then the household receives disutility from these additional hours of work (holding constant Q_F and Q_H).

[55] The assumption that the utility function is nonincreasing in t_H and t_L is not necessarily justified but we make it in order to obtain more definite results. It should be noted that Schreyer and Diewert (2013) do not make the nonincreasing in t_H assumption in their paper so the present paper is less general in this respect.

where t_H is the quantity of *household time spent in household production* (i.e., in producing Q_H using the household time input t_H) and q_S is the amount of *market purchases of the time of external workers* who could also produce Q_H. Thus we have:

(1) $Q_F = F(q_F,t_F)$;
(2) $Q_H = H(q_H,t_H+q_S)$

where we assume that F and H are continuous, concave, linearly homogeneous functions defined over the nonnegative orthant.[6] It is important to note that household time t_H and purchased time to undertake household work q_S are assumed to be perfect substitutes in equation (2) above. This perfect substitutes assumption will play a key role in the analysis to follow. We will assume that the household faces the fixed positive prices p_F for purchases of q_F, p_H for q_H and w_S for hiring units of labour to do paid hours of housework q_S. We also assume that the household faces an after tax wage rate w_L for each unit of labour supply t_L and spends at most *nonlabour income* Y on purchases of market goods and services.[7] There is also a household time constraint that must be satisfied; i.e., t_F plus t_H plus t_L cannot exceed $T > 0$ units of time. Using the above assumptions, the household's *budget constraint* and *time constraint* are (3) and (4) below:

(3) $p_Fq_F + p_Hq_H + w_Sq_S \leq Y + w_Lt_L$;
(4) $t_F + t_H + t_L \leq T$.

Our final assumption is that the observable vector of market goods and services purchases (q_F^*,q_H^*,q_S^*) and the observable time allocation vector (t_F^*,t_H^*,t_L^*) solves the following constrained utility maximization problem:

$$(5)\ u^* \equiv \max_{q_F\geq 0,q_H\geq 0,q_S\geq 0,t_F\geq 0,t_H\geq 0,t_L\geq 0} \{U[F(q_F,t_F),H(q_H, t_H+q_S),t_H,t_L] : Y+w_Lt_L - p_Fq_F - p_Hq_H - w_Sq_S \geq 0;\ T - t_F - t_H - t_L \geq 0\}.$$

It can be verified that the constrained maximization problem in (5) is a *concave programming problem*; i.e., the objective function and the two constraint functions are concave and the feasible region is a convex set. Thus by the Karlin (1959; 201-203) Uzawa (1958) *Saddle Point Theorem*, there exist multipliers $\lambda^* \geq 0$ and $\omega^* \geq 0$ such that $(\lambda^*,\omega^*,q_F^*,q_H^*,q_S^*,t_F^*,t_H^*,t_L^*)$ is a solution to the following min-max problem:[8]

$$(6)\ u^* \equiv \min_{\lambda\geq 0,\omega\geq 0} \max_{q_F\geq 0,q_H\geq 0,q_S\geq 0,t_F\geq 0,t_H\geq 0,t_L\geq 0} \{U[F(q_F,t_F),H(q_H, t_H+q_S),t_H,t_L] + \lambda(Y + w_Lt_L - p_Fq_F - p_Hq_H - w_Sq_S) + \omega(T - t_F - t_H - t_L)\}.$$

Note that the two linear constraints in (5) have been absorbed into the objective function of (6). In subsequent sections of this paper, we will assume that the functions U, F and H are differentiable

[6] We also assume that F and H are positive if their arguments are positive, which will imply that F and H are nondecreasing in their arguments. The assumption that F and H are linearly homogeneous is fairly standard in this literature; see Becker (1965).
[7] Y could be negative if the amount of labour supplied is sufficiently positive.
[8] In order to rigorously obtain the equivalence of (5) and (6), we assume that the Slater (1950) constraint qualification condition is satisfied; i.e., we assume that nonnegative vectors (q_F,q_H,q_S) and (t_F,t_H,t_L) exist such that the two constraints in (5) hold with a strict inequality.

and we will utilize the resulting first order conditions for the min-max problem defined by (6) in order to derive some useful results. However, in the present section, we can derive some useful results without assuming differentiability of U, F and H.[9]

Our concavity and monotonicity assumptions on U, F and H are sufficient to imply $\lambda^* > 0$ and $\omega^* > 0$ and that the inequality constraints in (5) will hold with equality at a solution to (5) or (6):

(7) $Y + w_Lt_L^* - p_Fq_F^* - p_Hq_H^* - w_Sq_S^* = 0$; $T - t_F^* - t_H^* - t_L^* = 0$.

Since λ^* and ω^* are both positive, we can define $w^* > 0$ as the following ratio:

(8) $w^* \equiv \omega^*/\lambda^*$.

The number w^* can be interpreted as the *imputed price of leisure time* t_F as we shall see. Now take the first equation in (7) and add to it the second equation in (7) multiplied by w^*. Rearranging terms in the resulting equation leads to the following equation:

(9) $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w^*t_H^* + w_Sq_S^* - (w_L - w^*)\, t_L^* = Y + w^*T \equiv F_I$

where F_I is defined as *imputed full income*, equal to nonlabour income expenditures Y plus the household's imputed value of time w^*T. Our imputed full income is an alternative to Becker's (1965) full income. In Becker's theoretical framework, the household utility function $U(Q_F,Q_H,t_H,t_L)$ is collapsed down to $U(Q_F,Q_H)$; i.e., there is no direct disutility of household work or market labour supply in Becker's theory.[10] In the Becker model, t_L in the household budget constraint can be replaced with $t_L = T - t_F - t_H$ and the constrained utility maximization problem (5) collapses down to the problem of maximizing $U[F(q_F,t_F),G(q_H,t_H)]$ subject to the single budget constraint:

(10) $p_Fq_F + w_Lt_F + p_Hq_H + w_Lt_H = Y + w_LT \equiv F_B$

where *Becker's full income* F_B is defined as nonlabour income expenditures Y plus the value of household time w_LT valued at the household's market wage rate w_L. Note that Becker's definition of full income has an advantage over our definition in that his definition depends only on observable data whereas our valuation of time involves the imputed price w^*. In the remainder of this paper, much of our attention will be focused on obtaining estimates or bounds for w^*. Our theoretical framework has the advantage of being more general and in particular, we can deal with households who do not offer any market labour supply.

Recall the max-min problem defined by (6) above and recall that we assumed that $(\lambda^*, \omega^*,q_F^*,q_H^*,q_S^*, t_F^*,t_H^*, t_L^*)$ was a solution to that problem. If we set $\omega = \omega^*$, then it can be seen that $(\lambda^*,q_F^*,q_H^*,q_S^*, t_F^*,t_H^*, t_L^*)$ is a solution to the following max-min problem:

(11) $u^* \equiv \min_{\lambda\geq 0}\ \max_{q_F\geq 0,q_H\geq 0,q_S\geq 0,t_F\geq 0,t_H\geq 0,t_L\geq 0}$

[9] In the nondifferentiable case, we assume that $U(Q_F,Q_H,t_H,t_L)$ is strictly increasing in Q_F and Q_H.
[10] Also q_S is missing from Becker's theoretical framework.

$$\{U[F(q_F,t_F),H(q_H,\, t_H+q_S),t_H,t_L] + \lambda(Y + w_Lt_L - p_Fq_F - p_Hq_H - w_Sq_S) + \omega^*(T - t_F - t_H - t_L)\}$$
$$= \min_{\lambda\geq 0}\ \max_{q_F\geq 0,q_H\geq 0,q_S\geq 0,t_F\geq 0,t_H\geq 0,t_L\geq 0}$$
$$\{U[F(q_F,t_F),H(q_H,\, t_H+q_S),t_H,t_L] + \lambda[Y + w_Lt_L - p_Fq_F - p_Hq_H - w_Sq_S + w^*(T - t_F - t_H - t_L)]\}$$

where w^* is defined as ω^*/λ^*. Now we can appeal to the Karlin-Uzawa Saddle Point Theorem in reverse and conclude that $(q_F^*,q_H^*,q_S^*,\, t_F^*,t_H^*,\, t_L^*)$ is a solution to the following utility maximization problem that involves only a single budget constraint:

$$(12)\ u^* = \max_{q_F\geq 0,q_H\geq 0,q_S\geq 0,t_F\geq 0,t_H\geq 0,t_L\geq 0}\ \{U[F(q_F,t_F),H(q_H,\, t_H+q_S),t_H,t_L] :$$
$$p_Fq_F + w^*t_F + p_Hq_H + w^*t_H + w_Sq_S - (w_L - w^*)\, t_L \leq Y + w^*T\}.$$

Thus *if* we are somehow able to determine the optimal imputed price of leisure time w^*, then this shadow price can be used in the single budget constraint in the constrained utility maximization problem (12) and (12) is a “classical” single constraint utility maximization problem for the household. Note that w^* is used to value household leisure time t_F and the value of household time w^*T in the single budget constraint in (12). Some other important points to notice about the utility maximization problems (5) and (12) are as follows:

- Our (Diewert-Schreyer) imputed full income $F_I = Y + w^*T$ is generally different from Becker’s full income $F_B = Y + w_LT$. Although our model of household behavior is more general, our measure of full income has the disadvantage that econometric estimation will in general be required in order to determine it; i.e., we need an estimate for the unobserved w^*.
- Our imputed value of an extra hour of time, w^*, is equal to the unobserved value of leisure time instead of the market wage w_L of the household.
- The household’s optimal allocation of leisure time t_F^* and of household work time t_H^* will generally be positive but the household’s optimal market labour supply t_L^* could be zero and its purchases of market labour services q_S^* that could substitute for its own household working time could also be zero. Market labour supply could be zero because the household consists of a retired worker or a “rich” individual who has sufficient non labour income to live on. Purchases of market labour services for doing household work could be zero for “frugal” households who simply prefer to do their own household work. Thus in general, it is necessary to consider the possibility of *corner solutions* for the household’s utility maximization problem (5).

In sections 3-6 below, we will consider the following four special cases for solutions to (5):[11]

- *Case 1*: $q_S^* > 0$; $t_L^* > 0$. This case corresponds to a household that purchases some market services q_S that can substitute for household work and the household also works at an external job.
- *Case 2*: $q_S^* = 0$; $t_L^* > 0$. This household supplies market labour but does not purchase any services that can substitute for household work.

[11] For all of these four cases, we assume that $q_F^* > 0$, $q_H^* > 0$, $t_F^* > 0$, $t_H^* > 0$, $\lambda^* > 0$ and $\omega^* > 0$. Cases 1 and 3 were considered by Schreyer and Diewert (2013) in their model, but they did not consider Cases 2 and 4.

- *Case 3*: $q_S^* > 0$; $t_L^* = 0$. This case corresponds to a household that does not work externally but purchases some services that can substitute for household work.
- *Case 4*: $q_S^* = 0$; $t_L^* = 0$. This case corresponds to a household that does not supply market labour services and does not purchase any services that can substitute for household work.

3. The Case of a Worker Household that Purchases Some Market Household Services

Assuming that U, F and G are once differentiable, the first order necessary (and sufficient) conditions for the interior solution $(\lambda^*, \omega^*, q_F^*, q_H^*, q_S^*, t_F^*, t_H^*, t_L^*)$ to solve (6) are as follows:[12]

$$(13)\ U_1[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]F_1(q_F^*,t_F^*) = \lambda^* p_F ;$$
$$(14)\ U_1[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]F_2(q_F^*,t_F^*) = \lambda^* w^* ;$$
$$(15)\ U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]H_1(q_H^*,t_H^*+q_S^*) = \lambda^* p_H ;$$
$$(16)\ U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]H_2(q_H^*,t_H^*+q_S^*) = \lambda^* w_S ;$$
$$(17)\ U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]H_2(q_H^*,t_H^*+q_S^*) + U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*] = \lambda^* w^* ;$$
$$(18)\ U_4[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*] = -\lambda^* (w_L - w^*) ;$$
$$(19)\ t_F^* + t_H^* + t_L^* = T ;$$
$$(20)\ p_F q_F^* + p_H q_H^* + w_S^* q_S^* = Y + w_L t_L^* .$$

Upon substituting (16) into (17), the resulting equation becomes:

$$(21)\ U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*] = -\lambda^* (w_S - w^*).$$

Since $\lambda^* > 0$ and $\omega^* > 0$, the definition of w^* as ω^*/λ^* implies that $w^* > 0$. Under our regularity assumptions on U, U_3 and U_4 are assumed to be nonpositive. Thus (18) and (21) imply that $w_L - w^* \geq 0$ and $w_S - w^* \geq 0$. Hence we have the following bounds on the imputed price of leisure time w^*:

$$(22)\ 0 < w^* \leq \min \{w_S, w_L\}.$$

This is an important new result: under the assumptions of Case 1, the imputed price of leisure time w^* is equal to or less than the market wage rate for the household w_L and equal to or less than the cost of hiring outside help to do household work w_S.[13]

If there is no direct disutility of household work so that $U(Q_F,Q_H,t_H,t_L) = U(Q_F,Q_H,t_L)$ or more generally, if $U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*] \equiv U_3^* = 0$, then (21) implies that the imputed price of leisure time, w^*, must equal the cost of hiring household work, w_S.[14] Similarly, if there is no direct disutility of market work so that $U(Q_F,Q_H,t_H,t_L) = U(Q_F,Q_H,t_H)$ or more generally, if

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[12] In these first order conditions, we have replaced ω^* wherever it occurs by $\lambda^* w^*$, which is just a relabelling of variables.

[13] It should be noted that this result depends on our assumption that U_3 and U_4 are nonpositive.

[14] Under this additional hypothesis that $U_3^* = 0$, we also require the condition that w_S be equal to or less than w_L, the market wage rate for this household. If the condition $w_S \leq w_L$ is not satisfied, then our conditions are not consistent; i.e., it must be the case that a corner solution holds and the present (interior equilibrium) case with U_3^* equal to zero cannot occur.

$U_4[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*] \equiv U_4^* = 0$, then (18) implies that the imputed price of leisure time, w^*, must equal the market wage rate, w_L.[15]

Conditions (18) and (21) are important in that they show that the household's imputed price of leisure time, w^*, is bounded from above by both the household's market wage rate, w_L, and the cost of hiring household help, w_S. Moreover, these equations show that the gap between w^* and the market wages w_L and w_S is determined by the *magnitudes* of the disutilities of household work (represented by the size of the partial derivative $U_3^* \leq 0$) and market labour supply (represented by the size of $U_4^* \leq 0$). Thus equation (18) implies that $w^* = w_L + (U_4^*/\lambda^*)$ so that the larger (in magnitude) is the disutility of market labour supply, the more w^* will be below w_L. Similarly, equation (21) implies that $w^* = w_S + (U_3^*/\lambda^*)$ so that the larger (in magnitude) is the disutility of doing household chores, the more w^* will be below w_S.[16]

It is possible to make use of the linear homogeneity of the household production functions, F and H, and convert the first order conditions (13)-(18) into a simpler, more intuitive form. However, in order to accomplish this task, a certain amount of background material must be explained.

First, we define the following *unit cost functions*, c^F and c^H, *that are dual to F and H* as follows. For positive prices p_F, w, p_H and w_S, define:

(23) $c^F(p_F,w) \equiv \min_{q_F\geq 0,t_F\geq 0} \{p_Fq_F + wt_F : F(q_F,t_F) = 1\}$;

(24) $c^H(p_H,w_S) \equiv \min_{q_H\geq 0,t_H\geq 0} \{p_Hq_F + w_St_H : H(q_H,t_H) = 1\}$.

Define the equilibrium *full price of a unit of leisure services* Q_F as P_F^* and the equilibrium *full price of a unit of household work services* Q_H as P_H^* as the unit cost of producing one unit of these services:

(25) $P_F^* \equiv c^F(p_F,w^*)$; $P_H^* \equiv c^H(p_H,w_S)$.

Since the household production functions F and H are linearly homogeneous, Euler's Theorem on homogeneous functions implies the following equations:

(26) $F_1(q_F^*,t_F^*)q_F^* + F_2(q_F^*,t_F^*)t_F^* = F(q_F^*,t_F^*) \equiv Q_F^*$;

(27) $H_1(q_H^*,t_H^*+q_S^*)q_F^* + H_2(q_H^*,t_H^*+q_S^*)(t_H^*+q_S^*) = H(q_H^*,t_H^*+q_S^*) \equiv Q_H^*$

where the household's *equilibrium full consumption of leisure services* is defined as $Q_F^* \equiv F(q_F^*,t_F^*)$ and its *equilibrium production of household work services* is defined as $Q_H^* \equiv H(q_F^*,t_F^*+q_s^*)$. The

[15] Under the hypothesis that $U_4^* = 0$, (18) implies that $w^* = w_L$ and the opportunity cost of leisure time w^* is equal to the market wage rate w_L; i.e., we are in a situation where Becker's model of the allocation of time is the correct one. Under these conditions, equation (21) becomes $U_3^* = -\lambda^*(w_S - w_L) \leq 0$ since $U_3^* \leq 0$. Thus we also require the condition that w_L be equal to or less than w_S. If the condition $w_L \leq w_S$ is not satisfied, then again, our conditions are not consistent; i.e., it must be the case that a corner solution holds and the present (interior equilibrium) case with U_4^* equal to zero cannot occur.

[16] Note that the conditions $w_S \neq w_L$, $U_3^* = U_4^* = 0$ are not consistent with the existence of an interior solution; i.e., under these conditions, we must have a corner solution where at least one of q_S^*, t_H^* or t_L^* is equal to zero.

significance of the prices P_F^* and P_H^* defined by (25) and the quantities Q_F^* and Q_H^* defined by (26) and (27) will be seen shortly.

Consider the following cost minimization problem where the household attempts to minimize the cost of achieving the leisure subutility level Q_F^* defined by (26):

(28) $\min_{q\geq 0,t\geq 0} \{p_Fq + w^*t : F(q,t) \geq Q_F^*\}$.

The first order necessary (and sufficient) conditions for this cost minimization problem are the existence of a $q^* \geq 0$, $t^* \geq 0$ and $\mu^* \geq 0$ such that the following conditions are satisfied:

(29) $F_1(q^*,t^*) = \mu^* p_F$;
(30) $F_2(q^*,t^*) = \mu^* w^*$;
(31) $F(q^*,t^*) = Q_F^*$.

Recalling the first order conditions (13) and (14) and definitions (25) and (27), it can be seen that $q^* \equiv q_F^*$, $t^* \equiv t_F^*$ and $\mu^* \equiv \lambda^*/U_1^*$ where $U_1^* \equiv U_1[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]$ satisfy the first order conditions for the cost minimization problem (28) and hence we have

(32) $P_F^*Q_F^* = c^F(p_F,w^*)F(q_F^*,t_F^*) = p_Fq_F^* + w^*q_F^*$.

Now consider the following cost minimization problem where the household attempts to minimize the cost of achieving the housework subutility level Q_H^* defined by (27):

(33) $\min_{q\geq 0,t\geq 0} \{p_Hq + w_St : H(q,t) \geq Q_H^*\}$.

The first order necessary (and sufficient) conditions for this cost minimization problem are the existence of a $q^* \geq 0$, $t^* \geq 0$ and $\mu^* \geq 0$ such that the following conditions are satisfied:

(34) $H_1(q^*,t^*) = \mu^* p_F$;
(35) $H_2(q^*,t^*) = \mu^* w^*$;
(36) $H(q^*,t^*) = Q_H^*$.

Recalling the first order conditions (15) and (16)[17] and definitions (25) and (26), it can be seen that $q^* \equiv q_H^*$, $t^* \equiv q_S^* + t_H^*$ and $\mu^* \equiv \lambda^*/U_2^*$ where $U_2^* \equiv U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,t_L^*]$ satisfy the first order conditions for the cost minimization problem (33) and hence we have

(37) $P_H^*Q_H^* = c^H(p_H,w_S)H(q_H^*,q_S^* + t_H^*) = p_Hq_H^* + w_S(q_S^* + t_H^*)$.

There are some significant points to note about the above algebra concerning the subutility cost minimization problems:

[17] In order to derive (37), it is important that the first order condition (16) hold where $q_S^* > 0$. This condition allows us to value household work time t_H^* at the opportunity cost wage w_S for hiring external help with housework. The corner solution case where $q_S^* = 0$ will be considered later.

- Time spent doing household work, t_H^*, should be valued at the opportunity cost of hiring external staff w_S to do this work provided that some staff is actually hired.
- Time spent in leisure activities, t_F^*, should be valued at the household's price of leisure time w^* which is in general unknown but is equal to or less than both w_S and the household's after tax market wage rate w_L.
- A close approximation to the price and quantity of household work, P_H^* and Q_H^* in equation (37) above, can be constructed without econometrically estimating the household production function, $H(q_H,q_S+t_H)$ or its dual unit cost function $c^H(p_H,w_S)$, if we use superlative index number techniques.[18]
- If we had an estimate for the price of household time spent in leisure activities w^*, then superlative index number techniques could again be used in order to construct close approximations to the price and quantity of household leisure, P_F^* and Q_F^* in equation (32) above.[19]

The above material can be used in order to simplify the first order conditions for the original max min problem, (13)-(18) above. Multiply both sides of (13) by q_F^* and multiply both sides of (14) by t_F^* and add the resulting equations. Using (25) and (26), it can be seen that the resulting equation simplifies to (38) below. Multiply both sides of (15) by q_H^* and multiply both sides of (16) by $q_S^* + t_H^*$ and add the resulting equations. Using (25) and (27), it can be seen that the resulting equation simplifies to (39) below. Equation (40) below is our old equation (21) and (41) is our old equation (18). Thus we have deduced that Q_F^*, Q_H^*, t_H^*, t_L^*, λ^* and $w^* \equiv \omega^*/\lambda^*$ satisfy the following equations:

(38) $U_1[Q_F^*,Q_H^*,t_H^*,t_L^*] = \lambda^* P_F^* > 0$;
(39) $U_2[Q_F^*,Q_H^*,t_H^*,t_L^*] = \lambda^* P_H^* > 0$;
(40) $U_3[Q_F^*,Q_H^*,t_H^*,t_L^*] = -\lambda^* (w_S - w^*) \leq 0$;
(41) $U_4[Q_F^*,Q_H^*,t_H^*,t_L^*] = -\lambda^* (w_L - w^*) \leq 0$.

Using (28)-(37), it can be seen that the budget constraint (9) can be rewritten as follows:

(42) $P_F^* Q_F^* + P_H^* Q_H^* - (w_S - w^*)t_H^* - (w_L - w^*)t_L^* = Y + w^*T \equiv F_I$.

[18] See Diewert (1976). The technique works as follows. Suppose that we can observe a household's price and quantity data pertaining to household work for T periods, say $p^t \equiv (p_H^t,w_S^t)$ and $q^t \equiv (q_H^t,q_S^t + t_H^t)$ for t = 1,...,T. Define the Fisher (1922) ideal price aggregate for period t as $P_H^t \equiv [p^t\cdot q^0/p^1\cdot q^0]^{1/2}[p^t\cdot q^t/p^1\cdot q^t]^{1/2}$ for t = 1,...,T. The *aggregate output of household work* for period t, Q_H^t, that corresponds to the *aggregate price of household work* in period t, P_H^t, is defined as $Q_H^t \equiv p^t\cdot q^t/P_H^t$ for t = 1,...,T. If the household production function $H(q_1,q_2)$ has the functional form $H(q_1,q_2) \equiv [a_{11}q_1^2 + 2a_{12}q_1q_2 + a_{22}q_2^2]^{1/2}$, then P_H^t and Q_H^t defined above using the Fisher price index and the observed data will exactly satisfy equation (37) where the data pertaining to period t is used in place of p_H, w_S, q_H^*,q_S^* and t_H^*. Diewert showed that this functional form for H is a flexible one (in the class of linearly homogeneous functions) and so even if H is not exactly equal to this assumed functional form, the Fisher aggregates P_H^t and Q_H^t defined above should approximate the true aggregates reasonably well.

[19] In particular, if we assume that $U_3^* = 0$ so that there is no separate disutility of household work, then w^* must equal the observable wage rate w_S for hiring workers to substitute for household work. In this case, good approximations to P_F^t and Q_F^t can be constructed using superlative index techniques as in the previous footnote.

Note that $P_F^*Q_F^* = p_Fq_F^* + w^*t_F^*$ is our estimate of the full value[20] of leisure consumption and $P_H^*Q_H^* = p_Hq_H^* + w_S(t_H^* + q_S^*)$ is the full value of household work activities (including purchased household labour). However, these full consumption values do not include adjustments for the direct disutility of household work t_H^* and of market labour supply t_L^*. Making these adjustments leads to the addition of the nonpositive *disutility terms* $-(w_S - w^*)t_H^* - (w_L - w^*)t_L^*$ to full consumption. Full consumption plus the disutility of work terms adds up to our concept of full income, $F_I \equiv Y + w^*T$.

Now consider the following single constraint utility maximization problem:

$$(43)\ \max_{Q_F \geq 0, Q_H \geq 0, t_H \geq 0, t_L \geq 0} \{U[Q_F,Q_H,t_H,t_L] : P_F^*Q_F + P_H^*Q_H - (w_S - w^*)t_H - (w_L - w^*)t_L \leq Y + w^*T\}.$$

It can be verified that the constrained maximization problem (43) is another concave programming problem and moreover, the quantities Q_F^*,Q_H^*,t_H^*,t_L^* and the multiplier λ^* that appeared in equations (38)-(42) will be a solution to (43).

The single constraint utility maximization problem (43) is almost a "standard" utility maximization problem that is treated in classical consumer demand theory: the only nonstandard aspects of it are that the utility function $U(Q_F,Q_H,t_H,t_L)$ is increasing in Q_F and Q_H and decreasing or at least nonincreasing in the two household time variables t_H and t_L.[21] Another nonstandard aspect of (43) is that a knowledge of w^* (the imputed price of household leisure time) is required in order to evaluate the budget constraint and to calculate the solutions Q_F^* and Q_H^*. In general, extra assumptions (such as $U_3^* = 0$) or econometric estimation will be required in order to calculate w^*. Possible econometric approaches are considered later in the paper.

We turn our attention to Case 2.

4. The Case of a Worker Household that Does not Purchase Any Market Labour Services

In this section, we analyze Case 2 where q_S^* equals 0 and labour supply t_L^* is positive. For this case, the household supplies market labour but does not purchase any services that can substitute for household work. Thus in this case, all equilibrium variables are assumed to be positive except we assume that $q_S^* = 0$. The Kuhn Tucker (1951) conditions which are necessary and sufficient for $\lambda^*,\omega^*,q_F^*,q_H^*,q_S^* = 0,t_F^*,t_H^*,t_L^*$ to solve (6) under these hypotheses are (13)-(15), (17)-(20) with $q_S^* = 0$ in these equations and the following condition which replaces (16):[22]

$$(44)\ U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*]H_2(q_H^*,t_H^*) \leq \lambda^*w_S\ .$$

Using the first order condition (14), the *imputed price of leisure time*, w^*, satisfies the following equation:

[20] Full values include the value of household time inputs in addition to purchased commodities.

[21] The prices for t_H and t_L in the household budget constraint, $-(w_L - w^*)t_H$ and $-(w_L - w^*)$, are also nonpositive instead of the usual property of being positive.

[22] Again, we have replaced ω^* wherever it occurs by λ^*w^*, which is just a relabelling of variables.

(45) $w^* = U_1[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*]F_2(q_F^*,t_F^*)/\lambda^* > 0$.

In the present case, the household's imputed price of time spent in household work can no longer be set equal to w_S, the market wage rate for hiring comparable household labour services. Thus we now define the (unobserved) *household's imputed price of time spent in household work*, w_H^*, as:

(46) $w_H^* \equiv U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*]H_2(q_H^*,t_H^*)/\lambda^* > 0$.

Thus in Case 2, *there are now two unobserved imputed prices of time*, w^* (the imputed price of household leisure time) and w_H^* (the imputed price of household working time), defined by (45) and (46). Inserting definition (46) into the inequality (44) and using $\lambda^* > 0$ leads to the following inequality:

(47) $0 < w_H^* \leq w_S$.

Thus when the household chooses not to hire any household market labour services, the imputed price of time for doing household work, w_H^*, cannot exceed the corresponding market wage rate, w_S.

Now substitute definition (45) into the first order condition (17) and we obtain the following equation:

(48) $U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*] = -\lambda^*(w_H^* - w^*) \leq 0$

where the inequality in (48) follows from the assumption that $U_3^* \leq 0$. Thus combining (47) and (48), w^* and w_H^* satisfy the following inequalities:

(49) $0 < w^* \leq w_H^* \leq w_S$.

The first order condition (18) is still valid for Case 2 and we rewrite this equation as follows:

(50) $U_4[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*] = -\lambda^*(w_L - w^*) \leq 0$

where the inequality in (50) follows from the assumption that $U_4^* \leq 0$. Thus (49) and (50) imply that the following inequalities must hold for Case 2:

(51) $0 < w^* \leq \min\{w_H^*,w_L\} \leq \min\{w_S,w_L\}$.

The inequalities in (51) are the Case 2 counterparts to the Case 1 inequalities (22). If $U_3^* = 0$, then $w^* = w_H^* \leq \min\{w_S,w_L\}$. If $U_4^* = 0$, then $w_S \geq w_H^* \geq w^* = w_L$.[23] If both $U_3^* = 0$ and $U_4^* = 0$ so that there is no marginal disutility of housework or market labour supply, then $w^* = w_H^* = w_L$, which is the Becker (1965) case.[24]

[23] Thus when $U_4^* = 0$, we also require that $w_S \geq w_L$ for the Case 2 corner solution to occur.
[24] Thus when $U_3^* = 0$ and $U_4^* = 0$, we require that $w_S \geq w_L$ for Case 2 to occur.

At the end of the previous section, we showed how the two constraint utility maximization problem (5) for Case 1 could be turned into the single constraint utility maximization problem (43), provided that we somehow knew what the equilibrium price of leisure time w^* was. A similar equivalence can be obtained for the Case 2 utility maximization problem provided that we know the equilibrium prices for both leisure time w^* and for household work time w_H^*. However, in order to do this, we need to redefine P_H^* and Q_H^*, the price and quantity of household work, which were defined earlier by (25) and (27) in the previous section. The new definitions for these variables are the following ones:

(52) $P_H^* \equiv c^H(p_H,w_H^*)$;
(53) $Q_H^* \equiv H(q_H^*,t_H^*) = H_1(q_H^*,t_H^*)q_H^* + H_2(q_H^*,t_H^*)t_H^*$.

Comparing the new definitions of P_H^* and Q_H^* with the old ones, it can be seen that we have replaced the observable market wage rate for household help w_S by the imputed price of time spent in household work w_H^* and $t_H^*+q_S^*$ has been replaced by t_H^* (since $q_S^* = 0$ for Case 2). With these new definitions, we can repeat the steps surrounding equations (38)-(42) and show that the following equations hold:

(54) $U_1[Q_F^*,Q_H^*,t_H^*,t_L^*] = \lambda^*P_F^* > 0$;
(55) $U_2[Q_F^*,Q_H^*,t_H^*,t_L^*] = \lambda^*P_H^* > 0$;
(56) $U_3[Q_F^*,Q_H^*,t_H^*,t_L^*] = -\lambda^*(w_H^* - w^*) \leq 0$;
(57) $U_4[Q_F^*,Q_H^*,t_H^*,t_L^*] = -\lambda^*(w_L - w^*) \leq 0$.
(58) $P_F^*Q_F^* + P_H^*Q_H^* - (w_H^* - w^*)t_H^* - (w_L - w^*)t_L^* = Y + w^*T \equiv F_I$.

Now consider the following single constraint utility maximization problem:

(59) $\max_{Q_F\geq 0,Q_H\geq 0,t_H\geq 0,t_L\geq 0}$ $\{U[Q_F,Q_H,t_H,t_L] : P_F^*Q_F + P_H^*Q_H - (w_H^*-w^*)t_H - (w_L-w^*)t_L \leq Y + w^*T\}$.

It can be verified that the constrained maximization problem (59) is a concave programming problem and moreover, the quantities Q_F^*,Q_H^*,t_H^*,t_L^* and the multiplier λ^* that appeared in equations (54)-(58) will be a solution to (59). Thus we have again derived a single constraint utility maximization problem (59) that is a counterpart to the two constraint utility maximization problem (5) for the Case 2 corner solution. As in the previous section, in order to define the prices and quantities that are used in (59), we need estimates for the two imputed prices of time, w^* and w_H^*.

At first sight, it might seem that the single constraint utility maximization problem (59) is of limited usefulness if we do not have a complete knowledge of the imputed prices, w^* and w_H^*. However, the real usefulness of the single constraint problem (59) is that it gives national income accountants some guidance on how to value leisure time and household work time in the System of National Accounts if a demand arises for these valuations. It can be shown[25] that the full value of household leisure services (including the value of household time inputs), $P_F^*Q_F^*$, and the full value of household work services (including time inputs), $P_H^*Q_H^*$, have the following decompositions:

(60) $P_F^*Q_F^* = p_Fq_F^* + w^*t_F^*$;

[25] Recall (32) and (37).

(61) $P_H^*Q_H^* = p_Hq_H^* + w_H^*t_H^*$.

Substituting (60) and (61) into the budget constraint (58) leads to the following equilibrium budget constraint for the household:

(62) $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w_{SI}^*t_H^* - (w_H^* - w^*)t_H^* - (w_L - w^*)t_L^* = Y + w^*T \equiv F_I$.

Thus in order to obtain the full value of leisure services, we need to add the value household leisure time $w^*t_F^*$ to the cost of market purchases of leisure type goods, $p_Fq_F^*$ and the full value of household work related activities is equal to market purchases of work related goods $p_Hq_H^*$ plus the value of household time spent in household work related activities $w_H^*t_H^*$. The disutility of household work is valued at $-(w_H^* - w^*)t_H^*$ and the disutility of external market labour supply is valued at $-(w_L - w^*)t_L^*$. The sum of these expenditures is equal to our measure of imputed full income, $Y + w^*T$.

We turn our attention to Case 3.

5. The Case of a Household that Purchases Market Labour Services but does not Supply Market Labour Services

In this section, we analyze Case 3, where the household purchases some services that can substitute for household work so that q_S^* is positive but the household does not work externally and so labour supply t_L^* is zero. Thus in this case, all equilibrium variables are assumed to be positive except we assume that $t_L^* = 0$. For the moment, we assume that the household *could* supply some labour at the wage rate $w_L > 0$, but chooses not to. The Kuhn Tucker conditions which are necessary and sufficient for $\lambda^*,\omega^*,q_F^*,q_H^*,q_S^*,t_F^*,t_H^*,t_L^* = 0$ to solve (6) under these hypotheses are (13)-(17), (19)-(20) and the following condition which replaces (18):

(63) $U_4[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,0] \leq -\lambda^* (w_L - w^*)$.

We can again substitute (16) into (17) and the resulting equation becomes:

(64) $U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*+q_S^*),t_H^*,0] = -\lambda^* (w_S - w^*) \leq 0$

where the inequality in (64) follows from the assumption that $U_3 \leq 0$. The inequality (63) does not in general constrain w^* so in this case, all we can deduce is the following implication of (64):

(65) $0 < w^* \leq w_S$.

In the case where the household is unable to offer any market labour supply due to disabilities or retirement, then we simply set t_L equal to zero in the consumer's utility maximization problem (5). In this case, the condition (63) is no longer relevant but the inequalities in (65) will still hold. Thus we end up with the same bounds on w^* for this case, no matter whether the household is capable of supplying labour services or not.

In order to derive the single constraint utility maximization problem that is equivalent to the original problem (5) and the derived problem (12) when $t_L = 0$, we note that a solution to (12) is also a solution to the following problem which is (12) except that we have set $t_L^* = 0$:

$$(66)\quad \max_{q_F\geq 0, q_H\geq 0, q_S\geq 0, t_F\geq 0, t_H\geq 0} \{U[F(q_F,t_F),H(q_H, t_H+q_S),t_H,0] : p_Fq_F + w^*t_F + p_Hq_H + w^*t_H + w_Sq_S \leq Y + w^*T\}.$$

Now we can use the analysis and definitions laid out in section 3 above except that wherever t_L^* occurs in section 3, replace t_L^* by 0. Using this analysis, it can be verified that (66) is equivalent to the following single constraint utility maximization problem:[26]

$$(67)\quad \max_{Q_F\geq 0, Q_H\geq 0, t_H\geq 0} \{U[Q_F,Q_H,t_H,0] : P_F^*Q_F + P_H^*Q_H - (w_S-w^*)t_H \leq Y + w^*T\}.$$

If $U_3^* = 0$ so that there is no disutility of household work, then we can set w^* equal to the observable wage rate for household labour, w_S, and the utility maximization problem (67) becomes an analogue to Becker's single constraint utility maximization problem except that we use the household hired labour wage rate, w_S, to value household time T in full income and in the production of household services Q_F and Q_H instead of the household labour supply after tax wage rate w_L (which is not relevant for a retired household).

We now turn our attention to Case 4.

6. The Case of a Household that does not Purchase Market Labour Services and does not Supply Market Labour Services

In this section, we analyze Case 4, the case of a frugal retired household. In this case, the household does not purchase any services that can substitute for household work so that q_S^* is zero and the household does not work externally and so labour supply t_L^* is also zero. Thus in this case, all equilibrium variables are assumed to be positive except we assume that $q_S^* = t_L^* = 0$. For the moment, we again assume that the household *could* supply some labour at the wage rate $w_L > 0$, but chooses not to. The Kuhn Tucker conditions which are necessary and sufficient for $\lambda^*,\omega^*,q_F^*,q_H^*,q_S^*,t_F^*,t_H^*,t_L^*$ to solve (6) under these hypotheses are (13)-(15), (17), (19)-(20) with $q_S^* = t_L^* = 0$ and the following inequality conditions which replace (16) and (18):

$$(68)\quad U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,t_L^*]H_2(q_H^*,t_H^*) \leq \lambda^* w_S\ ;$$
$$(69)\quad U_4[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,0] \leq -\lambda^*(w_L - w^*).$$

As in the previous section, the inequality (69) does not imply any inequality constraints on w^*. If the household is not able to offer any market labour supply, then we can simply set t_L equal to zero in the consumer's utility maximization problem (5) and hence, there will be no first order condition for this variable and so condition (69) can be dropped.

[26] Note that the household's equilibrium budget constraint in (66), $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w^*t_H^* + w_Sq_S^* = Y + w^*T$, can be rewritten as follows: $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w_S(q_S^* + t_H^*) - (w_S - w^*)t_H^* = Y + w^*T$. This last budget constraint matches up with the budget constraint in (67).

Using the first order condition (14), the *imputed price of leisure time*, w^*, satisfies the following equation:

$$(70)\ w^* = U_1[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,0]F_2(q_F^*,t_F^*)/\lambda^* > 0.$$

As in Case 2, the household's imputed price of time spent in household work can no longer be set equal to w_S, the market wage rate for hiring comparable household labour services. Thus we now define the (unobserved) *household's imputed price of time spent in household work*, w_H^*, as:

$$(71)\ w_H^* \equiv U_2[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,0]H_2(q_H^*,t_H^*)/\lambda^* > 0.$$

We can substitute (71) into (68) and the resulting inequality becomes:

$$(72)\ \lambda^*(w_S - w_H^*) \geq 0.$$

Thus the household's imputed price for household work, w_H^*, is bounded from above by its market counterpart, w_S. Now substitute definition (71) into the first order condition (17) and we obtain the following equation:

$$(73)\ U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,0] = -\lambda^*(w_H^* - w^*) \leq 0$$

where the inequality in (73) follows from the assumption that $U_3^* \leq 0$. Thus combining (72) and (73), w^* and w_H^* satisfy the following inequalities:

$$(74)\ 0 < w^* \leq w_H^* \leq w_S.$$

Note that the above inequalities are the same as the inequalities (49) that we obtained in our analysis of Case 2 above. Note also if $U_3^* \equiv U_3[F(q_F^*,t_F^*),H(q_H^*,t_H^*),t_H^*,0] = 0$ so that there is no direct disutility of household work at the observed equilibrium, then the imputed price of leisure w^* is equal to the imputed price of household work w_H^* and both of these prices are bounded from above by the market wage rate for doing household work, w_S.

We can repeat most of the algebra that was developed at the end of our analysis of Case 2, except that t_L^* is replaced by 0. The Case 2 definitions of P_F^*, P_H^*, Q_F^* and Q_H^* remain the same and equations (54) to (58) are replaced by the following equations:

$$(75)\ U_1[Q_F^*,Q_H^*,t_H^*,0] = \lambda^* P_F^* > 0\ ;$$
$$(76)\ U_2[Q_F^*,Q_H^*,t_H^*,0] = \lambda^* P_H^* > 0\ ;$$
$$(77)\ U_3[Q_F^*,Q_H^*,t_H^*,0] = -\lambda^*(w_H^* - w^*) \leq 0\ ;$$
$$(78)\ P_F^*Q_F^* + P_H^*Q_H^* - (w_H^* - w^*)t_H^* = Y + w^*T \equiv F_I.$$

Now consider the following single constraint utility maximization problem:

$$(79)\ \max_{Q_F\geq 0,Q_H\geq 0,t_H\geq 0}\ \{U[Q_F,Q_H,t_H,0] : P_F^*Q_F + P_H^*Q_H - (w_H^*-w^*)t_H \leq Y + w^*T\}.$$

It can be verified that the constrained maximization problem (79) is a concave programming problem and moreover, the quantities Q_F^*,Q_H^*,t_H^* and the multiplier λ^* that appeared in equations (75)-(78) will be a solution to (79). Thus we have again derived a single constraint utility maximization problem (79) that is a counterpart to the two constraint utility maximization problem (5) for the Case 4 corner solution. As was the case in our analysis of Case 2, in order to define the prices and quantities that are used in (79), *we need a knowledge of the two imputed prices of time,* w^* and w_H^*.

As in section 4 above, it can be shown that the full value of household leisure services (including the value of household time inputs), $P_F^*Q_F^*$, and the full value of household work services (including time inputs), $P_H^*Q_H^*$, have the decompositions (60) and (61). Substituting (60) and (61) into the budget constraint (78) leads to the following equilibrium budget constraint for the Case 4 household:[27]

(80) $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w_H^*t_H^* - (w_H^* - w^*)t_H^* = Y + w^*T \equiv F_I$.

This completes our analysis of the four cases of the household's general utility maximization problem (5) with two constraints that we have singled out for a more detailed analysis. In the following sections, we will suggest some possible methods that could be used to provide econometric estimates for the various imputed prices of household time that we have encountered in our four cases.

7. The Econometric Estimation of Preferences for Case 1: A Primal Approach

Recall the household's constrained utility maximization problem (5) in section 2 above. In section 3, we considered a special case of the general problem where the equilibrium quantities were all positive. In this case, it proves to be convenient to follow Becker's (1965) example and use the time constraint to solve for $t_L = T - t_F - t_H$ and substitute this equation into the objective function and the household budget constraint. This reduces the two constraint utility maximization problem down to the following single constraint utility maximization problem involving five decision variables rather than the six variables in (5):

$$(81)\ \max_{q_F\geq 0, q_H\geq 0, q_S\geq 0, t_F\geq 0, t_H\geq 0} \{U[F(q_F,t_F),H(q_H,q_S + t_H),t_H,T - t_F - t_H] : \\ p_Fq_F + w_Lt_F + p_Hq_H + w_Lt_H + w_Sq_S \leq Y + w_LT \equiv F_B\}.$$

Note that the utility maximization problem (81) uses Becker's definition of full income, F_B, where time is valued at the household after tax market wage rate w_L. Suppose that the household faces the positive prices p_F^τ, w_L^τ, p_H^τ, w_S^τ in period τ, spends nonlabour income Y^τ in period τ and the positive quantities q_F^τ, t_F^τ, q_H^τ, t_H^τ, q_S^τ solve the period τ utility maximization problem (81) with Kuhn Tucker multiplier $\lambda^\tau > 0$ using the period τ prices for $\tau = 1,...,\Upsilon$. We also assume that the following equations and inequalities hold:

[27] If $U(Q_F,Q_H,t_H,0) = U(Q_F,Q_H,0,0)$ so that there is no direct disutility of household work or, more generally, if $U_3^* \equiv U_3[Q_F^*,Q_H^*,t_H^*,0] = 0$, then $w^* = w_{SI}^*$ and the budget constraint (80) becomes $p_Fq_F^* + w^*t_F^* + p_Hq_H^* + w_H^*t_H^* = Y + w^*T$. This model is similar to Becker's model except that the household values its time at the unobserved price of leisure time w^* instead of the market wage rate w_L.

(82) $t_L^\tau = T - t_F^\tau - t_H^\tau > 0$; $\tau = 1,...,\Upsilon$.

The first order necessary and sufficient conditions for q_F^τ, t_F^τ, q_H^τ, t_H^τ, q_S^τ, t_H^τ, λ^τ to solve the period τ utility maximization problem (81) are as follows:

(83) $U_1^\tau F_1^\tau = \lambda^\tau p_F^\tau$;
(84) $U_1^\tau F_2^\tau - U_4^\tau = \lambda^\tau w_L^\tau$;
(85) $U_2^\tau H_1^\tau = \lambda^\tau p_H^\tau$;
(86) $U_2^\tau H_2^\tau = \lambda^\tau w_S^\tau$;
(87) $U_2^\tau H_2^\tau + U_3^\tau - U_4^\tau = \lambda^\tau w_L^\tau$;
(88) $p_F^\tau q_F^\tau + w_L^\tau t_F^\tau + p_H^\tau q_H^\tau + w_S^\tau q_S^\tau + w_L^\tau t_H^\tau = Y^\tau + w_L^\tau T \equiv F_B^\tau$

where $U_1^\tau \equiv U_1[F(q_F^\tau,t_F^\tau),H(q_H^\tau,q_S^\tau+t_H^\tau),t_H^\tau,T-t_F^\tau-t_H^\tau]$, $F_1^\tau \equiv F_1(q_F^\tau,t_F^\tau)$, $H_1^\tau \equiv H(q_H^\tau,q_S^\tau+t_H^\tau)$, etc. Multiply both sides of (83)-(87) by q_F^τ, t_F^τ, q_H^τ, q_S^τ and t_H^τ respectively and sum the resulting equations. Use this equation to solve for the marginal utility of income in period τ, λ^τ. Using the budget constraint (88), we find that:

(89) $\lambda^\tau = D^\tau/F_B^\tau$

where F_B^τ is Becker's full income for period τ and D^τ is defined as follows:

(90) $D^\tau \equiv U_1^\tau F_1^\tau q_F^\tau + [U_1^\tau F_2^\tau - U_4^\tau]t_F^\tau + U_2^\tau H_1^\tau q_H^\tau + U_2^\tau H_2^\tau q_S^\tau + [U_2^\tau H_2^\tau + U_3^\tau - U_4^\tau]t_H^\tau$
$\quad = U_1^\tau F^\tau + U_2^\tau H^\tau + U_3^\tau t_H^\tau + U_4^\tau t_L^\tau - U_4^\tau T$.

In order to derive the second equation in (90), we used equations (82) and the following definitions and identities for F^τ and H^τ:[28]

(91) $F^\tau \equiv F(q_F^\tau,t_F^\tau) = F_1^\tau q_F^\tau + F_2^\tau t_F^\tau$; $H^\tau \equiv H(q_H^\tau,q_S^\tau+t_H^\tau) = H_1^\tau q_H^\tau + H_2^\tau[q_S^\tau + t_H^\tau]$.

Return to equations (83)-(87). Multiply both sides of (83)-(87) by q_F^τ, t_F^τ, q_H^τ, q_S^τ and t_H^τ respectively. Replace λ^τ in these modified equations by the right hand side of (89), D^τ/F_B^τ, and we obtain the following system of *inverse demand functions in share form*:

(92) $p_F^\tau q_F^\tau/F_B^\tau = U_1^\tau F_1^\tau q_F^\tau/D^\tau$;
(93) $w_L^\tau t_F^\tau/F_B^\tau = [U_1^\tau F_2^\tau - U_4^\tau]t_F^\tau/D^\tau$;
(94) $p_H^\tau q_H^\tau/F_B^\tau = U_2^\tau H_1^\tau q_F^\tau/D^\tau$;
(95) $w_S^\tau q_S^\tau/F_B^\tau = U_2^\tau H_2^\tau q_S^\tau/D^\tau$;
(96) $w_L^\tau t_H^\tau/F_B^\tau = [U_2^\tau H_2^\tau + U_3^\tau - U_4^\tau]t_H^\tau/D^\tau$.

[28] The equations in (90) are the analogues to equations (26) and (27) applied to the period τ data; i.e., they follow from Euler's Theorem on homogeneous functions.

Note that the numerators on the left hand sides of (92)-(96) sum up to F_B^τ, Becker's full income for period τ. Thus for each period, the sum of the left hand sides of (92)-(96) sum up to unity (as do the right hand sides).

Equations (92)-(96) can be used as the starting point for an econometric model. Choose suitable differentiable functional forms for the "macro" utility function, $U(F,H,t_H,t_L)$, and the "micro" leisure and household work (linearly homogeneous) utility functions, $F(q_F,t_F)$ and $H(q_H,q_S + t_H)$. Calculate the partial derivatives that appear on the right hand sides of equations (92)-(96), add error terms these equations, drop any one of the resulting equations, and use nonlinear regression techniques to estimate the unknown parameters which appear in the functional forms for U, F and H.[29]

Finally, we need to indicate how the price of leisure time can be recovered from our econometric model. Recall that we denoted the price of leisure time in our general model explained in section 2 above by w^* and recall that our Case 1 model was explained in section 3 above. The first order conditions for the Case 1 model in section 3 were equations (13)-(20). In order to make these equations comparable to equations (83)-(88) in this section, we will replace q_F^*, q_H^*,q_S^*, t_F^* t_H^*, t_L^* and w^* in equations (1)-(20) by q_F^τ, q_H^τ,q_S^τ, t_F^τ t_H^τ, t_L^τ and w^τ. Thus our present task is to show how the econometric model presented in this section can generate estimates for the period τ price of leisure, w^τ.

Once the unknown parameters in the functional forms for U, F and H have been determined, the *period τ price of leisure* can be defined as follows:

$$(97)\ w^\tau \equiv U_1[F(q_F^\tau,t_F^\tau),H(q_H^\tau,q_S^\tau+t_H^\tau), t_H^\tau,t_L^\tau]F_2(q_F^\tau,t_F^\tau)/\lambda^\tau$$

where λ^τ is defined by (89). Using this definition, it can be seen that (13), (15) and (16) are equivalent to (83), (85) and (86). Using (97), $U_1^\tau F_2^\tau = \lambda^\tau w^\tau$ and this equation is equivalent to (14). Also using (97), (84) is equivalent to (18). Finally, (84) and (87) imply $U_2^\tau H_2^\tau + U_3^\tau - U_4^\tau = U_1^\tau F_2^\tau - U_4^\tau$ or $U_2^\tau H_2^\tau + U_3^\tau = U_1^\tau F_2^\tau = \lambda^\tau w^\tau$ using (97) again and so $U_2^\tau H_2^\tau + U_3^\tau = \lambda^\tau w^\tau$ which is equivalent to (17). Thus the first order conditions derived in this section are equivalent to the first order conditions for the Case 1 model derived in section 3 above.

8. The Econometric Estimation of Preferences for Case 2

Case 2 is where labour supply is positive (so that in period τ, $t^\tau > 0$) but the household does not purchase any market labour services to perform household work tasks (so that $q_S^\tau = 0$ but $t_H^\tau > 0$). This Case was considered in section 4 above.

The first order conditions for this problem are again equations (83)-(88) except (86) is dropped and q_S^t is set equal to 0. Equations (89)-(97) are still valid with $q_S^\tau \equiv 0$, except that the estimating

[29] Various cardinalizing normalizations on the utility functions U, F and G will have to be made in order to identify the remaining parameters. It should be noted that prices are regarded as the dependent variables and quantities are regarded as independent variables in this system of estimating equations. Thus it will not be easy to obtain reliable estimates of the unknown parameters in this very nonlinear and unconventional framework. However, for our present purposes, we simply want to make the point that it is not impossible to estimate our rather complex model of household behavior.

equation (95) is dropped. Thus there are only three independent estimating equations for this Case (whereas we had four independent estimating equations for Case 1).

Once the unknown parameters in the functional forms for U, F and H have been determined, the *period τ imputed price of leisure time* w^τ can be defined by (97) (with $q_S^\tau = 0$) and the *household's imputed price of time spent in household work*, w_H^τ, can be defined as follows:[30]

$$(98)\ w_H^\tau \equiv U_2[F(q_F^\tau,t_F^\tau),H(q_H^\tau,t_H^\tau),t_H^\tau,t_L^\tau]H_2(q_H^\tau,t_H^\tau)/\lambda^\tau > 0.$$

where λ^τ is defined by (89).

If information on the price of market labour w_S^τ is available during period τ, then the following Kuhn-Tucker condition should be checked once the unknown parameters in the functional forms for U, F and H have been estimated:

$$(99)\ U_2[F(q_F^\tau,t_F^\tau),H(q_H^\tau,t_H^\tau)]H_2(q_H^\tau,t_H^\tau) \le w_S^\tau\, D^\tau/F_B^\tau$$

where D^τ is defined by (90) with $q_S^\tau \equiv 0$ and F_B^τ is Becker's full income for period τ.

9. The Econometric Estimation of Preferences for Case 3

In this case, we assume that the household purchases some services that can substitute for household work so that in period τ, q_S^τ is positive but the household does not work externally and so labour supply t_L^τ is zero. Thus in this case, all equilibrium variables are assumed to be positive except we assume that $t_L^\tau = 0$. Unfortunately, the econometric estimating equations for this case are quite different from the estimating equations for the previous two cases so it will be necessary to develop some new algebra.

In this case, it proves to be convenient to use the time constraint to solve for household leisure time in terms of the total time available, T, and the amount of time spent in household work, t_H. Thus we set $t_F = T - t_H$ and substitute this equation into the objective function and the household budget constraint. Taking into account the fact that household labour supply t_L is equal to 0, this reduces the two constraint utility maximization problem down to the following period τ single constraint utility maximization problem involving four decision variables rather than the six variables in (5):

$$(100)\ \max_{q_F\ge 0,q_H\ge 0,q_S\ge 0,t_H\ge 0} \{U[F(q_F,T-t_H),H(q_H,q_S+t_H),t_H,0] : p_F^\tau q_F + p_H^\tau q_H + w_S^\tau q_S \le Y^\tau\}$$

where $Y^\tau > 0$ is the household's nonlabour income which it spends on market goods and services during period τ. Suppose that the positive quantities q_F^τ, q_H^τ, q_S^τ, t_H^τ (and $t_F^\tau = T - t_H^\tau$) solve the period τ utility maximization problem (100) with Kuhn Tucker multiplier $\lambda^\tau > 0$ using the period τ prices. The first order necessary and sufficient conditions for q_F^τ, q_H^τ, q_S^τ, t_H^τ, λ^τ to solve the period τ utility maximization problem (100) are as follows:

[30] This is the period τ counterpart to definition (46) above.

$$(101)\ U_1^\tau F_1^\tau = \lambda^\tau p_F^\tau ;$$
$$(102)\ U_2^\tau H_1^\tau = \lambda^\tau p_H^\tau ;$$
$$(103)\ U_2^\tau H_2^\tau = \lambda^\tau w_S^\tau ;$$
$$(104)\ U_2^\tau H_2^\tau - U_1^\tau F_2^\tau + U_3^\tau = 0 ;$$
$$(105)\ p_F^\tau q_F^\tau + p_H^\tau q_H^\tau + w_S^\tau q_S^\tau = Y^\tau$$

where $U_1^\tau \equiv U_1[F(q_F^\tau,t_F^\tau),H(q_H^\tau,q_S^\tau+t_H^\tau),t_H^\tau,0]$, $F_1^\tau \equiv F_1(q_F^\tau,t_F^\tau)$, $H_1^\tau \equiv H(q_H^\tau,q_S^\tau+t_H^\tau)$, etc. Substitute (103) into (104) and we obtain the following equation:

$$(106)\ U_3^\tau - U_1^\tau F_2^\tau = -\lambda^\tau w_S^\tau.$$

Multiply both sides of (101)-(103) and (106) by q_F^τ, q_H^τ, $q_S^\tau + t_H^\tau$ and t_H^τ respectively and sum the resulting equations. Use this equation to solve for the marginal utility of income in period τ, λ^τ. Using the budget constraint (105), we find that:

$$(107)\ \lambda^\tau = E^\tau/Y^\tau$$

where Y^τ is nonlabour income for period τ and E^τ is defined as follows:[31]

$$\begin{aligned}(108)\ E^\tau &\equiv U_1^\tau F_1^\tau q_F^\tau + U_2^\tau H_1^\tau q_H^\tau + U_2^\tau H_2^\tau[q_S^\tau + t_H^\tau] + U_3^\tau t_H^\tau - U_1^\tau F_2^\tau t_H^\tau \\ &= U_1^\tau F_1^\tau q_F^\tau + U_2^\tau H_1^\tau q_H^\tau + U_2^\tau H_2^\tau[q_S^\tau + t_H^\tau] + U_3^\tau t_H^\tau - U_1^\tau F_2^\tau[T - t_F^\tau] \\ &= U_1^\tau F^\tau + U_2^\tau H^\tau + U_3^\tau t_H^\tau - U_1^\tau F_2^\tau T.\end{aligned}$$

Return to equations (101)-(103) and (106). Multiply both sides of these equations by q_F^τ, q_H^τ, $q_S^\tau + t_H^\tau$ and t_H^τ respectively. Replace λ^τ in these modified equations by the right hand side of (107), E^τ/Y^τ, and we obtain the following system of *inverse demand functions in share form*:

$$(109)\ p_F^\tau q_F^\tau/Y^\tau = U_1^\tau F_1^\tau q_F^\tau/E^\tau ;$$
$$(110)\ p_H^\tau q_H^\tau/Y^\tau = U_2^\tau H_1^\tau q_H^\tau/E^\tau ;$$
$$(111)\ w_S^\tau[q_S^\tau + t_H^\tau]/Y^\tau = U_2^\tau H_2^\tau[q_S^\tau + t_H^\tau]/E^\tau ;$$
$$(112)\ -w_S^\tau t_H^\tau/Y^\tau = [U_3^\tau - U_1^\tau F_2^\tau]t_H^\tau/E^\tau.$$

Note that the numerators on the left hand sides of (109)-(112) sum up to Y^τ, nonlabour income for period τ. Thus for each period, the sum of the left hand sides of (109)-(112) sum up to unity (as do the right hand sides). Thus only three of the four share equations, (109)-(112), are independent and can be used as estimating equations.

Now choose suitable differentiable functional forms for the "macro" utility function, $U(F,H,t_H,0)$, and the "micro" leisure and household work (linearly homogeneous) utility functions, $F(q_F,t_F)$ and $H(q_H,q_S+t_H)$. Calculate the partial derivatives that appear on the right hand sides of equations (109)-(112), add error terms these equations, drop any one of the resulting equations, and use nonlinear regression techniques to estimate the unknown parameters which appear in the functional forms for U, F and H.

[31] We used equations (90) and $t_F^\tau = T - t_H^t$ to derive the equations in (108).

Once the unknown parameters in the functional forms for U, F and H have been determined, the *period τ price of leisure* w^τ can be defined as follows:[32]

$$(113)\ w^\tau \equiv U_1[F(q_F{}^\tau,t_F{}^\tau),H(q_H{}^\tau,q_S{}^\tau+t_H{}^\tau),\, t_H{}^\tau,0]F_2(q_F{}^\tau,t_F{}^\tau)/\lambda^\tau$$

where λ^τ is defined by (107) above.

10. The Econometric Estimation of Preferences for Case 4

In this case, we assume that the household does not offer any labour services and does not purchase any market services that can substitute for household work so that in period τ, $q_S{}^\tau = 0$ and $t_L{}^\tau = 0$. The econometric estimating equations for this case are somewhat different from the estimating equations for the previous cases so it will be necessary to develop some new algebra.

In this case, it again proves to be convenient to use the time constraint to solve for household leisure time in terms of the total time available, T, and the amount of time spent in household work, t_H. Thus we set $t_F = T - t_H$ and substitute this equation into the objective function and the household budget constraint. Taking into account the fact that household labour supply t_L and purchases of market labour services q_S are equal to 0, this reduces the two constraint utility maximization problem down to the following period τ single constraint utility maximization problem involving three decision variables rather than the six variables in (5):

$$(114)\ \max_{q_F \geq 0, q_H \geq 0, t_H \geq 0} \{U[F(q_F,T - t_H),H(q_H,t_H),t_H,0] : p_F{}^\tau q_F + p_H{}^\tau q_H \leq Y^\tau\}$$

where $Y^\tau > 0$ is the household's nonlabour income which it spends on market goods and services during period τ. Suppose that the positive quantities $q_F{}^\tau$, $q_H{}^\tau$, $t_H{}^\tau$ (and $t_F{}^\tau = T - t_H{}^\tau$) solve the period τ utility maximization problem (114) with Kuhn Tucker multiplier $\lambda^\tau > 0$ using the period τ prices. The first order necessary and sufficient conditions for $q_F{}^\tau$, $q_H{}^\tau$, $t_H{}^\tau$, λ^τ to solve the period τ utility maximization problem (114) under our regularity conditions are as follows:

$$(115)\ U_1{}^\tau F_1{}^\tau = \lambda^\tau p_F{}^\tau\ ;$$
$$(116)\ U_2{}^\tau H_1{}^\tau = \lambda^\tau p_H{}^\tau\ ;$$
$$(117)\ U_2{}^\tau H_2{}^\tau - U_1{}^\tau F_2{}^\tau + U_3{}^\tau = 0\ ;$$
$$(118)\ p_F{}^\tau q_F{}^\tau + p_H{}^\tau q_H{}^\tau = Y^\tau$$

where $U_1{}^\tau \equiv U_1[F(q_F{}^\tau,t_F{}^\tau),H(q_H{}^\tau,t_H{}^\tau),t_H{}^\tau,0]$, $F_1{}^\tau \equiv F_1(q_F{}^\tau,t_F{}^\tau)$, $H_1{}^\tau \equiv H(q_H{}^\tau,t_H{}^\tau)$, etc. Multiply both sides of (115) and (116) by $q_F{}^\tau$ and $q_H{}^\tau$ respectively, multiply both sides of (117) by $t_H{}^t$ and sum the resulting equations. Use the resulting equation to solve for the marginal utility of income in period τ, λ^τ. Using the budget constraint (118), we find that:

$$(119)\ \lambda^\tau = E^\tau/Y^\tau$$

[32] It should be the case that $w^\tau \leq w_S{}^\tau$ as is required under our assumptions.

where Y^τ is nonlabour income for period τ and E^τ is defined by (108) where $q_S^\tau \equiv 0$.

Return to equations (115)-(117). Multiply both sides of these equations by q_F^τ, q_H^τ and t_H^τ/E^τ respectively. Replace λ^τ in the modified equations (115) and (116) by the right hand side of (119), E^τ/Y^τ, and we obtain the following system of potential estimating equations:

(120) $p_F^\tau q_F^\tau/Y^\tau = U_1^\tau F_1^\tau q_F^\tau/E^\tau$;
(121) $p_H^\tau q_H^\tau/Y^\tau = U_2^\tau H_1^\tau q_H^\tau/E^\tau$;
(122) $0 = [U_2^\tau H_2^\tau - U_1^\tau F_2^\tau + U_3^\tau]/E^\tau$.

Note that the left hand sides of (120)-(121) sum up to unity, using (118), and the right hand sides of (120)-(121) also sum to unity using definition (108) with $q_S^\tau \equiv 0$. Thus only two of these three equations are independent and can be used as estimating equations. Equation (122) is the obvious equation that should be dropped.[33]

Now choose suitable differentiable functional forms for the "macro" utility function, $U(F,H,t_H,0)$, and the "micro" leisure and household work (linearly homogeneous) utility functions, $F(q_F,t_F)$ and $H(q_H,t_H)$. Calculate the partial derivatives that appear on the right hand sides of equations (120) and (121), add error terms these equations, and use nonlinear regression techniques to estimate the unknown parameters which appear in the functional forms for U, F and H.

Once the unknown parameters in the functional forms for U, F and H have been determined, the *period τ price of leisure time* w^τ can be defined as follows:

(123) $w^\tau \equiv U_1[F(q_F^\tau,t_F^\tau),H(q_H^\tau,q_S^\tau+t_H^\tau), t_H^\tau,0]F_2(q_F^\tau,t_F^\tau)/\lambda^\tau$

where λ^τ is defined by (119) above. Similarly, the *household's imputed price of time spent performing household work,* w_H^τ, can be defined as follows:[34]

(124) $w_H^\tau \equiv U_2[F(q_F^\tau,t_F^\tau),H(q_H^\tau,t_H^\tau),t_H^\tau,0]H_2(q_H^\tau,t_H^\tau)/\lambda^\tau > 0$.

The price w^τ is the "correct" price to value household time t_F^τ spent on leisure type activities during period τ and the price w_H^τ is the "correct" price to value household time spent doing housework activities during the period. Thus the household's full consumption valuation of leisure and household work activities in period τ is equal to $p_F^\tau q_F^\tau + w^\tau t_F^\tau + p_H^\tau q_H^\tau + w_H^\tau t_H^\tau$, which in turn is equal to $P_F^\tau Q_F^\tau + P_H^\tau Q_H^\tau$. However, as was explained in section 3, these full consumption values do not include an adjustment for the direct disutility of household work t_H. Making this adjustment leads to the addition of the nonpositive *disutility term* $-(w_H^\tau - w^\tau)t_H^\tau$ to full consumption. Full

[33] This is a rather unusual estimating equation to say the least! However, our model of utility maximizing behavior implies that this equation should hold. Equation (122) implies that $U_3^\tau = U_1^\tau F_2^\tau - U_2^\tau H_2^\tau = \lambda^\tau w^\tau - \lambda^\tau w_H^\tau$ where w^τ is the household's period τ imputed price of leisure time and w_H^τ is the corresponding imputed value of time spent doing household work. This equation ensures that household time is properly allocated among the two competing uses.

[34] If information w_S^τ is available on the relevant period τ market wage rate for purchased household labour services, then we need to check that the Kuhn-Tucker condition (44) is satisfied; i.e., we need to check that $w_H^\tau \leq w_S^\tau$.

consumption plus the direct disutility of household work adds up to our concept of full income, $F_I^\tau \equiv Y^\tau + w^\tau T$.

There will be many econometric challenges in attempting to estimate consumer preferences in this case. Hopefully, the analysis presented in this section (and in the previous 3 sections) will stimulate some interest in addressing these econometric problems.

Until econometric estimates of the imputed price of leisure and household working time are available, we will have to make some guesses to value household time in this Case. Perhaps the best that can be done under these circumstances is to postulate that there is no "extra" disutility of household work so that $U(F(q_F,t_F),H(q_H,t_H),t_H)$ becomes the simpler utility function, $U(F(q_F,t_F),H(q_H,t_H))$. Under this assumption, the imputed value of an hour of household leisure time w will be equal to the imputed value of household work time w_H. We know that w_H must be equal to or less than the corresponding market wage for the provision of household work services, w_S, so the national income accountant should simply make a guess that the household price of time $w = w_H$ is equal to some fraction of the corresponding market wage rate w_S.

11. Conclusion

Our paper is basically a generalization of Becker's (1965) classic paper on the allocation of household time between competing uses. Becker made two simplifying assumptions which we relax in this paper:

- The household provides market labour supply and the marginal wage rate provides Becker's valuation of household time.
- There is no direct disutility of household work and no direct disutility of providing market labour services.

Relaxing these assumptions leads to a much richer theoretical framework but estimating preferences in this more general framework will be much more challenging. Some of the advantages of our more general approach are as follows:

- Our more general framework can deal with households who are unable or unwilling to provide market labour services.
- Our approach attempts to reconcile two separate approaches to the valuation of household time: Becker's approach which uses the household's market wage rate to value household time and the approach used by national income accountants which values time doing household chores at the wage rates applicable for hired household help.
- Our approach finds that corner solutions are very probable and so that in general, there will be no single rule that always provides the correct valuation for household time. We analyzed four cases in some detail and found different valuation rules for each case.

There are some significant limitations of our analysis that should be addressed in future research:

- Our household had only one individual in it.
- Our model is highly aggregated.

- We assumed that household work undertaken by the household is a perfect substitute for hired household help.
- We assumed that there was no direct positive utility from undertaking household work or providing market labour services. This assumption may or may not be true.
- Our suggested econometric frameworks were based on the specification of primal utility functions. It would be useful to develop dual characterizations of our 4 models.

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