Sub-national PPPs: Methodology and Application by using CPI data

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Sub-national PPPs: Methodology and Application by using CPI data

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1. Introduction[1]

The need for constructing sub-national Purchasing Power Parities (PPPs) to carry out analyses on inter-area price level, living standards and real income comparisons in the context of poverty, rural-urban and regional (local) differences has been discussed since long time in literature (see for example Kokoski, 1991, Moulton, 1995, Bretell and Gardiner, 2002).
However, while considerable resources have been spent by the international and national statistical agencies on calculating PPPs rates between countries, the issue of intra-national PPPs has received much less attention.
In surveying international practice, it can be seen that an increasing number of scholars are conducting interesting experimental estimations of PPPs within country, but only few countries produce official indexes of spatial prices and without any international framework guiding the construction of PPPs, both from a methodological and an empirical point of view.
In 2010, the need of constructing sub-national PPPs in the framework of the International Comparison Program (ICP) and using the CPI data has been stressed and discussed during the 2nd Meeting of the Technical Advisory Group (TAG) of the ICP, also with the aim of implementing an international project in this field (Biggeri et al, 2010; McCarthy, 2010).

As we will illustrate in detail in the following section, all the analyses of available data and the experiments undertaken until now have highlighted significant differences in the level of prices and the cost of living within countries, both for large and medium size countries, reinforcing the need for the computation of sub-national PPPs. In any case, the main issues to face refer to the characteristics of data available and to the choice of the method to estimates the PPPs. Up to now, two main general approaches for computing sub-national PPPs has been proposed which are based, respectively, on:

(i) the consumer prices collected from retail outlets throughout the country for the compilation of the Consumer Price Indices (CPI), and the use of the GEKS method or the methods belonging to the family of the Country Product Dummy (CPD) models;
(ii) data collected using sample surveys on household consumption expenditure (Expenditure Consumer Surveys) and the adoption of a demand system based methodology.

In this context, the aim of this paper is twofold. First, to review the proposals and the experiments carried out in order to develop a framework for evaluating and addressing the problems of data collection and of the choice of the methods for computing and implementing temporally consistent spatial indexes. Second, to explore characteristics and reliability of PPPs estimated by using different CPD models and by carrying out some experiments on Italian data concerning the 20 regional chief towns.

[1] The authors wish to thank Federico Polidoro and Stefania Occhiobello working at the Italian Statistical Institute (Istat) for their support and assistance in interpreting and examining the price data collected for CPI computations and used for constructing spatial price indexes.

Working in the line of the ICP, and thinking to a general project under its umbrella, this paper focuses on the construction of spatial indices using CPI data. Indeed this kind of data may be readily available as every National Statistical Offices (NSOs) conducts on a regular basis the collection of price data with high frequency due to their important role in conjuntural analysis. This should allow to save resources by the requested integration between PPP and CPI construction (Rao, 2001; Biggeri and Laureti, 2009) and to compute sub-national PPPs and national and international PPPs in a consistent framework.
An important general issue on which various authors have focused (Hill and Hill, 2009; Hill and Syed, 2014) is that of obtaining reliable PPPs at the basic heading (BH) level. Thus, the role of the CPD method for constructing intra-national PPPs at the lowest level of aggregation is analysed and discussed in this paper. Specific issues which arise in the context of sub-national PPP construction are examined. In particular, we focus on the need to take into account the spatial autocorrelation among price relatives (Aten, 1996; Rao, 2004) that surely will be more important among the areas within a country than among countries. Bearing in mind the importance of use of CPI data, the various CPD model specifications, based on average and individual price data and including or not quality characteristics, are directly compared both on a methodological and on empirical point of view.
In order to evaluate the reliability of the sub-national PPPs the statistical uncertainty associated with the point estimates are presented and discussed.

The remainder of this paper is structured as follows. Section 2 reviews the proposals and the experiments carried out for constructing intra-national PPPs with the aim of obtaining some guidelines for future works, which will be briefly illustrated in Section3. Section 4 deals with the methodological and empirical problems related to the estimation of sub-national PPPs using CPD methods at the lowest level of aggregation where information on expenditures or quantities is not available.
In Section 5 five different CPD models are directly compared by using CPI ISTAT data for the estimation of PPP for the BH preserved milk and other products. The results obtained are presented and discussed. Section 6 provides some concluding remarks.

2. Review of proposals and experiments conducted for the computation of sub-national PPPs

In surveying works on the construction of sub-national spatial prices indexes, taking into account the aim of this paper, it is convenient to start with the official computation and dissemination of those indexes (i.e. the works carried out by the National Statistical Agencies).

Not-withstanding the first official measure of inter-area differences in the cost of living was developed in the 1940s in the US, using the standard budget of the Family Budget program of the Bureau of Labor Statistics, up to now only few countries produce official indexes of spatial prices or have carried out experiments to do it (US, Australia, UK and Italy).

In the US, the inter-area price level comparisons reached an interesting development in the mid-1990s as a result of the studies conducted by various researchers, belonging essentially to the Bureau of Labor Statistics and the Bureau of Economic Analysis (BEA), see for example Kokoski, 1991 and Kokoski, Moulton and Zieschang, 1999. Then further developments has been carried out in the mid of 2000s with many research conducted by Aten (see for example, 2005 and 2006). She has continued to work by improving researches in this field both as number of products and areas considered, arriving to compute Regional Price Partities (RPPs) for 50 states and the District of

Columbia and for 366 metropolitan areas. In the last years, BEA disseminated currently data on regional price parities and real personal income for States and Metropolitan Areas (Aten, Figueroa and Martin, 2014). These works used detailed data on prices collected for the CPI and data on the expenditure weights taken from the Consumer Expenditure Survey. The analysis of the characteristics of data and, above all, the methods of computation of PPPs at product level, Basic Headings (BH) and aggregated commodity group level were improved in the last decennium, up to include the measures in an economic approach framework (Aten and Reinsdorf, 2010). The RPPs compare the average price level of an area with the national average price level for all the areas. For the compilation both the GEKS and various kind and modified versions of CPD methods have been subsequently applied.

The UK Office for National Statistics (ONS) carried out in 2000 a one-off exercise to produce indicative figures on the variation in prices between regions. Then ONS published estimates of regional price level comparisons in 2004 for the year 2003. These first two exercises were done as a by-product of a survey conducted to provide data for Eurostat's Purchasing Power Parities (PPP) programme (Fenwick and O'Donoghue, 2003). In 2005, the ONS produced relative regional consumer price levels based on CPI price data (mainly food items, tobacco and drinks) supplemented with administrative data sources and, above all, with a purpose-designed regional price level survey for items of expenditure where suitable data was not available (mainly clothing, furniture, electrical goods and travel), as explained in Wingfield, Fenwick and Smith, 2005.
Then ONS produced Relative Regional Consumer Price Levels (RRCPLs) for 2010 with more adequate procedure (ONS, 2011) using data on price observations from the existing monthly CPI collection and regional price survey conducted for the computation of the Spatial Adjustment Factors (SAFs) requested by Eurostat-OECD PPP program. The methodology adopted by ONS in constructing RRCPLs is consistent with the approach used by Eurostat in the calculation of the PPPs for the mentioned program.

The Australian Bureau of Statistics (ABS) worked to compile and disseminate experimental indexes on living costs in the 8 capital cities using existing price data collected for CPIs and calculated spatial price using GEKS (Waschka, Milne, Khoo, Quirey and Zhao, 2003). ABS continued to compute and disseminate this kind of information, every time call for caution in the interpretation of the results.
In 2011, as written in the June CPI release, the ABS discontinued the publication of the Average Retail Prices of Selected Items in Eight Capital Cities.

The Italian Statistical Institute (ISTAT) conducted one experiment for the computation of regional PPPs. ISTAT started a research project in 2005 aimed at testing the possibility of using and integrating the statistical information currently supplied within the CPI surveys and only refers to some expenditure divisions: Food and beverages, Clothing and footwear and Furniture for the home, which represent about 34% of the total consumer expenditures.
In order to calculate sub-national PPPs for consumer prices for *20 cities* (the regional capitals), the procedure for international comparison, following the principle of strict comparability of the products, has been adopted. For this reason, a complicated analysis of data collected for CPIs was carried out to check if the characteristics of products included in that list were the same in the different cities. Regarding Food and beverage products ISTAT was able to use the CPI data (of about 1,300 products) after having achieved comparability; while for Clothing and Furniture products it was necessary to carry out an *ad hoc survey* in order to collect new data for strictly comparable products in the 20 cities. The computation of PPPs were made for the BHs of each mentioned division of expenditure and the results referring to year 2009 were published in 2010 (ISTAT, 2010).

Due to the cost of ad hoc survey, ISTAT has started to implement a new strategy to compute sub-national PPPs using essentially existing CPI data.

We have to mention that Statistics New Zealand (SNZ) has also investigated the possibility of carrying out spatial comparisons of prices since 2005. In fact the SNZ asked two experts to develop a methodology for constructing Spatial Cost of Living Indexes. The report prepared by the experts (Melser and Hill, 2005) is very exhaustive facing all the problems and giving solutions or suggestions both conceptual and practical for the construction of sub-national PPPs and can surely be considered a reference text for their implementation in different countries as well. However, to the authors' knowledge, SNZ has never disseminated data on sub national PPPs.

Besides the national statistical agencies, many researchers have conducted studies and experiments to estimate sub-national PPPs. Usually, the studies have been implemented through the cooperation or agreement with national statistical agency and/or international agencies that provided data for the experiments.
As far as these estimations of sub-national PPPs are concerned, in surveying international literature it can be seen that these works are referred essentially to large countries, such as Brazil, India, China and so on, where the differences in the standards of living and level of prices are likely to be substantial. Aside from differential costs incurred in transporting goods, the differences between regions are likely higher in countries with historically different regional patterns of development. However, it is obvious that the differences in intra country levels of prices can also be due to the differences in terms of climate and tastes and preferences.

As we have already mentioned, in these works two main general methods of computation of sub-national PPPs has been proposed, which are based, respectively, on:
1. the consumer prices collected from retail outlets throughout the country for the compilation of the Consumer Price Indices (CPI), and the use of the GEKS method or the methods that belonging to the family of the Country Product Dummy (CPD) methods;
2. data collected using sample surveys on household consumption expenditure (Expenditure Consumer Surveys) and adoption of a demand system based methodology.

Mentioning here only the most quoted papers, in the first group it is important to refer to the work developed by Aten (1999) using the CPD method for estimating the 1987 price levels on food products and the general price level in 10 cities in Brazil. At that time regional prices statistics were not widely available. She discussed the practices for price collection and examined three alternative index number methods for the construction of the spatial regional indexes: the Geary, Gini-Elteto-Koves-Szulc (GEKS) and Fisher averages methods. A first analysis was based on food prices only. In a second analysis the price of services were included to test if they affect the price levels in any systematic manner.

A study to compute regional purchasing power parities of China in the year 2002 has been conducted by three authors (Xiumin Li, Zhang and Du, 2005). The study was based on average prices data from 31 cities and districts within the provinces, and Shanghai was taken as the base (norm) province. The comparison study was based on the average prices on 127 goods and services across China. The computation carried out were very simple. The authors used two methods, namely Loven's formula that calculates the two weighted average values (prices) and Paasche's price formula that creates the indices. The final PPP is obtained from calculating the weight mean value of each subclass's PPP.

Other experimental projects have been conducted along the research lines of the international agencies and/or stimulated by them.

Eurostat have implemented the discussion on the topic in several meeting of its Working groups and supported the experiments in some countries as we have seen above for UK, NSO, enlarging regional price survey conducted for the computation of the Spatial Adjustment Factors (SAFs) requested by Eurostat-OECD PPP program.
The World Bank and Asian Development Bank developed a specific experiment in the Philippines (McCarthy, 2010; Dikhanov, Palanyandy and Capilit, 2011). This is a very important experiment because aims at analysing the plausibility of integrating ICP with the Philippines' CPI by computing subnational PPPs using regional prices and expenditure weights from the CPI. The study also aims at finding out whether prices collected for the CPI could be used to provide reliable estimates of price levels for a range of products in each region and to verify if the results are consistent with the information coming from the ICP process. The paper explains all the phases implemented for the construction of the regional PPPs pointing out the main issues and the ways to solve them, both for the data preparation and the methods of index computation and aggregation. The CPD method was used in elementary aggregation and the chain Tornkqvist index was adopted for the higher level aggregation. The work showed that the subnational price levels obtained from both the CPI and ICP processes are broadly similar.

The second approach to estimate the consumer PPPs using a demand system model applied to data collected with sample surveys on household consumption expenditure, acquired importance also for the estimation of regional PPPs with the work done by Coondoo, Ray and Majumder (2004), which presented an illustrative evidence for India. The study was carried out for a subset of 45 food items and 25 states. The Authors used a regression based procedure for the estimation of multilateral consumer price indexes using micro-level data (e.g. the household level data on commodity-prices/unit values). As illustrated in the paper, first, the authors specified and estimated the set of item-specific price equations using the pooled data for all regions and including in the regression a set of regional dummy variables for each equation. Second, the set of multilateral consumer price indexes are obtained , again using regression analysis, by relating the slope and intercepts terms of the estimated item-specific price equations. The Authors explained that their suggested procedure belongs, in a generic sense, to the CPD methodology as the price equations essentially shares the hedonic feature, with few basic differences.
Majumder, Ray and Sinha have continued to work on using this approach to compute the regional PPPs for India and also for Indonesia and Vietnam, enlarging and improving the analyses up to propose a unified framework for the estimation of intra and inter country purchasing power parities, and obtaining very interesting results (Majumder, Ray and Sinha, 2013, 2013 and 2014).

The approach followed by the authors is very interesting and it is important to verify its possible extension to other countries and to check the validity and kind of the results obtained comparing them with the results provided by the experiments done using other methods of computation.

However, taking into account the aims of this paper, from here on we will focus only on the results of the experiments conducted using the CPI data and the methods to compute the sub-national PPPs in lines with those used by the ICP and EUROSTAT-OECD Program. The availability, in most of the countries in the world, of price data collected, frequently on monthly basis, to compute CPI constrains to try to use these data also to compute the sub-national PPPs. This could allow both to save resources, as the ICP surveys could reduce the collection of data, and to obtain coherent sub-national and national PPPs.

3. The definition of an international framework to compute sub-national PPPs: the main issues for data preparation and for the choice of the methods of computation

From the national and international experiments examined, we can pick up some suggestions to prepare an international project to compute consumer sub-national PPPs based on the same general framework.

The main issues for the implementation of a project to construct the above PPPs using CPI and consumption expenditures data, as for inter country PPPs, concern (Biggeri et al, 2010):

(i) data preparation;
(ii) aggregation at the Basic Heading (BH) Level
(iii) aggregation above the BH level.

At the national level, the need of carrying out spatial price comparisons across sub-regions cannot be understated as they are essential both for research analysis and social-economic political interventions. Much research has been focused on exploring the possibility of using, as much as possible, existing price data collected through the national price surveys conducted as a part of the CPI computations.
Obviously, the estimates of regional price levels or changes are not a natural by-product of the CPI computation. However, although the CPI survey is not designed as an inter-area survey, it is possible to use its price observations and expenditure (or sampling) weights to obtain sub-national PPPs.
As the main purposes of the CPIs and spatial PPPs are quite different, the price data available from the CPI sources are usually not in a form suitable for spatial price comparisons. Though the issues of *comparability* and *representativity* are likely to be less serious in the context of sub-national PPPs, they need to be considered carefully in making use of the CPI data. However products collected for CPI may neither be comparable nor representative across different regions especially if the countries are large or diverse in terms of climate and tastes and preferences. On the other end, the current ICP methodology is not designed to make an efficient use of CPI price data if the items are not comparable within all the regions.
Therefore the first and most important stage in constructing PPPs is the data preparation based on CPI data and other existing data at country level.
In order to identify the most suitable methodology that allows using existing data in an efficient way, detailed information on the available data and related database must be provided. Specific information may be required depending on the objective of the PPPs computation, which can be carried out for the region-to-region and/or rural-urban comparisons.
The main information and technical requirements needed for the data preparation phase can be specified separately for CPI structure and price collection and for the estimation of the expenditure weights, along with the data that would be needed.
To this end, it is necessary to have both the prices and expenditure weights databases. Information on data structure and data descriptors of the databases is required.
Firstly it is necessary to have the prices database at a very detailed level, which include elementary price data collected for each product in each outlet.
If this kind of database with detailed data is not available, a less detailed database which considers the average prices (among outlets), of each product for each territorial area, can be used.
Moreover an analysis of the variability of territorial area price levels and of the relationships between product prices across product groups should be carry out.

A matrix of prices by products and territorial area must be prepared and the classification of products must be as much as possible comparable with that used at level of the ICP Basic Heading. Secondly, the matrix of expenditure weights at detailed level by products and territorial areas used for the compilation of CPIs is needed. Also in this case it is necessary to verify the correspondence of the system of weights both with the national accounts household data and the expenditure weights used for the aggregation above the BH level in the ICP exercise. It is worth nothing that the system of expenditure weights used for the CPI may be not split by territorial areas.
At the end of these processes the matrix of prices will be, almost surely, not complete and a lot of cells will not contain data due to the lack of identical or similar products in all the territorial areas. To compute the Sub-National PPPs at BH level it is necessary to impute the missing data, identifying the most appropriate method.

The previous suggestions are quite general but they can be specified and clarified only after many experiments.
In this paper we present a first experiment that we are carrying out in the context of a scientific cooperation with ISTAT. The issues which characterized the data preparation phase have been faced essentially by ISTAT researchers, while we focused on the methods for the computation of the sub-national PPPs.
Therefore, before presenting our experiment, the role of the CPD method for constructing reliable intra-national PPPs at the lowest level of aggregation (BH) has to be analysed and discussed in the following section. Specific advantages and issues which arise in the context of sub-national PPP construction must be also examined. In particular, we focus on the need to take into account the spatial autocorrelation among price relatives that surely will be more important among the areas within a country than among countries. Bearing in mind the importance of using CPI data, the various CPD model specifications, based on average and individual price data and including or not quality characteristics, are directly compared both on a methodological and on empirical point of view. Indeed, one of the concern of this paper is with regression-based parity estimators that incorporate quality adjustments for non-comparable products, primarily through the use of a hedonic framework (Silver, 2009).

4. The CPD methods for estimating sub-national PPPs

4.1 The CDP model using average prices: unweighted *vs* weighted approach

The most widely used statistical approach to the multilateral aggregation of prices at the first stage of aggregation is the Country Product Dummy (CPD) method, proposed by Summers (1973). This method for making international comparisons of prices can be viewed as a very simple type of hedonic regression model where the only characteristic of the commodity is the commodity itself[2]. The CPD method can also be viewed as an example of the stochastic approach to index numbers (Diewert, 2010).

Suppose that we are attempting to make an intra-national comparison of prices between R regions at the Basic Heading (BH) level (no expenditure weights are available for the price comparisons). For each BH let $\overline{p}_{nr}$ denote the average price of item *n* in region *R* (*n=1,2,...N; r=1,2,...,R*).

[2] In the 2005 ICP round it has been the recommended method of aggregation below the basic heading level. The CPD model has gained popularity as an aggregation method for aggregation above the basic heading level (see Rao, 2004, 2005; Diewert, 2005). Hill and Syed (2010) used detailed price data, provided by the Word Bank that also funding the project, with the aim of analysing how the CPD could improve PPP estimates in the context of the ICP.

The basic statistical model that is assumed is:

$$\bar{p}_{nr} = \alpha_r \beta_n u_{nr} \qquad (1)$$

where the α_r and β_n are unknown parameters to be estimated and u_{nr} are independently and identically distributed random variables. The parameter α_r is to be interpreted as the average level of prices (over all items in this group of items) in region *r* relative to other regions, and the parameter β_n is to be interpreted as the average (over all regions) multiplicative premium that item *n* is worth relative to an average item in this grouping of items.
Therefore, α_r are the basic heading region price levels that we want to determine while β_n are item or individual product effects. It is usually assumed that the disturbances are log-normally distributed or that $\ln u_{nr}$'s are normally distributed with mean 0 and a constant variance σ^2 .

Taking logarithms of both sides of (1) leads to a linear model:

$$\begin{aligned} \ln \bar{p}_{nr} &= \ln \alpha_r + \ln \beta_n + \ln u_{nr} \\ &= a_r + b_n + v_{nr} \end{aligned} \qquad (2)$$

where v_{nr} are random disturbance terms which are independently and identically (normally) distributed with zero mean and variance σ^2. The CPD model can be seen as a simple fixed effects model where region-effects provide estimates of purchasing power parities and commodity-specific effects provide estimates of national prices (Rao, 2004). A major advantage of the method is that standard errors are obtained for the parity estimates.
Thus model (2) can be written as

$$\ln \bar{p}_{nr} = \sum_{r=1}^{R} a_r D_r + \sum_{n=1}^{N} b_n D_n^* + v_{nr} \qquad (3)$$

where D_r (r=1,2,…,R) and D_n^* are respectively country and commodity dummy variables.

The intra-national PPP for region r is given by:

$$PPP_r = \exp(a_r) \qquad (4)$$

Usually, price observations used are annual average price for the commodity *n*. Recently, Dikhanov et al (2011) suggested a combined spatial-temporal model, called Country-Time-Product-Dummy method, to use quarterly CPI data.
Thus far in ICP the average price used has been the arithmetic mean. However, various authors have argued that the geometric means are better as they rely on weaker assumptions (Rao, 2004, Hill and Syed, 2014). It is possible to express it as the average of logarithm of price quotes:

$$\ln \bar{p}_{nr} = \frac{1}{K_{nr}} \sum_{k=1}^{K_{nr}} \ln p_{nrk} \qquad (5)$$

The CPD formulation (3) is unweighted and is estimated using OLS, which implies giving equal weight to the average prices of all regions. However, the various price observations may have different levels of reliability. Rao (2004) suggested that under the assumption that individual price quotes are independently and identically distributed with variance σ^2 a natural choice of weights

for use in Weighted Least Square (WLS) is simply $w_{nr}=K_{nr}$ as the CPD model (3) based on (5) will have heteroskedastic disturbances given that:

$$Var\left(\overline{p}_{nr}\right)=\frac{1}{K_{nr}}\sigma^2$$

Diewert (2005) and Rao (2002, 2004) have shown that specific weights used in weighted–least-squares (WLS) estimators for CPD parity estimates correspond to specific index number formulas.
As underlined by Silver (2009) the use of the CPD method as formulated in equation (3) suffers from both the use of grouped data and the inability to included quality variations[3].
Hill and Syed (2014) suggested to measure reliability by the variance of the log average prices which in the case of geometric mean can be calculated as follows:

$$\text{var}\left(\ln \overline{p}_{nr}\right)=\frac{\text{var}\left(\ln p_{nrk}\right)}{K_{nr}}$$

Thus weights can be expressed by:

$$w_{nr}=\frac{1}{\overline{\text{var}}_n+\text{var}\left(\ln \overline{p}_{nr}\right)} \qquad (6)$$

Where $\overline{\text{var}}_n$ is the average variance of the log average prices across the whole set of regions and is included to ensure that w_{nr} is still defined when for a particular region r $\text{var}\left(\ln \overline{p}_{nr}\right)=0$.

4. 2 CDP regression using price quotes: the hedonic approach

If data on prices for each product from each outlet (price quotes) in each region are available, intra-national PPPs can be estimated using CPD models for ungrouped data (Diewert, 2004, Silver, 2009). In this case, it would be possible to extend the CPD model to incorporate additional characteristics associated with each quotation, including information on the type of outlet and product brand.
Indeed, as a normal part of CPI calculations each of product price will have attached the detailed specifications which define the type of product (size, brand) and the type of outlet in which the product is sold.
Although representativity is not viewed as an issue in these subnational comparisons because a product will not be included in a country's CPI basket unless it is representative, the actual products priced for the CPI are not necessarily identical in different regions.
Different brands may be preferred in the various regions. In addition distribution channels are different and reflect different consumption habits.
Therefore, it is essential to include these information into the CPD model specification in order to improve PPP estimations.

Let $Z_1, Z_2, \ldots, Z_J$ represent a set of quality characteristics then the hedonic CPD model would be:

$$\ln p_{knr}=\sum_{r=1}^{R}a_r D_r+\sum_{n=1}^{N}b_n D_n^*+\sum_{j=1}^{J}\lambda_j Z_j+v_{knr} \qquad (7)$$

[3] Kmenta (1986) demonstrates that OLS parameter estimates based on group means are unbiased, though the disturbances are likely to be heteroskedastic and estimates inefficient, as a result of varying within group sample sizes. Such heteroskedasticity is avoided if the number of observations (outlets sampled for each product group in each region) is the same.

where p_{knr} is the price in outlet *k* of product *n* in region *r* .

This approach generalizes the usual CPD method in that the price variability is modeled right down to the level of the individual prices that are collected by the regions involved in the comparison (Diewert, 2004). Incorporating quality characteristics into the estimation of CPD price parities will improve the efficiency of the estimates, remove potential bias, and enable the inclusion of strictly non-comparable items[4].

Silver (2009) suggested that it is possible to improve on the specification of (7) by also introducing quality and product interaction terms.

4.3 The CPD Model with spatially auto-correlated error structure

When estimating sub-national PPPs an issue which can frequently arise is that of spatial autocorrelation. However, in the context of sub-national PPP construction using CPD models, the possible presence of autocorrelation among disturbances across regions for a specific commodity has not been investigated.

Referring to international comparison, evidence of spatial autocorrelation for a number of commodity groups among countries has been found by Aten (1996). Rao (2004) has drawn attention to the effects of spatial autocorrelation and the necessary adjustments to the estimates. Previously, Rao(2001) obtained estimated PPP using the CPD model after accounting for the presence of spatial autocorrelation for aggregation above the basic heading parities.

To test for the presence of spatial autocorrelation we can use the well-known Moran's I statistic. Rao (2004) suggested using residuals from an OLS CPD regression to test for spatially autocorrelated errors as the disturbance term for the n-th commodity in region r is the logarithm of price of commodity *n*, p_{nr}, converted using PPPr and expressed relative to the national average price of same commodity.

The Moran's I is essentially a cross product correlation measure that incorporates "space" by means of a spatial weight matrix **W**. Significance can be based on analytical derivations, or, more commonly, on a comparison to a reference distribution obtained by randomly permuting the observed values. This statistic can be expressed as follows:

$$I = \frac{R}{\sum_{r=1}^{R}\sum_{s=1}^{S} w_{rs}} \frac{\sum_{r=1}^{R}\sum_{s=1}^{S} w_{rs}\left(\hat{v}_{nr} - \bar{\hat{v}}\right)\left(\hat{v}_{ns} - \bar{\hat{v}}\right)}{\sum_{r=1}^{R}\left(\hat{v}_{nr} - \bar{\hat{v}}\right)^2} \qquad r \neq s$$

Measures of spatial autocorrelation for residuals take into account the dependence among observations by means of a spatial weight matrix **W**, which defines the structure of spatial relationships. For a set of R regions the spatial matrix **W** is a RxR matrix with the diagonal elements equal to zero; the other elements w_{rs} represent the intensity of the effects of the region r on region s (Anselin and Bera, 1998).

It is worth noting that since measures of spatial autocorrelations may depend on various specifications of **W** (Fischer and Getis, 2010), the sensitivity of the results should be assessed by using different representations of spatial structure. In literature there are very few formal guidelines concerning the choice of the most adequate spatial weights. However, variables that show a good

[4] The use of a CPD model incorporating quality adjustments was suggested in Kokoski, Moulton and Zieschang (1996) in the context of inter-area price comparisons.

deal of local spatial heterogeneity for the selected scale of analysis would probably be more appropriately modeled by few links in **W**, while a homogeneous or spatial trending variable would be better modeled by a **W** with many links. This implies that the scale characteristics of data are crucial elements in the creation of **W**. As spatial units become larger, the spatial dependence between the units tends to decrease.
Aten (1996) provided more details on the choice of weights in modelling spatial autocorrelation in the CPD context.
By focusing on the results of the Moran I test, it is possible to determine whether the classical CPD regression or a spatial-adapted CPD should be estimated.
Whenever spatial dependence is not negligible, a Spatial Error model (SEM) may be specified. Referring to the CPD model (3) and stacking all the R observations for each commodity into a vector, then the model containing spatial dependence in the disturbances can be expressed as follows:

$$\ln \overline{p}_n = \mathbf{x}_n \beta + v_n$$
$$v_n = \lambda \mathbf{W} v_n + \varepsilon_n$$

With the data generating process (LeSage and Pace, 2009):

$$\ln \overline{p}_n = \mathbf{x}_n \beta + \left(I_n - \lambda \mathbf{W}\right)^{-1} \varepsilon_n$$
$$\varepsilon_n \square N\left(0, \sigma^2 I_n\right)$$

5. A first exploration of the various CPD specifications using Italian data for regional capital cities

As a first analysis of the various CPD models to be used for constructing intra-national PPPs we refer to a specific dataset constructing by using CPI data.
This data set consists of both elementary price quotes and average prices referred to March 2013, to the products of consumer prices Italian basket belonging to the basic heading "Preserved milk and other milk products", which is composed of 5 products (UHT milk, dry milk, yogurt, organic yogurt double cream), for 20 regional chief towns.
The data set was provided by the Italian Statistical Institute (ISTAT), in the context of a scientific cooperation with the University of Florence and the University of Tuscia. The elementary price quotes were treated in order to respect the statistical confidentiality and the authors worked at ISTAT offices to test various hypotheses of CPD approach to subnational PPPs compilation.
PPPs are expressed as a percentage of the price level for the city of Rome, which is equal to 100.0.

Focusing on this BH we compare PPP estimation results obtained by using the following CPD specifications:

1. CPD model (3) with geometric mean for average prices expressed by (5), which is the standard (unweighted) CPD method estimated by using OLS;
2. CPD (wgm) which uses geometric means and is estimated using WLS with weights $w_{nr}=K_{nr}$;
3. CPD (wgm1) which uses geometric means and is estimated using WLS with weights specified in (6);
4. CPD (quote) which is based on the individual price quotes;
5. Hedonic CPD (quote) which uses individual price quotes and incorporates quality characteristics, i.e. brand and outlet type.

Firstly, let us examine the spatial variability of prices by using the box-plot. As an example, Figure 1 shows the box-plots for the price of preserved milk for March 2014 across the 20 regional capital cities .
A greater level of variability emerges for the cities of Florence, Genoa and Rome.

Figure 1 Price level variability for the 20 regional chief towns: UHT Milk

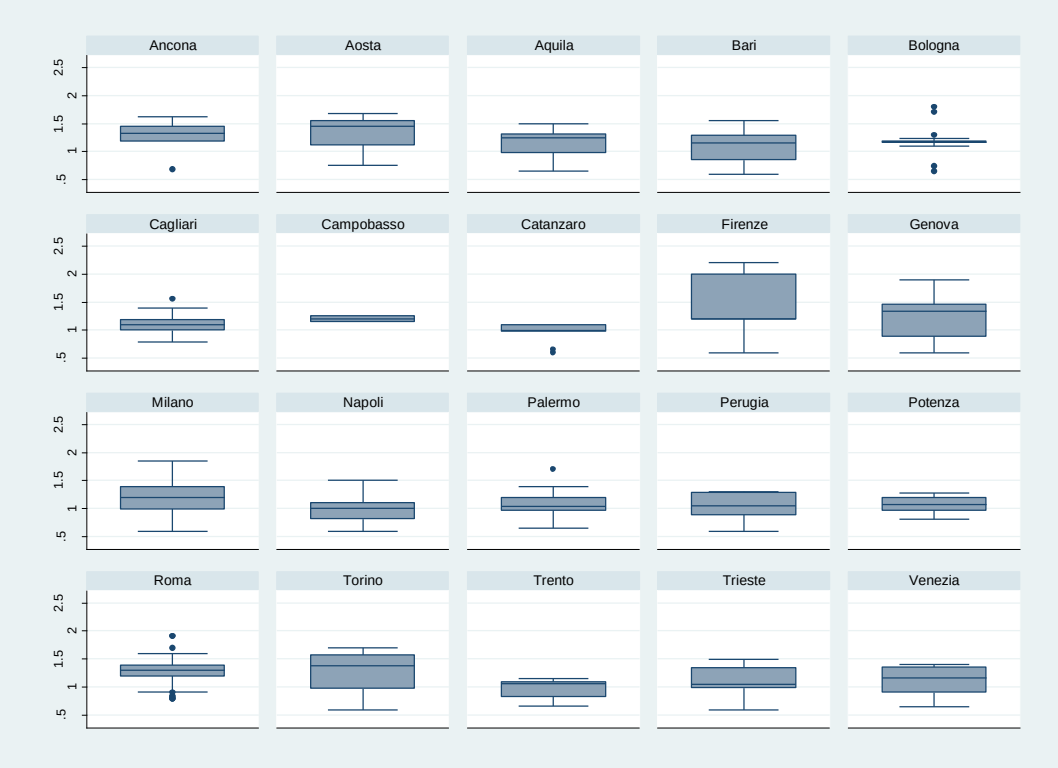


Table 1 shows the PPPs from weighted and unweighted CPD methods based on average prices (computed using geometric mean) and contrasts them with CPD model and the hedonic CPD based on individual price quotes.
Results in Table 1 show clear differences between PPPs from weighted and unweighted CPD models. The results form CPD (wgm1) which uses weights (6) do not significantly differ from those obtained from unweighted CPD. The weighted CPD methods are comparable to the CPD based on individual price quotes as it is clear from Figure 2.
Figure 3 highlights that results from the hedonic CPD appear to be generally lower than the weighed CPD with the exception of few cities (Ancona, Bologna, Genoa, Perugia and Trento).
By considering the RMSE it appears that the preferred method should be the weighted CPD. This result correspond to that found by others (Rao, 2001; Hill and Syed, 2014).

Table 1 Sub-national PPPs for the BH "Preserved milk and other products"-Rome=100

	CPD(gm)	CPD(wgm1)	CPD (wgm)	CPD (quotes)	Hedonic CPD
Ancona	103.64	101.87	102.47	102.89	98.19
Aosta	97.94	98.19	101.02	100.64	101.21
Aquila	90.64	89.39	94.39	94.97	100.86

Bari	95.85	97.14	92.62	92.93	87.96
Bologna	87.42	87.66	90.28	89.53	94.00
Cagliari	82.79	81.75	87.18	86.38	87.57
Campobasso	113.86	116.38	104.07	105.07	93.71
Catanzaro	78.37	79.07	79.70	80.19	77.20
Firenze	86.92	87.90	92.28	92.18	91.92
Genova	100.24	98.35	95.90	96.81	99.60
Milano	101.69	100.97	97.79	98.14	97.69
Napoli	103.11	102.89	92.79	93.86	91.42
Palermo	89.54	89.41	87.44	87.17	87.76
Perugia	83.01	83.26	79.96	80.05	83.98
Potenza	92.26	90.91	88.32	89.25	88.98
Torino	93.94	93.35	93.80	94.60	92.66
Trento	77.02	76.78	79.41	78.63	94.30
Trieste	89.70	89.34	90.38	89.35	92.33
Venezia	90.25	90.46	91.04	90.20	87.72
RMSE	0.14539	0.14438	0.11116	0.26352	0.19601

Figure 2 PPPs estimates: CPD(wgm) versus CPD(quotes)- Rome=100

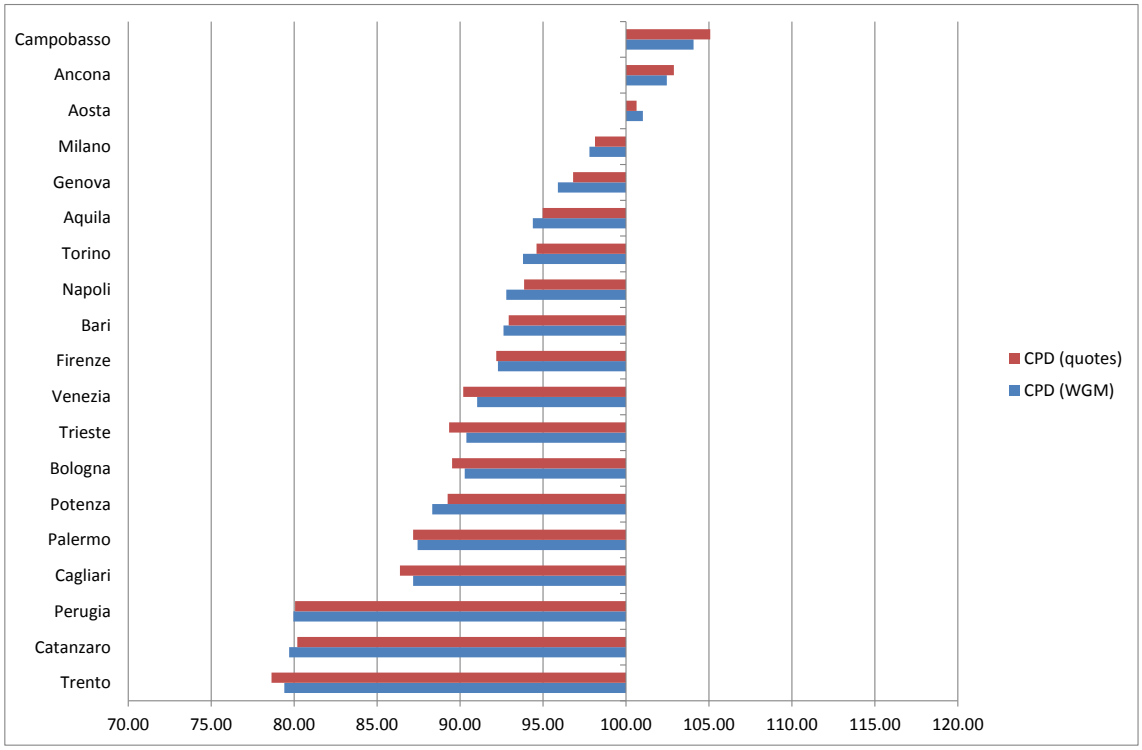
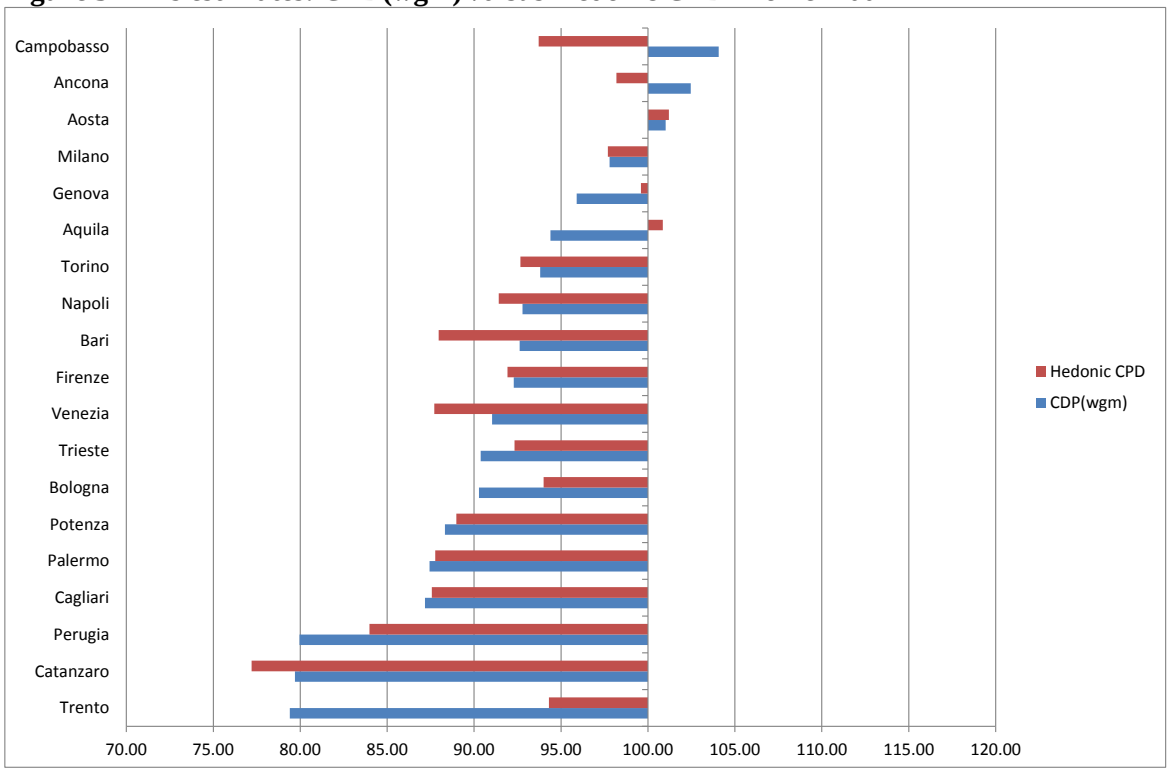


Figure 3 PPPs estimates: CPD(wgm) *versus* Hedonic CPD- Rome=100

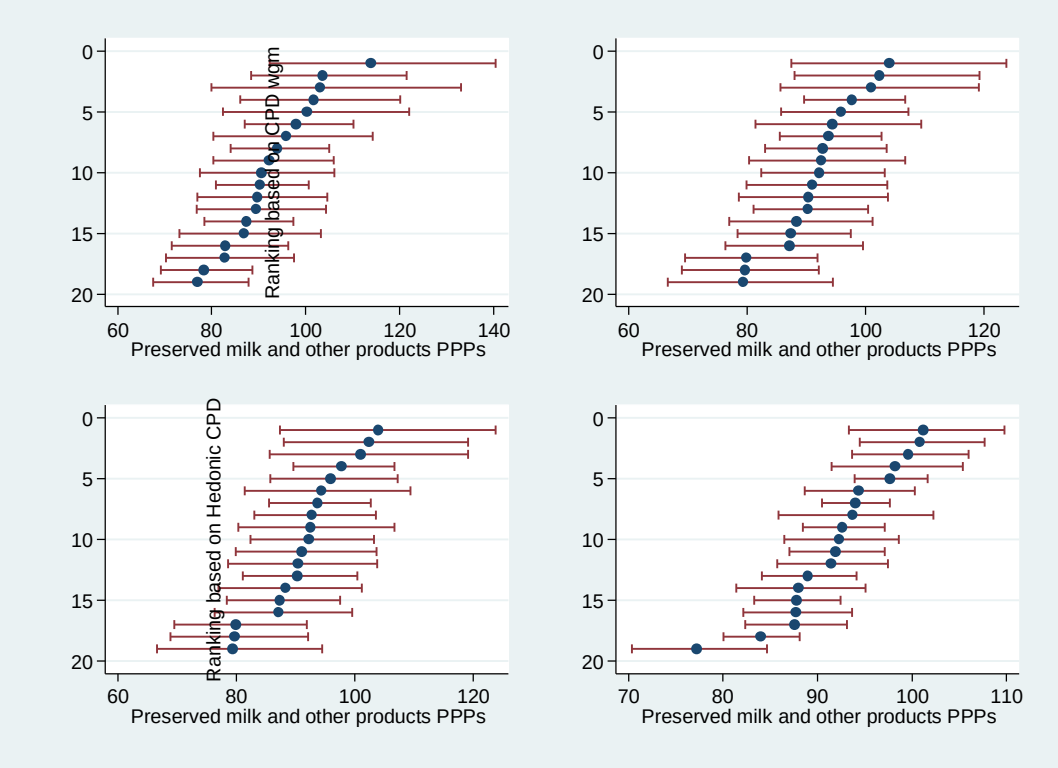


Besides the analysis of the various PPP estimates, it is important to verify their accuracy in order to determine if the purchasing power of a country's currency unit in a region can be distinguished from the others in that country significantly (i.e. purchasing power of euro in the various Italian regions represented by the regional chief towns).
This issue is particularly important considering the possible use of PPPs for converting household consumption expenditure or income levels.

Figure 4 shows the estimated PPPs (the blue dots in the figure below) for each regional chief towns obtained from the four CPD models together with the 95% confidence intervals associated with the point-estimates. Confidence intervals that do not overlap enable us to consider the purchasing power of a country's currency unit in a region significantly different from the others while a considerable overlapping in the confidence intervals suggests that there is a low level of precision in the estimates thus emphasizing a high level of uncertainty in the rankings.

Two main issues are observed by analysing the four rankings presented in Figure 4. The average lengths of the confidence intervals decrease for the hedonic CPD compared to the unweighed CPD (CPDgm). Moreover, significant changes in the magnitude and the distribution of the point estimates are observed since the *S-shaped* distribution of the estimated PPPs characterizing the unweighted CPD considerably increased for the hedonic CPD thus making the comparison among purchasing power in the various regions more reliable.

Figure 4. 95% confidence interval for PPPs: comparison between the four CPD methods



The presence of significant correlation in price structures across regions has been investigated for each of the different CPD methods considered.
In order to construct the W matrix, as a measure of spatial contiguity we considered geographical distance between cities by using their coordinates (latitude and longitude).
We found significant spatial autocorrelation only in the residuals of the CPD (wgm). Indeed, the calculated Moran's I-statistic, equal to I=0.017, is statistically significant at the 10% level.

6.Concluding remarks

The first provisional experiment, here presented, based on Italian CPI data to compare sub-national PPPs estimation results obtained by using five CPD specifications is very interesting and can already suggest the more suitable approach to be followed also for experiments to be conducted in other countries. However, our experiment must be extended to all different BHs and completed to arrive at the general sub-national consumer PPPs.

In any case, if the objective is to define a general framework to be followed from every country, it is important that a Working Group be established under the ICP umbrella in order to plan and implement a specific project on the field.

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