



Aggregate Labour Productivity Growth, TFP and Structural Change with EU-KLEMS

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Abstract

In this paper we present a decomposition of aggregate labour productivity growth (ALP) into three main components: sectoral total factor productivity (TFP), sectoral capital deepening and structural change (reallocation of labour across sectors). In order to disentangle the relative contribution of these factors, we adopt an econometric approach to estimate production functions for 28 sectors in the major European countries, Japan, Australia and the US (data are obtained from the EU-KLEMS project; period 1977-2007; 13 countries). Estimates of the production frontiers are obtained using a model which allows flexible identification of country specific time trends (temporal variation in individual heterogeneity) and time variation in the vector of slope coefficients of the production frontiers (bias in technical change). We find that the main driver of aggregate labour productivity growth is the sectoral factor deepening effect. TFP growth is very stable across years and across countries and it ranges between 0.2-0.5% per year, thus accounting for less than 30% of the growth in ALP in most countries. The structural change component has a slightly negative effect on ALP, pointing to the fact that the displacement of labour has been unfavorable to aggregate labour productivity.

1 Introduction

In this paper we consider the relative contribution of three main factors to aggregate labour productivity growth: sectoral total factor productivity (TFP), sectoral capital deepening and structural change (reallocation of labour across sectors). In order to disentangle the relative contribution of these factors, we adopt an econometric approach to estimate production frontiers for 28 sectors in the major European countries, Japan, Australia and the US (data are obtained from the EU-KLEMS project; period 1977-2007; 13 countries). Estimates of the production frontiers are obtained using a flexible translog specification which allows the identification of country specific time trends (temporal variation in individual heterogeneity) and time variation in the vector of slope coefficients of the production frontiers (bias in technical change). The framework can be thought as an extension of the fixed effects model and is especially useful when dealing with EU-KLEMS data because of their “long” panel nature (i.e., small number of countries for a relative long time span). The specification can be written as a standard state-space formulation and estimated using Kalman filtering and smoothing algorithms.

The decomposition of labour productivity growth is obtained as follow. *First*, we apply the econometric model to the 28 sectors in order to obtain estimates of the coefficients of the production frontiers. *Second*, we use this estimates to produce a decomposition of sectoral labour productivity growth into capital deepening and TFP. *Third*, we disentangle the effect of the increase of labour productivity at sectoral level (arising from capital deepening and TFP) from the effect of the displacement of labour across sectors. In fact, if the labour share of high productivity sectors increase with respect to low productivity sectors, aggregate labour productivity will improve because of this mobility (even if labour productivity growth at sectoral level is kept constant). This effect is what we call *structural change*. *Fourth*, we consider an enhanced decomposition of aggregate labour productivity in which the aggregate effect of sectoral capital deepening and sectoral TFP growth are separated. Our final decomposition of aggregate labour productivity growth will entail therefore three components: the contribution of sectoral TFP growth, the effect of sectoral capital deepening, and the effect of structural change (labour mobility across sectors).

Our results suggest that the main driver of aggregate labour productivity growth is the sectoral factor deepening effect. TFP growth is very stable across years and across countries and

it ranges between 0.2-0.5% per year. The structural change component has a slightly negative effect on aggregate labour productivity growth, pointing to the fact that the displacement of labour has been unfavorable to aggregate labour productivity.

The paper is organized as follow. In section 2 we present the basic production model and its state-space representation. In section 3, the econometric estimation strategy is presented. Section 4 introduces the decomposition of aggregate labour productivity growth into the aforementioned components. Section 5 describes the data used and the main empirical result. Finally, section 6 concludes.

2 The production model

The sectoral production technology is represented via a constant returns to scale (CRS) production function where a single output y_{it}^j (log of output) is produced by means of multiple inputs X_{it}^j where $i = 1, \dots, N$ indexes the number of countries, $t = 1, \dots, T$ indexes the number of time periods and $j = 1, \dots, J$ indexes the number of sectors. A translog specification can be accommodated by including the log of inputs, the squared log of inputs and interaction terms into the vector X_{it}^j :

$$y_{it}^j = \gamma_{it}^j + X_{it}^j \beta_t^j + \epsilon_{it}^j \quad (1)$$

Since we assume CRS the production function can be expressed in intensive form and all the variables can be normalized by the quantity of labour. Therefore equation (1) contains on the left-hand side the log of labour productivity (output per worker) and on the right-hand side the log of inputs over labour (for example, capital per worker). The intensive form production function defined in (1) is time varying due to the time varying coefficients $(\gamma_{it}^j, \beta_t^j)$. If one is willing to assume no productivity change then the production frontier becomes time invariant $y_{it}^j = \gamma_i^j + X_{it}^j \beta^j + \epsilon_{it}^j$. In this last equation the technology is fixed (no technical change as the parameters are time invariant) and the country specific intercept is a standard fixed effect unobserved heterogeneity component. It is possible to estimate such a model using standard panel data estimators (for a detailed discussion of this point see Schmidt and Sickles, 1984). Of course, the validity of such a procedure is predicated on the assumption that the production function parameters are time invariant and this is a tolerable assumption for “short” panels.

On the other hand, when T becomes larger the time invariant model becomes less appealing. In this instance the state-space representation provides the best option to elegantly estimate the time-varying model. In order to do so, we need to introduce some further assumption on the law of motion of the parameters of the model. We assume that both the country specific intercepts and the slope of the production function follow a random walk:

$$\begin{cases} \gamma_t^j = \gamma_{t-1}^j + \psi_t^j \\ \beta_t^j = \beta_{t-1}^j + \nu_t^j \end{cases} \quad (2)$$

where $\gamma_t^j = (\gamma_{1t}^j, \dots, \gamma_{Nt}^j)'$, $\psi_t \sim N(0, H_t = \sigma_{j\eta}^2 I_N)$ and $\nu_t \sim N(0, Q_t)$ are independently distributed random errors (Q_t is defined in the next section with more detail). In order to estimate the model via the Kalman filter we need to write it in the standard form. This can be easily done by defining the following matrices and vectors: $\alpha_t^j = \begin{pmatrix} a_t^j \\ \beta_t^j \end{pmatrix}$, $Z_t^j = [I_N, X_{it}^j]$, $D = \begin{bmatrix} I_N & \\ & I_K \end{bmatrix}$, $\eta_t^j = \begin{pmatrix} \psi_t^j \\ \nu_t^j \end{pmatrix}$. Equations (1) and (2) can now be represented using the following more compact notation:

$$y_t^j = Z_t^j \alpha_t^j + \epsilon_t^j \quad (3)$$

$$\alpha_t^j = D \alpha_{t-1}^j + \eta_t^j \quad (4)$$

where D will be omitted from here on as it is I_{N+K} . This is a standard state-space model and the α_t^j can be that can be estimated via the Kalman filter and smoothing algorithms given estimates of the variance-covariances Q_t and H_t . The first equation is called the measurement equation, while the second equation is called the transition equation. In the next section we present details on the estimation of the model in a Bayesian framework.

3 The Econometric Estimation

We estimate the state-space form derived in the previous section in a Bayesian framework (this differs from the classical estimation strategy followed by Peyrache and Rambaldi (2012) in the fact that it allows more flexibility in the specification of the covariance matrices). We initialize the state vector using a diffuse prior $\alpha_0 \sim N(a_0, P_0)$ and define the transition equation covariance matrix Q_t as a partitioned diagonal matrix where the partition divides the state vector into two components. The first component contains N stochastic trends (γ_t) and the second component contains K time-varying slopes (β_t). The specification is such that contemporaneous correlation is allowed within each component, although no correlation is allowed across the two components. Therefore the specification is of the form $Q_t = \text{diag}(\Omega_{\gamma,t}, \Omega_{\beta,t})$. We do not directly specify these diagonal sub-matrices, but use what is known as a *dynamic discounting* approach originating from the engineering and statistical literature. A detail treatment of this approach can be found in West and Harrison (1999), Ch.6. The limiting behaviour of the mean square error of the filter when using discounting has been studied by Triantafyllopoulos (2007). Here we only make a brief presentation as it is needed to establish notation and present the Bayesian estimation algorithm.

The conditional distribution of the state (posterior in a Bayesian context) at period $t - 1$ is given by $\alpha_{t-1} \sim N(a_{t-1|t-1}, P_{t-1|t-1})$, where $a_{t-1|t-1}$ is a filter estimate (given H_t and Q_t). The Kalman filter prediction equations provide the priors of the state in a Bayesian context. At period t , the prior distribution is $\alpha_{t|t-1} \sim N(a_{t|t-1}, P_{t|t-1})$, where, given the form of the econometric model in this study, are given by

$$\begin{aligned} a_{t|t-1} &= a_{t-1|t-1} \\ P_{t|t-1} &= P_{t-1|t-1} + Q_t \end{aligned} \tag{5}$$

A prediction of y_t can be made at time t given all known information at $t - 1$. We can write a distribution for the prediction $y_{t|t-1} \sim N(\hat{y}_{t|t-1}, F_{t|t-1})$, where,

$$\begin{aligned} \hat{y}_{t|t-1} &= Z_t a_{t|t-1} \\ F_{t|t-1} &= Z_t P_{t|t-1} Z_t' + H_t \end{aligned} \tag{6}$$

Once information for time t is available the updating of the state can occur. This forms a posterior in a Bayesian context, $\alpha_t \sim N(a_{t|t}, P_{t|t})$, where,

$$\begin{aligned}
a_{t|t} &= a_{t|t-1} + K_t \nu_t \\
\nu_t &= y_t - Z_t a_{t|t-1} \\
K_t &= P_{t|t-1} Z_t' F_{t|t-1}^{-1} \\
P_{t|t} &= P_{t|t-1} - K_t F_t K_t'
\end{aligned} \tag{7}$$

In (5), the precision of $\alpha_{t|t-1}$ is associated with $P_{t|t-1}^{-1}$, and that of $\alpha_{t-1|t-1}$ is $P_{t-1|t-1}^{-1}$. However, $P_{t-1|t-1}^{-1}$ is the precision of $\alpha_{t|t-1}$ if there is no stochastic change in the state vector from period $t-1$ to period t ($Q_t = 0$). In fact this is the global model (converging state). However, locally the dynamics are better captured by $Q_t \neq 0$. The discount literature specifies the precision $P_{t|t-1}^{-1}$ as a discounted $P_{t-1|t-1}$ by a proportion.

$$P_{t|t-1} = \Delta^{-1/2} P_{t-1|t-1} \Delta^{-1/2} \tag{8}$$

where $\Delta = \text{diag}(\delta_\gamma I_N, \delta_\beta I_K)$, $0 < \delta_\gamma \leq 1$ and $0 < \delta_\beta \leq 1$ are the discount factors associated with the country specific trends (γ_t) and slope parameters (β_t) in the model.

Using (8) and the second equation in (5), it is easy to see that $Q_t = (P_{t-1|t-1} - \Delta^{-1/2} P_{t-1|t-1} \Delta^{-1/2})$, which is a partitioned diagonal matrix as required, and $Q_t = 0$ if the discount factors are equal to 1.

The estimation of the country specific trends, γ_t , and slope parameters β_t is achieved by using the forward filter (equations (7) which provides estimates $\gamma_{t|t}$ and $\beta_{t|t}$ at each t). Subsequently, these estimates are revised to be made conditional on the complete sample by running a smoothing algorithm to compute $\gamma_{t|T}$ and $\beta_{t|T}$ (the equations of the classical smoother are presented in the Appendix) and their mean square error $P_{t|T}$. However, in order to obtain estimates using these two algorithms (forward filter and backward filter), we require values of additional parameters, which in this case are $\theta = [\sigma_\epsilon^2, \delta_\gamma, \delta_\beta]$. When θ is known, the Kalman filter and appropriate smoothing provide the minimum mean square estimator of the state and corresponding MSE (minimum linear estimator if normality is not assume), which is the posterior distribution of α_t in a Bayesian approach. However, these additional parameters are

seldom known and thus must be estimated. In a Classical setting they are considered fixed parameters and can be consistently estimated by evaluation of the likelihood (see Harvey (1989), Ch. 3 or Durbin and Koopman (2012), Ch. 2) and then used to replace the unknown parameters in the forward and backward filters. In a Bayesian framework, all parameters of the model are estimated using MCMC algorithms, typically the Gibbs sampler. This is a standard Bayesian estimation of a state space model, and thus we only briefly describe the algorithm here, and present some details in the Appendix. One noteworthy point is that in the conventional Bayesian literature the discount factors are not directly estimated, but calibrated as they are typically in the range of 0.9 - 0.99. Recent work by Koop and Korobilis (2013) proposes to use a model averaging approach over a range of values of the discount factors (they use the term forgetting factors in their work). We have not pursued this approach in this version of the paper. Instead, we use diagnostic tools to inform the calibration of the discount factors. A conventional diagnostic tool is to use standardised one-step ahead prediction errors. The one-step ahead prediction errors are ν_t (see (7)), which are standardised by constructing

$$u_t = L_t' \nu_t$$

where L_t is the lower Choleski factor of F_t^{-1} ($L_t L_t' = F_t^{-1}$)

These standardised prediction errors are conditionally independent and follow standard Normal distributions ($u_t \sim N(0, I)$)¹

From the above discussion it follows that given a set value for Δ , the Gibbs sampler has two blocks. The first block is to draw the state from $p(\alpha|y, h_\epsilon)$, where $\alpha = [\alpha'_1, \dots, \alpha'_T]'$ and $y = [y'_1, \dots, y'_T]$. In the second block we draw $h_\epsilon = \sigma_\epsilon^2$ from $p(h_\epsilon)p(y|h_\epsilon)$. This can be repeated for alternative values of Δ if u_t are found to deviate from the standard normal.

¹The one-step prediction errors analysis not only provides information of goodness of fit, but also addresses other key issues such as irregularities in the data series (outliers), model inadequacies (observational variance structure, choice of discount factor), or both (see for example West and Harrison (1999), Durbin and Koopman (2012)). The first can be examined via raw errors sequence whilst the latter is detected via investigating the observed deviation in the standardised error sequence.

4 Aggregate labour productivity decomposition

The purpose of this section is to establish a productivity decomposition at the aggregate level for each country in each time period. This decomposition exercise can be interpreted as a post-estimation exercise, given our estimates of the production model presented in section 3. In order to do so we follow a two-step decomposition procedure: in the first step we decompose sectoral labour productivity growth into the contribution of total factor productivity (TFP) and factor deepening (FD); in the second step we define aggregate labour productivity growth for each country and decompose it by aggregating TFP and FD across sectors. For each country, in each time period and for each sector labour productivity growth is given by the difference in the log of output per worker in two time periods; by using equation (1) we obtain:

$$y_{it+1}^j - y_{it}^j = \gamma_{it+1}^j + X_{it+1}^j \beta_{t+1}^j - \gamma_{it}^j - X_{it}^j \beta_t^j \quad (9)$$

This observed change can be imputed to three different effects: i) the shift in the country specific intercept, ii) the shift in the slope parameters and iii) the growth in the inputs. The first two effects are what one usually refers to as productivity change, while the last effect is an input growth effect. To isolate the productivity effect from the input growth effect we use a Malmquist logic, keeping some of the variables fixed while moving the others in order to separate the relative contribution of the different effects. TFP can be measured keeping the level of inputs at the base period level, obtaining the equivalent of the base period Malmquist productivity index:

$$TFP_1 = \gamma_{it+1}^j - \gamma_{it}^j + X_{it}^j (\beta_{t+1}^j - \beta_t^j) \quad (10)$$

Keeping the level of inputs at the comparison period value, we obtain the equivalent of the comparison period Malmquist productivity index:

$$TFP_2 = \gamma_{it+1}^j - \gamma_{it}^j + X_{it+1}^j (\beta_{t+1}^j - \beta_t^j) \quad (11)$$

and to avoid the arbitrariness of choosing the base, we use the geometric mean of these two

indexes in order to get an index of productivity growth:

$$\begin{aligned} TFP_{it}^j &= \frac{1}{2} (TFP_1 + TFP_2) = \\ &= \gamma_{it+1}^j - \gamma_{it}^j + \frac{1}{2} (X_{it}^j + X_{it+1}^j) (\beta_{it+1}^j - \beta_{it}^j) \end{aligned} \quad (12)$$

The input growth effect or factor accumulation effect (FD) can be computed using the same logic. The base period index is:

$$FD_1 = (X_{it+1}^j - X_{it}^j) \beta_{it}^j \quad (13)$$

The comparison period index is:

$$FD_2 = (X_{it+1}^j - X_{it}^j) \beta_{it+1}^j \quad (14)$$

Finally, we use the geometric mean as a measure of the input growth effect:

$$FD_{it}^j = \frac{1}{2} (FD_1 + FD_2) = \frac{1}{2} (\beta_{it}^j + \beta_{it+1}^j) (X_{it+1}^j - X_{it}^j) \quad (15)$$

It is easy to verify that the two effects are an exhaustive and mutually exclusive decomposition of the log change in output per worker:

$$y_{it+1}^j - y_{it}^j = TFP_{it}^j + FD_{it}^j \quad (16)$$

Therefore TFP has the standard interpretation of being the difference between output growth and input growth between two time periods. Moreover, since we allow for a time-varying slopes in the production function, the TFP measure defined in (12) incorporates any possible bias in technical change that could occur. All these components are country and time specific.

We now turn our attention to the aggregation of these quantities across sectors in order to obtain the effect of sectoral TFP growth onto aggregate labour productivity (ALP). Since we use EU-KLEMS data, output will be proxied by gross output and our input vector consist basically of 3 quantities: number of hours worked, capital services and intermediate inputs (II).

The following accounting identity will hold in each sector for each time period:

$$VA_{it}^j = GO_{it}^j - II_{it}^j$$

i.e. value added is equal to the difference between the value of gross output and the value of intermediate inputs. Aggregating across sectors one obtains:

$$VA_{it} = GO_{it} + II_{it} = \sum_{j=1}^J (GO_{it}^j - II_{it}^j)$$

Since we use a volume measure of gross output (Q_{it}^j) and a volume measure of intermediate inputs (M_{it}^j), the associated price indexes will be: $P_{it}^j = \frac{GO_{it}^j}{Q_{it}^j}$ and $W_{it}^j = \frac{II_{it}^j}{M_{it}^j}$. In each time period gross output for the economy is equal to

$$GO_{it} = P_{it} Q_{it} = \sum_{j=1}^J P_{it}^j Q_{it}^j$$

For the economy at aggregate level, we have aggregate labour equal to $L_{it} = \sum_j L_{it}^j$ and aggregate labour productivity:

$$Y_{it} = \frac{Q_{it}}{L_{it}} = \frac{\sum_j Q_{it}^j}{\sum_j L_{it}^j} = \sum_j \frac{Q_{it}^j}{L_{it}^j} \frac{L_{it}^j}{L_{it}} = \sum_j s_{it}^j Y_{it}^j$$

The change in aggregate labour productivity is:

$$\frac{Y_{it+1}}{Y_{it}} = \frac{\sum_j s_{it+1}^j Y_{it+1}^j}{\sum_j s_{it}^j Y_{it}^j}$$

This can be decomposed as:

$$ALP = \frac{Y_{it+1}}{Y_{it}} = \frac{\sum_j s_{it+1}^j Y_{it+1}^j}{\sum_j s_{it}^j Y_{it+1}^j} \frac{\sum_j s_{it}^j Y_{it+1}^j}{\sum_j s_{it}^j Y_{it}^j} = SC \cdot SLP$$

The first terms can be written as:

$$SC = \sum_j \delta_{it}^j \frac{s_{it+1}^j}{s_{it}^j}$$

where $\sum_j \delta_{it}^j = \sum_j \frac{s_{it}^j Y_{it+1}^j}{\sum_j s_{it}^j Y_{it+1}^j} = 1$. The second term can be expressed as:

$$SLP = \sum_j \pi_{it}^j \frac{Y_{it+1}^j}{Y_{it}^j} \quad (17)$$

with $\sum_j \pi_{it}^j = \sum_j \frac{s_{it}^j Y_{it}^j}{\sum_j s_{it}^j Y_{it}^j} = 1$. Since sectoral labour productivity growth is equal to the product of total factor productivity (TFP) and factor deepening (FD) we may further decompose the index in equation (17). In fact, TFP can be aggregated using gross output shares as weights (see Zelenyuk, 2006):

$$TFP_{it} = \sum \theta_{it}^j TFP_{it}^j$$

where $\theta_{it}^j = \frac{GO_{it}^j}{GO_{it}}$. The capital deepening component is obtained residually as the ratio of the previous two indexes. Therefore the overall aggregate decomposition for each country in each year will be:

$$ALP = SC \cdot SLP = SC \cdot TFP \cdot KD \quad (18)$$

5 Empirics

The EU-KLEMS dataset is an official project that collects input and output data on prices and quantities for 26 industrialized countries in the time span 1970-2007 (see O'Mahony and Timmer (2009)). For each industry the database provides value data on gross output, capital compensation, intermediate inputs (materials and energy) along with fixed base price and quantity index numbers (1995=100). We used the amount of total hours worked by persons engaged as a proxy for the quantity of labour (the alternative of using the number of persons engaged is less satisfactory). Since gross output, intermediate inputs and capital services are measured in local currencies we used PPPs to adjust for cross-sectional differential in the general level of prices. PPPs indexes use US as benchmark (US=100, 1995=100), are sector specific (i.e., each sector has different PPPs) and different for sectoral output, intermediate inputs and capital services (i.e., there are three sets of PPPs). Due to lack of data (missing values) we limit our attention to a subset of data, specifically 13 countries and 20 industrial sectors in the time span 1977-2007. The list of countries and sectors is reported in Table 1.

[Table 1 here]

We build a balanced panel dataset in the following way. Let $j = 1, \dots, 20$ be the sector and $i = 1, \dots, 13$ the country, then the index of sectoral output for country i at time t Y_{it}^j is:

$$Y_{it}^j = \frac{GO_{i1995}^j}{PPP_i^j} I_{it}^j \quad (19)$$

where GO_{i1995}^j is the value of gross output in 1995 for sector j in country i ; I_{it}^j is the fixed base index of sectoral output quantity change between time t and the base period 1995; PPP_i^j is the purchasing power parity of country i in sector j . With a similar procedure quantity index numbers are built for intermediate output (materials) and capital services. With this procedure we obtain a “true” balanced panel data set where cross-sectional (cross-country) comparability is built using PPPs and time comparability is built using fixed base quantity index numbers. This procedure guarantees that in each time period countries can be compared. The result is a quantity index number that proxies sectoral output production, a quantity index number proxying capital services, a quantity index number that proxies the level of materials used and the number of hours worked for the labour input. All the variables obtained with this procedure have been normalized by the sample minimum. The usual procedure of normalizing variables by sample mean is very unfruitful in our modeling setting. In fact, consider that the sign of the capital bias component depends on: the sign of $[\beta_{kt+1} - \beta_{kt}]$ that is common to all countries and the sign of $(\log k_{it+1} + \log k_{it})$ that is country specific. Now, if we normalize by the sample mean for a positive (negative) sign of $[\beta_{kt+1} - \beta_{kt}]$ the countries below the sample mean will have a negative (positive) capital bias, while the countries above the mean will have a positive (negative) capital bias. This is unreasonable, since although the magnitude of the capital bias should be proportional to the quantity of capital used, it should be monotonic for all the countries, i.e. the same for all the countries. This is guaranteed by a transformation by the sample minimum.

Our main empirical results are reported in Table 2. We report aggregate labour productivity growth (ALP) and its decomposition into the three components described in Section 4: structural change (SC), TFP and factor deepening (FD). We report the average growth rate for each component in each decade and for the whole period. With the exemption of Japan, TFP growth is very stable across the different decades for all countries at values between 0.2-

0.5%. Structural change has been slightly negative for most countries in these decades while the main contributor to labour productivity growth has been factor deepening. For example, by looking at US figures it is clear that SC and TFP are stable across decades and the main driver in the differences in labour productivity growth is a factor deepening effect. This trend is quite similar in the sample of countries considered in this study. In fact, the main changes in aggregate labour productivity growth are driven by the factor deepening component. This is clear by looking, for example, at the trends of Italy: TFP growth is quite stable (below 0.2%), as it is the structural change component (exception in the first decade). The dramatic drop in aggregate labour productivity growth from 2.9% in the second decade to 1.0% in the first decade is basically entirely explained by the factor deepening component. Japan is the only exception to this main trend, with the highest TFP growth of 0.7% on average on the entire period. It should also be noted that though at a low level, TFP growth is always positive for all countries in all decades. This points to the fact that even if TFP is not the main component of labour productivity growth, it is yet very stable at a positive rate.

[Table 2 here]

The parameter estimates for the 28 sectors as well as residual analysis are available from the authors and will be included in the next version of the paper.

6 Conclusions

In this paper we propose an econometric method to estimate sectoral level production functions for a group of 13 OECD countries in the years 1977-2006. We use the sectoral production functions to decompose sectoral labour productivity growth into a TFP component and a factor deepening component. We then aggregate these components to obtain a decomposition of aggregate labour productivity growth for the entire economy into three components: structural change, aggregate TFP and aggregate factor deepening. The structural change component represents the effect of labour displacement across the different sectors of the economy. The empirical results point to the fact that the factor deepening component accounts for the lion's share of aggregate labour productivity growth. Aggregate TFP growth (with the exception of Japan) ranges between 0.2-0.5% per annum across the whole period for all countries. Structural

change has had a slightly negative effect for some of the countries in the sample and does not represent a major contributor to long run aggregate labour productivity growth.

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Tables and Figures

Table 1: Descriptive Statistics

	Descriptive Statistics by sector (1977-2007)																			
	Output					Labour					Capital					Materials				
	Min	Mean	STD	Max		Min	Mean	STD	Max		Min	Mean	STD	Max		Min	Mean	STD	Max	
Wood and products of wood and cork	828	13,440	20,080	99,378	21	285	349	1,426	64	1,233	2,013	8,693	689	8,372	12,996	71,547				
Coke, refined petroleum and nuclear fuel	594	26,013	41,693	194,522	1	60	99	491	2	4,827	9,946	56,428	699	24,225	35,924	169,595				
Chemicals and chemical	2,396	72,095	95,269	461,750	29	486	645	2,629	352	12,673	21,886	101,700	1,335	42,621	57,827	282,576				
Rubber and plastics	960	29,991	36,060	170,767	21	421	559	2,103	115	3,475	5,760	29,137	564	16,357	21,961	109,391				
Other non-metallic mineral	1,478	25,253	24,045	96,428	22	376	402	1,515	155	3,580	5,386	22,047	790	12,112	12,117	52,665				
Machinery, nec	3,363	60,374	74,962	288,313	64	922	1,110	4,237	135	5,932	9,393	48,983	2,140	34,887	41,582	172,417				
Post and telecommunications	840	56,011	108,844	734,119	66	748	1,175	5,383	211	15,496	31,465	194,701	276	19,173	43,284	314,477				
Food , beverages and tobacco	5,235	91,169	115,655	550,211	62	1,037	1,161	4,218	394	10,913	17,076	71,771	3,900	65,003	85,400	436,668				
Textiles, textile , leather and footwear	959	31,519	40,667	166,466	14	926	1,183	4,901	93	2,511	3,819	15,639	784	22,634	26,644	109,259				
Pulp, paper, paper , printing and publishing	3,088	54,036	85,259	387,482	59	874	1,276	5,090	134	7,279	13,058	60,036	2,187	31,507	46,726	231,100				
Basic metals and fabricated metal	2,735	87,388	105,092	419,053	68	1,282	1,532	6,872	187	10,834	16,935	63,705	1,875	54,146	62,838	255,769				
Electrical and optical equipment	1,274	77,569	143,818	865,165	59	1,214	1,756	6,721	33	10,635	21,627	107,885	1,109	46,756	73,305	410,514				
Transport equipment	1,004	86,388	141,471	669,023	19	899	1,261	5,095	5	6,538	11,182	51,337	1,132	64,643	97,235	488,287				
Manufacturing nec; recycling	1,255	22,571	28,298	144,508	26	521	648	2,557	4	1,961	3,092	15,774	529	13,574	16,633	82,133				
Transport and storage	5,281	79,752	106,201	578,825	211	2,060	2,357	9,000	601	11,208	16,467	95,496	2,661	47,121	55,018	282,599				
Agriculture, hunting, forestry and fishing	3,912	40,804	61,179	302,858	111	2,482	2,853	16,180	33	10,390	17,425	70,888	3,169	27,188	37,504	166,401				
Mining and quarrying	128	22,024	43,684	179,254	5	250	501	2,652	43	7,034	14,528	68,419	150	10,456	21,917	118,322				
Electricity, gas and water supply	1,388	42,786	66,832	297,766	19	310	381	1,633	871	19,406	33,591	154,517	726	23,747	29,299	151,814				
Construction	10,950	161,265	175,377	816,186	226	3,953	4,669	19,975	72	11,843	13,984	67,926	7,269	91,119	90,980	491,109				
Hotels and restaurants	2,471	59,378	98,319	512,795	83	2,643	3,803	15,852	23	8,964	15,126	71,724	1,076	33,016	49,066	262,974				

SOURCE: EU-KLEMS Dataset. List of countries included: Australia, Austria, Belgium, Denmark, Spain, Finland, France, Germany, Italy, Japan, Netherlands, UK, USA.

Table 2: Decomposition of Aggregate Labour Productivity Growth

	1977-1987					1998-2006				
	ALP	SLP	SC	TFP	FD	ALP	SLP	SC	TFP	FD
AUS	1.4	1.9	-0.5	0.3	1.6	1.5	1.5	0.1	0.2	1.3
AUT	2.8	2.8	0.0	0.6	2.3	2.7	3.3	-0.7	0.3	3.0
BEL	2.4	2.7	-0.3	0.5	2.2	1.6	2.5	-0.8	0.3	2.2
DNK	2.3	2.3	-0.1	0.4	2.0	2.0	2.4	-0.4	0.2	2.2
ESP	2.1	2.3	-0.2	0.4	1.9	1.8	1.7	0.1	0.2	1.6
FIN	3.4	3.1	0.4	0.5	2.5	2.9	3.3	-0.3	0.2	3.1
FRA	3.1	2.9	0.2	0.6	2.2	2.6	3.0	-0.4	0.2	2.8
GER	1.8	1.8	0.0	0.4	1.5	2.3	2.6	-0.3	0.2	2.4
ITA	2.6	1.5	1.1	0.2	1.3	1.0	1.4	-0.4	0.2	1.2
JPN	3.3	3.0	0.3	0.9	2.0	2.0	2.0	0.0	0.5	1.5
NLD	0.5	1.0	-0.5	0.6	0.4	1.7	2.5	-0.7	0.3	2.1
UK	1.1	1.6	-0.6	0.3	1.3	2.2	2.8	-0.6	0.3	2.5
US	0.8	1.1	-0.3	0.4	0.7	2.5	3.1	-0.6	0.4	2.8
	1988-1997					1977-2006				
	ALP	SLP	SC	TFP	FD	ALP	SLP	SC	TFP	FD
AUS	3	3.8	-0.8	0.4	3.4	2	2.4	-0.4	0.3	2.1
AUT	2.4	2.5	0	0.5	2	2.6	2.9	-0.2	0.4	2.4
BEL	2.8	3	-0.2	0.3	2.7	2.3	2.7	-0.4	0.4	2.4
DNK	2.3	2.6	-0.3	0.3	2.3	2.2	2.4	-0.2	0.3	2.1
ESP	1.8	1.3	0.5	0.2	1.1	1.9	1.8	0.1	0.2	1.5
FIN	3.5	3.3	0.2	0.3	3	3.3	3.2	0.1	0.4	2.9
FRA	2.3	2.5	-0.2	0.4	2.1	2.7	2.8	-0.2	0.4	2.4
GER	3.2	3.1	0.1	0.2	2.9	2.4	2.5	-0.1	0.3	2.2
ITA	2.9	2.7	0.2	0.1	2.6	2.2	1.9	0.3	0.2	1.7
JPN	3.1	3	0.1	0.7	2.3	2.8	2.7	0.1	0.7	2
NLD	1.6	2.2	-0.6	0.4	1.8	1.3	1.8	-0.6	0.4	1.4
UK	2.5	3	-0.5	0.3	2.7	1.9	2.5	-0.6	0.3	2.1
US	1.5	2	-0.5	0.4	1.6	1.6	2	-0.5	0.4	1.6

A Appendix

A.1 Backward Filter- Classical Smoother

The posterior distribution is $\alpha_t \sim N(a_{t|T}, P_{t|T})$ where,

$$\begin{aligned}
a_{t|T} &= a_{t|t} + B_t(a_{t+1|T} - a_{t+1|t}) \\
P_{t|T} &= P_{t|t} - B_t(P_{t+1|t} - P_{t+1|T})B_t' \\
B_t &= P_{t|t}P_{t+1|t}^{-1} \\
a_{t+1|T} &= a_{t+1|t+1} \\
P_{t+1|T} &= P_{t+1|t+1} \quad \text{where } t = t-1, \dots, 1
\end{aligned} \tag{20}$$

A.2 MCMC algorithm

For many technical derivations it is easier to work with the inverse of the variances, error precisions, than with the variances and thus the vector θ , is redefined $\theta = [\delta_\gamma, \delta_\beta, h_\epsilon]$, where $h_\epsilon = \frac{1}{\sigma_\epsilon^2}$. Following the conventional Bayesian approach, priors for the state vector, α_t , and additional other parameters, here h_ϵ , need to be defined. The priors will be then updated via the likelihood function to obtain the conditional posterior densities. The distribution of the original state, α_0 , follows the literature and it is given by a diffuse prior $N(0, P_0)$ where P_0 is sufficiently large. The prior for h_ϵ is Gamma $G(s, v)$. It is noted that $\delta_\gamma, \delta_\beta$ are fixed throughout the running of the MCMC algorithm.

To summarise the priors used in this study are:

$$p(\alpha_0, h_\epsilon) = p(\alpha_0)p(h_\epsilon)$$

$$p(h_\epsilon) = f_G\left(\frac{s_0}{2}, \frac{v_0}{2}\right)$$

$$p(\alpha_0) = f_N(0, \kappa) \text{ where } \kappa \text{ is sufficiently large.}$$

The likelihood for the state space model is:

$$p(y_1, \dots, y_T | h_\epsilon) = p(y | h_\epsilon) = p(y_1) \prod_{t=2}^T p(y_t | Y_{t-1})$$

where $Y_{t-1} = (y'_1, \dots, y'_{t-1})'$

$$\log p(y_1, \dots, y_T | h_\epsilon) = -\frac{\sum_{t=1}^T N}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln |F_t| - \frac{1}{2} \sum_{t=1}^T (y_t - Z_t a_{t|t-1})' F_t^{-1} (y_t - Z_t a_{t|t-1})$$

The joint posterior is :

$$p(\alpha, h_\epsilon | y) = p(\alpha_0, h_\epsilon) p(y_1) \prod_{t=2}^T p(y_t | Y_{t-1}) \tag{21}$$

where $\alpha = [\alpha'_1, \dots, \alpha'_T]'$; $y = [y'_1, \dots, y'_T]'$

h_ϵ can be easily to sample from (21), but it is not the case for α . An alternative way to sample α_t is to use MCMC algorithm; and the following joint density function is adopted.

$$p(\alpha, h_\epsilon | y) = p(\alpha) p(h_\epsilon) p(y | \alpha, h_\epsilon) \tag{22}$$

where

$$p(y | \alpha, h_\epsilon) = \prod_{t=1}^T (2\pi h_\epsilon^{-N})^{-1/2} \exp\left(-\frac{1}{2} h_\epsilon (y_t - Z_t \alpha_t)' (y_t - Z_t \alpha_t)\right)$$

$$p(\alpha) = p(\alpha_0) \prod_{t=1}^n p(\alpha_t | \alpha_{t-1})$$

The full expression is $p(\alpha | \delta) = p(\alpha_0) \prod_{t=1}^T p(\alpha_t | \alpha_{t-1}, \Delta)$. We suppress Δ here for Δ is fixed throughout MCMC algorithm

- **Block 1:** Draw α from $p(\alpha|y, h_\epsilon)$

Many methods have been proposed in the literature to draw the state vector such as Carter and Kohn (1994), Fruhwirth-Schnatter (1994), DeJong and Shephard (1995) and Durbin and Koopman (2002). The Durbin and Koopman (2002) method with some modifications will be applied here.

- Step 1: Sample η_t^+ from $N(0, Q_t)$, α_0^+ from $N(\alpha_0, P_0)$, ϵ_t^+ from $N(0, H_t)$. Then use equation (3) and (4) to generate the y^+, α^+ .
- Step 2: Use the Kalman Filter (7) and Smoother (20) equations to compute the smoothed vector $\hat{\alpha} = E(\alpha|y)$, and $\hat{\alpha}^+ = E(\alpha^+|y^+)$ (where $\hat{\alpha}$ is the smoothed estimate given the actual observation y^* whereas $\hat{\alpha}^+$ is the smoothed estimate given the sampled observation y^+).
- Step 3: Following Durbin and Koopman (2002), $\tilde{\alpha} = \hat{\alpha} - \hat{\alpha}^+ + \alpha^+$ is considered as the draw α from $p(\alpha|y, h_\epsilon)$

- **Block 2:** Draw h_ϵ from $p(h_\epsilon)p(y|h_\epsilon)$ which is the Gamma distribution $G(\frac{s_t^\epsilon}{2}, \frac{v_t^\epsilon}{2})$ where

$$\begin{aligned} s_t^\epsilon &= 1 + s_{t-1}^\epsilon \\ v_t^\epsilon &= v_{t-1}^\epsilon + \frac{v_{t-1}^\epsilon}{s_{t-1}^\epsilon} [(y_t - Z_t a_{t|t-1})' F_t^{-1} (y_t - Z_t a_{t|t-1})] \end{aligned}$$

B The Estimated Model

The empirical model estimated in this paper is a translog function where the parameters of the square and cross-product terms are kept as time invariant. Thus, we can re-write the model (3) as

$$y_t = Z_t \alpha_t + M_t \beta + \epsilon_t \quad \epsilon_t \sim N(0, H_t) \quad (23)$$

$$\alpha_t = \alpha_{t-1} + \eta_t \quad \eta_t \sim N(0, Q_t) \quad (24)$$

where

y_t is $N \times 1$; $t = 1, \dots, T$

$Z_t^\dagger = [I_N \quad X_t]$; X_t is $N \times K_1$

$\alpha_t = \begin{bmatrix} \gamma_t \\ \beta_t^\dagger \end{bmatrix}$ is the $m \times 1$ state vector; $\alpha_0 \sim N(a_0, P_0)$; where $m = N + K_1$

M_t is $N \times K_2$, and $K_1 + K_2 = K$

β are the parameters attached to the regressors in M_t

$\beta_0 \sim N(b_0, P_0)$

$\epsilon_t \sim N(0, H_t)$; $H_t = \sigma_\epsilon^2 I_N$

$$\eta_t \sim N(0, Q_t)$$

To estimate this model we used a slightly modified algorithm that estimates β as a separate block in the Gibbs sampler.

B.1 The filter

Forward Filter

Posterior at period t-1

$$\alpha_{t-1} \sim N(a_{t-1|t-1}, P_{t-1|t-1})$$

Prior at period t

$$\begin{aligned} \alpha_{t|t-1} &\sim N(a_{t|t-1}, P_{t|t-1}) \\ a_{t|t-1} &= a_{t-1|t-1} \\ P_{t|t-1} &= P_{t-1|t-1} + Q_t \end{aligned} \tag{25}$$

Forecast at t

$$\begin{aligned} y_{t|t-1}^* &\sim N(\hat{y}_{t|t-1}, F_{t|t-1}) \\ \hat{y}_{t|t-1} &= Z_t^* a_{t|t-1} + M_t b \\ F_{t|t-1} &= Z_t^* P_{t|t-1} Z_t^{*'} + H_t \end{aligned} \tag{26}$$

Posterior at t

$$\begin{aligned} \alpha_t &\sim N(a_{t|t}, P_{t|t}) \\ a_{t|t} &= a_{t|t-1} + K_t \nu_t \\ \nu_t &= y_t^* - Z_t^* a_{t|t-1} - M_t b \\ K_t &= P_{t|t-1} Z_t^{*'} F_{t|t-1}^{-1} \\ P_{t|t} &= P_{t|t-1} - K_t F_t K_t' \end{aligned} \tag{27}$$

The backward filter remains as in A.1.

B.2 MCMC algorithm

The priors:

$$p(\alpha_0, h_\epsilon) = p(\alpha_0)p(\beta_0)p(h_\epsilon)$$

$$p(h_\epsilon) = f_G\left(\frac{s_0^\epsilon}{2}, \frac{v_0^\epsilon}{2}\right)$$

$$p(\alpha_0) = f_N(0, \kappa) \text{ where } \kappa \text{ is sufficiently large.}$$

$$p(\beta_0) = f_N(0, \kappa) \text{ where } \kappa \text{ is sufficiently large.}$$

The likelihood for the state space model is:

$$p(y_1, \dots, y_T | h_\epsilon) = p(y | h_\epsilon) = p(y_1) \prod_{t=2}^T p(y_t | Y_{t-1})$$

where $Y_{t-1} = (y_1', \dots, y_{t-1}')'$

$$\log p(y_1, \dots, y_T | h_\epsilon) = -\frac{\sum_{t=1}^T N}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^T \ln |F_t| - \frac{1}{2} \sum_{t=1}^T \nu_t' F_t^{-1} \nu_t;$$

$$\nu_t = y_t - Z_t a_{t|t-1} - M_t b$$

The joint posterior is :

$$p(\alpha, \beta, h_\epsilon | y) = p(\alpha_0, \beta_0, h_\epsilon) p(y_1) \prod_{t=2}^T p(y_t | Y_{t-1}) \quad (28)$$

where $\alpha = [\alpha'_1, \dots, \alpha'_T]'$; $y = [y'_1, \dots, y'_T]$

h_ϵ can be easily to sample from (28), but that is not the case for α and β . An alternative way to sample α_t is to use MCMC algorithm; and the following joint density function is adopted.

$$p(\alpha, h_\epsilon | y) = p(\alpha) p(\beta_0) p(h_\epsilon) p(y | \alpha, \beta, h_\epsilon) \quad (29)$$

where

$$p(y | \alpha, h_\epsilon) = \prod_{t=1}^T (2\pi h_\epsilon^{-N})^{-1/2} \exp\left(-\frac{1}{2} h_\epsilon (y_t - Z_t \alpha_t - M_t \beta)' (y_t - Z_t \alpha_t - M_t \beta)\right)$$

$$p(\alpha) = p(\alpha_0) \prod_{t=1}^n p(\alpha_t | \alpha_{t-1})$$

• **Block 1:** Draw α from $p(\alpha | y, \beta, h_\epsilon)$

- Step 1: Sample η_t^+ from $N(0, Q_t)$, α_0^+ from $N(\alpha_0, P_0)$, ϵ_t^+ from $N(0, H_t)$. Then use equation (3) and (4) to generate the y^+, α^+ .
- Step 2: Use the Kalman Filter (27) and Smoother (20) equations to compute the smoothed vector $\hat{\alpha} = E(\alpha | y)$, and $\hat{\alpha}^+ = E(\alpha^+ | y^+)$ (where $\hat{\alpha}$ is the smoothed estimate given the actual observation y^* whereas $\hat{\alpha}^+$ is the smoothed estimate given the sampled observation y^+).
- Step 3: Following Durbin and Koopman (2002), $\tilde{\alpha} = \hat{\alpha} - \hat{\alpha}^+ + \alpha^+$ is considered as the draw α from $p(\alpha | y, h_\epsilon)$

• **Block 2:** Draw β from $p(\beta_0) p(y | \alpha, \beta, h_\epsilon)$ which is the multivariate Normal distribution $N(b, P)$

$$P = (P_0^{-1} + h_\epsilon \sum_{t=1}^T M_t' M_t)^{-1}$$

$$b = \bar{P} \left[(P_0^{-1} b_0 + h_\epsilon \sum_{t=1}^T M_t' (y_t - Z_t \alpha_t)) \right]$$

• **Block 3:** Draw h_ϵ from $p(h_\epsilon) p(y | h_\epsilon)$ which is the Gamma distribution $G(\frac{s_t^\epsilon}{2}, \frac{v_t^\epsilon}{2})$ where

$$s_t^\epsilon = 1 + s_{t-1}^\epsilon$$

$$v_t^\epsilon = v_{t-1}^\epsilon + \frac{v_{t-1}^\epsilon}{s_{t-1}^\epsilon} \left[(y_t - Z_t a_{t|t-1} - M_t b)' F_t^{-1} (y_t - Z_t a_{t|t-1} - M_t b) \right]$$