An Economic Approach to Identifying the Drivers of Productivity Change in the Market Sectors of the Australian Economy

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AN ECONOMIC APPROACH TO IDENTIFYING THE DRIVERS OF PRODUCTIVITY CHANGE IN THE MARKET SECTORS OF THE AUSTRALIAN ECONOMY

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ABSTRACT. This paper uses a geometric Young TFP index to measure productivity change in eighteen sectors of the Australian economy over the period 1970 to 2007. One of the advantages of the geometric Young index is that it can be exhaustively decomposed into measures of technical change, environmental change, and various types of efficiency change. The paper identifies common assumptions concerning technologies, markets and firm behaviour that are sufficient for least squares estimators to be consistent estimators of these components. In this paper, a two-stage least squares estimator is used to overcome an endogeneity problem. The paper finds that the main driver of productivity change over the sample period has been scale-mix efficiency change. Scale-mix efficiency is a measure of how well a firm is capturing economies of scale and scope. In the case of the Australian mining industry, for example, significant changes in scale-mix efficiency in the last decade are found to have been associated with increases in the use of labour.

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1. INTRODUCTION

Total factor productivity (TFP) indexes are measures of output change divided by measures of input change. In the growth accounting literature, it is common to measure the output change component using the change in real gross domestic product (GDP). The input change component is usually measured using a Törnqvist index. For many growth accountants, the main advantage of the Törnqvist index is that it provides for a convenient multiplicative decomposition of total input change into changes in capital, labour and other inputs. The main disadvantage of the index is that it violates a transitivity axiom. Transitivity says that a direct comparison of two observations should yield the same measure of input change as an indirect comparison via a third observation. For example, if all inputs in period 2 were 20% higher than in period 1, and if all inputs in period 3 were 20% higher than in period 2, all direct and indirect comparisons should say that inputs in period 3 were $1.2 \times 1.2 - 1 = 44\%$ higher than in period 1. The fact that the Törnqvist index doesn't satisfy this axiom means it cannot be used to make reliable comparisons involving more than two observations. This paper measures the input change component of TFP change using a geometric Young index. This index satisfies nine important axioms from index number theory, including the transitivity axiom [9]. It can also be used to decompose total input change into changes in different inputs..

Most, if not all, TFP indexes can, by construction, be deterministically decomposed into changes in individual output and input quantities. These accounting-type decompositions are often used by statistical agencies to provide policy-makers with useful information on, for example, the relationship between "capital deepening" and total output. Economic decompositions are also possible. Specifically, most TFP indexes can be decomposed into measures of technical progress (a measure of outward movements in the production frontier due to the discovery of new technologies), environmental change (a measure of movements in the frontier due to changes in characteristics of the production environment), technical efficiency change (a measure of movements towards or away from the frontier) and scale and mix efficiency change (measures of movements around the frontier surface to capture economies of scale and scope) [11]. There are at least two reasons for wanting to identify these types of components. First, all other things being equal, productivity growth that is driven by technical progress, improvements in the production environment, and/or increases in technical efficiency will always be associated with higher net income. However, productivity growth that is driven by increases in scale and mix efficiency could easily be associated with lower net income. Thus, identifying the technical change, environmental change and efficiency change components of productivity change is critically important for determining whether productivity growth is associated with higher or lower net income (and therefore wealth). Second, different policies will generally have different effects on the different components of productivity change. For example, increases in R&D expenditure will generally have a larger impact on the rate of technical change than on scale efficiency, and policies that change input prices are likely to have a larger effect on input mix efficiency than on technical efficiency. This paper uses econometric methods to decompose the geometric Young TFP index into a combined measure of technical and environmental change, a measure of technical efficiency change, and measures of scale and mix efficiency change. This necessarily involves estimating a functional representation of the production technology.

The structure of the paper is as follows. Section 2 lists some weak assumptions concerning the production technology that are sufficient for a functional representation of the technology to exist. It also lists some stronger assumptions that can be used to motivate economic-theoretic productivity indexes. Section 3 lists some weak assumptions concerning markets and firm behaviour that are often used to explain changes in observed input-output choices. It then describes the optimisation problem of a firm that must make production choices before output prices are known. Section 4 lists common assumptions concerning data measurement errors and omitted variables. In practice, it is almost certain that these assumptions will not be satisfied. Section 5 explains how assumptions concerning technologies, markets, behaviour and data can be used to inform the choice of a meaningful TFP index. For example, if (i) the technology is a constant-returns-to-scale Cobb-Douglas technology, (ii) markets are perfectly competitive, (iii) firms maximise profits, and (iv) there are no data or measurement errors, then a meaningful measure of TFP change is the Solow [16] residual. Section 6 explains that if a functional representation of the production technology exists, then it can be written in the form of the stochastic production frontier model of Aigner, Lovell and Schmidt [2]. It also discusses the key statistical assumptions that must hold in order for least squares estimators to be consistent estimators of the components of TFP change. Section 7 uses EU-KLEMS data to compute and explain the drivers of productivity change in eighteen sectors of the Australian economy. No statistical evidence is found that the assumptions underpinning the model are not satisfied. Section 8 concludes.

2. TECHNOLOGIES

In this paper, a technology is defined as a technique, process, system or method for transforming inputs into outputs. Examples of technologies include the Bessemer process for making steel, and R-DNA technologies for producing human insulin. In practice, it is convenient to think of a technology as a book of instructions. The set of all technologies available in period t is referred to as the period-t metatechnology. If a technology is viewed as a book of instructions, then the metatechnology should be viewed as a library. Mathematically, the set of input-output combinations that are possible using the period-t metatechnology in an environment characterised by a vector z is $T^t(z) \equiv \{(x,q) : \text{inputs } x \in \mathbb{R}_+^{M^*} \text{ can produce outputs } q \in \mathbb{R}_+^{N^*} \text{ in period } t \text{ in environment } z \in \mathbb{R}_+^{J^*}\}$. The set of outputs that can be produced using x and the period-t metatechnology in an environment characterised by z is the output set $P^t(x,z) \equiv \{q : (x,q) \in T^t(z)\}$. The set of inputs capable of producing q in period t and environment z is the input set $L^t(q,z) \equiv \{x : (x,q) \in T^t(z)\}$. It is common to assume:

T1: $P^t(x,z) \equiv \{q : (x,q) \in T^t(z)\}$ is bounded for all $x \in \mathbb{R}_+^{M^*}$,
T2: $q \geq 0 \Rightarrow (0,q) \notin T^t(z)$ (weak essentiality),
T3: $(x,q) \in T^t(z)$ and $0 \leq \lambda \leq 1 \Rightarrow (x, \lambda q) \in T^t(z)$ (weak disposability of outputs),
T4: $(x,q) \in T^t(z)$ and $\lambda \geq 1 \Rightarrow (\lambda x, q) \in T^t(z)$ (weak disposability of inputs),
T5: $P^t(x,z)$ is closed for all $x \in \mathbb{R}_+^{M^*}$ and
T6: $L^t(q,z) \equiv \{x : (x,q) \in T^t(z)\}$ is closed for all $q \in \mathbb{R}_+^{N^*}$.

In this paper, these are the only assumptions that will not be subjected to empirical testing. Assumption T1 says there is a limit to what can be produced using a finite amount of inputs; T2 (weak essentiality) says that a positive amount of at least one input is required to produce a positive amount of output; T3 (weak disposability of outputs) says that if an input vector can be used to produce a given output vector then it can also be used to produce a scalar contraction of that output vector; T4 (weak disposability of inputs) says that if an output vector can be produced using a particular input vector then it can also be produced using a scalar magnification of that input vector (weak disposability of inputs); and T5 and T6 say that the output and input sets contain all the points on their boundaries. These assumptions are maintained throughout this paper, mainly to ensure that a functional representation of the production possibilities set exists: if T1–T6 are satisfied then equivalent representations of the technology are the output and input distance functions of Shephard [15, pp. 206, 207]:

$$D_O^t(x,q,z) = \inf\{\delta > 0 : (x, q/\delta) \in T^t(z)\} \tag{1}$$

and $$D_I^t(x,q,z) = \sup\{\rho > 0 : (x/\rho, q) \in T^t(z)\}. \tag{2}$$

Technically-feasible input-output combinations are characterised by $D_O^t(x,q,z) \leq 1$ and $D_I^t(x,q,z) \geq 1$. Technically efficient points correspond to $D_O^t(x,q,z) = D_I^t(x,q,z) = 1$.

If T1–T6 are satisfied then the output (resp. input) distance function is nonnegative (NN) and homogeneous of degree one (HD1) in outputs (resp. inputs). Additional assumptions concerning the production technology place additional structure on the these functions. For example, it is common to assume

T7: $(x,q) \in T^t(z)$ and $0 \leq q^1 \leq q \Rightarrow (x,q^1) \in T^t(z)$ (strong disposability of outputs),
T8: $(x,q) \in T^t(z)$ and $x^1 \geq x \Rightarrow (x^1,q) \in T^t(z)$ (strong disposability of inputs),
T9: $(x,q) \in T^t(z) \Leftrightarrow (\lambda x, \lambda^r q) \in T^t(z)$ for all $\lambda > 0$ (homogeneity of degree r),
T10: $P^t(x,z) = a(x,z,\bar{t},t)P^{\bar{t}}(x,z)$ for any $\bar{t} \in G_t$ (implicit Hicks output neutrality),
T11: $L^t(q,z) = b(q,z,\bar{t},t)L^{\bar{t}}(q,z)$ for any $\bar{t} \in G_t$ (implicit Hicks input neutrality),
T12: $P^t(x,z) = g(x,\bar{x},z,t)P^t(\bar{x},z)$ for any $\bar{x} \in \mathbb{R}_+^{M^*}$ (output homotheticity),
T13: $L^t(q,z) = h(q,\bar{q},z,t)L^t(\bar{q},z)$ for any $\bar{q} \in \mathbb{R}_+^{N^*}$ (input homotheticity),
T14: $T^t(z) = k(z,\bar{z},t)T^t(\bar{z})$ for any $\bar{z} \in \mathbb{R}_+^{J^*}$ (environmental homotheticity) and
T15: the boundary of $T^t(z)$ is a Cobb-Douglas function

where $a(.)$, $b(.)$, $g(.)$, $h(.)$ and $k(.)$ are scalar-valued functions with the property $a(x,z,s,s) = b(q,z,s,s) = g(\bar{x},\bar{x},z,t) = h(\bar{q},\bar{q},z,t) = k(\bar{z},\bar{z},t) = 1$. Assumption T7 (strong, or free, disposability of outputs) says it is possible to use the same inputs to produce fewer outputs, while T8 (strong, or free, disposability of inputs) says it is possible to produce the same outputs using more inputs. If outputs (resp. inputs) are strongly disposable then the output distance function is nondecreasing (resp. nonincreasing) in outputs (resp. inputs). Assumption T9 (homogeneity of degree r) says that a one percent increase in inputs allows for an r percent increase in outputs. If the technology is homogeneous of degree r then the output distance function, for example, is homogeneous of degree $-r$ in inputs. The Hicks neutrality and homotheticity properties have implications for the partial derivative and separability properties of distance functions. For example, T12 (output homotheticity) says that the outputs that can be produced using a given input vector and metatechnology in a particular production environment are a scalar multiple of the outputs that can be produced using an arbitrary input vector and the same metatechnology in the same environment. If the technology is output homothetic (OH) then marginal rates of technical substitution are independent of outputs and marginal rates of technical transformation are independent of inputs. If T9–T14 are satisfied then the distance function can be written as $D^t_O(x,q,z) = Q(q)/F(x,z,t)$ where $Q(q) \equiv D^{\bar{t}}_O(\bar{x},q,\bar{z})/D^{\bar{t}}_O(\bar{x},\bar{q},\bar{z})$ is an aggregate output, and $F(x,z,t)$ is a production function that is HDr in inputs. Finally, T15 (Cobb-Douglas) says the production frontier is the well known log-linear function of Cobb and Douglas [3]. If T15 is satisfied then the homotheticity and Hicks neutrality assumptions (T9-T14) will also be satisfied so long as all the slope coefficients in the function are time-invariant. To see exactly what this means, we need to introduce a firm subscript i and a time subscript t into the notation so that, for example, $x_{it} = (x_{1it},\ldots,x_{M^*it})'$ denotes the input vector of firm i in period t. If T9–T15 are satisfied then:

$$\ln D^t_O(x_{it},q_{it},z_{it}) = \sum_{n=2}^{N^*} \alpha_n \ln\left(\frac{q_{nit}}{q_{1it}}\right) - \gamma_t - \sum_{j=1}^{J^*} \rho_j \ln z_{jit} - \sum_{m=1}^{M^*} \beta_m \ln x_{mit} + \ln q_{1it}. \quad (3)$$

If outputs and inputs are strongly disposable (T7 and T8) then $\alpha_{nt} \geq 0$ and $\beta_{mt} \geq 0$ respectively. In this case, the technology is homogeneous of degree $r = \sum_m \beta_m$. The technology is said to exhibit decreasing returns to scale (DRS), constant returns to scale (CRS) or increasing returns to scale (IRS) as r is less than, equal to, or greater than one.

3. Markets and Behaviour

Economic policy-makers are interested in the reasons why some firms might rationally choose to be less productive than others. Equivalently, they are interested in why firms might rationally choose different input-output combinations from the production possibilities set. Explaining the choices made by firms involves assumptions about markets and firm behaviour. For example, it is common to assume:

- B1: markets are perfectly competitive,
- B2: firms have benefit functions that are strictly increasing in net returns, and
- B3: input prices and characteristics of the production environment are known at the time production decisions are made.

Assumption B1 says, among other things, that firms are price-takers in output and input markets. Assumption B2 merely says they prefer more income to less. Assumption B3 implies that firms know the prices they will be charged for their inputs, and that production choices are delayed until random variables directly involved in the production process are known (e.g., crop planting decisions are delayed until the arrival of monsoon rains). These assumptions allow for the possibility that output prices are unknown at the time production designs are made. Output prices will be unknown if, for example, the production process is time-consuming and markets are incomplete (e.g., there are no futures markets).

If B1–B3 are satisfied then the rational optimising firm will maximise expected profits. Mathematically, the period-t optimisation problem of firm i is:

$$B(p_{it}^e, w_{it}, z_{it}, t) = \max_{x \geq 0, q \geq 0} \{p_{it}^{e\prime} q - w_{it}^{\prime} x : D_O^t(x, q, z_{it}) \leq 1\} \tag{4}$$

where $p_{it}^e \geq 0$ is a vector of expected output prices, and $w_{it} > 0$ is a vector of known input prices. The output and input vectors that solve this problem are $q_{it}^* = q(p_{it}^e, w_{it}, z_{it}, t)$ and $x_{it}^* = x(p_{it}^e, w_{it}, z_{it}, t)$. The constraint in (4) will be binding at the optimum, which means $D_O^t(x_{it}^*, q_{it}^*, z_{it}) = 1$. If the firm forms naive expectations (i.e., it sets $p_{it}^e = p_{i,t-1}$) and chooses the outputs and inputs that solve (4) (i.e., it chooses $q_{it} = q_{it}^*$ and $x_{it} = x_{it}^*$) then

$$q_{it} = q(p_{i,t-1}, w_{it}, z_{it}, t), \tag{5}$$

$$x_{it} = x(p_{i,t-1}, w_{it}, z_{it}, t) \tag{6}$$

$$\text{and} \qquad D_O^t(x_{it}, q_{it}, z_{it}) = 1. \tag{7}$$

Even if the firm does not manage to solve the expected profit maximisation problem, observed outputs and inputs will still be functions of input prices in period t and output prices in period $t-1$. In practice, firms are often prevented from solving optimisation problems such as (4) because they are prevented from choosing inputs, outputs or technologies freely. For example, the U.S. Fair Labor Standards Act prevents firms in non-agricultural industries from employing children under the age of twelve, and, from 1980–1997, the Cohen-Boyer patents were used by Stanford University to prevent firms from using recombinant-DNA technology without the payment of a license fee.

4. Data and Aggregation

Consider a dataset containing observations on M inputs, N outputs and J environmental variables. Common assumptions concerning the data are:

D1: $M = M^*$, $N = N^*$ and $J = J^*$ (all variables are observed),
D2: $Q(q_{it}) = D_O^{\bar{t}}(\bar{x}, q_{it}, \bar{z}) / D_O^{\bar{t}}(\bar{x}, \bar{q}, \bar{z})$ (outputs are measured w/o error),
D3: $X(x_{it}) = D_I^{\bar{t}}(x_{it}, \bar{q}, \bar{z}) / D_I^{\bar{t}}(\bar{x}, \bar{q}, \bar{z})$, (inputs are measured w/o error) and

D4: $Z(z_{it}) = D_I^{\bar{t}}(\bar{x}, \bar{q}, z_{it})/D_I^{\bar{t}}(\bar{x}, \bar{q}, \bar{z})$ (env. variables are measured w/o error)

where $\bar{x}$ and $\bar{q}$ are reference input and output vectors, $\bar{z}$ is a reference vector of environmental variables, and $\bar{t}$ is reference time period. Assumption D1 says that all inputs, outputs and environmental variables involved in the production process are observed. Assumptions D2–D4 say that outputs inputs and environmental variables are measured without error. To see this, suppose assumptions T1–T15 and D1 hold. Then the output distance function is given by the Cobb-Douglas function (3) with $M = M^*$, $N = N^*$ and $J = J^*$. In this case, D2–D4 will be satisfied if and only if (respectively):

$$\ln Q(q_{it}) = \sum_{n=1}^{N} \alpha_n \ln(q_{nit}/\bar{q}_n), \quad (8)$$

$$\ln X(x_{it}) = \sum_{m=1}^{M} \lambda_m \ln(x_{mit}/\bar{x}_m) \quad (9)$$

$$\text{and} \qquad \ln Z(z_{it}) = \sum_{j=1}^{J} \xi_j \ln(z_{jit}/\bar{z}_j) \quad (10)$$

where $\lambda_m \equiv \beta_m/r$, $\xi_j \equiv \rho_j/\sum_k \rho_k$ and $\sum_n \alpha_n = \sum_m \beta_m = \sum_j \xi_j = 1$. In this case, the Cobb-Douglas function (3) can be rewritten as

$$\ln D_O^t(x_{it}, q_{it}, z_{it}) = \ln Q_{it} - \ln A(t) - \eta \ln Z_{it} - r \ln X_{it} \quad (11)$$

where $Q_{it} \equiv Q(q_{it})$ is an aggregate output, $X_{it} \equiv X(x_{it})$ is an aggregate output, $Z_{it} \equiv Z(z_{it})$ is an aggregate environmental variable, $A(t) \equiv 1/D_O^t(\bar{x}, \bar{q}, \bar{z})$ is variable that measures the rate of technical change, and $\eta \equiv \sum_k \rho_k$ is an elasticity that measures the percent increase in outputs associated with a one percent increase in all environmental variables. Note that equations (8)–(10) can be used to easily disaggregate outputs, inputs and environmental variables into finer categories. For example, let k_{it}, l_{it}, m_{it} and o_{it} denote vectors of capital, labour, materials and *all* other inputs involved in the production process. Then, the Cobb-Douglas function (3) can also be rewritten as

$$\begin{aligned} \ln D_O^t(x_{it}, q_{it}, z_{it}) = \ln Q_{it} - \ln A(t) - \eta \ln Z_{it} - \beta_K \ln K_{it} - \beta_L \ln L_{it} \\ - \beta_M \ln M_{it} - \beta_O \ln O_{it} \end{aligned} \quad (12)$$

where $K_{it} \equiv K(k_{it})$, $L_{it} \equiv L(l_{it})$, $M_{it} \equiv M(m_{it})$ and $O_{it} \equiv O(o_{it})$ are aggregate capital, labour, materials and other inputs respectively. Importantly, if all variables are observed and measured without error (i.e., if D1–D4 hold) then equations (3), (11) and (12) are all equivalent representations of a Cobb-Douglas technology.

5. Efficiency and Productivity Change

O'Donnell [11, pp. 256–257] defines the TFP of firm i in period t as:

$$TFP_{it} \equiv Q_{it}/X_{it} \tag{13}$$

where $Q_{it} \equiv Q(q_{it})$ is an aggregate output, $X_{it} \equiv X(x_{it})$ is an aggregate input, and $Q(.)$ and $X(.)$ are nonnegative (NN), nondecreasing (ND), linearly homogeneous (HD1) scalar aggregator functions. Measuring TFP as the ratio of an aggregate output to an aggregate input is an idea that can be traced back at least as far as Jorgenson and Griliches [6] and Nadiri [8]. However, the distinguishing feature of the O'Donnell [11] definition is that the aggregator functions are NN, ND and HD1. Other authors are silent about the properties of their aggregator functions, and often use aggregator functions that do not satisfy these properties. Such indexes are not 'proper' in the sense of O'Donnell [9]. For example, Nadiri [8, p. 1138, eq. (1)(b)] uses an input aggregator function that is not HD1. Consequently, his TFP index does not satisfy a proportionality axiom. The proportionality axiom says, for example, that if all inputs double and all outputs double then the productivity index should take the value one (indicating that average product has not changed).

In practice, information concerning technologies, markets and behaviour can be used to inform the choice of input and output aggregator functions. For example, if outputs and inputs are strongly disposable (T7 and T8) then the aggregator functions in D2 and D3 are NN, ND and HD1 in outputs and inputs respectively. If the technology assumptions T9–T15 are also satisfied then (the logarithms of) these aggregator functions are given by equations (8) and (9). If the market and behavioural assumptions B1–B3 are also satisfied then (the antilogarithms) of (8) and (9) become

$$Q(q_{it}) = \prod_{n=1}^{N} \left(\frac{q_{nit}}{\bar{q}_n} \right)^{r_n} \tag{14}$$

$$\text{and} \qquad X(x_{it}) = \prod_{m=1}^{M} \left(\frac{x_{mit}}{\bar{x}_m} \right)^{s_m} \tag{15}$$

where r_n and s_m are expected-profit-maximising revenue and cost shares. The fact that these shares (or weights) are observation-invariant means that associated output quantity, input quantity and TFP indexes are proper. Of course, in practice, observed revenue and cost shares are rarely observation-invariant, for several reasons. Aside from bounded rationality and/or regulations that may prevent firms from choosing inputs and outputs freely, revenue shares may vary because firms face different input prices and/or form different expectations about unknown future output prices. In such cases, the menu of admissible aggregator functions (i.e., those that are NN, ND and HD1) includes

$$Q(q_{it}) = \prod_{n=1}^{N} \left(\frac{q_{nit}}{\bar{q}_n} \right)^{\bar{r}_n} \tag{16}$$

$$\text{and} \qquad X(x_{it}) = \prod_{m=1}^{M} \left(\frac{x_{mit}}{\bar{x}_m} \right)^{\bar{s}_m} \tag{17}$$

where $\bar{r}_n$ and $\bar{s}_m$ are representative revenue and cost shares (e.g., as the notation suggests, sample mean shares). These aggregator functions can be used to construct proper output and input quantity indexes. On the input side, for example, the index that compares the inputs of firm i in period t with the inputs of firm k in period s is:

$$XI_{ksit} \equiv \frac{X(x_{it})}{X(x_{ks})} = \prod_{m=1}^{M} \left(\frac{x_{mit}}{x_{mks}} \right)^{\bar{s}_m}. \tag{18}$$

This is the quantity analogue of a geometric Young price index. It it is a proper index in the sense that it satisfies the nine basic axioms listed in O'Donnell [9], including transitivity, identity and circularity. If sample arithmetic averages are used as representative cost shares and there are only two observations in the sample then it collapses to a binary Törnqvist index. The Törnqvist index is not a proper index for multiple comparisons because it does not satisfy the circularity axiom. For details see O'Donnell [9].

One of the aims of this paper is to measure efficiency and productivity change. The index that compares the TFP of firm i in period t with the TFP of firm k in period s is:

$$TFPI_{ksit} = \frac{TFP_{it}}{TFP_{ks}} = \frac{Q_{it}/X_{it}}{Q_{ks}/X_{ks}} = \frac{QI_{ksit}}{XI_{ksit}} \tag{19}$$

where $QI_{ksit} \equiv Q_{it}/Q_{ks}$ and $XI_{ksit} \equiv X_{it}/X_{ks}$ are output and input quantity indexes respectively. All TFP indexes that can be written in terms of aggregate quantities in this way can be decomposed into measures of technical change, environmental change and various types of efficiency change. For details, see O'Donnell [10][11][12][13]. To illustrate, if assumptions T1–T15 and D1-D4 hold then the production technology can be represented by the aggregate production function (12). Equivalently:

$$Q_{it} = A(t) Z_{it}^{\eta} K_{it}^{\beta_K} L_{it}^{\beta_L} M_{it}^{\beta_M} O_{it}^{\beta_O} e^{-u_{it}} \tag{20}$$

where $e^{-u_{it}} \equiv D_O^t(x_{it}, q_{it}, z_{it}) \leq 1$ is an output-oriented measure of technical efficiency that can be traced back as far as Debreu [4] and Farrell [5].[1] If inputs are aggregated using the geometric Young aggregator function (17), for example, then $X_{it} \propto K_{it}^{\bar{s}_K} L_{it}^{\bar{s}_L} M_{it}^{\bar{s}_M} O_{it}^{\bar{s}_O}$ and

$$TFPI_{ksit} = \frac{A(t)}{A(s)} \left(\frac{Z_{it}}{Z_{ks}}\right)^{\eta} \left(\frac{K_{it}}{K_{ks}}\right)^{\delta_K} \left(\frac{L_{it}}{L_{ks}}\right)^{\delta_L} \left(\frac{M_{it}}{M_{ks}}\right)^{\delta_M} \left(\frac{O_{it}}{O_{ks}}\right)^{\delta_O} \left(\frac{e^{-u_{it}}}{e^{-u_{ks}}}\right) \tag{21}$$

where $\delta_K \equiv (\beta_K - \bar{s}_K)$, $\delta_L \equiv (\beta_L - \bar{s}_L)$, $\delta_M \equiv (\beta_M - \bar{s}_M)$ and $\delta_O \equiv (\beta_O - \bar{s}_O)$ are the differences between output elasticities and representative cost shares. The first two terms on the right hand side of (21) are measures of technical change (ΔT) and environmental change (ΔZ): these components measure movements in the production frontier. The last term on the right hand side of (21) is a measure of output-oriented technical efficiency change (ΔOTE): this component measures movements towards or away from the frontier. The middle terms are the contributions of capital, labour, materials and other inputs to output-oriented scale-mix efficiency change (ΔOSME= ΔK×ΔL×ΔM×ΔO): these components measure movements around the frontier surface to capture economies of scale and scope.

Equation (21) includes the growth accounting measure of TFP change as a special case. To see this, suppose the technology is a variable returns to scale homothetic Cobb-Douglas technology (i.e., T9 and T12 hold with $r \neq 1$), that technical change is Hicks neutral (T10 and T11), and that all variables involved in the production process are observed and measured without error (D1–D4). Under these assumptions, the relationship between inputs, outputs and environmental variables is given by (11). If there are no environmental variables involved in the production process (i.e., $\eta = 0$) and firms are technically efficient (e.g., because they maximise profit) then (11) says $0 = \ln Q_{it} - \ln A(t) - r \ln X_{it}$. It follows that $\ln(Q_{it}/X_{it}) = \ln A(t) + (r-1)\ln X_{it}$ and that

$$TFPI_{ksit} = \frac{A(t)}{A(s)} \left(\frac{X_{it}}{X_{ks}}\right)^{r-1}. \tag{22}$$

If $r = 1$ then and only then TFP change is equal to technical change: $TFPI_{ksit} = A(t)/A(s)$. This is the Solow [16] residual that forms the basis for measuring productivity change in the growth accounting literature. If $r \neq 1$ then $A(t)/A(s) = TFPI_{ksit}(X_{it}/X_{ks})^{(1-r)}$ and the Solow residual is a biased measure of TFP change.

[1] The input-oriented measure is $1/D_I^t(x_{it}, q_{it}, z_{it}) \leq 1$. Debreu [4] refers to ITE_{it} as the coefficient of resource utilisation.

6. ECONOMETRICS

The TFP index on the left-hand side of (21) can be calculated without knowing anything about the production frontier. However, identifying the technical, environmental and efficiency change components on the right-hand side of (21) requires knowledge of the frontier parameters $A(t)$, η, β_K, β_L, β_M and β_O. The growth accounting approach to estimating these parameters involves assumptions that are so restrictive that they are almost certainly false (i.e., T9–T12, T15, D1–D4, profit maximisation, and $r \neq 1$). Under these assumptions, η, β_K, β_L, β_M and β_O can be computed using index number methods, and $A(t)$ can be computed as a residual. There is little or no formal testing of the assumptions, and changes in the residual are interpreted as changes in TFP.

In contrast, the econometric approach to estimating the unknown prameters does not require any restrictive assumptions about technologies, markets or behaviour. The only assumptions that are required are assumptions that ensure the output distance function exists (i.e., T1–T6). To see this, and with a view to the empirical application in Section 7, observe that equation (20) can be rewritten in the form:

$$\ln Q_{it} = \ln A(t) + \eta \ln Z_{it} + \beta_K \ln K_{it} + \beta_L \ln L_{it} + \beta_M \ln M_{it} + v_{it} - u_{it} \qquad (23)$$

where $u_{it} \equiv -\ln D^t_O(x_{it}, q_{it}, z_{it}) \geq 0$ is an output oriented technical inefficiency effect, and v_{it} is a residual that accounts for any specification and/or measurement errors involved in deriving (20). In the present case, this so-called noise component is $v_{it} \equiv \ln Q_{it} - \ln A(t) - \eta \ln Z_{it} - \beta_K \ln K_{it} - \beta_L \ln L_{it} - \beta_M \ln M_{it} - \ln D^t_O(x_{it}, q_{it}, z_{it})$. If assumptions T7–T15, B1–B3 and D1–D4 are true then this noise component vanishes and equation (23) collapses to the deterministic frontier of Aigner and Chu [1]. However, if any of these assumptions are not true, then v_{it} is non-zero.

The least squares approach to estimating $A(t)$, η, β_K, β_L and β_M involves finding a flexible model specification (i.e., finding a weak set of assumptions) that will make v_{it} reasonably small. It is common to assume that one or more variables in the model are stochastic. Then v_{it} is stochastic and (23) corresponds to the stochastic frontier model of Meeusen and van den Broeck [7] and Aigner, Lovell and Schmidt [2]. To estimate the parameters of stochastic frontier models by least squares methods it is common to assume that the composite error term $\varepsilon_{it} \equiv v_{it} - u_{it}$ is a random variable with the following properties:

E1: $E(\varepsilon_{it}) = -\mu \leq 0$ for all i and t (constant mean),
E2: $Var(\varepsilon_{it}) = \sigma^2_\varepsilon$ for all i and t (homoskedasticity),

E3: $Cov(\varepsilon_{it}, \varepsilon_{ks}) = 0$ if $i \neq k$ or $t \neq s$ (serially and cross-sectionally uncorrelated) and
E4: $E(\varepsilon_{it}|Z_{it}) = E(\varepsilon_{it}|K_{it}) = E(\varepsilon_{it}|L_{it}) = E(\varepsilon_{it}|M_{it}) = -\mu$ (exogeneity).

Assumptions E1 and E2 say that the composite error terms have the same mean and variance. Assumption E3 says they are independently distributed. Assumption E4 implies that the explanatory variables are uncorrelated with the error terms (i.e., they are exogenous). If these assumptions hold then the ordinary least squares (OLS) estimator is the best linear unbiased estimator of the slope parameters in (23). If E2 or E3 do not hold then it is still unbiased but no longer efficient. However, if E4 is not satisfied then OLS is biased and inconsistent. Thus, the key assumption that needs to be satisfied in order to use the OLS estimator is E4. This assumption can be tested using a Hausman test. If E4 is rejected then consistent estimates of the unknown parameters can be obtained using two-stage least squares (2SLS).

7. Productivity Change in the Australian Economy

This section reports estimates the drivers of TFP change in $I = 18$ sectors of the Australian economy for the $T = 38$ years from 1970 to 2007. The data were drawn from the EU-KLEMS database, and least squares methods were used to estimate the unknown parameters in equation (23). The technical change component was modelled as $\ln A(t) = \gamma_0 + \gamma_1 t$. There were no environmental variables in the dataset. Therefore, to account for the fact that environmental variables differ across sectors (e.g., rainfall is involved in the production of wheat but not generally involved in the production of steel), sectoral dummy variables were introduced into the model. The introduction of sectoral dummy variables is equivalent to setting $Z_{it} = Z_i$ in (23). Technologies also differ across sectors (e.g., the techniques used to transform wheat into flour differ from the techniques used to transform coal into electricity). The sectoral dummy variables also account for these differences in technologies. With this treatment of technical change and environmental effects, equation (23) becomes

$$\ln Q_{it} = \gamma_0 + \gamma_1 t + \sum_{j=2}^{I} \rho_j S_{jit} + \beta_K \ln K_{it} + \beta_L \ln L_{it} + \beta_M \ln M_{it} + v_{it} - u_{it} \qquad (24)$$

where $\rho_i \equiv \eta \ln Z_i$, and $S_{jit} = 1$ if $j = i$, and 0 otherwise. The associated measure of TFP change is:

$$TFPI_{ksit} = \frac{\exp\left(\gamma_1 t + \sum_{j=2}^{I} \rho_j S_{jit}\right)}{\exp\left(\gamma_1 s + \sum_{j=2}^{I} \rho_j S_{jks}\right)} \left(\frac{K_{it}}{K_{ks}}\right)^{\delta_K} \left(\frac{L_{it}}{L_{ks}}\right)^{\delta_L} \left(\frac{M_{it}}{M_{ks}}\right)^{\delta_M} \left(\frac{e^{-u_{it}}}{e^{-u_{ks}}}\right) \left(\frac{e^{v_{it}}}{e^{v_{ks}}}\right). \tag{25}$$

The first component on the right-hand side of this equation is a combined measure of technical and environmental change (ΔTZ). The remaining components are the capital, labour and materials components of scale-mix efficiency change (ΔK×ΔL×ΔM), a measure of output-oriented technical efficiency change (ΔOTE), and the change in statistical noise.

7.1. **Data.** With the exception of the price of capital, all data were drawn from the March 2011 release of the EU-KLEMS database. The capital price is the average of the annual yield on Australian government 10-year bonds reported by the Reserve Bank of Australia. For details on the construction of the EU-KLEMS database, see O'Mahony and Timmer [14]. Variables in the dataset are listed in Table 1 (EU-KLEMS variable names are in parentheses). Output and input quantity series were derived by dividing values by prices. Sector codes are listed in Table 2.

7.2. **OLS Estimates.** OLS stepwise regression was used to determine if any sectoral dummy variables should be omitted from the model given by (24). The OLS estimates at the end of the stepping sequence are reported in the first three columns of Table 3. If assumption E4 does not hold then the OLS estimator is biased and inconsistent. Common sources of so-called endogeneity bias are (i) omitted variables (D1), (ii) measurement errors (D2–D4), and (iii) simultaneity (e.g., that inputs and outputs are determined within a system of equations of the type described in Section 3). This hypothesis was tested using a Hausman test. The optimisation problem described in Section 3 suggested that input prices in period t and output prices in period $t-1$ should be used as instruments. Using these instruments, the Hausman test leads to the rejection of E4 at very small levels of significance (the p-value is less than 0.0000), leading to the conclusion that assumptions T1–T15 and D1–D4 do not all hold. Thus, the OLS estimates are unreliable.

7.3. **2SLS Estimates.** Two-stage least squares estimates of the unknown parameters are reported in the last three columns of Table 3. The estimated coefficient of the time trend is negative (but not statistically significant) suggesting that steady productivity gains through technical progress have been more than offset by steady deteriorations in the production environment. The estimated capital, labour and materials elasticities are positive, which is consistent with strong disposability of inputs and outputs (i.e., T7 and T8 would not be rejected at levels of significance less than 0.0000). The estimated elasticity of scale is $r = 0.6888$ indicating that the technology exhibits decreasing returns to scale. The null hypothesis of constant returns to scale (i.e., T9 with $r = 1$) is rejected at levels of significance less than 0.0000. The implication is that the Solow residual is an unreliable estimate of TFP change.

7.4. **Estimates of Productivity and Efficiency Change.** The sample average input cost shares for capital, labour and materials are $\bar{s}_K = 0.0209$, $\bar{s}_L = 0.3624$ and $\bar{s}_M = 0.6167$. These representative cost shares and the 2SLS parameter estimates reported in Table 3 were used to compute the technical, environmental and scale-mix efficiency change components in (25). The measure of technical efficiency change was calculated as the change in corrected ordinary least squares (COLS) estimates of output oriented technical efficiency. The noise component was then calculated as a residual. An artefact of this COLS-based procedure is that the estimated noise component is zero. Thus, the estimated technical efficiency change component reported in this section could also be viewed as a combined estimate of changes in both technical efficiency and noise.

Estimates of TFP change, technical and environmental change, output oriented scale-mix efficiency change, and technical efficiency change are presented in Figures 1 to 4. All the measures presented in these figures are indexes that use agriculture ($i = 1$) in 1970 ($t = 1$) as the reference sector/period. The estimates presented in Figures 1 to 4 suggest that (respectively)

(1) the community, social and personal services sector ($i = 18$) was the least productive sector in the Australian economy from 1970 to 2007; the hotel/restaurant sector ($i = 13$) was the most productive sector during the 1970s; and the electricity/gas/water sector ($i = 11$) was the most productive sector from the early 1990s until almost the end of the sample period;

(2) hotels and restaurants operate in the most favourable production environments and/or use the most productive technologies; mining/quarrying firms and elecricity/gas/water suppliers operate in the least favourable production environments and/or use the least productive technologies;
(3) the mining/quarrying sector was the most scale-mix efficient sector from the early 1970s until the early 1990s, and the electricity/gas/water sector was the most scale-mix efficient sector from the early 1990s until almost the end of the sample period; and
(4) hotels and restaurants were among the most technically efficient firms in the 1970s, and were among the least technically efficient firms from the early 1990s until the end of the sample period.

For comparison purposes, Figures 1 to 4 were deliberately drawn with the same vertical scale. A comparison the four figures reveals that the most important source of productivity differences across sectors has been differences in scale-mix efficiency (i.e., returns to scale and scope). For many sectors, changes in scale mix-efficiency have also been the most important driver of productivity change over time. To see this, the components of TFP change in the mining/quarrying sector have be consolidated into Figure 5. It is evident from this figure that changes in the productivity of mining/quarrying firms were driven by changes in both technical and scale-mix efficiency in the early 1970s, by changes in technical efficiency during the late-1970s and 1980s, and by changes in scale-mix efficiency since the mid-1990s until the end of the sample period.

To obtain further insights into these results, the components of scale-mix efficiency change in the mining/quarrying sector are presented in Figure 6. It is evident from this figure that long-term changes in scale-mix efficiency are being driven by 'capital deepening', but short- to medium-term changes are being driven by changes in the labour input. Even deeper insights into the drivers of scale-mix efficiency change (and ultimately TFP change) are provided in Figure 7. This figure presents capital and labour input quantity indexes (KI and LI) as well as the capital and labour components of scale-mix efficiency change (dK and dL). It is evident from this figure (and the capital index series that is off the scale) that capital investment is positively correlated with the capital component of scale-mix efficiency change (the correlation coefficient between KI and dK is 0.67). On the other hand, the labour input is strongly negatively correlated with labour component of scale-mix efficiency change (the correlation coefficient is -0.79).

This suggests that higher employment, not higher capital investment, has been driving the decline in productivity in this sector.[2]

8. Conclusion

TFP indexes are important measures of economic performance. Common economic assumptions concerning technologies, markets, behaviour and data are often used to inform the selection of meaningful index formulas. For example, if (i) technologies are constant-returns-to-scale Cobb-Douglas technologies, (ii) markets are perfectly competitive, (iii) firms maximise profits, and (v) all inputs and outputs involved in the production process are observed and measured without error, then a meaningful measure of productivity change is the Solow [16] residual. The Solow residual is the bedrock of the growth accounting approach to measuring productivity change. This paper shows that if the assumptions underpinning such an index are not true, then the index will be a biased estimator of productivity change. The practical implication is that, as far as possible, the assumptions that are used to motivate different productivity indexes should be subjected to empirical testing.

This paper uses a geometric Young TFP index to measure productivity change in eighteen sectors of the Australian economy over the period 1970 to 2007. One of the advantages of this index is that it can be computed without making any assumptions concerning technologies, markets, behaviour or data. However, these types of assumptions are needed if the index is to be decomposed into measures of technical change, environmental change and efficiency change. This paper tests but does not reject assumptions that are sufficient to ensure consistent estimation of these components. On the other hand, key assumptions underpinning the use of the Solow [16] residual as a measure of TFP change are rejected at even the smallest levels of signficance.

The empirical results reveal that changes in scale-mix efficiency have been the main driver of TFP change over the sample period. Scale-mix efficiency is a measure of how well a firm is capturing economies of scale and scope. Changes in scale-mix efficiency usually occur when rational optimising firms change the scale of their operations and/or their output and input mixes in response to changes in prices. In the case of the Australian mining industry, for example, significant changes in scale-mix efficiency in the last decade are found to have been associated with increases in the use of labour. This

[2]Computationally, this is because the geometric Young input quantity index is giving relatively little weight to the capital input, albeit more weight than many other input indexes.

finding is consistent with the view that TFP declines in the sector have been driven by rational optimising behaviour on the part of profit-seeking firms. The policy implication is that no policy response is necessary.

TABLE 1. Variables in the KLEMS Dataset

Variable	Description
OBS	observation
SECT	sector code
YR	year
R	value of output (AUD mill.) (GO)
CK	capital compensation (AUD mill.) (CAP)
CL	labour compensation (AUD mill.) (LAB)
CM	materials, energy and services input cost (AUD mill.) (II)
P	output price index (GO_P)
WK	yield on 10-year government bonds (%)
WL	labour compensation per hour worked (LAB_AVG)
WM	materials, energy and services price index (II_P)

TABLE 2. Sector Codes in the KLEMS Dataset

SECT	Sector
1	Agriculture, hunting, forestry and fishing
2	Mining and quarrying
3	Food, beverages and tobacco
4	Textiles, leather and footwear
5	Wood and of wood and cork
6	Pulp, paper, printing and publishing
7	Chemical, rubber, plastics and fuel
8	Other non-metallic minerals
9	Basic metals and fabricated metal
10	Machinery n.e.c
11	Electricity, gas and water supply
12	Wholesale and retail trade
13	Hotels and restaurants
14	Transport and storage
15	Post and telecommunications
16	Financial intermediation
17	Real estate, renting and business activities
18	Community, social and personal services

TABLE 3. Parameter Estimates

Variable	OLS Estimate	OLS St. Error	OLS p-value	2SLS Estimate	2SLS St. Error	2SLS p-value
Constant	1.0850	0.0460	0.0000	1.5015	0.1505	0.0000
T	0.0059	0.0011	0.0000	-0.0032	0.0028	0.2644
S2	-0.0395	0.0142	0.0056	-0.1178	0.0236	0.0000
S4	0.0767	0.0138	0.0000	0.0117	0.0194	0.5477
S5	0.1311	0.0160	0.0000	0.0561	0.0248	0.0239
S8	0.1455	0.0162	0.0000	0.0500	0.0246	0.0431
S10	0.0888	0.0140	0.0000	0.0350	0.0191	0.0667
S11	-0.0306	0.0145	0.0351	-0.1054	0.0205	0.0000
S12	-0.0876	0.0209	0.0000	0.0536	0.03371	0.1120
S13	0.0965	0.0134	0.0000	0.1173	0.0244	0.0000
S14	-0.0894	0.0140	0.0000	-0.0368	0.0181	0.0427
S16	-0.0431	0.0131	0.0011	0.0063	0.0201	0.7530
S17	-0.0464	0.0161	0.0041	0.0529	0.0241	0.0286
S18	-0.1844	0.0221	0.0000	-0.0095	0.0383	0.8046
$\ln K$	0.0049	0.0063	0.4383	0.0654	0.0233	0.0052
$\ln L$	0.0758	0.0081	0.0000	0.0052	0.0145	0.7223
$\ln M$	0.6297	0.0069	0.0000	0.6183	0.0397	0.0000
r				0.6888	0.0247	0.0000
$1-r$				0.3111	0.0247	0.0000

FIGURE 1. TFP Change

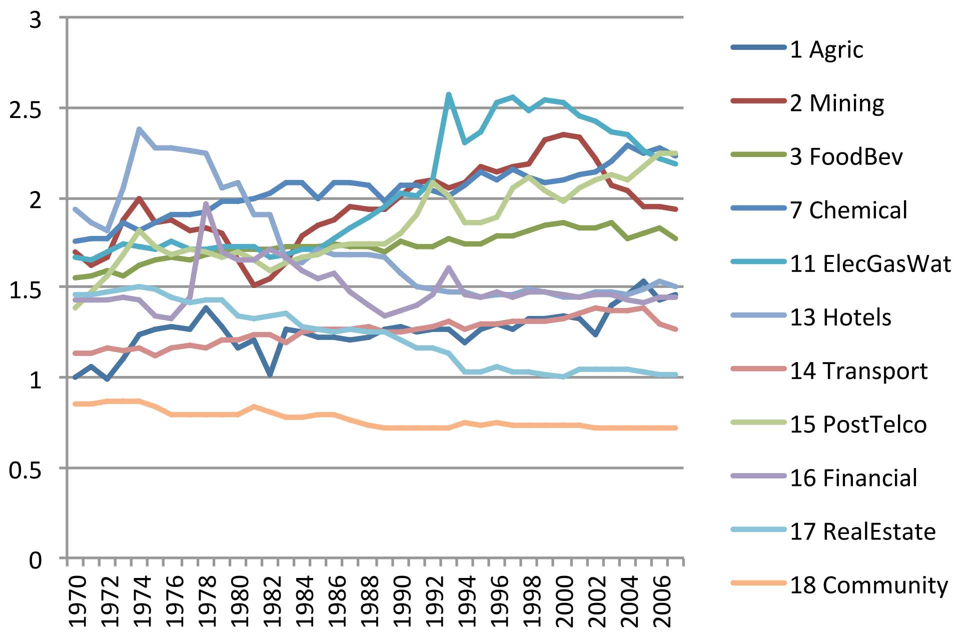


FIGURE 2. Technical and Environmental Change

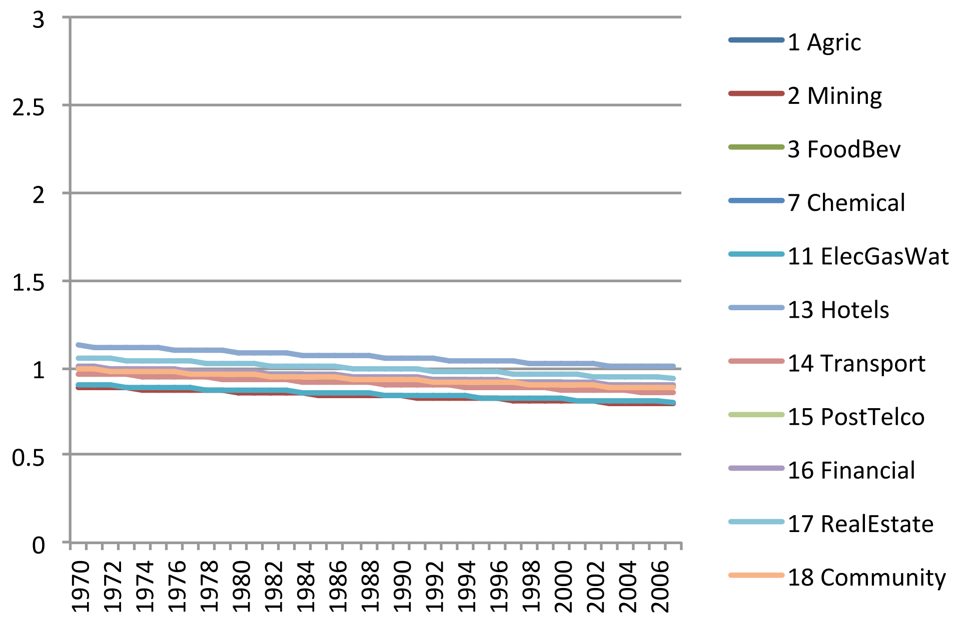


FIGURE 3. Scale-Mix Efficiency Change

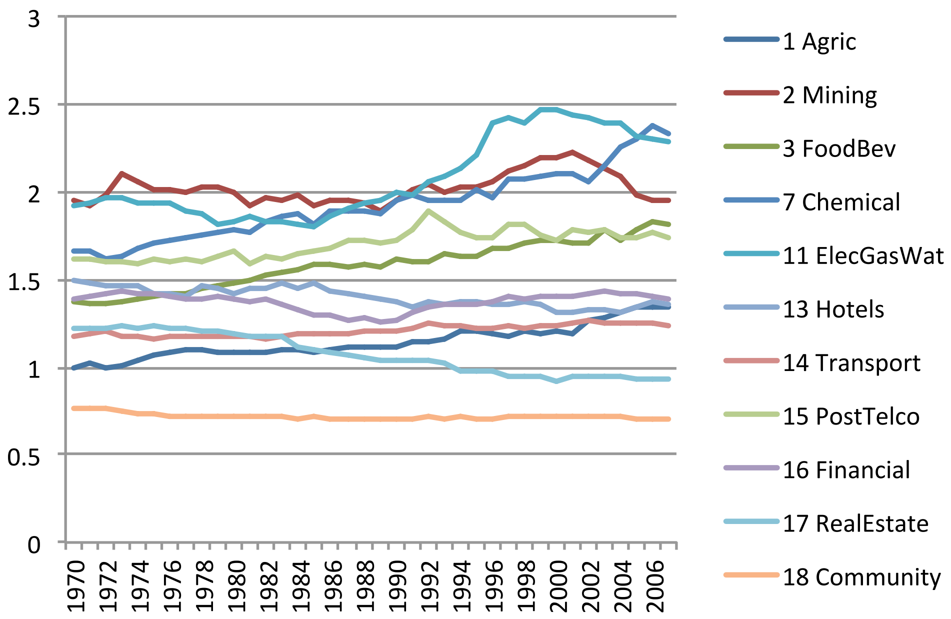


FIGURE 4. Technical Efficiency Change

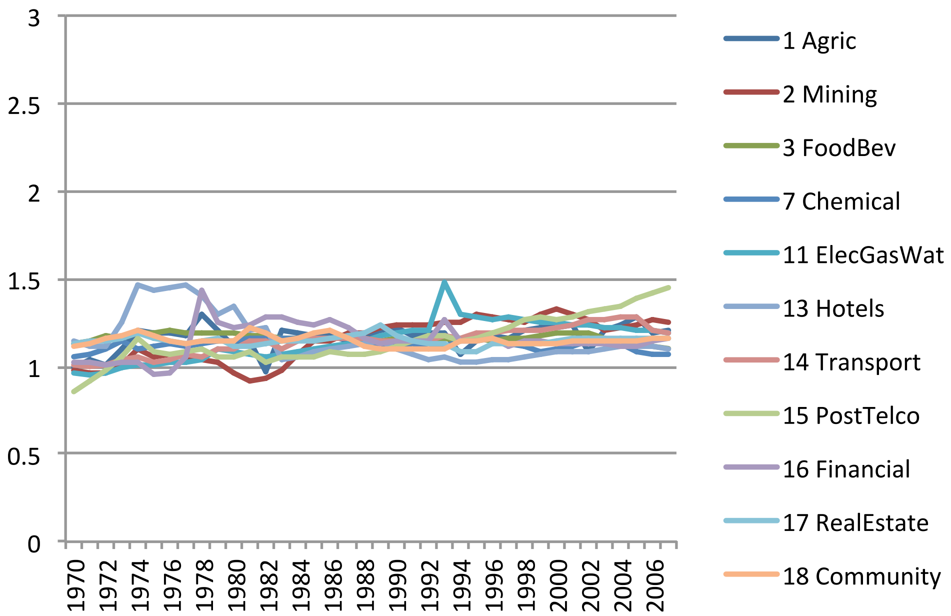


FIGURE 5. Components of TFP Change: Mining and Quarrying

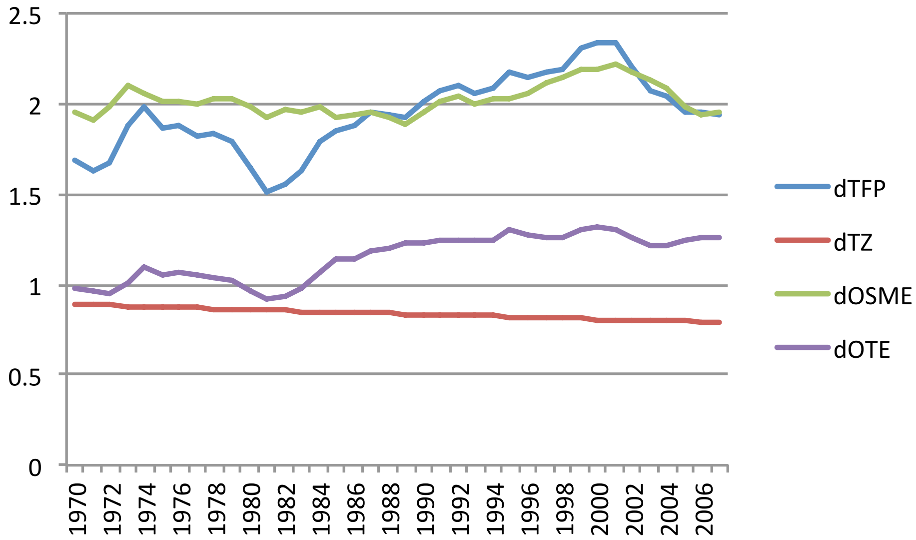


FIGURE 6. Components of OSME Change: Mining and Quarrying

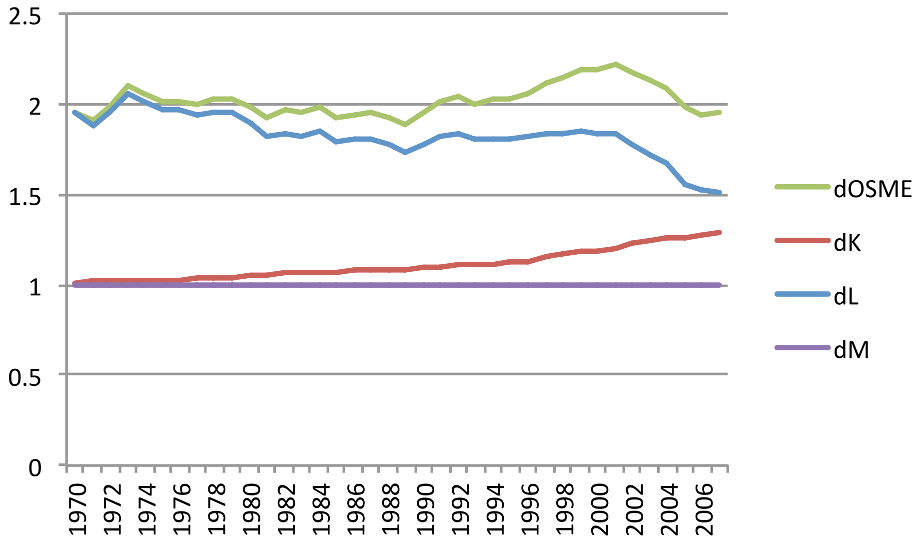
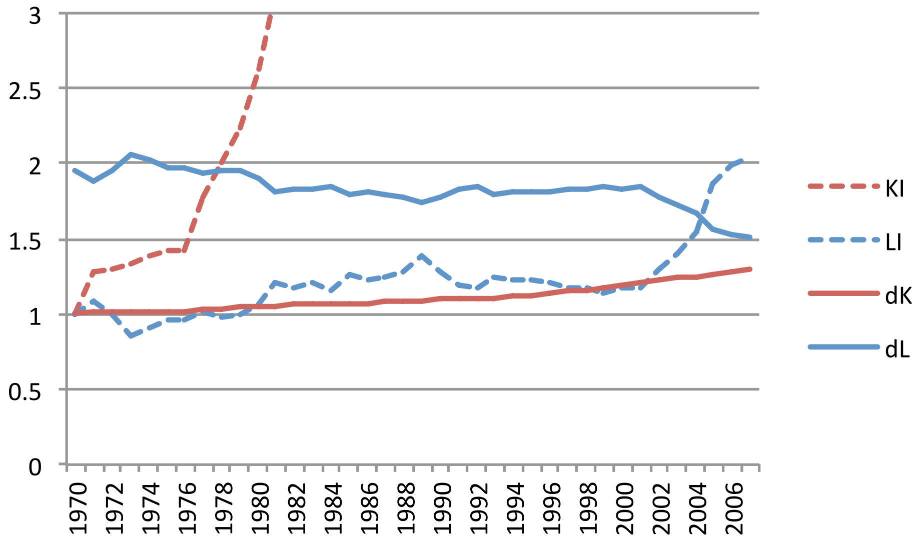


FIGURE 7. Capital and Labour Change: Mining and Quarrying



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