



Measuring Impoverishment: An Overlooked Dimension of Fiscal Incidence

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Abstract

The effect of taxes and benefits on the poor is usually measured using standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity and horizontal inequity. However, these measures can fail to capture an important aspect: that some of the poor are made poorer (or some of the non-poor made poor) by the tax-benefit system. We call this impoverishment and formally establish the relationships between impoverishment, stochastic dominance tests, horizontal inequity, and progressivity measures. The directional mobility literature provides a useful framework to measure impoverishment. We propose using a transition matrix and income loss matrix, and establish a mobility dominance criterion to compare alternate tax-benefit systems. We illustrate with data from Brazil.

Keywords: stochastic dominance, poverty, fiscal incidence, mobility

JEL: I32, H22

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1. Introduction

The effect of taxes and benefits on the poor is usually measured using standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity and vertical and horizontal inequity. Here we argue that the equity assessment of a tax and benefit system needs to incorporate another dimension: the extent of impoverishment induced by it. Stochastic dominance tests (Atkinson, 1987; Foster and Shorrocks, 1988) do not take into account individuals' initial position, so it is possible for poverty to unambiguously fall, while at the same time some pre-tax poor (nonpoor) are further impoverished (made poor) by the fiscal system. Thus, standard measures can fail to capture impoverishment caused by a tax and benefit system because decreases in income of some poor may be (more than) compensated by income increases of other poor.

We posit that the extent to which a tax and transfer system impoverishes the poor (or makes non-poor people poor) is valuable information for the analyst and the policymaker. Policymakers can use this information to modify government interventions or introduce new mechanisms that reduce impoverishment, if not completely eliminate it. In the next section, we formally define impoverishment and establish its relationship with traditional measures of inequality and poverty, horizontal inequity, stochastic dominance tests, and progressivity. In Section 3, we propose using directional mobility measures to assess the degree of impoverishment of a tax-benefit system. Although there are many measures of directional income mobility (see Fields (2008) for a survey), we begin with a Markovian transition matrix (first used to measure income mobility in Champernowne (1953) and mobility induced by taxes in Atkinson (1980)) and an income loss matrix because the information they contain is straightforward and easy to convey to policymakers. Since these matrices are sensitive to the choice of the number of groups and the cut-offs that delineate groups, we propose complementing them with a downward mobility curve (Foster and Rothbaum, 2012) that measures impoverishment over a range of poverty lines. In Section 4, we use household survey data from Brazil to illustrate the failures of standard measures to capture impoverishment and to apply the transition and income loss matrices, as well as the downward mobility curves. Section 5 explores extensions of our results in two areas: social welfare and multidimensional poverty. Section 6 presents concluding remarks.

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2. Impoverishment

Standard measures of the effect of taxes and transfers on the poor, such as poverty indicators (including distribution-sensitive measures such as the squared poverty gap), inequality indicators, stochastic dominance tests, Lorenz dominance tests, measures of progressivity, and measures of vertical and horizontal inequity leave out important information about how the poor are affected by fiscal policy. For example, we can observe: first order stochastic dominance—which implies unambiguous reductions in poverty, second and higher order stochastic dominance, and generalized Lorenz dominance; the distribution of income becoming less unequal; progressive net taxes (i.e., taxes minus transfers); low or no horizontal inequity; but some of the poor becoming substantially poorer. The extent to which a tax and transfer system impoverishes the poor or makes non-poor people poor is valuable information for the analyst and policymaker.

In this section we formally define impoverishment and establish its relationship with traditional measures of poverty, progressivity, and horizontal inequity, as well as stochastic dominance tests, which are frequently used to show that poverty and inequality have unambiguously fallen (see Atkinson, 1987; Foster and Shorrocks, 1988).

First, in Proposition 1 we show that there can be weak stochastic dominance of the post-tax distribution with respect to the pre-tax distribution over the domain from zero to the maximum poverty line even when impoverishment has occurred. This can occur only if there is re-ranking among the poor. When the losses of some poor are compensated by gains of other even poorer individuals, impoverishment goes unnoticed by stochastic dominance tests (and hence, also by standard poverty measures) because they are anonymous with respect to initial income. Thus, stochastic dominance is not a sufficient condition to establish the absence of impoverishment. Proposition 2 shows that if the post-tax and transfer distribution does not weakly first order stochastically dominate the pre-tax and transfer distribution over the relevant domain, there was impoverishment of at least one poor person or of some non-poor person into poverty. Thus, the absence of weak stochastic dominance of the post-tax and transfer distribution over the pre-tax and transfer distribution is a sufficient condition to establish that impoverishment has occurred. In order to be sure that no impoverishment has occurred, a sufficient condition is the simultaneous fulfillment of weak stochastic dominance (of the post-tax and transfer distribution over the pre-tax and transfer distribution) *and* no re-ranking over the domain from zero to the maximum poverty line. This is shown in proposition 3.

Formally, denote the well-being space Ω . For ease of exposition, we will take income as our measure of well-being, where income takes non-negative values and is bounded: $\Omega \subset \mathbb{R}_+$ and $\sup\{\Omega\} < \infty$. However, additional measures of well-being could be incorporated into the analysis to accommodate multi-dimensional poverty measures; see section 5.2. Denote individual income before taxes and transfers by $y_i^0 \in \Omega$ and individual income after taxes and transfers by $y_i^1 \in \Omega$ for each $i \in \mathcal{S}$ where \mathcal{S} is the set of individuals in society. The set Ω can represent household per capita incomes or can account for differences in need among individuals (adjusting, for example, for different caloric needs based on age and economies of scale within households), in which case we assume there exists a function $\varphi: \mathbb{R}_+ \rightarrow \Omega$ which maps household per capita income into equalized income. The cumulative

distributions of the before and after taxes and transfers income concepts are non-decreasing functions $F_0: \Omega \rightarrow [0,1]$ and $F_1: \Omega \rightarrow [0,1]$.

We create the vectors \mathbf{y}^0 and \mathbf{y}^1 which contain as elements each individual's income before and after taxes and transfers, respectively.² In both vectors, individuals are ranked in ascending order of pre-tax income (in other words, if individual i occupies position k of \mathbf{y}^0 , that same individual will also occupy position k of \mathbf{y}^1 ; even if re-ranking occurs, the order of the \mathbf{y}^1 vector reflects the pre-tax income ranking). To be clear, the cumulative distribution function F_1 does re-rank individuals (unlike the vector \mathbf{y}^1): denoting $F(\mathbf{y}) \equiv \mathbf{p}$, if there is re-ranking caused by taxes and transfers, a certain value of \mathbf{p} will not necessarily correspond to the same individual in each distribution. The poverty line, which lies in the well-being space, will be denoted $\mathbf{z} \in \Omega$.³

Definition 1. There is *impoverishment* if $y_i^1 < y_i^0$ and $y_i^1 < z$ for some individual i . In other words, the individual could have been poor before taxes and transfers and been made poorer by the fiscal system, or non-poor before taxes and transfers but poor after.

Definition 2. *Horizontal inequity* can be defined in two ways: the classical definition or the re-ranking definition. There is *classical horizontal inequity* if equals are treated unequally by the tax and transfer system. Classical horizontal inequity occurs if $y_i^0 = y_j^0$ and $y_i^1 \neq y_j^1$ for some pair of individuals (i, j) who, from an ethical viewpoint, should be treated equally by the fiscal system based on their characteristics. There is *re-ranking* if $y_i^0 \geq y_j^0$ and $y_i^1 < y_j^1$ for some such (i, j) pair. There is horizontal inequity *among the poor* if the above conditions hold and $y_k^n < z$ for all $k \in \{i, j\}$ and for some $n \in \{0, 1\}$.

Definition 3. The post-tax and transfer income distribution F_1 weakly *first order stochastic dominates* the pre-tax and transfer income distribution F_0 if $F_1(\mathbf{y}) \leq F_0(\mathbf{y})$ for all $\mathbf{y} \in \Omega$. A less restrictive condition is that F_1 first order stochastic dominates F_0 *among the poor*, or $F_1(\mathbf{y}) \leq F_0(\mathbf{y})$ for all $\mathbf{y} \in [0, z]$. Note that, by the definition of cumulative distribution functions, first order stochastic dominance is an anonymous concept.

Proposition 1. If there is re-ranking among the poor, first order stochastic dominance of F_1 over F_0 among the poor is not a sufficient condition for the absence of impoverishment.

Proof. Consider the example where $\mathbf{y}^0 = (5, 8, 20)$, $\mathbf{y}^1 = (9, 6, 18)$, $\mathbf{z} = 10$. F_1 first order stochastic dominates F_0 among the poor and there is impoverishment.

Since first order stochastic dominance implies higher order dominance, proposition 1 also tells us that we cannot use second order dominance to conclude that there has been no

² Note that the sum of the elements of \mathbf{y}^0 need not equal the sum of the elements of \mathbf{y}^1 for various reasons. Taxes could exceed transfers if tax revenues are spent on other things as well (e.g., defense). Transfers could exceed taxes if they are financed by other sources (e.g., oil revenues, debt). Finally, taxes could equal transfers in household per capita terms, but not in equalized terms on Ω .

³ Note that a single poverty line in Ω can still account for differences in needs across individuals, since household per capita incomes are mapped into equalized incomes in Ω by the function φ .

impoverishment. Furthermore, since first order stochastic dominance among the poor implies an unambiguous reduction in poverty according to any poverty measure in a broad class of additively separable measures⁴ (Atkinson, 1987), this also tells us that poverty measures will not necessarily capture impoverishment. (This fact can also be seen with simple examples; consider the example in the proof of proposition 3.)

Inequality measures are similarly anonymous with respect to initial income and are therefore also likely to overlook impoverishment. First order stochastic dominance implies second order stochastic dominance, which is equivalent to generalized Lorenz dominance (Foster and Shorrocks, 1988). When $\int_{\Omega} y^0 dF_0(y) = \int_{\Omega} y^1 dF_1(y)$ (i.e., the distributions have equal means), generalized Lorenz dominance implies Lorenz dominance. Since first order stochastic dominance is not a sufficient condition for the absence of impoverishment (provided that re-ranking has occurred), it follows that Lorenz dominance tests can show unambiguously lower inequality in spite of impoverishment.

Proposition 2. If F_1 does not weakly first order stochastically dominate F_0 among the poor, then impoverishment has occurred.

Proof. F_1 does not weakly first order stochastically dominate F_0 among the poor implies that there exists a $\hat{y} \in [0, z]$ such that $F_1(\hat{y}) > F_0(\hat{y})$. By the definition of cumulative distribution functions, this implies that the proportion of individuals with $y_i^1 < \hat{y}$ is higher than the proportion of individuals with $y_i^0 < \hat{y}$. Since the total number of individuals is identical in the pre-tax and post-tax distributions, this implies that there exists some individual j such that $y_j^0 > \hat{y}$ and $y_j^1 < \hat{y}$. Since $\hat{y} \leq z$, this implies $y_j^1 < y_j^0$ and $y_j^1 < z$, implying impoverishment has occurred.

Proposition 3. If there is no re-ranking among the poor and F_1 first order stochastically dominates F_0 among the poor, then impoverishment has not occurred.

Proof. By contrapositive: impoverishment among the poor implies that $y_i^1 < y_i^0$ and $y_i^1 < z$ for some individual i . If there does not exist an individual j with $y_j^0 < y_i^0 \leq y_j^1$ then $F_1(y_i^1) > F_0(y_i^1)$ which implies F_1 does not first order stochastically dominate F_0 on the interval $[0, z]$. If there does exist such an individual j , then re-ranking has occurred.

While stochastic dominance tests and standard measures of poverty fail to capture impoverishment because they are anonymous with respect to initial income, measures of horizontal equity—which by definition take into account individuals' pre-tax positions—can

⁴ Specifically, let the poverty function $p(y, z)$ be defined on all of $\Omega \times \Omega$ with $p(y, z) = 0$ whenever $y \geq z$. First order stochastic dominance implies a reduction in poverty using any poverty measure from the class of additively separable measures P such that there exists a monotonic transformation $G(P) = \int_{\Omega} p(y, z) dF(y)$ satisfying $G'(P) < 0$ (Atkinson 1987). This class of poverty measures includes the headcount, poverty gap, and squared poverty gap indices (indeed, it includes any member of the class of poverty measures proposed by Foster et al. (1984)), as well as the Watts (1968) measure, and the second measure proposed by Clark et al. (1981).

also fail to capture impoverishment.⁵ Even if some pre-tax poor are impoverished by the tax system, the ranking among the poor may not have changed (so there is no horizontal inequity due to re-ranking) and pre-tax equals may be impoverished to the same degree (so there is no classical horizontal inequity—i.e., violation of the principle that pre-tax equals should be treated equally). Neither does the presence of horizontal inequity necessarily imply impoverishment, because there could be re-ranking among the poor or unequal treatment among pre-tax equals when the tax-benefit system lifts incomes of some of the poor without decreasing incomes of any poor (i.e., no impoverishment). Horizontal inequity among the poor is therefore neither a necessary nor a sufficient condition for the presence of impoverishment, as shown in proposition 4. Thus, measures that account for horizontal inequity among the poor (e.g., Bibi and Duclos, 2007) will not necessarily capture this form of inequity of a tax system.

Proposition 4. Horizontal inequity is neither a necessary nor a sufficient condition for impoverishment.

Proof. Not sufficient: consider the example with four individuals where

$\mathbf{y}^0 = (5, 5, 6, 20)$, $\mathbf{y}^1 = (5, 7, 6, 18)$, $z = 10$. Horizontal inequity among the poor has occurred (by both the classical and re-ranking definitions), but impoverishment has not. Not necessary: consider the example with three individuals where $\mathbf{y}^0 = (5, 8, 20)$, $\mathbf{y}^1 = (6, 7, 20)$, $z = 10$. Impoverishment has occurred, but neither classical horizontal inequity nor re-ranking has occurred.

Standard measures of progressivity and redistributive effect, despite being non-anonymous with respect to income before taxes and transfers (like horizontal inequity), can indicate that a tax-benefit system is progressive even when it impoverishes a substantial proportion of the poor. We illustrate this in two ways. First, we apply the strictest definition of progressivity which is used in theoretical work but rarely in practice, and show that even with these high standards for progressivity, a system can impoverish a proportion of the poor while being everywhere progressive. We then reiterate the result using less strict summary measures of progressivity: the commonly used progressivity indicator from Kakwani (1977) and the common measure of redistributive effect from Reynolds and Smolensky (1977).

In the theoretical literature, it is often assumed that net taxes paid are determined by a non-stochastic and continuously differentiable function. In this case, net taxes paid (in other words, taxes minus benefits), are a function of pre-tax income and other characteristics; denote this function $N(\mathbf{y}^0, \cdot)$, where $\mathbf{y}^1 = \mathbf{y}^0 - N(\mathbf{y}^0, \cdot)$. Note that the function $N(\mathbf{y}^0, \cdot)$ will be negative for individuals who receive more in transfers than they pay in taxes. A tax and transfer system is progressive when net taxes paid, relative to income, increase in income. Specifically:

⁵ The view that the unequal treatment of equals—classical horizontal inequity—or re-ranking are unfair is accepted by a wide range of economists and philosophers, egalitarian or not. Horizontal inequity brings in the *status quo ante*; its measurement relies on comparisons that are “non-anonymous” in the language of Bourguignon (2011). Duclos (2008) reviews horizontal and vertical equity.

Definition 4. Denote net taxes paid relative to income as $n(y^0, \cdot) \equiv N(y^0, \cdot)/y^0$ and denote its partial derivative with respect to y^0 at a given pre-tax and transfer income point \hat{y} as $n'(\hat{y})$. The tax-benefit system is *locally progressive* at \hat{y} if $n'(\hat{y}) > 0$. The tax-benefit system is *globally progressive* if $n'(y^0) > 0$ for all $y^0 \in \Omega$.⁶

Proposition 5. Global progressivity of a tax and transfer system is neither a necessary nor sufficient condition for no impoverishment.

Proof. Not sufficient: consider the example where $z = 4$ and $n(y^0) = 3y^0/(y^0 + 1) - 2$, shown in figure 1. It is easy to see that this tax system is globally progressive since $n'(y^0) > 0$ for all $y^0 \in \Omega$. However, the curve $n(y^0)$ is only negative for $y^0 \in [0, 2)$; hence poor individuals with pre-tax incomes greater than 2 but below the poverty line are made poorer by the fiscal system. Not necessary: consider the example where $z = 4$ and $n(y^0) = (y^0 - 1)^2/(.5(y^0)^2 + 1) - 1$, shown in figure 2. This type of tax-benefit schedule could arise in practice if the government were committed to not impoverishing any poor, but had trouble targeting its benefit programs to the poorest of the poor, who might be marginalized. Since $n(y^0) < 0$ for all $y^0 \in (0, z)$, all the poor are net beneficiaries from the fiscal system. Since $y^1 = y^0(1 - n(y^0)) > z$ for all $y^0 > z$, none of the non-poor become poor as a result of the fiscal system. These two conditions imply that there is no impoverishment. However, the $n(y^0)$ function is decreasing on the interval $(0, 1)$, meaning that the global progressivity condition does not hold.

Figure 1. Counterexample proving “not sufficient”

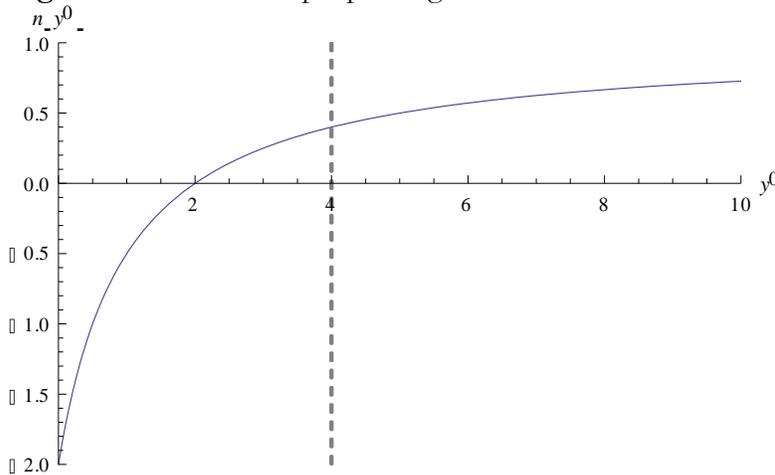
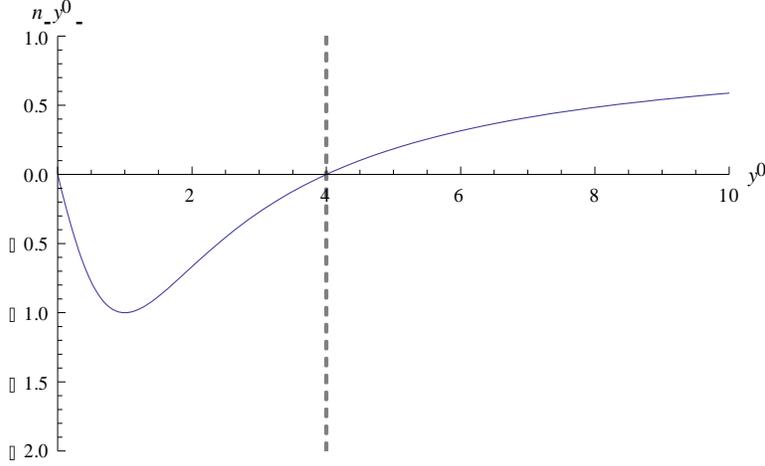


Figure 2. Counterexample proving “not necessary”

⁶ This definition of global progressivity coincides with the tax-redistribution approach to measuring global progressivity (Duclos, 2008).



In practice, global progressivity rarely holds due to the complexity of the various tax and transfer mechanisms and their variable impacts on household behavior. As a result, summary measures of progressivity are often used to assess a fiscal system's progressivity. Kakwani (1977) proposed an index of progressivity for taxes. Lambert (1985) proposed a Kakwani index for transfers analogous to the negative of the Kakwani index for taxes, so that a positive Kakwani index implies progressivity in both cases. Attempting to define a Kakwani index for the net fiscal system is problematic because net taxes (i.e., taxes minus benefits) are positive for some individuals and negative for others, which creates a concentration curve of net taxes that is not well-behaved (Lambert, 2001). Nevertheless, it is reasonable to define a tax-benefit system as conclusively progressive if both its taxes and its benefits have positive Kakwani indices. A commonly used measure of the redistributive effect the tax-benefit system as a whole comes from Reynolds and Smolensky (1977). These and other summary measures of progressivity, although less demanding than global progressivity, also fail to capture whether impoverishment has occurred.

Definition 5. Suppose that the population is large enough that F_0 and F_1 can be approximated by continuous and differentiable functions. Suppose they are also invertible so that $F_0(y) = p$ implies $y = F_0^{-1}(p)$. The *Lorenz curve of pre-tax and transfer income* is $L_0(p) \equiv \int_0^{F_0^{-1}(p)} y^0 dF_0(y) / \mu^0$ where $\mu^0 \equiv \int_{\Omega} y^0 dF_0(y)$ is mean income before taxes and transfers. The *Gini coefficient of pre-tax and transfer income* is defined as $G_0 \equiv 1 - 2 \int_0^1 L_0(p) dp$

Definition 6. The *concentration curve of post-tax and transfer income with respect to pre-tax and transfer income* is $c_1(p) \equiv \int_0^{F_0^{-1}(p)} y^1 dF_0(y) / \mu^1$ where $\mu^1 \equiv \int_{\Omega} y^1 dF_1(y)$ is mean income after taxes and transfers. The *concentration index of post-tax and transfer income with respect to pre-tax and transfer income* is $C_1 \equiv 1 - 2 \int_0^1 c_1(p) dp$.

Definition 7. Denote the taxes paid (benefits received) by individual i as t_i (b_i). By definition, $y_i^1 = y_i^0 - t_i + b_i$. Denote total taxes paid (benefits received) as a fraction of total pre-tax and transfer income in society as \bar{t} (\bar{b}). The *concentration curve of taxes* is $c_t(p) \equiv$

$\int_0^{F_0^{-1}(p)} t dF_0(y) / \bar{t}\mu^0$ and the *concentration index of taxes* is $C_t \equiv 1 - 2 \int_0^1 c_t(p) dp$. The concentration curve and index of benefits are defined analogously.

Definition 8. The *Reynolds-Smolensky index of post-tax and transfer income with respect to pre-tax and transfer income* is $R \equiv 2 \int_0^1 c_1(p) - L_0(p) dp = G_0 - C_1$. The fiscal system is progressive if $R \in (0, G_0]$.

Definition 9. The *Kakwani index of taxes* is $K_t \equiv 2 \int_0^1 L_0(p) - c_t(p) dp = C_t - G_0$ and the *Kakwani index of transfers* is $K_b \equiv 2 \int_0^1 c_b(p) - L_0(p) dp = G_0 - C_b$. Taxes are progressive if $K_t \in (0, 1 - G_0]$ and benefits are progressive if $K_b \in (0, 1 + G_0]$.

Proposition 6. A progressive tax-benefit system, as measured by the Kakwani coefficients for taxes and transfers and the Reynolds-Smolensky index of redistributive effect, is neither a necessary nor sufficient condition for no impoverishment.

Proof. Not sufficient: consider the example where $\mathbf{y}^0 = (5, 8, 20)$, $\mathbf{y}^1 = (9, 6, 14)$, $z = 10$, taxes are $\mathbf{t} = (1, 4, 7)$ and benefits $\mathbf{b} = (5, 2, 1)$. We have $R = 0.28$, $K_t = 0.05$, $K_b = 0.95$, which indicates a progressive net fiscal system, but impoverishment has occurred. Not necessary: consider an example with no impoverishment where $\mathbf{y}^0 = (8, 13, 20)$, $\mathbf{y}^1 = (8, 11, 22)$, $z = 10$, $\mathbf{t} = (1, 3, 1)$, $\mathbf{b} = (1, 1, 3)$. We have $R = -0.05$, $K_t = -0.29$, $K_b = -0.11$, indicating a regressive net fiscal system.

3. Measuring Impoverishment using Directional Mobility

Having seen that our standard measures can overlook impoverishment, a natural starting point is to ask what percent of the total population experiences impoverishment.

Definition 10. The *impoverishment headcount ratio* $h(\mathbf{y}^0, \mathbf{y}^1)$ simply measures the proportion of the population that are impoverished (i.e., that are either poor before taxes and transfers and made even poorer by the fiscal system, or non-poor before taxes and transfers and made poor by the fiscal system). Denoting the indicator function—which has a value of 1 if its argument is true and 0 otherwise—as $\mathbb{I}(\cdot)$, and denoting the cardinality of the set \mathcal{S} of individuals in society as $|\mathcal{S}|$, we have $h(\mathbf{y}^0, \mathbf{y}^1) = |\mathcal{S}|^{-1} \sum_{i=1}^{|\mathcal{S}|} \mathbb{I}(y_i^1 < y_i^0) \mathbb{I}(y_i^1 < z)$.

After determining the proportion of the population that is impoverished, the directional mobility literature provides a useful framework to further analyze impoverishment and convey this information to policymakers.⁷ We begin by defining two matrices which, despite their shortcomings, provide a useful first assessment of impoverishment and present information in a way that is easy to convey to policymakers.

Definition 11. A *fiscal mobility matrix* measures the directional movement between the before and after net taxes situations among k pre-defined income categories. It can be represented

⁷ Directional mobility is a subcategory of the “mobility as movement” definition (as opposed to the time independence definition). See Fields (2008) for a survey of the income mobility literature.

by the $k \times k$ transition matrix P , where the ij th element of P , denoted p_{ij} , can be interpreted as the probability of moving to income group j after taxes and transfers for individuals who were in income group i before taxes and transfers. Hence, P is a row stochastic matrix with $\sum_{j=1}^k p_{ij} = 1$ for all $i \in \{1, \dots, k\}$. Transition matrices were first used to compare pre- and post-tax income distributions in Atkinson (1980).

Define \mathbf{z} as a vector of poverty lines between \underline{z} (the lowest reasonable poverty line) and \bar{z} (the highest reasonable poverty line). In other words, \mathbf{z} is an ordered vector whose component values define tranches of income ranges which demarcate varying degrees of poverty severity.⁸ These poverty lines will determine a subset r of the k income categories ($r < k$) for which p_{ij} denotes the probability of moving into more severe poverty (poverty) after net taxes, for individuals who were less poor (not poor) before net taxes. For example, we could let $k = 4, r = 2$, where the income groups are extreme poor, moderate poor, near poor, and non-poor.

Definition 12. There is *downward mobility among the poor* if $\sum_{i=1}^r \sum_{j:j < i} p_{ij} > 0$. There is *downward mobility of some non-poor into poverty* if $\sum_{i=r+1}^k \sum_{j:j < r} p_{ij} > 0$.

Note that the fiscal mobility matrix does not capture the impoverishment of poor individuals who remain in their income group. Moreover, we are interested in knowing not only what percentage of the poor (non-poor) becomes poorer (poor) but also how much they lose on average.

Definition 13. Let L be the $k \times k$ matrix of proportional income losses, with element l_{ij} equal to the average percent decrease in income of those who began in group i and lost income due to taxes and transfers, ending in group $j \leq i$. By construction, L is negative semidefinite and weakly lower-triangular. There is impoverishment if and only if $l_{ij} < 0$ for some $j \leq r$.

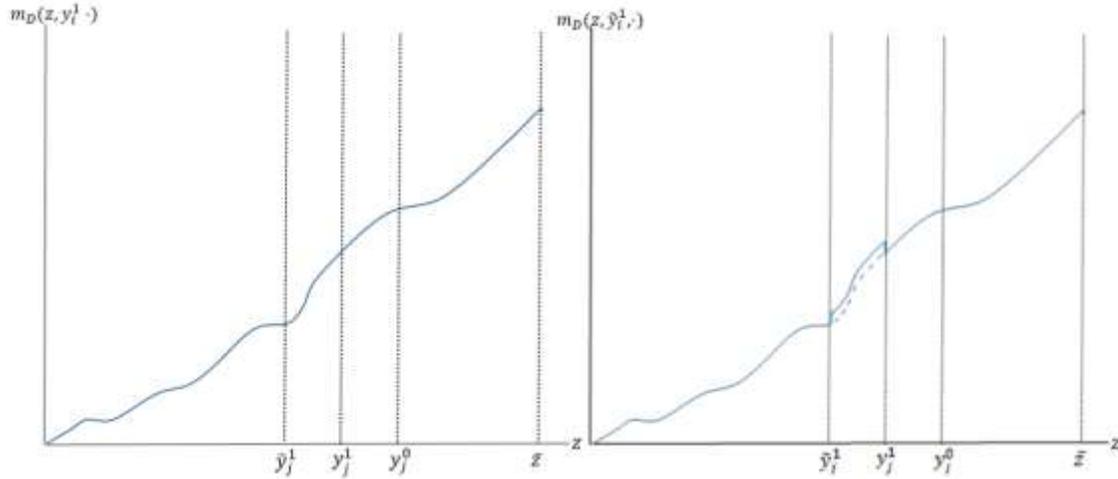
Another drawback of the fiscal mobility matrix is that the degree of downward mobility it captures may be sensitive to the number of income groups chosen and the selection of the cut-offs separating groups. Although these limitations disappear in the limit (as $k \rightarrow \infty$), using a large number of income groups is impractical and undermines the simplicity that makes the transition matrix an attractive tool. These limitations are overcome by a graphical measure of downward mobility proposed by Foster and Rothbaum (2012); this measure can be used in combination with the fiscal mobility and income loss matrices to assess the extent of impoverishment caused by the fiscal system. For each possible poverty line z , their measure is simply the percent of the population that experienced downward mobility across that line; in other words, it is the proportion of the population with $y_i^1 < z < y_i^0$.

Definition 14. The downward mobility function from Foster and Rothbaum (2012) is defined as $m_D(z, \cdot) = |S|^{-1} \sum_{i=1}^{|S|} \mathbb{I}(y_i^0 > z) \mathbb{I}(y_i^1 < z)$. Since we are concerned with impoverishment, we will restrict it to being defined only on the domain $z \in [0, \bar{z}]$.

⁸ We are grateful to Peter Lambert for suggesting this interpretation of the vector of poverty lines.

Note that this function captures both the presence of impoverishment (like the fiscal mobility matrix) and the amount lost (like the income loss matrix). It is obvious that it captures the presence of impoverishment; to see that it captures the amount lost, consider an impoverished individual i with $y_i^1 < y_i^0 < \bar{z}$. Now consider an alternative scenario where the individual is impoverished to a greater extent, and is left with income after taxes and transfers of \tilde{y}_i^1 where $\tilde{y}_i^1 < y_i^1 < y_i^0 < \bar{z}$. The higher degree of impoverishment in the alternative scenario will not be overlooked by the function $m_D(z, \cdot)$: we have $m_D(z, \tilde{y}_i^1, \cdot) = m_D(z, y_i^1, \cdot) + |\mathcal{S}|^{-1}$ for all $z \in [\tilde{y}_i^1, y_i^1]$ (i.e., for all poverty lines between \tilde{y}_i^1 and y_i^1 , downward mobility will be slightly higher—by the amount $|\mathcal{S}|^{-1}$ —in the alternative scenario) while $m_D(z, \tilde{y}_i^1, \cdot) = m_D(z, y_i^1, \cdot)$ (i.e., the curves coincide) for all other z in the domain. This is illustrated in figure 3.

Figure 3. The downward mobility curve captures the extent of impoverishment



Note: the right panel shows the alternative situation where one individual is impoverished to a greater extent (to income level \tilde{y}_i^1 instead of y_i^1 , where $\tilde{y}_i^1 < y_i^1$).

Foster and Rothbaum (2012) also propose a partial downward mobility ordering. Their dominance criteria involves first and second order stochastic dominance tests of the downward mobility curves. It is worth emphasizing that these tests are unrelated to the stochastic dominance tests discussed previously, which tested for dominance of cumulative income distribution functions rather than downward mobility curves.

Definition 14. Consider two alternative income distributions after taxes and transfers: F_1^A and F_1^B , both generated from the same pre-tax and transfer income distribution F_0 . Situation A *first order downward mobility dominates* (i.e., exhibits less downward mobility than) situation B if $m_D(z, \mathbf{y}_A^1, \cdot) \leq m_D(z, \mathbf{y}_B^1, \cdot)$ for all $z \in [0, \bar{z}]$ with strict inequality for at least one such z . Situation A *second order downward mobility dominates* situation B if $\int_0^c m_D(z, \mathbf{y}_A^1, \cdot) dz \leq \int_0^c m_D(z, \mathbf{y}_B^1, \cdot) dz$ for all $c \in [0, \bar{z}]$ with strict inequality for at least one such c .

4. An Illustration with Brazilian Data

In this section we use results of a tax and benefits incidence analysis for Brazil to illustrate the importance of measuring impoverishment as an indicator of the impact of fiscal policy on the poor.⁹ In particular, we show that standard poverty and inequality indicators, stochastic dominance tests, and measures of progressivity would lead us to conclude that Brazil's tax and transfer system is overall favorable to the poor. However, we also show that in spite of all these positive outcomes, the extent of impoverishment is significant.¹⁰

Following the methods and income definitions outlined in Lustig and Higgins (2013), the incidence analysis compares market income (before taxes and transfers) to post-fiscal income (after direct and indirect taxes, direct cash and food transfers, and indirect subsidies). Market income includes labor income (including fringe benefits, vacation pay, etc.) capital income (rents, profits, interest, and dividends), private transfers (alimony, remittances, etc.), income from contributory pensions,¹¹ imputed rent for owner-occupied housing, and goods produced for own consumption. Post-fiscal income is calculated as market income plus direct transfers and indirect subsidies, minus direct and indirect taxes. Direct transfers include conditional cash transfers (CCTs) from Brazil's flagship anti-poverty program Bolsa Família, non-contributory pensions, public scholarships, unemployment benefits, special circumstances pensions, milk transfers from Programa de Aquisição de Alimentos (PAA), and other direct transfers. Indirect subsidies include energy subsidies for low-income households. Direct taxes include the individual income tax, payroll taxes, and property taxes. Indirect taxes include consumption taxes at the state and national level.¹²

The Brazilian data illustrates many of the points made in section 2. The Gini falls from 0.57 before taxes and transfers to 0.54 after. The after taxes and transfers income distribution Lorenz dominates the before taxes and transfers distribution, so inequality unambiguously falls. The post-tax distribution first order stochastic dominates the pre-tax distribution on the interval between zero and slightly above \$3 PPP per day, so poverty unambiguously falls below this income level (figure 4). (To illustrate, the headcount index at \$2.50 PPP per day falls from 15.4% to 14.3%, and the squared poverty gap from 3.8% to 2.3%.)

⁹ Note that our methods of measuring impoverishment can be applied to two types of data: data in which actual taxes and benefits are enumerated, or data in which they are computed from a tax-benefit microsimulation model.

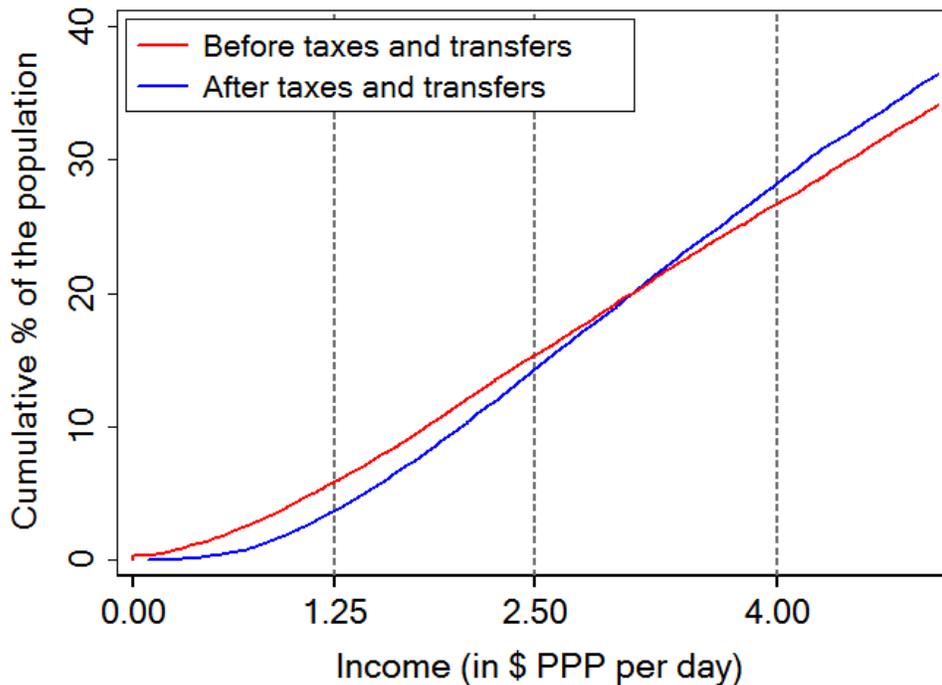
¹⁰ This analysis uses the *Pesquisa de Orçamentos Familiares* (Family Expenditure Survey; POF) 2008-2009.

¹¹ As described in Lustig, Pessino, and Scott (forthcoming), there is no agreement in the literature on whether contributory social security pensions should be treated as market income (similar to income from personal savings) or as a government transfer. Higgins and Pereira (forthcoming) analyze the tax and transfer system under both scenarios; for brevity, the illustration in this paper makes a choice between the two options, treating contributory pensions as market income.

¹² The consumption taxes included are the state-level Imposto sobre Circulação de Mercadorias e Serviços (ICMS) and the federal-level Imposto sobre Produtos Industrializados (IPI). Our measures of post-fiscal income poverty and inequality differ from those in Higgins and Pereira (forthcoming) because that study include two additional indirect taxes levied at the federal level: Programa de Integração Social (PIS) and Contribuição para o Financiamento da Seguridade Social (COFINS), relying on secondary source estimates of these taxes' effective rates by decile. Because we do not include these federal indirect taxes in this illustration, our estimates of impoverishment and downward fiscal mobility in Brazil are likely to be lower bound estimates.

Furthermore, the post-tax distribution second order stochastically dominates the pre-tax distribution on the entire interval of reasonable poverty lines from zero to \$4 PPP per day, which implies (Atkinson, 1987) an unambiguous reduction in poverty over this range of poverty lines using most poverty measures (with one exception being crude headcount index). More specifically, any poverty measure P of the form given in footnote 4 with the additional restrictions that the poverty function $p(y, z)$ be continuous, non-decreasing, and weakly concave in y will show a reduction in poverty for any reasonable poverty line.¹³ Thus, these measures will mask the significant impoverishment occurring among those who live on less than \$4 PPP per day.

Figure 4. Cumulative distribution functions before and after taxes and transfers in Brazil



Source: Authors' calculations based on POF (2008-2009).

Common summary indicators of progressivity also indicate that the tax-benefit system is progressive: we have $R = 0.05$, $K_t = 0.04$, $K_b = 0.54$. Note that these indices are calculated with respect to the before tax and transfer income distribution and are thus non-anonymous. However, they still mask impoverishment.

In Brazil, 12.3% of the population is impoverished by the tax and transfer system. This easily calculated impoverishment headcount ratio has already provided us with important information on the extent of impoverishment in Brazil, which could be relevant information for policymakers but was not captured by any of the traditional measures.

¹³ These additional restrictions on the poverty function preclude the headcount index, but do not preclude any of the other measures mentioned in footnote 4, such as the poverty gap index and squared poverty gap index.

Table 1 shows the fiscal mobility matrix P for Brazil; the added row (column) labeled “percent of population” give population shares for the market income (post-fiscal income) groups, while the last column gives the mean market income (in purchasing-power parity adjusted US dollars per day) of members of that market income group. Our income groups in this example are four in total. The poor are divided into two groups: those with less than \$2.50 PPP per day (the extreme poor) and between \$2.50 and \$4 PPP per day (the moderately poor). The two non-poor groups are: between \$4 and \$10 PPP per day (the vulnerable) and above \$10 PPP per day.¹⁴ As a result of (mainly indirect) taxes and insufficient transfers, 10.6% of those vulnerable to poverty become poor and 11.4% percent of the moderate poor become extremely poor. The impact of indirect taxes on the poor is partly due to the fact that exemptions on consumption taxes are almost non-existent in Brazil (Corbacho, Cibils, and Lora 2013). As shown above, this impoverishment is not captured by the standard measures of inequality, poverty, progressivity, and incidence.

Table 1. Fiscal Mobility Matrix for Brazil

		Post-tax and transfer income groups				Percent of population	Mean income
		<2.5	2.5-4	4-10	>10		
Pre-tax and transfer income groups	<2.5	84.7%	10.3%	3.8%	1.3%	15.4%	\$1.45
	2.5-4	11.4%	77.5%	10.5%	0.6%	11.3%	\$3.24
	4-10	0.0%	10.5%	86.2%	3.2%	33.5%	\$6.67
	>10	0.0%	0.0%	13.4%	86.6%	39.8%	\$28.41
	Percent of population	14.3%	13.9%	36.0%	35.8%	100.0%	\$14.14

Note: Mean incomes are measured in pre-tax income and are in US\$ PPP per day. Rows may not sum to exactly 100% due to rounding. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.

Source: Authors’ calculations based on POF (2008-2009).

Now that we have established that taxes and transfers induce downward mobility among the poor, the next step is to ask *how much* the impoverished lose. For this, we use the income loss matrix L , shown in Table 2. The income loss matrix shows us the average loss of losers, by their pre- and post-taxes and transfers income groups, as a proportion of their before taxes and transfers incomes. In addition, we include the average before taxes and transfers incomes of each of these groups below the percent income loss. The last column shows the average income loss and market income of everyone who paid more taxes than they received benefits in the corresponding market income group. The extreme poor who are impoverished have before transfers income of \$1.93 PPP per day on average and lose 9.6%

¹⁴ The \$2.50 and \$4 PPP per day poverty lines are commonly used as extreme and moderate poverty lines for Latin America, and roughly correspond to the median official extreme and moderate poverty lines in those countries (CEDLAS and World Bank, 2010). The \$10 PPP per day line is the upper bound of those vulnerable to falling into poverty in three Latin American countries, calculated by Lopez-Calva and Ortiz-Juarez (2013) and the lower bound of the middle class used by Kharas (2010) and Ferreira et al. (2013).

of their income on average. The moderately poor who become extremely poor have before transfers income of \$2.72 PPP per day and lose 16.8% of their income on average.

Table 2. Income Loss Matrix for Brazil

		Post-tax and transfer income groups				Percent of population	Group average
		<2.5	2.5-4	4-10	>10		
Pre-tax and transfer income groups	<2.5	-9.6% \$1.93				15.4%	-9.6% \$1.93
	2.5-4	-16.8% \$2.72	-10.7% \$3.38			11.3%	-11.7% \$3.28
	4-10		-18.1% \$4.37	-15.8% \$7.03		33.5%	-16.2% \$6.70
	>10			-20.6% \$11.02	-20.5% \$31.80	39.8%	-20.5% \$28.85
	Percent of population	14.3%	13.9%	36.0%	35.8%	100.0%	

Note: All monetary amounts are measured in pre-tax income and are in PPP-adjusted dollars per day. Zeroes are omitted from the matrix for enhanced readability. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.

Source: Authors' calculations based on POF (2008-2009).

The fiscal mobility matrix can also be used to compare the actual fiscal system to alternative scenarios such as proposed reforms. To illustrate, we compare the actual system to a tax-benefit model in which transfers received are still as observed in our data, while the current (progressive) tax system is replaced by a neutral tax system that generates the same amount of tax revenue as the current system. As before, if we denote the total taxes collected divided by total pre-tax and transfer income by \bar{t} , everyone pays taxes proportional to their income at rate \bar{t} in the neutral tax system. Hence, the neutral tax system is horizontally equitable and neither progressive nor regressive. *Ex ante*, it is difficult to determine whether the neutral tax system will entail more or less impoverishment than the actual tax system. Table 3 shows the fiscal mobility matrix where post-tax and transfer income is calculated assuming the neutral tax system instead of the actual tax system.

Table 3. Fiscal Mobility Matrix for Brazil under Neutral Tax System

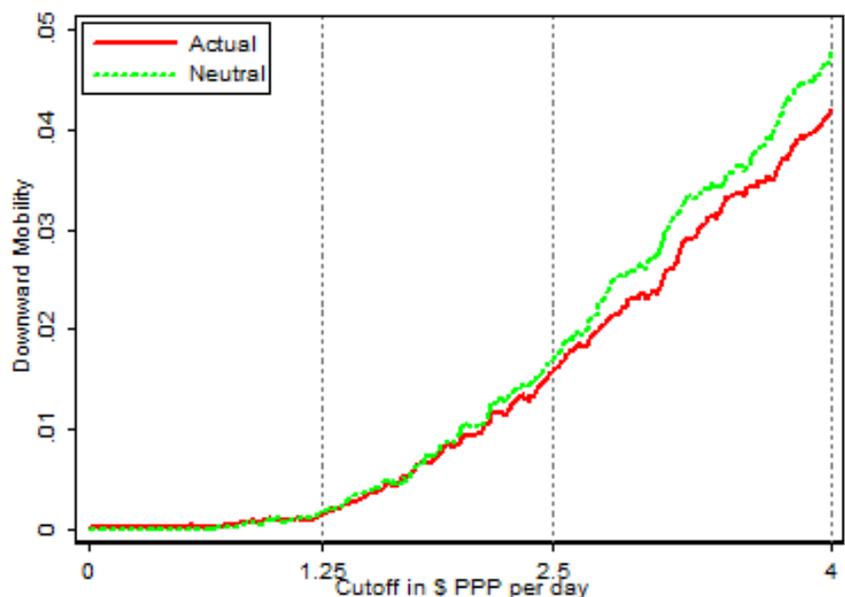
		Post-tax and transfer income groups				Percent of population	Mean income
		<2.5	2.5-4	4-10	>10		
Pre-tax and transfer income groups	<2.5	84.6%	9.5%	4.4%	1.4%	15.4%	\$1.45
	2.5-4	16.1%	72.9%	9.9%	1.1%	11.3%	\$3.24
	4-10	0.0%	14.5%	82.1%	3.4%	33.5%	\$6.67
	>10	0.0%	0.0%	16.5%	83.5%	39.8%	\$28.41
	Percent of population	14.8%	14.6%	35.9%	34.7%	100.0%	\$14.14

Note: Mean incomes are measured in pre-tax income and are in US\$ PPP per day. Rows may not sum to exactly 100% due to rounding. Differences in group shares between the before and after taxes and transfers distributions are all statistically significant from zero at the 0.1% significance level.
 Source: Authors' calculations based on POF (2008-2009).

From the matrix, it appears that downward fiscal mobility is higher in the actual tax scenario than the neutral tax system. However, this comparison highlights one of the drawbacks of the fiscal mobility matrix: those who begin in the poorest group and are impoverished do not contribute to the downward mobility measured by the matrix. The downward mobility curve overcomes this problem by measuring downward mobility at each possible poverty line. Figure 5 graphs the downward mobility curve for all possible cut-offs from zero to the highest poverty line of \$4 PPP per day in both the actual scenario and under the neutral tax. For a given poverty line, the curve tells us what portion of the population crosses that line from above as a result of the fiscal system. For example, slightly over 4% of Brazil's population has income before taxes and transfers above \$4 PPP per day but income after taxes and transfers below that line; under the neutral tax, this would happen to closer to 5% of the population, meaning that over one million additional people would be pushed into poverty by the fiscal system under the neutral tax scenario.¹⁵

Figure 5. Downward Mobility Curves for Brazil under Actual and Neutral Tax Systems

¹⁵ Note that the percentages shown in the mobility curve are percentages of the *total* population, which is why they appear lower than the numbers in the mobility matrix. For example, the mobility matrix tells us that 10.5% of the pre-tax and transfer *vulnerable* group (those with pre-tax and transfer incomes between \$4 and \$10 PPP per day, who in turn make up 33.5% of the total population) become moderately poor (i.e., have post-tax and transfer income between \$2.50 and \$4 PPP per day); the curve tells us that slightly over 4% of the *total* population begins non-poor (i.e., has pre-tax and transfer income above the \$4 PPP per day poverty line) and becomes poor as a result of taxes and transfer (i.e., has post-tax and transfer income below the \$4 PPP per day poverty line). The 4% of the population that begin non-poor and are made poor by the fiscal system are a *subset* of the set of impoverished individuals (12.3% of the population); the latter also includes those who were poor before taxes and transfers and made poorer by the tax and transfer system. Hence, if we consider the possibility of calculating the impoverishment headcount index at various poverty lines, we have $m_D(z, \mathbf{y}^1, \mathbf{y}^2) \leq h(z, \mathbf{y}^1, \mathbf{y}^2)$ for any z .



Source: Authors' calculations based on POF (2008-2009).

Using Foster and Rothbaum's (2012) criteria for mobility curve dominance, we can conclude that the poorest of the poor are less impoverished by the *neutral* tax: the neutral tax downward mobility curve second order stochastically dominates the actual tax system's downward mobility curve for all poverty lines on the interval between \$0 and \$1.84 PPP per day. However, the remaining poor are less impoverished by the *actual* tax system: the actual tax downward mobility curve first order stochastically dominates the neutral tax system for all poverty lines on the domain between \$1.80 and \$4 PPP per day. Like the fiscal mobility matrix and income loss matrix, an analysis of the downward mobility curves has exposed important information that would not be available using standard measures. It has also allowed us to compare the extent of impoverishment in the actual tax and transfer system to that of a possible alternative or proposed reform.

5. Extensions

5.1 Social Welfare

In section 3, we showed that first order stochastic dominance tests fail to capture impoverishment. Since first order stochastic dominance implies second order dominance and second order dominance is equivalent to generalized Lorenz dominance (Foster and Shorrocks, 1988), how do we reconcile these results with the traditional result that generalized Lorenz dominance implies higher social welfare¹⁶ (Shorrocks, 1983)? It is worth noting that these results use anonymous social welfare functions: individual utilities and the social welfare function for the post-tax and transfer distribution are independent of what the pre-tax and transfer income distribution would have looked like, how much different

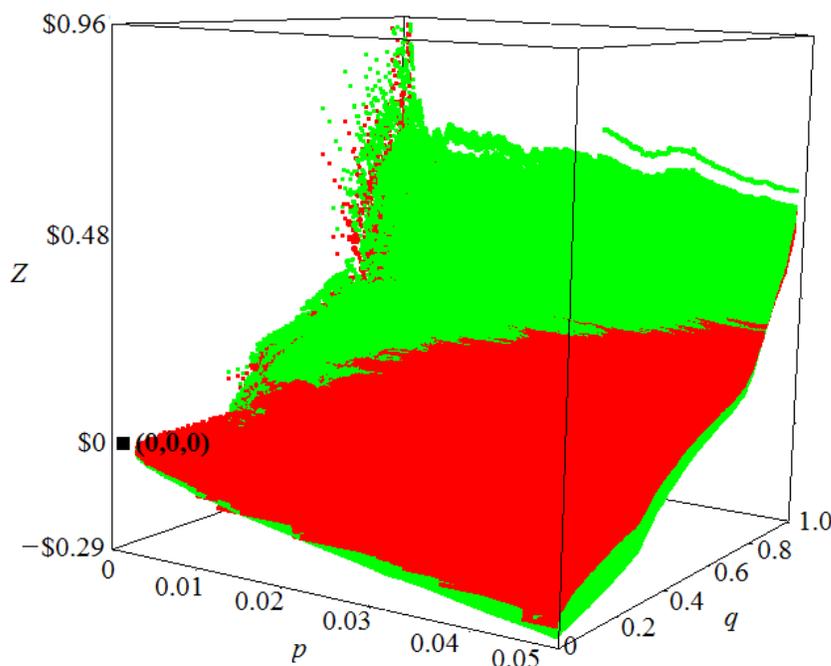
¹⁶ Provided that the social welfare function is non-decreasing and S-concave, and utility functions are concave, increasing, and identical for all individuals.

individuals gained or lost from the tax-benefit system, and whether some of the individuals who lost were already unable to afford basic necessities. If individual utility functions or social welfare are dependent on the amount individuals gain or lose as well as their income level (as in Bourguignon, 2011), the traditional results no longer hold.

Specifically, Bourguignon (2011) posits that individual utility is dependent on both “status quo” income y_i^0 and the amount gained or lost, $y_i^1 - y_i^0$. This contrasts with standard social welfare measurement where utility depends only on income after taxes and transfers y_i^1 . The status quo in Bourguignon’s framework is usually post-tax and transfer income before some proposed reform to the fiscal system, against which post-tax and transfer income under two potential reforms are compared. However, he also mentions its applicability to the scenario to which we are applying it here, where a planner is comparing two distributions on their distance from the market income distribution. Given such a utility function and the standard utilitarian social welfare criterion, social welfare in the actual tax and transfer system can be compared to that of the tax and transfer system in an alternative scenario or under a proposed reform. He defines the surface $Z^X(p, q)$, the mean income change for the $q \in [0,1]$ most impoverished (or least enriched) proportion of the $p \in [0,1]$ poorest proportion of individuals before taxes and transfers, in scenario $X \in \{A, B\}$. Social welfare is (weakly) higher under tax and transfer system A than under system B if $Z^A(p, q) \geq Z^B(p, q)$ for all $(p, q) \in [0,1] \times [0,1]$.

We apply Bourguignon’s (2011) measure to the comparison of the actual tax and transfer system in Brazil to the neutral tax from section 4. Because the surfaces cross, no conclusion can be made about social welfare in the two scenarios. The conclusion about impoverishment is similar to the conclusion we reached using the downward mobility curve: the poorest of the poor are more impoverished off under the neutral system—as seen by the fact that $Z^{actual}(p, q) \geq Z^{neutral}(p, q)$ for about the 30% most impoverished (or least enriched) of those with very low incomes. Figure 6 shows the $Z^X(p, q)$ surfaces for the actual and neutral tax systems, zoomed in to only the ultra-poor (who make up just over 5% of the population, hence the domain of the p axis from 0 to just over 0.05). The green surface is the neutral system—it lies below the red actual system surface for all $p \in [0,1]$ and $q \in [0,0.3]$, but for higher values of q the curves cross, indicating a lack of social welfare dominance.

Figure 6. Testing for Bourguignon’s welfare dominance among the ultra-poor.



Note: The green surface is the neutral tax and transfer scenario. The red surface is the actual tax and transfer scenario. The Z-axis is measured in PPP-adjusted US dollars per day. The origin is marked with a black square. Source: Authors' calculations based on POF (2008-2009).

Furthermore, if individuals are loss averse (Kahneman and Tversky, 1979), it is easy to show that a transfer satisfying the Pigou-Dalton transfer principle from a poor individual to a poorer one can lead to not only impoverishment but also lower social welfare. However, a Pigou-Dalton transfer from a rich individual to a poorer one could also lead to lower social welfare. In the presence of loss aversion, the relevant question which has not been adequately studied is whether the way a poor individual experiences loss aversion is significantly different from the way a non-poor individual does, given the poor person's struggle to meet basic needs. Further exploration into this question would be an interesting avenue for further research.

5.2 Multidimensional Poverty

The framework in section 2 and measures in section 3 can be easily extended to multidimensional measures of poverty that take into account more than just income. The well-being space Ω would no longer be a subset of the non-negative real numbers; instead, it would be the Cartesian product of the sets of possible outcomes in each dimension. Provided that the outcomes in each dimension can be ranked by their desirability, the univariate cumulative distribution functions from section 2 are easily generalized to multivariate cumulative distribution functions.

Instead of income vectors \mathbf{y}^0 and \mathbf{y}^1 we would then be interested in the matrices $\mathbf{Y}^0 = [y_{ij}^0]$ and $\mathbf{Y}^1 = [y_{ij}^1]$ where y_{ij}^n denotes individual i 's outcome in dimension j for $n \in \{0,1\}$. If we adopt the "counting" approach to multidimensional poverty measurement from Alkire

and Foster (2011) and choose d dimensions with which we will measure poverty, instead of a one-dimensional poverty line z , we would have a poverty line in each dimension, which could be used to construct the vector $\mathbf{z} = (z_1, \dots, z_d)$.

(unfinished)

6. Concluding Remarks

We have shown that a country can perform well by standard indicators of inequality, poverty, Lorenz dominance, first order stochastic dominance, and progressivity despite having impoverishment and a non-trivial sub-section of the poor experience downward fiscal mobility into a lower income group (and having a non-trivial sub-section of the non-poor experience downward mobility into poverty). Standard indicators, such as the Gini, headcount, poverty gap, and squared poverty gap indices, as well as dominance criteria using Lorenz curves and cumulative distribution functions, overlook impoverishment because they do not concern themselves with who the before transfers poor are. Non-anonymous indicators of horizontal inequity, progressivity, and redistributive effect also overlook impoverishment.

The relationship between first order stochastic dominance, re-ranking, and impoverishment can be summarized as follows. If the post-tax and transfer distribution does not weakly first order stochastically dominate the pre-tax and transfer distribution, then impoverishment has occurred. If, on the other hand, it does dominate, one must check whether re-ranking occurred. If the tax-benefit system was rank-preserving and the post-tax distribution first order stochastically dominates the pre-tax distribution, no impoverishment has occurred. If, however, re-ranking took place, dominance tests cannot be used to determine whether there was impoverishment. In this case, the fiscal mobility matrix, income loss matrix, and downward mobility curves can be used to determine if impoverishment occurred and measure it.

Fiscal mobility matrices are a useful tool for identifying how much downward fiscal mobility occurs among the poor. In the case of Brazil we saw that 11% of the vulnerable become poor and 11% of the moderate poor become extremely poor despite any cash transfers they receive. Some of those who begin in the extremely poor group are also impoverished, losing 10% of their already low incomes on average. Meanwhile, we would not have been aware of this impoverishment and downward fiscal mobility if we relied on standard tools; extreme poverty and inequality decline, there is first order stochastic dominance to the left of \$3 PPP per day, second order stochastic dominance over the domain of reasonable poverty lines, and taxes and transfers are progressive.

Although here we apply the notion of impoverishment to assess tax and benefit systems, its usage can be extended to any “before-after” situation. For example, it can be applied to analyze the changes in trade policy, rising food prices, a depreciation of the currency, or fiscal austerity measures.

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