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On the Distributional Impact of Embodied Technological Progress in Developing Countries *

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Abstract

This paper revisits the development problem under embodiment studied by Boucekkine *et al.* (2006) by introducing labour heterogeneity, sectoral allocation of capital and a certain degree of inefficiency into this model. We first show that the economy has a stronger tendency to specialize in the production of investment goods because what we label as the *efficient marginal productivity of capital effect* is higher once these changes are introduced. Using a calibrated version of the model for a representative developing country, we then show the optimality of an immediate and massive increase of adoption efforts when foreign technological acceleration occur. This result challenges the main outcome reported so far in the literature. Then, it is suggested that a policy package oriented to facilitating adoption and increasing the number of educated people might be a good strategy to make less problematic the catching up process in the South. The performance of the economy looks wealthy and vigorous, either in the short- or in the long run. In particular, we report that simultaneous gains in growth and equality are significant. With this strategy, Kuznets' hypothesis (1955) would not hold. However, robustness analysis suggests all these desirable achievements may be delayed if inefficiency in adoption is high.

JEL: E22, E60, O15, O40.
Key words: Embodiment, Distributional Effects, Economic Development

1 Introduction

This paper is concerned with technology and technological diffusion in developing countries. This issue deserves attention because it is well-recognized that technological progress is at the heart of human progress and development. The recently specific issue on the subject published by the World Bank (2008) highlights consciously this relevance. How technological progress can occur (or be created) is a matter of concern for many scholars and policy makers around the globe. In developing countries, this process

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is made in practice by adopting and adapting new foreign technologies, particularly pre-existing ones. Central to understanding how technology contributes to GDP are the domestic factors that determine how successfully developing countries are able to absorb and apply those technologies. While this connection has been widely studied -as theoretically as empirically- by assuming that technological progress is disembodied in nature, scarce efforts have been conducted on this topic when this progress is embodied. Moreover, the study of the propagation mechanisms by which technology adoption spurs or harms other important development goals that are not well captured by GDP alone has been poorly documented in the literature, independently of the nature of technological progress. This paper aims to contribute precisely in this line. Broadly speaking, we concentrate on the effects of technology adoption under embodiment on primary welfare, aggregates and other "development variables". Then, we focus on some indirect effects of this adoption, particularly on income inequality. To this end, we construct a framework strongly based on Boucekkine *et al.* (2006). We do this on purpose for two reasons. First, to the best of our knowledge, we identify this work as the sole contribution specifically related to our general objective: to study the economic development problem under embodiment in developing countries. Second, we do believe that the main result reported by these authors is very striking (or controversial) in nature, namely: the sub-optimality of an immediate increase in the adoption efforts in response to acceleration in the pace of embodied technical progress abroad. Then, in order to challenge this outcome (and others reported in the same work) we use a different but very comparable model.

Of course, this paper also recognizes (direct or indirect) contributions (or influences) from previous works. On the academic side, the seminal and well-known works by Kuznets (1955) and Nelson and Phelps (1966) are on the top. In addition, Krusell (1998), Benhabib and Hobijn (2002) and Boucekkine *et al.* (2003, 2005) are important to understand the implications of embodiment technological change on the long run growth. On the empirical side, there are not so far works that estimate the contribution of investment-specific technological advance to economic growth in developing countries -which should be the end goal according to the traditional growth accounting estimates of investment-specific (or neutral technological progress). We do thing the issue that we barely know about the real impact of embodied technological progress on the South should have large implications that warrant future research. This view is supported on experiences reported by empirical works by Greenwood *et al.* (1997, 2007) and Sakellaris and Wilson (2004) -among others- conducted respectively on the US economy and European countries which distinguish the fact that a large portion of technological progress is embodied in new capital goods which impacts largely on productivity growth.

The basic structure of the model analysed in this paper is as follows. There are three sectors in the economy: the sector producing final goods, the sector producing intermediate (capital) goods and the sector producing technology. Labour is assumed heterogeneous (skilled and unskilled workers) and sectoral allocation of capital stock is allowed. The technology sector is borrowed from Fuentes *et al.* (2012). This sector adopts or imitates new innovations from abroad. Like these authors, we keep the central assumption that the technological gap can only vanish asymptotically. In addition, a certain degree of inefficiency in adoption is assumed which may represent any barrier to the diffusion of technology across the economy like the abandon of new capital, lack of capabilities, brain drain and so on. Using a calibrated version of this set-up for a representative developing economy, we study how the optimal assignation of resources compatible with social welfare maximization under technological acceleration abroad and other internal shocks draws the optimal paths of different endogenous variables.

In terms of contributions, we first challenge the main conclusion reported in the literature. Concretely, we defend the optimality of an immediate and massive increase of adoption efforts when foreign technological accelerations occur. In addition, we offer a nicer way to explain the transition dynamics that appears when the economy is faced with shocks. More precisely, we discuss in detail how different shocks propagate inside the economy throughout the two competitive mechanisms present in the model, namely, *the efficient marginal productivity of capital effect* and *the obsolescence one*. The first effect appears as an important determinant of consumption-savings choice which may lead the economy to produce more final goods, particularly investment ones. According to Boucekkine *et al.* (2003), the second effect has to with the tendency of the economy to specialize in producing intermediate (capital) goods because they are progressively cheaper under embodiment. We demonstrate that when capital is introduced in the adoption sector the economy tends to be driven by the first effect. Furthermore, we highlight the connection between inefficiency in the technology-adopting sector and development. We shall see that the higher the degree of this inefficiency the lower the level of development archived in the long run. However, it is suggested that a policy package focused on increasing both sectoral productivities and the number of educated people might be a good strategy not only to make less problematic the catching up process in the South but also for development in broader sense. Indeed, we found that our particular view of generating embodied technological progress helps to sophisticate the productive capacity of the economy without harming important development issues such as income equality and the technological gap in environments where that package is implemented, either in the short- or in the long run. Therefore, in order to accelerate the rhythm of racing to development via embodiment, the study of the transitional dynamics created from the choice of the right combination of correct *pro*-industrialization policies should be at the top of any program on development. Finally, we offer conscious quantitative analyses of the effects of different shocks which allows us to put in perspective the pros. and cons. of each one of them. This exercise is the main input used for designing the policy package above-mentioned .

The paper is organized as follows. Section 2 presents the model. Section 3 is devoted to the central planner problem. Section 4 deals with the steady state solution and its properties. Section 5 deals with the analysis of the short run dynamics. Section 6 includes a robustness study and Section 7 concludes.

2 The model

As mentioned, the model to be developed here is strongly based on Boucekkine *et al.* (2006), and it introduces a technology adoption sector very similar to Fuentes *et al.* (2012). We begin by lying out the three-sector model:

$$Y_t = A_t(\mu_{k,t}K_t)^\alpha L_{y,t}^\theta H_{y,t}^\Psi \tag{1}$$

$$K_t = q_t I_t + (1-\delta)K_{t-1} \tag{2}$$

$$q_t = (1-\delta_{q,t}) \cdot q_{t-1} + d_t \left[(1-\mu_{k,t})K_t\right]^a L_{q,t}^b H_{q,t}^c (q_t^0 - q_t) \tag{3}$$

Equation (1) gives the production function in the final good sector at any date t. This good can be used either for consumption (non durable goods) or investment (durable goods). Technology here is Cobb-Douglas on capital and labour. A fraction $\mu_{k,t}$ of the total capital (K_t) is used in this sector. We assume heterogeneous abilities which influence the marginal product each worker produces. Thus (L_y) units of **unskilled** labour and (H_y) units of **skilled** labour are also required to produce one unit of final good (Y_t). The supply labour is assumed inelastic. As usual, we assume constant returns-to-scale in this technology, which simply means that the respective "elasticities" in production obey the equation $\alpha + \theta + \Psi = 1$. The parameter α is the elasticity of output with respect to physical capital, θ that with respect to unskilled labour and Ψ that with respect to skilled labour. In addition, we assume that the technological progress in this sector (A_t) is the nature disembodied, i.e., it is independent of the pace of capital accumulation.

Equation (2) gives the law of motion of capital or the production function in the capital sector. It has exactly the same structure proposed by Boucekkine *et al.* (2006). Accordingly, the final good I_t is the sole input in this sector at time t; q_t is the embodied technological progress in capital goods which is endogenously determined and δ is the rate of depreciation of capital. As usual in this kind of framework, the growth rate of q is precisely the rate of decline of the relative price of capital. More concretely, this price equals to $1/q_t$, i.e., under scenarios in which q is an increasing function of time, capital goods turn cheaper than consumption ones across time which diminishes the growth rate of consumption. This effect was labelled as *the obsolescence effect* by Boucekkine *et al.* (2003).

Equation (3) gives the technology used in the imitation (or technology adoption) sector which determines the level of embodied technical progress q_t at any time t. It is also a Cobb-Douglas function on capital and labour which requires a fraction $(1 - \mu_{k,t})$ of the total capital (K_t), $(L_{A,t})$ units of **unskilled** labour and $H_{A,t}$ units of **skilled** labour as inputs. Like Fuentes *et al.* (2012), we also assume the presence of inefficiencies in the adoption process which are captured by the parameter $0 < \delta_q < 1$. As said in section (1), the sources of such inefficiencies can be varied: lack of capabilities, brain drain, abandon of physical capital, and so on. We assume constant returns-to-scale in this technology. Thus the condition $a + b + c = 1$ is imposed for the respective input intensities. Following the literature, we also include the variable d_t which captures any potential shock to this sector. For example, d_t may represent an institutional improvement of the productivity of labour in the imitation sector or a trade policy reform easing technology transfers[1]. To make things tractable and comparable with Boucekkine *et al.* (2006), we assume that this variable follows an entirely exogenous path with no specific trend. The whole equation (3) describes the diffusion of the frontier technology imported from the North, here formalized by the exogenous variable q_t^0, through the imitation (or adoption) process. More precisely, equation (2) specifies the technological catching-up process at work in the South. As such, it's a more involved reformulation of Nelson and Phelps' mechanism.

As usual, the technological gap at t, say TG_t, is defined as $\frac{q_t^0 - q_t}{q_t}$, which from equation (2) takes the form

$$TG_t = \frac{1}{d_t \left[(1 - \mu_{k,t})K_t\right]^a L_{q,t}^b H_{q,t}^c} \left(1 - (1 - \delta_{q,t}) \cdot \frac{q_{t-1}}{q_t}\right) \quad (*)$$

As noted in the literature, it follows that the technological gap depends on the adoption efforts (L_q, H_q), the exogenous productivity variable d_t and the rate of inefficiency $\delta_{q,t}$ of the embodied technological progress. However, we observe that it also depends on the fraction of capital used in the technology sector $[(1 - \mu_{k,t})K_t]$. Compared with pre-existing studies, we shall see later that this factor is an important novel ingredient that participates in the evolution of the technological gap across time. As usual in this kind of framework, the technological gap is always strictly positive and it can only vanish asymptotically if labour assignments (L_q, H_q) , or the exogenous variable d_t or the respective fraction of capital go to infinity when time tends to infinity.

We assume here perfect mobility of labour across sectors and that both the unskilled and skilled labour resources are limited at any time. More precisely, we assume that the aggregate endowment of each type of labour is given by:

[1] More technical details on d_t can be seen in Boucekkine *et al.* (2006).

$$L_{q,t} + L_{y,t} = L_t \tag{4}$$

$$H_{q,t} + H_{y,t} = H_t \tag{5}$$

where L and H being exogenously determined. Finally, since the final good can be used either for consumption or investment, the common closing equation holds:

$$Y_t = C_t + I_t \tag{6}$$

How distributional aspects do affect the optimal allocation of resources (labour plus capital) to the production of final goods versus the imitation sector? Which are the direct and indirect effects of this allocation on selected variables defining development, including income inequality? How inefficiency in the adoption process may affect economic development? We shall address these questions in the next sections.

3 The central planner problem

In this paper, we assume all agents have the same preferences for the sole consumption good that is produced in the economy. Accordingly, we consider the usual optimal growth problem:

$$\max_{\{K_t, q_t, L_{q,t}, L_{y,t}, H_{q,t}, H_{y,t}, C_t, I_t, \mu_{k,t}\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \qquad (P1)$$

This subject to equations (1)-(6), given q_{-1} and K_{-1}, and the corresponding positive constraints (particularly $0 \leq \mu_{k,t} \leq 1$). $U(\cdot)$ is a standard utility function and $\beta < 1$ is the traditional time discounting factor.

The Lagrangian for this maximization problem reads as follows after substituting equation (1) into (6):

$$\begin{aligned} \pounds = \sum_{t=0}^{\infty} \beta^t U(C_t) + \sum_{t=0}^{\infty} \beta^t \lambda_{q,t} \left\{(1-\delta_{q,t})q_{t-1} + d_t\left[(1-\mu_{k,t})K_t\right]^a L_{q,t}^b H_{q,t}^c (q_t^0 - q_t) - q_t\right\} \\ + \sum_{t=0}^{\infty} \beta^t \omega_{H,t} \left\{H - H_{q,t} - H_{y,t}\right\} + \sum_{t=0}^{\infty} \beta^t \omega_{L,t} \left\{L - L_{q,t} - L_{y,t}\right\} \\ + \sum_{t=0}^{\infty} \beta^t \lambda_{y,t} \left\{A_t(\mu_{k,t}K_t)^{\alpha} L_{y,t}^{\theta} H_{y,t}^{\Psi} - C_t - I_t\right\} + \sum_{t=0}^{\infty} \beta^t \lambda_{k,t} \left\{q_t I_t + (1-\delta)K_{t-1} - K_t\right\} \end{aligned}$$

Here, the set $\{\lambda_q, \lambda_y, \lambda_k, \omega_L, \omega_H\}$ represents the multipliers associated with the respective maximization problem restrictions. As usual, we shall use respectively the shadow price of unskilled and skilled labour (ω_L, ω_H) to quantify wage inequality hereafter.

The corresponding first order conditions can be characterized as follows after tedious but simple (and canonical) algebra:

$$U'(C_t) = \lambda_{y,t} \tag{7}$$

$$\lambda_{k,t} = \frac{\lambda_{y,t}}{q_t} = \frac{U'(C_t)}{q_t} \tag{8}$$

$$\omega_{L,t} = \lambda_{q,t} d_t [(1-\mu_{k,t})K_t]^a b L_{q,t}^{b-1} H_{q,t}^c (q_t^0 - q_t) = \lambda_{q,t} \frac{b(q_t - (1-\delta_{q,t})q_{t-1})}{L_{q,t}} \tag{9}$$

$$\omega_{L,t} = \lambda_{y,t} \theta A_t [\mu_{k,t} K_t]^\alpha L_{y,t}^{\theta-1} H_{y,t}^\Psi \tag{10}$$

$$\omega_{H,t} = \lambda_{q,t} d_t [(1-\mu_{k,t})K_t]^a L_{q,t}^b c H_{q,t}^{c-1} (q_t^0 - q_t) = \lambda_{q,t} \frac{c(q_t - (1-\delta_{q,t})q_{t-1})}{H_{q,t}} \tag{11}$$

$$\omega_{H,t} = \lambda_{y,t} \Psi A_t [\mu_{k,t} K_t]^\alpha L_{y,t}^\theta H_{y,t}^{\Psi-1} \tag{12}$$

$$\begin{aligned} &-\lambda_{q,t} d_t a \left[(1-\mu_{k,t})K_t\right]^{a-1} K_t L_{q,t}^b H_{q,t}^c (q_t^0 - q_t) + \\ &+ \lambda_{y,t} \alpha A_t [\mu_{k,t} K_t]^{\alpha-1} K_t L_{y,t}^\theta H_{y,t}^\Psi = 0 \end{aligned} \tag{13}$$

$$\begin{aligned} &\lambda_{q,t} d_t a \left[(1-\mu_{k,t})K_t\right]^{a-1} (1-\mu_{k,t}) L_{q,t}^b H_{q,t}^c (q_t^0 - q_t) - \lambda_{k,t} + \\ &+ \lambda_{y,t} \alpha A_t [\mu_{k,t} K_t]^{\alpha-1} \mu_{k,t} L_{y,t}^\theta H_{y,t}^\Psi + \beta(1-\delta)\lambda_{k,t+1} = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &-\lambda_{q,t} \left\{1 + d_t \left[(1-\mu_{k,t})K_t\right]^a L_{q,t}^b H_{q,t}^c\right\} + \\ &+ \lambda_{k,t} I_t + \beta \cdot \lambda_{q,t+1}(1-\delta_{q,t}) = 0 \end{aligned} \tag{15}$$

Equation (7) gives the optimal condition for C_t. Equation (8) gives the same for I_t. Equations (9) to (12) are the optimality conditions with respect to $(L_{q,t}, L_{y,t}, H_{q,t}, H_{y,t})$, respectively. Perfect labour mobility across sectors implies that equation (9) is equal to equation (10) and equation (11) is equal to equation (12). From this, it is easy to check that the following equation expressing educated-not-educated labour equilibrium allocation across sectors holds:

$$\frac{\theta \cdot L_{q,t}}{b \cdot L_{y,t}} = \frac{\Psi \cdot H_{q,t}}{c \cdot H_{y,t}} \tag{16}$$

By dividing now (12) to (10), the level of wage inequality in the economy reads as:

$$\Delta\omega_t = \frac{\omega_{H,t}}{\omega_{L,t}} = \frac{\Psi L_{y,t}}{\theta H_{y,t}} \tag{17}$$

Even though equations (16)-(17) formally have the same structure defined in Fuentes *et al.* (2012), we make clear that the effects caused by exogenous shocks and the mechanisms throughout which they propagate inside the economy are different in nature from those reported by these authors, particularly those related to the evolution of income inequality across time[2].

[2] At this point, it is worth recalling here that we are dealing with a closed economy under embodied technological progress and domestic investment, whilst Fuentes *et al.* (2012) assume an open economy under disembodied technological progress with no domestic investment

Equation (13) gives the optimal condition for $\mu_{k,t}$. We can rewrite this last equation in a simpler way. In fact, by substituting equation (7) into (10) and equalizing then equation 10 to the right side of equation (9), the multiplier (shadow price of q_t) $\lambda_{q,t}$ reads as follows:

$$\lambda_{q,t} = U'(C_t) \frac{\theta L_{q,t}}{b L_{Y,t}} \frac{A_t \left[\mu_{k,t} K_t\right]^{\alpha} L_{y,t}^{\theta} H_{y,t}^{\Psi}}{d_t \left[(1-\mu_{k,t}) K_t\right]^{a} L_{q,t}^{b} H_{q,t}^{c} (q_t^0 - q_t)} \tag{18}$$

By substituting now equations (1) and (18) into (13), the optimal condition for $\mu_{k,t}$ is rewritten in the following form:

$$\frac{\mu_{k,t}}{1-\mu_{k,t}} = \frac{b \alpha L_{y,t}}{\theta a L_{q,t}} \qquad (13')$$

This last equation reflects the very usual allocation of resources across sectors which depends exclusively on the respective intensities[3].

Equations (14) gives the optimal condition for K_t. By substituting equations (1), (7), (8) and (18) into (14), this condition can be rewritten in a more appropriate way as follows:

$$\begin{aligned} \frac{U'(C_{t+1})}{U'(C_t)} &= \frac{1}{\beta(1-\delta)} \frac{q_{t+1}}{q_t} \left[1 - q_t \alpha \frac{Y_t}{K_t} - a \frac{\theta}{b} q_t \frac{L_{q,t} Y_t}{L_{y,t} K_t}\right] \\ &= \frac{1}{\beta(1-\delta)} \frac{q_{t+1}}{q_t} \left[1 - q_t PMK_t (1+\eta_t)\right] \qquad (14') \end{aligned}$$

Where $PMK_t = \alpha Y_t / K_t$ is the marginal productivity of capital and $\eta_t = (a\theta/\alpha b)/(L_{q,t}/L_{y,t}) > 0$, $\forall t$. With the exception of the factor η, this equation looks very similar to found it in the literature, particularly in Boucekkine *et al.* (2006). Therefore, the two effects shaping the trajectories of consumption and saving across time apply also here. On the one hand, we see that the higher the incentives to save by increasing the efficient marginal productivity of capital $q_t PMK_t$, the higher the growth rate of consumption. We shall label this property as the *efficient marginal productivity of capital effect*. When compared with these authors, we observe, however, that this effect appears intensified in our model by the factor $\eta > 0$ which means that consumption-saving paths are also affected by the allocation of labour across sectors when capital stock is introduced in the adoption technology. In total, this efficient marginal productivity effect decreases both current consumption and present social welfare. On the other hand, we also observe that the growth rate of consumption tends be lower in presence of the obsolescence effect mentioned in Section 2, i.e., for higher values of q. This effect diminishes progressively the relative price of capital goods $(1/q_t)$ which raises current consumption and in turn current social welfare. Overall, the presence of η in our model increases the gap between the first over the second effect which privileges, in principle, future than present gains in welfare. As a result, while Boucekkine *et al.* (2006) claim that the rate of decline of the relative price of capital is the crucial determinant of the output composition, we say that sectoral distribution of labour is also important in determining that composition. Of course, the time preference β and the rate of depreciation of capital δ play also an active role in sketching the final pattern of this

[3]Note that if we combine equation (3) with equation (16) we get $\frac{\mu_{K,t}}{1-\mu_{K,t}} = \frac{b\alpha L_{y,t}}{\theta a L_{q,t}} = \frac{c\alpha H_{y,t}}{\Psi a H_{q,t}}$. If technologies were the same, the allocation of resources would be the identical across sectors.

variable, as we see from equation (14').

Equation (15) is the optimal condition for q_t. As the previous equations, it can also be rewritten in a different way. In fact, by substituting equations (2) and (8) into (15) we get:

$$\frac{U'(C_t)}{q_t}\frac{K_t-(1-\delta)K_{t-1}}{q_t} = \lambda_{q,t}\left(1+d_t[(1-\mu_{k,t})K_t]^a L_{q,t}^b H_{q,t}^c\right) - \beta(1-\delta_q)\lambda_{q,t+1} \tag{15'}$$

Recalling that $\lambda_{q,t}$ is the shadow price of q_t , this equation represents the equality between marginal revenue (left side) to the present value of marginal cost (right side) in terms of welfare under a marginal increase in q_t. Unlike Boucekkine *et al.* (2006), we observe that the marginal cost of q_t at time t depends also on capital stock in the technology sector and the inter-temporal term includes the parameter δ_q which comes entirely from the specification of the adoption sector.

Let us now to investigate the steady state growth paths.

4 The steady-state paths

From here, we assume a logarithmic utility function. Equations (1)-(6) plus equations (13'), (14'), (15') and (16) define the following long term system (we shall omit the upper-bar symbol usually designed for long run levels just to unburden the presentation):

$$Y = A[\mu_k K]^\alpha L_y^\theta H_y^\Psi \tag{19}$$
$$\delta K = qI \tag{20}$$
$$\delta_q q = d[(1-\mu_k)K]^a L_q^b H_q^c (q^0 - q) \tag{21}$$
$$L_y + L_q = L \tag{22}$$
$$H_y + H_q = H \tag{23}$$
$$Y = C + I \tag{24}$$
$$\tfrac{\mu_k}{1-\mu_k} = \tfrac{b\alpha}{\theta a}\tfrac{L_Y}{L_q} \tag{25}$$
$$\tfrac{\theta L_q}{bL_y} = \tfrac{\Psi}{c}\tfrac{H_q}{H_y} \tag{26}$$
$$1-\beta(1-\delta) = \tfrac{qY}{K}\left(\alpha + \tfrac{a\theta}{b}\tfrac{L_q}{L_y}\right) \tag{27}$$
$$\delta_q \tfrac{b}{a}\delta = \tfrac{qY}{K}\tfrac{L_q}{L_y}\left[1 + d[(1-\mu_k)K]^a L_q^b H_q^c - \beta(1-\delta_q)\right] \tag{28}$$

Notice that this set of equations contains 10 equations for the 10 unknowns $\{L_q, L_y, H_q, H_y, K, I, C, Y, q, \mu\}$. The above system has two characteristics. It's highly non-linear (see for example the last equation of the system above) and non-recursive which makes it impossible to reduce its dimension significantly. However, an important insight can be obtained from equations (20),(25) and (27). In fact, these equations can be easily combined to get the following expression for the investment ratio:

$$s = \frac{I}{Y} = \frac{\delta(\alpha+\gamma\eta)}{1-\beta(1-\delta)} = \frac{\delta\alpha}{[1-\beta(1-\delta)]}\frac{1}{\mu_k} \tag{29}$$

Where $\gamma = a\theta/b$ and $\eta = L_q/L_y$[4]. Equation (29) makes an important difference with the related literature (particularly with Boucekkine *et al.* (2006)). While these authors define a balanced growth path from which this rate is an exogenous function of the growth rate of the level of the embodied technological progress abroad, we see that the composition of output is conducted here by an endogenous and local factor, namely, the allocation of capital across sectors. Therefore, the comparative static of this ratio is *a priori* ambiguous because it will depend on how the value of μ_k is affected by each particular shock. Unfortunately, due to the complex nature of the steady sate solution, nothing more can be said about until looking at the simulations.

On the other hand, by defining $z_1 = \frac{\theta a}{b\alpha}$ and $z_2 = \frac{\psi a}{c\alpha}$, equations (17), (25), (26) can be easily combined to get the following expression for the long term level of income inequality:

$$\triangle w = \frac{c}{b}\frac{L}{H}\left[\frac{(1-\mu_k) + z_2\mu_k}{(1-\mu_k) + z_1\mu_k}\right] \tag{30}$$

This last equation is very important because it clearly emphasizes that, under embodiment, the long run income inequality resulting from welfare maximization is affected in our model by three different and visible factors (or mechanisms). First, we see that technological parameters such as elasticities matter. This is not new in the literature. Indeed, Bertola (1993) and García-Peñalosa *et al.* (2005) have already identified this dependence under disembodied technical progress. However, the nature of this relationship is quite different in our model. While these authors found that the labour share in production is an important factor determining long term income inequality, we found that this latter variable is strongly influenced by the ratio between the labour share of skilled to unskilled labour in *adoption*, say $\frac{c}{b}$. An important question raises from this finding: could a change in the economy structure be implemented in order to influence the distribution of income through a change in the ratio $\frac{c}{b}$? Unfortunately, the answer to this question seems to be *no*, at least at the current state of knowledge. Indeed, it is well known that elasticities -like preferences- are very hard to modify, mainly perhaps because we know very few on how to make any influence on them through policy or private decisions made inside firms in order to cause changes in factor returns over time[5]. Moreover, both theoretical and empirical works dealing with the evolution of factor shares across time and its impact on the functioning of the economy have been focused on the production (final) sector[6]. Therefore, this lack of knowledge is still larger when we talk about technology adoption. Although strong, this is not the main reason justifying our response, though. Even considering an ideal scenario where this ratio is manipulable, one would expect that, in order to have an efficient adoption sector, the skilled labour share there (c) is always bigger than the respective unskilled one (b), at least in the long run. Therefore, the expected value for such a ratio should be always bigger than one. As a result, we have identified a *technical* source of long run income inequality very difficult to manage which should serve, at least, to refocus policy on income distribution towards more effective (and still unfounded) instruments. The second factor determining long run income inequality is the ratio L/H -which is totally exogenous in our model. As well known, this factor implicitly reflects the key issue that inequality is closely linked to educational and skill levels. In particular, it has been argued by who emphasize technology as the driving cause of widening wage inequality around the world that this is

[4]With the exception of η, all the remaining parameters are constant.

[5]We refer to that, even though recent literature on factor shares shows that there is a variation both over time and across countries, our understanding about functional distribution of income and its determinants remains still in question among economists, particularly from the economic development point of view.

[6]Useful insights into the labour share can be found in Guerreiro (2012) and Schneider (2011). For the capital share, see Jones (2003) and Ortega *et al.* (2006)

due to "skill biased technical change" has caused a rapid shift in the demand for skilled relative to unskilled labour. Other things constant, our model suggests the first-best response against this phenomenon should consider the increase of the number of educated people relative to non-educated ones, causing the equilibrium skilled/unskilled income ratio, $\triangle w$, to down. At first glance, this first-best reaction may sound too simple. Indeed, according to modern vintage capital theory, it is. However, from an economic development point of view it suggests in a quite intuitive way something expected to occur in developing countries: the sophistication of the productive capacity of the economy should be accomplished by the increase in the number (and quality) of people able to manage such a more sophisticated economy. And this puts pressure on policy -particularly on educational policy- in the sense that it is expected that it contributes strongly and decidedly to provide an economic environment where most people can achieve high level of education. The third factor determining long run income inequality is endogenous and related to a particular weighted distribution of capital across sectors, namely, $f(\mu_k) = \frac{(1-\mu_k)+z_2\mu_k}{(1-\mu_k)+z_1\mu_k} > 0$ where the weighting factors depend on labour elasticities in both production and adoption. More specifically, we can easily deduce from this latter equation that when $z_2 < z_1$, i.e, when $\frac{\psi}{\theta} < \frac{c}{b}$, $f(\mu_k) < 1$. As a result, this factor helps then to diminish the long run level of income inequality. On the contrary, when $z_2 > z_1$, $f(\mu_k) > 1$ and it contributes to increase that level. Overall, we see that, under embodiment, the distribution of labour intensities in production *relative* to that one in adoption also matters in long run income inequality. As argued before, there is no much room for policy if one would like to modify these structural parameters, though[7]. In the particular case where $z_1 = z_2$, it is possible to make the following proposition:

Proposition 1 *By defining* $x = \frac{L}{H}$ *and if* $\frac{\psi}{\theta} = \frac{c}{b}$,

i) $\triangle w = \frac{c}{b}x$,

ii) $\frac{\partial \triangle w}{\partial x} = \frac{c}{b} > 0$,

iii) $\frac{\partial \triangle w}{\partial L} = \frac{x}{H} > 0$,

iv) $\frac{\partial \triangle w}{\partial H} = -\frac{xL}{H^2} < 0$,

Proof: Since $\frac{\psi}{\theta} = \frac{c}{b}$ implies that $z_1 = z_2$, it is easily verified from equation (30) that $\triangle w = \frac{c}{b}x$. Therefore, ii)-iv) are directly derived from this result.■

First of all, we would like to highlight that the condition $z_1 = z_2$ does not mean that both technologies, production and adoption, are the same [8]. Actually, the capital share in each technology remains unaltered (and different) under this condition. Just the distribution of the share of labour are equal across sectors. In such a situation, long run income inequality becomes fully exogenous and it depends mainly on the ratio between the total unskilled labour force to the total skilled labour one. In other words, embodiment plays no much role in determining the long run level of inequality. Not surprisingly, we see that the this level can be reduced only by increasing the level of educated people, H.

The last factor determining long run inequality that is also stemmed from $f(\mu_k)$ has to do with μ_k itself. Of course, this is an endogenous variable, reason for which any comparative static respect to this variable makes no sense. However, one thing is clear. The higher the long run value of μ_k the higher the long run level of income inequality. This result should not be unexpected at all. Indeed, it may sound rare (or counter-factual) that capital stock was mostly allocated to the intermediate sector, at least in the long run. The problem then is to know how and to what long run income inequality can be reduced from

[7] Note that the effect of the productivity of capital (α) on income inequality, even if present through z_1 and z_2, appears quite diminished mainly because it appears as in the numerator as in the denominator

[8] The educated guess $\frac{c}{b} > 1$ mentioned above would imply that $\psi > \theta$, i.e, the share of skilled labour in production should be higher than the respective share of unskilled one. This condition could be counter-factual if we assume that the economy is consisted just of "workers and engineers", i.e, of low- and high-income workers. However, if we incorporate middle-income workers (technicians, for example) into the skilled people, such a condition makes sense, particularly because we are at the steady state point, situation at which the sophistication of the economy is expected to be higher.

the steady state situation by exogenous shocks that alter the reassignment of capital across sectors as a result of social welfare maximization. The following findings can be advanced in this regard. Suppose the economy is faced with an exogenous shock on the arbitrary variable "y" different from H and L[9]. Then, it is easy to verify from equation (30) that, other things constant,

$$\frac{\partial \triangle w}{\partial y} = \underbrace{\frac{c}{b}\frac{L}{H}\frac{a}{\alpha}\frac{1}{[(1-\mu_k)+z_1\mu_k]^2}}_{>0}\left[\frac{\psi}{c}-\frac{\theta}{b}\right]\frac{\partial \mu_k}{\partial y} \tag{31}$$

From equation (31), we observe that the distribution of the labour shares across sectors also matters in determining the long run *change* in the level of inequality. More specifically, other things constant, if $\frac{\psi}{\theta} > \frac{c}{b}$, this condition contributes to increase that level. By contrast, if $\frac{\psi}{\theta} < \frac{c}{b}$, that level is reduced. On the other hand, equation (31) shows that the key endogenous mechanism generating changes in the long run level of inequality in our model is related to the reallocation of capital stock across sectors once shocks occur. We do think that this is an important input (result) for policy. If, for any reason, a society wants to prioritize the final (or the intermediate) sector by increasing there (decreasing) the long run level of capital $\frac{\partial \mu_k}{\partial y} > 0 \left(\frac{\partial \mu_k}{\partial y} < 0\right)$, the resulting effect of the underlying policy on inequality will depend on how structural technological parameters such as (ψ, θ, c, b) have been fixed. Therefore, our model suggests that the race to development based on the sophistication of the economy via embodiment might eventually contribute to reduce long run inequality. However, a serious implementation of such strategy requires a complete knowledge of the technologies involved.

4.1 Parametrized Example

Consider the following calibration of the model which is made for a representative developing country. A first set of parameters if fixed a priori to what we view as reasonable values given the empirical evidence available (see Table 1). The rate of depreciation of capital is 15% and the psychological discount factor is 0.98. Since we do not have data on the rate of inefficiency in adoption we set this parameter arbitrarily at 10%.

A second set of parameters is fixed in order to match a series of moments of the steady state we consider. The levels of skilled and unskilled labour are such that the skilled population is $3,2\%$ of total population (roughly the share of workers with higher education in developing countries[10]) and the ratio of the two wages is 15 (the ratio of elite workers wage to unskilled workers wage). The parameters (θ, Ψ) are such that the share of total labour (skilled plus unskilled) in the final sector is near 0.68. The same global criterion is used for choosing the parameters (b, c) in the technological sector, i.e., the share of total labour in this sector is fixed at 0.8. The policy variable d and the level of the frontier technology imported from the North are fixed at 0.2 and 1, respectively, to have a ratio consumption to output of 0.7 and a capital to output ratio of 1.18.

Proposition 2 *If $\beta < 1$, $0 < \delta < \frac{[1-\beta(1-\delta)]}{\theta}$, there is a unique steady state solution.* A detailed proof of this statement is reported in the Appendix.

[9]We have ruled out the possibility of a shock on L because it would be an undesirable goal in economic development to increase the number of unskilled people, either in absolute or relative terms.

[10]Boucekkine and De la Croix (2003) fix this value at 10% for developed countries.

Parameter	Symbol	Value
Capital share in the final sector	α	0.32
Unskilled labour share in the final sector	θ	0.476
Skilled labour share in the final sector	Ψ	0.204
Capital share in the technological sector	a	0.20
Unskilled labour share in the technological sector	b	0.24
Skilled labour share in the technological sector	c	0.56
Total factor productivity in the final sector	A	2.67
Productivity in the technology sector	d	0.2
Total amount of unskilled labour	L	30
Total amount of skilled labour	H	1
Time discounting factor	β	0.98
Rate of depreciation of capital	δ	0.15
Rate of inefficiency in adoption	δ_q	0.10
Level of technological progress in the North	q^0	1

Table 1: **Parameters**

To highlight the the importance of inefficiency in adoption, we additionally consider another calibration which differs from the benchmark in the value of δ_q. Concretely, we consider the case where δ_q equals three times the benchmark value. The long run results of the two parametrization are shown in table 2:

	Benchmark model	When δ_q is high $\delta_q = 0.3$
Lq	1.73801	2.72935
Ly	28.262	27.2706
Hq	0.250832	0.35271
Hy	0.749168	0.64729
μ_k	0.929169	0.88963
q	0.575588	0.367473
K	50.1083	25.2522
I	13.0584	10.3078
C	29.1559	21.5965
Y	42.2143	31.9043
SW	3.37266	3.07253
TG	0.737355	1.72129
$\triangle w$	16.1676	18.0559
K/Y	1.187	0.791497
s	0.309336	0.323084

Table 2: **Long run effects of the calibrations**

Several lessons can be drawn from Table 2. Generally speaking, we observe that the economy with a higher level of inefficiency in adoption (higher δ_q) shows a much weaker long run performance in variables such as aggregates, inequality, and the technological gap. More specifically, it is verified that adoption efforts (and capital) are much higher in the weaker economy. This issue is a direct consequence of equation (21): $\delta_q q = d[(1-\mu)K]^a L_q^b H_q^c (q^0 - q)$. In fact, the product $\delta_q q$ increases when δ_q goes up. Then, it is clear that under the assumption that d and q^o are constant and that gains in $(q^o - q)$ are not big enough to balance the equation, resources devoted to adoption must be incremented in order to correct the imbalance created by the rise in δ_q. Therefore, this reallocation of resources in favour of the intermediate sector implies a lower level of activity in the economy and then of welfare. What is particularly striking of this reassignment is the issue that it is driven by augmenting more the amount of *unskilled* labour than that of the skilled one. In a certain way, one could then say that in those economies where the inefficiency in adoption is high, the shortage of skills obligates the economy to engage significantly unskilled people in adoption processes which could eventually generate a proliferation of inefficiency in this sector and, by implication, in the economy as a whole[11]. On the other hand. it is particularly interesting that this type of inefficiency has also distributional effects on the economy. We see that the higher is δ_q the higher is the long run level of inequality. According to our model, an increase of labour efforts in adoption is necessarily and simultaneously followed by a decrease in labour efforts in production because total labour is constant. Therefore, income inequality is decreased due to the fall in the amount of skilled labour in production (around 13%) is much bigger than the respective fall in the amount of unskilled labour (around 3.5%), just as equation (17) predicts. Overall, we do thing that this observable connection between inefficiency in adoption and economic performance opens a window to be explored in the future which will require necessarily to endogenize δ_q. By now, our model suggest that developing countries should assess (optimize) seriously the efficiency of their catching up processes in order to permit the liberalization of more resources towards the final sector for then improving the long run performance of the economy under embodiment.

5 Dynamics

In this section, we will present the results of dynamics simulations conducted on the calibrated model described above. We shall show the numerical exercises in three groups. The first group contains two separated technological shocks, one being of the embodied type and the other of the disembodied type. The second group involves independent institutional shocks. The third group brings together an exercise involving simultaneous shocks with a robustness analyses. The magnitude of the shocks is set equal to 1%. All the shocks will be permanent. The dynamic system is reported in Appendix A. The solution paths are displayed in Figures 1 to 6[12] . Each solution path represents the evolution of the percentage deviation of a given variable with respect to its initial steady state value (vertical axis). Horizontal axis represents periods

Before presenting the simulations, we recall that, for fixed values of β and δ, optimal social welfare paths are strongly influenced, among other things, by the two opposing underlying forces at work in the model (see Section 3): the efficient marginal productivity effect and the obsolescence one; the first effect decreasing the incentives to privilege current welfare and the second one spurring them. Under scenarios in which q is an increasing (and convergent) function of time -which is actually the case assumed in this paper-, the obsolescence effect will be increasingly rich as time goes on, independently of the type of shock. However, under rational expectations, we expect that this increment is much more intense during some periods until converging to a flatter pattern[13]. While the obsolescence effect pushes the economy to specialize in the production of capital goods via enhancing the q-sector, the other effect may lead the economy to the production of investment goods which requires strength necessarily the final sector with more resources. *A priori*, one could anticipate that the problem is solved if the economy devotes exclusively to the production of investment goods which are required as inputs in the intermediate sector. However,

[11]This finding is not actually far from reality. It is a very common practice observed specially in those countries where adoption is incipient. There, unskilled people receive just a short-lived instruction before being engaged in production

[12]We use Dynare, the package developed by Juilliard (2012), for the simulations and stability assessment of non-linear forward-looking variables

[13]This is because we have assumed, by construction, a Cobb-Douglas function for q.

we will see that the solution for this trade-off is far from being easy, particularly under technological acceleration abroad. Keeping these issues in mind helps to understand in a more effective way how different shocks propagate throughout the economy drawing thus the optimal trajectories consistent with social welfare maximization.

5.1 Optimal responses under technological acceleration

Here, we study how the economic system reacts under two separated technological acceleration. The frontier technological parameter q^0 is first increased permanently by 1%, as reflected in Figure 1. Then, the total factor productivity A is raised by 1%, as showed in Figure 2. The main lessons drawn from the experiments are as follows.

Unlike Boucekkine et al. (2006), an external technological acceleration does induce a massive intensification of the adoption efforts in the short run[14]. See Figures 1(a)-(c). In addition, we see that this response is accompanied by an increase in the fraction of capital devoted to adoption. See Figure 1 (e). This inter-sectoral reallocation of resources (adoption efforts plus the fraction of capital) in favour of the q-technology lasts 8 periods. As expected, the level of embodied technological progress goes up, either in the short- or the long run. As a result, we note that this temporal reassignment of resources makes fall output for 4 periods, consumption for 6 periods and investment for 3 periods. See Figures 1 (h)-(j). Consequently, welfare is below its initial state value for 6 periods. See Figure 1 (k). In addition, the technological gap and income inequality are both increased during 5 and 8 periods, respectively. Quantitatively, income inequality rises by 0.8% (at the peak) whilst the technological gap achieves a maximum of 1.8% over its initial steady state value. See Figure 1 (l)-(m). Overall, we observe that the optimal short run responses are unfavourable for development in many aspects during the catching up process under technological acceleration abroad. However, we will see that this unfortunate situation is notoriously reverted in the long run. In fact, we first note that the allocation of resources achieves its long run condition from period 9. This final position is characterized by a distribution of resources slightly favourable to production. See again Figures 1 (b)-(d). In addition, capital stock is significantly increased by 1.8%. See Figure 1(g). As a consequence, aggregates are all incremented by 0.5% (on average); both capital to output ratio and welfare rise by 1.1% and 0.18%, respectively. The technological gap is reduced by 0.25%. However, this final reallocation of resources is not sufficient to alter significantly both income inequality and the investment ratio. See Figures 1(m)-(o). Compared with Boucekkine *et al.* (2006), we observe this long run scenario differs in all the aspects considered by them. In fact, they reported that, under embodied technological acceleration, as adoption efforts as the technological gap and the investment ratio were all incremented in the long run. More importantly, while they claimed that long term social welfare maximization is incompatible with sharp adoption efforts, we argue the opposite. Therefore, the introduction of capital in the technology sector joint with the assumption of labour heterogeneity implies a sharp difference in the outcomes when this shock occur, either in the short- or in the long run.

In searching for a rationale for such behaviours, let us convoke again the two opposite effects mentioned before. In the short run, it is clear that the obsolescence effect carries the economy for a while because the relative price of capital goods is progressively and significantly cheaper[15]. Therefore, the optimal short run strategy would be then to specialize the economy in the production of intermediate goods by increasing rapidly the level of the embodied technological progress q. For this, both massive adoption efforts and more capital are required. However, the previous strategy is not sustainable because capital obeys to the law of diminishing marginal product and social welfare cannot be sacrificed for a long time. Therefore, the optimal long run strategy is to incentive saving by increasing the production of

[14]Since labour resources are limited, the opposite is observed in labour allocated to production.

[15]In fact, q grows faster than investment at any time. See Figures 1(f) and 1(h)

final goods, particularly investment ones. Since investment is the sole input required in the production of capital goods, it is clear that this strategy is also compatible with the decline of the relative price of these goods which incentives its production[16]. Consequently, labour is slightly reallocated in favour of the final good sector which implies that gains in output and the reduction of the technological gap are both driven mostly by the sharp rise in capital stock. Then, the efficient marginal productivity effect leads eventually to the economy towards its long term position. Finally, the issue that the investment ratio keeps unaltered in the long run can be explained from equation (29). From it, it deduced that an increase in q^0 does not affect the long run distribution of labour across sectors, an important mechanism defining the long run output composition in our model.

Summarizing, we see that it would be optimum that the economy engages immediately in catching up the leader when an embodied technological acceleration occurs abroad even though it must pay short run costs in terms of welfare and others, including the rise in income inequality. The long run scenario so performed looks not bad, which gives theoretical support to the view of embodiment in economic development, at least in the form assumed in this paper.

From now, we will study what happens when the economy is benefited from technological improvements in the final sector throughout permanent changes in the **disembodied** technological progress A.

We first note similar qualitative behaviour in the reallocation of resources as in the previous shock. In fact, we observe a significant short run reassignment of resources (particularly labour efforts) to the q-sector for 7 periods which increases the production of capital goods. See Figures 2(a)-(c). However, this redistribution is not as massive as the previous one (around 5 times lower at the peak). What is completely different here, though, is that this response is accompanied by a boom in the production of investment goods (it goes up almost 2% at the peak) which comes to reinforce the production of capital goods. To be more precise, what happens here is exactly the opposite. Since investment rises 10 times more than q, it clear that is the efficient marginal productivity effect who leads the economy during the short run and the impact of the obsolescence effect, although present, is secondary and lower. As an expected consequence, short run performance looks wealthy and vigorous in terms of aggregates such as consumption, investment and output. See Figures 2(h)-(j). However, welfare shows a moderate rate of growth across time. See Figure 2(k). On the other hand, income inequality is raised by 0.15% (at the peak) and capital to output ratio is below its steady state value for a significant part of the total period which means that output increases faster than capital. See Figures 2(m)-(n). In addition, while the ratio investment to output keeps unaltered (even in the long run), the technological gap is reduced from the beginning of the shock. See Figures 2(l) and 2(o). From period 7, the economy begins to converge to its final point, which is mainly characterized by a moderate reallocation of resources to the final sector and a high rate of growth of aggregates (near 1.6%, on average). At this point, it is worth mentioning that the long run growth rate of capital stock is almost the same reported under an embodied (q^0) technological shock. The reason is simple. Under embodiment, this rate is mainly driven by increasing the level of q (q-sector) while, under disembodiment, it is driven mainly by increasing investment (final sector). On the other hand, social welfare is increased by 0.48%; the technological gap is reduced by 0.22%; capital to output ratio is increased by 0.1% and income inequality is reduced by 0.02%. It is important now to understand why a disembodied shock produces the outcomes we have reported.

Following the arguments in the literature, but from a different perspective, we know that an increase in A raises, on the one hand, the marginal productivity of capital which encourages saving throughout the production of investment goods. On the other, an increase in A raises at the same time the marginal

[16] In very simple words, this strategy is well crystallized in the common saying, "to kill two birds with one stone"

productivity of labour in the final sector which incentives the reassignment of labour to the q-sector in order to produce capital goods. This produces a progressive (and weak) fall (0.1% at the maximum) in the relative price of these goods which enhances a bit more its production. We already know that this might be made by increasing both q and investment for a while but not forever due to the trade-offs present in the model. This is exactly what happens here. Since the shock on A is permanent, it is expected that the efficient marginal productivity effect (by means the permanent increase in the marginal productivity of capital) leads the economy either in the short- or in the long run. In other words, the shock on A impacts more significant and durably on the marginal productivity of capital than on its relative price. For this reason, the reallocation of labour to adoption is moderate and shortly whilst investment is highly boosted either in the short- or in the long run. However, we observe that this boom is not able to modify the investment ratio output in the long run. Moreover, it is not able to modify it at any time. The reason is quite straight-forward and lies again in equation (14'). Since a shock on A has not a significant impact on the relative price of capital goods at any time, the factor η representing the distribution of unskilled labour across sectors is not affected by the shock.

Compared with Boucekkine *et al.* (2006), we bring to light the following differences. First, the magnitude of the reallocation of labour efforts to the q-sector in the short run is much more significant in our case. Second, while they reported an immobile labour allocation from period 10, we have observed a reassignment of them to the final sector starting from period 7. Third, while they reported an immobile technological gap we have seen that it is reduced from the beginning of the shock. Forty, while they reported a short-lived investment boom we observe that this boom is present as in the short- as in the long run. In short, the introduction of capital in the technological sector turns less trickier the outcomes produced by an A-shock![17].

5.2 Optimal responses under institutional and policy improvements

In this second group of simulations, policy variables d and H are increased separated and permanently by 1%, as reflected in Figures 3 and 4. Following the arguments in the literature, the shock on d could represent any improvement in the quality of education and/or trade policy and institutions favourable to adoption of foreign technologies. Likewise, the shock on H could be the result of the expansion of national coverage of education order to generate more educated people. The main lessons to be drawn from the experiments are as follows:

Once d is permanently increased, we basically observe the same qualitative short run behaviour as in the case of the previous shocks but with some noticeable quantitative differences in some variables. The first best short-run response is to reallocate resources in favour of the q-sector which goes up rapidly during the first periods in detriment of the final sector. However, this reassignment, is neither so massive nor so prolonged. Quantitatively speaking, it is as moderate as the shock on A (around 0.5% at the peak for labour and 0.05% at the peak for the fraction of capital) and lasts 3 periods (less than half of the previous periods). See Figures 3 (a)-(d). As a result, all aggregates and social welfare fall below their initial steady state values for a short while but this fall is much less acute than that observed under technological acceleration abroad. For instance, output falls by 0.01%; investment by 0.055% and consumption and social welfare fall by less than 0.01% (on average). See Figures 3(h)-(k). On the other hand, short run income inequality is increased (at the peak) by 0.12% (almost the same value obtained under a A-shock and an eighth of the value obtained under a q^0-shock). See Figure 3(m). Like the shock on A, the technological gap is reduced from the beginning of the shock. However, its rate of decline is almost 4 times bigger here. See Figure 3(l).Going now to the long run, we note from Figures 3(a)-(d) that the redistribution of

[17] The authors themselves claimed that their respective results are "much trickier" when compared with those outcomes produced under technological acceleration abroad.

resources to the final sector starts from period 4 and the economy gradually converges to its definite final situation from period 10. Like the shock on q^0, the weak short run performance in terms of aggregates and others is fully reverted in the long run. However, there are significant quantitative differences. Here, output, consumption and investment go up, on average, 2.5 times less and social welfare raises 3 times less. By contrast, the technological gap is reduced by 7 times more and income inequality is moved falling by 0.1%. Respect to long run ratios, capital to output ratio increases 3.3 times less. Likewise, while the investment ratio keeps unaltered under a q^0-shock, it is slightly reduced here by 0.05%. On the other hand, compared now with the shock on A, while aggregates such as output, consumption and investment increase by 8 times less (on average) under a d-shock, social welfare makes the same by 7. In addition, both the technological gap and income inequality are reduced by 4 times more. Capital to output is increased by 3 times more and the investment ratio is reduced here by 0.02% instead of keeping constant.

In short, -and speaking now in plain English- we notice that whereas the optimal short run scenario performed by a d-shock looks much better when compared to the shock on q^0, it looks much weaker when compared to the shock on A. Therefore, short run adoption is less problematic, in relative terms, under an A- than a d-shock or than a q^0-shock. However, the situation turns more mixed when we go to the long term. Briefly, we observe a lower level of activity but with more equality and with a stronger reduction in the technological gap under a permanent shock on d.

Compared with the literature (especially, Boucekkine *et al.* (2006)), we do not note significant qualitative differences neither the short- nor in the long run under a permanent shock on d . However, our quantitative results differ markedly from them. For instance, short run adoption efforts are 3.0 times higher (at the peak) than those reported by these authors. In addition the technological gap is reduced at a rate 100 times higher than those reported by them. On the other hand, both long run adoption efforts and the technological gap are reduced by 200 and 95 times more in our simulations, respectively. These numbers bolster emphasize another dimension of the effects caused by the introduction of capital and labour heterogeneity in Boucekkine *et al.* (2006)'s model. It is not difficult to understand these differences. A shock on d can be seen like a permanent improvement in the "total factor productivity" of the q-sector which boost both the marginal productivity of capital and labour in this sector. Therefore, one would expect a release of resources toward the final sector. This effectively happens, but just from period 4. In other words, since the shock on d is permanent, the first impulses conducted to specialize the economy in the production of capital goods by increasing the level of embodied technological progress q are promptly reoriented to saving via increasing investment. Then, the efficient marginal productivity effect dominates over the obsolescence one during a significant part of the total period. Finally, since the long run impact of d on the relative price of capital goods is moderate, the production of final goods and in turn the level of activity is modest here but sufficient to reduce significantly both income inequality and the technological gap. Finally, according to equation (29), the long run investment ratio decreases a bit because the efficient marginal productivity effect causes that the factor $\eta = L_q/L_Y$ is lower. In other words, the long run composition of output is slightly altered in favour of consumption

From now, the institutional variable H is separately shocked. As said, this shock could be the result of the expansion of national coverage of education in order to generate more educated people. Taking into account the insights gained from the previous tests, we will make the exposure as concise and clear as possible. Accordingly, we also observe here a massive reassignment of resources to the q-sector, which is particularly driving by increasing the level of skilled labour in this sector. See Figures 4(a) and 4(c). This implies a rapid growth in the level of embodied technological progress which reduces progressively the price of capital goods. See Figure 4(f). However, this reallocation is not made (on average) in detriment of the final sector like the previous experiments. Indeed, even though the short run level of unskilled labour in production decreases, the level of skilled labour there is significantly raised. Figures 4(b) and

4(d). Therefore, the economy takes advantages from this shock and the planner is in a better position to solve the inherent trade-offs in resources distribution. See Figures 4(b) and 4(d). Like the shock on d, the reallocation to the q-sector is very shortly (4 periods) and once it is gone, the economy starts its way to its long run scenario, which is achieved at period 12, two periods later than that observed under the shock on d. At this point, it is worth explaining this little difference in timing. The shock on d affects more directly the fall in the relative price of capital goods $1/q$ which is more accelerated than that caused by the shock on H. In other words, the duration of the obsolescence effect is more extensive under this latter shock. Similarly to the shock on d, we see that the optimal strategy to produce capital goods via q is rapidly replaced by one incorporating the production of final goods. It is then crystal that the efficient marginal productivity effect also leads the economy towards its convergence point. Overall, we observe a good performance in all relevant variables, at least in the long run. Output, consumption, investment and social welfare are all of them increased either in the short- or in the long run. See Figures 4(h)-(k). In addition, both income inequality and the technological gap are reduced from the beginning of the shock. See See Figures 4(l)-(m). Finally, capital to output ratio falls for a while but ends up with a value above its initial steady state one; investment to output ratio goes up markedly in the short run to finish slightly reduced in the long run. See Figures 4(m)-(o).

Summarizing, the most relevant qualitative differences with the previous policy experiment (d-shock) are threefold. Social welfare and output increase whilst income inequality decreases at *any* time. On the other hand, when compared to the same shock, the current long run scenario appears more varied in quantitative terms. On the side of improvements, all aggregates and welfare are twice bigger whilst income inequality is reduced by a factor of ten. By contrast, the technological gap and capital to output ratio are both almost 1.7 times lower.

At this point, it should be clear for the reader that each of the shock here analysed has its pros. and cons. respect to the others. In order to put this in perspective, Table 3 shows a survey of the resulted reported above, particularly those related to development. In this table we have indicated with two arrows those more significant impacts (in relative terms).

	SHORT RUN				**LONG RUN**			
Variable-shock	q^0	**A**	**d**	**H**	q^0	**A**	**d**	**H**
Welfare	↓ (7)	↑	↓(4)	↑	↑	↑↑	↑	↑
Output	↓ (4)	↑	↓(3)	↑	↑	↑	↑	↑
Investment	↓ (3)	↑↑	↓(3)	↑	↑	↑↑	↑	↑
Technological gap	↑ (5)	↓	↓↓	↓(a bit)	↓	↓↓	↓	↓↓
Inequality	↑ (8)	↑ (5)	↑(3)	↓	neutral	↓(a bit)	↓	↓↓
Investment to output ratio	↓ (2)	neutral	↓(3)	↑	neutral	neutral	↓(a bit)	↓(a bit)
Capital to output ratio	↓ (4)	↓ (11)	↓(3)	↓(5)	↑	↑ (a bit)	↑	↑

Table 3: **Short- and long run effects of separated shocks**; (x) means periods; (a bit) means the change is not significant

From this table, we would like to highlight two issues. First, it is clearly verified that models incorporating embodied technological progress are goods at boosting investment. Moreover, this property is stressed when distributional aspects are included into. In other words, the factor η in equation (14') plays an important role in promoting a strategy for capital accumulation via investment (efficient marginal productivity effect) more than throughout increasing the level of embodied technological progress, i.e., by reducing the price of capital goods (obsolescence effect). This applies for any type of shock. Second, the results suggest that, under adoption of new foreign technologies, income inequality emerges like an operational result from welfare maximization, particularly in the short run. Then, institutional changes are strongly required in the economy in order to reduce it significantly. Our model allows us identify two key aspects related with this goal: *massivity* and *velocity*. Put in another way, it seems to be that income inequality is a matter of *momentum*, said in physical terms[18]. It is known that by augmenting the number of educated people (spurring massivity) is a desirable issue that might promote more equality inside the economy. On the other hand, by increasing simultaneously the "total factor productivity" in the adoption sector would help print velocity to this promotion. Then, the study of the transitional dynamics towards the long run situation should occupy an important place in any plan (strategy) to achieve growth with equality, which implies to build appropriated institutions serving to this purpose. How much time a society is willing to wait to see a significant reduction in income inequality, for example? Even thought the answer to this question is far from the scope of this paper, we propose a final experiment involving simultaneous shocks on all the exogenous variables listed above. Armed with the information gathered in Table 3, we just want to show how a policy package[19] is required for engaging the economy more properly in the catching up process in order to achieve development. For this, q^0, A, d and H will be increased simultaneously and permanently by 1%. Figure 5 reports the quantitative results. Table 4 shows (summarizes) the qualitative results.

Variable-Timing	**SHORT RUN**	**LONG RUN**
Welfare	↑	↑
Output	↑↑	↑↑
Investment	↑↑	↑↑
Technological gap	↑(2)	↓↓
Inequality	↓	↓↓
Investment to output ratio	↑↑	↓(a bit)
Capital to output ratio	↓	↑↑

Table 4: **Short- and long run effects of combined shocks**; (x) means periods; (a bit) means the change is not significant

From both this table and Figure 5 we observe that under technological acceleration abroad both the short and the long run performance of the economy are notoriously improved when shocks on A, d and H are simultaneously combined with it. For instance, and looking at the long run, it is noted that while variables such output, consumption and investment increase (on average) by 2.5%, social welfare goes up by 0.8%. See Figures 5 (h)-(k). In addition, the technological gap and income inequality are both reduced by 1.6% and 1.2%, respectively. See Figures 5(l)-(m). Respect to ratios, capital to output ratio is significantly increased and the investment ratio is slightly reduced.

6 Robustness

In this section we study the robustness of the results listed above to change in the parameter δ_q. Concretely, inefficiency in adoption is raised by a factor of 3 ($\delta_q = 0.3$). Since the long run differences are quiet similar to those reported in section 4.1, the analysis is focused on the transitional dynamics towards the respective steady state values. The benchmark model chosen here is that containing simultaneously all the shocks made before. We do this because we want to stress the idea that to study the effects of a policy package is much better (effective) than to study the effects of a single policy. Table 4 shows the

[18] In physics, *momentum* is defined as the product of mass times velocity.

[19] This term is frequently used by politicians when pro-*growth* agendas are promoted.

long run scenarios and Figures 6 reports the transitional dynamics.

When δ_q is high, we observe almost the same qualitative pattern reported in the benchmark case with two striking exceptions: the technological gap is reduced from the beginning of time and not from period 3 and the long run investment ratio is slightly above its initial steady state value. See Figures 6(l) and 6(o). Quantitatively, four important facts are apparent when δ_q is big: adoption efforts are relatively less massive; growth rates of aggregates and ratios are higher either in the short- or in the long run; the rise in social welfare is similar in both economies; the optimal timing of the phase of adoption via intensification of resources in the q-sector (labor plus capital) is prolonged in one period which means that the transitional dynamic is a bit softer. See Figures 6(a)-(d).

What's explains this behaviour? The reason lies in equation (14'). When δ_q is high, the factor $\eta_t = (a\theta/\alpha b)/(L_{q,t}/L_{Y,t}) > 0$ is bigger which comes to intensify *even more* the efficient marginal productivity effect. Therefore, saving is stronger encouraged than in the benchmark case, which is done mainly throughout the production of investment goods. In few words, that is to say economies with similar endowments but different degree of efficiency in the adoption sector are both mainly driven by the efficient marginal productivity effect when the policy package is implemented. This spurs economic growth and reduces both income equality and the technological gap. However, gains in social welfare are not so different between the economies. This striking issue opens another branch to be more explored when the catching up process is carried out under embodiment.

7 Conclusions

In this paper, the authors, centering around Boucekkine *et al.* (2006)'s model and introducing labour heterogeneity, sectoral allocation of capital and a certain degree of inefficiency in adoption have discussed various effects of the resulting embodiment technical progress on economic development, made comparative analysis with the results reported by these authors, and finally investigated about how a developing economy could cope with a technological acceleration abroad. We have found that labour allocation across sectors affects consumption-saving patterns. Concretely, we have demonstrated that this allocation amplify the efficient marginal productivity of capital which leads the economy to specialize sooner than later in the production of final goods, particularly investment ones. In other words, the obsolescence effect found by Boucekkine *et al.* (2003) -which would be inherent to embodied technological change and enhances the production of intermediate goods- is relatively diminished when the mentioned changes are introduced. In addition, we have found that the distribution of capital across sectors is the key endogenous mechanism determining income inequality in the long run: the higher the long run value of this factor in production, the higher the long run level of inequality. Structural parameters such as the distribution of the labour shares between skilled and unskilled workers in production and adoption also matters in the determination of such inequality. Using a calibrated version of the model for a representative developing country, we then show the optimality of an immediate and massive increase of adoption efforts when foreign technological acceleration occur, which challenges the main result reported so far in the literature. Moreover, we claim that social welfare maximization is indeed compatible with the reduction of the technological gap in the long run, which does not hold particularly in Boucekkine *et al.* (2006). Furthermore, simulations show that a policy package focused mainly on increasing both productivity in the adoption sector and the number of educated people should be part of an appropriate strategy for promoting development under technological acceleration abroad. It is suggested that this strategy would make less problematic the catching up process in the South. Apart from affecting positively welfare and aggregates, we report that gains in the reduction of both income inequality and the technological gap are significant. More precisely, with this strategy Simon Kuznets' hypothesis -which states that as a country develops there is a ”natural cycle” of economic inequality- would not hold.

Finally, we show that, for any reason, when the adoption sector is not efficient at all, the long run performance of the economy tends to worst when this inefficient if high, including income inequality. Even though the transitional dynamics shows here a more vigorous economy we eventually find that this kind of inefficiency enhances a long-lived specialization in the production of intermediate goods when external technological acceleration occur which ends up undermining the final goods sector. We suggest that this type of inefficiency should be taken (and tackled) very seriously in the South in order to accelerate its race to development, race that -in our opinion- should also consider particular attention on the reduction of the very high levels of income inequality observed in this zone. Overall, we have shown that, in order to have success in this matter, any development strategy based on embodied technological progress requires a complete knowledge of the technologies involved as in production as in adoption.

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8 Appendix

1. Proof of Proposition 1

For simplicity, let us denote by:

$$z_1 = \frac{\theta a}{b\alpha}, \quad z_2 = \frac{\psi a}{c\alpha}, \quad z_3 = \frac{1-\beta(1-\delta)}{\alpha}, \quad z_4 = \frac{\delta\theta}{1-\beta(1-\delta)}, \quad z_5(\mu_k) = \frac{\mu_k}{(1-\mu_k)}$$

From these definitions, equations (19)-(28) are sufficient to demonstrate that the following endogenous variables can be written as a function of μ_k:

$$L_q(\mu_k) = \frac{L}{1+z_1z_5}, \quad L_y(\mu_k) = \frac{z_1z_5L}{1+z_1z_5}, \quad H_q(\mu_k) = \frac{H}{1+z_2z_5}, \quad H_y(\mu_k) = \frac{z_2z_5H}{1+z_2z_5}, \quad \frac{qY}{K}(\mu_k) = z_3\mu_k,$$
$$q(\mu_k) = q^o\left[1 - \frac{(1-\mu_k)}{z_4}\right], \quad K(\mu_k) = \frac{1}{\mu_k}\left(\frac{A[L_y(\mu_k)]^\theta[H_y(\mu_k)]^\psi q(\mu_k)}{z_3}\right)^{\frac{1}{1-\alpha}}$$

Then, by means of successive substitutions, it is possible to reduce the system of ten equilibrium restrictions to a single implicit equation involving $\mu_k \in (0,1)$:

$$G(\mu_k) = \frac{[1-\beta(1-\delta)]}{\theta}(1-\mu_k)\left[1 + d[(1-\mu_k)K(\mu_k)]^a[L_q(\mu_k)]^b[H_q(\mu_k)]^c\right] - \delta = 0 \tag{32}$$

It should be easy to check that $G(\mu_k)$ is a decreasing and concave function with

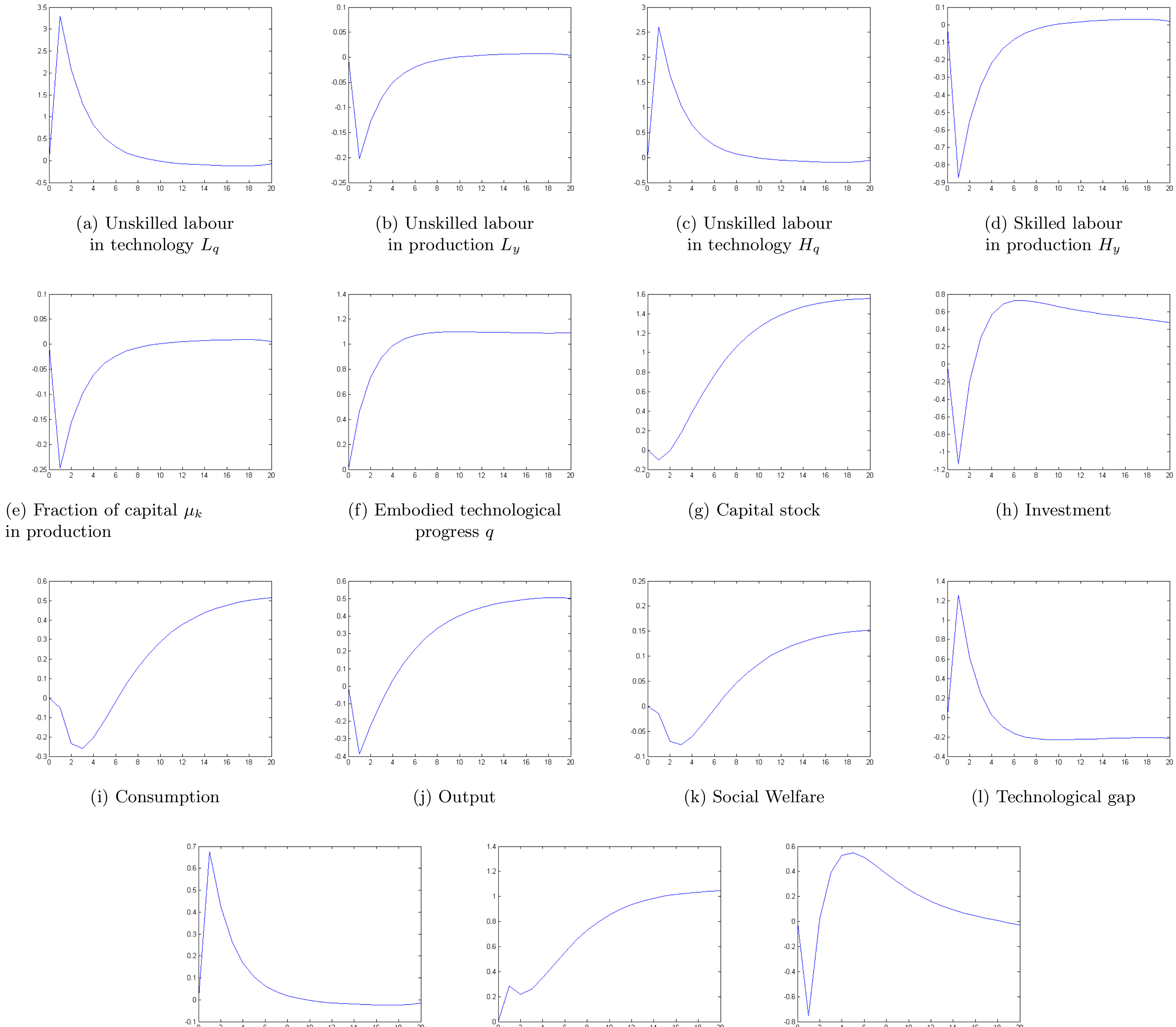
$$lim_{\mu_k\to 0}G(\mu_k) = \frac{[1-\beta(1-\delta)]}{\theta} - \delta > 0 \quad as \quad \delta < \frac{[1-\beta(1-\delta)]}{\theta} \tag{33}$$

$$lim_{\mu_k\to 1}G(\mu_k) = -\delta < 0, \ \ as \quad \delta > 0, \tag{34}$$

Thus, there exists a unique $\mu_k^* \in (0,1)$ which satisfies $G(\mu_q) = 0$.

2. The model's dynamic system

$$\begin{aligned}
Y_t &= A_t(\mu_{k,t}K_t)^\alpha L_{y,t}^\theta H_{y,t}^\Psi \\
K_t &= q_tI_t + (1-\delta)K_{t-1} \\
q_t &= (1-\delta_{q,t})\cdot q_{t-1} + d_t\left[(1-\mu_{k,t}K_t)\right]^a L_{q,t}^b H_{q,t}^c(q_t^0 - q_t) \\
L_{q,t} + L_{y,t} &= L_t \\
H_{q,t} + H_{y,t} &= H_t \\
\frac{\theta L_{q,t}}{bL_{y,t}} &= \frac{\Psi H_{q,t}}{cH_{y,t}} \\
Y_t &= C_t + It \\
\frac{\mu_{k,t}}{1-\mu_{k,t}} &= \frac{b\alpha}{\theta a}\frac{L_{Y,t}}{L_{q,t}} \\
\frac{C_t}{C_{t+1}} &= \frac{1}{\beta(1-\delta)}\frac{q_{t+1}}{q_t}\left[1 - q_t\alpha\frac{Y_t}{K_t} - a\frac{\theta}{b}q_t\frac{L_{q,t}}{L_{Y,t}}\frac{Y_t}{K_t}\right] \\
\frac{1}{C_t}\frac{K_t-(1-\delta)K_{t-1}}{q_t^2} &= \frac{1}{C_t}\frac{\theta}{b}\frac{Y_t}{[q_t-(1-\delta_q)q_{t-1}]}\frac{L_{q,t}}{L_{y,t}}\left(1 + d_t[(1-\mu_{k,t})K_t]^aL_{q,t}^bH_{q,t}^c\right) - \\
&\beta(1-\delta_q)\frac{q}{C_{t+1}}\frac{\theta}{b}\frac{Y_{t+1}}{[q_{t+1}-(1-\delta_q)q_t]}\frac{L_{q,t+1}}{L_{y,t+1}}
\end{aligned}$$



(a) Unskilled labour in technology L_q

(b) Unskilled labour in production L_y

(c) Unskilled labour in technology H_q

(d) Skilled labour in production H_y

(e) Fraction of capital μ_k in production

(f) Embodied technological progress q

(g) Capital stock

(h) Investment

(i) Consumption

(j) Output

(k) Social Welfare

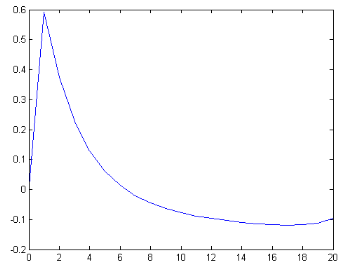
(l) Technological gap

(m) Inequality

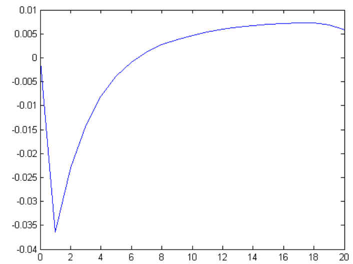
(n) Capital to output ratio K/Y

(o) Investment ratio I/Y

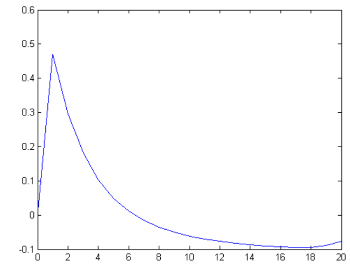
Figure 1: Growth rate of different variables under a permanent shock of 1% in q^0



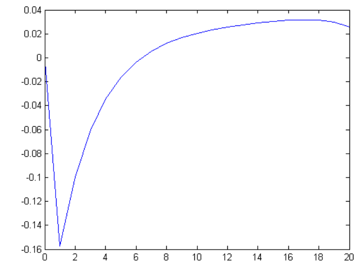
(a) Unskilled labour
in technology L_q



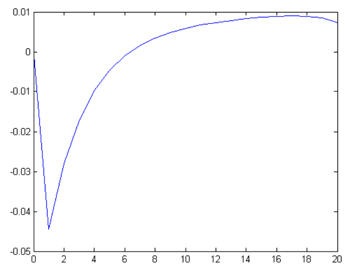
(b) Unskilled labour
in production L_y



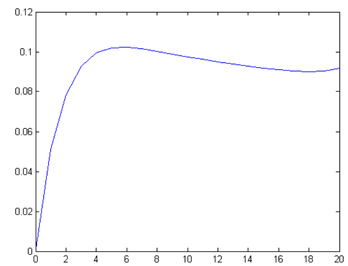
(c) Unskilled labour
in technology H_q



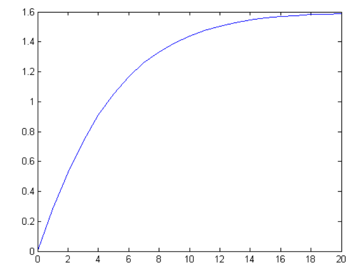
(d) Skilled labour
in production H_y



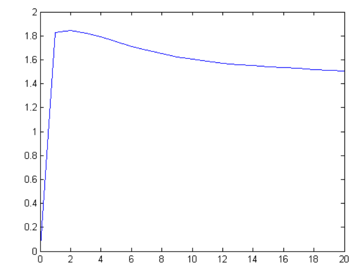
(e) Fraction of capital μ_k
in production



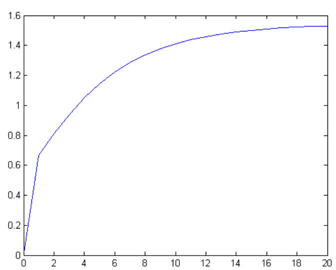
(f) Embodied technological
progress q



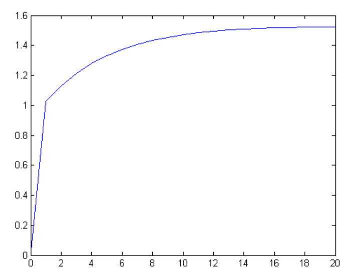
(g) Capital stock



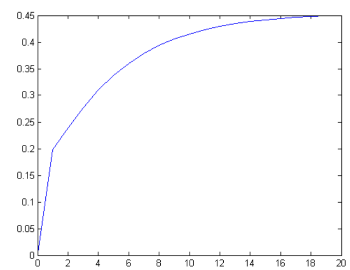
(h) Investment



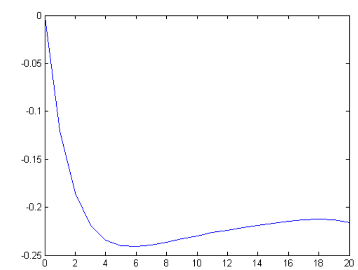
(i) Consumption



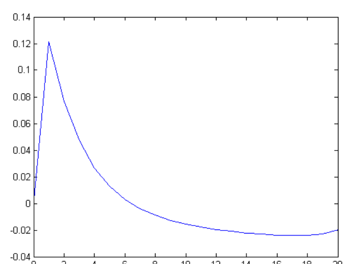
(j) Output



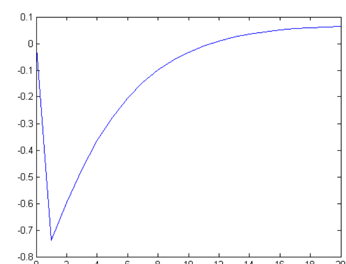
(k) Social Welfare



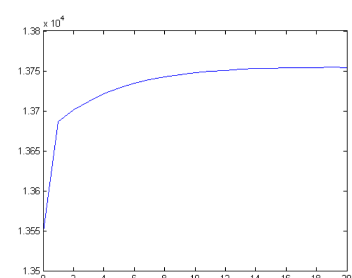
(l) Technological gap



(m) Inequality

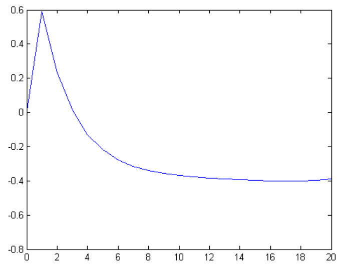


(n) Capital to output ratio K/Y

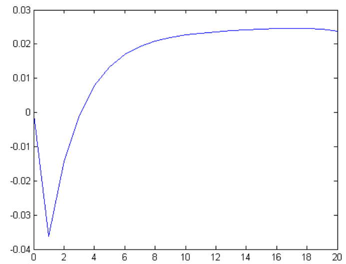


(o) Investment ratio I/Y

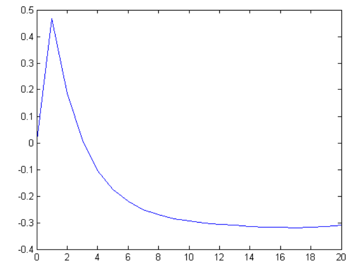
Figure 2: Growth rate of different variables under a permanent shock of 1% in A



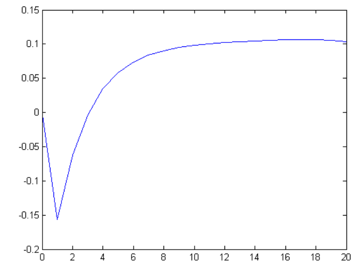
(a) Unskilled labour in technology L_q



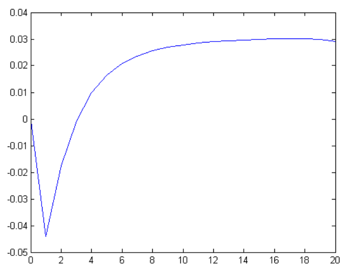
(b) Unskilled labour in production L_y



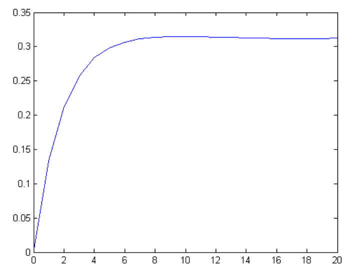
(c) Unskilled labour in technology H_q



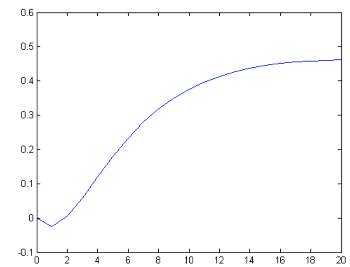
(d) Skilled labour in production H_y



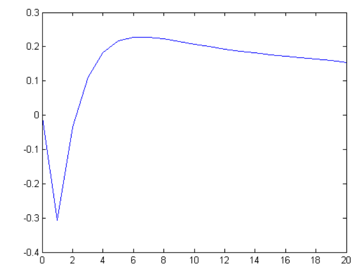
(e) Fraction of capital μ_k in production



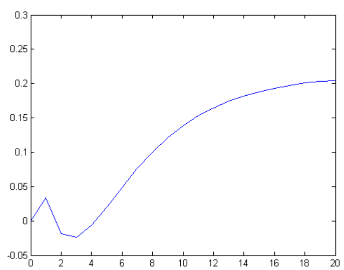
(f) Embodied technological progress q



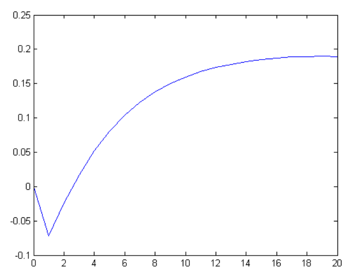
(g) Capital stock



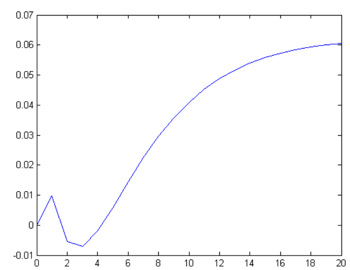
(h) Investment



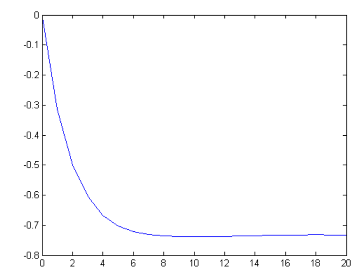
(i) Consumption



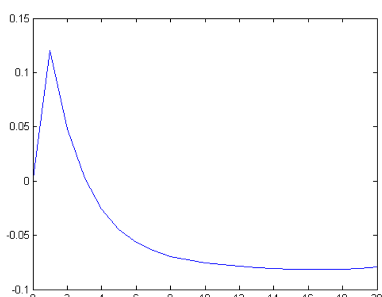
(j) Output



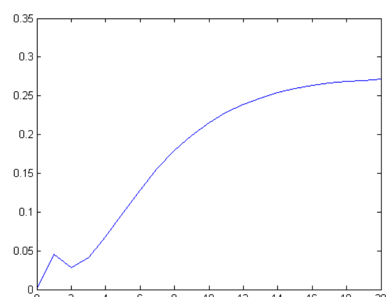
(k) Social Welfare



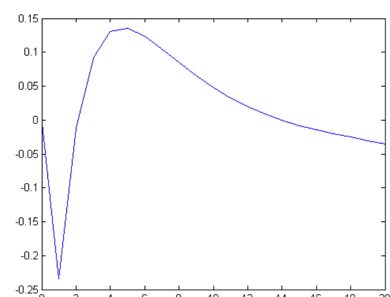
(l) Technological gap



(m) Inequality



(n) Capital to output ratio K/Y



(o) Investment ratio I/Y

Figure 3: Growth rate of different variables under a permanent shock of 1% in d

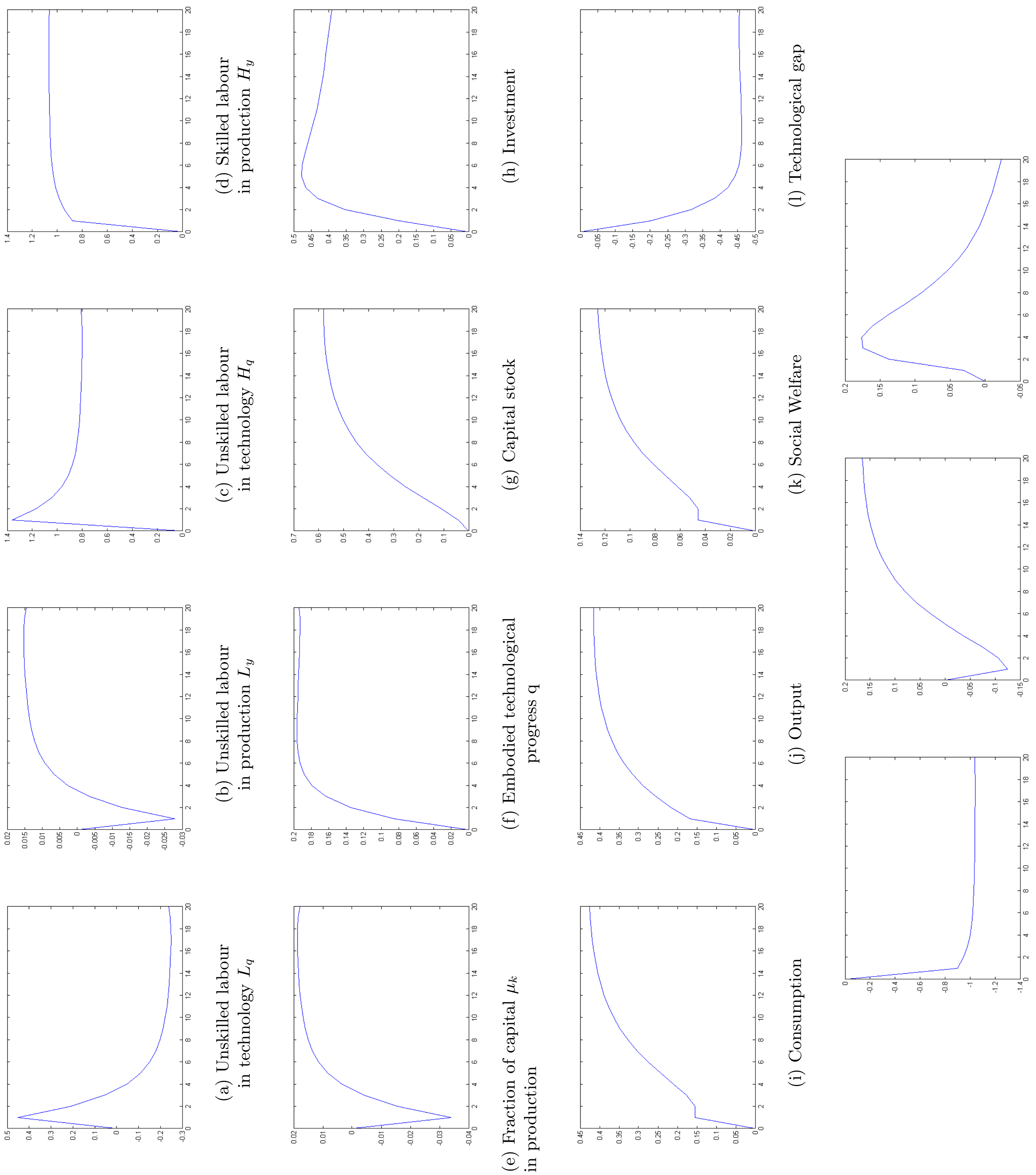
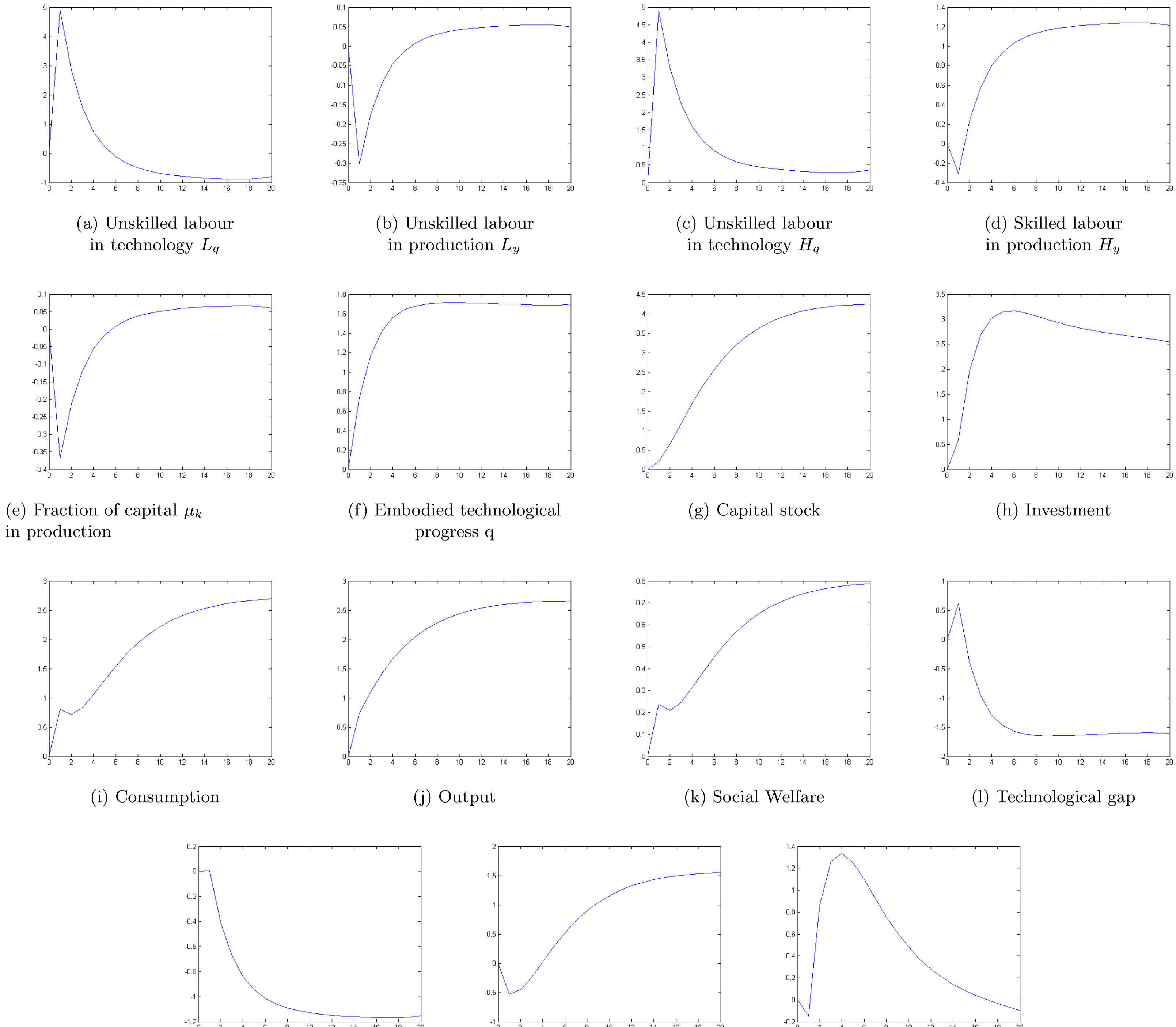


Figure 4: Growth rate of different variables under a permanent shock of 1% in H



(a) Unskilled labour in technology L_q

(b) Unskilled labour in production L_y

(c) Unskilled labour in technology H_q

(d) Skilled labour in production H_y

(e) Fraction of capital μ_k in production

(f) Embodied technological progress q

(g) Capital stock

(h) Investment

(i) Consumption

(j) Output

(k) Social Welfare

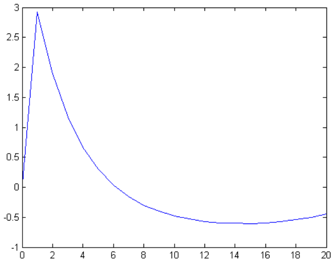
(l) Technological gap

(m) Inequality

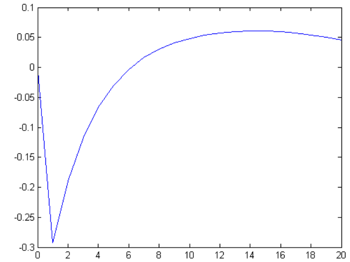
(n) Capital to output ratio K/Y

(o) Investment ratio I/Y

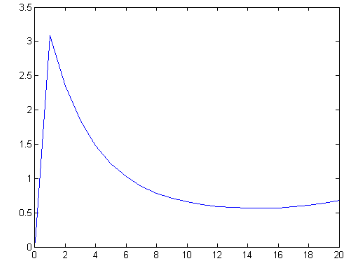
Figure 5: Growth rate of different variables under a permanent shock of 1% in q^0, A, d, H.



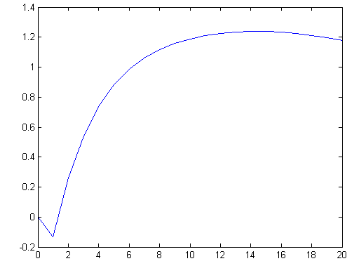
(a) Unskilled labour in technology L_q



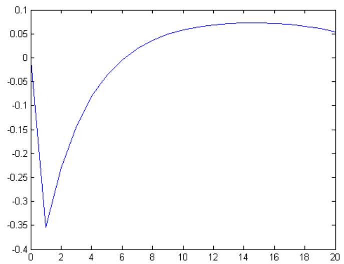
(b) Unskilled labour in production L_y



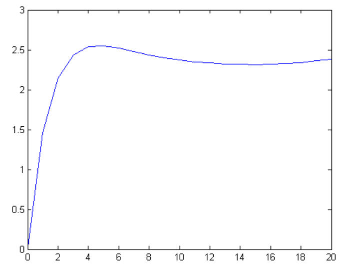
(c) Unskilled labour in technology H_q



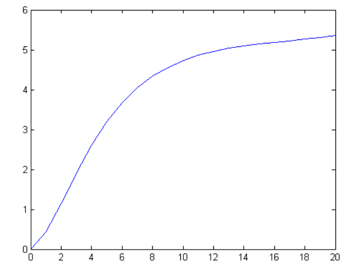
(d) Skilled labour in production H_y



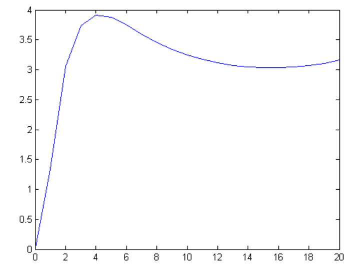
(e) Fraction of capital μ_k in production



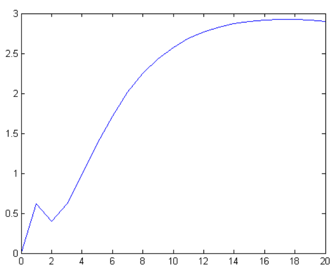
(f) Embodied technological progress q



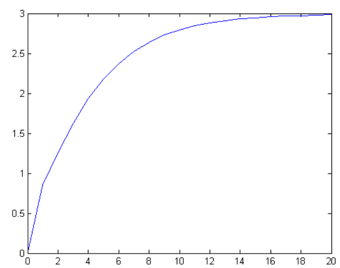
(g) Capital stock



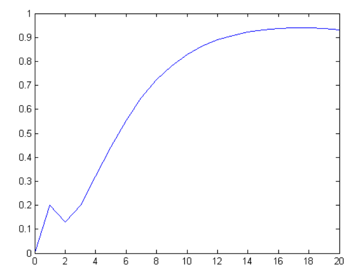
(h) Investment



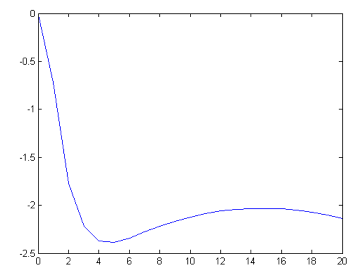
(i) Consumption



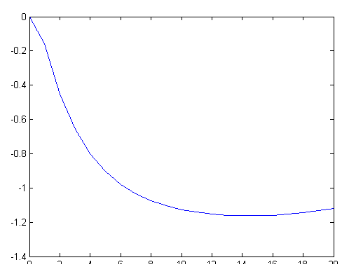
(j) Output



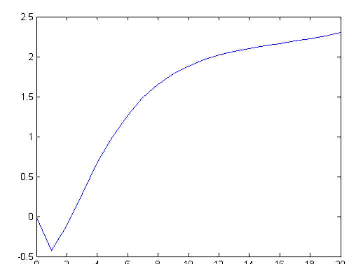
(k) Social Welfare



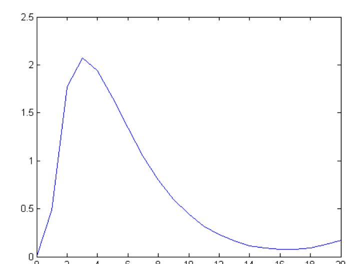
(l) Technological gap



(m) Inequality



(n) Capital to output ratio K/Y



(o) Investment ratio I/Y

Figure 6: Growth rate of different variables under a permanent shock in q^0, A, d, H. Robustness