Opportunity-sensitive poverty measurement

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1. Motivation

- Poverty analysts have long found it desirable for their toolkit to include measures that are sensitive to inequality among the poor.

"Given other things, a pure transfer of income from a person below the poverty line to *anyone* who is richer must increase the poverty measure"

Transfer axiom in Sen (1976, p. 219)

Foster, Greer, Thorbecke (1984) combined sensitivity to relative deprivation with sub-group consistency and decomposability.

1. Motivation

- But sensitive to which inequality?
  - Sen (1980): “Equality of What?” – Tanner Lectures
  - Dworkin (1981): “What is equality? Part 2: Equality of resources”
  - Cohen (1989): “On the currency of egalitarian justice”
  - Roemer (1998): “Equality of Opportunity”
  - Van de Gaer (1993): “Equality of Opportunity and Investment in Human Capital”

- Inequality of opportunity is now typically understood as that inequality associated with pre-determined circumstances, over which individuals have no control.
  - Ex-ante: inequality in the opportunity sets across circumstance-homogeneous groups (between types).
  - Ex-post: inequality among people exerting the same degree of effort (within tranches).

1. Motivation

The value of (outcome) inequality-sensitive poverty measures is to distinguish between poverty in distributions such as B and C.

(z=5)	A	B	C	D
I	9	9	9	9
II	8	8	8	8
III	7	7	7	7
IV	6	7	7	7
V	4	3	4	4
VI	3	3	3	3
VII	2	2	1	1
VIII	1	1	1	1
FGT (0)	0.5	0.5	0.5	0.5
FGT (1)	0.25	0.275	0.275	0.275
FGT (2)	0.15	0.165	0.185	0.185

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2. Two classes of opportunity-sensitive poverty measures

- Notation and set-up:
  - *H* individuals indexed by *h*.
  - Individuals fully described by a list of circumstances *c*, belonging to a finite set $\Omega = \{c_1, \ldots, c_n\}$ and a scalar effort level *e*.
  - We partition the population into *n* types $T_i \in \Gamma$, such that $h \in T_i \Leftrightarrow c^h = c_i$
  - Income is generated by a function $x_{h \in i} = g(c_i, e_h)$, $g : \Omega \times \Re_+ \rightarrow \Re_+$
  - Each type-specific income distribution is denoted $F_i(x)$
  - The societal income distribution is $F(x) = \sum_{i=1}^{n} q_i^F F_i(x)$
  - Poverty: the set of poor individuals in each type is $T_i^z := \{h \in T_i \mid x_h \leq z\}$
- Key assumption: There is agreement on an ordering of types: $T_{i+1} \succeq T_i$

2. Two classes of opportunity-sensitive poverty measures

- Axioms:

A1: Monotonicity: *P* is non-increasing in *x.*

$$\frac{\partial P}{\partial x_h \,|h \in \mathbf{T}_i^z} \leq 0, \forall i \in \{1,...,n\}, \forall h \in \mathbf{T}_i^z$$

A2: Focus: For all *F, G* in D:

$$P\left(F(x), z, \mathcal{T}\right) = P\left(G(x), z, \mathcal{T}\right) \qquad \text{if } F_i(x) = G_i(x), \ \ \forall x \leq z, \forall i.$$

A3: Additivity: There exist functions $p_i : \Re \rightarrow \Re$ for all $i \in \{1,...,n\}$, assumed to be twice differentiable, such that

$$P\left(F(x), z, \mathcal{T}\right) = \sum_{i=1}^{n} q_i^F \int_{x_h, h \in \mathbf{T}_i^z} p_i(x,z) f_i\left(x\right) dx \qquad \textit{for all } F \in D.$$

2. Two classes of opportunity-sensitive poverty measures

- Axioms:

A4: Anonymity within types: For all F in D, and for all $i \in \{1,\dots,n\}$

$$P\left(F_1(x),\dots,F_i(x),\dots,F_n(x);z\right) = P\left(F_1(x),\dots,F_i(y),\dots,F_n(x);z\right), \forall i \in \{1,\dots,n\}$$

whenever $y = \Pi x$ *and* Π *is any permutation matrix.*

A5: Inequality of opportunity aversion (IOA)

$$\frac{\partial P}{\partial x_h \big| h \in T_i^z} \le \frac{\partial P}{\partial x_k \big| h \in T_{i+1}^z} \le 0, \forall i \in \{1,\dots,n\}, \forall h \in T_i^z, \forall k \in T_{i+1}^z,$$

2. Two classes of opportunity-sensitive poverty measures

A6: Inequality neutrality within types (INW)

$$\frac{\partial^2 P}{\partial x_h^2 \big| h \in T_i^z} = 0, \forall i \in \{1,\ldots,n\}$$

A7: Inequality aversion within types (IAW)

$$\frac{\partial^2 P}{\partial x_h^2 \big| h \in T_i^z} \geq 0, \forall i \in \{1,\ldots,n\}$$

2. Two classes of opportunity-sensitive poverty measures

The <u>narrow</u> class of opportunity-sensitive poverty measures:

$$\mathbf{P}^N := \{P : D \times \mathbb{R}_+ \times \Im \rightarrow \mathbb{R}_+ \mid P \text{ satisfies } ADD,\ ANON,\ MON,\ FOCUS,\ IOA,\ INW\ \}$$

Remark 1 *$P \in \mathbf{P}^N$ if and only if, for all $F \in D$, $P(F(x), z) = \sum_{i=1}^{n} q_i^F \int p_i(x) f_i(x)\, dx$, where the functions $p_i : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfy the following conditions:*

P1. $p_i(x, z) = 0$, $\forall x > z$ and $\forall i \in 1, ..., n$

P2. $p_i(x, z) \geq 0$, $\forall x \leq z$ and $\forall i \in 1, ..., n$

P3. $\frac{\partial p_i(x,z)}{\partial x} \leq \frac{\partial p_{i+1}(x,z)}{\partial x} \leq 0\ \forall x, \forall i = 1, ..., n-1$

P4. $\frac{\partial^2 p_i(x,z)}{\partial x^2} = 0$, $\forall x$ and $\forall i \in 1, ..., n$

2. Two classes of opportunity-sensitive poverty measures

The broad class of opportunity-sensitive poverty measures:

$$\mathbf{P}^B := \{P : D \times \mathbb{R}_+ \times \Im \rightarrow \mathbb{R}_+ \mid P \textit{ satisfies ADD, ANON, MON, FOCUS, IOA, IAW } \}$$

Remark 2 *$P \in \mathbf{P}^B$ if and only if, for all $F \in D$,*
$P(F(x), z) = \sum_{i=1}^{n} q_i^F \int p_i(x) f_i(x)\, dx$, where the functions $p_i : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfy conditions P1 - P3 and P4′:

p4′. $\frac{\partial^2 p_i(x,z)}{\partial x^2} \geq 0, \forall x \text{ and } \forall i \in 1, ..., n$

3. Partial orderings: opportunity-sensitive poverty dominance

- OSPD in the broad class:

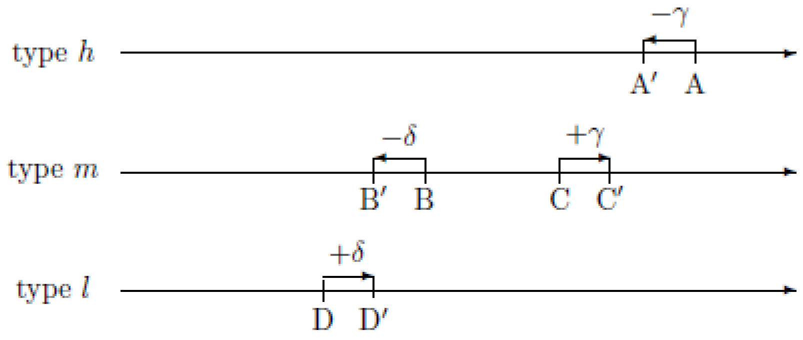
  Theorem 1 (Jenkins and Lambert, 1993, Chambaz and Maurin, 1998): For all distributions F(x), G(x) in D and a poverty line z, $P(G(x),z) \geq P(F(x),z), \forall P \in P^B$ if and only if the following condition is satisfied:

$$\sum_{i=1}^{j}(q_i^G G_i(x) - q_i^F F_i(x)) \geq 0, \quad \forall x \leq z, \quad and \quad \forall j \in 1, ..., n \qquad (1)$$

4. Complete orderings: a specific sub-class of poverty indices

- There is a potential tension between A5 (IOA) and A7 (IAW).
  - A sequence of type-progressive transfers may increase inequality within types.

Figure 1: example 1: 3 types 2 transfers



  - Does the broad class collapse to the narrow?

4. Complete orderings: a specific sub-class of poverty indices

1. No. A sub-class of poverty indices, P_{FGT}^{B} , can be shown to satisfy both IOA and IAW:

$$P_{FGT}(F, z, \mathcal{T}) = \frac{1}{n} \sum_{i=1}^{n} q_i^F (n + 1 - rk(i)) \int_0^z \left(\frac{z - x}{z} \right)^\alpha f_i(x)\, dx$$

2. Notice that this is obtained from the general class we had earlier, $P(F(x), z) = \sum_{i=1}^{n} q_i^F \int_0^z p_i(x) f_i(x) dx$ by writing $p_i(x) = w_i p(x)$ and using the classic FGT formulation for the individual poverty index, and the inverse type rank as the weight w_i.

3. For α = 0 or α = 1, $P_{FGT} \subset P^N$

5. Empirical application: OSP in Europe

- Do poverty comparisons across countries vary when a concern with inequality of opportunity is taken into account?

- Consider 18 European countries: Austria (AT), Belgium (BE), Cyprus (CY), Czech Republic (CZ), Germany (DE), Estonia (EE), Spain (ES), Finland (FI), France (FR), Greece (GR), Hungary (HU), Italy (IT), Lithuania (LT), Luxembourg (LU), Latvia (LV), the Netherlands (NL), Poland (PL) and Slovakia (SK).

- EU SILC data from the 2005 round
  - Special model on intergenerational transmission of poverty

5. Empirical application: OSP in Europe

- Three circumstance variables:
  - Gender (M, F)
  - parental education when respondent was aged 12-16 (upper secondary or more; other)
  - parental occupation (high-skill non-manual; lower-skill non-manual; skilled manual; elementary occupations)
- Hence 16 types.
- Types ranked by poverty headcount: $p_i(x) = \int_0^z f_i(x)\,dx$
- Poverty line: follow Decanq et al. (2013) and use 60% of median equivalent household income of the combined population of the 18 countries:
  - Euro 9,275 per annum, at PPP exchange rates, in 2004 prices.

5. Empirical application: OSP in Europe

- Opportunity-sensitive poverty dominance in the P^B class.

Table 1: Dominance conditions associated with Theorem 1

	AT	BE	CY	CZ	DE	EE	ES	FI	FR	GR	HU	IT	LT	LU	LV	NL	PL	SK
AT	.	>																
BE	<	.																
CY			.															
CZ	>	>	>	.														
DE	<			<	.													
EE	>***	>***	>***	>***	>***	.												
ES	>	>**	>***	>	>	<***	.											
FI			>	<		<***	<***	.										
FR		<	<	<	<	<***	<***		.									
GR	>***	>***	>***	>	>***	<	>	>***	>***	.								
HU	>	>	>***	>**	>	<	>	>*	>***	<	.							
IT	>	>	>***	>	>	<***		>***	>*	<		.						
LT	>	>***	>***	>*	>***			>***	>***		>	>**	.					
LU	<	<	<	<	<	<***	<***	<	<	<***	<*	<***	<***	.				
LV	>***	>***	>***	>***	>***	>	>***	>***	>***	>	>	>***		>***	.			
NL				<		<***	<***	<	<	<***	<**	<***	<***	>	<***	.		
PL	>*	>***	>***	>***	>***		>	>***	>***		>	>	<	>***	<	>***	.	
SK	>	>***	>***	>**	>***	<		>***	>***			>	<	>***	<	>***		.

Source: Authors' calculation from EU-SILC (2005). ">" in row i and column j means that poverty is higher in country i than in country j. The dominance condition for each pair is obtained checking the sequential conditions in Theorem 1 (equation (1)). 90%, 95%, 99% confidence intervals are based on the quantile distribution of 200 bootstrap re-sampling with replacement.

5. Empirical application: OSP in Europe

- Poverty ranks across eighteen European countries: standard headcount against opportunity-sensitive headcount

Figure 2: Poverty headcount and opportunity-sensitive poverty headcount

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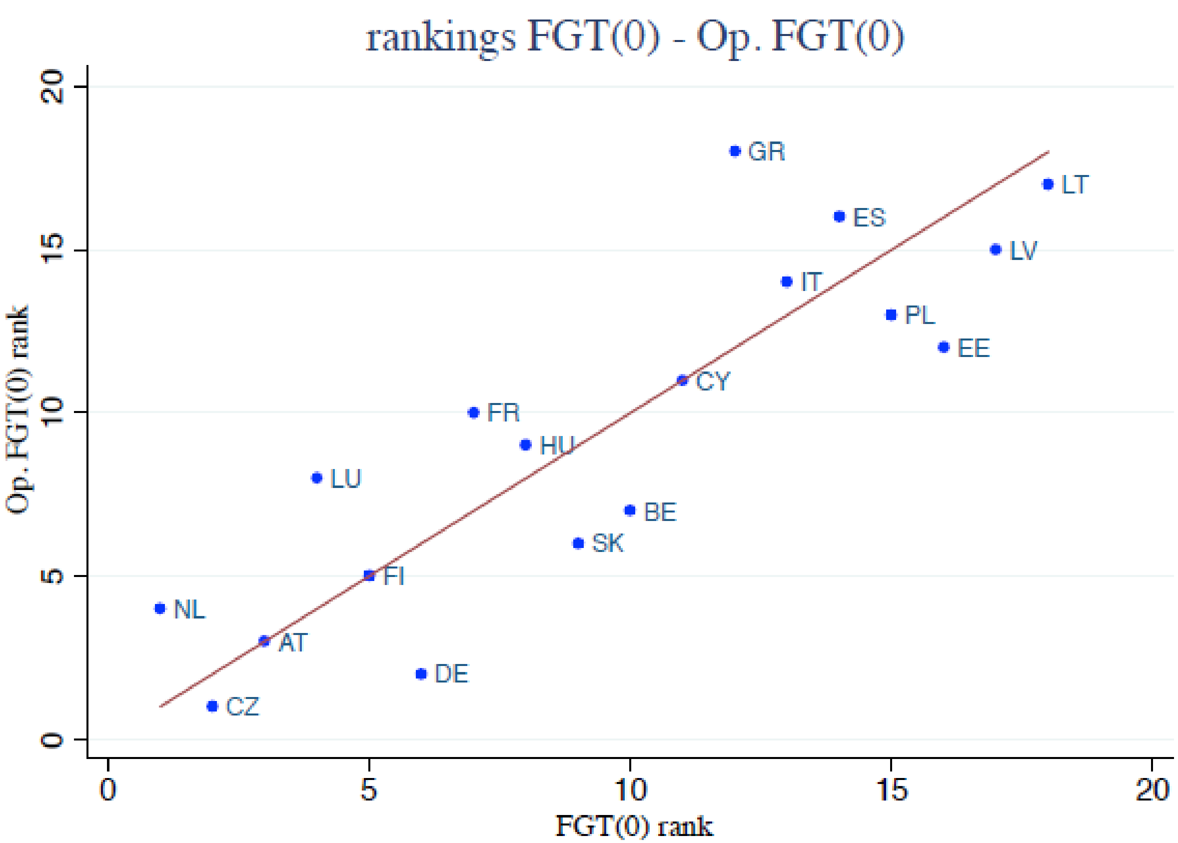

5. Empirical application: OSP in Europe

- Poverty levels across eighteen European countries: standard headcount against opportunity-sensitive headcount

Figure 3: Ranking of $FGT(0)$ and $OpFGT(0)$

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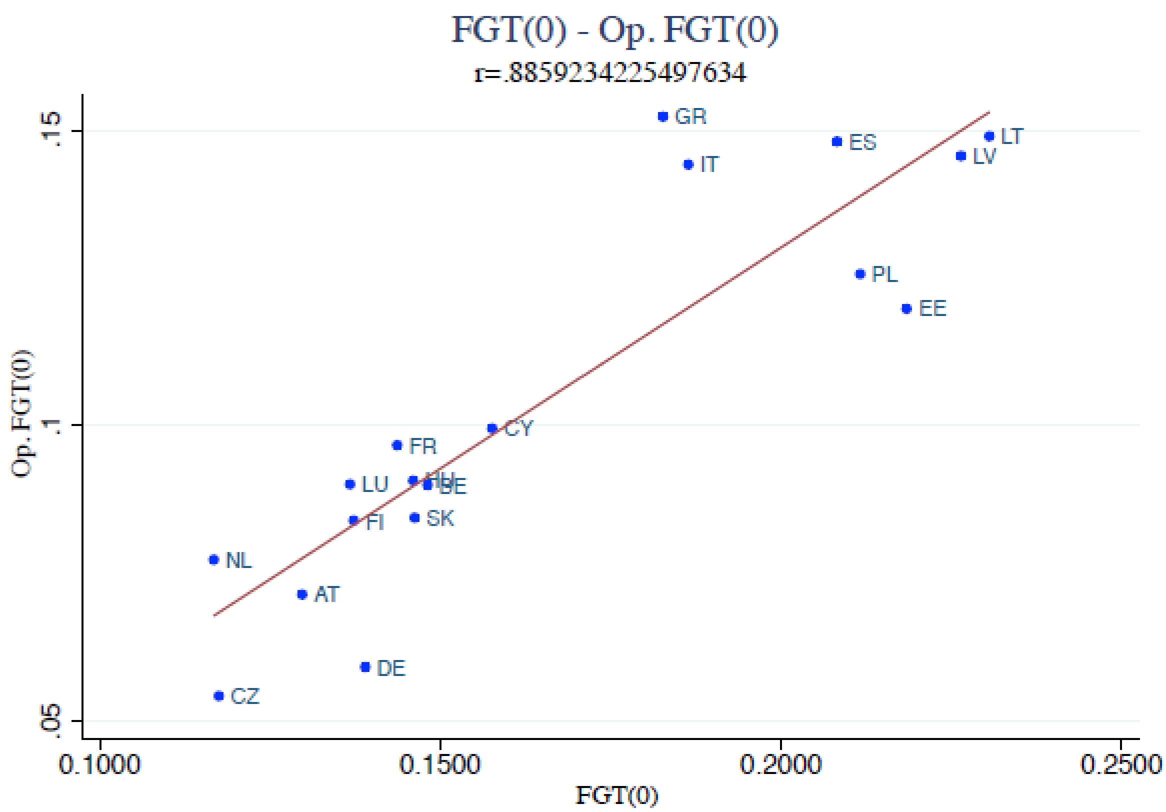

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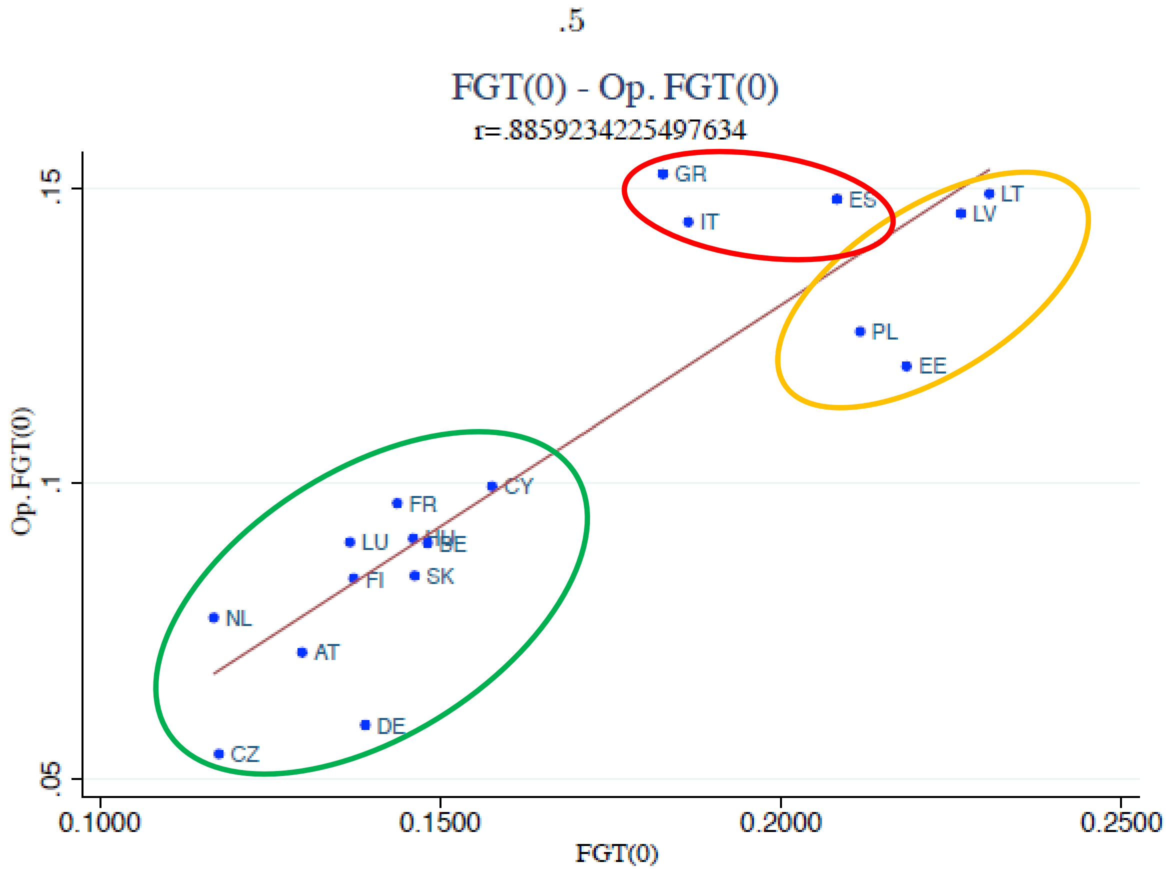

6. Conclusions

1. It has often been judged important to have poverty measures that are sensitive to inequality among the poor.

2. Many now feel that inequality in the space of opportunities is at least as important as inequality in outcomes.

3. This paper proposed two classes of opportunity-sensitive poverty measures
   - Depending on whether one wishes to impose strict neutrality to inequality within types, or to allow for some aversion.
   - Dominance conditions were derived.

4. An FGT-inspired sub-class of OSP was derived, and shown to resolve the tension between aversion to inequality of opportunity and to outcome inequality.

5. Using EU-SILC data for eighteen European countries around 2005, we show that accounting for I. Op. induces a number of country re-rankings.
   - These re-rankings often reflect the concentration of poverty across types; and their population shares.