Decompositions of Productivity Growth into Sectoral Effects

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Abstract

The paper provides some new decompositions of labour productivity growth and Total Factor Productivity (TFP) growth into sectoral effects. These new decompositions draw on the earlier work of Tang and Wang (2004). The economy wide labour productivity growth rate turns out to depend on the sectoral productivity growth rates, real output price changes and changes in sectoral labour input shares. The economy wide TFP growth decomposition into explanatory factors is similar but some extra terms due to real input price change make their appearance in the decomposition.

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Key Words

Total Factor Productivity, labour productivity, index numbers, sectoral contributions to growth.

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1. Introduction

Denison (1962) pointed out that measures of economy wide productivity change cannot be obtained as a simple weighted sum of the corresponding industry measures; he showed that changes in the allocation of resources across the industries also played an important role in contributing to aggregate productivity change. However, the Denison decomposition of aggregate labour productivity change into explanatory effects ignored the role of changes in industry output prices. In an important paper, Tang and Wang (2004; 426) extended the Denison decomposition to take into account changes in real output prices in their decomposition of economy wide labour productivity into explanatory contribution effects. However, Tang and Wang combined the effects of changes in real output prices with the effects of changes in input shares and so in section 2, we rework their methodology in order to provide a decomposition of aggregate labour productivity growth into separate contribution terms due to sectoral productivity growth, changes in input shares and changes in real output prices. In section 3, we present some alternative interpretations of the basic decomposition. In section 4, we generalize the results in sections 2 and 3 in order to provide a decomposition of economy wide Total Factor Productivity (or Multifactor Productivity) growth into industry explanatory factors. Section 5 concludes. This extension to measuring aggregate MFP growth adds a fourth set of explanatory terms: the effects of changes in real input prices. An Appendix illustrates the decompositions using Australian data for 16 industries over the period 1995-2012.

2. The Tang and Wang Methodology Reworked

Let there be N sectors or industries in the economy.[2] Suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t [3] and *labour input* L_n^t for n = 1,...,N. We assume that these labour inputs can be added across sectors and that the *economy wide labour input* in period t is L^t defined as follows:

$$(1)\ L^t \equiv \sum_{n=1}^{N} L_n^t\ ; \qquad t = 0,1.$$

Industry n labour productivity in period t, X_n^t, is defined as industry n real output divided by industry n labour input:

$$(2)\ X_n^t \equiv Y_n^t/L_n^t\ ; \qquad t = 0,1\ ;\ n = 1,...,N.$$

[2] The material in this section follows Diewert (2004).

[3] These industry real output aggregates Y_n^t and the corresponding prices P_n^t are *indexes* of the underlying micro net outputs produced by industry n. The exact functional form for these indexes does not matter for our analysis but we assume the indexes satisfy the property that for each t and n, $P_n^tY_n^t$ equals the industry n nominal value added for period t.

It is not entirely clear how aggregate labour productivity should be defined since the outputs produced by the various industries are measured in heterogeneous units, which are in general, not comparable. Thus we need to weight these heterogeneous outputs by their prices, sum the resulting period t values and then divide by an appropriate *output price index*, say P^t for period t, in order to make the economy wide nominal value of aggregate output comparable in real terms across periods.[4] Thus with an appropriate choice for the aggregate output price index P^t, the period t *economy wide labour productivity*, X^t, is defined as follows:[5]

$$(3)\ X^t \equiv \sum_{n=1}^N P_n^t Y_n^t / P^t L^t = \sum_{n=1}^N (P_n^t/P^t) Y_n^t / L^t = \sum_{n=1}^N p_n^t Y_n^t / L^t ; \qquad t = 0,1$$

where the *period t industry n real output price*, p_n^t, is defined as the industry t output price P_n^t, divided by the aggregate output price index for period t, P^t; i.e., we have the following definitions:[6]

$$(4)\ p_n^t \equiv P_n^t/P^t ; \qquad n = 1,...,N ; t = 0,1.$$

Using definitions (2) and (3), it is possible to relate the period t aggregate productivity level X^t to the industry productivity levels X_n^t as follows:[7]

$$\begin{aligned}(5)\ X^t &\equiv \sum_{n=1}^N P_n^t Y_n^t / P^t L^t \\ &= \sum_{n=1}^N p_n^t [Y_n^t/L_n^t][L_n^t/L^t] \\ &= \sum_{n=1}^N p_n^t s_{Ln}^t X_n^t \qquad \text{using definitions (2)}\end{aligned}$$

where the *share of labour used by industry n in period t*, s_{Ln}^t, is defined in the obvious way as follows:

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[4] The main problem is how exactly should the aggregate period t output level, Y^t, be defined? In practice, a sequence of value added output levels Y^t (and the corresponding value added output price indexes P^t) will be defined by aggregating individual industry outputs (and intermediate inputs entered with negative signs) into economy wide output levels, using bilateral Laspeyres, Paasche, Fisher (1922) or other indexes as building blocks. In the Appendix, we use chained Fisher output indexes over the market industry volume levels Y_n^t with the corresponding Australian Bureau of Statistics (ABS) official industry price indexes P_n^t used as price weights for the Y_n^t. Thus for each year t, our Fisher index product P^tY^t will equal the ABS sum of industry price times volumes for year t, $\sum_{n=1}^N P_n^t Y_n^t$, which in turn is equal to market sector nominal value added for year t. Thus our $Y^t = \sum_{n=1}^N P_n^t Y_n^t / P^t$ and so equation (3) can be rewritten as $X^t = Y^t/L^t$. We note that the ABS uses chained Laspeyres volume indexes to aggregate value added over industries to form aggregate market sector real value added but the resulting differences in the Y^t are very small.

[5] This follows the methodological approach taken by Tang and Wang (2004; 425). As noted in the previous footnote, the aggregate output price index P^t can be formed by applying an index number formula to the industry output prices (or value added deflators) for period t, $(P_1^t,...,P_N^t)$, and the corresponding real output quantities (or industry real value added estimates) for period t, $(Y_1^t,...,Y_N^t)$. The application of chained superlative indexes would be appropriate in this context but again, the exact form of index does not matter for our analysis as long as P^tY^t equals period t aggregate nominal value added.

[6] These definitions follow those of Tang and Wang (2004; 425).

[7] Equation (5) corresponds to equation (2) in Tang and Wang (2004; 426).

(6) $s_{Ln}{}^t \equiv L_n{}^t/L^t$; $\quad n = 1,...,N$; $t = 0,1$.

Thus aggregate labour productivity for the economy in period t is a weighted sum of the sectoral labour productivities where the weight for industry n is $p_n{}^t$, the real output price for industry n in period t, times $s_{Ln}{}^t$, the share of labour used by industry n in period t.

Up to this point, our analysis follows that of Tang and Wang (2004; 425-426) but now our analysis will diverge from theirs.[8]

First, we define the *value added or output share of industry n in total value added for period t*, $s_{Yn}{}^t$, as follows:

$$(7)\ s_{Yn}{}^t \equiv P_n{}^tY_n{}^t/\textstyle\sum_{i=1}^N P_i{}^tY_i{}^t \qquad t = 0,1 ;\ n = 1,...,N$$
$$= p_n{}^tY_n{}^t/\textstyle\sum_{i=1}^N p_i{}^tY_i{}^t \qquad \text{using definitions (4).}$$

Note that the product of the sector n real output price times its labour share in period t, $p_n{}^ts_{Ln}{}^t$, with the sector n labour productivity in period t, $X_n{}^t$, equals the following expression:

$$(8)\ p_n{}^ts_{Ln}{}^tX_n{}^t = p_n{}^t[L_n{}^t/L^t][Y_n{}^t/L_n{}^t] ; \qquad t = 0,1 ;\ n = 1,...,N$$
$$= p_n{}^tY_n{}^t/L^t .$$

Now we are ready to develop an expression for the rate of growth of economy wide labour productivity. Using definition (3) and equation (5), *aggregate labour productivity growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$(9)\ X^1/X^0 = \textstyle\sum_{n=1}^N p_n{}^1 s_{Ln}{}^1 X_n{}^1/\sum_{n=1}^N p_n{}^0s_{Ln}{}^0X_n{}^0$$
$$= \textstyle\sum_{n=1}^N [p_n{}^1/p_n{}^0][s_{Ln}{}^1/s_{Ln}{}^0][X_n{}^1/X_n{}^0][p_n{}^0Y_n{}^0/L^0]/\sum_{i=1}^N [p_i{}^0Y_i{}^0/L^0] \qquad \text{using (8)}$$
$$= \textstyle\sum_{n=1}^N [p_n{}^1/p_n{}^0][s_{Ln}{}^1/s_{Ln}{}^0][X_n{}^1/X_n{}^0]\ s_{Yn}{}^0 \qquad \text{using definitions (7).}$$

Thus overall economy wide labour productivity growth, X^1/X^0, is an output share weighted average of three *growth factors* associated with industry n. The three growth factors are:

- $X_n{}^1/X_n{}^0$, (one plus) the rate of growth in the labour productivity of industry n;
- $s_{Ln}{}^1/s_{Ln}{}^0$, (one plus) the rate of growth in the share of labour being utilized by industry n and
- $p_n{}^1/p_n{}^0 = [P_n{}^1/P_n{}^0]/[P^1/P^0]$ which is (one plus) the rate of growth in the real output price of industry n.

[8] Tang and Wang (2004; 425-426) *combined* the effects of the real price for industry n for period t, $p_n{}^t$, with the industry n labour share $s_{Ln}{}^t$ for period t by defining the relative size of industry n in period t, $s_n{}^t$, as the product of $p_n{}^t$ and $s_{Ln}{}^t$; i.e., they defined the industry n *weight* in period t as $s_n{}^t \equiv p_n{}^ts_{Ln}{}^t$. They then rewrote equation (5), $X^t = \sum_{n=1}^N p_n{}^ts_{Ln}{}^tX_n{}^t$, as $X^t = \sum_{n=1}^N s_n{}^tX_n{}^t$. Thus their analysis of the effects of the changes in the weights $s_n{}^t$ did not isolate the separate effects of changes in industry real output prices and industry labour input shares.

Thus in looking at the contribution of industry n to overall (one plus) labour productivity growth, we start with a straightforward share weighted contribution factor, $s_{Yn}{}^{0}[X_n{}^{1}/X_n{}^{0}]$, which is the period 0 output or value added share of industry n in period 0, $s_{Yn}{}^{0}$, times the industry n rate of labour productivity growth (plus one), $X_n{}^{1}/X_n{}^{0}$. This straightforward contribution factor will be augmented if real output price growth is positive (if $p_n{}^{1}/p_n{}^{0}$ is greater than one) and if the share of labour used by industry n is growing (if $s_{Ln}{}^{1}/s_{Ln}{}^{0}$ is greater than one). The decomposition of overall labour productivity growth given by the last line of (9) seems to be intuitively reasonable and fairly simple as opposed to the decomposition obtained by Tang and Wang (2004; 426) which does not separately distinguish the effects of real output price change from changes in the industry's labour share.

3. Alternative Expressions and Discussion

The literature on aggregate labour productivity decompositions [9] has focused on decompositions that decompose aggregate labour productivity growth into explanatory factors that are functions of growth rates (percentage changes in variables) rather than growth factors (one plus the growth rates). Thus in this section, we will rewrite (9) in growth rate form as opposed to its present contribution factor form.

Define the *aggregate labour productivity growth rate* Γ, the *sectoral labour productivity growth rates* γ_n, the *sectoral real output price growth rates* ρ_n and the *sectoral labour input share growth rates* σ_n between periods 0 and 1 as follows:

$$(10)\ \Gamma \equiv (X^1/X^0) - 1\ ;$$
$$(11)\ \gamma_n \equiv (X_n{}^1/X_n{}^0) - 1\ ; \qquad n = 1,\ldots,N;$$
$$(12)\ \rho_n \equiv (p_n{}^1/p_n{}^0) - 1\ ; \qquad n = 1,\ldots,N;$$
$$(13)\ \sigma_n \equiv (s_{Ln}{}^1/s_{Ln}{}^0) - 1\ ; \qquad n = 1,\ldots,N.$$

Now substitute definitions (10)-(13) into (9) and we obtain the following decomposition for the *aggregate labour productivity growth rate* Γ:[10]

$$\begin{aligned}(14)\ \Gamma &= \textstyle\sum_{n=1}^{N} s_{Yn}{}^{0}\,\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\}\\ &= \textstyle\sum_{n=1}^{N} s_{Yn}{}^{0}\,\{\gamma_n + \rho_n + \sigma_n + \gamma_n\rho_n + \gamma_n\sigma_n + \rho_n\sigma_n + \gamma_n\rho_n\sigma_n\}\\ &= \textstyle\sum_{n=1}^{N} s_{Yn}{}^{0}\gamma_n + \sum_{n=1}^{N} s_{Yn}{}^{0}\rho_n + \sum_{n=1}^{N} s_{Yn}{}^{0}\sigma_n\\ &\quad + \textstyle\sum_{n=1}^{N} s_{Yn}{}^{0}\gamma_n\rho_n + \sum_{n=1}^{N} s_{Yn}{}^{0}\gamma_n\sigma_n + \sum_{n=1}^{N} s_{Yn}{}^{0}\rho_n\sigma_n + \sum_{n=1}^{N} s_{Yn}{}^{0}\gamma_n\rho_n\sigma_n.\end{aligned}$$

The above *exact* expressions for aggregate labour productivity growth tell us that Γ is a *quadratic function* in the industry growth rates for labour productivity γ_n, real output price growth rates ρ_n and industry labour input share growth rates σ_n. The *total contribution to the overall growth rate Γ from industry n* is the nth term in the first equation in (14), $s_{Yn}{}^{0}\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\}$. This expression is relatively easy to

[9] See Tang and Wang (2004) and de Avillez (2012) for references to this literature.

[10] We use the fact that the industry period 0 value added output shares $s_{Yn}{}^{0}$ sum to one.

interpret. If industry n's real output price and labour input share remained constant, then the growth rates ρ_n and σ_n would be 0 and the contribution of industry n to economy wide labour productivity growth would be its output share in period 0, s_{Yn}^0, times $[1+\gamma_n][1+0][1+0] - 1$ which is equal to γ_n. Thus under these conditions, the contribution of industry n to economy wide labour productivity growth is equal to the industry n labour productivity growth rate γ_n times its period 0 value added share in economy wide value added, s_{Yn}^0. This is an entirely sensible result. In the case where ρ_n and σ_n are positive, one plus the industry n labour productivity growth, $1+\gamma_n$, is *augmented* by the industry n real output price growth factor $1+\rho_n$ and further *augmented* by the industry n labour input share growth factor $1+\sigma_n$ and then 1 is subtracted from the product of these factors to give us the *industry n augmented labour productivity growth factor*, $[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1$. Augmenting $1+\gamma_n$ by $1+\sigma_n$ is reasonable since the increased labour share for industry n in period 1 will indicate a relative increase in labour input into the industry and increase the importance of industry n in overall labour productivity. Augmenting $1+\gamma_n$ by $1+\rho_n$ is more difficult to explain but the increase in the real price of industry n's output will increase the importance of the output of industry n in the economy wide aggregate output index and evidently, multiplying $1+\gamma_n$ by $1+\rho_n$ will reflect this increased importance.

The last equation in (14) tells us that Γ is equal to an output share weighted average of the industry productivity growth rates, $\sum_{n=1}^N s_{Yn}^0\gamma_n$, plus a share weighted average of the industry real output price growth rates, $\sum_{n=1}^N s_{Yn}^0\rho_n$, plus a share weighted average of the industry labour input share growth rates, $\sum_{n=1}^N s_{Yn}^0\sigma_n$, plus the quadratic terms in the industry growth rates, $\sum_{n=1}^N s_{Yn}^0\gamma_n\rho_n + \sum_{n=1}^N s_{Yn}^0\gamma_n\sigma_n + \sum_{n=1}^N s_{Yn}^0\rho_n\sigma_n$, plus the cubic terms in the industry growth rates, $\sum_{n=1}^N s_{Yn}^0\gamma_n\rho_n\sigma_n$. Since growth rates are generally small, the first three sets of terms on the right hand side of (15) will generally be the dominant ones. The last four sets of terms represent second and third order *interaction terms*.

It is possible to give interpretations for the first three sets of terms on the right hand side of the last equation in (14). The first set of terms, $\sum_{n=1}^N s_{Yn}^0\gamma_n$, can be interpreted as economy wide labour productivity growth provided that all real output price growth rates ρ_n are equal to zero and all labour input share growth rates σ_n are equal to zero. This set of terms is just the straightforward aggregation of industry productivity growth rates and could be called the *direct effect*. The second set of terms, $\sum_{n=1}^N s_{Yn}^0\rho_n$, can be interpreted as economy wide labour productivity growth provided that all industry labour productivity growth rates γ_n are equal to zero and all labour input share growth rates σ_n are equal to zero. Thus even if all industry labour productivity levels remain constant and all labour input shares remain constant, economy wide labour productivity growth can change due to changes in industry real output prices. As mentioned above, this effect is due to the changes in output prices leading to changes in the price weights for the industry output growth rates, which in turn affects aggregate labour productivity growth. This effect could be called the *output price weighting effect*. The third set of terms, $\sum_{n=1}^N s_{Yn}^0\sigma_n$, can be interpreted as economy wide labour productivity growth provided that all industry labour productivity growth rates γ_n are equal to zero and all real output growth rates ρ_n are equal to zero. Thus even if all industry labour productivity levels remain constant and all industry real output prices remain constant, economy wide labour productivity growth can change due to changes in industry labour input shares. This effect could be called the *labour input reallocation effect*.[11] Note that it is not possible to see this reallocation effect in the industry n contribution term, $s_{Yn}{}^0\sigma_n$. This term correctly gives the contribution to economy wide labour productivity growth of an increase in industry n's labour share (so that σ_n is greater than 0 in this case) but the overall effect of this increase in n's labour share is offset by a decrease in other industry labour shares and it is the net effect of the change in n's labour share that gives rise to the reallocation effect. However, if N is greater than two, it is not possible to determine precisely how the increase in labour share for industry n is offset by decreases in shares for the other industries. Nevertheless, it is possible to determine the overall labour input reallocation effect. Similarly, although the overall output price weighting effect can be determined as the weighted sum $\sum_{n=1}^{N} s_{Yn}{}^0\rho_n$ of the industry output price changes ρ_n, one cannot interpret the industry n contribution term $s_{Yn}{}^0\rho_n$ as the *independent* effect of a change in industry n's real output price because an increase in industry n's nominal price $P_n{}^1$ will affect the economy wide price index P^t and thus the industry real prices $p_n{}^t = P_n{}^t/P^t$ cannot vary independently, just as the industry labour input shares $s_{Ln}{}^t$ cannot vary independently.[12]

The general decomposition formula (14) can be specialized to give Denison's (1962) decomposition formula. Suppose that the real output price growth rates ρ_n are all equal to 0. Then (14) reduces to the following decomposition of aggregate labour productivity growth:

$$
\begin{aligned}
(15)\ \Gamma &= \sum_{n=1}^{N} s_{Yn}{}^0 \{[1+\gamma_n][1+\sigma_n] - 1\} \\
&= \sum_{n=1}^{N} s_{Yn}{}^0 \{\gamma_n + \sigma_n + \gamma_n\sigma_n\} \\
&= \sum_{n=1}^{N} s_{Yn}{}^0\gamma_n + \sum_{n=1}^{N} s_{Yn}{}^0\sigma_n + \sum_{n=1}^{N} s_{Yn}{}^0\gamma_n\sigma_n.
\end{aligned}
$$

De Avillez (2012) calls the decomposition given by (15) the *traditional labour productivity decomposition*. However, it only provides an exact under the restrictive assumption that all $\rho_n = 0$.[13]

The reader may have noticed that there seems to be a slight asymmetry in the labour productivity growth formula (9) in that the output value added shares for the *base period*, $s_{Yn}{}^0$, are used as weights for the symmetric growth factors $p_n{}^1/p_n{}^0$, $s_{Ln}{}^1/s_{Ln}{}^0$ and $X_n{}^1/X_n{}^0$. However, it is possible to develop a decomposition for the reciprocal of aggregate productivity growth where the period 1 value added shares, $s_{Yn}{}^1$, are used as weights in this alternative decomposition. Thus using the same notation and steps that were used to establish (9), we can establish the following decomposition:[14]

[11] This type of effect was first noticed by Denison (1962). It is called the *reallocation level effect* by de Avillez (2012).
[12] On the other hand, the industry productivity growth rates γ_n can vary independently.
[13] De Avillez (2012) called the first term in the last line of (15) the *within effect*, the second term the *reallocation level effect* and the third term the *reallocation growth effect*.
[14] This derivation of (16) is due to Balk (2008) who noticed the asymmetry in (9).

$$(16)\ X^0/X^1 = \sum_{n=1}^N p_n^0 s_{Ln}^0 X_n^0/\sum_{n=1}^N p_n^1 s_{Ln}^1 X_n^1$$
$$= \sum_{n=1}^N [p_n^0/p_n^1][s_{Ln}^0/s_{Ln}^1][X_n^0/X_n^1][p_n^1 Y_n^1/L^1]/\sum_{i=1}^N [p_i^1 Y_i^1/L^1] \qquad \text{using (8)}$$
$$= \sum_{n=1}^N [p_n^0/p_n^1][s_{Ln}^0/s_{Ln}^1][X_n^0/X_n^1]\, s_{Yn}^1.$$

Now take reciprocals of both sides of (16) and using definitions (12)-(15), we obtain the following *alternative decomposition for aggregate productivity growth* Γ:

$$(17)\ \Gamma = [\sum_{n=1}^N s_{Yn}^1 \{(1+\gamma_n)(1+\rho_n)(1+\sigma_n)\}^{-1}]^{-1} - 1$$

Both (14) and (17) provide exact decompositions of aggregate labour productivity growth using the industry growth rates γ_n, ρ_n and σ_n and the industry value added shares s_{Yn}^t as explanatory variables. But it can be seen that the decomposition given by (14) is much easier to interpret so we will not discuss (17) further.

We return to the productivity decomposition given by the last line of (14). There are 7 sets of explanatory terms on the right hand side of this last equation. Each of these sets of terms could be reported for each industry as contributions to overall aggregate labour productivity growth but practical economic analysts will find it a bit burdensome to consider so many explanatory factors. Moreover, in most situations, only the first order terms, $\sum_{n=1}^N s_{Yn}^0\gamma_n + \sum_{n=1}^N s_{Yn}^0\rho_n + \sum_{n=1}^N s_{Yn}^0\sigma_n$, will be important numerically with the second and third order terms being very small in magnitude. Thus we propose to consolidate these seven sets of terms into the following three sets of terms. For n = 1,...,N, define the *industry n contributions* (to overall labour productivity growth) *due to the change in industry n labour productivity*, ΔX_n; *due to the change in industry n real output prices*, Δp_n; and *due to changes in industry n labour input shares*, Δs_{Ln}, as follows, for n = 1,...,N:

$$(18)\ \Delta X_n \equiv s_{Yn}^0 \gamma_n\{1 + (1/2)\rho_n + (1/2)\sigma_n + (1/3)\rho_n\sigma_n\};$$
$$(19)\ \Delta p_n \equiv s_{Yn}^0 \rho_n\{1 + (1/2)\gamma_n + (1/2)\sigma_n + (1/3)\gamma_n\sigma_n\};$$
$$(20)\ \Delta s_{Ln} \equiv s_{Yn}^0 \sigma_n\{1 + (1/2)\gamma_n + (1/2)\rho_n + (1/3)\gamma_n\rho_n\}.$$

It can be shown that the above contributions sum up to the aggregate labour productivity growth rate Γ defined by (14); i.e., we have

$$(21)\ \Gamma = \sum_{n=1}^N \Delta X_n + \sum_{n=1}^N \Delta p_n + \sum_{n=1}^N \Delta s_{Ln}\ .$$

Essentially, we have simplified (14) by assigning the second and third order terms in (14) to corresponding first order terms in a symmetric, even handed manner.[15]

[15] A similar allocation has been applied to the Bennett (1920) decomposition of a value difference, say $p_n^1q_n^1 - p_n^0q_n^0$, into the two terms, $(1/2)(p_n^1 + p_n^0)\Delta q_n$ and $(1/2)(q_n^1 + q_n^0)\Delta p_n$, where $\Delta q_n \equiv q_n^1 - q_n^0$ and $\Delta p_n \equiv p_n^1 - p_n^0$. The quantity change term has the decomposition $(1/2)(p_n^1 + p_n^0)\Delta q_n = p_n^0\Delta q_n + (1/2)\Delta p_n\Delta q_n$ while the price change term has the decomposition $(1/2)(q_n^1 + q_n^0)\Delta p_n = q_n^0\Delta p_n + (1/2)\Delta p_n\Delta q_n$. Note that the value change can also be written as the sum of the following 3 terms: $p_n^0\Delta q_n + q_n^0\Delta p_n + \Delta p_n\Delta q_n$. Thus ½ of the second order interaction term $\Delta p_n\Delta q_n$ is assigned to overall quantity change and the other ½ is assigned

In the following section, we will show how the analysis presented in sections 2 and 3 can be generalized to provide a decomposition of economy wide Total Factor Productivity growth.

4. An Extension to a Decomposition of Aggregate Multifactor Productivity Growth

Again, let there be N sectors or industries in the economy and again suppose that for period t = 0,1, the *output* (or real value added or volume) of sector n is Y_n^t with corresponding period t *price* P_n^t. However, we now assume that each sector uses many inputs and index number techniques are used to form industry input aggregates Z_n^t with corresponding aggregate industry input prices W_n^t for n = 1,...,N and t = 0,1.[16]

Industry n Total Factor Productivity (*TFP*) *in period t*, X_n^t, is defined as industry n real output Y_n^t divided by industry n real input Z_n^t:

$$(22)\ X_n^t \equiv Y_n^t/Z_n^t\ ; \qquad t = 0,1\ ;\ n = 1,\ldots,N.$$

As in section 2, *economy wide real output in period t*, Y^t, is defined as total value added divided by the economy wide output price index P^t. Thus we have[17]

$$(23)\ Y^t = \sum_{n=1}^N P_n^tY_n^t/P^t = \sum_{n=1}^N p_n^tY_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real output price* is defined as $p_n^t \equiv P_n^t/P^t$ for n = 1,...,N and t = 0,1.

We define economy wide real input in an analogous manner. Thus we form an *economy wide input price index* for period t, W^t, in one of two ways:[18]

- By aggregating over all industry micro economic input prices using microeconomic input quantities as weights to form W^t (single stage aggregation of inputs) or

to overall price change in the Bennett decomposition of value change. We are following a similar symmetric assignment scheme in allocating higher order interaction terms to the first order terms. For more on the properties of the Bennett decomposition, see Harberger (1971), Diewert (2005) and Diewert and Mizobuchi (2009).

[16] These industry input aggregates Z_n^t and the corresponding price indexes W_n^t are *indexes* of the underlying micro inputs utilized by industry n. The exact functional form for these indexes does not matter for our analysis but we assume the indexes satisfy the property that for each t and n, $W_n^tZ_n^t$ equals the industry n input cost for period t.

[17] As in sections 2 and 3, $P_n^tY_n^t$ is nominal industry n value added in period t so that Y_n^t is deflated (by the industry n value added price index P_n^t) industry n value added. It need not be the case that $P_n^tY_n^t$ is equal to $W_n^tZ_n^t$; i.e., it is not necessary that value added equal input cost for each industry.

[18] In either case, we assume that the product of the total economy input price index for period t, W^t, times the corresponding aggregate input quantity or volume index, Z^t, is equal to total economy nominal input cost. If Laspeyres or Paasche price indexes are used throughout, then the two stage and single stage input aggregates will coincide. If superlative indexes are used throughout, then the two stage and single stage input aggregates will approximate each other closely using annual data; see Diewert (1978).

- By aggregating the industry aggregate input prices W_n^t (with corresponding input quantities or volumes Z_n^t) into the aggregate period t input price index W^t using an appropriate index number formula (two stage aggregation of inputs).

Economy wide real input in period t, Z^t, is defined as economy wide input cost divided by the economy wide input price index W^t:[19]

$$(24)\ Z^t = \sum_{n=1}^N W_n^t Z_n^t / W^t = \sum_{n=1}^N w_n^t Z_n^t\ ; \qquad t = 0,1$$

where the *period t industry n real input price* is defined as:

$$(25)\ w_n^t \equiv W_n^t / W^t; \qquad n = 1,\ldots,N \text{ and } t = 0,1.$$

The *economy wide level of TFP* (or *MFP*) *in period t*, X^t, is defined as aggregate real output divided by aggregate real input:

$$(26)\ X^t \equiv Y^t / Z^t\ ; \qquad t = 0,1.$$

We denote the output share of industry n in period t, s_{Yn}^t, by (7) again and we define the *input share of industry n in economy wide cost in period t*, s_{Zn}^t, as follows:

$$\begin{aligned}(27)\ s_{Zn}^t &\equiv W_n^t Z_n^t / \textstyle\sum_{i=1}^N W_i^t Z_i^t \qquad t = 0,1\ ;\ n = 1,\ldots,N \\ &= w_n^t Z_n^t / \textstyle\sum_{i=1}^N w_i^t Z_i^t\end{aligned}$$

where the second equation in (27) follows from the definitions $w_n^t \equiv W_n^t/W^t$.

Substitute (23) and (24) into definition (26) and we obtain the following expression for the economy wide level of TFP in period t:

$$\begin{aligned}(28)\ X^t &= \textstyle\sum_{n=1}^N p_n^t Y_n^t / \sum_{i=1}^N w_i^t Z_i^t && t = 0,1 \\ &= \textstyle\sum_{n=1}^N p_n^t (Y_n^t/Z_n^t) Z_n^t / \sum_{i=1}^N w_i^t Z_i^t \\ &= \textstyle\sum_{n=1}^N (p_n^t/w_n^t) X_n^t w_n^t Z_n^t / \sum_{i=1}^N w_i^t Z_i^t && \text{using (22)} \\ &= \textstyle\sum_{n=1}^N (p_n^t/w_n^t) X_n^t s_{Zn}^t && \text{using (27).}\end{aligned}$$

Now we are ready to develop an expression for the rate of *growth* of economy wide Total Factor Productivity. Using (28), *aggregate TFP growth* (plus 1) going from period 0 to 1, X^1/X^0, is equal to:

$$\begin{aligned}(29)\ X^1/X^0 &= \textstyle\sum_{n=1}^N (p_n^1/w_n^1) X_n^1 s_{Zn}^1 / \sum_{i=1}^N (p_i^0/w_i^0) X_i^0 s_{Zi}^0 \\ &= \textstyle\sum_{n=1}^N (p_n^1/p_n^0)(w_n^0/w_n^1)(X_n^1/X_n^0)(s_{Zn}^1/s_{Zn}^0)(p_n^0/w_n^0) X_n^0 s_{Zn}^0 / \sum_{i=1}^N (p_i^0/w_i^0) X_i^0 s_{Zi}^0 \\ &= \textstyle\sum_{n=1}^N s_{Yn}^0 (p_n^1/p_n^0)(w_n^0/w_n^1)(s_{Zn}^1/s_{Zn}^0)(X_n^1/X_n^0).\end{aligned}$$

The last equation in (29) follows from the following equations for n = 1,...,N:

[19] Note that $W^t Z^t$ equals period t total economy input cost for each t.

(30) $(p_n^0/w_n^0)X_n^0 s_{Zn}^0 = (p_n^0/w_n^0)(Y_n^0/Z_n^0)(w_n^0 Z_n^0/\sum_{i=1}^N w_i^0 Z_i^0)$ using (22) and (27)
$= p_n^0 Y_n^0/\sum_{i=1}^N w_i^0 Z_i^0$.

Thus one plus economy wide TFP growth, X^1/X^0, is equal to an output share weighted average (with the base period weights s_{Yn}^0) of one plus the industry TFP growth rates, times an augmentation factor, which is the product $(p_n^1/p_n^0)(w_n^0/w_n^1)(s_{Zn}^1/s_{Zn}^0)$. Thus formula (29) is very similar to our previous labour productivity growth formula (9) except we have an additional multiplicative contribution factor, which is w_n^0/w_n^1, the *reciprocal* of one plus the rate of growth of real input prices for sector n.

We can use definitions (11) and (12) in section 3 to rewrite the decomposition (29), which is in contribution factor form, into a growth rate form. We need to add the following definitions:

(31) $\sigma_n \equiv (s_{Zn}^1/s_{Zn}^0) - 1$; n = 1,...,N.
(32) $\omega_n \equiv (w_n^0/w_n^1) - 1$; n = 1,...,N.

Note that $1 + \omega_n$ equals w_n^0/w_n^1 so that ω_n is a *reciprocal growth rate of real input prices for sector n*. Now substitute (11), (12), (31) and (32) into (29) and we obtain the following *decomposition for economy wide TFP growth* $\Gamma \equiv (X^1/X^0) - 1$ into explanatory industry contribution terms:

(33) $\Gamma = \sum_{n=1}^N s_{Yn}^0 \{[1+\gamma_n][1+\rho_n][1+\omega_n][1+\sigma_n] - 1\}$.

Thus the contribution term for industry n is the nth term in the above summation, which depends only on industry n TFP growth γ_n, the output share s_{Yn}^0, the real output price growth ρ_n, the real reciprocal input price growth ω_n and industry n input cost share growth σ_n.

The industry n contribution term $s_{Yn}^0 [1+\gamma_n][1+\rho_n][1+\omega_n][1+\sigma_n] - 1\}$ can be written out as the sum of 4 first order terms, 6 second order interaction terms, 4 third order interaction terms and 1 fourth order interaction term or 15 separate contribution effects in all. As was the case with the labour productivity contribution formula (14), this will be too many terms for analysts to handle and so we suggest the following counterpart to the labour productivity decomposition defined earlier by (21): for n = 1,...,N, define the *industry n contributions* (to overall MFP or TFP growth) *due to the changes in industry n TFP*, ΔX_n; *due to the changes in industry n real output prices*, Δp_n; *due to the changes in industry n real reciprocal input prices*, Δw_n; and *due to the changes in industry n input cost shares*, Δs_{Zn}, as follows, for n = 1,...,N:

(34) $\Delta X_n \equiv s_{Yn}^0 \gamma_n\{1+(½)\rho_n+(½)\omega_n+(½)\sigma_n+(⅓)\rho_n\omega_n+(⅓)\rho_n\sigma_n+(⅓)\rho_n\sigma_n+(¼)\rho_n\omega_n\sigma_n\}$;
(35) $\Delta p_n \equiv s_{Yn}^0 \rho_n\{1+(½)\gamma_n+(½)\omega_n+(½)\sigma_n+(⅓)\gamma_n\omega_n+(⅓)\gamma_n\sigma_n+(⅓)\omega_n\sigma_n+(¼)\gamma_n\omega_n\sigma_n\}$;
(36) $\Delta w_n \equiv s_{Yn}^0 \omega_n\{1+(½)\gamma_n+(½)\rho_n+(½)\sigma_n+(⅓)\gamma_n\rho_n+(⅓)\gamma_n\sigma_n+(⅓)\rho_n\sigma_n+(¼)\gamma_n\rho_n\sigma_n\}$;
(37) $\Delta s_{Zn} \equiv s_{Yn}^0 \sigma_n\{1+(½)\gamma_n+(½)\rho_n+(½)\omega_n+(⅓)\gamma_n\rho_n+(⅓)\gamma_n\omega_n+(⅓)\rho_n\omega_n+(¼)\gamma_n\rho_n\omega_n\}$;

It can be shown that the above contributions sum up exactly to the aggregate MFP growth rate Γ defined by (29); i.e., we have

$$(38)\ \Gamma = \sum_{n=1}^{N} \Delta X_n + \sum_{n=1}^{N} \Delta p_n + \sum_{n=1}^{N} \Delta w_n + \sum_{n=1}^{N} \Delta s_{Zn} .$$

Our empirical results for Australia indicate that while individual industry terms for any of the above contribution terms on the right hand side of (38) can be quite significant, when we sum the last three sets of terms in (38), we find that the sum over industries is close to zero; i.e., for each time period, $\sum_{n=1}^{N} \Delta p_n \approx \sum_{n=1}^{N} \Delta w_n \approx \sum_{n=1}^{N} \Delta s_{Zn} \approx 0$. Thus while the real price contributions and the industry share contributions, Δp_n, Δw_n and Δs_{Zn} can be individually quite substantial for many industries n, when we sum these effects over all industries, the overall sum of these contributions is close to zero for each time period. Thus the individual industry MFP contribution terms, ΔX_n, are generally more important in contributing to aggregate productivity growth than the price and input reallocation terms.

5. Conclusion

If one wishes to decompose aggregate labour productivity growth into explanatory factors that depend on the industries in the aggregate, then the decompositions given by (9) and the first line in (14) appear to be the simplest ones in the literature to date. These decompositions have the advantage that only four industry variables need to be reported in order to explain each industry contribution term: the industry shares of aggregate value added in the base period s_{Yn}^0, the industry labour productivity growth rates γ_n, the industry real output price growth rates ρ_n and the industry labour input share growth rates σ_n between the two periods under consideration. The overall industry n contribution term is $s_{Yn}^0\{[1+\gamma_n][1+\rho_n][1+\sigma_n] - 1\}$.

If a decomposition of aggregate TFP growth is desired, then the decompositions given by (29) and (33) are also very simple. In this framework, the overall industry n contribution term is $s_{Yn}^0\{[1+\gamma_n][1+\rho_n][1+\omega_n][1+\sigma_n] - 1\}$ where $1+\omega_n$ equals w_n^0/w_n^1, which in turn is the reciprocal of one plus the real input price growth for industry n. However, if a decomposition of aggregate TFP growth into explanatory factors is desired that depend on a sum of terms involving γ_n, ρ_n, ω_n and σ_n separately, then we recommend the decomposition of aggregate labour productivity growth defined by (18)-(21) and the decomposition of TFP growth defined by (34)-(38). In our empirical example using Australian data, we found that the main driver for the TFP decomposition was the first set of terms in the decomposition (38); i.e., in the aggregate, the sum of the effects involving the ρ_n, ω_n and σ_n was close to zero. However, using the Australian data, this unimportance result did not hold for the labour productivity decomposition defined by (21): the effects on Market Sector labour productivity growth of changes in industry shares of labour input proved to be significant.

Appendix: Empirical Application to Australian Market Sector Data; 1995-2012

We apply the productivity decompositions given by (21) (for aggregate labour productivity growth) and (38) (for TFP or MFP growth) using official industry data from the Australian Bureau of Statistics (ABS) for the June years 1995-2012.[20] From Table 9 of the ABS (2012), we can obtain indexes of industry real value for the following 16 Market Sector industries:

1. Agriculture, Forestry and Fishing;
2. Mining;
3. Manufacturing;
4. Electricity, Gas, Water and Waste Services;
5. Construction;
6. Wholesale Trade;
7. Retail Trade;
8. Accommodation and Food Services;
9. Transport, Postal and Warehousing;
10. Information, Media and Telecommunications;
11. Financial and Insurance Services;
12. Rental, Hiring and Real Estate Services;
13. Professional, Scientific and Technical Services;
14. Administrative and Support Services;
15. Arts and Recreation Services and
16. Other Services.

From Tables 9 and 10 of the ABS (2012), we can obtain quality adjusted indexes of labour input and capital services for the same 16 industries for the June years 1995-2012. From Table 14 of the same publication, we obtained the input cost share of labour and capital by the 16 industries. Finally, from the ABS (2013), estimates of value added by industry in current dollars for the 16 industries were obtained. Dividing these nominal value added estimates by industry by the corresponding indexes of industry real value added gives us implicit price indexes for each industry output. These price indexes were normalized to equal 1 in 1995 and these normalized industry price indexes, P_n^t, are listed in Table 1 below. The industry n year t nominal value added was divided by the corresponding normalized price index P_n^t in order to obtain an estimate of real value added for industry n for year t, Y_n^t. The Y_n^t are listed in Table 2 below. The units of measurement are in billions of constant 1995 dollars.

Table 1: Industry Value Added Output Prices P_n^t 1995-2012

P_1^t	P_2^t	P_3^t	P_4^t	P_5^t	P_6^t	P_7^t	P_8^t	P_9^t	P_{10}^t	P_{11}^t	P_{12}^t	P_{13}^t	P_{14}^t	P_{15}^t	P_{16}^t
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.9851	1.0383	1.0138	0.9619	1.0316	1.0224	1.0412	1.0576	0.9659	1.0268	1.0402	1.0544	1.0814	1.0747	1.0822	1.0188
0.9178	1.0174	1.0169	0.9636	1.0703	1.0101	1.0661	1.0309	0.9646	1.0323	1.0358	1.0909	1.1041	1.1075	1.1199	1.0247
0.9088	1.0412	1.0565	0.9905	1.0175	0.9678	1.0629	1.0828	0.9782	1.0613	1.0687	1.0876	1.1451	1.1386	1.2403	1.0278
0.8753	1.0348	1.0441	0.9921	0.9989	0.9848	1.0610	1.0966	1.0262	1.0530	1.0541	1.0314	1.1578	1.1637	1.2269	1.0629
0.8921	1.0870	1.0656	0.9866	1.0492	0.9861	1.0696	1.1157	1.0152	1.0510	1.0900	1.0692	1.2237	1.2185	1.3750	1.1128
1.0288	1.2681	1.0596	1.0458	1.0672	1.0191	1.0828	1.1397	1.0179	1.0757	1.1493	1.1347	1.2550	1.2618	1.3828	1.0955
1.2045	1.3025	1.0693	1.1118	1.1226	1.0837	1.1176	1.1558	1.0507	1.0895	1.1364	1.1591	1.2676	1.2691	1.4187	1.1878

[20] A June year is an aggregation of a rolling year of four quarters of data ending in June of the indicated year.

1.1908	**1.3054**	**1.1264**	**1.1814**	**1.0958**	**1.1578**	**1.1281**	**1.1475**	**1.0737**	**1.0788**	**1.1481**	**1.2413**	**1.3414**	**1.3474**	**1.4466**	**1.2249**
1.0915	**1.2715**	**1.1960**	**1.2453**	**1.1797**	**1.1834**	**1.1485**	**1.2195**	**1.1303**	**1.1041**	**1.1400**	**1.2316**	**1.3811**	**1.4672**	**1.4884**	**1.2962**
1.0566	**1.6245**	**1.2370**	**1.3269**	**1.2442**	**1.1886**	**1.1419**	**1.2414**	**1.1769**	**1.1392**	**1.1959**	**1.2895**	**1.4741**	**1.5609**	**1.4780**	**1.3776**
1.0512	**2.2551**	**1.2874**	**1.3581**	**1.2512**	**1.1934**	**1.1824**	**1.2790**	**1.2006**	**1.1036**	**1.2449**	**1.3210**	**1.5609**	**1.6518**	**1.4859**	**1.4124**
1.0938	**2.4195**	**1.2990**	**1.3925**	**1.3482**	**1.2371**	**1.2299**	**1.3396**	**1.3154**	**1.1014**	**1.2733**	**1.5442**	**1.7293**	**1.7921**	**1.4027**	**1.4463**
1.1670	**2.5724**	**1.3326**	**1.4658**	**1.3731**	**1.3206**	**1.2628**	**1.4452**	**1.3254**	**1.1099**	**1.2846**	**1.7567**	**1.8949**	**1.9056**	**1.4805**	**1.4763**
1.0450	**3.4128**	**1.3767**	**1.4725**	**1.4384**	**1.2979**	**1.3095**	**1.5699**	**1.3534**	**1.1379**	**1.4141**	**1.6783**	**2.0095**	**1.9643**	**1.4154**	**1.5515**
1.0423	**2.6241**	**1.3422**	**1.5490**	**1.5324**	**1.3113**	**1.3137**	**1.6956**	**1.3689**	**1.1919**	**1.4480**	**1.7480**	**2.0208**	**2.1059**	**1.4804**	**1.6204**
1.1024	**3.6669**	**1.3418**	**1.7287**	**1.5632**	**1.4058**	**1.3764**	**1.7765**	**1.4319**	**1.1620**	**1.5080**	**1.8389**	**2.0643**	**2.2189**	**1.4945**	**1.6949**
1.0500	**3.7267**	**1.3205**	**1.8435**	**1.5450**	**1.4129**	**1.4202**	**1.8260**	**1.4100**	**1.1729**	**1.5881**	**1.9545**	**2.1659**	**2.3319**	**1.5247**	**1.6454**

Table 2: Industry Value Added Output Volumes Y_n^t 1995-2012

Y_1^t	Y_2^t	Y_3^t	Y_4^t	Y_5^t	Y_6^t	Y_7^t	Y_8^t	Y_9^t	Y_{10}^t	Y_{11}^t	Y_{12}^t	Y_{13}^t	Y_{14}^t	Y_{15}^t	Y_{16}^t
15.15	**21.78**	**67.10**	**15.24**	**29.74**	**24.71**	**24.57**	**12.21**	**26.32**	**17.57**	**34.71**	**11.19**	**19.84**	**9.36**	**4.28**	**10.31**
18.37	**23.75**	**68.61**	**15.40**	**30.28**	**26.35**	**25.7**	**12.27**	**28.41**	**18.57**	**36.12**	**11.67**	**20.19**	**9.39**	**4.27**	**10.77**
19.86	**24.06**	**69.71**	**15.34**	**31.10**	**27.75**	**26.99**	**12.97**	**29.57**	**19.92**	**38.76**	**12.29**	**21.17**	**9.54**	**4.38**	**11.37**
19.88	**24.89**	**71.64**	**15.92**	**34.20**	**29.59**	**27.97**	**13.50**	**30.22**	**21.54**	**42.11**	**12.72**	**22.31**	**10.50**	**4.59**	**11.58**
21.78	**24.74**	**73.03**	**16.26**	**37.32**	**30.57**	**29.38**	**14.64**	**31.06**	**23.09**	**46.68**	**13.02**	**24.65**	**11.56**	**4.86**	**11.95**
23.15	**25.92**	**73.60**	**16.60**	**39.74**	**32.13**	**30.52**	**15.45**	**32.16**	**23.76**	**50.22**	**13.57**	**25.83**	**12.16**	**5.04**	**12.28**
24.03	**28.04**	**75.13**	**16.91**	**34.02**	**31.92**	**31.37**	**16.02**	**33.40**	**24.60**	**51.42**	**13.34**	**28.65**	**13.03**	**5.26**	**12.90**
24.68	**28.14**	**76.75**	**17.07**	**38.12**	**32.76**	**33.44**	**15.97**	**34.47**	**25.29**	**55.24**	**13.66**	**29.88**	**13.65**	**5.26**	**13.01**
19.43	**28.33**	**79.85**	**17.29**	**44.32**	**34.31**	**34.98**	**16.46**	**36.56**	**26.86**	**56.44**	**14.99**	**30.38**	**13.70**	**5.47**	**13.58**
24.39	**27.50**	**80.72**	**17.34**	**47.33**	**35.92**	**36.82**	**17.09**	**37.73**	**28.07**	**61.99**	**15.16**	**31.84**	**13.66**	**5.84**	**14.12**
25.35	**28.94**	**79.75**	**17.44**	**49.46**	**37.27**	**38.95**	**17.91**	**39.94**	**28.76**	**67.24**	**15.21**	**32.27**	**13.79**	**6.15**	**13.82**
26.06	**29.49**	**79.46**	**17.74**	**53.53**	**38.51**	**39.50**	**18.41**	**41.17**	**29.94**	**71.12**	**15.93**	**33.55**	**14.34**	**6.28**	**13.77**
22.08	**31.99**	**81.00**	**17.91**	**56.39**	**39.33**	**41.61**	**18.70**	**43.54**	**31.80**	**79.69**	**15.09**	**34.32**	**14.88**	**6.69**	**14.05**
23.61	**32.63**	**84.24**	**17.95**	**60.33**	**40.50**	**43.60**	**18.65**	**45.92**	**33.79**	**85.98**	**14.59**	**35.39**	**15.69**	**6.84**	**14.31**
27.78	**33.63**	**79.93**	**18.71**	**62.93**	**41.17**	**43.65**	**18.21**	**45.56**	**34.17**	**85.38**	**15.49**	**37.02**	**14.71**	**7.37**	**14.68**
27.26	**36.27**	**80.30**	**19.21**	**63.10**	**42.58**	**44.44**	**17.89**	**46.40**	**34.67**	**85.35**	**15.80**	**40.06**	**14.59**	**7.39**	**14.65**
29.17	**35.76**	**80.34**	**19.56**	**66.11**	**42.34**	**44.98**	**18.33**	**47.87**	**35.78**	**88.81**	**15.86**	**43.06**	**15.47**	**7.48**	**14.75**
30.99	**38.16**	**79.60**	**19.30**	**68.93**	**44.98**	**46.19**	**18.89**	**49.47**	**35.62**	**91.21**	**16.45**	**45.03**	**15.14**	**7.78**	**15.27**

We assume that value added is equal to input cost by industry so that with our estimates of nominal value added by industry and the ABS cost shares, we can obtain estimates for the value of labour input and capital services input by industry. These nominal cost values can be divided by the ABS index estimates of real labour and capital services input to give us implicit price indexes for labour and capital services by industry. These price indexes were normalized to equal 1 in 1995 and are listed in Tables 3 and 5 below as W_{Ln}^t and W_{Kn}^t for industry n in year t. Finally, nominal industry n labour cost in year t was divided by W_{Ln}^t to give the industry n quantity of labour input in year t, Q_{Ln}^t, and nominal industry n capital services cost in year t was divided by W_{Kn}^t to give the industry n quantity of capital services input in year t, Q_{Kn}^t. The Q_{Ln}^t and Q_{Kn}^t are listed in Tables 4 and 6 below.

Table 3: Industry Quality Adjusted Wages W_{Ln}^t 1995-2012

W_{L1}^t	W_{L2}^t	W_{L3}^t	W_{L4}^t	W_{L5}^t	W_{L6}^t	W_{L7}^t	W_{L8}^t	W_{L9}^t	W_{L10}^t	W_{L11}^t	W_{L12}^t	W_{L13}^t	W_{L14}^t	W_{L15}^t	W_{L16}^t
1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
0.8622	**1.1790**	**1.1037**	**1.0931**	**1.0907**	**1.0217**	**1.0949**	**1.0713**	**0.9624**	**1.0503**	**1.0850**	**1.1874**	**1.0037**	**1.0327**	**1.1739**	**0.9667**
1.1229	**1.2057**	**1.0840**	**1.1955**	**1.1538**	**1.1302**	**1.2271**	**1.1047**	**1.1005**	**1.0232**	**1.1508**	**1.1878**	**1.0235**	**1.0879**	**1.2028**	**1.0872**
1.1387	**1.2479**	**1.1183**	**1.1179**	**1.1874**	**1.2047**	**1.2673**	**1.2148**	**1.0318**	**1.2120**	**1.3221**	**1.3668**	**1.0560**	**1.1011**	**1.2553**	**1.1361**
1.1248	**1.2321**	**1.1673**	**1.0728**	**1.1637**	**1.2474**	**1.2820**	**1.2689**	**1.0527**	**1.2515**	**1.3513**	**1.3221**	**1.1601**	**1.1462**	**1.2795**	**1.1842**
1.1516	**1.1866**	**1.1803**	**1.1148**	**1.1586**	**1.2657**	**1.3066**	**1.2583**	**1.0677**	**1.1640**	**1.5043**	**1.3201**	**1.1990**	**1.1892**	**1.3291**	**1.2591**
1.3365	**1.2690**	**1.2917**	**1.1933**	**1.1107**	**1.3788**	**1.3376**	**1.2744**	**1.1038**	**1.1769**	**1.5911**	**1.3072**	**1.2943**	**1.2027**	**1.3531**	**1.3623**
1.4668	**1.3775**	**1.3007**	**1.2199**	**1.2860**	**1.4683**	**1.3632**	**1.3515**	**1.2429**	**1.2673**	**1.6737**	**1.3418**	**1.4453**	**1.3219**	**1.3506**	**1.4011**
1.4157	**1.3142**	**1.3607**	**1.2448**	**1.2421**	**1.5287**	**1.3861**	**1.3681**	**1.2744**	**1.1258**	**1.7000**	**1.4812**	**1.4804**	**1.3536**	**1.3459**	**1.3620**
1.5780	**1.2930**	**1.4066**	**1.2284**	**1.4558**	**1.5559**	**1.4666**	**1.4202**	**1.3356**	**1.2123**	**1.8449**	**1.5822**	**1.5180**	**1.4424**	**1.4992**	**1.5120**
1.6401	**1.2785**	**1.4915**	**1.3363**	**1.4812**	**1.6505**	**1.4586**	**1.5153**	**1.4048**	**1.2603**	**2.0613**	**1.7276**	**1.5144**	**1.4907**	**1.4655**	**1.6981**
1.7610	**1.2767**	**1.5947**	**1.3111**	**1.4311**	**1.6854**	**1.5176**	**1.5505**	**1.4963**	**1.3136**	**2.1464**	**1.8119**	**1.5419**	**1.6431**	**1.3589**	**1.7604**
1.5849	**1.3927**	**1.7036**	**1.3308**	**1.3523**	**1.7208**	**1.5591**	**1.7021**	**1.6113**	**1.3053**	**2.4060**	**1.9844**	**1.6602**	**1.9609**	**1.5820**	**1.9383**

1.7604	1.5895	1.7764	1.3301	1.7602	1.9922	1.6633	1.7381	1.6167	1.5111	2.4901	2.1009	1.7901	2.2206	1.4834	1.6926
1.6663	1.5752	1.8074	1.2615	1.7369	1.9361	1.7795	1.8583	1.4687	1.5222	2.4810	2.2534	1.8332	2.1328	1.4254	2.1050
1.5439	1.5805	1.8212	1.2942	1.8312	1.9099	1.8943	1.8478	1.5881	1.6186	2.5570	2.3614	1.7869	2.0566	1.5310	2.1999
1.7510	1.5579	1.8826	1.2808	1.9668	2.1709	1.9816	2.0151	1.6814	1.6625	2.7450	2.2121	1.8893	2.2512	1.6091	2.3827
1.8034	1.6336	1.9824	1.3557	2.0354	2.3342	2.1189	2.1726	1.7478	1.8073	2.7947	2.3444	1.9825	2.3373	1.7217	2.4996

Table 4: Industry Quality Adjusted Labour Input Q_{Ln}^{t} 1995-2012

L_1^t	L_2^t	L_3^t	L_4^t	L_5^t	L_6^t	L_7^t	L_8^t	L_9^t	L_{10}^t	L_{11}^t	L_{12}^t	L_{13}^t	L_{14}^t	L_{15}^t	L_{16}^t
7.42	6.10	38.92	5.33	21.71	16.06	17.69	9.89	16.84	7.56	16.66	4.36	18.46	8.70	3.00	7.94
7.55	6.07	38.44	5.01	21.76	16.35	17.84	9.93	17.39	7.99	17.31	3.94	20.22	9.19	3.03	8.51
7.79	6.09	39.24	4.33	21.64	15.87	17.82	10.29	17.63	8.44	17.44	4.40	21.23	9.13	3.14	8.36
8.09	6.02	39.25	4.37	22.57	16.16	18.06	10.35	18.05	7.54	17.36	4.55	22.74	10.21	3.27	8.59
7.63	6.03	38.54	4.66	23.70	16.41	18.72	10.50	18.47	7.58	18.21	4.67	23.37	11.15	3.36	8.47
8.07	5.70	38.54	4.70	26.27	17.27	19.73	11.09	18.65	8.58	18.56	4.84	24.25	11.83	3.34	8.68
7.77	5.60	37.59	4.89	24.84	16.52	19.55	11.46	19.41	9.00	18.94	5.09	25.55	12.85	3.28	8.30
7.90	5.59	36.60	5.14	25.29	16.44	20.02	11.20	18.65	8.26	19.50	5.19	24.37	12.06	3.54	8.71
6.70	6.19	38.34	5.42	26.59	16.89	21.35	11.19	18.79	9.01	19.82	5.53	25.05	12.82	3.59	8.80
6.58	6.76	37.06	5.63	29.15	17.21	21.34	11.59	19.80	8.69	20.30	6.14	25.78	12.92	3.59	8.96
6.37	7.35	37.70	5.89	30.75	17.18	22.56	11.88	20.41	9.36	21.07	6.13	27.01	13.28	3.78	8.74
6.07	8.85	36.57	6.43	32.29	17.18	22.78	11.70	20.48	9.56	22.28	6.50	29.21	13.27	4.12	8.73
6.40	9.45	36.44	6.56	34.86	18.66	23.30	11.48	20.97	9.93	23.62	6.81	30.38	12.92	4.21	8.70
6.42	10.03	37.92	7.12	35.77	17.99	24.16	11.94	22.58	9.43	23.95	6.71	31.85	12.66	4.58	9.49
6.62	11.66	36.53	8.08	36.48	18.49	23.45	11.69	23.93	9.20	23.84	6.46	31.65	12.47	4.76	9.20
6.63	12.04	35.51	8.28	36.43	19.59	22.50	11.98	23.20	8.68	24.17	6.08	33.52	13.60	4.50	8.85
6.24	14.31	35.50	9.24	37.30	19.19	23.12	12.28	23.64	8.75	24.40	6.86	34.34	14.33	4.72	9.13
5.96	17.41	34.46	9.45	36.63	19.06	22.91	12.07	23.15	9.01	25.92	7.00	35.42	14.35	4.75	8.95

Table 5: Industry Prices of Capital Services W_{Kn}^{t} 1995-2012

W_{K1}^t	W_{K2}^t	W_{K3}^t	W_{K4}^t	W_{K5}^t	W_{K6}^t	W_{K7}^t	W_{K8}^t	W_{K9}^t	W_{K10}^t	W_{K11}^t	W_{K12}^t	W_{K13}^t	W_{K14}^t	W_{K15}^t	W_{K16}^t
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.4897	1.0616	0.9240	0.9291	0.8805	1.1523	1.0002	0.9934	1.1114	0.9672	1.0137	1.0475	0.9600	0.7913	0.7612	1.0388
1.2146	0.9804	0.9427	0.9396	0.9692	1.0835	0.9097	0.8053	0.9185	0.9818	1.0268	1.0353	0.8907	0.7197	0.7335	0.8658
1.1262	0.9787	1.0153	1.0320	0.9049	0.9299	0.8554	0.7720	1.0832	1.0639	1.0683	0.8617	0.7266	0.6627	0.9546	0.6357
1.3337	0.9241	0.9661	1.0195	1.0594	0.9251	0.8329	0.9452	1.1984	1.0815	1.1289	0.7527	0.6042	0.5558	0.8974	0.7112
1.4451	1.0702	0.9839	0.9822	1.2013	0.8965	0.7549	1.0668	1.1861	1.0003	1.1551	0.7605	0.9463	0.5372	1.2561	0.6527
1.8224	1.4123	0.9096	1.0149	0.9071	0.8642	0.8074	1.1547	1.1467	0.9720	1.1769	0.7182	0.9757	0.6482	1.3429	0.6196
2.3047	1.4118	0.9922	1.0550	1.0375	0.9739	1.0066	1.0208	1.1543	0.9877	1.1678	0.6936	0.8273	0.8344	1.2116	0.6571
1.7554	1.3658	1.0406	1.0979	1.5095	1.1347	0.9206	1.0509	1.2872	1.0404	1.1354	0.7516	1.0471	0.6132	1.3331	0.8520
2.0611	1.1914	1.1619	1.1212	1.2460	1.1742	0.9622	1.2137	1.2779	1.0781	1.1516	0.5781	1.2541	0.7166	1.3472	0.7807
2.0278	1.6434	1.0517	1.1120	1.4108	1.0895	0.9447	1.1060	1.3597	1.0424	1.2136	0.5164	1.5636	0.8131	1.3750	0.6231
2.0259	2.2225	1.0190	1.0898	1.7411	1.0710	0.9247	1.3451	1.3087	0.9629	1.2807	0.4726	1.5506	0.8330	1.3758	0.5562
1.6815	2.3589	0.9555	1.0611	2.3053	0.9890	1.0681	1.3074	1.5476	0.9627	1.3518	0.4433	1.7027	0.5370	0.9544	0.4215
1.9253	2.2463	0.9449	1.0212	1.4593	0.9667	0.9941	1.4006	1.5049	0.9489	1.4829	0.4598	1.7193	0.6661	1.0965	0.5368
2.1037	2.8352	0.9101	0.9859	1.8012	0.9329	1.0052	1.5123	1.5511	0.9661	1.7573	0.4229	2.6014	0.8073	1.1370	0.3305
2.0937	2.0311	0.8849	1.0189	1.8719	0.9515	0.9949	1.7803	1.4897	1.0213	1.7352	0.4680	3.1391	0.9100	1.2128	0.3787
2.3937	2.5923	0.8346	1.1091	1.7928	0.9002	0.9905	1.6816	1.5432	0.9779	1.8418	0.4672	3.3894	0.6439	1.0281	0.2707
2.3915	2.2837	0.7448	1.0901	1.8600	0.9293	1.0318	1.7794	1.5045	0.8939	1.9649	0.5057	3.6555	0.5178	1.0137	0.2158

Table 6: Industry Capital Services Volumes Q_{Kn}^{t} 1995-2012

7.72	15.68	28.18	9.91	8.03	8.65	6.88	2.32	9.47	10.02	18.05	6.83	1.39	0.65	1.28	2.37
7.77	16.49	29.36	10.04	8.51	8.89	7.22	2.35	9.63	11.04	18.53	7.28	1.59	0.77	1.40	2.64
7.80	17.48	30.08	10.23	8.59	9.31	7.59	2.49	9.94	12.15	19.55	7.90	1.84	0.88	1.54	2.96
7.86	18.80	31.31	10.54	8.85	9.85	7.99	2.65	10.10	12.89	20.64	8.83	2.11	1.08	1.67	3.37
7.86	19.67	32.36	10.92	9.15	10.41	8.61	2.89	10.37	13.71	21.79	9.64	2.36	1.21	1.86	3.75
7.86	20.01	33.48	11.34	9.37	10.95	9.08	3.07	10.74	14.98	23.22	10.69	2.67	1.38	1.99	4.19
7.87	20.14	34.13	11.68	9.60	11.29	9.68	3.16	10.97	16.34	24.61	11.80	2.95	1.52	2.11	4.56
7.87	20.51	34.74	12.06	9.90	11.66	10.03	3.25	11.30	17.29	25.80	12.79	3.20	1.66	2.22	4.94
7.78	21.12	36.30	12.47	10.30	12.25	10.71	3.39	11.89	18.10	27.40	13.86	3.50	1.81	2.31	5.47
7.88	22.01	38.22	13.10	10.75	13.39	11.43	3.61	12.68	18.97	28.84	15.50	3.86	1.96	2.45	6.10
8.06	22.88	40.33	13.74	11.34	14.64	12.24	3.82	13.48	20.11	30.48	17.47	4.26	2.12	2.58	6.72
8.25	24.83	43.17	14.37	11.92	15.87	13.13	4.03	14.35	21.28	31.80	19.59	4.73	2.28	2.71	7.34
8.33	27.23	45.15	15.28	12.53	16.73	13.89	4.22	15.17	22.92	33.03	22.08	5.23	2.48	2.85	8.20

8.44	**30.27**	**47.52**	**16.49**	**13.62**	**18.26**	**14.95**	**4.43**	**16.18**	**24.50**	**34.26**	**25.09**	**5.85**	**2.69**	**3.05**	**9.45**
8.55	**34.00**	**48.36**	**17.60**	**15.08**	**18.90**	**15.35**	**4.54**	**17.09**	**25.76**	**35.04**	**27.05**	**6.29**	**2.86**	**3.21**	**10.34**
8.69	**37.49**	**48.72**	**18.69**	**16.01**	**19.36**	**15.84**	**4.60**	**17.91**	**26.71**	**35.61**	**28.32**	**6.70**	**3.04**	**3.34**	**11.28**
8.87	**41.98**	**49.08**	**19.81**	**16.72**	**19.84**	**16.25**	**4.65**	**18.65**	**27.64**	**36.36**	**29.97**	**7.08**	**3.20**	**3.48**	**12.01**
9.12	**49.82**	**49.39**	**20.88**	**17.18**	**20.51**	**16.53**	**4.65**	**19.47**	**28.51**	**36.86**	**31.16**	**7.47**	**3.41**	**3.63**	**12.81**

In order to calculate industry measures of Multifactor productivity, we will need estimates of aggregate input prices and quantities (or volumes) by industry. Thus we computed chained Fisher (1922) price and quantity indexes, W_n^t and Z_n^t for industry n in year t and these indexes are listed in Tables 7 and 8 respectively.[21]

Table 7: Industry Input Price Indexes W_n^t 1995-2012

W_1^t	W_2^t	W_3^t	W_4^t	W_5^t	W_6^t	W_7^t	W_8^t	W_9^t	W_{10}^t	W_{11}^t	W_{12}^t	W_{13}^t	W_{14}^t	W_{15}^t	W_{16}^t
1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
1.1813	**1.0939**	**1.0271**	**0.9851**	**1.0328**	**1.0676**	**1.0680**	**1.0564**	**1.0158**	**1.0025**	**1.0481**	**1.0993**	**1.0006**	**1.0150**	**1.0469**	**0.9835**
1.1721	**1.0410**	**1.0240**	**1.0247**	**1.1032**	**1.1120**	**1.1349**	**1.0471**	**1.0352**	**0.9998**	**1.0863**	**1.0916**	**1.0136**	**1.0598**	**1.0576**	**1.0309**
1.1360	**1.0503**	**1.0752**	**1.0673**	**1.1091**	**1.1012**	**1.1465**	**1.1287**	**1.0505**	**1.1237**	**1.1882**	**1.0409**	**1.0299**	**1.0666**	**1.1676**	**1.0038**
1.2332	**1.0047**	**1.0801**	**1.0450**	**1.1354**	**1.1256**	**1.1497**	**1.2083**	**1.1053**	**1.1494**	**1.2347**	**0.9524**	**1.1143**	**1.0971**	**1.1634**	**1.0609**
1.3025	**1.1081**	**1.0953**	**1.0311**	**1.1700**	**1.1257**	**1.1419**	**1.2264**	**1.1104**	**1.0654**	**1.3183**	**0.9572**	**1.1826**	**1.1343**	**1.3291**	**1.0944**
1.5843	**1.3964**	**1.1195**	**1.0776**	**1.0554**	**1.1814**	**1.1803**	**1.2583**	**1.1190**	**1.0519**	**1.3690**	**0.9230**	**1.2719**	**1.1584**	**1.3782**	**1.1535**
1.8919	**1.4197**	**1.1645**	**1.1141**	**1.2184**	**1.2804**	**1.2649**	**1.2891**	**1.2096**	**1.0933**	**1.3999**	**0.9156**	**1.3912**	**1.2872**	**1.3245**	**1.1938**
1.5842	**1.3693**	**1.2195**	**1.1519**	**1.3193**	**1.3843**	**1.2513**	**1.3108**	**1.2798**	**1.0822**	**1.3924**	**1.0004**	**1.4492**	**1.2880**	**1.3698**	**1.2449**
1.8217	**1.2287**	**1.3040**	**1.1633**	**1.4023**	**1.4173**	**1.3198**	**1.3880**	**1.3136**	**1.1364**	**1.4633**	**0.9007**	**1.5092**	**1.3808**	**1.4697**	**1.3122**
1.8311	**1.5728**	**1.2888**	**1.1895**	**1.4654**	**1.4318**	**1.3082**	**1.4349**	**1.3878**	**1.1270**	**1.5911**	**0.8950**	**1.5480**	**1.4371**	**1.4608**	**1.3577**
1.8815	**2.0110**	**1.3208**	**1.1662**	**1.5180**	**1.4419**	**1.3397**	**1.5224**	**1.4217**	**1.0887**	**1.6670**	**0.8828**	**1.5703**	**1.5747**	**1.3958**	**1.3629**
1.6137	**2.1443**	**1.3358**	**1.1521**	**1.6111**	**1.4206**	**1.4214**	**1.6261**	**1.5886**	**1.0858**	**1.8187**	**0.9045**	**1.6955**	**1.8063**	**1.3601**	**1.3979**
1.8245	**2.1096**	**1.3633**	**1.1238**	**1.6804**	**1.5539**	**1.4596**	**1.6784**	**1.5738**	**1.1369**	**1.9322**	**0.9491**	**1.8103**	**2.0558**	**1.3582**	**1.3181**
1.8837	**2.5525**	**1.3570**	**1.0779**	**1.7611**	**1.5067**	**1.5379**	**1.7987**	**1.5066**	**1.1529**	**2.0940**	**0.9506**	**1.9912**	**2.0067**	**1.3393**	**1.4311**
1.8258	**1.9427**	**1.3480**	**1.1110**	**1.8486**	**1.5027**	**1.6052**	**1.8672**	**1.5491**	**1.2213**	**2.1120**	**1.0216**	**2.0427**	**1.9609**	**1.4350**	**1.5184**
2.0816	**2.3638**	**1.3445**	**1.1692**	**1.9184**	**1.6122**	**1.6573**	**1.9630**	**1.6252**	**1.1981**	**2.2545**	**0.9866**	**2.1719**	**2.0781**	**1.4004**	**1.5427**
2.1009	**2.1505**	**1.3328**	**1.1798**	**1.9868**	**1.7124**	**1.7601**	**2.1069**	**1.6444**	**1.1677**	**2.3496**	**1.0564**	**2.2962**	**2.1275**	**1.4603**	**1.5659**

Table 8: Industry Input Volume (Quantity) Indexes Z_n^t 1995-2012

Z_1^t	Z_2^t	Z_3^t	Z_4^t	Z_5^t	Z_6^t	Z_7^t	Z_8^t	Z_9^t	Z_{10}^t	Z_{11}^t	Z_{12}^t	Z_{13}^t	Z_{14}^t	Z_{15}^t	Z_{16}^t
15.15	**21.78**	**67.10**	**15.24**	**29.74**	**24.71**	**24.57**	**12.21**	**26.32**	**17.57**	**34.71**	**11.19**	**19.85**	**9.36**	**4.28**	**10.31**
15.32	**22.55**	**67.72**	**15.03**	**30.24**	**25.24**	**25.06**	**12.28**	**27.02**	**19.02**	**35.85**	**11.19**	**21.81**	**9.95**	**4.42**	**11.16**
15.55	**23.52**	**69.23**	**14.43**	**30.18**	**25.2**	**25.35**	**12.77**	**27.56**	**20.57**	**36.96**	**12.29**	**23.06**	**9.97**	**4.64**	**11.30**
15.90	**24.67**	**70.39**	**14.77**	**31.38**	**26.00**	**25.93**	**12.95**	**28.15**	**20.34**	**37.88**	**13.29**	**24.80**	**11.21**	**4.88**	**11.86**
15.46	**25.48**	**70.60**	**15.44**	**32.83**	**26.74**	**27.11**	**13.29**	**28.84**	**21.15**	**39.85**	**14.10**	**25.61**	**12.26**	**5.13**	**11.97**
15.86	**25.42**	**71.61**	**15.88**	**35.64**	**28.14**	**28.58**	**14.05**	**29.41**	**23.43**	**41.53**	**15.16**	**26.73**	**13.06**	**5.21**	**12.49**
15.60	**25.46**	**71.11**	**16.41**	**34.40**	**27.54**	**28.78**	**14.51**	**30.38**	**25.16**	**43.17**	**16.39**	**28.27**	**14.19**	**5.27**	**12.25**
15.71	**25.82**	**70.48**	**17.04**	**35.13**	**27.73**	**29.55**	**14.32**	**29.95**	**25.20**	**44.84**	**17.30**	**27.22**	**13.46**	**5.63**	**12.94**
14.61	**27.01**	**73.76**	**17.73**	**36.81**	**28.69**	**31.53**	**14.41**	**30.67**	**26.77**	**46.54**	**18.60**	**28.12**	**14.33**	**5.78**	**13.37**
14.61	**28.46**	**74.03**	**18.57**	**39.82**	**29.99**	**32.04**	**15.01**	**32.47**	**27.27**	**48.30**	**20.73**	**29.13**	**14.51**	**5.91**	**13.95**
14.63	**29.89**	**76.54**	**19.46**	**42.00**	**30.94**	**34.00**	**15.49**	**33.87**	**29.07**	**50.54**	**21.92**	**30.73**	**14.98**	**6.22**	**14.02**
14.56	**33.07**	**77.46**	**20.66**	**44.12**	**31.87**	**34.87**	**15.47**	**34.77**	**30.35**	**53.12**	**23.83**	**33.35**	**15.05**	**6.69**	**14.27**
14.97	**36.09**	**78.77**	**21.65**	**47.19**	**34.25**	**36.00**	**15.41**	**36.05**	**32.26**	**55.79**	**25.77**	**35.00**	**14.76**	**6.90**	**14.54**
15.10	**39.79**	**82.35**	**23.42**	**49.30**	**34.42**	**37.72**	**16.06**	**38.67**	**32.99**	**57.16**	**27.01**	**37.05**	**14.55**	**7.46**	**16.03**
15.41	**44.96**	**81.09**	**25.55**	**51.40**	**35.47**	**37.17**	**15.90**	**40.93**	**33.73**	**57.65**	**27.35**	**37.36**	**14.40**	**7.79**	**15.91**
15.56	**49.00**	**79.96**	**26.78**	**52.31**	**37.15**	**36.37**	**16.24**	**41.00**	**33.84**	**58.52**	**27.03**	**39.63**	**15.67**	**7.63**	**15.63**
15.45	**55.47**	**80.19**	**28.92**	**53.87**	**36.92**	**37.36**	**16.59**	**42.18**	**34.71**	**59.41**	**29.56**	**40.92**	**16.51**	**7.98**	**16.21**
15.49	**66.14**	**78.87**	**30.15**	**53.60**	**37.11**	**37.27**	**16.37**	**42.42**	**35.78**	**61.65**	**30.44**	**42.48**	**16.60**	**8.12**	**16.04**

The above Tables provide the data that are necessary to calculate Industry and Market Sector MFP estimates. However, in order to calculate Industry and Market Sector Labour Productivity estimates, we will require an additional Table. From the ABS (2012), Table

[21] The price and quantity (volume) series listed in Tables 3-6 were the inputs into the Fisher index number formula for each industry.

9, we can obtain *indexes* of *hours worked* in the 16 industries as well as for the Market Sector. Denote these index series for year t as H_1^t-H_{16}^t and H^t. We ran an ordinary least squares regression of H on H_1-H_{16} (with no constant term) in order to recover the actual industry hours worked by industry (up to a factor of proportionality).[22] Denote the estimated industry regression coefficients by α_1-α_{16} and define Market Sector hours worked in year t by $L^t \equiv H^t$ and the corresponding year t industry n measures of hours worked by $L_n^t \equiv \alpha_n H_n^t$ for n = 1,...,16. The resulting measures of Labour input are listed in Table 9 below. Note that for each year t, $L^t = \sum_{n=1}^{16} L_n^t$.

Table 9: Market Sector Hours Worked L^t and Industry Hours Worked L_n^t 1995-2012

t	L^t	L_1^t	L_2^t	L_3^t	L_4^t	L_5^t	L_6^t	L_7^t	L_8^t	L_9^t	L_{10}^t	L_{11}^t	L_{12}^t	L_{13}^t	L_{14}^t	L_{15}^t	L_{16}^t
1995	**79.11**	**5.92**	**1.4**	**14.65**	**1.15**	**7.6**	**5.05**	**9.42**	**5.94**	**5.59**	**2.43**	**3.77**	**1.65**	**5.87**	**2.58**	**1.46**	**4.64**
1996	**80.24**	**6.01**	**1.38**	**14.41**	**1.07**	**7.6**	**5.12**	**9.49**	**5.96**	**5.76**	**2.56**	**3.88**	**1.48**	**6.40**	**2.71**	**1.47**	**4.95**
1997	**80.94**	**6.18**	**1.38**	**14.61**	**0.91**	**7.54**	**4.95**	**9.45**	**6.16**	**5.81**	**2.70**	**3.87**	**1.65**	**6.69**	**2.68**	**1.52**	**4.84**
1998	**82.29**	**6.39**	**1.36**	**14.52**	**0.91**	**7.84**	**5.02**	**9.56**	**6.19**	**5.92**	**2.40**	**3.81**	**1.69**	**7.14**	**2.99**	**1.57**	**4.97**
1999	**83.11**	**6.00**	**1.35**	**14.17**	**0.97**	**8.21**	**5.08**	**9.88**	**6.28**	**6.03**	**2.40**	**3.97**	**1.73**	**7.30**	**3.25**	**1.61**	**4.88**
2000	**86.18**	**6.33**	**1.27**	**14.08**	**0.97**	**9.08**	**5.33**	**10.39**	**6.62**	**6.07**	**2.70**	**4.01**	**1.79**	**7.55**	**3.43**	**1.59**	**4.98**
2001	**85.66**	**6.07**	**1.24**	**13.64**	**1.00**	**8.56**	**5.08**	**10.27**	**6.83**	**6.28**	**2.82**	**4.06**	**1.87**	**7.92**	**3.71**	**1.56**	**4.74**
2002	**84.84**	**6.15**	**1.23**	**13.23**	**1.05**	**8.71**	**5.03**	**10.48**	**6.66**	**6.02**	**2.58**	**4.15**	**1.90**	**7.53**	**3.47**	**1.68**	**4.97**
2003	**86.69**	**5.19**	**1.37**	**13.80**	**1.10**	**9.16**	**5.13**	**11.15**	**6.64**	**6.05**	**2.79**	**4.18**	**2.02**	**7.71**	**3.68**	**1.70**	**5.01**
2004	**88.15**	**5.08**	**1.49**	**13.29**	**1.15**	**10.05**	**5.20**	**11.12**	**6.87**	**6.36**	**2.68**	**4.24**	**2.23**	**7.90**	**3.69**	**1.70**	**5.10**
2005	**90.49**	**4.91**	**1.62**	**13.47**	**1.20**	**10.60**	**5.16**	**11.73**	**7.03**	**6.54**	**2.87**	**4.37**	**2.22**	**8.25**	**3.78**	**1.78**	**4.97**
2006	**91.84**	**4.66**	**1.96**	**13.01**	**1.30**	**11.14**	**5.13**	**11.81**	**6.91**	**6.55**	**2.92**	**4.58**	**2.35**	**8.88**	**3.76**	**1.94**	**4.94**
2007	**94.58**	**4.93**	**2.08**	**12.93**	**1.33**	**12.05**	**5.54**	**12.07**	**6.81**	**6.69**	**3.01**	**4.83**	**2.46**	**9.25**	**3.67**	**1.98**	**4.96**
2008	**97.57**	**4.95**	**2.21**	**13.41**	**1.45**	**12.39**	**5.31**	**12.51**	**7.11**	**7.18**	**2.83**	**4.87**	**2.43**	**9.71**	**3.60**	**2.16**	**5.44**
2009	**97.73**	**5.12**	**2.57**	**12.88**	**1.64**	**12.67**	**5.43**	**12.14**	**7.00**	**7.59**	**2.75**	**4.83**	**2.34**	**9.66**	**3.55**	**2.25**	**5.30**
2010	**97.63**	**5.14**	**2.65**	**12.48**	**1.69**	**12.70**	**5.72**	**11.66**	**7.21**	**7.33**	**2.59**	**4.87**	**2.21**	**10.25**	**3.88**	**2.14**	**5.13**
2011	**100.00**	**4.86**	**3.14**	**12.44**	**1.89**	**13.05**	**5.57**	**11.99**	**7.44**	**7.45**	**2.60**	**4.89**	**2.49**	**10.53**	**4.10**	**2.25**	**5.32**
2012	**100.27**	**4.65**	**3.81**	**12.04**	**1.93**	**12.86**	**5.50**	**11.90**	**7.37**	**7.27**	**2.68**	**5.18**	**2.55**	**10.89**	**4.12**	**2.27**	**5.25**

With the above industry data listed, we can now use equations (2) in the main text to calculate the labour productivity level of industry n in year t, X_n^t as Y_n^t/L_n^t, where the Y_n^t are listed in Table 2 and the L_n^t are listed in Table 9 above. We normalize these industry labour productivity levels by dividing each X_n^t by X_n^{1995} for t = 1995,...,2012 and n = 1,...,16. Denote the normalized industry labour productivities by $X_n^{t*} \equiv X_n^t/X_n^{1995}$. These normalized labour productivity estimates are listed in Table 10 below.

The next step is to construct a measure of aggregate market sector real value added, Y^t for each year t. We will use Fisher chained indexes of the industry outputs Y_n^t (with price weights P_n^t) in order to construct the Y^t. The P_n^t are listed in Table 1 and the corresponding Y_n^t are listed in Table 2. The chained Fisher Market Sector output price index that corresponds to Y^t is denoted by P^t. The P^t and Y^t are listed in Table 15 below. Note that these indexes satisfy the identity $Y^t = \sum_{n=1}^{16} P_n^t Y_n^t / P^t$ for each t. With Y^t and L^t defined, the year t Market Sector labour productivity level is defined as $X^t \equiv Y^t/L^t$ for t = 1995,...,2012. We normalize these Market Sector labour productivity levels by dividing

[22] The R^2 for the regression turned out to be 1.0000 so we are confident that we recovered the actual hours worked by industry from our procedure (up to a factor of proportionality).

each X^t by X^{1995} so that $X^{t*} \equiv X^t/X^{1995}$ for t = 1995,...,2012. These normalized Market Sector labour productivity estimates are listed in the second column of Table 9 below.

Table 10: Market Sector Aggregate X^{t*} and Industry Labour Productivity Levels X_n^{t*} Relative to 1995 Levels: 1995-2012.

t	X^{t*}	X_1^{t*}	X_2^{t*}	X_3^{t*}	X_4^{t*}	X_5^{t*}	X_6^{t*}	X_7^{t*}	X_8^{t*}	X_9^{t*}	X_{10}^{t*}	X_{11}^{t*}	X_{12}^{t*}	X_{13}^{t*}	X_{14}^{t*}	X_{15}^{t*}	X_{16}^{t*}
1995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1996	1.032	1.194	1.106	1.039	1.086	1.018	1.052	1.038	1.001	1.048	1.003	1.011	1.162	0.933	0.956	0.992	0.979
1997	1.064	1.256	1.121	1.042	1.272	1.054	1.146	1.095	1.024	1.081	1.020	1.088	1.099	0.936	0.981	0.984	1.057
1998	1.099	1.216	1.176	1.077	1.320	1.115	1.205	1.122	1.061	1.084	1.241	1.201	1.109	0.924	0.968	0.998	1.049
1999	1.148	1.419	1.178	1.125	1.265	1.161	1.230	1.140	1.134	1.094	1.330	1.277	1.110	0.999	0.981	1.031	1.102
2000	1.154	1.429	1.312	1.141	1.291	1.118	1.232	1.126	1.135	1.125	1.217	1.360	1.118	1.012	0.977	1.081	1.110
2001	1.184	1.547	1.453	1.203	1.276	1.016	1.284	1.171	1.141	1.130	1.206	1.376	1.051	1.070	0.968	1.149	1.225
2002	1.242	1.568	1.471	1.267	1.227	1.118	1.331	1.224	1.166	1.216	1.355	1.446	1.060	1.174	1.085	1.068	1.178
2003	1.255	1.463	1.329	1.263	1.186	1.236	1.367	1.203	1.206	1.284	1.331	1.467	1.094	1.166	1.027	1.097	1.220
2004	1.292	1.876	1.186	1.326	1.138	1.203	1.412	1.269	1.210	1.260	1.448	1.588	1.002	1.192	1.021	1.171	1.247
2005	1.300	2.018	1.148	1.293	1.097	1.192	1.476	1.273	1.239	1.297	1.386	1.671	1.010	1.157	1.006	1.179	1.252
2006	1.321	2.185	0.967	1.333	1.030	1.228	1.534	1.282	1.296	1.335	1.418	1.687	0.999	1.118	1.052	1.105	1.254
2007	1.335	1.750	0.988	1.368	1.016	1.196	1.451	1.322	1.336	1.382	1.461	1.792	0.905	1.097	1.118	1.153	1.275
2008	1.350	1.864	0.949	1.371	0.934	1.244	1.559	1.336	1.276	1.358	1.651	1.918	0.886	1.078	1.202	1.081	1.184
2009	1.364	2.120	0.841	1.355	0.861	1.269	1.550	1.379	1.266	1.275	1.718	1.920	0.976	1.134	1.143	1.118	1.247
2010	1.395	2.073	0.880	1.405	0.857	1.270	1.521	1.461	1.207	1.345	1.851	1.904	1.054	1.156	1.037	1.178	1.285
2011	1.396	2.346	0.732	1.410	0.781	1.294	1.554	1.438	1.198	1.365	1.903	1.973	0.939	1.209	1.040	1.134	1.248
2012	1.437	2.605	0.644	1.443	0.754	1.370	1.671	1.488	1.247	1.445	1.838	1.913	0.951	1.223	1.014	1.169	1.309

Over the 18 year period, there was a 43.7% increase in Market Sector labour productivity. It can be seen that industries 2, 4 and 12 experienced negative labour productivity growth over the sample period.

We turn now to our decomposition of aggregate labour productivity growth into explanatory factors. For t = 1996,1997,...,2012, define the year t aggregate and industry rates of growth of labour productivity by the following extensions of definitions (10) and (11) to the case of many periods: $\Gamma^t \equiv (X^t/X^{t-1}) - 1$ and $\gamma_n^t \equiv (X_n^t/X_n^{t-1}) - 1$ for n = 1,...,16. These aggregate and industry rates of growth of labour productivity (times 100) are listed in Table 11 below.

Table 11: Growth Rates for the Market Sector Aggregate Labour Productivity Γ^t and the Industry Labour Productivity Growth Rates γ_n^t, 1996-2012 (in percentage points)

t	Γ^t	γ_1^t	γ_2^t	γ_3^t	γ_4^t	γ_5^t	γ_6^t	γ_7^t	γ_8^t	γ_9^t	γ_{10}^t	γ_{11}^t	γ_{12}^t	γ_{13}^t	γ_{14}^t	γ_{15}^t	γ_{16}^t
1996	3.16	19.44	10.63	3.94	8.57	1.8	5.21	3.84	0.11	4.76	0.33	1.12	16.24	-6.71	-4.40	-0.80	-2.05
1997	3.17	5.16	1.31	0.21	17.15	3.55	8.90	5.45	2.29	3.20	1.68	7.58	-5.49	0.32	2.67	-0.85	7.90
1998	3.24	-3.20	4.94	3.40	3.76	5.76	5.15	2.42	3.57	0.30	21.63	10.37	0.98	-1.25	-1.33	1.48	-0.75
1999	4.45	16.67	0.16	4.47	-4.15	4.19	2.09	1.66	6.92	0.88	7.19	6.37	0.05	8.06	1.30	3.28	5.03
2000	0.51	0.77	11.35	1.43	2.07	-3.70	0.17	-1.24	0.09	2.88	-8.53	6.52	0.72	1.33	-0.38	4.89	0.74
2001	2.62	8.24	10.8	5.37	-1.17	-9.20	4.26	4.00	0.51	0.38	-0.84	1.13	-5.96	5.72	-0.92	6.30	10.38
2002	4.93	1.36	1.19	5.33	-3.85	10.14	3.65	4.47	2.25	7.67	12.34	5.10	0.84	9.70	12.07	-7.06	-3.84
2003	1.03	-6.69	-9.63	-0.26	-3.34	10.54	2.68	-1.70	3.40	5.53	-1.78	1.44	3.19	-0.69	-5.38	2.74	3.59
2004	2.93	28.22	-10.7	4.97	-4.04	-2.66	3.28	5.54	0.30	-1.82	8.81	8.28	-8.39	2.27	-0.60	6.70	2.15
2005	0.62	7.56	-3.23	-2.52	-3.64	-0.92	4.57	0.29	2.43	2.92	-4.33	5.24	0.79	-2.95	-1.45	0.68	0.40
2006	1.67	8.29	-15.8	3.16	-6.11	2.97	3.93	0.73	4.61	2.94	2.34	0.92	-1.08	-3.39	4.59	-6.31	0.22
2007	1.06	-19.9	2.22	2.56	-1.32	-2.61	-5.43	3.06	3.06	3.53	3.04	6.25	-9.47	-1.82	6.27	4.35	1.66
2008	1.10	6.50	-3.98	0.28	-8.05	4.06	7.44	1.09	-4.51	-1.73	12.99	7.00	-2.12	-1.75	7.52	-6.21	-7.12
2009	1.01	13.73	-11.4	-1.22	-7.89	2.00	-0.58	3.18	-0.79	-6.13	4.09	0.12	10.23	5.14	-4.92	3.42	5.25
2010	2.28	-2.23	4.61	3.69	-0.36	0.03	-1.84	5.99	-4.65	5.45	7.72	-0.85	7.96	1.97	-9.26	5.41	3.11
2011	0.11	13.15	-16.8	0.37	-8.94	1.96	2.13	-1.56	-0.70	1.51	2.81	3.63	-10.9	4.63	0.31	-3.76	-2.90

2012	2.90	11.06	-12.0	2.36	-3.38	5.81	7.57	3.46	4.06	5.90	-3.43	-3.05	1.30	1.13	-2.57	3.11	4.90
Mean	2.16	6.36	-2.14	2.21	-1.45	1.98	3.13	2.39	1.35	2.24	3.89	3.95	-0.06	1.28	0.21	1.02	1.69

Thus on average, Market Sector labour productivity grew at about 2.16% per year.

The extension of definitions (12) and (13) to the case of many time periods is $\rho_n^t \equiv (p_n^t/p_n^{t-1}) - 1$ (the rate of growth of the industry n real output price for year t) and $\sigma_n^t \equiv (s_{Ln}^t/s_{Ln}^{t-1}) - 1$ (the rate of growth of the industry n share of total hours worked for year t) for n = 1,...,N. The real output price for industry n, p_n^t is defined as P_n^t/P^t where the industry output prices P_n^t are listed in Table 1 above and the Market Sector output prices P^t are listed in Table 15 below. The industry labour input shares s_{Ln}^t are defined as industry n hours worked in year t L_n^t divided by Market Sector hours worked in year t, L^t. The L^t and L_n^t are listed in Table 9 above. The industry nominal value added shares of Market Sector value added are defined as $s_{Yn}^t \equiv P_n^tY_n^t/P^tY^t$ for t = 1995,...,2012 and n = 1,...,16. We will not list the ρ_n^t, σ_n^t and s_{Yn}^t since these numbers can readily be calculated using the information in the listed Tables. The year t *labour productivity contribution terms* due to industry *productivity growth* (ΔX_n^t), due to changes in industry *real output prices* (Δp_n^t) and due to changes in industry *labour input shares* (Δs_{Ln}^t) are defined by (A1)-(A3) below:

(A1) $\Delta X_n^t \equiv s_{Yn}^{t-1}\,\gamma_n^t\{1 + (1/2)\rho_n^t + (1/2)\sigma_n^t + (1/3)\rho_n^t\sigma_n^t\}$;
(A2) $\Delta p_n^t \equiv s_{Yn}^{t-1}\,\rho_n^t\{1 + (1/2)\gamma_n^t + (1/2)\sigma_n^t + (1/3)\gamma_n^t\sigma_n^t\}$;
(A3) $\Delta s_{Ln}^t \equiv s_{Yn}^{t-1}\,\sigma_n^t\{1 + (1/2)\gamma_n^t + (1/2)\rho_n^t + (1/3)\gamma_n^t\rho_{nt}^t\}$.

Table 12 below lists the industry productivity growth contribution terms ΔX_n^t (times 100) along with the year by year sum of these terms, $\text{Sum}^t = \sum_{n=1}^N \Delta X_n^t$ (times 100).

Table 12: Contributions of Industry Productivity Growth to Aggregate Labour Productivity Growth ΔX_n^t and Sum of Industry Contributions $\sum_n \Delta X_n^t$, 1996-2012 (percentage points)

t	ΔX_1^t	ΔX_2^t	ΔX_3^t	ΔX_4^t	ΔX_5^t	ΔX_6^t	ΔX_7^t	ΔX_8^t	ΔX_9^t	ΔX_{10}^t	ΔX_{11}^t	ΔX_{12}^t	ΔX_{13}^t	ΔX_{14}^t	ΔX_{15}^t	ΔX_{16}^t	Sum
1996	0.84	0.67	0.75	0.35	0.16	0.37	0.28	0	0.36	0.02	0.11	0.50	-0.41	-0.12	-0.01	-0.06	3.80
1997	0.25	0.09	0.04	0.63	0.30	0.63	0.40	0.08	0.24	0.09	0.76	-0.20	0.02	0.07	-0.01	0.23	3.63
1998	-0.15	0.31	0.63	0.14	0.49	0.36	0.18	0.13	0.02	1.09	1.07	0.03	-0.08	-0.04	0.02	-0.02	4.19
1999	0.70	0.01	0.81	-0.16	0.36	0.15	0.12	0.25	0.07	0.40	0.71	0	0.51	0.04	0.05	0.15	4.14
2000	0.03	0.65	0.25	0.07	-0.33	0.01	-0.09	0	0.21	-0.49	0.74	0.02	0.09	-0.01	0.07	0.02	1.24
2001	0.38	0.70	0.88	-0.04	-0.80	0.29	0.28	0.02	0.03	-0.05	0.14	-0.19	0.40	-0.03	0.09	0.29	2.38
2002	0.07	0.09	0.85	-0.15	0.77	0.25	0.32	0.08	0.53	0.64	0.62	0.03	0.69	0.39	-0.11	-0.12	4.95
2003	-0.34	-0.70	-0.04	-0.13	0.86	0.19	-0.12	0.12	0.38	-0.10	0.17	0.10	-0.05	-0.19	0.04	0.11	0.29
2004	1.10	-0.72	0.80	-0.15	-0.25	0.23	0.39	0.01	-0.13	0.45	0.95	-0.29	0.17	-0.02	0.10	0.07	2.69
2005	0.32	-0.22	-0.40	-0.13	-0.09	0.31	0.02	0.08	0.21	-0.23	0.62	0.02	-0.22	-0.05	0.01	0.01	0.27
2006	0.32	-1.47	0.47	-0.22	0.28	0.26	0.05	0.15	0.21	0.11	0.12	-0.03	-0.26	0.15	-0.09	0.01	0.07
2007	-0.79	0.22	0.36	-0.04	-0.26	-0.36	0.20	0.10	0.25	0.14	0.79	-0.30	-0.14	0.21	0.05	0.04	0.48
2008	0.20	-0.41	0.04	-0.27	0.40	0.46	0.07	-0.15	-0.13	0.56	0.90	-0.07	-0.14	0.26	-0.08	-0.19	1.46
2009	0.43	-1.40	-0.16	-0.26	0.20	-0.04	0.20	-0.03	-0.45	0.18	0.02	0.29	0.41	-0.17	0.04	0.13	-0.60
2010	-0.07	0.53	0.44	-0.01	0	-0.11	0.38	-0.16	0.37	0.34	-0.12	0.23	0.17	-0.33	0.06	0.08	1.81
2011	0.39	-2.20	0.04	-0.31	0.20	0.13	-0.10	-0.02	0.10	0.12	0.48	-0.35	0.41	0.01	-0.04	-0.08	-1.21
2012	0.34	-1.75	0.25	-0.12	0.59	0.45	0.22	0.13	0.39	-0.14	-0.43	0.04	0.10	-0.09	0.04	0.12	0.13
Mean	0.24	-0.33	0.35	-0.05	0.17	0.21	0.16	0.05	0.16	0.18	0.45	-.009	0.10	0.005	0.01	0.05	1.75

Note that the overall mean of the industry labour productivity growth rates to overall market sector labour productivity growth is 1.75 percentage points per year. The productivity growth of Industry 11 (Financial and Insurance Services) contributes the most to overall Market Sector Labour Productivity growth—about 0.45 percentage points per year. The most negative contribution comes from Industry 2 (Mining; −0.33 percentage points per year) followed by Industry 4 (Electricity, Gas, Water and Waste Services; −0.05 percentage points per year).

Table 13 below lists the industry real output price change contribution terms Δp_n^t (times 100) along with the year by year sum of these terms, $Sum^t = \sum_{n=1}^{N} \Delta p_n^t$ (times 100).

Table 13: Contributions of Industry Real Output Price Changes to Aggregate Labour Productivity Growth Δp_n^t and Sum of Industry Contributions $\sum_n \Delta p_n^t$, 1996-2012 (percentage points)

t	Δp_1^t	Δp_2^t	Δp_3^t	Δp_4^t	Δp_5^t	Δp_6^t	Δp_7^t	Δp_8^t	Δp_9^t	Δp_{10}^t	Δp_{11}^t	Δp_{12}^t	Δp_{13}^t	Δp_{14}^t	Δp_{15}^t	Δp_{16}^t	Sum
1996	-0.19	0.09	-0.21	-0.27	0.06	-0.02	0.12	0.11	-0.45	0.01	0.15	0.10	0.32	0.13	0.07	-0.02	0
1997	-0.36	-0.15	0.01	0	0.3	-0.11	0.16	-0.10	-0.03	0.01	-0.07	0.11	0.11	0.08	0.04	0.01	0
1998	-0.11	0.06	0.46	0.05	-0.56	-0.41	-0.12	0.13	0	0.08	0.19	-0.06	0.14	0.04	0.12	-0.03	0
1999	-0.16	-0.03	-0.18	0.01	-0.14	0.14	0	0.06	0.37	-0.03	-0.13	-0.17	0.09	0.07	-0.01	0.11	0
2000	-0.03	0.15	-0.08	-0.11	0.21	-0.17	-0.12	-0.03	-0.26	-0.15	0.10	0.03	0.20	0.07	0.13	0.06	0
2001	0.52	0.80	-0.70	0.08	-0.16	-0.02	-0.17	-0.05	-0.24	-0.07	0.21	0.08	-0.08	0	-0.05	-0.15	0
2002	0.70	-0.03	-0.35	0.11	0.16	0.21	0.01	-0.06	0.01	-0.10	-0.52	-0.03	-0.16	-0.09	-0.01	0.15	0
2003	-0.18	-0.16	0.44	0.13	-0.42	0.29	-0.11	-0.11	-0.02	-0.18	-0.17	0.14	0.23	0.12	-0.01	0.02	0
2004	-0.51	-0.35	0.51	0.09	0.41	-0.05	-0.08	0.11	0.16	-0.03	-0.43	-0.12	0	0.19	0	0.09	0
2005	-0.35	1.29	-0.23	0.06	0.05	-0.31	-0.38	-0.11	-0.06	-0.09	0	-0.01	0.13	0.05	-0.08	0.04	0
2006	-0.23	2.34	-0.16	-0.10	-0.44	-0.32	-0.11	-0.07	-0.22	-0.41	-0.13	-0.08	0.05	0.02	-0.06	-0.07	0.01
2007	-0.03	0.21	-0.57	-0.08	0.25	-0.08	-0.06	-0.01	0.31	-0.24	-0.34	0.33	0.41	0.11	-0.14	-0.07	0
2008	0.08	0.21	-0.20	0.04	-0.22	0.16	-0.09	0.12	-0.24	-0.15	-0.42	0.27	0.41	0.08	0.02	-0.05	0
2009	-0.57	2.53	-0.39	-0.18	-0.16	-0.5	-0.17	0.07	-0.29	-0.17	0.45	-0.33	-0.03	-0.11	-0.13	-0.03	0.01
2010	0.03	-2.97	-0.18	0.20	0.78	0.13	0.09	0.29	0.15	0.26	0.47	0.15	0.14	0.27	0.07	0.14	0
2011	-0.04	3.14	-0.80	0.13	-0.53	0	-0.15	-0.08	-0.17	-0.42	-0.40	-0.06	-0.43	-0.06	-0.07	-0.06	0.01
2012	-0.21	-0.01	-0.35	0.16	-0.30	-0.07	0.09	0.04	-0.22	-0.03	0.48	0.13	0.29	0.11	0	-0.12	0
Mean	-0.10	0.42	-0.17	0.02	-0.04	-0.07	-0.06	0.02	-0.07	-0.10	-0.03	0.03	0.11	0.06	-.006	.0001	.0018

Note that the overall mean of the industry contributions to overall market sector labour productivity growth due to changes in real industry output prices is practically zero (0.0018 percentage points per year). The changes in the real output prices of Industry 2 (Mining) contribute the most to overall Market Sector Labour Productivity growth: about 0.42 percentage points per year, followed by Industry 11, Finance and Insurance Services (0.11 percentage points per year). The most negative contribution comes from Industry 3 (Manufacturing; −0.17 percentage points per year) followed by Industry 1 (Agriculture; −0.10 percentage points per year). These effects flow from increasing real output prices for Industries 2 and 11 and decreasing real output prices for Industries 1 and 3.

Table 14 below lists the industry contributions of changes in the industry share of labour input in hours to overall labour productivity growth Δs_{Ln}^t (times 100) along with the year by year sum of these terms, $Sum^t = \sum_{n=1}^{N} \Delta s_{Ln}^t$ (times 100).

Table 14: Contributions of Changes in Industry Labour Input Shares to Aggregate Labour Productivity Growth Δs_{Ln}^t and Sum of Industry Contributions $\sum_n \Delta s_{Ln}^t$, 1996-2012 (percentage points)

t	Δs_{L1}^t	Δs_{L2}^t	Δs_{L3}^t	Δs_{L4}^t	Δs_{L5}^t	Δs_{L6}^t	Δs_{L7}^t	Δs_{L8}^t	Δs_{L9}^t	Δs_{L10}^t	Δs_{L11}^t	Δs_{L12}^t	Δs_{L13}^t	Δs_{L14}^t	Δs_{L15}^t	Δs_{L16}^t	Sum
1996	0	-0.19	-0.60	-0.37	-0.12	0	-0.05	-0.04	0.12	0.20	0.15	-0.41	0.43	0.10	-0.01	0.15	-0.64

1997	**0.09**	**-0.06**	**0.10**	**-0.68**	**-0.14**	**-0.31**	**-0.10**	**0.09**	**0**	**0.24**	**-0.12**	**0.35**	**0.22**	**-0.06**	**0.03**	**-0.10**	**-0.46**
1998	**0.08**	**-0.20**	**-0.43**	**-0.06**	**0.20**	**-0.02**	**-0.04**	**-0.04**	**0.02**	**-0.75**	**-0.35**	**0.03**	**0.30**	**0.27**	**0.02**	**0.03**	**-0.95**
1999	**-0.33**	**-0.11**	**-0.63**	**0.21**	**0.32**	**0.01**	**0.17**	**0.02**	**0.06**	**-0.06**	**0.36**	**0.04**	**0.08**	**0.23**	**0.02**	**-0.08**	**0.31**
2000	**0.08**	**-0.59**	**-0.74**	**-0.13**	**0.57**	**0.08**	**0.10**	**0.06**	**-0.22**	**0.45**	**-0.31**	**-0.01**	**-0.02**	**0.06**	**-0.07**	**-0.05**	**-0.73**
2001	**-0.17**	**-0.12**	**-0.43**	**0.13**	**-0.44**	**-0.29**	**-0.04**	**0.14**	**0.29**	**0.27**	**0.22**	**0.16**	**0.39**	**0.28**	**-0.02**	**-0.13**	**0.24**
2002	**0.13**	**0.01**	**-0.34**	**0.22**	**0.22**	**0**	**0.22**	**-0.06**	**-0.23**	**-0.44**	**0.39**	**0.08**	**-0.31**	**-0.20**	**0.13**	**0.17**	**-0.02**
2003	**-0.94**	**0.60**	**0.33**	**0.09**	**0.25**	**-0.01**	**0.29**	**-0.09**	**-0.12**	**0.30**	**-0.17**	**0.13**	**0.02**	**0.12**	**-0.01**	**-0.04**	**0.74**
2004	**-0.17**	**0.43**	**-0.90**	**0.10**	**0.70**	**-0.02**	**-0.14**	**0.06**	**0.24**	**-0.30**	**-0.03**	**0.27**	**0.06**	**-0.05**	**-0.02**	**0**	**0.23**
2005	**-0.26**	**0.38**	**-0.20**	**0.06**	**0.26**	**-0.24**	**0.19**	**-0.01**	**0.01**	**0.22**	**0.05**	**-0.10**	**0.13**	**-0.01**	**0.03**	**-0.16**	**0.35**
2006	**-0.27**	**1.49**	**-0.75**	**0.23**	**0.34**	**-0.14**	**-0.05**	**-0.11**	**-0.10**	**0.01**	**0.41**	**0.13**	**0.44**	**-0.07**	**0.10**	**-0.06**	**1.59**
2007	**0.10**	**0.30**	**-0.51**	**-0.02**	**0.48**	**0.31**	**-0.05**	**-0.15**	**-0.06**	**0**	**0.31**	**0.05**	**0.09**	**-0.19**	**-0.01**	**-0.07**	**0.58**
2008	**-0.09**	**0.30**	**0.07**	**0.18**	**-0.03**	**-0.47**	**0.03**	**0.04**	**0.29**	**-0.42**	**-0.31**	**-0.13**	**0.14**	**-0.18**	**0.07**	**0.16**	**-0.35**
2009	**0.11**	**1.72**	**-0.54**	**0.38**	**0.21**	**0.13**	**-0.21**	**-0.06**	**0.39**	**-0.13**	**-0.13**	**-0.12**	**-0.06**	**-0.05**	**0.05**	**-0.07**	**1.60**
2010	**0.02**	**0.37**	**-0.37**	**0.10**	**0.04**	**0.33**	**-0.26**	**0.10**	**-0.24**	**-0.27**	**0.13**	**-0.17**	**0.53**	**0.30**	**-0.06**	**-0.08**	**0.46**
2011	**-0.26**	**1.73**	**-0.31**	**0.29**	**0.03**	**-0.31**	**0.02**	**0.02**	**-0.05**	**-0.09**	**-0.27**	**0.29**	**0.03**	**0.11**	**0.03**	**0.03**	**1.31**
2012	**-0.15**	**2.60**	**-0.37**	**0.06**	**-0.18**	**-0.09**	**-0.06**	**-0.04**	**-0.19**	**0.11**	**0.76**	**0.06**	**0.29**	**0.01**	**0.01**	**-0.04**	**2.77**
Mean	**-0.12**	**0.51**	**-0.39**	**0.05**	**0.16**	**-0.06**	**.002**	**-.003**	**0.01**	**-0.04**	**0.06**	**0.04**	**0.16**	**0.04**	**0.02**	**-0.02**	**.414**

The overall mean of the industry contributions to overall market sector labour productivity growth due to changes in the industry shares of labour hours is substantial at 0.414 percentage points per year. Note that the sum of the final columns in Tables 12-14 equals the first column in Table 11; i.e., the decomposition of overall Market Sector labour productivity growth given by (21) is exact. The changes in the labour share of Industry 2 (Mining) contribute the most to overall Market Sector Labour Productivity growth—about 0.51 percentage points per year, followed by Industries 5 and 13 (Construction and Professional, Technical and Scientific Services) at 0.16 percentage points per year. The most negative contribution comes from Industry 3 (Manufacturing; −0.39 percentage points per year) followed by Industry 1 (Agriculture; −0.12 percentage points per year).

An increase in labour productivity of a production unit is not a reliable guide in determining whether the efficiency of the unit has increased, since an increase in output with no change in labour input could be due to an increase in capital input. Multifactor Productivity growth takes into account the growth of all inputs and thus is a better indicator of efficiency improvement than labour productivity growth. Thus the decomposition of MFP growth into explanatory factors should be of more interest to economic analysts.

Recall that equations (22) defined the industry n MFP or TFP level in period t as $X_n^t \equiv Y_n^t/Z_n^t$ where the *industry output volumes* Y_n^t are listed in Table 2 and the *industry input volumes* Z_n^t are listed in Table 8 above.[23] The corresponding industry output and input indexes were defined as P_n^t and W_n^t and are listed in Tables 1 and 7 above. As noted above, the Market Sector value added output volume indexes for period t, Y^t, and the corresponding price indexes P^t were defined as chained Fisher indexes using the industry output prices and volumes, $P_1^t,...,P_{16}^t$ and $Y_1^t,...,Y_{16}^t$ as the input series into the index number formulae. The resulting P^t and Y^t are listed in Table 15 below. The Market Sector input volume indexes for period t, Z^t, and the corresponding price indexes W^t were defined as chained Fisher indexes using the industry input prices and volumes for both labour and capital, $W_{L1}^t,...,W_{L16}^t$; $W_{K1}^t,...,W_{K16}^t$ and $Q_{L1}^t,...,Q_{L16}^t$; $Q_{K1}^t,...,Q_{K16}^t$ as the input series into the Fisher index number formula (see Tables 3-6 for a listing of these

[23] Note that X_n^t is now defined as industry n TFP or MFP for year t (instead of labour productivity).

labour and capital input price and quantity series). The resulting W^t and Z^t are listed in Table 15 below. Note that for each year t, we have $Z^t = \sum_{n=1}^{16} (W_n^t/W^t)Z_n^t = \sum_{n=1}^{16} w_n^t Z_n^t$ where the year t real input price for industry n is defined as $w_n^t \equiv W_n^t/W^t$ for n = 1,...,16. Finally, the year t Market Sector MFP is defined as $X^t \equiv Y^t/Z^t$. The X^t are listed in Tables 15 and 16.[24] The industry n MFP levels, $X_n^t \equiv Y_n^t/Z_n^t$, are listed in Table 16.

Table 15: Market Sector MFP X^t, Value Added Output Y^t, Input Z^t, Labour Input L^t, Capital Services Input K^t and Price Indexes for Aggregate Output, Input, Labour and Capital

Year t	X^t	Y^t	Z^t	L^t	K^t	P^t	W^t	W_L^t	W_K^t
1996	1.0000	344.08	344.08	206.64	137.44	1.0000	1.0000	1.0000	1.0000
1997	1.0177	360.04	353.79	210.41	143.41	1.0248	1.0428	1.0568	1.0221
1998	1.0338	374.70	362.44	212.61	150.12	1.0274	1.0622	1.1185	0.9804
1999	1.0510	393.29	374.21	216.83	158.11	1.0414	1.0945	1.1682	0.9884
2000	1.0756	414.90	385.72	221.18	165.73	1.0391	1.1177	1.1995	1.0006
2001	1.0753	432.41	402.12	229.82	173.69	1.0656	1.1459	1.2251	1.0318
2002	1.0792	441.06	408.70	230.34	180.44	1.1044	1.1919	1.2854	1.0587
2003	1.1124	458.38	412.06	228.43	186.56	1.1390	1.2670	1.3731	1.1172
2004	1.1068	473.20	427.54	235.76	195.09	1.1676	1.2923	1.3891	1.1535
2005	1.1174	495.25	443.23	241.18	206.24	1.2018	1.3428	1.4760	1.1598
2006	1.1065	511.55	462.30	248.91	218.51	1.2611	1.3955	1.5441	1.1935
2007	1.0959	527.86	481.66	255.29	232.84	1.3268	1.4541	1.5951	1.2591
2008	1.0906	549.35	503.73	263.87	247.52	1.3933	1.5194	1.6888	1.2919
2009	1.0861	572.96	527.55	271.30	266.19	1.4505	1.5754	1.8293	1.2578
2010	1.0692	579.69	542.18	272.43	282.42	1.5437	1.6505	1.8421	1.3916
2011	1.0673	592.28	554.91	273.75	296.04	1.5276	1.6304	1.8805	1.3173
2012	1.0554	607.33	575.47	281.03	311.09	1.6380	1.7287	1.9942	1.3964
1996	1.0528	626.64	595.20	283.99	331.65	1.6660	1.7540	2.0917	1.3568

Table 16: Market Sector MFP X^t and Industry MFP Levels X_n^t Relative to 1995 Levels: 1995-2012.

t	X^t	X_1^t	X_2^t	X_3^t	X_4^t	X_5^t	X_6^t	X_7^t	X_8^t	X_9^t	X_{10}^t	X_{11}^t	X_{12}^t	X_{13}^t	X_{14}^t	X_{15}^t	X_{16}^t
1995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1996	1.018	1.199	1.054	1.013	1.024	1.001	1.044	1.026	0.999	1.052	0.976	1.008	1.043	0.925	0.944	0.967	0.965
1997	1.034	1.277	1.023	1.007	1.063	1.031	1.101	1.064	1.016	1.073	0.969	1.049	1.001	0.918	0.957	0.944	1.006
1998	1.051	1.250	1.009	1.018	1.078	1.090	1.138	1.079	1.042	1.074	1.059	1.112	0.957	0.899	0.937	0.941	0.977
1999	1.076	1.409	0.971	1.034	1.053	1.137	1.143	1.084	1.102	1.077	1.092	1.171	0.923	0.962	0.943	0.948	0.998
2000	1.075	1.460	1.019	1.028	1.045	1.115	1.142	1.068	1.099	1.094	1.014	1.209	0.895	0.966	0.931	0.967	0.984
2001	1.079	1.540	1.101	1.057	1.030	0.989	1.159	1.090	1.104	1.099	0.978	1.191	0.813	1.013	0.918	0.997	1.053
2002	1.112	1.571	1.090	1.089	1.002	1.085	1.182	1.132	1.115	1.151	1.004	1.232	0.790	1.097	1.014	0.934	1.005
2003	1.107	1.330	1.049	1.083	0.975	1.204	1.196	1.109	1.142	1.192	1.003	1.213	0.806	1.080	0.956	0.947	1.016
2004	1.117	1.669	0.966	1.090	0.934	1.189	1.198	1.149	1.138	1.162	1.029	1.284	0.731	1.093	0.941	0.987	1.012
2005	1.107	1.733	0.968	1.042	0.896	1.178	1.205	1.146	1.156	1.179	0.989	1.330	0.694	1.050	0.921	0.988	0.986
2006	1.096	1.790	0.892	1.026	0.859	1.213	1.208	1.133	1.190	1.184	0.986	1.339	0.668	1.006	0.953	0.939	0.965
2007	1.091	1.475	0.886	1.028	0.827	1.195	1.148	1.156	1.214	1.208	0.986	1.428	0.586	0.980	1.008	0.970	0.967
2008	1.086	1.563	0.820	1.023	0.767	1.224	1.177	1.156	1.161	1.187	1.024	1.504	0.540	0.955	1.079	0.917	0.893
2009	1.069	1.803	0.748	0.986	0.732	1.224	1.161	1.174	1.146	1.113	1.013	1.481	0.566	0.991	1.022	0.946	0.922
2010	1.067	1.752	0.740	1.004	0.717	1.206	1.146	1.222	1.101	1.132	1.025	1.459	0.584	1.011	0.931	0.969	0.937
2011	1.055	1.888	0.645	1.002	0.676	1.227	1.147	1.204	1.105	1.135	1.031	1.495	0.537	1.052	0.937	0.937	0.910
2012	1.053	2.001	0.577	1.009	0.640	1.286	1.212	1.239	1.154	1.166	0.996	1.480	0.540	1.060	0.912	0.958	0.952

Thus Market Sector MFP increased by 5.3% over the sample period (and has steadily declined since hitting a peak level of 1.117 in 2004). The industries which had the highest

[24] It was not necessary to normalize the MFP levels, X^t and X_n^t, (as was done for the labour productivity levels) since in 1995, for each sector, the value of inputs equals the value of outputs both in real and nominal terms.

rates of growth of MFP over the sample period were Industry 1 (Agriculture with X_1^{2012} = 2.001) and Industry 11 (Finance and Insurance Services with X_{11}^{2012} = 1.480). The industries which had absolute declines in MFP were 2 (Mining with X_2^{2012} = 0.577), 4 (Electricity, Gas, Water and Waste Services with X_4^{2012} = 0.640), 10 (Information, Media and Telecom Services with X_{10}^{2012} = 0.996), 12 (Rental, Hiring and Real Estate Services with X_{12}^{2012} = 0.540), 14 (Administrative and Support Services with X_{14}^{2012} = 0.912), 15 (Arts and Recreational Services with X_{15}^{2012} = 0.958) and 16 (Other Services with X_{16}^{2012} = 0.952). Thus the MFP measures paint a very different efficiency picture compared to the labour productivity measures.[25]

Table 17 presents essentially the same information as is in Table 16 except in Table 17, we list the rates of growth of MFP (times 100 in order to convert into percentage points) for the market sector, $\Gamma^t \equiv (X^t/X^{t-1}) - 1$, and for industries 1-16, $\gamma^t \equiv (X_n^t/X_n^{t-1}) - 1$ for n = 1,...,16.

Table 17: Growth Rates for Market Sector MFP Γ^t and for the Industry MFP Growth Rates γ_n^t, 1996-2012 (in percentage points)

t	Γ^t	γ_1^t	γ_2^t	γ_3^t	γ_4^t	γ_5^t	γ_6^t	γ_7^t	γ_8^t	γ_9^t	γ_{10}^t	γ_{11}^t	γ_{12}^t	γ_{13}^t	γ_{14}^t	γ_{15}^t	γ_{16}^t
1996	**1.77**	**19.92**	**5.35**	**1.31**	**2.41**	**0.11**	**4.41**	**2.58**	**-0.12**	**5.16**	**-2.36**	**0.76**	**4.26**	**-7.47**	**-5.55**	**-3.26**	**-3.46**
1997	**1.59**	**6.49**	**-2.88**	**-0.6**	**3.83**	**2.95**	**5.43**	**3.77**	**1.69**	**2.06**	**-0.80**	**4.08**	**-4.02**	**-0.78**	**1.32**	**-2.38**	**4.21**
1998	**1.66**	**-2.12**	**-1.41**	**1.06**	**1.33**	**5.75**	**3.36**	**1.33**	**2.63**	**0.06**	**9.31**	**6.02**	**-4.36**	**-2.03**	**-2.11**	**-0.32**	**-2.92**
1999	**2.34**	**12.71**	**-3.75**	**1.65**	**-2.25**	**4.28**	**0.45**	**0.46**	**5.7**	**0.29**	**3.09**	**5.35**	**-3.51**	**7.00**	**0.65**	**0.73**	**2.19**
2000	**-0.03**	**3.64**	**5.00**	**-0.63**	**-0.78**	**-1.89**	**-0.12**	**-1.48**	**-0.23**	**1.55**	**-7.12**	**3.25**	**-3.05**	**0.41**	**-1.26**	**1.94**	**-1.46**
2001	**0.35**	**5.47**	**8.01**	**2.79**	**-1.40**	**-11.3**	**1.55**	**2.1**	**0.43**	**0.51**	**-3.55**	**-1.51**	**-9.14**	**4.87**	**-1.39**	**3.11**	**7.06**
2002	**3.08**	**1.99**	**-1.01**	**3.06**	**-2.75**	**9.74**	**1.92**	**3.83**	**1.03**	**4.71**	**2.63**	**3.42**	**-2.89**	**8.29**	**10.49**	**-6.33**	**-4.54**
2003	**-0.50**	**-15.3**	**-3.77**	**-0.58**	**-2.70**	**10.92**	**1.19**	**-2.00**	**2.42**	**3.55**	**-0.03**	**-1.55**	**2.02**	**-1.56**	**-5.75**	**1.42**	**1.11**
2004	**0.96**	**25.46**	**-7.88**	**0.71**	**-4.19**	**-1.26**	**0.18**	**3.6**	**-0.37**	**-2.50**	**2.59**	**5.84**	**-9.25**	**1.15**	**-1.55**	**4.28**	**-0.39**
2005	**-0.97**	**3.84**	**0.19**	**-4.45**	**-4.04**	**-0.92**	**0.57**	**-0.3**	**1.56**	**1.47**	**-3.87**	**3.65**	**-5.10**	**-3.90**	**-2.17**	**0.09**	**-2.65**
2006	**-0.96**	**3.28**	**-7.90**	**-1.54**	**-4.21**	**3.01**	**0.31**	**-1.11**	**2.98**	**0.41**	**-0.29**	**0.64**	**-3.72**	**-4.20**	**3.54**	**-4.95**	**-2.09**
2007	**-0.49**	**-17.6**	**-0.61**	**0.23**	**-3.64**	**-1.50**	**-4.96**	**2.00**	**1.97**	**1.99**	**-0.06**	**6.67**	**-12.3**	**-2.54**	**5.73**	**3.23**	**0.16**
2008	**-0.41**	**5.97**	**-7.47**	**-0.52**	**-7.34**	**2.41**	**2.46**	**0.01**	**-4.32**	**-1.67**	**3.90**	**5.31**	**-7.76**	**-2.56**	**7.03**	**-5.39**	**-7.63**
2009	**-1.55**	**15.29**	**-8.80**	**-3.65**	**-4.52**	**0.04**	**-1.34**	**1.60**	**-1.35**	**-6.26**	**-1.08**	**-1.55**	**4.84**	**3.72**	**-5.30**	**3.14**	**3.31**
2010	**-0.17**	**-2.82**	**-1.02**	**1.89**	**-2.02**	**-1.47**	**-1.28**	**4.05**	**-3.89**	**1.66**	**1.13**	**-1.51**	**3.18**	**2.02**	**-8.85**	**2.44**	**1.59**
2011	**-1.12**	**7.79**	**-12.9**	**-0.23**	**-5.70**	**1.73**	**0.07**	**-1.46**	**0.35**	**0.29**	**0.62**	**2.50**	**-8.20**	**4.08**	**0.58**	**-3.33**	**-2.86**
2012	**-0.24**	**5.97**	**-10.5**	**0.73**	**-5.38**	**4.78**	**5.68**	**2.93**	**4.42**	**2.75**	**-3.43**	**-1.04**	**0.74**	**0.76**	**-2.58**	**2.21**	**4.56**
Mean	**0.31**	**4.71**	**-3.02**	**0.07**	**-2.55**	**1.61**	**1.17**	**1.29**	**0.88**	**0.94**	**0.04**	**2.37**	**-3.43**	**0.43**	**-0.42**	**-0.20**	**-0.22**

Thus the average rate of TFP growth for the market sector over the sample period was only 0.31 percentage points per year.

We turn now to the decomposition of Market sector MFP growth in year t, $\Gamma^t \equiv (X^t/X^{t-1}) - 1$, into explanatory factors; i.e., recall the decomposition (38) in the main text. The definitions of the year t industry n share of Market sector value added, $s_{Yn}{}^t \equiv P_n{}^tY_n{}^t/P^tY^t$, and the rates of growth of the industry n real output prices for year t, $\rho_n{}^t \equiv (p_n{}^t/p_n{}^{t-1}) - 1$, remain unchanged. The year t industry rates of MFP are defined as $\gamma_n{}^t \equiv (X_n{}^t/X_n{}^{t-1}) - 1$ for n = 1,...,16 but of course, now the $X_n{}^t$ are industry n levels of MFP instead of levels of labour productivity. The industry n share of Market Sector total cost in year t, $s_{Zn}{}^t$, is

[25] However, many of the declining sectors have outputs which are difficult to measure and so measurement error may explain some of these sectoral declines in MFP.

defined as $W_n{}^tZ_n{}^t/W^tZ^t$ for n = 1,...,16[26] and $\sigma_n{}^t \equiv s_{Zn}{}^t/s_{Zn}{}^{t-1}$ is defined as the year t rate of input cost growth for industry n for t = 1996,...,2012. The industry n real input price for year t is defined as $w_n{}^t \equiv W_n{}^t/W^t$ for n = 1,...,16 and t = 1995,...,2012 and the year t reciprocal rate of growth in these real input prices is defined as $\omega_n{}^t \equiv (w_n{}^{t-1}/w_n{}^t) - 1$ for t = 1996,...,2012. The extension of the decomposition terms (34)-(37) to the case of many periods is given by definitions (A4)-(A7) below for n = 1,...,16 and t = 1996,...,2012:

$$\text{(A4)}\ \Delta X_n{}^t \equiv s_{Yn}{}^{t-1}\,\gamma_n{}^t\{1+(\tfrac{1}{2})\rho_n{}^t+(\tfrac{1}{2})\omega_n{}^t+(\tfrac{1}{2})\sigma_n{}^t+(\tfrac{1}{3})\rho_n{}^t\omega_n{}^t +(\tfrac{1}{3})\rho_n{}^t\sigma_n{}^t+(\tfrac{1}{3})\rho_n{}^t\sigma_n{}^t+(\tfrac{1}{4})\rho_n{}^t\omega_n{}^t\sigma_n{}^t\};$$

$$\text{(A5)}\ \Delta p_n{}^t \equiv s_{Yn}{}^{t-1}\,\rho_n{}^t\{1+(\tfrac{1}{2})\gamma_n{}^t+(\tfrac{1}{2})\omega_n{}^t+(\tfrac{1}{2})\sigma_n{}^t+(\tfrac{1}{3})\gamma_n{}^t\omega_n{}^t +(\tfrac{1}{3})\gamma_n{}^t\sigma_n{}^t+(\tfrac{1}{3})\omega_n{}^t\sigma_n{}^t+(\tfrac{1}{4})\gamma_n{}^t\omega_n{}^t\sigma_n{}^t\};$$

$$\text{(A6)}\ \Delta w_n{}^t \equiv s_{Yn}{}^{t-1}\,\omega_n{}^t\{1+(\tfrac{1}{2})\gamma_n{}^t+(\tfrac{1}{2})\rho_n{}^t+(\tfrac{1}{2})\sigma_n{}^t+(\tfrac{1}{3})\gamma_n{}^t\rho_n{}^t +(\tfrac{1}{3})\gamma_n{}^t\sigma_n{}^t+(\tfrac{1}{3})\rho_n{}^t\sigma_n{}^t+(\tfrac{1}{4})\gamma_n{}^t\rho_n{}^t\sigma_n{}^t\};$$

$$\text{(A7)}\ \Delta s_{Zn}{}^t \equiv s_{Yn}{}^{t-1}\,\sigma_n{}^t\{1+(\tfrac{1}{2})\gamma_n{}^t+(\tfrac{1}{2})\rho_n{}^t+(\tfrac{1}{2})\omega_n{}^t+(\tfrac{1}{3})\gamma_n{}^t\rho_n{}^t +(\tfrac{1}{3})\gamma_n{}^t\omega_n{}^t+(\tfrac{1}{3})\rho_n{}^t\omega_n{}^t+(\tfrac{1}{4})\gamma_{nt}\rho_n{}^t\omega_n{}^t\}.$$

The above contributions sum up exactly to the year t Market Sector MFP growth rate $\Gamma^t \equiv (X^t/X^{t-1}) - 1$; i.e., we have

$$\text{(A8)}\ \Gamma^t = \textstyle\sum_{n=1}^{16} \Delta X_n{}^t + \sum_{n=1}^{16} \Delta p_n{}^t + \sum_{n=1}^{16} \Delta w_n{}^t + \sum_{n=1}^{16} \Delta s_{Zn}{}^t .$$

Tables 18-21 below list the industry contribution terms to overall Market Sector TFP growth defined by (A4)-(A7) above (times 100). The final column in each Table lists the sum over industries of the individual industry contribution terms.

Table 18: Contributions of Industry MFP Growth to Market Sector MFP Growth $\Delta X_n{}^t$ and Sum of Industry Contributions $\sum_n \Delta X_n{}^t$, 1996-2012 (percentage points)

t	$\Delta X_1{}^t$	$\Delta X_2{}^t$	$\Delta X_3{}^t$	$\Delta X_4{}^t$	$\Delta X_5{}^t$	$\Delta X_6{}^t$	$\Delta X_7{}^t$	$\Delta X_8{}^t$	$\Delta X_9{}^t$	$\Delta X_{10}{}^t$	$\Delta X_{11}{}^t$	$\Delta X_{12}{}^t$	$\Delta X_{13}{}^t$	$\Delta X_{14}{}^t$	$\Delta X_{15}{}^t$	$\Delta X_{16}{}^t$	Sum
1996	0.85	0.34	0.25	0.1	0.01	0.32	0.18	0	0.38	-0.12	0.08	0.14	-0.46	-0.16	-0.04	-0.11	1.77
1997	0.31	-0.19	-0.11	0.15	0.25	0.39	0.28	0.06	0.15	-0.04	0.42	-0.14	-0.05	0.04	-0.03	0.12	1.59
1998	-0.10	-0.09	0.20	0.05	0.48	0.24	0.10	0.09	0	0.49	0.63	-0.15	-0.13	-0.06	0	-0.09	1.66
1999	0.54	-0.24	0.30	-0.09	0.36	0.03	0.03	0.20	0.02	0.17	0.59	-0.12	0.44	0.02	0.01	0.06	2.35
2000	0.16	0.29	-0.11	-0.03	-0.17	-0.01	-0.11	-0.01	0.11	-0.41	0.37	-0.10	0.03	-0.04	0.03	-0.04	-0.03
2001	0.26	0.52	0.46	-0.05	-0.99	0.10	0.15	0.02	0.04	-0.2	-0.18	-0.30	0.34	-0.05	0.05	0.2	0.35
2002	0.11	-0.07	0.49	-0.10	0.74	0.13	0.27	0.04	0.33	0.14	0.41	-0.09	0.59	0.34	-0.10	-0.14	3.08
2003	-0.82	-0.26	-0.09	-0.10	0.88	0.08	-0.14	0.08	0.24	0	-0.19	0.06	-0.11	-0.20	0.02	0.03	-0.51
2004	0.99	-0.52	0.12	-0.16	-0.12	0.01	0.25	-0.01	-0.18	0.13	0.67	-0.32	0.08	-0.05	0.06	-0.01	0.96
2005	0.16	0.01	-0.71	-0.15	-0.09	0.04	-0.02	0.05	0.10	-0.20	0.43	-0.16	-0.29	-0.07	0	-0.08	-0.97
2006	0.13	-0.70	-0.23	-0.15	0.28	0.02	-0.08	0.10	0.03	-0.01	0.08	-0.11	-0.32	0.12	-0.07	-0.06	-0.97
2007	-0.68	-0.06	0.03	-0.12	-0.15	-0.33	0.13	0.06	0.14	0	0.83	-0.40	-0.20	0.19	0.04	0	-0.49
2008	0.19	-0.78	-0.07	-0.24	0.24	0.16	0	-0.14	-0.12	0.17	0.69	-0.25	-0.21	0.24	-0.07	-0.21	-0.41
2009	0.47	-1.06	-0.48	-0.14	0	-0.08	0.10	-0.04	-0.46	-0.05	-0.21	0.14	0.30	-0.18	0.04	0.08	-1.57
2010	-0.09	-0.12	0.23	-0.06	-0.15	-0.08	0.25	-0.13	0.11	0.05	-0.21	0.09	0.17	-0.31	0.03	0.04	-0.17
2011	0.24	-1.65	-0.03	-0.20	0.18	0	-0.09	0.01	0.02	0.03	0.33	-0.25	0.36	0.02	-0.04	-0.07	-1.14
2012	0.18	-1.49	0.08	-0.19	0.48	0.33	0.18	0.14	0.18	-0.14	-0.14	0.02	0.07	-0.09	0.02	0.11	-0.24
Mean	0.17	-0.36	0.02	-0.09	0.13	0.08	0.09	0.030	0.07	.0004	0.27	-0.11	0.04	-0.01	-.003	-.009	0.31

[26] Since we have set industry input cost equal to the nominal value added output for each industry in each year, it turns out that the input cost shares $s_{Zn}{}^t$ are equal to the industry's value added output share $s_{Yn}{}^t$.

Table 19: Contributions of Industry Real Output Price Changes to Aggregate MFP Growth Δp_n^t and Sum of Industry Contributions $\sum_n \Delta p_n^t$, 1996-2012 (percentage points)

t	Δp_1^t	Δp_2^t	Δp_3^t	Δp_4^t	Δp_5^t	Δp_6^t	Δp_7^t	Δp_8^t	Δp_9^t	Δp_{10}^t	Δp_{11}^t	Δp_{12}^t	Δp_{13}^t	Δp_{14}^t	Δp_{15}^t	Δp_{16}^t	Sum
1996	-0.19	0.09	-0.21	-0.27	0.06	-0.02	0.12	0.11	-0.45	0.01	0.15	0.09	0.32	0.13	0.07	-0.02	0
1997	-0.36	-0.15	0.01	0	0.30	-0.11	0.16	-0.10	-0.03	0.01	-0.07	0.11	0.11	0.08	0.04	0.01	0
1998	-0.11	0.06	0.46	0.05	-0.55	-0.41	-0.12	0.13	0	0.08	0.19	-0.06	0.14	0.04	0.12	-0.03	0
1999	-0.16	-0.03	-0.18	0.01	-0.14	0.14	0	0.05	0.37	-0.03	-0.13	-0.17	0.09	0.07	-0.01	0.11	0
2000	-0.03	0.15	-0.08	-0.11	0.21	-0.16	-0.12	-0.03	-0.26	-0.15	0.10	0.03	0.20	0.07	0.13	0.06	0
2001	0.51	0.80	-0.69	0.08	-0.16	-0.02	-0.17	-0.05	-0.23	-0.07	0.21	0.07	-0.08	0	-0.05	-0.15	0.01
2002	0.69	-0.03	-0.35	0.11	0.16	0.21	0.01	-0.06	0.01	-0.10	-0.52	-0.03	-0.15	-0.09	-0.01	0.15	0
2003	-0.18	-0.15	0.43	0.13	-0.42	0.29	-0.11	-0.11	-0.02	-0.18	-0.17	0.14	0.23	0.12	-0.01	0.02	0
2004	-0.51	-0.35	0.51	0.09	0.41	-0.05	-0.08	0.11	0.16	-0.03	-0.43	-0.12	0	0.19	0	0.08	0
2005	-0.35	1.29	-0.23	0.05	0.05	-0.31	-0.38	-0.11	-0.06	-0.09	0	-0.01	0.12	0.05	-0.08	0.04	0.01
2006	-0.22	2.33	-0.16	-0.10	-0.43	-0.31	-0.11	-0.07	-0.22	-0.40	-0.13	-0.08	0.05	0.02	-0.06	-0.07	0.02
2007	-0.03	0.21	-0.56	-0.08	0.25	-0.08	-0.06	-0.01	0.31	-0.24	-0.34	0.32	0.41	0.11	-0.14	-0.07	0
2008	0.08	0.21	-0.20	0.04	-0.22	0.16	-0.09	0.12	-0.24	-0.15	-0.42	0.27	0.40	0.07	0.02	-0.05	0
2009	-0.56	2.51	-0.38	-0.18	-0.16	-0.49	-0.17	0.07	-0.29	-0.16	0.45	-0.32	-0.03	-0.11	-0.13	-0.03	0.02
2010	0.03	-2.96	-0.18	0.19	0.77	0.13	0.09	0.29	0.15	0.25	0.46	0.15	0.14	0.27	0.07	0.14	-0.02
2011	-0.04	3.13	-0.79	0.13	-0.52	0	-0.15	-0.08	-0.17	-0.41	-0.39	-0.06	-0.43	-0.06	-0.07	-0.06	0.02
2012	-0.21	-0.01	-0.34	0.16	-0.29	-0.07	0.09	0.03	-0.22	-0.03	0.48	0.13	0.28	0.11	0	-0.11	0
Mean	-0.10	0.42	-0.17	0.02	-0.04	-0.07	-0.06	0.02	-0.07	0.10	-0.03	0.03	0.11	0.06	-.006	.0002	.0028

Table 20: Contributions of Industry Real Reciprocal Input Price Changes to Aggregate MFP Growth Δw_n^t and Sum of Industry Contributions $\sum_n \Delta w_n^t$, 1996-2012 (percentage points)

t	Δw_1^t	Δw_2^t	Δw_3^t	Δw_4^t	Δw_5^t	Δw_6^t	Δw_7^t	Δw_8^t	Δw_9^t	Δw_{10}^t	Δw_{11}^t	Δw_{12}^t	Δw_{13}^t	Δw_{14}^t	Δw_{15}^t	Δw_{16}^t	Sum
1996	-0.59	-0.31	0.29	0.24	0.08	-0.17	-0.17	-0.05	0.20	0.20	-0.05	-0.18	0.24	0.07	0	0.18	0
1997	0.13	0.45	0.40	-0.08	-0.41	-0.16	-0.31	0.10	0	0.11	-0.18	0.09	0.03	-0.07	0.01	-0.09	0
1998	0.28	0.13	-0.35	-0.04	0.21	0.29	0.15	-0.16	0.11	-0.48	-0.65	0.27	0.09	0.07	-0.09	0.17	0
1999	-0.27	0.41	0.30	0.16	-0.02	-0.01	0.13	-0.17	-0.22	-0.01	-0.20	0.36	-0.38	-0.02	0.03	-0.10	0
2000	-0.13	-0.44	0.19	0.14	-0.05	0.17	0.23	0.04	0.15	0.56	-0.47	0.06	-0.23	-0.03	-0.16	-0.02	0
2001	-0.75	-1.30	0.29	-0.02	1.18	-0.06	0.04	0.05	0.22	0.28	0.02	0.24	-0.24	0.06	0	-0.04	-0.01
2002	-0.64	0.32	0.35	0.10	-0.66	-0.13	-0.06	0.14	-0.12	0.12	0.48	0.22	-0.21	-0.15	0.15	0.08	-0.01
2003	0.98	0.38	-0.42	-0.05	-0.51	-0.41	0.22	0.01	-0.26	0.16	0.30	-0.22	-0.15	0.06	-0.02	-0.07	0.02
2004	-0.44	0.93	-0.47	0.10	-0.21	0.11	-0.11	-0.07	0.09	-0.06	-0.13	0.47	-0.02	-0.11	-0.05	-0.04	0
2005	0.14	-1.38	0.79	0.06	-0.05	0.20	0.33	0.02	-0.12	0.24	-0.55	0.14	0.10	0	0.06	0.01	-0.02
2006	0.06	-1.73	0.25	0.21	0.06	0.23	0.12	-0.06	0.12	0.37	-0.07	0.17	0.20	-0.17	0.12	0.11	-0.03
2007	0.70	-0.20	0.46	0.19	-0.15	0.38	-0.10	-0.07	-0.49	0.22	-0.56	0.06	-0.25	-0.32	0.09	0.05	0.01
2008	-0.28	0.53	0.21	0.20	-0.06	-0.34	0.06	0.01	0.34	-0.04	-0.32	-0.04	-0.23	-0.33	0.05	0.25	0
2009	0.05	-1.65	0.66	0.27	0	0.48	-0.04	-0.07	0.64	0.14	-0.45	0.13	-0.40	0.24	0.07	-0.09	-0.02
2010	0.06	3.09	-0.07	-0.14	-0.63	-0.06	-0.35	-0.16	-0.28	-0.31	-0.28	-0.25	-0.33	0.04	-0.10	-0.18	0.05
2011	-0.23	-1.64	0.69	0.03	0.22	-0.07	0.17	0.03	0.07	0.34	-0.09	0.28	-0.02	0	0.10	0.11	-0.02
2012	0.02	1.46	0.24	0.02	-0.21	-0.28	-0.29	-0.18	0.02	0.16	-0.37	-0.16	-0.38	-0.03	-0.03	0	0
Mean	-0.05	-0.06	0.23	0.08	-0.07	0.01	.001	-0.04	0.03	0.12	-0.21	0.10	-0.13	-0.04	0.01	0.02	-.002

Table 21: Contributions of Changes in Industry Input Cost Shares to Aggregate MFP Growth Δs_{Zn}^t and Sum of Industry Contributions $\sum_n \Delta s_{Zn}^t$, 1996-2012 (percentage points)

t	Δs_{Z1}^t	Δs_{Z2}^t	Δs_{Z3}^t	Δs_{Z4}^t	Δs_{Z5}^t	Δs_{Z6}^t	Δs_{Z7}^t	Δs_{Z8}^t	Δs_{Z9}^t	Δs_{Z10}^t	Δs_{Z11}^t	Δs_{Z12}^t	Δs_{Z13}^t	Δs_{Z14}^t	Δs_{Z15}^t	Δs_{Z16}^t	Sum
1996	0.51	0.36	-0.66	-0.42	-0.18	0.12	0.11	-0.03	-0.21	0.06	0.1	0.08	0.15	0.02	0.01	-0.02	0
1997	-0.17	-0.33	-0.44	-0.18	0.18	-0.02	0.22	-0.04	-0.03	0.17	0.25	0.15	0.16	0.01	0.02	0.05	0
1998	-0.33	-0.03	0.07	0.01	-0.15	-0.29	-0.22	0.10	-0.19	0.24	0.57	-0.11	0.17	0.18	0.12	-0.12	0
1999	0.01	-0.39	-0.80	-0.11	0.15	-0.01	-0.03	0.16	0.18	0.06	0.43	-0.26	0.39	0.20	-0.01	0.04	0
2000	0.06	0.18	-0.67	-0.19	0.40	-0.11	-0.15	0.02	-0.31	-0.22	0.47	0.03	0.24	0.09	0.12	0.02	0
2001	0.60	1.19	-0.68	0.08	-1.60	-0.20	-0.11	0.01	-0.11	0.01	0.25	-0.04	0.52	0.16	-0.01	-0.06	0
2002	0.63	-0.28	-0.63	0.01	0.76	0.12	0.19	-0.22	-0.04	-0.16	-0.11	-0.07	-0.13	-0.06	-0.06	0.06	0
2003	-1.51	-0.33	0.56	0.06	0.59	0.39	-0.02	-0.12	0.17	-0.03	-0.30	0.33	0.12	0.02	0	0.05	-0.01
2004	0.29	-0.82	-0.06	-0.07	0.60	-0.05	-0.04	0.08	0.06	-0.04	0.15	-0.23	0.01	0.03	0.03	0.06	0

2005	**-0.32**	**1.41**	**-0.93**	**-0.04**	**0.16**	**-0.27**	**-0.21**	**-0.05**	**0.12**	**-0.13**	**0.59**	**-0.10**	**-0.01**	**-0.03**	**-0.05**	**-0.12**	**0.01**
2006	**-0.24**	**2.22**	**-0.68**	**-0.15**	**0.02**	**-0.30**	**-0.22**	**-0.08**	**-0.23**	**-0.36**	**0.18**	**-0.04**	**0.10**	**0.05**	**-0.08**	**-0.17**	**0.02**
2007	**-0.76**	**0.62**	**-0.86**	**-0.18**	**0.37**	**-0.20**	**0.02**	**-0.09**	**0.42**	**-0.14**	**0.61**	**0.04**	**0.27**	**0.10**	**-0.11**	**-0.12**	**0**
2008	**0.16**	**-0.01**	**-0.24**	**-0.09**	**0.04**	**0.08**	**-0.06**	**-0.03**	**-0.16**	**-0.06**	**0.03**	**0.04**	**0.32**	**0.11**	**-0.01**	**-0.11**	**0**
2009	**-0.07**	**2.72**	**-1.20**	**-0.09**	**0.15**	**-0.46**	**-0.23**	**-0.05**	**-0.43**	**-0.17**	**0.20**	**-0.18**	**0.24**	**-0.37**	**-0.05**	**0**	**0.02**
2010	**-0.10**	**-2.33**	**-0.38**	**0.21**	**0.57**	**0.20**	**0.06**	**0.16**	**0.13**	**0.22**	**0.17**	**0.15**	**0.63**	**0.17**	**0.04**	**0.08**	**-0.02**
2011	**0.09**	**2.67**	**-1.07**	**0.11**	**-0.30**	**-0.19**	**-0.23**	**-0.08**	**-0.13**	**-0.39**	**-0.20**	**-0.12**	**-0.01**	**0.05**	**-0.09**	**-0.11**	**0.02**
2012	**-0.11**	**0.45**	**-0.77**	**0.01**	**-0.19**	**0.10**	**0.06**	**0.03**	**-0.21**	**-0.18**	**0.41**	**0.15**	**0.41**	**-0.07**	**0.01**	**-0.11**	**0**
Mean	**-0.07**	**0.43**	**-0.56**	**-0.06**	**0.09**	**-0.06**	**-0.05**	**-.014**	**-0.06**	**-0.06**	**0.22**	**-0.01**	**0.21**	**0.04**	**-.006**	**-0.03**	**.0024**

Thus the final column in Table 18 lists the sum of the industry TFP contribution terms, $\sum_{n=1}^{16} \Delta X_n^t$ (times 100), the final column in Table 19 lists the sum of the real output price terms, $\sum_{n=1}^{16} \Delta p_n^t$ (times 100) the final column in Table 20 lists the sum of the real input price terms, $\sum_{n=1}^{16} \Delta w_n^t$ (times 100), and the final column in Table 21 lists the sum of the industry change in input cost share terms, $\sum_{n=1}^{16} \Delta s_{Zn}^t$ (times 100). The sample averages for these four sets of terms turned out to be: 0.31, 0.003, −0.002 and 0.002. Thus while the contribution terms Δp_n^t, Δw_n^t and Δs_{Zn}^t can be fairly large for some industries n for some years t, when we sum these contribution terms over all of the industries, it turns out that the overall effect of real input and output price changes and changes in input cost shares is close to zero in each year![27] Thus these contribution terms simply reallocate the effects of the individual industry MFP contribution terms among the industries but do not change the fact that when aggregating over industries, what counts is the first set of terms, $\sum_{n=1}^{16} \Delta X_n^t$, in the productivity growth decomposition defined by (A8) above. This result is quite different from the results obtained for our decomposition of Market Sector labour productivity growth.

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[27] Note also that the sum of the entries in the last columns of Tables 18-21 is equal to the entries in the first column of Table 17. This is just a check that our exact decomposition of Γ^t given by (A8) does in fact hold.

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