Extended Business Sector Data on Outputs and Inputs for the U.S.: 1987-2011

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Abstract

Using recent data from the Bureau of Economic Analysis (BEA), the Bureau of Labour Statistics (BLS), the Board of Governors of the Federal Reserve System (or Federal Reserve Board or FRB) and the US Department of Agriculture (USDA), the paper constructs a top down data set that covers the outputs produced and inputs used by an Extended Business Sector of the US economy for the years 1987-2011. The Extended Business Sector consists of the entire US economy less the inputs used and outputs produced by the Public Administration sector and less the US housing sector, both rented and owned. The constructed data set is suitable for measuring the Total Factor Productivity (TFP) or Multifactor Productivity (MFP) of this Extended Business Sector.

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Keywords

Total Factor Productivity, Multifactor Productivity, measurement of capital, measurement of inventory change, user costs, real interest rates.

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1. Introduction

The *Total Factor Productivity* (TFP) of a production sector is defined as the real output produced by the sector divided by the real input utilized by the sector. *TFP growth* is defined as (one plus) output growth divided by (one plus) input growth; i.e., it is usually defined as an *output quantity index* divided by an *input quantity index*.[2]

When national statistical agencies construct TFP growth estimates for an aggregate of firms or industries such as the entire private sector of the economy, they take a *bottom up* approach; i.e., they form estimates for the real output produced and real input used by the industry and then take the ratio of output to input to form an estimate of the industry's TFP. The industry TFP growth rates are then weighted appropriately and aggregated up to form national estimates of TFP growth.

It is also possible to form estimates of national TFP growth using a *top down* approach. This approach uses final demand data for the economy to form national output measures and uses economy wide labour and capital services inputs to form national input measures. Thus the top down approach provides no industrial detail.

From a purely methodological point of view, the bottom up approach is preferable. If wage rates and user costs were constant across industries, there would be little difference between the two approaches. However, wage rates for the "same' type of labour generally differ across industries and user costs for the same type of capital also differ across industries (due to differing costs of capital across industries) and the bottom up approach can adjust for these variations while the top down approach cannot. Thus the top down approach will usually be subject to some aggregation bias.[3]

However, there are two reasons why it is useful to implement a top down approach to measuring a nation's productivity performance:

- Using the top down approach, it is easy to measure the effects of changes in a country's *terms of trade*; i.e., using this approach, it is possible to measure changes in the prices of exports and imports on the real income generated by the production sector of the economy.[4] The effects of productivity growth and growth of primary inputs can be measured along with these terms of trade effects in a unified framework.

[2] This index number definition of TFP growth dates back to Jorgenson and Griliches (1967). The BLS (1983) (2012) calls TFP growth "Multifactor Productivity growth" (MFP growth). For a comprehensive Manual on how to measure Multifactor Productivity, see Schreyer (2001).
[3] Gu (2012) explains the problem in some detail. There will generally be some unit value aggregation bias in the labour and capital services estimates constructed by the top down approach; see Jorgenson (2012), Jorgenson and Schreyer (2012), Diewert (2012) and Schreyer (2012) for additional discussion on this point. For a comprehensive discussion of unit value bias, see Diewert and von der Lippe (2010).
[4] Studies that use variants of this approach include Diewert and Morrison (1986), Morrison and Diewert (1990), Kohli (1990) (2004) (2006), Fox and Kohli (1998), Diewert and Lawrence (2000) (2006), Diewert, Mizobuchi and Nomura (2005), Cho, Kim and Schreyer (2012) and Diewert and Yu (2012).

- The top down approach can generate more reliable measures of a nation's TFP than the bottom up approach if there is a considerable amount of measurement error in the sectoral data.[5] The problem is that it is difficult to construct accurate sectoral data on inputs. Statistical agencies generally do not have accurate surveys for the prices and quantities of intermediate inputs used by industries. Furthermore, the allocation of labour and capital to industries is not a trivial task. Inaccurate estimates of sectoral inputs can be largely avoided by the use of the top down approach since the intersectoral allocation of inputs does not matter using this approach.

In this paper, we will construct a suitable data base so that the top down approach can be implemented for the U.S. economy for the years 1987-2011.[6] In this paper, we will construct estimates of TFP growth (or Multifactor Productivity growth) for our concept of an *Extended Market Sector* for these years and compare our top down estimates with the bottom up estimates for the *Private Business Sector* provided by the Bureau of Labor Statistics (2012); see section 7 below. Our Extended Market Sector covers the entire economy except the Public Administration Sector (or General Government Sector) and the Housing Sector (including both rented and owned housing).[7] Note that the output concept used by the BLS for the Private Business Sector is equal to GDP less the output of household workers, nonprofit institutions, the gross housing product of Owner Occupied Housing, the rental value of nonprofit institutional real estate and the output of Government Enterprises.[8]

In section 2 below, we develop preliminary estimates for consumption, investment, deliveries of our Extended Market Sector to the General Government sector, exports and imports. In section 3, we use more detailed BEA data to disaggregate exports and imports of goods. In section 4, the construction of our labour input data is described while section 5 describes the construction of reproducible capital stocks, inventory stocks and business and farm land. Section 6 constructs user costs of capital. Section 7 constructs TFP estimates for our Extended Market Sector and compares them to the corresponding BLS Multifactor Productivity estimates for the U.S. Private Business Sector. Section 8 concludes.

Our primary source of data is the Bureau of Economic Analysis (BEA) but we also make use of Bureau of Labor Statistics (BLS) data, U.S. Department of Agriculture data and balance sheet data from the Board of Governors of the Federal Reserve.

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[5] A sufficient condition for the existence of sectoral measurement error is the existence of unreasonable balancing Internal Rates of Return for the sector.

[6] NAICS data for these years is readily available on the internet.

[7] The BLS private business sector covers rented housing but not the services of Owner Occupied Housing (OOH). We have excluded OOH as well as rented housing because accurate information on the stock of rental housing and the land occupied by rental housing is not available. Information on the stock of all residential housing and the value of residential land is available but not a breakdown into the rented and owned parts.

[8] See the BLS (2012).

2. Estimates of U.S. Final Demand Expenditures for the Extended Business Sector: Consumption, Investment, Deliveries to General Government, Exports and Imports

Our primary data source is the interactive website of the Bureau of Economic Analysis (2012). Using this source, from Table1.1.5: Gross Domestic Product, (last revised on October 26, 2012), we can find annual data on the Gross Domestic Product for the U.S., V_{GDP}^t, for the years t = 1987-2011. This same table has value data for the expenditure components of GDP: V_{PC}^t = personal consumption expenditures; V_{PI}^t = gross private domestic investment; V_{GCI}^t = government consumption expenditures and government gross investment; V_{XG}^t = exports of goods; V_{XS}^t = exports of services; V_{MG}^t = imports of goods and V_{MS}^t = imports of services. Price indexes for the above value aggregates, P_{GDP}^t, P_{PC}^t, P_{PI}^t, P_{GCI}^t, P_{XG}^t, P_{XS}^t, P_{MG}^t and P_{MS}^t, can be found in BEA Table 1.1.4: Price Indexes for Gross Domestic Product; Index numbers.[9] The corresponding quantity indexes, Q_{GDP}^t, Q_{PC}^t, Q_{PI}^t, Q_{GCI}^t, Q_{XG}^t, Q_{XS}^t, Q_{MG}^t and Q_{MS}^t, can be obtained by dividing each value series by the matching price index.

Note that investment that ends up in the government sector is in the government value aggregate V_{GCI}^t. We will remove these investment expenditures from this government value aggregate. In order to accomplish this task, we use BEA Table 3.9.5: Government Consumption Expenditures and Gross Investment. This Table has value information on government consumption expenditures, V_{GC}^t, that are produced by the government and valued at their cost of production. This Table also lists government investment expenditures, V_{GI}^t, for the years 1987-2011. Corresponding quantity series for government investment expenditures for the years 1987-1997 can be found in BEA 5.8.3A: Real Gross Government Fixed Investment by Type, Quantity Indexes. Another quantity series for government investment expenditures that covers the years 1997-2011 can be found in BEA Table 5.8.6B: Real Gross Government Fixed Investment by Type, Chained Dollars. The two quantity series were linked to form the quantity series Q_{GI}^t that covered the entire sample period, 1987-2011. The implicit price series, P_{GI}^t, was formed by dividing V_{GI}^t by Q_{GI}^t. Having constructed the private sector investment price and quantity series, P_{PI}^t and Q_{PI}^t, and the government sector investment price and quantity series, P_{GI}^t and Q_{GI}^t, we can construct an aggregate price of investment P_I^t and an aggregate quantity of investment Q_I^t series as chained Fisher aggregates of the two component investment series. The corresponding aggregate value of investment, V_I^t, is set equal to $P_I^tQ_I^t$. We can also construct indirect price and quantity series for government consumption expenditures, P_{GC}^t and Q_{GC}^t, as follows: construct chained Fisher index aggregates of P_{GCI}^t and Q_{GCI}^t and P_{GI}^t and $-Q_{GI}^t$, which we denote by P_{GC}^t and Q_{GC}^t.[10]

Thus we have the following *final expenditure decomposition* of U.S GDP in year t:

(1) $V_{GDP}^t = V_{PC}^t + V_I^t + V_{GC}^t + V_{XG}^t + Q_{XS}^t - Q_{MG}^t - Q_{MS}^t$

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[9] These price indexes coincide with their chained Fisher (1922) index counterparts, which are available for the period 1995-2011 in various BEA Tables.

[10] We note that the corresponding indirect value of government consumption series, $P_{GC}^tQ_{GC}^t$, agrees with the government value of consumption series, V_{GC}^t, which was downloaded earlier.

and we have companion price and quantity series for each value aggregate in equation (1).

The BEA provides the following alternative *aggregate production sector decomposition* of GDP for year t which we will find useful:

(2) $V_{GDP}^t = V_{BB}^t + V_{HP}^t + V_{NI}^t + V_{GG}^t$

where V_{BB}^t = gross domestic product excluding gross value added of households and institutions and of general government (and thus this is a *broad measure of business sector output*), V_{HP}^t = the value of household production, V_{NI}^t = the value of production for nonprofit institutions and V_{GG}^t = value added of general government; i.e., this aggregate is equal to the compensation of general government employees plus general government consumption of fixed capital. Information on these production sector components of GDP are available for 1987-2011 from BEA Table 1.3.5: Gross Value Added by Sector. This same Table has as an Addendum item, the Value of Gross Housing Value Added, V_H^t.[11] Chained quantity indexes that correspond to these value aggregates, Q_{BB}^t, Q_{HP}^t, Q_{NI}^t, Q_{GG}^t and Q_H^t may be found in BEA Table 1.3.6: Real Gross Value Added by Sector, Chained Dollars. The corresponding implicit price indexes, P_{BB}^t, P_{HP}^t, P_{NI}^t, P_{GG}^t and P_H^t can be calculated by dividing each value aggregate by the matching quantity index.

What is the relationship of the value of government consumption expenditures, V_{GC}^t, and the value added of the general government sector, V_{GG}^t? The value of government consumption expenditures should equal general government value added V_{GG}^t plus the value of intermediate input purchases by the general government sector,[12] V_G^t; i.e., we have:

(3) $V_{GC}^t = V_G^t + V_{GG}^t$.

We have price and quantity series for V_{GC}^t and V_{GG}^t and so it is possible to construct implicit chained Fisher price and quantity indexes for the government intermediate input value aggregate, P_G^t and Q_G^t, by forming chained Fisher indexes using the two price series, P_{GC}^t and P_{GG}^t, and the two quantity series, Q_{GC}^t and $-Q_{GG}^t$. We note that $V_G^t \equiv P_G^tQ_G^t$ also equals the value aggregate V_G^t defined implicitly in equation (3). In what follows, we will interpret V_G^t as the (net) value of deliveries of goods and services to the general government sector by the rest of the economy.

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[11] This value aggregate includes the value of housing rents and the imputed rental services of Owner Occupied Housing. The imputed services of OOH and most rents are included in V_B^t but some housing rents are included in the other sectors.

[12] Actually, V_G^t is equal to intermediate input purchases by the general government sector, less the value of goods and services sold to the rest of the economy by the general government sector. Thus V_G^t is equal to *net* general government intermediate input purchases from the rest of the economy. Conversely, V_G^t is equal to the net value of goods and services supplied to the general government sector from the rest of the economy.

We require one additional definition before we define the scope of our extended business sector aggregate. Note that the gross value of housing expenditures, V_H^t, is part of the value of personal consumption expenditures, V_{PC}^t. Our consumption aggregate, V_C^t, will net out these housing expenditures:

(4) $V_C^t \equiv V_{PC}^t - V_H^t$.

As usual, we have price and quantity series for V_{PC}^t and V_H^t and so it is possible to construct implicit chained Fisher price and quantity indexes for our consumption aggregate that excludes housing services, P_C^t and Q_C^t, by forming chained Fisher indexes using the two price series, P_{PC}^t and P_H^t, and the two quantity series, Q_{PC}^t and $-Q_H^t$. We note that $V_C^t \equiv P_C^t Q_C^t$ also equals the value aggregate V_C^t defined by equation (4).

We define the value of our *extended business sector output*, V_B^t, to be the GDP of the entire economy in year t less the gross value of housing services V_H^t less general government value added V_{GG}^t; i.e.,[13]

$$
\begin{aligned}
(5)\ V_B^t &\equiv V_{GDP}^t - V_H^t - V_{GG}^t \\
&= V_{PC}^t + V_I^t + V_{GC}^t + V_{XG}^t + Q_{XS}^t - Q_{MG}^t - Q_{MS}^t - V_H^t - V_{GG}^t && \text{using (1)} \\
&= (V_{PC}^t - V_H^t) + V_I^t + (V_{GC}^t - V_{GG}^t) + V_{XG}^t + Q_{XS}^t - Q_{MG}^t - Q_{MS}^t \\
&= V_C^t + V_I^t + V_G^t + V_{XG}^t + Q_{XS}^t - Q_{MG}^t - Q_{MS}^t && \text{using (3) and (4).}
\end{aligned}
$$

We have indicated how price and quantity indexes for each of the value aggregates on the right hand side of the last equation in (5) can be constructed. These value aggregates are the basic building blocks for our extended business sector measure of output.

The prices that are matched to quantities in the subaggregates that make up V_B^t are final demand prices. From the viewpoint of production theory, it is more appropriate to adjust these final demand prices into prices that producers face for their outputs and inputs.[14] This involves subtracting various commodity taxes from the price of consumption and adding taxes on imports of goods. Thus we now make a brief detour and describe various taxes that are available in BEA tables.

From BEA Table 3.2: Federal Government Current Receipts and Expenditures, we can read the annual year t values following types of Federal taxes: V_{TFP}^t = Federal personal current taxes; V_{TFE}^t = Federal excise taxes; V_{TFCD}^t = Federal customs duties; V_{TFCI}^t = Federal taxes on corporate income and VT_{FSI}^t = Federal contributions for government social insurance. From BEA Table 3.1: Government Current Receipts and Expenditures, we can obtain information on taxes paid by all levels of government for the following types of tax: V_{TP}^t = Total personal taxes; V_{TPI}^t = Total taxes on production and imports; V_{TCI}^t = Total taxes on corporate income and V_S^t = Value of subsidies. Finally we can obtain state and local government tax information from BEA Table 3.3: State and Local

[13] This is actually a preliminary definition; we will make some commodity tax adjustments to this definition shortly.
[14] This observation dates back to Jorgenson and Griliches (1972).

Government Current Receipts and Expenditures: V_{TSLPT}^t = State and local government property taxes.

We will define the value of property taxes in year t as $V_{TPROP}^t = V_{TSLPT}^t$ and we define taxes on the imports of goods as $V_{TMG}^t = V_{TFCD}^t$. We will assume that total commodity taxes on our consumption aggregate, V_{TC}^t, are equal to total taxes on production and imports less customs duty less property taxes; i.e.,

(6) $V_{TC}^t = V_{TPI}^t - V_{TMG}^t - V_{TPROP}^t$.

The key series on tax revenues and subsidies are listed below in Table 1.

Table 1: Key Tax Series for the US Economy; 1987-2011.

Year t	V_{TC}^t	V_{TPROP}^t	V_{TMG}^t	V_{TP}^t	V_{TCI}^t	V_{TFSI}^t	V_S^t
1987	205.6	126.4	15.5	489.1	127.1	317.4	30.3
1988	221.6	136.5	16.4	504.9	137.2	354.8	29.5
1989	231.5	149.9	17.5	566.1	141.5	378.0	27.4
1990	246.0	161.5	17.5	592.7	140.6	402.0	27.0
1991	264.2	176.1	16.8	586.6	133.6	420.6	27.5
1992	280.4	184.7	18.3	610.5	143.1	444.0	30.1
1993	296.0	187.3	19.8	646.5	165.4	465.5	36.7
1994	324.4	199.4	21.4	690.5	186.7	496.2	32.5
1995	335.5	202.6	19.8	743.9	211.0	521.9	34.8
1996	349.2	212.4	19.2	832.0	223.6	545.4	35.2
1997	368.5	223.5	19.6	926.2	237.1	579.4	33.8
1998	388.9	231.0	19.6	1026.4	239.2	617.4	36.4
1999	411.6	242.8	19.2	1107.5	248.8	654.8	45.2
2000	432.8	254.7	21.1	1232.3	254.7	698.6	45.8
2001	439.1	268.0	20.6	1234.8	193.5	723.3	58.7
2002	453.5	289.4	19.9	1050.4	181.3	739.3	41.4
2003	478.6	306.8	21.4	1000.3	231.8	762.8	49.1
2004	513.4	326.7	23.3	1047.8	292.0	807.6	46.4
2005	558.0	346.9	25.3	1208.6	395.9	852.6	60.9
2006	590.0	370.1	26.7	1352.4	454.2	904.6	51.4
2007	602.4	396.0	28.8	1488.7	420.6	945.3	54.6
2008	601.1	408.3	29.2	1435.7	281.0	973.1	52.9
2009	568.9	431.2	23.1	1144.6	245.9	949.1	59.7
2010	592.9	433.5	28.6	1194.8	349.5	969.8	57.0
2011	626.2	439.8	31.9	1398.0	351.8	905.5	61.6

A consumption tax rate can be generated as $\tau_C^t \equiv V_{TCS}^t/V_C^t$ for t = 1987,...,2011 and a new producer price series for our consumption aggregate for year t can be set equal to $(1-\tau_C^t)P_C^t$.[15] The quantity of consumption remains the same at Q_C^t and the new value of

[15] This is only an approximation to a best practice methodology which would disaggregate consumption into "tax homogeneous" categories, make adjustments for consumption taxes for each category and then aggregate up the components with the new more appropriate producer prices into an overall consumption aggregate. However, since U.S. consumption taxes are so small in general, the aggregation error involved in our approximate procedure is probably small.

consumption is equal to $(1-\tau_C{}^t)P_C{}^tQ_C{}^t$. We will abuse notation and denote the new producer consumption price by $P_C{}^t$ in Table 2 below (and the new producer value of consumption will also be denoted by $V_C{}^t$).

In a similar manner, we can adjust the price index for imported goods for trade taxes. Define an aggregate import tax rate on goods as $\tau_{MG}{}^t \equiv V_{TMG}{}^t/V_{MG}{}^t$ for t = 1987,...,2011 and a new producer price series for our imported good aggregate for year t can be set equal to $(1+\tau_{MG}{}^t)P_{MG}{}^t$.[16] The quantity of imported goods remains the same at $Q_{MG}{}^t$ and the new value of imported goods is equal to $(1+\tau_{MG}{}^t)P_{MG}{}^tQ_{MG}{}^t$. We will abuse notation and denote the new producer import price series by $P_{MG}{}^t$ in Table 2 below (and the new producer value of imported goods series will be denoted by $V_{MG}{}^t$).

Tables 2 and 3 below list the price and quantity indexes for the output components of our Extended Business Sector of the U.S. Economy.[17]

Table 2: Price Indexes for Output Components of the U.S. Extended Business Sector; 1987-2011

Year t	$P_C{}^t$	$P_I{}^t$	$P_G{}^t$	$P_{XG}{}^t$	$P_{XS}{}^t$	$P_{MG}{}^t$	$P_{MS}{}^t$	$P_H{}^t$	$P_{GC}{}^t$	$P_{GG}{}^t$
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.03914	1.02119	1.00221	1.06439	1.01997	1.04652	1.04871	1.04698	1.02913	1.03914
1989	1.08660	1.04511	1.01384	1.07825	1.04729	1.07575	1.04315	1.09302	1.06351	1.08185
1990	1.13572	1.06559	1.04711	1.06803	1.10101	1.09402	1.12125	1.14879	1.11068	1.13410
1991	1.17434	1.08201	1.04905	1.06690	1.15601	1.07249	1.16219	1.18214	1.15368	1.19213
1992	1.20916	1.07975	1.04502	1.04979	1.18558	1.06655	1.19473	1.21744	1.18849	1.24123
1993	1.23494	1.09453	1.04263	1.04386	1.20056	1.05448	1.20483	1.25238	1.21902	1.28378
1994	1.25540	1.11128	1.04834	1.05500	1.21532	1.06029	1.22841	1.28751	1.25280	1.32779
1995	1.28288	1.12674	1.07263	1.08006	1.08320	1.08294	1.26676	1.32818	1.28675	1.36529
1996	1.31084	1.12511	1.09429	1.05207	1.10394	1.05364	1.29879	1.36978	1.32078	1.40389
1997	1.33402	1.12220	1.10627	1.02311	1.11523	1.00870	1.29840	1.40475	1.34857	1.43764
1998	1.34314	1.11269	1.09534	0.99083	1.11451	0.94649	1.27294	1.45241	1.36891	1.47024
1999	1.36464	1.11428	1.11248	0.97740	1.12763	0.94480	1.31361	1.48735	1.41680	1.53050
2000	1.40124	1.12849	1.17917	0.98974	1.16158	0.99005	1.32993	1.52600	1.47800	1.58853
2001	1.42637	1.13921	1.18750	0.98361	1.16303	0.96194	1.32975	1.59130	1.52234	1.64837
2002	1.44127	1.14575	1.18135	0.97724	1.16506	0.94422	1.35952	1.65618	1.56753	1.71759
2003	1.46931	1.15857	1.21933	0.99633	1.19508	0.97173	1.44440	1.69931	1.64318	1.81045
2004	1.50887	1.19870	1.26721	1.03198	1.23459	1.01796	1.50973	1.72134	1.71531	1.89298
2005	1.55309	1.25613	1.36592	1.06432	1.29169	1.08352	1.57930	1.75564	1.81351	1.98698
2006	1.59457	1.31120	1.43770	1.09964	1.33969	1.12823	1.63926	1.80710	1.90069	2.07911
2007	1.64043	1.34414	1.49526	1.13660	1.38421	1.16747	1.70169	1.86138	1.98846	2.18004
2008	1.69599	1.36787	1.62374	1.19357	1.44125	1.30194	1.80008	1.92737	2.08909	2.26087
2009	1.69592	1.35820	1.50461	1.11249	1.40777	1.14172	1.76502	1.96451	2.06918	2.29957
2010	1.73672	1.34469	1.57333	1.16891	1.45640	1.21915	1.81181	1.94011	2.12768	2.34868
2011	1.78075	1.36761	1.67865	1.25789	1.51088	1.32658	1.86535	1.97227	2.20201	2.40101

Table 3: Quantity Indexes for the Output Components of the U.S. Extended Business Sector; 1987-2011

[16] Again, there will be some aggregation bias in our procedure unless all imported goods are taxed at the same rate (which is not the case).

[17] All price indexes have been normalized to equal unity in 1987 and the quantity indexes have also been normalized so that the product of price times quantity equals value (in billions of U.S. dollars). The last 3 columns in Tables 2 and 3 list some supplementary series of interest.

Year t	Q_C^t	Q_I^t	Q_G^t	Q_{XG}^t	Q_{XS}^t	Q_{MG}^t	Q_{MS}^t	Q_H^t	Q_{GC}^t	Q_{GG}^t
1987	2505.9	969.3	223.6	257.5	106.2	430.2	93.9	385.5	815.1	591.5
1988	2611.0	986.8	221.7	306.1	115.8	447.6	97.2	396.7	828.7	606.8
1989	2686.9	1026.3	227.6	342.6	127.8	467.0	101.8	405.7	849.0	621.3
1990	2739.9	1010.4	232.4	371.3	141.2	480.5	108.5	415.9	869.7	637.2
1991	2731.6	945.6	236.6	397.0	149.7	482.7	105.7	429.8	880.5	643.9
1992	2826.7	1007.5	237.8	426.8	157.7	528.0	102.9	442.4	883.8	646.1
1993	2935.1	1071.4	234.2	440.6	163.0	580.8	105.6	449.5	882.2	647.5
1994	3045.9	1186.2	237.5	483.5	173.3	658.6	111.2	468.0	885.1	647.6
1995	3126.6	1221.7	239.2	540.1	211.0	717.9	114.5	482.4	887.0	648.1
1996	3244.6	1319.3	242.5	587.7	225.8	784.4	120.5	490.1	890.9	649.1
1997	3367.9	1462.4	254.7	672.2	239.1	897.3	131.0	504.4	906.2	654.1
1998	3562.0	1594.1	265.4	687.2	244.9	1004.1	145.3	513.1	922.8	661.5
1999	3761.3	1731.1	289.7	713.3	259.0	1129.2	155.1	536.7	948.5	667.5
2000	3959.1	1840.1	295.0	792.4	265.9	1280.5	172.0	556.9	965.2	679.1
2001	4070.0	1741.5	324.0	743.4	254.9	1239.4	170.7	568.3	1001.4	691.3
2002	4199.9	1737.3	362.7	716.6	259.8	1285.6	173.8	563.0	1046.2	705.3
2003	4347.1	1800.1	381.0	729.5	262.9	1348.7	177.2	552.1	1069.1	713.7
2004	4485.8	1952.9	395.4	791.7	294.2	1498.2	196.9	574.4	1084.6	718.1
2005	4630.4	2041.4	396.4	851.3	308.9	1599.6	202.5	600.4	1090.6	723.0
2006	4753.8	2099.0	404.3	931.6	333.4	1694.1	216.8	625.8	1101.3	727.2
2007	4858.1	2047.2	410.1	1022.3	361.0	1738.6	219.8	645.0	1115.3	736.0
2008	4796.4	1889.6	420.6	1087.1	381.1	1670.9	227.8	674.3	1139.7	751.0
2009	4690.4	1513.9	464.2	957.0	371.3	1410.7	220.2	673.1	1189.0	766.2
2010	4779.6	1667.9	470.5	1093.8	388.6	1620.4	225.8	681.4	1199.4	771.4
2011	4914.1	1707.4	439.1	1172.2	410.2	1704.4	232.1	685.5	1171.4	767.3

In the following section, we will disaggregate the components of exports and imports of goods into additional components using various BEA Tables.

3. Disaggregated Components of U.S. Exports and Imports of Goods

We can obtain the values for 8 classes of *exports of goods* for the years 1987-2011 from BEA Table 4.2.5: Exports and Imports of Goods and Services by Type of Product. The 8 classes of export product are as follows:

V_{X1} = Value of foods, feeds and beverages;
V_{X2} = Value of industrial supplies and materials;
V_{X3} = Value of capital goods: civilian aircraft, engines and parts;
V_{X4} = Value of capital goods: computers, peripherals and parts;
V_{X5} = Value of capital goods: other;
V_{X6} = Value of capital goods: automotive vehicles, engines and parts;
V_{X7} = Value of consumer goods, except automotive;
V_{X8} = Value of other exports.

The corresponding price indexes, P_{Xn}^t, can be found in BEA Table 4.2.4: Price Indexes for Exports and Imports of Goods and Services by Type of Product; Index numbers. Export quantity indexes, Q_{Xn}^t, can be obtained by dividing each value series by the corresponding price index; i.e., $Q_{Xn}^t \equiv V_{Xn}^t/P_{Xn}^t$ for n = 1,...,8 and t = 1987,...,2011. The disaggregated price and quantity series for exports are listed below in Tables 4 and 5.[18] It can be seen that there is some degree of variability in the component price indexes.

Table 4: Price Indexes for Eight Classes of Goods Exports; 1987-2001

Year t	P_{X1}^t	P_{X2}^t	P_{X3}^t	P_{X4}^t	P_{X5}^t	P_{X6}^t	P_{X7}^t	P_{X8}^t
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.21815	1.10775	1.03053	0.93359	1.03106	1.02015	1.04007	1.07825
1989	1.25837	1.12836	1.07814	0.85309	1.03071	1.04419	1.07851	1.11213
1990	1.18019	1.13876	1.13629	0.75736	0.99019	1.07399	1.11479	1.13392
1991	1.17838	1.11104	1.23107	0.68262	0.98243	1.10392	1.15193	1.15337
1992	1.17160	1.07988	1.26989	0.59564	0.95345	1.12512	1.17665	1.16003
1993	1.18556	1.08563	1.30480	0.51906	0.93687	1.13484	1.19048	1.17866
1994	1.22596	1.15718	1.34360	0.47411	0.91653	1.14565	1.19463	1.21504
1995	1.33139	1.31058	1.38927	0.42228	0.87842	1.15993	1.21086	1.27440
1996	1.48989	1.25023	1.44466	0.35444	0.82817	1.17267	1.22687	1.27224
1997	1.37479	1.24506	1.49405	0.30491	0.79357	1.18208	1.23595	1.25903
1998	1.25205	1.17865	1.51324	0.26796	0.78231	1.18331	1.23604	1.23024
1999	1.19424	1.16128	1.54670	0.24298	0.77593	1.19060	1.23179	1.22322
2000	1.17503	1.23775	1.62201	0.23106	0.77121	1.20048	1.23686	1.24735
2001	1.18042	1.20171	1.71443	0.22367	0.76762	1.20460	1.23243	1.24330
2002	1.20980	1.18405	1.76135	0.20983	0.75974	1.21069	1.22615	1.24232
2003	1.31927	1.26597	1.82105	0.20489	0.74394	1.21929	1.23313	1.27566
2004	1.44586	1.40931	1.88514	0.20193	0.73696	1.22888	1.24478	1.33160
2005	1.42761	1.56597	1.96839	0.18632	0.74344	1.24278	1.25862	1.38635
2006	1.47964	1.71128	2.04939	0.17794	0.74996	1.25793	1.27469	1.44071
2007	1.73886	1.83952	2.14176	0.16452	0.74291	1.27245	1.30153	1.50232
2008	2.09711	2.02961	2.24483	0.15073	0.74105	1.28804	1.32926	1.60936
2009	1.90278	1.65415	2.35514	0.14050	0.73871	1.29476	1.33216	1.50392
2010	1.97884	1.89453	2.41917	0.13687	0.74315	1.30165	1.34660	1.58548
2011	2.33001	2.21971	2.50702	0.12970	0.75049	1.32359	1.36479	1.71883

Table 5: Quantity Indexes for Eight Classes of Goods Exports; 1987-2001

Year t	Q_{X1}^t	Q_{X2}^t	Q_{X3}^t	Q_{X4}^t	Q_{X5}^t	Q_{X6}^t	Q_{X7}^t	Q_{X8}^t
1987	25.2	67.4	16.4	18.8	57.5	27.6	20.3	24.3
1988	27.7	76.0	20.6	25.7	71.7	32.7	26.0	26.2
1989	28.8	84.5	25.0	28.3	83.3	33.6	33.3	26.7
1990	29.7	89.5	28.3	34.2	95.8	33.7	39.0	23.7
1991	30.3	95.6	29.7	40.0	104.5	36.1	40.5	24.7
1992	34.4	97.4	29.7	48.4	115.3	41.7	43.5	24.1
1993	34.2	94.8	25.1	56.4	128.7	45.5	45.8	23.6
1994	34.6	99.7	23.4	70.2	153.7	50.2	50.0	24.2
1995	38.2	107.6	18.8	94.0	191.9	52.9	53.0	24.7
1996	37.6	113.0	21.3	123.3	216.6	54.9	56.5	26.3
1997	37.8	122.9	27.7	162.0	258.5	62.1	62.3	28.9
1998	37.4	121.6	35.4	168.7	256.9	61.3	64.2	31.8
1999	38.5	122.6	34.2	192.2	272.7	63.2	65.7	33.8
2000	40.8	134.6	29.7	240.2	328.6	67.0	72.3	34.6
2001	41.8	129.2	30.7	212.8	288.7	62.6	71.6	33.0
2002	41.0	129.6	28.6	184.0	265.2	65.2	68.8	35.0
2003	41.7	132.9	25.6	194.7	278.2	66.1	72.9	30.8
2004	39.1	141.6	24.5	212.0	323.9	72.6	82.9	30.8
2005	41.3	145.3	28.4	244.2	345.7	79.2	91.6	34.3

[18] All of the price indexes have been normalized to equal unity in 1987 with offsetting normalizations to the quantity indexes to preserve values.

2006	**44.6**	**156.2**	**31.5**	**267.5**	**389.2**	**85.3**	**101.3**	**35.3**
2007	**48.5**	**171.9**	**34.1**	**276.6**	**423.3**	**95.3**	**112.2**	**40.8**
2008	**51.6**	**190.6**	**33.0**	**291.2**	**458.5**	**94.3**	**121.3**	**38.4**
2009	**49.3**	**177.4**	**31.8**	**268.3**	**377.7**	**63.1**	**112.2**	**36.3**
2010	**54.4**	**205.1**	**29.8**	**320.0**	**446.7**	**86.0**	**122.7**	**36.2**
2011	**54.2**	**218.2**	**32.0**	**373.2**	**485.9**	**100.6**	**128.2**	**36.4**

We can obtain values for 9 classes of *imports of goods* for the years 1987-2011 from BEA Table 4.2.5: Exports and Imports of Goods and Services by Type of Product. The 9 classes of imports are as follows:

V_{M1} = Value of foods, feeds and beverages;
V_{M2} = Value of industrial supplies and materials, except petroleum and products;
V_{M3} = Value of petroleum and products;
V_{M4} = Value of capital goods: Civilian aircraft, engines and parts;
V_{M5} = Value of capital goods: Computers, peripherals and parts;
V_{M6} = Value of capital goods: Other;
V_{M7} = Value of automotive vehicles, engines, and parts;
V_{M8} = Value of consumer goods, except automotive;
V_{M9} = Value of other imports.

The corresponding price indexes, P_{Mn}^{t*}, can be found in BEA Table 4.2.4: Price Indexes for Exports and Imports of Goods and Services by Type of Product; Index numbers. Import quantity indexes, Q_{Mn}^{t}, can be obtained by dividing each value series by the corresponding price index; i.e., $Q_{Mn}^{t} \equiv V_{Mn}^{t}/P_{Mn}^{t*}$ for n = 1,...,8 and t = 1987,...,2011. Recall that we defined an aggregate import tax rate on goods in the previous section as $t_{MG}^{t} \equiv VT_{MG}^{t}/V_{MG}^{t}$ for t = 1987,...,2011. We use that tax rate here and define new tax adjusted price series for our disaggregated imported good components P_{Mn}^{t} equal to $(1+t_{MG}^{t})P_{Mn}^{t*}$ for n = 1,...,9. The disaggregated price and quantity series for imports are listed below in Tables 6 and 7.[19]

Table 6: Price Indexes for Nine Classes of Goods Imports; 1987-2001

Year t	P_{M1}^{t}	P_{M2}^{t}	P_{M3}^{t}	P_{M4}^{t}	P_{M5}^{t}	P_{M6}^{t}	P_{M7}^{t}	P_{M8}^{t}	P_{M9}^{t}
1987	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**
1988	**1.04525**	**1.14612**	**0.83348**	**1.03366**	**0.94349**	**1.06579**	**1.05804**	**1.06863**	**1.07533**
1989	**1.02060**	**1.20489**	**0.98997**	**1.08126**	**0.87349**	**1.03919**	**1.07869**	**1.09723**	**1.10263**
1990	**1.04089**	**1.17574**	**1.19666**	**1.13377**	**0.82836**	**0.98708**	**1.08538**	**1.12961**	**1.11865**
1991	**1.08084**	**1.15809**	**0.99662**	**1.25874**	**0.73111**	**0.96282**	**1.12681**	**1.13798**	**1.13560**
1992	**1.07871**	**1.14761**	**0.95064**	**1.28962**	**0.64300**	**0.94320**	**1.14709**	**1.17215**	**1.15316**
1993	**1.07598**	**1.13836**	**0.86603**	**1.32744**	**0.57907**	**0.93800**	**1.16453**	**1.18184**	**1.16743**
1994	**1.16672**	**1.17043**	**0.80645**	**1.36479**	**0.52695**	**0.94460**	**1.20008**	**1.18828**	**1.19410**
1995	**1.21373**	**1.26677**	**0.89275**	**1.39094**	**0.48030**	**0.93388**	**1.22895**	**1.19956**	**1.23390**
1996	**1.18532**	**1.24592**	**1.06649**	**1.44865**	**0.41138**	**0.81023**	**1.23401**	**1.20062**	**1.23239**
1997	**1.19402**	**1.24252**	**1.00874**	**1.50079**	**0.35495**	**0.71206**	**1.23453**	**1.18403**	**1.22309**
1998	**1.15492**	**1.17993**	**0.66862**	**1.52475**	**0.29394**	**0.68176**	**1.23485**	**1.16654**	**1.21577**
1999	**1.11389**	**1.17037**	**0.88117**	**1.54786**	**0.25556**	**0.67067**	**1.24001**	**1.15526**	**1.20954**
2000	**1.09788**	**1.28116**	**1.47895**	**1.59570**	**0.24027**	**0.65999**	**1.24698**	**1.14546**	**1.22576**
2001	**1.06562**	**1.26860**	**1.23805**	**1.65843**	**0.21787**	**0.65183**	**1.24687**	**1.13726**	**1.22087**

[19] As usual, we normalized all price series to equal unity in 1987.

2002	**1.07737**	**1.18637**	**1.26549**	**1.69131**	**0.20046**	**0.63718**	**1.24949**	**1.12415**	**1.20751**
2003	**1.12347**	**1.28501**	**1.52535**	**1.73280**	**0.18719**	**0.63924**	**1.25620**	**1.12150**	**1.23257**
2004	**1.18092**	**1.43170**	**1.93786**	**1.78958**	**0.17491**	**0.64007**	**1.27617**	**1.12630**	**1.27062**
2005	**1.24787**	**1.57640**	**2.64936**	**1.85450**	**0.16094**	**0.65021**	**1.28886**	**1.13570**	**1.31427**
2006	**1.29401**	**1.65484**	**3.24031**	**1.92597**	**0.14934**	**0.65687**	**1.29299**	**1.13932**	**1.34478**
2007	**1.39303**	**1.74798**	**3.61594**	**2.02547**	**0.14098**	**0.66278**	**1.30628**	**1.15488**	**1.37090**
2008	**1.53717**	**2.02092**	**5.16158**	**2.14772**	**0.13289**	**0.67460**	**1.33843**	**1.18287**	**1.50219**
2009	**1.49287**	**1.64933**	**3.13235**	**2.26851**	**0.12556**	**0.66564**	**1.35006**	**1.17976**	**1.47115**
2010	**1.63040**	**1.82814**	**4.11912**	**2.32949**	**0.12317**	**0.66119**	**1.35932**	**1.17966**	**1.51235**
2011	**1.86992**	**2.01435**	**5.48569**	**2.42014**	**0.11714**	**0.67279**	**1.40042**	**1.20006**	**1.56836**

Table 7: Quantity Indexes for Nine Classes of Goods Imports; 1987-2001

Year t	Q_{M1}^t	Q_{M2}^t	Q_3^t	Q_{M4}^t	Q_{M5}^t	Q_{M6}^t	Q_{M7}^t	Q_{M8}^t	Q_{M9}^t
1987	**25.7**	**68.6**	**44.5**	**6.8**	**15.4**	**66.1**	**88.4**	**92.1**	**22.6**
1988	**24.7**	**69.3**	**49.3**	**7.9**	**20.2**	**73.9**	**86.1**	**93.5**	**23.4**
1989	**25.3**	**67.7**	**53.3**	**9.0**	**25.5**	**81.1**	**83.9**	**97.8**	**25.3**
1990	**26.2**	**68.8**	**53.8**	**9.5**	**28.6**	**86.9**	**84.0**	**96.2**	**29.4**
1991	**25.1**	**67.5**	**53.6**	**9.6**	**36.7**	**89.5**	**78.4**	**97.8**	**30.0**
1992	**26.5**	**74.2**	**56.1**	**10.1**	**50.9**	**99.2**	**82.5**	**107.9**	**31.0**
1993	**26.8**	**80.5**	**61.4**	**8.8**	**67.8**	**114.3**	**90.6**	**116.9**	**31.6**
1994	**27.4**	**92.5**	**65.7**	**8.5**	**90.5**	**139.3**	**101.6**	**126.7**	**35.1**
1995	**28.1**	**96.9**	**64.3**	**7.9**	**120.2**	**170.5**	**103.3**	**136.6**	**35.9**
1996	**30.8**	**102.4**	**69.8**	**9.0**	**153.0**	**194.8**	**106.7**	**147.1**	**37.1**
1997	**34.0**	**110.8**	**72.7**	**11.3**	**202.2**	**239.4**	**115.5**	**168.5**	**42.7**
1998	**36.5**	**123.0**	**77.7**	**14.6**	**251.9**	**262.9**	**122.8**	**191.3**	**49.9**
1999	**39.8**	**129.0**	**78.3**	**15.7**	**324.8**	**289.2**	**147.0**	**214.6**	**58.7**
2000	**42.6**	**137.8**	**82.7**	**16.8**	**380.1**	**355.8**	**159.8**	**252.3**	**66.1**
2001	**44.5**	**133.0**	**85.2**	**19.3**	**345.7**	**301.3**	**154.9**	**256.3**	**67.4**
2002	**46.9**	**136.7**	**83.2**	**15.4**	**381.4**	**292.4**	**165.8**	**281.0**	**69.6**
2003	**50.5**	**139.0**	**88.7**	**14.1**	**415.5**	**311.4**	**170.0**	**306.1**	**66.3**
2004	**53.4**	**160.6**	**94.6**	**13.8**	**514.4**	**367.4**	**181.5**	**340.1**	**66.3**
2005	**55.4**	**171.3**	**96.5**	**14.1**	**588.3**	**408.4**	**188.5**	**367.6**	**69.7**
2006	**58.7**	**178.5**	**94.6**	**14.9**	**688.0**	**448.0**	**201.3**	**397.1**	**70.5**
2007	**59.4**	**171.6**	**97.3**	**17.2**	**757.0**	**469.1**	**199.4**	**420.0**	**70.9**
2008	**59.6**	**160.0**	**93.5**	**16.8**	**771.8**	**483.8**	**176.6**	**417.0**	**55.5**
2009	**56.3**	**121.1**	**86.7**	**13.7**	**761.1**	**379.8**	**119.6**	**371.0**	**52.1**
2010	**57.6**	**138.5**	**87.1**	**13.6**	**966.4**	**463.2**	**168.4**	**418.5**	**59.5**
2011	**58.7**	**147.6**	**85.5**	**14.8**	**1036.4**	**539.9**	**184.9**	**437.4**	**51.5**

The BEA uses chained Fisher (1922) indexes to form higher level aggregates. We will use chained Törnqvist indexes[20] to form higher level aggregates. However, the chained Törnqvist price index of the import components listed in Tables 6 and 7 above is very close to the corresponding chained Fisher index and hence to the aggregate index P_{MG}^t that was reported in Table 2 above and similarly, the chained Törnqvist price index of the export components listed in Tables 4 and 5 above is very close to the corresponding chained Fisher index and hence to the aggregate index P_{XG}^t that was also reported in Table 2 above.[21]

[20] See Diewert (1976) for a definition of these indexes and their connection with production theory.

[21] Diewert (1978) showed theoretically why superlative indexes (like the Fisher and Törnqvist) approximate each other so closely. Empirically, chained Fisher indexes generally closely approximate their chained Törnqvist counterparts when using annual macroeconomic data.

We now turn our attention to the development of measures of labour input into our Extended Business Sector.

4. Measures of Labour Input for the Extended Business Sector

The BEA makes estimates of the compensation of employees (paid workers) across all sectors of the economy so that it is possible to obtain estimates of labour compensation for our definition of the Extended Business Sector. However, the BEA does not make estimates for the value of self employment labour input or the input of unpaid family workers. The BLS does make estimates for both employee labour income and impute labour income for the self employed and it provides a quality adjusted wage rate for all types of labour. Thus the BLS measures of labour input and compensation would be ideal for our purpose except that the definition of the BLS Private Business Sector does not coincide with our definition of the Extended Business Sector.

The BLS (2012) defines the scope of its private business sector as follows:

BLS Private Business Sector = Gross Domestic Product (GDP)
− output of general government
− output of household workers − output of nonprofit institutions
− gross housing product of owner occupied dwellings
− rental value of nonprofit institutional real estate
− output of government enterprises.

The scope of our business sector is defined as follows:

Extended Business Sector = Gross Domestic Product (GDP)
− output of general government
− gross housing product of all residential dwellings, both rented and owned.

Thus the scope of our business sector is broader than the BLS scope in that we include the outputs of households, nonprofits and government enterprises but it is narrower in that we exclude all housing services, not just owner occupied housing services.[22]

Our starting point for developing estimates for the price and quantity of labour input into the Extended Business Sector is the BLS estimates for the price and quantity of labour input into their Private Business Sector. For later comparison purposes, we will also list the BLS output and capital services measures in this section.

The basic source is following Table on the BLS (2012) website: Table; Net Multifactor Productivity and Cost, 1948-2011; SIC 1948-87 linked to NAICS 1987-2011; Private Business Sector Excluding Government Enterprises; Basic Measures; Levels. From this

[22] There is good reason to exclude the services of owner occupied housing from a productivity study since imputed output in this sector will be equal to input and hence there is no possibility of productivity gains in this sector. We exclude the services of rented dwelling units because of the difficulties in obtaining accurate information on the price and quantity of land and structure inputs used in the rental sector.

Table, we can download information for 1987-2011 on: Multifactor Productivity for year t MFP^t; real value added output $Q_{YBLS}{}^t$; current dollar output $V_{YBLS}{}^t$, labour input $Q_{LBLS}{}^t$; labour compensation in current dollars $V_{LBLS}{}^t$; capital services $Q_{KBLS}{}^t$ and capital income in current dollars $V_{KBLS}{}^t$.[23] Implicit price indexes for output and the two inputs for the years 1987-2010 can be obtained by dividing the values by the corresponding quantity indexes; i.e., $P_{YBLS}{}^t \equiv V_{YBLS}{}^t/Q_{YBLS}{}^t$, $P_{LBLS}{}^t \equiv V_{LBLS}{}^t/Q_{LBLS}{}^t$ and $P_{KBLS}{}^t \equiv V_{KBLS}{}^t/Q_{KBLS}{}^t$. The resulting price series were normalized to equal one in 1987 and the units of measurement for the quantity indexes were changed so as to preserve values. The resulting BLS price and quantity series along with the BLS Multifactor Productivity series are listed in Table 8 below.

Table 8: BLS Price and Quantity Indexes for U.S. Output, Labour and Capital Services Input and Multifactor Productivity: 1987-2011

Year t	$P_{YBLS}{}^t$	$P_{LBLS}{}^t$	$P_{KBLS}{}^t$	$Q_{YBLS}{}^t$	$Q_{LBLS}{}^t$	$Q_{KBLS}{}^t$	$V_{YBLS}{}^t$	$V_{LBLS}{}^t$	$V_{KBLS}{}^t$	MFP^t
1987	1.00000	1.00000	1.00000	3595.1	2262.8	1111.8	3595.1	2262.8	1111.8	81.427
1988	1.03072	1.05387	1.00596	3750.4	2339.0	1153.8	3865.6	2465.0	1160.6	82.073
1989	1.06830	1.08184	1.07632	3889.8	2414.4	1198.8	4155.5	2612.0	1290.3	82.289
1990	1.10650	1.14336	1.08872	3949.7	2411.7	1235.3	4370.3	2757.4	1344.9	82.796
1991	1.14042	1.18158	1.04864	3917.3	2387.2	1268.5	4467.3	2820.7	1330.2	81.978
1992	1.16138	1.23841	1.08096	4073.4	2411.7	1295.3	4730.8	2986.7	1400.1	84.092
1993	1.18410	1.26337	1.11780	4209.0	2487.0	1335.6	4983.9	3142.0	1492.9	84.263
1994	1.20430	1.27999	1.16635	4417.4	2599.0	1382.6	5319.9	3326.7	1612.7	84.884
1995	1.22508	1.30605	1.17531	4545.0	2668.1	1442.0	5567.9	3484.6	1694.8	84.636
1996	1.24446	1.35197	1.21461	4753.5	2721.0	1507.7	5915.6	3678.7	1831.3	86.087
1997	1.26409	1.39284	1.22074	5001.0	2827.6	1587.3	6321.8	3938.4	1937.7	86.781
1998	1.27264	1.47226	1.16094	5252.3	2891.7	1685.9	6684.3	4257.3	1957.2	88.042
1999	1.28280	1.53361	1.16503	5547.8	2960.6	1798.2	7116.6	4540.4	2095.0	89.663
2000	1.30658	1.63566	1.12042	5800.6	2996.6	1912.1	7578.9	4901.5	2142.4	91.217
2001	1.32842	1.70552	1.10758	5855.0	2943.9	2000.3	7777.9	5021.0	2215.5	91.940
2002	1.33854	1.76674	1.11702	5970.2	2886.6	2064.3	7991.3	5099.8	2306.2	94.114
2003	1.35654	1.83698	1.17465	6162.5	2874.0	2117.8	8359.7	5279.4	2487.7	96.653
2004	1.39131	1.90377	1.29738	6412.5	2906.3	2172.6	8921.8	5532.9	2818.6	98.987
2005	1.43668	1.96156	1.39820	6633.8	2963.7	2242.6	9530.6	5813.6	3135.5	100.000
2006	1.47768	2.03217	1.43613	6839.5	3034.5	2312.1	10106.6	6166.6	3320.5	100.451
2007	1.51586	2.11251	1.47378	6981.5	3069.8	2376.1	10583.0	6485.1	3501.8	100.801
2008	1.54326	2.17342	1.42667	6889.0	3025.9	2434.1	10631.5	6576.5	3472.7	99.564
2009	1.55249	2.18900	1.35084	6558.6	2830.1	2449.1	10182.1	6195.1	3308.3	98.812
2010	1.57228	2.23542	1.46225	6817.9	2846.2	2460.5	10719.6	6362.5	3597.8	102.175
2011	0	0	0	6964.3	2901.1	2504.0	0	0	0	102.472
GAG	1.01987	1.03559	1.01666	1.02793	1.01041	1.03441	1.04865	1.04597	1.05238	1.00962

The last row in Table 8 gives (one plus) the Geometric Average Growth (GAG) rate for the variable in that column going from 1987 to 2010 for the price and value series and going from 1987 to 2011 for the quantity series. These rates of growth apply to the Private Business Sector of the BLS and can be compared later with the corresponding rates of growth for our Extended Business Sector listed in Tables 29 and 30 below.

Looking at the last 3 columns in Table 8, it can be seen that the BLS estimate for the value of output in year t, $V_{YBLS}{}^t$, is somewhat greater than the value of inputs for the

[23] The three value series do not contain estimates for 2011.

same year, $V_{LBLS}{}^t + V_{KBLS}{}^t$.[24] The BLS generally uses balancing Internal Rates of Return to make the value of inputs equal to the value of outputs for each industry. However, if the industry IRR looks unreasonable, an exogenous IRR is substituted for it.[25] Thus it may be the case that several industries had unreasonably high IRRs and so the BLS substituted lower rates in their user cost formulae, leading to lower estimates of the total value of inputs as compared to the corresponding total value of outputs.

We want to extend the BLS wage rate series to 2011 and we will use the movement in a related wage index to accomplish this task.

From BEA Table 6.2C: Compensation of Employees by Industry, we can find the compensation of employees for all domestic industries for the years 1987-2000. BEA Table 6.2D continues this compensation series for the years 2000-2011. BEA Table 6.5D: Full-Time Equivalent Employees by Industry, has an employment series for the years 2000-2011 for all domestic industries. Thus we can form an economy wide wage rate for employees by dividing total compensation of employees by the number of full time equivalent employees for the years 2000-2011. We use the rate of change in this wage rate to extend the BLS wage rate series $P_{LBLS}{}^t$ to 2011. We will relabel the resulting wage rate series as $P_L{}^t$ and we will use it as our measure of quality adjusted wage rates for the Extended Business Sector.

The remaining problem is to find estimates for the value of labour input for the household, nonprofit and government enterprise sectors. We will add estimates for these value aggregates to the BLS labour value aggregate, $V_{LBLS}{}^t$, in order to obtain an estimate of the value of labour services for the Extended Business Sector, which we will denote by $V_L{}^t$.

From BEA Tables 6.2C and 6.2D, we can obtain the compensation of employees by industry for the two periods 1987-2000 and 2000-2011. As parts of these Tables, the compensation of federal government employees and state and local employees of government enterprises is listed. We sum up these estimates and denote the resulting compensation series as $V_{LG}{}^t$ for general government employees and $V_{LGE}{}^t$ for government enterprise employees. The latter series will be our measure of labour compensation for the government enterprise sector.

From BEA Table 1.3.5: Gross Value Added by Sector, we obtained estimates of the value added generated by households and nonprofit institutions serving households, $V_{HP}{}^t$ and $V_{NI}{}^t$ respectively; recall equation (2) above. BEA Table 6.2C: Compensation of Employees by Industry, has an Addendum item: Compensation of Employees by Industry; Households and institutions for the years 1987-2000, which we label as $V_{LHI}{}^t$. Unfortunately, this addendum series was not continued in BEA Table 6.2D. However, when we formed the ratio of $V_{LHI}{}^t$ to $V_{HP}{}^t + V_{NI}{}^t$ for the years 1987-2000, we found that the ratio was quite stable at 0.37. Thus the ratio of employment compensation in the household and nonprofits industries to value added in those industries was reasonably

[24] The excess value of output is $220.4 billion in 1987 and this increases to $759.3 billion in 2011.

[25] See Harper, Nakamura and Zhang (2012) for a description of the BLS treatment of abnormal IRRs.

stable at 37%. We extended V_{LHI}^t to the years 2001-2011 by setting it equal to $0.37(V_{HP}^t + V_{NI}^t) \equiv V_{LHI}^t$.

Finally, we approximate[26] the total value of labour input used by the Extended Business Sector, V_L^t, as follows:

(7) $V_L^t \equiv V_{LBLS}^t + V_{LHI}^t + V_{LGE}^t$.

Thus the value of labour input into the Extended Business Sector (EBS) is set equal to the BLS value of labour services V_{LBLS}^t plus an estimate of the value of labour input by the household and nonprofit institution sectors V_{LHI}^t plus the value of labour input into the government enterprise sector V_{LGE}^t. We approximate the EBS wage rate by the BLS Private Sector wage rate P_L^t. The quantity of labour input into the EBS is set equal to $Q_L^t \equiv V_L^t/P_L^t$.

The price, quantity and value series for EBS labour input can be found in Table 9 below along with the value of labour input for the household and nonprofit institution, the government enterprise and the general government sectors.

Table 9: Extended Market Sector Wage Rate, Quantity and Value of Labour Input, Value of Household and Nonprofit Institutions, Government Enterprise and General Government Labour Input

Year t	P_L^t	Q_L^t	V_L^t	V_{LHI}^t	V_{LGE}^t	V_{LG}^t
1987	1.00000	2492.8	2492.8	171.0	58.9	558.4
1988	1.05387	2580.9	2719.9	190.8	64.0	596.3
1989	1.08184	2670.0	2888.5	209.2	67.3	634.4
1990	1.14336	2677.8	3061.7	230.8	73.5	683.2
1991	1.18158	2663.6	3147.2	249.3	77.2	724.7
1992	1.23841	2696.9	3339.8	271.3	81.8	759.0
1993	1.26337	2779.2	3511.2	286.1	83.0	783.7
1994	1.27999	2901.7	3714.1	300.7	86.8	810.9
1995	1.30605	2981.2	3893.6	320.0	89.0	832.5
1996	1.35197	3036.7	4105.5	335.1	91.7	858.4
1997	1.39284	3149.4	4386.6	353.4	94.8	887.0
1998	1.47226	3216.1	4734.9	379.6	98.0	918.7
1999	1.53361	3289.5	5044.8	402.8	101.6	964.7
2000	1.63566	3325.2	5439.0	429.2	108.3	1020.3
2001	1.70552	3277.2	5589.3	456.2	112.2	1079.3
2002	1.76674	3224.3	5696.4	480.3	116.3	1150.4
2003	1.83698	3208.7	5894.4	498.5	116.5	1225.9
2004	1.90377	3248.8	6184.9	526.8	125.2	1291.1
2005	1.96156	3313.2	6499.0	557.4	128.1	1355.8
2006	2.03217	3391.8	6892.7	593.1	133.0	1420.3
2007	2.11251	3430.3	7246.5	623.8	137.7	1499.0
2008	2.17342	3398.2	7385.7	668.1	141.1	1579.5
2009	2.18900	3205.9	7017.6	682.6	139.9	1631.4
2010	2.23542	3215.3	7187.4	684.9	140.0	1673.5
2011	2.29767	3267.4	7507.4	700.1	141.7	1693.1

[26] A better approximation would subtract off the labour input associated with the rentals of dwelling units.

We turn now to the development of capital stock data for the Extended Business Sector.

5. Reproducible Capital Stock and Land Estimates for the Extended Business Sector

Our source of information on the stocks of reproducible assets held by the EBS is the BEA, which has done an excellent job in providing information on reproducible capital stock components for the U.S. economy.

Our main source of information for the values of net capital stocks held by the Private Sector at the end of each year (or the beginning of the following year) is BEA Table 2.1: Current Cost Net Stock of Private Fixed Assets, Equipment and Software and Structures by Type in Billions of dollars. Using this Table, we obtained beginning of the year estimates for the for the Private Sector of the U.S. economy for the following capital stock components for t = 1988,...,2012:

V_{K1}^t = Capital stock value of computers and peripheral equipment;
V_{K2}^t = Capital stock value of software;
V_{K3}^t = Capital stock value of communication equipment;
V_{K4}^t = Capital stock value of medical equipment and instruments;
V_{K5}^t = Capital stock value of nonmedical instruments;
V_{K6}^t = Capital stock value of photocopy and related equipment;
V_{K7}^t = Capital stock value of office and accounting equipment;
V_{K8}^t = Capital stock value of industrial equipment;
V_{K9}^t = Capital stock value of light trucks (including utility vehicles);
V_{K10}^t = Capital stock value of other trucks, buses, and truck trailers;
V_{K11}^t = Capital stock value of autos;
V_{K12}^t = Capital stock value of aircraft;
V_{K13}^t = Capital stock value of ships and boats;
V_{K14}^t = Capital stock value of railroad equipment;
V_{K15}^t = Capital stock value of other equipment;
V_{K16}^t = Capital stock value of office buildings;
V_{K17}^t = Capital stock value of health care buildings including hospitals;
V_{K18}^t = Capital stock value of multi-merchandise shopping buildings;
V_{K19}^t = Capital stock value of food and beverage establishments;
V_{K20}^t = Capital stock value of warehouses;
V_{K21}^t = Capital stock value of other commercial buildings;
V_{K22}^t = Capital stock value of manufacturing structures;
V_{K23}^t = Capital stock value of power and communication structures;
V_{K24}^t = Capital stock value of mining exploration, shafts and wells;
V_{K25}^t = Capital stock value of other structures;
V_{KRS}^t = Capital stock value of residential structures and
V_{KNS}^t = Capital stock value of nonresidential structures (equals sum of V_{K16}^t to V_{K25}^t).

We will temporarily define $V_{K26}^t \equiv V_{KRS}^t$ (the value of Private Sector Residential Structures) and $V_{K27}^t \equiv V_{KNS}^t$ (the value of Private Sector Nonresidential Structures). The starting capital stocks for 1987 are missing in BEA Table 2.1. We will obtain estimates for these missing stocks later. The capital stocks that are listed in Table 2.1 are for the Private Sector of the U.S. economy. Thus only the capital stocks for the Government Sector are excluded in this Table. However, the Government Sector consists of the General Government Sector (we want to exclude the capital stocks used by this sector) and the Government Enterprise Sector (we want to include the capital services used by this sector[27]).

There is additional information on various subcomponents of the above capital stock components V_{Kn}^t listed in BEA Table 2.1.[28] For some components of the capital stock, we used the most disaggregated data in Table 2.1 while for other components, we used higher level aggregates. The reason for this asymmetric treatment of asset classes is as follows. We started with relatively few capital stock components but when we compared the component capital stock price with the corresponding investment price for the same component, we sometimes found large differences in the two price series. When this occurs, this is a sign of asset heterogeneity; i.e., the prices of the item asset classes in the component did not move proportionally over time, leading to differences between the component investment price index and the component stock price index. Thus when this difference in asset prices versus investment prices occurred for an aggregated asset class in Table 2.1, we abandoned the aggregate and worked with the component series listed in Table 2.1.[29]

Quantity indexes that correspond to the above capital stock values are available in BEA Table 2.2: Chain-Type Quantity Indexes for Net Stock of Private Fixed Assets, Equipment, Software and Structures by Type; Index numbers, 2005=100; Last Revised on August 15, 2012. Table 2.2 listed the quantity indexes Q_{Kn}^t for the 27 asset classes listed above and corresponding price indexes P_{Kn}^t that covered the years 1988-2012 were calculated as $P_{Kn}^t \equiv V_{Kn}^t/Q_{Kn}^t$ for n = 1,...,27 and t = 1988,...,2012.

The value of annual investments in the above assets V_{In}^t that are delivered to the private sector for the years t = 1987,...,2011 can be found in BEA Table 2.7: Investment in Private Fixed Assets, Equipment and Software and Structures by Type. Chained Fisher quantity indexes Q_{In}^t for these investments can be found in BEA Table 2.8: Chain-Type Quantity Indexes for Investment in Private Fixed Assets, Equipment and Software and Structures by Type; Index numbers, 2005=100. The corresponding implicit investment price indexes P_{In}^t can be formed by dividing V_{In}^t by Q_{In}^t; i.e., $P_{In}^t \equiv V_{In}^t/Q_{In}^t$ for n = 1,...,27 and t = 1987, ..., 2011.

We compared the investment prices P_{In}^t to the corresponding asset prices P_{Kn}^t for the years t = 1988,...,2011 and they corresponded sufficiently well that we set the asset prices for 1987 equal to the corresponding investment prices; i.e., we set $P_{Kn}^{1987} \equiv P_{In}^{1987}$ for n =

[27] We will form estimates for the capital stocks used by the Government Enterprise Sector later.
[28] Table 2.1 lists data on some 75 components and subcomponents of the U.S. Private Sector reproducible capital stock.
[29] The asset price series for the first 26 assets listed above ended up being quite close to the corresponding investment price series for the same assets.

1,...,27. However, we still need to determine the starting stock values V_{Kn}^{1987} and quantities Q_{Kn}^{1987} for 1987.

The value of annual depreciation in the above assets V_{Dn}^{t} for the years 1988-2011 can be found in Table 2.4: Current Cost Depreciation of Private Fixed Assets, Equipment and Software and Structures by Type in Billions of dollars. Annual depreciation rates by type of asset, δ_n^t, can be obtained by dividing V_{Dn}^{t} by the corresponding beginning of the year asset value V_{Kn}^{t}; i.e., define δ_n^t as follows:

(8) $\delta_n^t \equiv V_{Dn}^t/V_{Kn}^t$; $n = 1,...,27$; $t = 1988,...,2011$.

We set the depreciation rates for 1987 equal to the corresponding depreciation rates for 1988; i.e., $\delta_n^{1987} \equiv \delta_n^{1988}$ for n = 1,...,27. These annual asset specific depreciation rates are listed in Tables 10-12 below. The arithmetic average of the annual depreciation rates is reported in the last row of each table.

Table 10: Private Sector Annual Depreciation Rates for Reproducible Assets 1-9; 1987-2011

Year t	δ_1^t	δ_2^t	δ_3^t	δ_4^t	δ_5^t	δ_6^t	δ_7^t	δ_8^t	δ_9^t
1987	0.34605	0.47339	0.11998	0.16988	0.15054	0.20501	0.35484	0.09762	0.21868
1988	0.34605	0.47339	0.11998	0.16988	0.15054	0.20501	0.35484	0.09762	0.21868
1989	0.36485	0.47903	0.12050	0.16935	0.15217	0.20551	0.35821	0.09696	0.21135
1990	0.35754	0.48750	0.11966	0.17211	0.15275	0.20238	0.35106	0.09695	0.20729
1991	0.36127	0.48588	0.11996	0.16845	0.15113	0.20189	0.35465	0.09488	0.21281
1992	0.37411	0.47078	0.11944	0.17160	0.15089	0.20251	0.36306	0.09513	0.20976
1993	0.38084	0.48942	0.11932	0.16928	0.14946	0.19859	0.37255	0.09537	0.22099
1994	0.39861	0.47648	0.11957	0.16684	0.14943	0.20069	0.37419	0.09646	0.23639
1995	0.40724	0.48585	0.12062	0.16667	0.14988	0.20032	0.37179	0.09806	0.22734
1996	0.38524	0.48537	0.12256	0.16545	0.14955	0.18760	0.36538	0.09738	0.23386
1997	0.38122	0.50714	0.12289	0.16535	0.14854	0.18951	0.36774	0.09734	0.23167
1998	0.37415	0.51167	0.12000	0.16846	0.14747	0.18899	0.35897	0.09775	0.22088
1999	0.39365	0.52930	0.12286	0.16820	0.14691	0.19355	0.34899	0.09756	0.22222
2000	0.39599	0.52194	0.12599	0.16905	0.14790	0.19103	0.35821	0.09769	0.21563
2001	0.35911	0.48764	0.12198	0.16997	0.14811	0.18400	0.36290	0.09692	0.20781
2002	0.37729	0.47491	0.11944	0.17152	0.14792	0.18202	0.38136	0.09596	0.20643
2003	0.38366	0.47612	0.11782	0.17260	0.14736	0.18939	0.41600	0.09598	0.21720
2004	0.39036	0.48230	0.11916	0.17090	0.14864	0.18699	0.40000	0.09708	0.20752
2005	0.38262	0.48549	0.12022	0.17265	0.14909	0.18786	0.38333	0.09746	0.21302
2006	0.39097	0.48681	0.11929	0.17285	0.15008	0.18692	0.38342	0.09806	0.21664
2007	0.39343	0.48166	0.11336	0.17359	0.15004	0.19601	0.34783	0.09793	0.21276
2008	0.38942	0.47999	0.11698	0.17283	0.14887	0.19417	0.35519	0.09818	0.19847
2009	0.38171	0.46719	0.11221	0.16953	0.14620	0.18910	0.34706	0.09396	0.19417
2010	0.39514	0.46624	0.11211	0.16908	0.14564	0.19388	0.33557	0.09431	0.20226
2011	0.39626	0.47177	0.10962	0.16946	0.14764	0.19718	0.34677	0.09557	0.21626
Average	0.38027	0.48549	0.11902	0.16982	0.14907	0.19441	0.36456	0.09673	0.21520

Table 11: Private Sector Annual Depreciation Rates for Reproducible Assets 10-18; 1987-2011

Year t	δ_{10}^t	δ_{11}^t	δ_{12}^t	δ_{13}^t	δ_{14}^t	δ_{15}^t	δ_{16}^t	δ_{17}^t	δ_{18}^t
1987	0.20714	0.20865	0.08108	0.06444	0.06221	0.15455	0.02608	0.02116	0.02781

1988	**0.20714**	**0.20865**	**0.08108**	**0.06444**	**0.06221**	**0.15455**	**0.02608**	**0.02116**	**0.02781**
1989	**0.21370**	**0.20389**	**0.08046**	**0.06332**	**0.06165**	**0.15438**	**0.02609**	**0.02113**	**0.02778**
1990	**0.20575**	**0.20961**	**0.08333**	**0.06263**	**0.06069**	**0.15389**	**0.02585**	**0.02107**	**0.02730**
1991	**0.20492**	**0.21635**	**0.08341**	**0.06277**	**0.06117**	**0.15034**	**0.02541**	**0.02081**	**0.02701**
1992	**0.21437**	**0.21116**	**0.08298**	**0.06318**	**0.06034**	**0.14989**	**0.02532**	**0.02077**	**0.02689**
1993	**0.21077**	**0.21560**	**0.08183**	**0.06402**	**0.06158**	**0.15094**	**0.02563**	**0.02097**	**0.02727**
1994	**0.21109**	**0.21757**	**0.08108**	**0.06263**	**0.06187**	**0.15231**	**0.02572**	**0.02080**	**0.02748**
1995	**0.21248**	**0.20836**	**0.08260**	**0.06332**	**0.06224**	**0.15296**	**0.02556**	**0.02074**	**0.02703**
1996	**0.20018**	**0.21133**	**0.08165**	**0.06346**	**0.06065**	**0.15218**	**0.02556**	**0.02059**	**0.02713**
1997	**0.19826**	**0.20516**	**0.08130**	**0.06250**	**0.05897**	**0.15252**	**0.02587**	**0.02084**	**0.02730**
1998	**0.20621**	**0.20590**	**0.08245**	**0.06410**	**0.06154**	**0.15339**	**0.02600**	**0.02076**	**0.02733**
1999	**0.20460**	**0.20987**	**0.08460**	**0.06356**	**0.06110**	**0.15333**	**0.02605**	**0.02095**	**0.02723**
2000	**0.19897**	**0.21347**	**0.08596**	**0.06458**	**0.06122**	**0.15335**	**0.02606**	**0.02085**	**0.02738**
2001	**0.19085**	**0.21053**	**0.08516**	**0.06400**	**0.05951**	**0.15283**	**0.02578**	**0.02088**	**0.02736**
2002	**0.19343**	**0.21107**	**0.08166**	**0.06471**	**0.05965**	**0.15223**	**0.02544**	**0.02051**	**0.02723**
2003	**0.19178**	**0.20669**	**0.08171**	**0.06542**	**0.06075**	**0.15265**	**0.02533**	**0.02075**	**0.02699**
2004	**0.20047**	**0.21393**	**0.08150**	**0.06372**	**0.06178**	**0.15474**	**0.02623**	**0.02139**	**0.02798**
2005	**0.20190**	**0.21899**	**0.08057**	**0.06419**	**0.06354**	**0.15522**	**0.02643**	**0.02146**	**0.02802**
2006	**0.20389**	**0.20971**	**0.08009**	**0.06441**	**0.06280**	**0.15547**	**0.02616**	**0.02129**	**0.02779**
2007	**0.19481**	**0.21137**	**0.07982**	**0.06432**	**0.06167**	**0.15454**	**0.02569**	**0.02088**	**0.02745**
2008	**0.19087**	**0.21182**	**0.08074**	**0.06442**	**0.06155**	**0.15631**	**0.02564**	**0.02073**	**0.02731**
2009	**0.18603**	**0.21078**	**0.07671**	**0.06403**	**0.05892**	**0.14949**	**0.02456**	**0.01964**	**0.02588**
2010	**0.18968**	**0.24441**	**0.07893**	**0.06447**	**0.05974**	**0.15048**	**0.02482**	**0.02019**	**0.02630**
2011	**0.19735**	**0.26839**	**0.07895**	**0.06394**	**0.06122**	**0.15374**	**0.02530**	**0.02051**	**0.02680**
Average	**0.20147**	**0.21453**	**0.08159**	**0.06386**	**0.06114**	**0.15305**	**0.02571**	**0.02083**	**0.02728**

Table 12: Private Sector Annual Depreciation Rates for Reproducible Assets 19-27; 1987-2011

Year t	$\delta_{19}{}^{t}$	$\delta_{20}{}^{t}$	$\delta_{21}{}^{t}$	$\delta_{22}{}^{t}$	$\delta_{23}{}^{t}$	$\delta_{24}{}^{t}$	$\delta_{25}{}^{t}$	$\delta_{26}{}^{t*}$	$\delta_{27}{}^{t*}$
1987	**0.02734**	**0.02336**	**0.02893**	**0.03260**	**0.02340**	**0.07921**	**0.02409**	**0.01554**	**0.02927**
1988	**0.02734**	**0.02336**	**0.02893**	**0.03260**	**0.02340**	**0.07921**	**0.02409**	**0.01554**	**0.02927**
1989	**0.02718**	**0.02376**	**0.02882**	**0.03280**	**0.02323**	**0.07487**	**0.02411**	**0.01553**	**0.02909**
1990	**0.02705**	**0.02300**	**0.02876**	**0.03282**	**0.02286**	**0.07664**	**0.02408**	**0.01544**	**0.02892**
1991	**0.02712**	**0.02258**	**0.02825**	**0.03239**	**0.02277**	**0.07350**	**0.02390**	**0.01539**	**0.02857**
1992	**0.02653**	**0.02298**	**0.02826**	**0.03230**	**0.02290**	**0.07400**	**0.02393**	**0.01554**	**0.02828**
1993	**0.02718**	**0.02297**	**0.02849**	**0.03255**	**0.02311**	**0.07181**	**0.02429**	**0.01558**	**0.02834**
1994	**0.02727**	**0.02341**	**0.02870**	**0.03265**	**0.02316**	**0.07230**	**0.02410**	**0.01553**	**0.02814**
1995	**0.02716**	**0.02364**	**0.02869**	**0.03263**	**0.02317**	**0.07537**	**0.02417**	**0.01545**	**0.02816**
1996	**0.02738**	**0.02356**	**0.02861**	**0.03269**	**0.02289**	**0.07231**	**0.02423**	**0.01553**	**0.02799**
1997	**0.02740**	**0.02374**	**0.02880**	**0.03263**	**0.02301**	**0.07805**	**0.02428**	**0.01557**	**0.02839**
1998	**0.02761**	**0.02359**	**0.02880**	**0.03292**	**0.02289**	**0.07828**	**0.02447**	**0.01559**	**0.02869**
1999	**0.02737**	**0.02360**	**0.02847**	**0.03261**	**0.02301**	**0.07220**	**0.02453**	**0.01567**	**0.02828**
2000	**0.02709**	**0.02328**	**0.02843**	**0.03259**	**0.02329**	**0.07700**	**0.02468**	**0.01565**	**0.02843**
2001	**0.02712**	**0.02322**	**0.02837**	**0.03258**	**0.02314**	**0.08413**	**0.02463**	**0.01568**	**0.02882**
2002	**0.02716**	**0.02302**	**0.02812**	**0.03209**	**0.02319**	**0.08333**	**0.02420**	**0.01538**	**0.02887**
2003	**0.02690**	**0.02289**	**0.02767**	**0.03173**	**0.02315**	**0.07681**	**0.02427**	**0.01557**	**0.02876**
2004	**0.02774**	**0.02368**	**0.02876**	**0.03306**	**0.02414**	**0.07931**	**0.02482**	**0.01594**	**0.02970**
2005	**0.02796**	**0.02338**	**0.02851**	**0.03302**	**0.02352**	**0.08777**	**0.02483**	**0.01571**	**0.03038**
2006	**0.02743**	**0.02361**	**0.02815**	**0.03279**	**0.02383**	**0.08674**	**0.02485**	**0.01542**	**0.03122**
2007	**0.02694**	**0.02316**	**0.02773**	**0.03283**	**0.02341**	**0.08047**	**0.02460**	**0.01502**	**0.03092**
2008	**0.02715**	**0.02305**	**0.02768**	**0.03306**	**0.02404**	**0.08355**	**0.02452**	**0.01491**	**0.03145**
2009	**0.02574**	**0.02190**	**0.02637**	**0.03116**	**0.02229**	**0.07051**	**0.02340**	**0.01481**	**0.02903**
2010	**0.02632**	**0.02241**	**0.02682**	**0.03197**	**0.02396**	**0.08040**	**0.02373**	**0.01516**	**0.03015**
2011	**0.02680**	**0.02271**	**0.02738**	**0.03246**	**0.02376**	**0.08293**	**0.02401**	**0.01532**	**0.03092**
Average	**0.02713**	**0.02319**	**0.02826**	**0.03254**	**0.02326**	**0.07803**	**0.02427**	**0.01546**	**0.02920**

Note that we denoted the depreciation rates for assets 26 and 27 with asterisks; i.e., δ_{26}^{t*} and δ_{27}^{t*}. This is due to the fact that we will replace these assets with other assets for our final list of assets that are inputs into our Extended Business Sector. Thus these depreciation rates for residential housing and for all nonresidential structures are listed only for general interest purposes.

The BEA uses the geometric model for depreciation. Thus the starting capital stock for year t+1 for asset n, Q_{Kn}^{t+1}, can be determined by the year t depreciation rate δ_n^t, the year t starting capital stock for asset n, Q_{Kn}^t, and the year t investment for asset n, Q_{In}^t, according to the following equation:

$$(9)\ Q_{Kn}^{t+1} = (1-\delta_n^t)Q_{Kn}^t + Q_{In}^t ; \qquad t = 1987,...,2012;\ n = 1,...,27.$$

Set t = 1987 and use equations (9) to solve for Q_{Kn}^{1987} for n = 1,...,N. Thus equations (9) along with investment and depreciation information for 1987 and starting capital stock information for 1988 can be used to determine the starting capital stocks for 1987.

The 27 asset prices P_{Kn}^t are listed in Tables 13-15 below and the corresponding beginning of the year BEA Private Sector capital stocks are listed in Tables 16-18 below.[30]

Table 13: BEA Private Sector Capital Stock Prices for Assets 1-9: 1987-2012

Year t	P_{K1}^t	P_{K2}^t	P_{K3}^t	P_{K4}^t	P_{K5}^t	P_{K6}^t	P_{K7}^t	P_{K8}^t	P_{K9}^t
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	0.92780	0.98932	1.00036	1.01866	1.02741	1.02917	1.01602	1.04638	1.02807
1989	0.86819	0.97464	0.98955	1.03718	1.06906	1.06588	1.04002	1.09228	1.08792
1990	0.81101	0.94308	0.99455	1.06084	1.10390	1.09276	1.02848	1.13902	1.12262
1991	0.74228	0.94746	0.99053	1.09947	1.13302	1.12247	1.03281	1.19779	1.18213
1992	0.66276	0.91471	0.98972	1.11372	1.15768	1.15286	1.02835	1.22134	1.27628
1993	0.56847	0.87921	0.98080	1.14274	1.19050	1.14941	1.04123	1.24058	1.29793
1994	0.48812	0.87675	0.97307	1.15483	1.20914	1.11389	1.05871	1.25936	1.34229
1995	0.42564	0.86002	0.94419	1.16564	1.22989	1.11397	1.07035	1.29236	1.33982
1996	0.34630	0.86083	0.91844	1.18318	1.25387	1.10088	1.07072	1.33511	1.33991
1997	0.26893	0.84024	0.91438	1.17476	1.27455	1.00469	1.05656	1.34940	1.36907
1998	0.20920	0.82380	0.88667	1.16150	1.28785	0.90309	1.06115	1.35405	1.35107
1999	0.15809	0.82056	0.83479	1.15810	1.29710	0.83391	1.04689	1.36351	1.35915
2000	0.13419	0.84406	0.79565	1.14954	1.30461	0.80274	1.04834	1.37097	1.34424
2001	0.11872	0.87057	0.76868	1.14885	1.31755	0.78233	1.04116	1.37938	1.30082
2002	0.09943	0.86497	0.74228	1.15648	1.32978	0.74548	1.02278	1.38392	1.28167
2003	0.08928	0.85294	0.71875	1.16971	1.34732	0.72277	1.02486	1.38916	1.28986
2004	0.08240	0.83658	0.67707	1.18224	1.36293	0.71507	1.02728	1.40381	1.46178
2005	0.07577	0.83310	0.65662	1.18275	1.37810	0.71447	1.04533	1.45226	1.41621
2006	0.06731	0.84074	0.64902	1.18181	1.40101	0.70698	1.02412	1.49403	1.37832
2007	0.06076	0.85551	0.67287	1.18467	1.43106	0.70669	1.01748	1.55567	1.35066
2008	0.05530	0.86249	0.60694	1.18656	1.46161	0.71928	1.03677	1.61539	1.34324
2009	0.05086	0.87011	0.59095	1.19726	1.49929	0.73510	1.05764	1.68169	1.29190
2010	0.04759	0.86206	0.57871	1.19990	1.51658	0.72879	1.06115	1.68984	1.35837
2011	0.04645	0.85997	0.57499	1.20550	1.52483	0.72621	1.05787	1.72600	1.37930
2012	0.04453	0.86264	0.54135	1.21188	1.56042	0.74330	1.09200	1.77336	1.41206

[30] As usual, we normalized all stock prices P_{Kn}^t to equal unity for t = 1987 (and adjusted the units for the Q_{Kn}^t as well to preserve values).

Table 14: BEA Private Sector Capital Stock Prices for Assets 10-18: 1987-2012

Year t	P_{K10}^{t}	P_{K11}^{t}	P_{K12}^{t}	P_{K13}^{t}	P_{K14}^{t}	P_{K15}^{t}	P_{K16}^{t}	P_{K17}^{t}	P_{K18}^{t}
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.01768	1.02054	1.02520	1.03719	1.00645	1.04273	1.03556	1.03661	1.03109
1989	1.02976	1.04469	1.07225	1.08528	1.06932	1.08796	1.07147	1.07277	1.06669
1990	1.06160	1.06116	1.12570	1.13134	1.12647	1.12783	1.10893	1.11037	1.10406
1991	1.08902	1.08159	1.19659	1.16264	1.15756	1.16951	1.13674	1.13784	1.13192
1992	1.11447	1.11334	1.25572	1.18837	1.16358	1.19143	1.14050	1.14167	1.13535
1993	1.20605	1.13830	1.29718	1.21774	1.16140	1.21499	1.16632	1.16757	1.16109
1994	1.24544	1.17219	1.33065	1.26558	1.19715	1.23920	1.20857	1.20998	1.20321
1995	1.28885	1.20729	1.37563	1.28199	1.23993	1.27152	1.26365	1.26492	1.25805
1996	1.33154	1.22385	1.43895	1.31087	1.31690	1.30630	1.29430	1.29532	1.28811
1997	1.31546	1.24014	1.47824	1.34931	1.31414	1.32762	1.32713	1.33041	1.32374
1998	1.31512	1.22939	1.48169	1.36902	1.29415	1.33963	1.39122	1.37677	1.37281
1999	1.35809	1.22192	1.48893	1.38419	1.29852	1.35024	1.45825	1.43468	1.44102
2000	1.37381	1.21371	1.52542	1.40927	1.30629	1.36274	1.52555	1.49586	1.48994
2001	1.38579	1.21417	1.61798	1.45666	1.31106	1.37504	1.57833	1.55918	1.54761
2002	1.38530	1.20707	1.66952	1.48460	1.30344	1.38989	1.62605	1.61492	1.60884
2003	1.40941	1.18749	1.70480	1.54155	1.31119	1.40650	1.67264	1.65294	1.66332
2004	1.41744	1.16754	1.79914	1.60556	1.35408	1.42436	1.71708	1.70507	1.70469
2005	1.47690	1.17442	1.91296	1.64880	1.49831	1.47835	1.88085	1.84770	1.86802
2006	1.51652	1.18332	1.99575	1.71080	1.62915	1.52058	2.08670	2.02212	2.03872
2007	1.57698	1.20237	2.07997	1.78933	1.72824	1.56655	2.26642	2.16037	2.19230
2008	1.62346	1.17481	2.14192	1.83275	1.77118	1.60494	2.35968	2.22803	2.28325
2009	1.68853	1.17376	2.27897	1.89648	1.80639	1.68898	2.51393	2.24152	2.43669
2010	1.72196	1.11001	2.24489	1.92396	1.77995	1.68522	2.41541	2.08144	2.32117
2011	1.76369	1.07057	2.29843	1.96891	1.79250	1.70173	2.41867	2.05843	2.33447
2012	1.80858	1.15813	2.37294	2.00466	1.85307	1.75534	2.51137	2.08278	2.42887

Table 15: BEA Private Sector Capital Stock Prices for Assets 19-25 and Aggregate Nonresidential and Residential Structures: 1987-2012

Year t	P_{K19}^{t}	P_{K20}^{t}	P_{K21}^{t}	P_{K22}^{t}	P_{K23}^{t}	P_{K24}^{t}	P_{K25}^{t}	P_{KRS}^{t}	P_{KNS}^{t}
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.04211	1.03602	1.03607	1.03688	1.04528	1.09971	1.03588	1.03486	1.04290
1989	1.07828	1.07183	1.07313	1.07295	1.10688	1.21274	1.06624	1.06859	1.08712
1990	1.11637	1.10998	1.11054	1.11066	1.15889	1.24056	1.10246	1.09940	1.12652
1991	1.14372	1.13679	1.13899	1.13834	1.17331	1.36740	1.12753	1.11785	1.15679
1992	1.14781	1.14158	1.14306	1.14203	1.18351	1.24764	1.13417	1.12367	1.15565
1993	1.17350	1.16724	1.16893	1.16795	1.20482	1.26363	1.16155	1.16304	1.18059
1994	1.21564	1.20926	1.21256	1.21015	1.24893	1.23874	1.21158	1.20627	1.22161
1995	1.27121	1.26367	1.26965	1.26530	1.29712	1.24127	1.25872	1.26046	1.27096
1996	1.30195	1.29438	1.30087	1.29556	1.34175	1.32419	1.29181	1.28616	1.30744
1997	1.33768	1.33024	1.33584	1.33202	1.36399	1.38386	1.33066	1.32250	1.34230
1998	1.38729	1.37613	1.38505	1.37760	1.39734	1.55822	1.36789	1.36156	1.39324
1999	1.45591	1.44743	1.45173	1.43998	1.40409	1.62107	1.41488	1.41163	1.44329
2000	1.50604	1.49632	1.50139	1.49088	1.43578	1.58185	1.46699	1.47396	1.48991
2001	1.56456	1.55465	1.55816	1.54426	1.48499	1.78035	1.52699	1.53311	1.55232
2002	1.62577	1.61657	1.61884	1.60209	1.51987	2.10091	1.58425	1.60369	1.61682
2003	1.68112	1.67045	1.67296	1.64541	1.55854	2.48899	1.62048	1.65977	1.67497
2004	1.72230	1.71234	1.71474	1.66742	1.61021	2.58586	1.66814	1.74804	1.72103
2005	1.88762	1.86748	1.88049	1.79916	1.79526	2.94655	1.78459	1.89687	1.88009
2006	2.05996	2.00851	2.05027	1.93360	1.92473	3.89149	1.91899	2.05318	2.06958
2007	2.20986	2.16615	2.19781	2.04012	2.05729	4.54015	2.04591	2.13292	2.23231
2008	2.30346	2.25569	2.28932	2.14324	2.14568	4.68007	2.11707	2.12310	2.32020

2009	**2.46090**	**2.40661**	**2.43987**	**2.31915**	**2.29439**	**4.84199**	**2.17960**	**2.06917**	**2.44178**
2010	**2.34532**	**2.29357**	**2.32759**	**2.19333**	**2.22180**	**4.21062**	**2.08014**	**2.02323**	**2.30974**
2011	**2.35862**	**2.30689**	**2.34266**	**2.19987**	**2.39044**	**4.47243**	**2.08094**	**2.02328**	**2.35614**
2012	**2.45436**	**2.40014**	**2.43699**	**2.27500**	**2.51365**	**4.82073**	**2.12876**	**2.02956**	**2.44873**

Note that the asset prices listed in the last two columns of Table 15 are $P_{KRS}{}^t$ and $P_{KNS}{}^t$, the stock price of residential structures and the aggregate stock price of nonresidential structures respectively. The capital services of residential structures will not be an input into our EBS and the services of the components (assets 16-25 listed above) of nonresidential structures will be inputs into our EBS so strictly speaking, it appears that tabling the price and quantity of these assets is not necessary. However, we will find that information on these assets will be useful in developing estimates for land services used by the EBS.

Table 16: BEA Beginning of the Year Private Sector Capital Stocks for Assets 1-9: 1987-2012

Year t	$Q_{K1}{}^t$	$Q_{K2}{}^t$	$Q_{K3}{}^t$	$Q_{K4}{}^t$	$Q_{K5}{}^t$	$Q_{K6}{}^t$	$Q_{K7}{}^t$	$Q_{K8}{}^t$	$Q_{K9}{}^t$
1987	70.5	49.5	221.5	52.6	42.8	41.5	23.3	689.6	45.5
1988	81.9	55.1	234.1	56.1	45.3	42.7	21.4	699.0	45.8
1989	95.0	63.6	249.9	59.8	47.3	44.3	19.3	711.0	47.0
1990	110.4	76.3	263.8	63.5	51.0	46.1	18.3	727.2	46.4
1991	119.0	89.7	277.7	68.0	54.9	47.2	16.7	737.4	44.9
1992	127.0	102.9	288.5	72.7	58.4	48.4	15.3	741.9	43.3
1993	146.9	118.3	300.8	78.1	61.8	49.5	14.7	746.3	41.8
1994	176.8	133.3	314.6	84.1	64.8	52.3	14.6	756.5	43.8
1995	21.0	147.9	335.4	88.5	67.8	55.6	14.6	772.5	51.9
1996	285.6	162.8	362.5	92.5	71.5	57.1	14.6	795.3	61.3
1997	396.0	183.3	395.1	97.3	75.0	58.8	14.7	819.2	74.7
1998	562.1	218.5	434.2	103.7	76.9	60.3	14.7	842.4	92.1
1999	797.0	259.9	481.7	112.4	78.7	63.2	14.2	867.5	107.3
2000	1114.1	313.2	541.7	121.4	80.3	63.9	12.8	890.8	122.8
2001	1437.8	367.2	629.3	131.6	82.5	63.9	11.9	920.8	133.9
2002	1647.4	400.9	693.7	142.7	84.9	59.7	11.5	939.8	138.3
2003	1810.0	422.3	723.9	156.0	87.1	54.8	12.2	950.2	138.9
2004	1964.8	445.6	752.4	169.8	88.9	51.6	15.1	959.7	140.1
2005	2142.0	475.7	785.4	183.6	91.5	48.4	17.2	965.1	147.5
2006	2337.1	505.0	816.3	200.7	94.6	45.4	18.8	976.8	160.4
2007	2656.4	532.2	860.0	217.4	98.7	42.6	20.3	997.8	176.4
2008	3009.2	562.1	914.1	237.0	102.5	43.0	17.7	1021.4	185.3
2009	3333.0	590.2	954.6	253.2	105.4	42.4	16.1	1038.5	175.4
2010	3456.7	609.8	977.2	264.7	105.0	40.3	14.0	1031.5	143.0
2011	3569.1	628.3	1013.7	277.6	105.7	39.1	11.7	1024.6	133.8
2012	3788.8	655.9	1038.1	290.7	106.6	38.7	10.2	1029.7	138.8

Table 17: BEA Beginning of the Year Private Sector Capital Stocks for Assets 10-18: 1987-2012

Year t	$Q_{K10}{}^t$	$Q_{K11}{}^t$	$Q_{K12}{}^t$	$Q_{K13}{}^t$	$Q_{K14}{}^t$	$Q_{K15}{}^t$	$Q_{K16}{}^t$	$Q_{K17}{}^t$	$Q_{K18}{}^t$
1987	81.3	93.9	84.1	44.5	66.6	404.5	403.9	228.0	162.8
1988	82.5	90.6	86.6	43.4	63.9	407.7	425.8	237.0	170.9
1989	82.3	93.4	89.3	42.2	62.2	412.6	447.2	247.0	178.9
1990	82.0	92.2	89.5	40.9	61.4	422.3	467.4	256.4	185.8
1991	78.4	92.7	92.2	39.7	60.7	429.4	484.6	266.1	192.9

1992	73.7	96.6	95.0	38.6	59.8	426.1	495.2	274.1	196.5
1993	72.4	100.2	98.0	37.2	58.7	420.4	501.8	281.9	199.0
1994	75.3	105.9	100.1	36.6	58.1	423.3	508.4	290.1	202.6
1995	79.6	116.9	98.6	35.7	58.3	430.4	514.0	297.3	205.9
1996	85.2	122.6	99.6	34.9	58.9	441.2	522.9	303.6	208.9
1997	87.4	128.1	99.8	34.4	59.4	453.4	533.1	310.2	213.0
1998	88.1	129.6	101.5	34.2	60.3	470.6	547.4	318.4	215.9
1999	92.9	131.0	106.4	34.1	61.8	493.2	566.0	326.1	219.1
2000	99.1	137.0	116.7	34.1	63.8	514.4	586.0	333.5	223.1
2001	101.0	142.4	127.0	34.3	65.4	537.7	609.6	341.0	226.7
2002	96.7	143.7	137.2	34.4	65.6	555.8	628.6	347.3	230.5
2003	93.2	143.4	141.4	34.7	65.3	571.5	637.3	355.7	233.9
2004	90.1	137.7	141.2	35.2	64.5	588.9	641.8	364.7	236.9
2005	92.6	134.5	142.1	35.9	64.1	608.8	645.8	373.2	240.7
2006	98.3	135.8	141.4	36.3	64.5	634.1	650.2	380.9	245.3
2007	105.1	140.5	140.3	36.5	65.7	661.3	657.8	390.3	252.6
2008	103.9	145.5	142.3	37.3	67.0	683.6	667.7	400.5	261.4
2009	98.4	140.7	143.6	38.7	68.6	699.1	677.0	411.2	268.0
2010	90.0	120.9	141.7	39.5	68.7	692.8	677.2	421.2	268.6
2011	85.6	119.4	140.5	39.7	68.3	691.1	671.7	428.8	266.9
2012	88.9	122.4	139.2	40.3	68.5	698.4	665.2	435.6	265.5

Table 18: BEA Beginning of the Year Private Sector Capital Stocks for Assets 19-25 and Aggregate Nonresidential and Residential Structures: 1987-2012

Year t	Q_{K19}^t	Q_{K20}^t	Q_{K21}^t	Q_{K22}^t	Q_{K23}^t	Q_{K24}^t	Q_{K25}^t	Q_{KRS}^t	Q_{KNS}^t
1987	86.8	78.8	117.4	485.8	612.0	205.3	815.7	4892.0	3192.8
1988	91.3	82.6	123.4	491.1	625.6	202.1	829.4	5043.9	3275.4
1989	95.5	86.4	129.3	497.2	633.9	200.5	840.2	5188.1	3351.7
1990	99.3	90.1	134.6	507.4	641.7	197.7	851.3	5320.3	3426.0
1991	103.2	93.5	139.9	520.8	651.3	197.0	864.5	5434.9	3506.9
1992	105.1	95.3	142.4	531.3	660.3	196.1	873.1	5527.7	3561.8
1993	106.6	97.0	144.1	539.2	668.1	192.9	879.1	5634.5	3601.5
1994	108.6	98.9	146.5	541.6	677.7	192.0	886.9	5766.6	3645.0
1995	110.1	100.4	151.0	547.5	682.1	191.3	893.9	5911.4	3684.9
1996	112.2	104.9	155.8	557.3	690.4	189.0	904.0	6053.0	3740.4
1997	114.6	110.8	161.2	568.2	697.9	188.0	919.1	6212.8	3807.0
1998	117.5	117.1	167.9	577.7	703.4	189.4	938.1	6373.5	3883.2
1999	120.5	123.0	174.2	587.7	711.8	188.8	959.6	6551.7	3966.4
2000	122.6	129.2	180.4	592.7	723.7	186.4	980.5	6741.6	4047.2
2001	124.9	135.7	187.8	598.2	739.2	186.9	1002.4	6932.5	4141.3
2002	126.8	142.4	193.3	603.0	754.5	190.2	1019.7	7121.0	4225.0
2003	128.2	146.4	198.7	597.6	770.6	188.8	1032.3	7323.8	4276.3
2004	129.8	150.4	202.8	591.4	784.8	189.7	1043.4	7548.8	4321.4
2005	130.7	153.5	207.1	585.8	793.3	194.5	1053.7	7792.2	4366.0
2006	131.0	156.0	209.7	581.9	800.1	201.7	1061.1	8027.6	4409.9
2007	131.0	159.5	211.7	580.8	809.6	212.4	1072.7	8266.0	4478.6
2008	131.1	163.5	214.6	582.9	829.7	223.8	1090.1	8432.7	4573.4
2009	131.0	167.0	216.1	589.5	852.6	236.4	1111.8	8520.5	4676.5
2010	129.6	167.3	214.6	597.5	875.5	238.1	1124.5	8562.2	4728.1
2011	128.1	166.1	212.0	597.9	891.1	242.9	1126.8	8596.5	4748.8
2012	127.0	165.1	209.5	598.3	902.9	252.4	1127.2	8627.5	4773.3

The effects of the Great Recession can be seen in the above capital stock tables with the fall in the stocks of light trucks (asset 9) after 2008 being notable.[31]

We need to obtain estimates for the stock of assets used by the Government Enterprise Sector of the economy. We can obtain information on the value of investments in the Government Enterprise sector $V_{I26}{}^t$ from BEA Table 5.8.5A: Gross Government Fixed Investment by Type; Billions of dollars; Government enterprise gross fixed investment for the years 1987-1997. This Table has a further breakdown of government enterprise investment into two subcategories: Structures and Equipment and software. Structure investment makes up about 80% of the total investment. The value of government enterprise investment series $V_{I26}{}^t$ is continued for the years 1997-2011 in BEA Table 5.8.5A: Gross Government Fixed Investment by Type; Government enterprise gross fixed investment.[32] We could not find an investment deflator for government enterprise investment so we used the general nonresidential structure deflator $P_{KNR}{}^t$ (see Table 15 above for a listing of this series); i.e., we set $P_{I26}{}^t = P_{KNR}{}^t$. We then defined an implicit quantity index for government enterprise investment, $Q_{I26}{}^t \equiv V_{I26}{}^t/P_{I26}{}^t$. We note that the average geometric growth rate for $Q_{I26}{}^t$ over the sample period was about ¾ of a percentage point.

We need to determine a depreciation rate for the government enterprise capital stock. We will approximate the true rates by a constant rate of 5%; i.e., we set $\delta_{26}{}^t = 0.05$ for all t.[33] Now suppose that investments up until the start of 1987 had been growing at the rate $\gamma \equiv 0.0075$ and suppose the geometric model of depreciation is valid for this sector with an annual depreciation rate of $\delta \equiv 0.05$. Then it can be seen that an approximation to the starting government enterprise capital stock at the beginning of 1987, $Q_{K26}{}^{1987}$, will be equal to the following expression:[34]

$$(10)\ Q_{K26}{}^{1987} = [Q_{I26}{}^{1987}/(1+\gamma)][1 + g + g^2 + g^3 + ...] \qquad \text{where } g \equiv (1-\delta)/(1+\gamma)$$
$$= [Q_{I26}{}^{1987}/(1+\gamma)][1/(1-g)]$$
$$= Q_{I26}{}^{1987}/(\delta+\gamma).$$

Once the 1987 starting capital stock $Q_{K26}{}^{1987}$ for the Government Enterprise sector has been determined by (10), the beginning of the year capital stocks for subsequent years can be determined recursively by using the following standard geometric capital stock equations:

$$(11)\ Q_{K26}{}^{t+1} = (1-\delta)Q_{K26}{}^t + Q_{I26}{}^t\ ; \qquad t = 1987,...,2011.$$

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[31] The stocks of residential dwelling units and nonresidential structures ($Q_{KRS}{}^t$ and $Q_{KNR}{}^t$) essentially stopped growing after 2008.
[32] The two series are consistent for the overlap year 1997.
[33] Suppose that the government enterprise capital stock is 80% structures and 20% equipment. Assume that the structure depreciation rate is 3% and the equipment depreciation rate is 13%. Then a weighted average of these rates is equal to 0.8(0.03) + 0.2(0.13) = 0.05; i.e., the average depreciation rate is 5%. It can be seen that our methodology for constructing a capital stock for the government enterprise sector is a rather rough approximation.
[34] This methodology for obtaining starting capital stocks dates back to Ulrich Kohli.

The price and quantity series for government enterprise capital, $P_{K26}{}^{t}$ and $Q_{K26}{}^{t}$, are listed below in Table 21.[35] Note that this new asset 26 replaces our old preliminary asset 26.

We turn our attention to obtaining estimates for the starting stocks of inventories. This information for the BEA Private Sector is available from BEA Table 5.9: Changes in Net Stock of Produced Assets (Fixed Assets and Inventories) in Billions of dollars; last revised on August 2, 2012. This Table provides information on the opening balances $V_{K27}{}^{t}$ for Private inventories;[36] estimates for government inventory stocks do not seem to be available.[37]

We require deflators for the stocks of Private Inventories and these deflators are available on a quarterly basis for 1987-1999 in BEA Table 5.7.9A: Implicit Price Deflators for Private Inventories by Industry; Index numbers, 2005=100; Seasonally adjusted. For the years 1988-2000, we used the quarterly deflator for the 4th quarter of the previous year.[38] Denote the resulting deflators as $P_{K27}{}^{t}$. These quarterly implicit price deflators are as of the end of the quarter and are consistent with indexes for the corresponding inventory stocks. Price deflators for Private Inventory stocks by quarter (from the Q4 of 1996 through to Q4 of 2011) are available in BEA Table 5.7.9B: Implicit Price Deflators for Private Inventories by Industry; Index numbers, 2005=100; Seasonally adjusted. The deflators in this Table are consistent with the deflators in Table 5.7.9A for the quarters that overlap.[39] Thus we use information in Table 5.7.9B to continue the series $P_{K27}{}^{t}$ through to the beginning of 2012. Implicit quantity indexes for Private Inventory stocks, $Q_{K27}{}^{t}$, can be formed by dividing $V_{K27}{}^{t}$ by $P_{K27}{}^{t}$. The inventory stock price and quantity series, $P_{K27}{}^{t}$ and $Q_{K27}{}^{t}$, were normalized so that $P_{K27}{}^{1987} = 1$ and these normalized series are listed in Table 21 below.

We now address the problem of finding estimates for the price and quantity of agricultural land and business land.

The BLS provides estimates for the price and quantity of the two types of land stock for the years 1987-2010 from the BLS (2012) website: Table: Capital and Related Measures from the Three-Digit Database, 1987 – 2010. Using this Table (sections 4.1 and 8.1), we can download the following variables:

$Q_{KBLSA}{}^{t}$ = Quantity of farm or agricultural land in 2005 dollars at the beginning of year t;
$Q_{KBLSB}{}^{t}$ = Quantity of private non-farm business land in 2005 dollars;
$P_{KBLSA}{}^{t}$ = Price deflator for the stock of agricultural land; Index = 1 in 2005;
$P_{KBLSB}{}^{t}$ = Price deflator for private non-farm business land; Index = 1 in 2005.

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[35] As usual, the units of measurement have been changed so that $P_{K26}{}^{1987} = 1$.
[36] Note that this inventory asset replaces our previous $V_{K27}{}^{t}$ asset.
[37] We require only the inventory stocks held by government enterprises. These stocks are likely to be very small and so we will neglect them.
[38] The inventory stock deflator $P_{K27}{}^{1987}$ for 1987 was set equal to the deflator obtained from Table 5.7.9A for the first quarter of 1987.
[39] The BEA measures real inventory change by differencing real inventory stocks. This is the treatment of inventory change that was recommended by Diewert (2005b).

Value series for the stocks of agricultural land and private non-farm business land in billions of dollars can be formed as $V_{KBLSA}^t \equiv P_{KBLSA}^t Q_{KBLSA}^t$ and $V_{KBLSB}^t \equiv P_{KBLSB}^t Q_{KBLSB}^t$ for t = 1987-2010. The resulting asset price, quantity and value series are listed below in Table 19.[40]

Table 19: BLS Estimates of the Price, Quantity and Value of Agricultural and Private Non-Farm Business Land: 1987-2010

Year t	P_{KBLSA}^t	P_{KBLSB}^t	Q_{KBLSA}^t	Q_{KBLSB}^t	V_{KBLSA}^t	V_{KBLSB}^t
1987	1.00000	1.00000	498.1	1444.7	498.1	1444.7
1988	0.86535	1.02403	591.7	1479.9	512.1	1515.5
1989	0.82594	1.06285	639.9	1519.1	528.6	1614.6
1990	0.86043	1.07948	639.6	1559.2	550.3	1683.1
1991	0.88670	1.10166	633.7	1586.9	561.9	1748.3
1992	0.90640	1.09982	630.3	1605.0	571.3	1765.2
1993	0.91297	1.14233	648.7	1621.3	592.2	1852.0
1994	0.91297	1.18299	673.3	164.02	614.7	1942.5
1995	0.93596	1.22921	683.7	1667.9	639.9	2050.2
1996	1.00821	1.25878	665.2	1693.2	670.7	2131.4
1997	1.07061	1.29945	654.3	1721.8	700.5	2237.4
1998	1.12644	1.36229	650.4	1752.4	732.6	2387.2
1999	1.18227	1.40665	648.6	1783.3	766.8	2508.5
2000	1.24959	1.46580	650.7	1816.7	813.1	2662.9
2001	1.32677	1.51941	652.1	1851.8	865.2	2813.7
2002	1.41544	1.56007	642.0	1878.9	908.7	2931.3
2003	1.50082	1.60444	639.1	1898.2	959.2	3045.6
2004	1.57471	1.70610	698.8	1917.4	1100.3	3271.3
2005	1.64204	1.84843	793.9	1938.2	1303.6	3582.7
2006	1.87685	1.99630	748.0	1962.5	1403.9	3917.7
2007	2.24959	2.10351	650.9	1992.1	1464.3	4190.4
2008	2.53695	2.18299	590.2	2027.8	1497.4	4426.6
2009	2.79967	2.18115	530.8	2053.9	1486.1	4479.9
2010	2.87685	2.12939	539.0	2064.3	1550.7	4395.7

Some problems emerge when we study the entries in Table 19:

- The quantity of agricultural land Q_{KBLSA}^t fluctuates too much to be believable for some years and
- The price of non-farm business land does not drop enough after the downturn in 2008; i.e., the price series P_{KBLSB}^t seems to be too smooth.[41]

Thus we will attempt to find alternative price and quantity land measures. We first address the problem of finding alternative series for agricultural land. We look at US Department of Agriculture (USDA) published series on farm land prices and quantities in the US. From the Productivity Table on the USDA (2012b) website (see Table 1. Indices

[40] As usual, we normalize the prices to equal unity in 1987.
[41] The BLS (2012) describes its land series as follows: "Land in the farm sector is based on published USDA data. Nonfarm land is derived from land-structure ratios estimated from 2001 Ohio data." In other words: the quality of the BLS non-farm land data is very poor. And as we shall see later, using an alternative USDA land series gives rise to a much smoother quantity series for agricultural land.

of farm output, input, and total factor productivity for the United States, 1948-2009; Indices are relative to 2005 = 1), we can obtain quantity indexes $Q_{K28}{}^t$ for the input of land into the US farm sector for the years t = 1987,...,2009. We extend this series for 2010-2012 by assuming that the quantity of land for those years equaled the 2009 quantity. We note that the USDA farm land series varied in a very smooth fashion as one would expect. Using USDA (2004), we can obtain the following series on the price of agricultural land for the years 1987-200: Farm Real Estate: Average Value per Acre; 48 States; US Average Farm Real Estate Value; Dollars per Acre; 1982-2003. The same series is continued to 2012 in USDA (2009) and USDA (2012a). We normalize this series to equal 1 in 1987 and denote the resulting series as $P_{K28}{}^t$. The price and quantity series for the stock of agricultural land, $P_{K28}{}^t$ and $Q_{K28}{}^t$, are listed in Table 21 below.

Finally, we need price and quantity series, $P_{K29}{}^t$ and $Q_{K29}{}^t$, for business sector land (excluding all farm, government and housing land). Our main source for these estimates are the national balance sheets estimated by the Board of Governors of the Federal Reserve System (2012). Using this source, from the Flow of Funds accounts, page 104, Table title: B.100 Balance Sheet of Households and Nonprofit Organizations; Billions of dollars, we can find a series $V_{NPRE}{}^t$ for the market value of Real Estate held by Nonprofit Organizations at the beginning of year t for the years 1987-2012. The same Table also contains two additional series as memo items: the replacement cost value of residential structures for Nonprofit Organizations, $V_{NPRS}{}^t$, and the replacement cost value of nonresidential structures for Nonprofit Organizations, $V_{NPNS}{}^t$. We define the value of Nonprofit Organizations Land, $V_{NPL}{}^{t*}$, as the value of Nonprofit real estate less the value of residential and nonresidential structures; i.e.,

(12) $V_{NPL}{}^{t*} \equiv V_{NPRE}{}^t - V_{NPRS}{}^t - V_{NPNS}{}^t$.

The nonprofits estimated land series $V_{NPL}{}^{t*}$ is listed in Table 20 below. It can be seen that this series is not entirely credible; the series has a big valley between the years 1992 and 1999. We will interpolate a constant rate of growth for the series between these two years to eliminate this valley. There is a peak in 2008 that seems to be too high so we linearly interpolate over this year. Finally, there is a very deep valley in the years 2010 and 2011 so we linearly interpolate over this valley. Denote the resulting series for the value of Nonprofit Organization land as $V_{NPL}{}^t$; this series is also listed in Table 20.

We now attempt to estimate the value of corporate (nonresidential) land. Our source is again the Board of Governors of the Federal Reserve System (2012). Using this source, from the Flow of Funds accounts, page 105, Table title: B.102 Balance Sheet of Nonfinancial Corporate Business; Billions of dollars, we can find a series $V_{CRE}{}^t$ for the market value of Real Estate held by the Nonfinancial Corporate sector at the beginning of year t for the years 1987-2012. The same Table also contains two additional series as memo items: the replacement cost value of residential structures for Nonfinancial Corporations, $V_{CRS}{}^t$ (this is very small) and the replacement cost value of nonresidential structures for Nonfinancial Corporations, $V_{CNS}{}^t$. We define the value of Corporate Land, $V_{CL}{}^{t*}$, as the value of Corporate real estate less the value of residential and nonresidential structures; i.e.,

(13) $V_{CL}{}^{t*} \equiv V_{CRE}{}^{t} - V_{CRS}{}^{t} - V_{CNS}{}^{t}$.

The Corporate estimated land series $V_{CL}{}^{t*}$ is listed in Table 20 below. Again, the peaks and valleys in this series are too extreme. We will interpolate a constant rate of growth for the series between 1992 and 1999 to eliminate the valley between these two years. There is a peak in 2008 and we linearly interpolate over this year. Finally, there is a very deep valley in the years 2010 and 2011 so we linearly interpolate over this valley. Denote the resulting series for the value of Nonfinancial Corporations land as $V_{CL}{}^{t}$; this series is also listed in Table 20.

Finally, we estimate the value of corporate (nonresidential) land. Our source is again the Board of Governors of the Federal Reserve System (2012). Using this source, from the Flow of Funds accounts, page 106, Table title: B.103 Balance Sheet of Nonfinancial Noncorporate Business; Billions of dollars, we can find a series $V_{NCNRE}{}^{t}$ for the market value of nonresidential Real Estate held by the Nonfinancial Noncorporate sector at the beginning of year t for the years 1987-2012. The same Table also contains the replacement cost value of nonresidential structures for the Nonfinancial Noncorporate sector, $V_{NCNS}{}^{t}$. We define the value of Nonfinancial Noncorporate Land, $V_{NCL}{}^{t*}$, as the value of Nonfinancial Noncorporate nonresidential real estate less the value of nonresidential structures; i.e.,

(14) $V_{NCL}{}^{t} \equiv V_{NCNRE}{}^{t} - V_{NCNS}{}^{t}$.

The Nonfinancial Noncorporate estimated land series $V_{NCL}{}^{t}$ is listed in Table 20 below.

The above Federal Reserve Board Balance Sheets also contain similar information on the total value of residential real estate and also separate estimates of the market value of residential structures. We use this information to construct a series for the value of residential land which we take to be the value of residential real estate less the market value of residential structures. The resulting residential land series is denoted as $V_{RL}{}^{t}$ and is listed in Table 20.[42]

Our estimate for the total value of Extended Business Sector Land (including agricultural land but excluding residential land and government land) is $V_{BAL}{}^{t}$ defined as the sum of nonprofits land, corporate land and noncorporate land; i.e.,

(15) $V_{BAL}{}^{t} \equiv V_{NPL}{}^{t} + V_{CL}{}^{t} + V_{NCL}{}^{t}$.

The series $V_{BAL}{}^{t}$ is also listed in Table 20 along with the BLS estimates for the value of agricultural land, $V_{KBLSA}{}^{t}$, and the BLS estimates for the value of nonagricultural private land, $V_{KBLSB}{}^{t}$, for comparison purposes.

[42] This series does not look to be particularly accurate but we use it only to allocate property taxes across structures and land values.

Table 20: Sectoral Land Estimates Based on Federal Reserve Balance Sheets and the BLS Estimates for the Value of Agricultural Land and Private Business Sector Nonfarm Land

Year t	V_{NPL}^{t*}	V_{NPL}^{t}	V_{CL}^{t*}	V_{CL}^{t}	V_{NCL}^{t}	V_{BAL}^{t}	V_{KBLSA}^{t}	V_{KBLSB}^{t}	V_{RL}^{t}
1987	201.8	201.8	817.4	817.4	674.2	1693.4	498.1	1444.7	2890.3
1988	207.7	207.7	848.9	848.9	706.3	1762.9	512.1	1515.5	3151.3
1989	226.8	226.8	891.1	891.1	747.5	1865.4	528.6	1614.6	3499.8
1990	242.2	242.2	956.7	956.7	792.2	1991.1	550.3	1683.1	3853.7
1991	220.6	220.6	983.7	983.7	848.2	2052.5	561.9	1748.3	3807.2
1992	163.3	163.3	848.5	848.5	777.1	1788.9	571.3	1765.2	3792.1
1993	89.8	175.2	586.2	862.5	698.8	1736.5	592.2	1852.0	3695.2
1994	62.1	188.0	242.3	876.7	694.7	1759.4	614.7	1942.5	3612.7
1995	68.4	201.7	101.2	891.1	730.4	1823.2	639.9	2050.2	3545.9
1996	72.8	216.4	111.0	905.8	760.0	1882.2	670.7	2131.4	2854.6
1997	55.5	232.1	14.8	920.7	756.4	1909.3	700.5	2237.4	2890.3
1998	209.1	249.0	679.0	935.9	987.2	2172.1	732.6	2387.2	3090.6
1999	267.2	267.2	951.3	951.3	1088.1	2306.6	766.8	2508.5	3545.8
2000	274.7	274.7	972.2	972.2	1127.4	2374.3	813.1	2662.9	4092.3
2001	369.1	369.1	1362.7	1362.7	1288.1	3019.9	865.2	2813.7	5293.3
2002	307.4	307.4	1027.3	1027.3	1237.8	2572.5	908.7	2931.3	6142.0
2003	353.6	353.6	1117.0	1117.0	1267.2	2737.8	959.2	3045.6	7072.3
2004	391.2	391.2	1254.8	1254.8	1386.9	3032.9	1100.3	3271.3	8061.1
2005	575.5	575.5	195.0	1951.0	1743.0	4269.5	1303.6	3582.7	9736.0
2006	711.9	711.9	2319.2	2319.2	1991.9	5023.0	1403.9	3917.7	12156.4
2007	782.7	782.7	2752.4	2752.4	2230.3	5765.4	1464.3	4190.4	11702.1
2008	1043.7	735.4	4094.5	2661.2	2705.9	6102.5	1497.4	4426.6	9196.1
2009	688.1	688.1	2570.0	2570.0	2264.1	5522.2	1486.1	4479.9	5245.1
2010	84.6	599.7	-14.7	2096.4	1525.8	4221.9	1550.7	4395.7	4749.3
2011	304.9	511.2	761.4	1622.9	1840.5	3974.6	0	0	4235.7
2012	422.8	422.8	1149.3	1149.3	2068.0	3640.1	0	0	2890.3

From viewing Table 20, it can be seen that our estimates (based on Federal Reserve Board Balance Sheet information) for the value of business land including farm land, V_{BAL}^{t}, are smaller than the sum of the BLS land value series, V_{KBLSA}^{t} for farm land, and V_{KBLSB}^{t}, for other private sector business land (excluding the land services associated with Owner Occupied Housing). Our Extended Business sector also excludes land used in the rental housing sector so it is not surprising that our land value is smaller than the BLS land value.

Define the value of nonfarm Extended Business Sector land at the beginning of year t, V_{K29}^{t}, as the total value of EBS land V_{BAL}^{t} (including farm land but excluding housing and government land) less our USDA based value of farm land V_{K28}^{t}:

(16) $V_{K29}^{t} \equiv V_{BAL}^{t} - V_{K28}^{t}$.

We will assume that the quantity of EBS land excluding farm land, Q_{K29}^{t}, is constant[43] so we set $Q_{K29}^{t} = V_{K29}^{1987}$ for t = 1987-2012. The price of EBS land is defined as $P_{K29}^{t} = V_{K29}^{t}/V_{K29}^{1987}$ for t = 1987-2012. The series P_{K29}^{t}, Q_{K29}^{t} and V_{K29}^{t} are listed in Table 21.[44]

[43] It is very likely that the quantity of land used in manufacturing has declined while commercial land has increased but accurate information on the size of the declines and increases is not readily available.
[44] Of course, the corresponding value series are $V_{Kn}^{t} \equiv P_{Kn}^{t}Q_{Kn}^{t}$ for n = 1,...,29 and t = 1987,...,2012.

Table 21: The Price and Quantity of Government Enterprise Capital, Inventory Stocks, Agricultural Land and Business (Nonfarm, Non Housing) Land: 1987-2012

Year t	P_{K26}^t	P_{K27}^t	P_{K28}^t	P_{K29}^t	Q_{K26}^t	Q_{K27}^t	Q_{K28}^t	Q_{K29}^t
1987	1.00000	1.00000	1.00000	1.00000	619.1	858.0	498.1	1195.3
1988	1.0420	1.03383	1.05509	1.03933	623.8	894.0	493.4	1195.3
1989	1.08712	1.09726	1.11519	1.10028	626.6	911.1	493.4	1195.3
1990	1.12652	1.11680	1.14023	1.19959	629.1	935.1	488.7	1195.3
1991	1.15679	1.14133	1.17362	1.24192	633.6	948.0	484.0	1195.3
1992	1.15565	1.11650	1.19032	1.01463	638.6	946.9	484.0	1195.3
1993	1.18059	1.12662	1.23539	0.95252	645.7	960.9	484.0	1195.3
1994	1.22161	1.14072	1.33222	0.92721	651.6	978.3	488.7	1195.3
1995	1.27096	1.15695	1.40902	0.94922	653.3	1032.5	488.7	1195.3
1996	1.30744	1.18874	1.48080	0.96920	655.4	1057.6	488.7	1195.3
1997	1.34230	1.18734	1.54591	0.95917	658.2	1082.0	493.4	1195.3
1998	1.39324	1.16179	1.62604	1.14602	661.8	1142.5	493.4	1195.3
1999	1.44329	1.11939	1.71953	1.22669	665.4	1198.5	488.7	1195.3
2000	1.48991	1.14421	1.81970	1.24237	671.5	1252.1	488.7	1195.3
2001	1.55232	1.17299	1.91987	1.74908	677.3	1299.2	484.0	1195.3
2002	1.61682	1.14270	2.02003	1.34216	684.8	1266.6	479.3	1195.3
2003	1.67497	1.16655	2.12020	1.44862	694.6	1276.5	474.6	1195.3
2004	1.72103	1.19813	2.23706	1.64911	706.8	1290.1	474.6	1195.3
2005	1.88009	1.25303	2.68781	2.51525	716.9	1341.9	469.9	1195.3
2006	2.06958	1.30669	3.05509	3.01326	723.7	1381.0	465.2	1195.3
2007	2.23231	1.34297	3.35559	3.51741	729.4	1427.5	465.2	1195.3
2008	2.32020	1.43355	3.62270	3.70971	734.8	1449.2	460.5	1195.3
2009	2.44178	1.42476	3.52254	3.27667	742.2	1420.8	455.8	1195.3
2010	2.30974	1.44016	3.67279	2.13152	748.4	1312.1	455.8	1195.3
2011	2.35614	1.53914	3.98998	1.80366	759.1	1351.9	455.8	1195.3
2012	2.44873	1.63460	4.42404	1.35829	763.8	1376.2	455.8	1195.3

It can be seen that our estimates for the value of nonfarm, nonresidential business land,V_{K29}^t, are probably not very accurate. The jump in P_{K29}^t in 2008 seems unusually large and the drop in this series from the 2008 peak seems to be too large.[45]

Using the price and quantity data on asset prices P_{Kn}^t and the corresponding capital stocks Q_{Kn}^t for n = 1,...,29 listed in Tables 13-18 and 21, we construct an aggregate capital stock price index for year t, P_{KW}^t, as a Törnqvist price index of the P_{Kn}^t and Q_{Kn}^t. The corresponding implicit Törnqvist wealth stock quantity index is denoted as Q_{KW}^t; these series are listed in Tables 29 and 30 below. The year t value of the capital stock used by the EBS is $V_{KW}^t \equiv P_{KW}^t Q_{KW}^t$ and this series is listed in Table 31 in section 7 below.

The next section develops user costs for the above 29 Extended Business Sector assets.

6. Primary Input Tax Rates, User Costs and Balancing Internal Rates of Return

Nonresidential structures (office buildings, factories, etc.) and business land have to pay property taxes on these inputs whereas machinery and equipment and inventory stocks are generally exempt from paying these taxes. Thus it is necessary to take into account

[45] It would be highly desirable for the BEA to develop better land estimates in the future.

property taxes when constructing user costs of capital for business nonresidential structures and business land. BEA information on total property taxes paid was listed in Table 1 in section 2; see the series V_{TPROP}^t.

We approximate the asset base on which these taxes fall as the total beginning of the year national value of business land, residential structures and business nonresidential structures. Data on these values has been constructed in the previous section for the years 1987-2012; see the series for V_{K26}^t (consists mainly of government enterprise structures), V_{K28}^t (agricultural land) and V_{K29}^t (business nonfarm nonresidential land) which are listed in Table 21, the series V_{KRS}^t (residential structures) and V_{KNS}^t (nonresidential nongovernment structures) which can be constructed using Tables 15 and 18 and finally, V_{RL}^t (residential land), which is listed in Table 20. These series were summed and the sum was used as the tax base for the property tax series V_{TPROP}^t that was listed in Table 1 in section 2; i.e., we define the year t *property tax rate* τ_P^t as follows:

(17) $\tau_P^t \equiv V_{TPROP}^t/(V_{K26}^t + V_{K27}^t + V_{K29}^t + V_{KRS}^t + V_{KNR}^t + V_{RL}^t)$; t = 1987,...,2011.

The resulting *property tax rates* are reported as the series τ_P^t in Table 22 below and it will be used in the construction of the user costs of farm and other business sector land and nonresidential structures.[46] Our estimated property tax rate starts at 0.95%, trends up to 1.14% in 1998, then trends down to 0.84% in 2006 and finishes up at 1.14% in 2011. We will apply this property tax rate to assets 16-26 and 28-29. Thus define the *asset specific property tax rates* for year t and asset n, τ_{Pn}^t, as follows:

(18) $\tau_{Pn}^t \equiv 0$ for n = 1-15 and 27 and $\tau_{Pn}^t \equiv \tau_P^t$ for n = 16-26 and 28-29.

We list some other tax rates for general interest in Table 22. The consumption tax rate τ_C^t and the import tax rate on goods τ_{MG}^t are listed; these rates were defined in section 2 above.

Labour tax rates, τ_L^t, and business tax rates on assets employed, τ_K^t, are also listed in Table 22. Recall that the total value of personal income taxes collected in year t, V_{TP}^t, were listed in Table 1 in section 2 along with the year t value of Federal Social Insurance payments (which fall on labour), V_{TFSI}^t. We will assume that 80% of personal income taxes are a tax on labour earnings while the remaining 20% fall on the return to financial capital. Thus we define the labour tax rate in year t, τ_L^t, as follows:

(19) $\tau_L^t \equiv (0.8V_{TP}^t + V_{TFSI}^t)/(V_L^t + V_{LG}^t)$

where V_L^t is our estimate of labour earnings in the Extended Business Sector and V_{LG}^t is the value of employee compensation in the general government sector (which is not part of the EBS). The series V_L^t and V_{LG}^t are listed in Table 9 above.

[46] This is a very rough approximation to the actual property tax rates on business sector land and nonresidential structures since actual property tax rates are different across different sectors and assets. For example, business sector property assets are generally taxed more heavily than household property assets.

We assume that corporate income taxes, V_{TCI}^t (listed in Table 1), are a tax on the return to financial capital and we add 20% of personal income taxes as an additional tax on capital. We also assume that subsidies, V_S^t (also listed in Table 1), are an offset to capital taxes.[47] Thus our overall estimated business tax rate on assets employed in the EBS in year t, τ_K^t, is defined as follows:[48]

(20) $\tau_K^t \equiv (0.2V_{TP}^t + V_{TCI}^t - V_S^t)/V_{KW}^t$

where $V_{KW}^t \equiv \sum_{n=1}^{29} P_{Kn}^t Q_{Kn}^t$ is the sum of the asset values for the capital stocks used by the Extended Business Sector. This wealth stock series is listed below in Table 31.

Table 22: Tax Rates on Outputs and Inputs for the US Economy and the Balancing After Tax Rate of Return on Assets r^t: 1987-2011

Year t	τ_C^t	τ_{MG}^t	τ_L^t	τ_P^t	τ_K^t	τ_{BI}^t	r^t
1987	0.07583	0.03738	0.23226	0.00951	0.02323	0.31991	0.04938
1988	0.07551	0.03628	0.22879	0.00961	0.02354	0.32854	0.04811
1989	0.07347	0.03609	0.23585	0.00984	0.02411	0.30804	0.05416
1990	0.07326	0.03444	0.23396	0.00993	0.02327	0.30774	0.05233
1991	0.07609	0.03354	0.22983	0.01053	0.02142	0.30085	0.04978
1992	0.07582	0.03359	0.22748	0.01110	0.02287	0.29102	0.05573
1993	0.07550	0.03341	0.22881	0.01102	0.02461	0.29656	0.05837
1994	0.07820	0.03161	0.23173	0.01134	0.02685	0.30617	0.06085
1995	0.07719	0.02613	0.23635	0.01105	0.02840	0.32686	0.05849
1996	0.07588	0.02378	0.24396	0.01163	0.02960	0.32241	0.06221
1997	0.07579	0.02213	0.25037	0.01176	0.03121	0.32760	0.06406
1998	0.07518	0.02106	0.25444	0.01139	0.03080	0.35158	0.05680
1999	0.07424	0.01833	0.25639	0.01114	0.03049	0.34639	0.05754
2000	0.07237	0.01693	0.26078	0.01087	0.03100	0.37041	0.05269
2001	0.07032	0.01758	0.25660	0.01014	0.02363	0.30994	0.05260
2002	0.06970	0.01667	0.23071	0.01031	0.02155	0.26214	0.06066
2003	0.06971	0.01660	0.21952	0.01013	0.02261	0.25654	0.06551
2004	0.07050	0.01551	0.22015	0.00992	0.02567	0.26621	0.07077
2005	0.07200	0.01481	0.23164	0.00905	0.02854	0.30454	0.06517
2006	0.07221	0.01417	0.23897	0.00836	0.03007	0.34002	0.05837
2007	0.07028	0.01439	0.24427	0.00848	0.02700	0.32963	0.05491
2008	0.06881	0.01361	0.23665	0.00897	0.01981	0.27547	0.05210
2009	0.06674	0.01455	0.21560	0.01036	0.01564	0.22335	0.05438
2010	0.06667	0.01469	0.21732	0.01113	0.02174	0.24677	0.06636
2011	0.06678	0.01431	0.21998	0.01140	0.02299	0.25120	0.06852
Average	0.07272	0.02286	0.23530	0.01036	0.02523	0.30280	0.05799

Using the asset specific property tax rates τ_{Pn}^t, the general business tax rates τ_K^t, the depreciation rates δ_n^t for n = 1-29 that were listed in the previous section along with the

[47] This is only an approximation; some subsidies are specific and should be added to the prices of the relevant specific revenue streams. However, subsidies are a small proportion of GDP and our approximation errors here will not materially affect our productivity measures.

[48] Of course, our treatment of business income taxes is only a very rough approximation to the complexity of the U.S. tax code.

asset prices $P_{Kn}{}^t$ that are listed in section 5, we can define *user costs* $U_n{}^t$ for our 29 asset classes as follows:[49]

(21) $U_n{}^t \equiv [r^t + \tau_K{}^t + \tau_{Pn}{}^t + \delta_n{}^t]P_{Kn}{}^t$;   n = 1,...,29 ; t = 1987,...,2011

where r^t is suitable real after tax cost of capital that applies to the entire extended business sector in year t. In the present study, we will follow national income accounting conventions and will take r^t to be the *balancing real rate of return*;[50] i.e., it is the rate of return that is consistent with the year t value of business sector net output being equal to the value of primary inputs used by the business sector in year t, where the user costs defined by (21) are used as prices for the beginning of the year capital inputs. Thus r^t can be determined as the solution to the following linear in r^t equation for t = 1961,...,2011:

$$(22)\ P_C{}^tQ_C{}^t + P_I{}^tQ_I{}^t + P_G{}^tQ_G{}^t + P_X{}^tQ_X{}^t - P_M{}^tQ_M{}^t$$
$$= P_L{}^tQ_L{}^t + \sum_{n=1}^{29} [r^t + \tau_K{}^t + \tau_{Pn}{}^t + \delta_n{}^t]P_{Kn}{}^tQ_{Kn}{}^t\ ; \qquad t = 1987,...,2011$$

where the various price and quantity series are defined in the above Tables.[51] The resulting series of balancing real rates of return are listed in Table 21 above. It should be noted that r^t can be interpreted as a real interest rate; i.e., it is the income earned by the business sector in year t relative to the starting capital stock, valued at the average investment prices for the period. This explains why we have not included a capital gains term in the user cost formulae defined by (14).[52]

The r^t defined implicitly by equations (22) and the asset tax rates $\tau_K{}^t$ defined by (20) can be used in order to define the sequence of *business income tax rates* $\tau_{BI}{}^t$ defined as follows:

(23) $\tau_{BI}{}^t \equiv \tau_K{}^t/(\tau_K{}^t + r^t)$;   t = 1987,...,2011.

Thus the rate $\tau_{BI}{}^t$ is the fraction of the year t gross return to capital employed by the EBS, $\tau_K{}^t + r^t$, that is paid out in taxes. The income tax rates $\tau_{BI}{}^t$ are also listed in Table 22.

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[49] For additional material on user costs and many historical references, see Jorgenson (1963) (1989) (1996a) (1996b) (2012), Hall and Jorgenson (1967), Christensen and Jorgenson (1969), Harper, Berndt and Wood (1980), Diewert (1980) (2005a), Schreyer (2009), Gu (2012) and Diewert (2012).

[50] For most purposes, it is probably preferable to use an exogenous real rate of return in the user costs (14) since the resulting prices will probably approximate market rental prices better. However, when we used the sample average after tax real rate of return (5.80%) in the user costs defined by (14), our results were basically unchanged: the constant r^t TFP level ended up at 1.31611 in 2011 as opposed to the level of 1.31797 for our present variable r^t model. The geometric average rates of TFP growth for the two models are 1.151% per year for the constant r^t model versus 1.157% per year for the variable r^t model. When the constant r^t model is used, the value of inputs is not longer equal to the value of outputs so that TFP^{1987} is not equal to unity for this model. Hence we normalized the constant r^t TFP series by dividing by the 1987 level so that (TFP^{2011}/TFP^{1987}) is equal to 1.31797 which in turn equals $(1.0151)^{24}$.

[51] $P_X{}^t$ and $Q_X{}^t$ are chained Törnqvist aggregates of our 9 classes of exports and $P_M{}^t$ and $Q_M{}^t$ are chained Törnqvist aggregates of our 10 classes of imports of goods. These series are listed in Table 24 below.

[52] We have essentially absorbed the capital gains (or losses) term into r^t.

Diewert and Yu (2012) did a similar productivity study for Canada and Gu (2012) explained why the Diewert and Yu methodology was not as good as the BLS and Statistics Canada methodologies, which allowed for balancing nominal rates of return for each sector. Diewert (2012) in a response to Gu agreed that it is methodologically preferable to work with balancing sectoral rates of return (because different sectors face different costs of capital) rather than work with an economy wide balancing rate of return. But using a better methodological approach is only better if the sectoral balancing rates of return are reasonably accurate. Diewert (2012) argued that the Statistics Canada sectoral rates were far from being reasonable and so he argued for the use of an overall sectoral rate of return for the Canadian data. The situation in the U.S. is different; the BLS publishes its sector IRRs (Internal Rates of Return) and they are quite reasonable. Thus one of the main purposes of the present paper is to compare the U.S. Multifactor Productivity growth rates obtained using the present top down approach with the bottom up approach used by the BLS and other statistical agencies. For the case of Canada, Diewert and Yu (2012) showed that there were major differences between the two approaches but Diewert (2012) attributed most of the differences to the fact that the Statistics Canada IRRs were not reasonable. It can be seen that our economy wide IRRs for the US (the r^t listed in Table 22) are quite reasonable and fairly similar to the BLS sector IRRs so we would expect the top down approach applied to the US economy to give results that are fairly similar to the BLS bottom up results.[53]

The user costs defined by equations (21) were constructed and then normalized to equal one in 1987. The resulting normalized user costs, also denoted by U_n^t, are listed in Tables 23-25 below. The units of measurement for the capital stocks, Q_{Kn}^t, were changed to offset the change in units to the user costs and these transformed capital quantities are listed as the K_n^t in Tables 26-28 below.

Table 23: User Costs for Assets 1-10: 1987-2011

Year t	U_1^t	U_2^t	U_3^t	U_4^t	U_5^t	U_6^t	U_7^t	U_8^t	U_9^t	U_{10}^t
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	0.92566	0.98757	0.99535	1.01460	1.02297	1.02559	1.01373	1.04045	1.02466	1.01417
1989	0.91891	0.99481	1.02130	1.05912	1.10399	1.08953	1.06199	1.12436	1.08168	1.07474
1990	0.83906	0.97262	1.00834	1.08368	1.12963	1.09418	1.02658	1.15455	1.09025	1.06768
1991	0.76676	0.96668	0.98316	1.08658	1.12884	1.10414	1.02894	1.16857	1.15257	1.07488
1992	0.71667	0.92038	1.01775	1.14915	1.19060	1.16737	1.06255	1.24648	1.26346	1.16715
1993	0.62980	0.92173	1.03028	1.18881	1.24009	1.16579	1.10964	1.29980	1.35445	1.26643
1994	0.56700	0.90595	1.04726	1.21224	1.28491	1.15712	1.14402	1.36244	1.49345	1.33022
1995	0.50237	0.90214	1.01733	1.21885	1.30495	1.15245	1.14855	1.40410	1.44533	1.37924
1996	0.39460	0.90999	1.02231	1.25526	1.35620	1.10798	1.14522	1.48383	1.49806	1.38981
1997	0.30608	0.92705	1.03578	1.26259	1.39255	1.03060	1.14446	1.52680	1.53663	1.38026
1998	0.23073	0.90418	0.95579	1.22652	1.35667	0.89975	1.10863	1.47435	1.43082	1.38124
1999	0.18189	0.92776	0.91411	1.22372	1.36563	0.84580	1.07033	1.48654	1.44761	1.42061
2000	0.15375	0.93623	0.86623	1.19810	1.35392	0.79434	1.08377	1.46072	1.38127	1.38807
2001	0.12345	0.89907	0.79112	1.16644	1.32459	0.73333	1.06962	1.40306	1.26846	1.32304
2002	0.10913	0.88258	0.77719	1.21007	1.37135	0.70952	1.10920	1.44844	1.27000	1.36493
2003	0.10061	0.88144	0.76858	1.25766	1.42178	0.72249	1.20869	1.50236	1.35200	1.41018
2004	0.09581	0.88676	0.75799	1.30343	1.49692	0.73005	1.19310	1.59593	1.52540	1.50442

[53] Of course the scope of our Extended Business Sector is a bit different from the scope of the BLS Private Sector and so we would expect some differences due to this difference in scope.

2005	**0.08621**	**0.88375**	**0.72937**	**1.29916**	**1.49943**	**0.72463**	**1.16659**	**1.63087**	**1.49126**	**1.56061**
2006	**0.07708**	**0.88579**	**0.70006**	**1.27346**	**1.49754**	**0.70124**	**1.13053**	**1.63687**	**1.44359**	**1.58474**
2007	**0.06899**	**0.88303**	**0.68222**	**1.24821**	**1.48747**	**0.70744**	**1.02292**	**1.64346**	**1.36631**	**1.55988**
2008	**0.06094**	**0.87181**	**0.59529**	**1.19758**	**1.44610**	**0.68939**	**1.03593**	**1.61408**	**1.24683**	**1.52499**
2009	**0.05488**	**0.85610**	**0.55916**	**1.18274**	**1.45273**	**0.68611**	**1.03198**	**1.61995**	**1.17171**	**1.54548**
2010	**0.05493**	**0.87522**	**0.60160**	**1.27257**	**1.58853**	**0.74023**	**1.05176**	**1.81071**	**1.35401**	**1.70981**
2011	**0.05412**	**0.88718**	**0.60047**	**1.29734**	**1.63412**	**0.75515**	**1.08466**	**1.89679**	**1.45731**	**1.82109**

Table 24: User Costs for Assets 11-20: 1987-2011

Year t	U_{11}^{t}	U_{12}^{t}	U_{13}^{t}	U_{14}^{t}	U_{15}^{t}	U_{16}^{t}	U_{17}^{t}	U_{18}^{t}	U_{19}^{t}	U_{20}^{t}
1987	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**
1988	**1.01704**	**1.01876**	**1.02988**	**0.99924**	**1.03830**	**1.02543**	**1.02594**	**1.02118**	**1.03204**	**1.02560**
1989	**1.04803**	**1.10741**	**1.12123**	**1.10977**	**1.11425**	**1.13303**	**1.13718**	**1.12650**	**1.13762**	**1.13951**
1990	**1.07607**	**1.16409**	**1.14109**	**1.13876**	**1.13940**	**1.13996**	**1.14472**	**1.13134**	**1.14654**	**1.14041**
1991	**1.10577**	**1.20373**	**1.13649**	**1.13650**	**1.14056**	**1.11275**	**1.11645**	**1.10698**	**1.12503**	**1.11081**
1992	**1.14700**	**1.32022**	**1.22942**	**1.19918**	**1.19843**	**1.20099**	**1.20991**	**1.19272**	**1.20735**	**1.20837**
1993	**1.20842**	**1.39108**	**1.30620**	**1.24535**	**1.25118**	**1.28362**	**1.29439**	**1.27481**	**1.29344**	**1.28869**
1994	**1.27227**	**1.46133**	**1.38825**	**1.32816**	**1.30932**	**1.38901**	**1.40010**	**1.38011**	**1.39837**	**1.40008**
1995	**1.26734**	**1.51704**	**1.40508**	**1.37153**	**1.34255**	**1.43982**	**1.45187**	**1.42717**	**1.45053**	**1.45537**
1996	**1.31906**	**1.62407**	**1.48516**	**1.48922**	**1.40309**	**1.53931**	**1.55270**	**1.52569**	**1.55260**	**1.55606**
1997	**1.32466**	**1.69831**	**1.55331**	**1.50343**	**1.44819**	**1.62903**	**1.64737**	**1.61572**	**1.64175**	**1.64960**
1998	**1.28291**	**1.63945**	**1.51540**	**1.43165**	**1.42121**	**1.60146**	**1.59105**	**1.57123**	**1.59915**	**1.59443**
1999	**1.29421**	**1.67241**	**1.53104**	**1.43634**	**1.43464**	**1.68565**	**1.66738**	**1.65397**	**1.68095**	**1.68361**
2000	**1.28230**	**1.68377**	**1.52459**	**1.40400**	**1.42198**	**1.69643**	**1.66758**	**1.64787**	**1.66914**	**1.66774**
2001	**1.23792**	**1.69907**	**1.49049**	**1.32003**	**1.38656**	**1.63147**	**1.61476**	**1.59651**	**1.61782**	**1.61107**
2002	**1.25864**	**1.78007**	**1.59148**	**1.37147**	**1.43441**	**1.77362**	**1.76900**	**1.75330**	**1.77894**	**1.77250**
2003	**1.24471**	**1.88386**	**1.72706**	**1.44785**	**1.49078**	**1.92284**	**1.91916**	**1.90668**	**1.93463**	**1.93227**
2004	**1.28841**	**2.08310**	**1.87638**	**1.58917**	**1.57502**	**2.13443**	**2.14270**	**2.11223**	**2.13995**	**2.14337**
2005	**1.30569**	**2.16920**	**1.89960**	**1.74755**	**1.62001**	**2.28959**	**2.26932**	**2.26437**	**2.29776**	**2.27839**
2006	**1.25440**	**2.18852**	**1.90809**	**1.82763**	**1.63274**	**2.42321**	**2.36640**	**2.35980**	**2.38817**	**2.34515**
2007	**1.25374**	**2.18873**	**1.90913**	**1.84048**	**1.63059**	**2.47095**	**2.36809**	**2.38739**	**2.40655**	**2.37146**
2008	**1.18514**	**2.12745**	**1.82315**	**1.75334**	**1.61244**	**2.33247**	**2.20123**	**2.25601**	**2.28299**	**2.23200**
2009	**1.17184**	**2.17576**	**1.85496**	**1.72760**	**1.63210**	**2.40923**	**2.14327**	**2.32701**	**2.35773**	**2.30505**
2010	**1.31226**	**2.43969**	**2.14178**	**1.95179**	**1.76991**	**2.76359**	**2.40366**	**2.64422**	**2.68477**	**2.64097**
2011	**1.36988**	**2.54914**	**2.23317**	**2.03055**	**1.83720**	**2.86261**	**2.45893**	**2.75023**	**2.79174**	**2.74544**

Table 25: User Costs for Assets 21-29: 1987-2011

Year t	U_{21}^{t}	U_{22}^{t}	U_{23}^{t}	U_{24}^{t}	U_{25}^{t}	U_{26}^{t}	U_{27}^{t}	U_{28}^{t}	U_{29}^{t}
1987	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**	**1.00000**
1988	**1.02622**	**1.02737**	**1.03477**	**1.09272**	**1.02554**	**1.03469**	**1.02008**	**1.04397**	**1.02838**
1989	**1.13179**	**1.13271**	**1.17018**	**1.22329**	**1.12887**	**1.13731**	**1.18280**	**1.19650**	**1.18050**
1990	**1.14140**	**1.14456**	**1.18848**	**1.24400**	**1.13645**	**1.15400**	**1.16281**	**1.18757**	**1.24940**
1991	**1.11551**	**1.12078**	**1.14834**	**1.30325**	**1.10884**	**1.14346**	**1.11912**	**1.16796**	**1.23593**
1992	**1.20300**	**1.20384**	**1.25124**	**1.25409**	**1.20260**	**1.21215**	**1.20868**	**1.30015**	**1.10825**
1993	**1.28331**	**1.28257**	**1.33138**	**1.28839**	**1.28858**	**1.28049**	**1.28761**	**1.41412**	**1.09033**
1994	**1.39006**	**1.38434**	**1.44216**	**1.30551**	**1.40082**	**1.37200**	**1.37785**	**1.60680**	**1.11831**
1995	**1.44519**	**1.43739**	**1.48693**	**1.32662**	**1.44563**	**1.41897**	**1.38447**	**1.68040**	**1.13204**
1996	**1.54277**	**1.53312**	**1.60297**	**1.43149**	**1.55020**	**1.51220**	**1.50311**	**1.86515**	**1.22076**
1997	**1.63224**	**1.61929**	**1.68037**	**1.57983**	**1.64509**	**1.59037**	**1.55789**	**2.01473**	**1.25005**
1998	**1.58780**	**1.57812**	**1.60814**	**1.70257**	**1.58537**	**1.56362**	**1.40170**	**1.96019**	**1.38153**
1999	**1.66561**	**1.65116**	**1.62389**	**1.71086**	**1.64693**	**1.62480**	**1.35711**	**2.07657**	**1.48140**
2000	**1.65775**	**1.64767**	**1.59975**	**1.67421**	**1.64395**	**1.62448**	**1.31873**	**2.09515**	**1.43043**
2001	**1.60516**	**1.59714**	**1.53700**	**1.88053**	**1.59272**	**1.59819**	**1.23151**	**2.01931**	**1.83967**
2002	**1.75893**	**1.74046**	**1.66847**	**2.29072**	**1.74328**	**1.74337**	**1.29373**	**2.27568**	**1.51202**
2003	**1.90778**	**1.87440**	**1.80629**	**2.70395**	**1.88345**	**1.88688**	**1.41577**	**2.53658**	**1.73311**

2004	2.11442	2.05250	2.02241	2.99357	2.09195	2.05563	1.59147	2.89742	2.13591
2005	2.26342	2.16711	2.19199	3.52213	2.18759	2.20359	1.61710	3.36307	3.14716
2006	2.35426	2.22812	2.25080	4.49039	2.24832	2.33687	1.59166	3.60119	3.55189
2007	2.37305	2.22484	2.25670	4.85583	2.25338	2.40156	1.51491	3.69305	3.87114
2008	2.24540	2.13839	2.14438	4.79234	2.11120	2.30699	1.41977	3.56797	3.65367
2009	2.31612	2.23031	2.20597	4.48191	2.10567	2.39020	1.37394	3.44771	3.20706
2010	2.63421	2.50304	2.59313	4.67312	2.40552	2.60147	1.74732	4.43779	2.57549
2011	2.74282	2.59201	2.86983	5.13862	2.48582	2.71922	1.93965	4.99965	2.26008

Table 26: Quantities of Capital Services for Assets 1-10: 1987-2011

Year t	K_1^t	K_2^t	K_3^t	K_4^t	K_5^t	K_6^t	K_7^t	K_8^t	K_9^t	K_{10}^t
1987	29.5	27.0	42.7	12.8	9.6	11.5	10.0	117.4	13.3	22.7
1988	34.3	30.1	45.1	13.6	10.1	11.9	9.1	119.0	13.3	23.1
1989	39.8	34.7	48.1	14.5	10.6	12.3	8.2	121.0	13.7	23.0
1990	46.2	41.7	50.8	15.4	11.4	12.8	7.8	123.8	13.5	22.9
1991	49.8	49.0	53.5	16.5	12.3	13.1	7.1	125.5	13.1	21.9
1992	53.2	56.2	55.6	17.6	13.0	13.4	6.5	126.3	12.6	20.6
1993	61.5	64.6	57.9	18.9	13.8	13.7	6.3	127.0	12.2	20.3
1994	74.0	72.8	60.6	20.4	14.5	14.5	6.2	128.8	12.8	21.1
1995	89.6	80.8	64.6	21.5	15.1	15.4	6.2	131.5	15.1	22.3
1996	119.6	88.9	69.8	22.4	16.0	15.9	6.2	135.4	17.9	23.8
1997	165.8	100.1	76.1	23.6	16.7	16.3	6.3	139.5	21.8	24.5
1998	235.3	119.3	83.6	25.1	17.2	16.7	6.3	143.4	26.8	24.6
1999	333.7	141.9	92.8	27.3	17.6	17.5	6.1	147.7	31.3	26.0
2000	466.4	171.0	104.3	29.4	17.9	17.7	5.5	151.6	35.8	27.7
2001	601.9	200.5	121.2	31.9	18.4	17.7	5.1	156.7	39.0	28.3
2002	689.7	218.9	133.6	34.6	18.9	16.6	4.9	160.0	40.3	27.1
2003	757.8	230.6	139.4	37.8	19.4	15.2	5.2	161.8	40.5	26.1
2004	822.6	243.3	144.9	41.2	19.8	14.3	6.5	163.4	40.8	25.2
2005	896.8	259.7	151.3	44.5	20.4	13.4	7.4	164.3	43.0	25.9
2006	978.4	275.7	157.2	48.7	21.1	12.6	8.0	166.3	46.7	27.5
2007	1112.1	290.6	165.6	52.7	22.0	11.8	8.7	169.9	51.4	29.4
2008	1259.8	306.9	176.0	57.5	22.9	11.9	7.6	173.9	54.0	29.1
2009	1395.4	322.2	183.8	61.4	23.5	11.8	6.9	176.8	51.1	27.5
2010	1447.2	332.9	188.2	64.2	23.4	11.2	6.0	175.6	41.7	25.2
2011	1494.2	343.1	195.2	67.3	23.6	10.9	5.0	174.4	39.0	23.9

Table 27: Quantities of Capital Services for Assets 11-20: 1987-2011

Year t	K_{11}^t	K_{12}^t	K_{13}^t	K_{14}^t	K_{15}^t	K_{16}^t	K_{17}^t	K_{18}^t	K_{19}^t	K_{20}^t
1987	26.4	12.9	6.1	9.0	91.9	39.9	21.4	16.3	8.7	7.6
1988	25.5	13.3	5.9	8.6	92.6	42.0	22.2	17.2	9.1	7.9
1989	26.3	13.7	5.8	8.4	93.7	44.1	23.2	18.0	9.5	8.3
1990	25.9	13.8	5.6	8.3	95.9	46.1	24.0	18.7	9.9	8.6
1991	26.1	14.2	5.4	8.2	97.5	47.8	25.0	19.4	10.3	9.0
1992	27.2	14.6	5.3	8.1	96.8	48.9	25.7	19.7	10.5	9.1
1993	28.2	15.1	5.1	7.9	95.5	49.5	26.4	20.0	10.7	9.3
1994	29.8	15.4	5.0	7.8	96.2	50.2	27.2	20.3	10.9	9.5
1995	32.9	15.2	4.9	7.9	97.8	50.7	27.9	20.7	11.0	9.6
1996	34.5	15.3	4.8	7.9	100.2	51.6	28.5	21.0	11.2	10.1
1997	36.0	15.3	4.7	8.0	103.0	52.6	29.1	21.4	11.5	10.6
1998	36.5	15.6	4.7	8.1	106.9	54.0	29.9	21.7	11.7	11.2
1999	36.8	16.4	4.7	8.3	112.0	55.9	30.6	22.0	12.0	11.8
2000	38.5	17.9	4.7	8.6	116.9	57.8	31.3	22.4	12.3	12.4
2001	40.1	19.5	4.7	8.8	122.1	60.2	32.0	22.8	12.5	13.0
2002	40.4	21.1	4.7	8.8	126.3	62.0	32.6	23.1	12.7	13.7

2003	**40.3**	**21.7**	**4.8**	**8.8**	**129.8**	**62.9**	**33.4**	**23.5**	**12.8**	**14.0**
2004	**38.7**	**21.7**	**4.8**	**8.7**	**133.8**	**63.3**	**34.2**	**23.8**	**13.0**	**14.4**
2005	**37.8**	**21.8**	**4.9**	**8.6**	**138.3**	**63.7**	**35.0**	**24.2**	**13.1**	**14.7**
2006	**38.2**	**21.7**	**5.0**	**8.7**	**144.0**	**64.2**	**35.7**	**24.6**	**13.1**	**15.0**
2007	**39.5**	**21.6**	**5.0**	**8.9**	**150.2**	**64.9**	**36.6**	**25.4**	**13.1**	**15.3**
2008	**40.9**	**21.9**	**5.1**	**9.0**	**155.3**	**65.9**	**37.6**	**26.2**	**13.1**	**15.7**
2009	**39.6**	**22.1**	**5.3**	**9.2**	**158.8**	**66.8**	**38.6**	**26.9**	**13.1**	**16.0**
2010	**34.0**	**21.8**	**5.4**	**9.3**	**157.4**	**66.8**	**39.5**	**27.0**	**13.0**	**16.1**
2011	**33.6**	**21.6**	**5.4**	**9.2**	**157.0**	**66.3**	**40.2**	**26.8**	**12.8**	**15.9**

Table 28: Quantities of Capital Services for Assets 21-29: 1987-2011

Year t	K_{21}^t	K_{22}^t	K_{23}^t	K_{24}^t	K_{25}^t	K_{26}^t	K_{27}^t	K_{28}^t	K_{29}^t
1987	**11.9**	**51.1**	**58.8**	**31.2**	**78.9**	**75.9**	**62.3**	**40.9**	**98.2**
1988	**12.5**	**51.7**	**60.1**	**30.7**	**80.2**	**76.5**	**64.9**	**40.5**	**98.2**
1989	**13.1**	**52.3**	**60.9**	**30.4**	**81.2**	**76.8**	**66.2**	**40.5**	**98.2**
1990	**13.7**	**53.4**	**61.6**	**30.0**	**82.3**	**77.1**	**67.9**	**40.1**	**98.2**
1991	**14.2**	**54.8**	**62.5**	**29.9**	**83.6**	**77.7**	**68.8**	**39.7**	**98.2**
1992	**14.5**	**55.9**	**63.4**	**29.8**	**84.4**	**78.3**	**68.8**	**39.7**	**98.2**
1993	**14.6**	**56.7**	**64.1**	**29.3**	**85.0**	**79.2**	**69.8**	**39.7**	**98.2**
1994	**14.9**	**57.0**	**65.1**	**29.1**	**85.8**	**79.9**	**71.0**	**40.1**	**98.2**
1995	**15.3**	**57.6**	**65.5**	**29.0**	**86.4**	**80.1**	**75.0**	**40.1**	**98.2**
1996	**15.8**	**58.6**	**66.3**	**28.7**	**87.4**	**80.4**	**76.8**	**40.1**	**98.2**
1997	**16.4**	**59.8**	**67.0**	**28.5**	**88.9**	**80.7**	**78.6**	**40.5**	**98.2**
1998	**17.0**	**60.8**	**67.5**	**28.8**	**90.7**	**81.1**	**83.0**	**40.5**	**98.2**
1999	**17.7**	**61.8**	**68.3**	**28.7**	**92.8**	**81.6**	**87.0**	**40.1**	**98.2**
2000	**18.3**	**62.4**	**69.5**	**28.3**	**94.8**	**82.3**	**90.9**	**40.1**	**98.2**
2001	**19.1**	**62.9**	**71.0**	**28.4**	**96.9**	**83.0**	**94.3**	**39.7**	**98.2**
2002	**19.6**	**63.4**	**72.4**	**28.9**	**98.6**	**84.0**	**92.0**	**39.4**	**98.2**
2003	**20.2**	**62.9**	**74.0**	**28.7**	**99.8**	**85.2**	**92.7**	**39.0**	**98.2**
2004	**20.6**	**62.2**	**75.3**	**28.8**	**100.9**	**86.7**	**93.7**	**39.0**	**98.2**
2005	**21.0**	**61.6**	**76.2**	**29.5**	**101.9**	**87.9**	**97.4**	**38.6**	**98.2**
2006	**21.3**	**61.2**	**76.8**	**30.6**	**102.6**	**88.7**	**100.3**	**38.2**	**98.2**
2007	**21.5**	**61.1**	**77.7**	**32.2**	**103.7**	**89.4**	**103.6**	**38.2**	**98.2**
2008	**21.8**	**61.3**	**79.7**	**34.0**	**105.4**	**90.1**	**105.2**	**37.8**	**98.2**
2009	**21.9**	**62.0**	**81.9**	**35.9**	**107.5**	**91.0**	**103.2**	**37.4**	**98.2**
2010	**21.8**	**62.9**	**84.1**	**36.1**	**108.7**	**91.8**	**95.3**	**37.4**	**98.2**
2011	**21.5**	**62.9**	**85.6**	**36.9**	**109.0**	**93.1**	**98.2**	**37.4**	**98.2**

In the following section, we list various output and input aggregates that are suitable for measuring the Multifactor or Total Factor Productivity[54] of the U.S. Extended Business Sector.

7. U.S. Aggregate Inputs, Outputs and Total Factor Productivity Levels for the Extended Business Sector

In this section, we list the main outputs and inputs produced and utilized by the Extended Business Sector.

The price indexes for consumption, investment and net deliveries of the EBS to the general government sector, P_C^t, P_I^t and P_G^t, were listed in section 2 in Table 2 and the

[54] Jorgenson and Griliches (1967) used the term Total Factor Productivity (TFP) while the Bureau of Labor Statistics (1983) used the term Multifactor Productivity (MFP) for the same basic concept, which is aggregate output divided by the corresponding aggregate input.

corresponding quantity indexes, Q_C^t, Q_I^t and Q_G^t, were listed in Table 3 above. We now define P_D^t, the year t price of domestic EBS production, as the chained Törnqvist price index of P_C^t, P_I^t and P_G^t. P_D^t is listed in Table 29 below and the corresponding implicit Törnqvist quantity index Q_D^t and the value $V_D^t \equiv P_D^tQ_D^t$ are listed in Tables 30 and 31 below.

Table 2 above lists the price indexes for the export of services, P_{XS}^t, and Table 3 lists the corresponding quantity indexes, Q_{XS}^t. Table 4 lists the 8 price series for the exports of various classes of goods, P_{X1}^t,...,P_{X8}^t, and Table 5 lists the corresponding export quantity series, P_{X1}^t,...,P_{X8}^t. Define P_X^t as the chained Törnqvist price index of P_{X1}^t,...,P_{X8}^t and P_{XS}^t. This aggregate export price series is listed in Table 29 below and the corresponding implicit Törnqvist quantity index Q_X^t and the value $V_X^t \equiv P_X^tQ_X^t$ are listed in Tables 30 and 31 below.

Table 2 above lists the price indexes for the import of services, P_{MS}^t, and Table 3 lists the corresponding quantity indexes, Q_{MS}^t. Table 6 lists the 9 price series for the imports of various classes of goods, P_{M1}^t,...,P_{M9}^t, and Table 7 lists the corresponding import quantity series, P_{M1}^t,...,P_{M9}^t. Define P_M^t as the chained Törnqvist price index of P_{M1}^t,...,P_{M9}^t and P_{MS}^t. This aggregate import price series is listed in Table 29 below and the corresponding implicit Törnqvist quantity indexes Q_M^t and the value series $V_M^t \equiv P_M^tQ_M^t$ are listed in Tables 30 and 31 below.

We define an EBS aggregate output price index for year t, P_Y^t, as a chained Törnqvist price index of the price indexes P_D^t, P_X^t, P_M^t with quantity weights Q_D^t, Q_X^t, $-Q_M^t$. Denote the corresponding implicit Törnqvist output index as Q_Y^t and the output value aggregate as $V_Y^t \equiv P_Y^tQ_Y^t$. The series P_Y^t, Q_Y^t and V_Y^t are listed in Tables 29-31.

We turn our attention to aggregate input series. Table 9 above lists our estimates for the price, quantity and value of EBS labour services, P_L^t, Q_L^t and V_L^t. These series are repeated in Tables 29-31 below.

In section 5 above, we explained how a price, quantity and value series for capital stocks used in the Extended Business Sector were constructed. These wealth type series were denoted by P_{KW}^t, Q_{KW}^t and V_{KW}^t and they are listed in Tables 29-31 respectively.

In section 6, we constructed user costs U_1^t,...,U_{29}^t for the 29 assets that we distinguish along with the corresponding capital services quantity inputs, K_1^t,...,K_{29}^t; see Tables 23-28 above. Denote Q_K^t as the direct chained Törnqvist quantity index which aggregates the quantities K_1^t,...,K_{29}^t using the user costs U_1^t,...,U_{29}^t as price weights.[55] Let P_K^t denote the corresponding implicit chained Törnqvist price index and let $V_K^t \equiv P_K^tQ_K^t$ denote the

[55] The reader may be puzzled as to why we suddenly switched to direct Törnqvist aggregation of quantities whereas up to now, we aggregated prices directly and defined quantity indexes implicitly. The reason for this switch is that in a subsequent paper, we want to apply the methodology developed by Diewert and Morrison (1986) and Kohli (1990) that allows nominal GDP to be decomposed into explanatory factors. In order to apply this methodology, we need to aggregate primary inputs using direct Törnqvist quantity indexes.

corresponding year t value of EBS capital services. The series P_K^t, Q_K^t and V_K^t are listed in Tables 29-31 respectively.

We construct an EBS input aggregate quantity index, Q_Z^t, as the direct chained Törnqvist quantity index which aggregates the Törnqvist quantity indexes Q_L^t and Q_K^t using the implicit Törnqvist price indexes P_L^t and P_K. Denote the corresponding implicit aggregate input price index as P_Z^t and the corresponding aggregate EBS input value as V_Z^t. The series P_Z^t, Q_Z^t and V_Z^t are listed in Tables 29-31 respectively.

Finally, we follow Jorgenson and Griliches (1967) and construct an estimate of the Total Factor Productivity of the EBS in year t, TFP^t, as the output index Q_Y^t divided by the corresponding input index Q_Z^t; i.e.,

(24) $TFP^t \equiv Q_Y^t/Q_Z^t$; $\qquad$ t = 987,...,2011.

The series TFP^t is listed in Table 29.

Table 29: Price Indexes for EBS Output and Input Aggregates and TFP

Year t	P_C^t	P_D^t	P_X^t	P_M^t	P_L^t	P_K^t	P_{KW}^t	P_Y^t	P_Z^t	TFP^t
1987	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1988	1.03914	1.03228	1.05178	1.04691	1.05387	1.02590	1.03808	1.03224	1.04564	1.01299
1989	1.08660	1.07156	1.06933	1.06993	1.08184	1.12253	1.08417	1.07134	1.09365	1.02083
1990	1.13572	1.11239	1.07676	1.09905	1.14336	1.13475	1.12646	1.10968	1.14071	1.02797
1991	1.17434	1.14331	1.09107	1.08888	1.18158	1.12223	1.15872	1.14456	1.16379	1.01680
1992	1.20916	1.16674	1.08663	1.08955	1.23841	1.17016	1.13006	1.16751	1.21794	1.04320
1993	1.23494	1.18815	1.08639	1.08098	1.26337	1.21965	1.13951	1.19059	1.25027	1.05013
1994	1.25540	1.20685	1.09857	1.08947	1.27999	1.28506	1.16439	1.21005	1.28139	1.05896
1995	1.28288	1.23098	1.08105	1.11446	1.30605	1.30854	1.19883	1.22741	1.30668	1.06458
1996	1.31084	1.25063	1.06691	1.09333	1.35197	1.37502	1.22906	1.24897	1.35872	1.08787
1997	1.33420	1.26608	1.04879	1.05391	1.39284	1.41856	1.24584	1.26930	1.40038	1.10327
1998	1.34314	1.26868	1.02476	0.99588	1.47226	1.37294	1.28736	1.27994	1.44205	1.12666
1999	1.36464	1.28413	1.01836	0.99947	1.53361	1.39349	1.31398	1.29409	1.49088	1.15206
2000	1.40124	1.31565	1.03637	1.04173	1.63566	1.36962	1.34133	1.32032	1.55341	1.17654
2001	1.42637	1.33580	1.03215	1.01669	1.70552	1.34185	1.43004	1.34705	1.59159	1.18153
2002	1.44127	1.34722	1.02792	1.00456	1.76674	1.38448	1.41251	1.36143	1.64684	1.20963
2003	1.46931	1.37169	1.04996	1.03921	1.83698	1.46922	1.45382	1.38267	1.72264	1.24588
2004	1.50887	1.41201	1.08655	1.08825	1.90377	1.59629	1.50354	1.41953	1.81117	1.27589
2005	1.55309	1.46335	1.12557	1.15516	1.96156	1.71258	1.68922	1.46371	1.89005	1.29127
2006	1.59457	1.51066	1.16430	1.20223	2.03217	1.77158	1.84496	1.50877	1.95715	1.29718
2007	1.64043	1.55366	1.20332	1.24466	2.11251	1.78887	1.98908	1.55054	2.01585	1.30009
2008	1.69599	1.60482	1.26041	1.37655	2.17342	1.70822	2.06762	1.57976	2.02634	1.28269
2009	1.69592	1.59477	1.19252	1.23083	2.18900	1.67878	2.08388	1.59363	2.02526	1.27085
2010	1.73672	1.62461	1.24686	1.30486	2.23542	1.82502	1.92438	1.61688	2.11064	1.30538
2011	1.78075	1.66735	1.32699	1.40688	2.29767	1.89415	1.93947	1.65133	2.17640	1.31797
GAG	1.02433	1.02153	1.01186	1.01433	1.03527	1.02697	1.02799	1.02112	1.03293	1.01157

The last row of Tables 29 and 30 list the Geometric Average Growth (GAG) rate for each of the series in the corresponding column. From the last row and column of Table 29, it can be seen that TFP for the EBS grew at the rate of 1.157% per year over the period 1987-2012, which is a very satisfactory rate of TFP growth for an advanced economy.

Table 30: Quantity Indexes for EBS Output and Input Aggregates and Real Capital Output Ratios

Year t	Q_C^t	Q_D^t	Q_X^t	Q_M^t	Q_L^t	Q_K^t	Q_{KW}^t	Q_Y^t	Q_Z^t	Q_{KW}^t/Q_Y^t
1987	2505.9	3698.8	363.7	524.1	2492.8	1045.6	8378.7	3538.4	3538.4	2.37
1988	2611.0	3819.8	422.0	544.9	2580.9	1068.9	8540.6	3697.3	3649.9	2.31
1989	2686.9	3940.9	470.5	568.6	2670.0	1095.0	8696.4	3843.5	3765.1	2.26
1990	2739.9	3984.0	512.6	588.8	2677.8	1123.4	8857.9	3907.9	3801.6	2.27
1991	2731.6	3917.7	546.7	588.1	2663.6	1147.8	9002.0	3875.1	3811.1	2.32
1992	2826.7	4074.9	584.4	629.8	2696.9	1165.0	9094.8	4028.3	3861.5	2.26
1993	2935.1	4243.2	603.6	684.2	2779.2	1186.0	9200.0	4164.1	3965.3	2.21
1994	3045.9	4467.0	656.0	766.5	2901.7	1215.8	9348.7	4360.6	4117.8	2.14
1995	3126.6	4585.1	751.0	827.6	2981.2	1253.3	9545.1	4508.4	4234.9	2.12
1996	3244.6	4799.9	813.2	898.9	3036.7	1296.0	9752.5	4714.0	4333.2	2.07
1997	3367.9	5067.9	910.0	1020.2	3149.4	1345.6	9992.7	4959.8	4495.5	2.01
1998	3562.0	5398.3	930.8	1139.9	3216.1	1407.5	10292.1	5209.1	4623.5	1.98
1999	3761.3	5750.2	971.3	1271.2	3289.5	1476.7	10610.0	5488.5	4764.1	1.93
2000	3959.1	6059.4	1055.1	1436.7	3325.2	1555.2	10952.2	5732.6	4872.5	1.91
2001	4070.0	6119.2	995.6	1395.9	3277.2	1634.3	11298.9	5777.3	4889.7	1.96
2002	4199.9	6288.6	975.7	1443.6	3224.3	1681.8	11498.1	5894.5	4873.0	1.95
2003	4347.1	6515.6	991.2	1507.3	3208.7	1713.4	11646.3	6083.6	4883.0	1.91
2004	4485.8	6806.2	1086.4	1674.6	3248.8	1743.8	11790.9	6318.0	4951.8	1.87
2005	4630.4	7036.7	1159.6	1777.3	3313.2	1781.1	11963.8	6524.0	5052.4	1.83
2006	4753.8	7224.5	1263.7	1885.3	3391.8	1820.9	12134.9	6706.5	5170.0	1.81
2007	4858.1	7295.2	1381.1	1931.2	3430.3	1870.4	12360.0	6831.4	5254.6	1.81
2008	4796.4	7105.1	1465.0	1878.4	3398.2	1918.7	12579.8	6749.9	5262.3	1.86
2009	4690.4	6715.2	1331.1	1624.3	3205.9	1953.5	12738.0	6461.5	5084.4	1.97
2010	4779.6	6945.6	1479.4	1827.4	3215.3	1948.7	12704.2	6644.9	5090.4	1.91
2011	4914.1	7090.8	1578.3	1915.0	3267.4	1961.7	12781.7	6796.4	5156.8	1.88
GAG	1.02846	1.02749	1.06307	1.05547	1.01134	1.02656	1.01775	1.02757	1.01582	0.99045

Looking at the last row of Table 30, it can be seen that aggregate consumption (C) produced by the EBS over the sample period grew faster than domestic output (C+I+G) produced by the EBS; Q_C^t grew at the annual geometric rate of 2.846% whereas Q_D^t grew at the rate of 2.749% per year. It can also be seen that exports grew faster than imports over the sample period: a 6.307% annual growth rate for Q_X^t versus a 5.547% growth rate for Q_M^t. Aggregate labour input grew at an annual rate of 1.134%, which is much less than the rate of growth of capital services, 2.656% per year. It is interesting to note that the capital stock utilized by the EBS grew at the rate of 1.775% per year, which is 0.881 percentage points below the rate of capital services growth. This difference between the growth of capital stocks and the corresponding services can be explained by the fact that inputs of machinery and equipment (particularly computers) with relatively high depreciation rates grow much faster than inputs of structures, which have relatively low depreciation rates. Thus the since the stock weights for machinery and equipment growth are much smaller than the corresponding capital services weights, capital services will generally tend to grow much faster than capital stocks.[56] Aggregate EBS output grew at the geometric average rate of 2.757% per year, about 1.17 percentage points higher than the average rate of input growth, which was 1.582% per year. The last column of Table 30 lists the EBS real capital output ratios, Q_{KW}^t/Q_Y^t. It can be seen that there was an

[56] This observation dates back to Jorgenson and Griliches (1972).

overall downward trend in the real capital output ratio which was 2.37 in 1987 and 1.88 in 2011.

Table 31: Values for EBS Output and Input Aggregates and Nominal Capital Output Ratios

Year t	V_C^t	V_D^t	V_X^t	V_M^t	V_L^t	V_K^t	V_{KW}^t	V_Y^t	V_Z^t	V_{KW}^t/V_Y^t
1987	2505.9	3698.8	363.7	524.1	2492.8	1045.6	8378.7	3538.4	3538.4	2.37
1988	2713.2	3943.1	443.8	570.4	2719.9	1096.6	8540.6	3816.5	3816.5	2.32
1989	2919.6	4222.9	503.1	608.3	2888.5	1229.2	8696.4	4117.7	4117.7	2.29
1990	3111.7	4431.8	551.9	647.2	3061.7	1274.8	8857.9	4336.5	4336.5	2.30
1991	3207.8	4479.2	596.5	640.4	3147.3	1288.1	9002.0	4435.3	4435.3	2.35
1992	3417.9	4754.3	635.0	686.2	3339.9	1363.2	9094.8	4703.1	4703.1	2.19
1993	3624.7	5041.5	655.7	739.6	3511.2	1446.5	9200.0	4957.7	4957.7	2.11
1994	3823.8	5391.0	720.6	835.1	3714.1	1562.4	9348.7	5276.6	5276.6	2.06
1995	4011.1	5644.2	811.8	922.4	3893.6	1640.0	9545.1	5533.6	5533.6	2.07
1996	4253.1	6002.9	867.6	982.8	4105.5	1782.1	9752.5	5887.6	5887.6	2.04
1997	4493.5	6416.3	954.3	1075.2	4386.6	1908.8	9992.7	6295.5	6295.5	1.98
1998	4784.3	6848.7	953.9	1135.2	4734.9	1932.4	10292.1	6667.4	6667.4	1.99
1999	5132.9	7384.0	989.1	1270.5	5044.8	2057.8	10610.0	7102.6	7102.6	1.96
2000	5547.7	7972.0	1093.5	1496.6	5438.9	2130.0	10952.2	7568.9	7568.9	1.94
2001	5805.3	8174.0	1027.6	1419.2	5589.3	2193.0	11298.9	7782.3	7782.3	2.08
2002	6053.2	8472.2	1003.0	1450.2	5696.5	2328.5	11498.1	8025.0	8025.0	2.02
2003	6387.3	8937.3	1040.7	1566.4	5894.3	2517.3	11646.3	8411.6	8411.6	2.01
2004	6768.5	9610.5	1180.4	1822.3	6185.0	2783.6	11790.9	8968.6	8968.6	1.98
2005	7191.5	10297.1	1305.2	2053.1	6499.0	3050.2	11963.8	9549.3	9549.3	2.12
2006	7580.2	10913.7	1471.3	2266.5	6892.7	3225.8	12134.9	10118.5	10118.5	2.21
2007	7969.3	11334.3	1661.9	2403.8	7246.5	3345.9	12360.0	10592.4	10592.4	2.32
2008	8134.7	11402.3	1846.6	2585.6	7385.7	3277.5	12579.8	10663.2	10663.2	2.44
2009	7954.6	10709.2	1587.3	1999.3	7017.7	3279.5	12738.0	10297.2	10297.2	2.58
2010	8300.8	11283.9	1844.6	2384.5	7187.5	3556.4	12704.2	10743.9	10743.9	2.28
2011	8750.8	11822.9	2094.4	2694.1	7507.4	3715.8	12781.7	11223.2	11223.2	2.21

Note that the value of EBS output, V_Y^t, is equal to the corresponding value of input, V_Z^t, for each year t. This is a consequence of our user cost methodology which sets the balancing real rate of return for the entire Extended Business Sector, r^t, to a value which makes the value of inputs equal to the value of outputs in each year t. The last column of Table 31 lists the nominal capital output ratio for the EBS for each year t, V_{KW}^t/V_Y^t. It can be seen that the nominal capital output ratio does not fall as much as the real capital output ratio over the sample period; the nominal ratio essentially fluctuates around its average value of 2.17.

Finally, we compare our TFP estimates for our EBS (TFP^t, the last column of Table 29) with the BLS MFP estimates for the U.S. Private Sector (MFP^t, the last column of Table 8) in Table 32 and Chart 1 below.[57]

Table 32: Extended Business Sector Total Factor Productivity and BLS Multifactor Productivity for the U.S. Private Sector

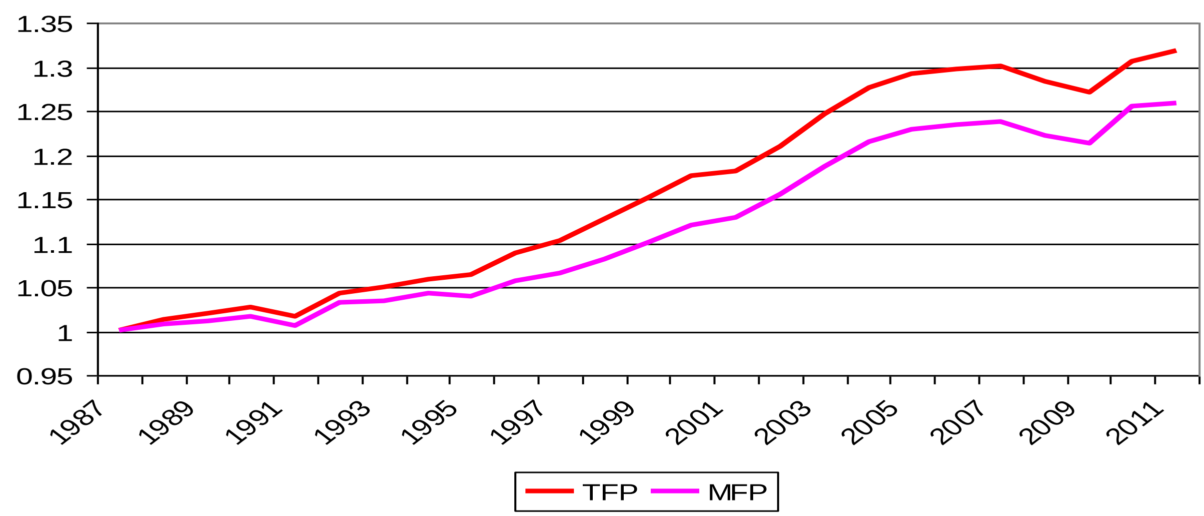
[57] The MFP^t series listed in Table 32 and Chart 1 is actually MFP^t/MFP^{1987}; i.e., we have normalized the BLS Multifactor productivity series listed in Table 8 above to equal unity in 1987.

Year t	TFP^t	MFP^t
1987	1.00000	1.00000
1988	1.01299	1.00793
1989	1.02083	1.01059
1990	1.02797	1.01681
1991	1.01680	1.00677
1992	1.04320	1.03273
1993	1.05013	1.03483
1994	1.05896	1.04246
1995	1.06458	1.03941
1996	1.08787	1.05723
1997	1.10327	1.06575
1998	1.12666	1.08124
1999	1.15206	1.10115
2000	1.17654	1.12023
2001	1.18153	1.12911
2002	1.20963	1.15581
2003	1.24588	1.18699
2004	1.27589	1.21565
2005	1.29127	1.22809
2006	1.29718	1.23363
2007	1.30009	1.23793
2008	1.28269	1.22274
2009	1.27085	1.21350
2010	1.30538	1.25480
2011	1.31797	1.25845
GAG	1.01157	1.00962

The Geometric Average Growth (GAG) rates over the sample period are listed in the last row of Table 32. It can be seen that our TFP growth in the EBS averaged 1.157% per year, somewhat higher than BLS MFP growth in the Private Sector which averaged 0.962% per year, a rate that is about 20% lower. Although on average, these growth rates are not that far apart, Chart 1 shows that the year to year fluctuations, while always in the same direction, can be fairly substantial for some years. Determining the exact causes of these differences is a task for future research.[58]

[58] The most likely sources of the differences are: (i) different scopes, in particular our exclusion of rented housing services and inclusion of the government enterprise sector; (ii) some unit value type aggregation bias in the aggregation of capital in the top down method (see the discussion in Jorgenson (2012; 12), Gu (2012) and Diewert (2012) on this source of bias); (iii) differing land series and (iv) different user cost formulae.

Chart 1: TFP and MFP for the U.S. Economy: 1987-2012



8. Conclusion

Our top down method for constructing the Total Factor Productivity for an Extended U.S. Business Sector generated an annual average growth rate of 1.157% over the period 1987-2011 as compared to the average BLS Multifactor Productivity growth rate for the U.S. Private Business Sector of 0.962% per year for the same period. Thus the top down method gives *roughly* the same productivity growth measures as the BLS bottom up method but there are some significant differences.[59] The exact sources of these differences is a topic for future research.

While constructing our data series, we made some very rough approximations that could be improved using existing data sources.[60] However, in some cases, we found that some aspects of the U.S. data were weak:

- Information on labour input by age, sex, type of worker and education level does not seem to be readily available. It would be good if the BEA could cooperate with the BLS to produce timely (published) quarterly series of disaggregated labour input by industry.[61]
- Accurate information on the price and quantity of business and residential land does not seem to be available and given the huge value of land in the U.S. economy, this is a serious data gap.

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[59] Jorgenson (2012) has some information on U.S. MFP growth over the period 1995-2010 as well as industry information for 65 industries. However, there are no detailed data tables, only charts, so it is difficult to make comparisons of his bottom up estimates with our top down estimates.

[60] In particular, our treatment of taxes was not as thorough as it could have been. Also, we have neglected the roles of infrastructure capital, R&D and other forms of intangible capital and subsoil mineral assets.

[61] The problems involved in making imputations for the labour input of the self employed (and unpaid family workers) should also be explicitly addressed.

- Estimates of R&D stocks and other intangible capital stocks should be integrated into the BEA reproducible capital accounts along with various resource stocks.

The latest international version of the System of National Accounts, *SNA 2008*, recognized the role of capital services in the production accounts. This is a big step forward since it allows inputs in the SNA production accounts to be decomposed into price and quantity components and hence the new SNA guidelines will facilitate the development of productivity accounts for each country that implements the revised SNA. The BEA is moving forward in this respect; see Jorgenson and Landefeld (2006) and Jorgenson (2012). As national statistical agencies make productivity accounts a part of their regular production of the national accounts, there will be benefits to the statistical system as a whole since a natural output of the new system of accounts will be balancing real (or nominal) rates of return by sector or industry. These balancing real rates of return will provide a check on the accuracy of the sectoral data: if the rates are erratic or very large or very small, this can indicate measurement error in the sectoral data and hence will give the statistical agency an early indication of problems with the data.

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