Equality of Educational Opportunity Employing PISA Data: Taking Both Achievement and Access into Account

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Paper Prepared for the IARIW-IBGE Conference
on Income, Wealth and Well-Being in Latin America

Rio de Janeiro, Brazil, September 11-14, 2013

Session 6: Inequality of Opportunity

Time: Friday, September 13, 11:00-12:30

Equality of educational opportunity employing PISA data: Taking both achievement and access into account*

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August, 2013

Abstract: While PISA datasets have been used for measuring equality of opportunity in education they have a limitation: samples cover a relatively limited fraction of developing countries' cohorts of 15-year-olds, casting doubts on the reliability of comparisons. To address that, we employ a bidimensional index, with achievement-EOp and access-EOp as the dimensions, and a Cobb-Douglas functional form. Employing recent PISA data for six Latin-American countries, rank reversals are observed with respect to orderings based upon a single dimension. The index can be generalized to include further dimensions, such as average scores.

Keywords: equality of opportunity, multidimensional measures, PISA, Latin America.

JEL Classification: I24, O54.

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* The authors would like to thank financial support provided by CAPES/PVE, CNPq/UFF ("Programa de Iniciação Científica") and the Universidad del Rosario. We are grateful to comments made by attendants of seminars at Universidade de São Paulo, Universidade Federal Fluminense (Niterói, Brazil), and at the ZEW Workshop on "Equality of Opportunity in Education" (Mannheim, Germany). We are particularly grateful to Erwin Ooghe, Johannes Kunz and Jérémie Gignoux. Finally, we thank the research assistance provided by Kamila Correa and Erika Londoño.

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1. Introduction

Equality of opportunity (EOp) is a liberal-egalitarian theory of justice according to which while inequalities due to different circumstances are intolerable, inequalities due to choices made by the individuals are acceptable (Roemer, 1998). Different methodologies have been proposed attempting to translate it into measuring procedures. (e.g., Checchi & Peragine, 2010; Dunnzlauf et al. 2010). Two recent surveys are available documenting the vast literature produced since 1998 (Pignataro, 2012; Ramos & Van de gaer, 2012).

Measuring inequality of opportunity (InEOp) in education has been the focus of recent contributions, which concentrate either on opportunity for access to a given level of studies (e.g., Paes de Barros et al. 2009; Vega et al. 2010), or on opportunity in terms of educational achievement (e.g., Checchi & Peragine, 2005; Ferreira & Gignoux, 2011;; Gamboa & Waltenberg, 2012).

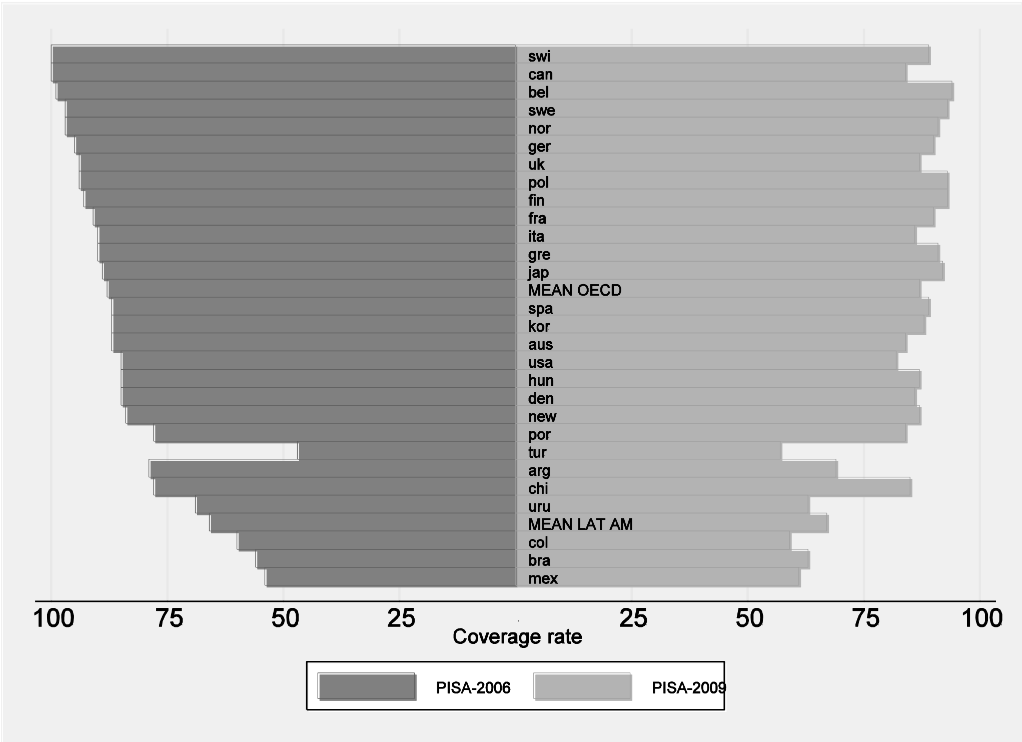
Pupils' educational achievement is usually measured by standardized test scores, such as those made available by OECD's *Programme for International Student Assessment* (PISA). While PISA datasets present many well-known virtues, they have some limitations, particularly in terms of the fraction of the population of individuals represented by the samples. There are different reasons for excluding individuals from evaluations, which can be divided in two categories: (i) exclusions based on technical or fortuitous reasons: due to logistic difficulties in the application of the test (e.g., remote regions), and to school-level exclusions related to pupils' physical or intellectual deficiencies, and (ii) exclusions based on recurrent grade repetition or dropout.

While the exclusions for fortuitous or technical reasons affect all countries to a certain extent, they are meaningless from the point of view of equality of educational opportunities; moreover they are not of a substantial magnitude. As for exclusions based on recurrent grade repetition and dropout, while they do not pose major obstacles to equality of educational opportunity in developed countries, they clearly do so in developing countries. Indeed while in developed countries, the coverage rate is systematically larger than 80% – sometimes approaching 100% –, the samples cover a relatively limited fraction of developing countries' cohorts of 15-year-olds, and are not uniform across countries and PISA waves (see Figure 1).[1]

These limitations cast doubts on the reliability of PISA-based measures of inequality of educational opportunities in developing countries, let alone international comparisons.

[1] Along this paper, we shall call this fraction the "coverage rate", defined as the ratio in a given country and year between the *weighted estimate of eligible non-excluded 15-year-olds from the student sample* and the *national 15-year-old population*.

Figure 1: Coverage rates at PISA 2006-2009



Previous attempts of addressing this issue have been ingeniously performed by Ferreira & Gignoux (2011), who have tried to explicitly reconstruct a full sample. Our strategy is of a different nature. Instead of attempting to reconstruct a full sample, we explicitly acknowledge that there are *two different dimensions of educational opportunity*: (i) access and (ii) achievement conditional on access.

Aggregating the two dimensions is not trivial, among other reasons because the relative weight attributed to each dimension will certainly vary according to normative preferences, to the objective of each study, and other reasons. Both dimensions might be viewed as important aspects of equality of educational opportunities: on the one hand, being enrolled (preferably not having repeated too many grades) might be viewed as a necessary condition for learning; on the other hand, access will not be sufficient for learning, if we consider that pupils should not only be at school, but should actually learn something there. Neither objective has been fully attained in the Latin American countries studied in this paper. It can also be argued that in middle-income countries access to schools is more crucial than the relative performance obtained by those individuals for which such advantage is accessible. This is so for those who consider that schools' role is not only to impart abilities and skills that will be converted into high test scores, but also think that schools are a socializing institution, fostering non-cognitive skills among pupils too (Barr, 2012). Being aware that different views are inevitable, we have opted for aggregating the dimensions by means of a Cobb-Douglas specification.

We illustrate our approach for six Latin-American countries that took part in PISA 2006 and 2009. Depending on the relative weight attributed to each dimension, the index we introduce ranks the six countries differently than does a conventional index, exclusively focused on achievement-EOp. In our view the different rankings produced represent a wider spectrum of preferences regarding the relative importance of achievement-EOp with respect to access-EOp than would do a conventional index that focused exclusively on either dimension.

The paper is organized as follows. Section 2 is devoted to explaining the motivation for this study. In Section 3 we describe our approach. Section 4 reports an application for six Latin-American countries. In Section 5 we generalize our method, allowing for further dimensions and weights. Section 6 contains final remarks.

2. PISA's coverage rate problem and attempts to circumvent it

OECD's PISA datasets, collected every three years since 2000, include test scores of representative samples of students in dozens of countries in three subjects – mathematics, sciences and reading – as well as detailed information on students' background and schools' personnel and functioning conditions. The fourth wave (2009), contains samples of about 520 thousand students representing pupils of more than 70 countries (OECD, 2012: 25).

Two related limitations affecting PISA samples should be mentioned. First, samples do not necessarily represent the national population of 15-year-olds in every participating country. The reasons fall into two categories: (i) some eligible pupils do not take the exams for logistic or fortuitous reasons (e.g., pupils living in a remote region, or pupils who were sick the day the exam took place); local managers of PISA exams might also exclude some pupils for physical or intellectual deficiencies (the accepted cases of which are carefully detailed in PISA manuals), and (ii) individuals who are enrolled in a very low grade ("grade 6" or below[2]) or who are not enrolled in schools are not assessed by PISA, being considered "ineligible". While the exclusions for fortuitous or technical reasons affect all countries to a certain extent, they are meaningless from the point of view of educational-EOp. The second category is more problematic. As a consequence, empirical findings based on PISA should not be taken as valid for cohorts of 15-year-old individuals, but for individuals who: (a) have not abandoned the educational system, and (b) have not repeated too many grades. While the actual and the ideal

[2] PISA's "grade 6" corresponds with different names in different countries. PISA technical reports (e.g., OECD, 2009 and 2012) provide tables containing country-by-country corresponding labels.

samples do not differ very much in developed countries, they diverge considerably in developing countries.

The second limitation is a corollary of the first: the proportion of the cohort of 15-year-old individuals which has been excluded is not uniform across countries, or over time. As shown in Figure 1, differences can be substantial casting doubts on the reliability of cross-country comparisons. The coverage rates in 2006 and 2009 are a good example: Argentina (79%, 69%), Brazil (55%, 63%), Chile (78%, 85%), Colombia (60%, 59%), Mexico (54%, 61%) and Uruguay (69%, 63%). These figures reveal that: (i) coverage rates are not high on average, (ii) although all countries come from the same region, cross-country dispersion is substantial, with a range of around 25 percentage points in each year, (iii) there are within-country oscillations across waves (6.5 percentage points on average).

For example, if Mexico turns out to show lower InEop in achievement than Argentina in 2006, it will be unclear whether that reflects larger unfair educational achievement inequality in the latter than in the former, or whether such result is driven by a more homogeneous sample in Mexico (lowest coverage rate in that year) than in Argentina (highest rate). In other words, had all 15-year-old Mexicans taken the exam, would Mexico's good educational EOp record as compared to Argentina's remain?

To address PISA coverage problems, we view at least three paths. The first is simply to be cautious when interpreting results of studies employing PISA for developing countries. That is simple, but risky, since readers (and policymakers) might basically overlook the call for caution. Gamboa & Waltenberg (2012) opt for that path, presenting results based on PISA limited samples and emphasizing that caution is necessary in their interpretation.

A second path consists of explicitly reconstructing full samples. Recently Ferreira & Gignoux (2011) have done so for four countries: Brazil, Indonesia, Mexico, and Turkey. Due to the absence of information in PISA samples about non-participant pupils, it is not possible to perform a correction such as Heckman's procedure (except through appropriate instrumental variables and complementary datasets). They have turned to ancillary datasets (i.e. household surveys), which, however, do not contain information on test scores, and they have then imposed some assumptions in order to undertake two kinds of simulation. The first consists of re-weighting test scores observations in PISA using information taken from the ancillary databases on the fraction of different types of individuals in the population.[3] The second one consists of imputing into the dataset pupils who were not evaluated, ascribing to them scores

[3] They adapt the methodology proposed by DiNardo et al. (1996).

equal to the lowest score obtained by individuals very similar to them according to observable characteristics.

The first simulation, which relies on more conventional assumptions, provides results almost equal to the original results, both in terms of inequality of achievement and of opportunities for achievement. While such somewhat unexpected finding might offer relief for those employing PISA datasets, applying the procedure to other countries/years could lead to more substantial changes. The second simulation leads to substantial differences with respect to the naive calculations, but as Ferreira & Gignoux (2011) themselves acknowledge, their assumptions are "admittedly extreme".

We would like to view our approach as complementary to theirs. Yet, we believe sample corrections still present high costs (especially being data-intensive) and limited benefits (results based on strong assumptions). For these reasons we tend to favor a different path.

3. A bidimensional approach: taking both achievement and access into account

We explicitly assume there are *two dimensions of educational opportunity* – namely, (i) access, and (ii) achievement conditional on access – each of which might rely on a different branch of the EOp literature and thus be measured according to a specific procedure before being aggregated into a composite index.

1. Inequality of opportunity in achievement. We restrict the calculation of InEOp in achievement to the available PISA samples employing Ferreira & Gignoux's (2011) regression-based index of inequality of educational opportunity (hereafter: IO_{FG}). This is calculated as the proportion of the variance of test scores that is explained by a set of circumstances, ranging from $IO_{FG} = 0$ (perfect equality of opportunity) to $IO_{FG} = 1$ (perfect inequality of opportunity). The exercise undertaken is essentially a static decomposition of inequality (as expressed by the variance) into unfair inequality (the R-squared) and fair – more precisely, "residual" – inequality (1 – R-squared).[4]

2. Inequality of opportunity in access: to the best of our knowledge, no consensus has emerged concerning how to adequately measure inequality of opportunity in terms of access, so we propose three methods.

   a. The first consists of employing countries' coverage rates available in PISA technical reports, which might range from 0 (no coverage) to 1 (full coverage). Following the notation employed in Paes de Barros et al. (2009) and Vega et al. (2010) we denote the overall coverage rate by $\overline{p}$, with $0 \leq \overline{p} \leq 1$. The rationale is to sanction each country

[4] Since the R-squared never goes down when new variables are added to the regression, it can be interpreted in the present context as a lower bound for achievement-InEOp. For details see Ferreira & Gignoux (2011).

according to the extent to which its educational system excludes individuals from PISA exams.

b. The second method consists of employing enrollment rates of 15-year-olds, which also range from 0 to 1. The logic is to sanction each country according to the extent to which its educational system excludes individuals from schools (generally).[5]

c. The third method can be seen as an extension to method (a) or (b). It consists of taking into account not only the overall coverage rate for each country (as in method a), or the overall enrolment rate (as in method b), but also *the coverage rate (or the enrolment rate) for different types of a given population*, which might vary across types (e.g., it can be 65% overall, 70% for girls, and 60% for boys). Paes de Barros et al. (2009) compute a "Human Opportunity Index" (*HOI*), as follows: $HOI = \bar{p}.(1-D)$, where: $\bar{p}$ is defined as in (a); and $0 \leq D \leq 1$ stands for a dissimilarity index, which aggregates the distance between the average coverage rate $\bar{p}$ and each type's coverage rate weighted by the relative frequency of each type in the population, with $0 \leq HOI \leq 1$.

Employing the coverage rate or the enrolment rate as a measure of access (methods a and b) might be defended by those who view access exclusively as a function of circumstances, such that any deviation from full coverage should be sanctioned, irrespectively of which groups are excluded, in which case the greater the distance from full coverage ($\bar{p}$ = 1) the larger the inequality of opportunity in that country.

Others might object that what matters for EOp is not the coverage rate (or the enrolment rate) for the whole population, but rather the extent to which different types are excluded – which is exactly the rationale of the term *D* in method (c). The index *HOI* possesses the advantage that while it does not ignore overall opportunities (through the term $\bar{p}$), it is not mute with respect to cross-types differential opportunities, since the term *D* captures cross-types inequalities.[6] However, objections are addressed on the basis that employing *HOI* could mean implicitly assuming that observed inequalities are partly fair, which would be

[5] To that rate could be added information, if available, on the proportion of 15-year-olds who have repeated too many grades, assuming that multiple grade repetition is not only a barrier to being eligible to PISA exams, but also a predictor of restricted life opportunities.

[6] The use of HOI – as opposed to Yalonetsky's (2012) recently proposed dissimilarity index – is endorsed both by Yalonetsky himself and in Pignataro's (2012) literature survey whenever the variable of interest is binary with a clear hierarchy, which is the case here: access (= 1) is more desirable than lack-of-access (= 0). Yalonetsky's (2012) index is indicated for multinomial contexts and for situations in which there is no clear advantage of one case over the other.

inconsistent with EOp theory if one considers that being out of schools (or PISA) are not choices for which teenagers should be held responsible.

Methods (a) and (b) rely on information which can be easily gathered, since coverage rates are readily available in PISA reports and information on enrolment rates can be found, for example, in international reports. Not requiring the handling of further datasets might be an attractive feature for those willing to make international/intertemporal comparisons.

The first method also presents a further advantage when it comes to measuring educational EOp. Since the coverage rate is provided in PISA technical reports, the data employed in both dimensions (achievement and access) come from the same source, which means more clarity and uniformity in definitions.

The remaining step is to aggregate such dimensions into one composite index, which we generically call a "Bidimensional Index of Educational EOp" (or *BIE*). Being aware that different views on the relative importance of each dimension will never be "scientifically resolved", we have opted for aggregating the dimensions by means of a Cobb-Douglas specification allowing different choices of relative weights to each dimension.[7]

Since we have three methods for calculating the access dimension, we obtain three versions of our index (BIE_1, BIE_2, and BIE_3) successively described below.

$$BIE_1 = \underbrace{\bar{p}^a}_{access-EOp} \cdot \underbrace{\left(1 - IO_{FG}\right)^b}_{achievement-EOp} \tag{3.1}$$

with: $0 \le \bar{p} \le 1$, $0 \le IO_{FG} \le 1$, $0 \le BIE_1 \le 1$, $0 \le a \le 1$, $0 \le b \le 1$.

The index BIE_1 is increasing in $\bar{p}$ and decreasing in IO_{FG} as would be desirable. Since IO_{FG} ranges in the interval [0,1], with 0 standing for perfect equality of opportunity in achievement, $(1 - IO_{FG})$ will equal 1 in the case where circumstances are completely unrelated to outcomes for pupils who have taken PISA exams. In such limiting case, BIE_1 will depend solely upon the coverage rate: the higher it is, the larger the educational opportunities offered to 15-year-olds of a given country will be. Conversely, in a case of full coverage, where $\bar{p} = 1$, the index BIE_1 will depend solely upon inequality of opportunity in achievement – a situation that might make sense for analyses of educational EOp in developed countries, where $\bar{p}$ does actually tend to 1.

[7] In a previous version of this study, we had employed fuzzy sets transformations and the results were qualitatively equal to those reported here.

The second version of the index, BIE_2, has exactly the same specification of BIE_1 with enrolment rate replacing $\overline{p}$, so its presentation is skipped.

BIE_3 relies on *HOI* instead of $\overline{p}$ and the achievement dimension remains as it was before:

$$BIE_3 = \underbrace{HOI^a}_{access-EOp} \cdot \underbrace{\left(1 - IO_{FG}\right)^b}_{achievement-EOp} \qquad (3.2)$$

with: $0 \le HOI \le 1$, $0 \le IO_{FG} < 1$, $0 \le BIE_3 \le 1$, $0 \le a \le 1$, $0 \le b \le 1$.

And since $HOI = \overline{p}.(1-D)$, we can rewrite it as:[8]

$$BIE_3 = \underbrace{[\ \underbrace{\overline{p}}_{overall} \ . \underbrace{(1-D)}_{across-types}\ ]^a}_{access-EOp} \cdot \underbrace{\left(1 - IO_{FG}\right)^b}_{achievement-EOp} \qquad (3.3)$$

with: $0 \le \overline{p} \le 1$, $0 \le D < 1$, $0 \le IO_{FG} \le 1$, $0 < BIE_3 \le 1$, $0 \le a \le 1$, $0 \le b \le 1$.

BIE_3 is decreasing in *D* (sanctioning cross-types access-InEOp in a given country). When $\overline{p} \to 1$, most people have access to school (or to PISA exams), which implies full access for each type, such that *D* will tend to zero. Then, BIE_3 will depend essentially upon achievement-EOp. Such case is relevant for advanced countries (e.g., in Switzerland or Canada in 2006,). It should be noticed that while that situation is not what is observed in most developing countries it is implicitly assumed in conventional calculations of inequality of opportunity in achievement, which often include such countries. When $IO_{FG} \to 0$, BIE_3 would depend almost entirely upon access-EOp, which, in turn, would depend upon its two sub-dimensions. Lastly, two countries could show similar IO_{FG} and $\overline{p}$, but could differ in terms of their relative cross-types dissimilarity with respect to access (*D*). Such case might apply to pair-wise comparisons of EOp among countries from a given region.

4. Application

We compare rankings of educational-EOp for six Latin-American countries that took part in PISA 2006 and 2009 using BIE_1 and a ranking obtained from a conventional index that only takes into account achievement-EOp.

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[8] Alternatively, *HOI* could be expressed with the enrolment rate in the place of the coverage rate.

In calculating the achievement dimension, we follow Ferreira & Gignoux (2011), employing PISA's "plausible values" for Mathematics as a measure of test scores and a set of circumstances including: mother and father education, father occupation, stock of educational capital, city size and ownership of specific durables. In this first application, the weights have been set to $a = b = 1$. Results are reported in Table 1.

Table 1. Unidimensional and bidimensional indices of (in)equality of educational opportunity –

Country	Panel A: 2006			
	$\overline{p}$	IO_{FG}	**(1-IO_{FG})**	**BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1**
Argentina	0,7902	0,4974	**0,5026**	**0,3971**
Brazil	0,5551	0,3247	**0,6753**	**0,3748**
Chile	0,7841	0,3175	**0,6825**	**0,5351**
Colombia	0,5988	0,1994	**0,8006**	**0,4795**
Mexico	0,5423	0,3239	**0,6761**	**0,3667**
Uruguay	0,6917	0,2858	**0,7142**	**0,4940**

Country	Panel B: 2009			
	$\overline{p}$	IO_{FG}	**(1-IO_{FG})**	**BIDIMENSIONAL equality of opportunity (achievement and access) BIE_1**
Argentina	0,686	0,550	**0,449**	**0,308**
Brazil	0,632	0,238	**0,762**	**0,481**
Chile	0,852	0,269	**0,731**	**0,623**
Colombia	0,585	0,302	**0,697**	**0,408**
Mexico	0,607	0,308	**0,692**	**0,419**
Uruguay	0,631	0,387	**0,613**	**0,387**

Source of data: PISA 2006 and 2009.

Both in 2006 and in 2009 rank reversals are observed when switching from an EOp index that focuses exclusively on achievement ($1 - IO_{FG}$) to an index that encompasses both achievement-EOp and in access-EOp (BIE_1) and weighs them equally.[9] Argentina is the most opportunity-unequal country in 2006 in terms of achievement, but after taking into account its relatively good coverage rate, it moves to the third position in educational EOp. Chile also

[9] Bootstrapped standard errors indicate that the difference between the countries' calculated indices are statistically significant. Results are not reported here, but are available upon request.

moves up, from the third to the first position. Colombia and Mexico do the opposite movement. In 2009, Brazil and Chile, as well as Colombia and Mexico exchange their positions.

Our results can be plotted over a map of iso-opportunity curves, as do Barros et al., 2009, simultaneously showing where a country stands in either dimension (in each axis), and in terms of overall educational EOp (the level curves). Figure 2a contains iso-opportunity curves for PISA 2006. Points representing Brazil and Mexico, which lie below the curve "BIE_1=0,4", are to be contrasted with Chile's, which is above the curve "BIE_1=0,5". It is interesting to compare Argentina and Colombia: while the former performs relatively well in the access-EOp dimension and poorly in the achievement-EOp dimension, the latter presents the opposite situation. With equal weights, Colombia reaches a higher iso-opportunity curve.

Figure 2b: Iso-opportunity curves and EOp in Latin-American countries employing BIE1 , with weights a = b = 1.

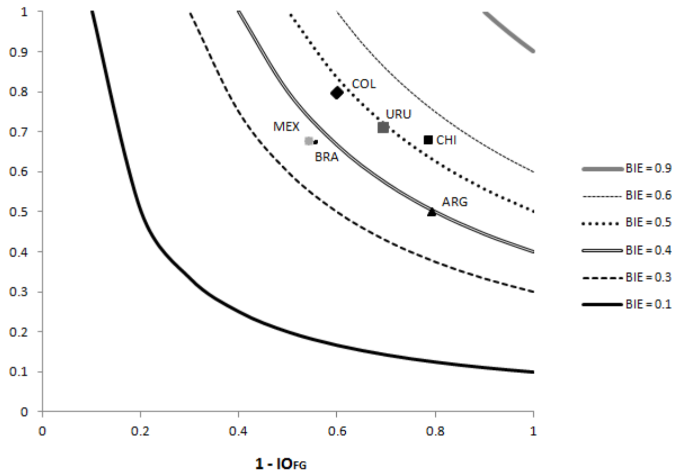


FIGURE 2a.

Figure 2b. Iso-opportunity curves and EOp in Latin-American countries employing BIE1, with weights a = ¾; b = ¼. >

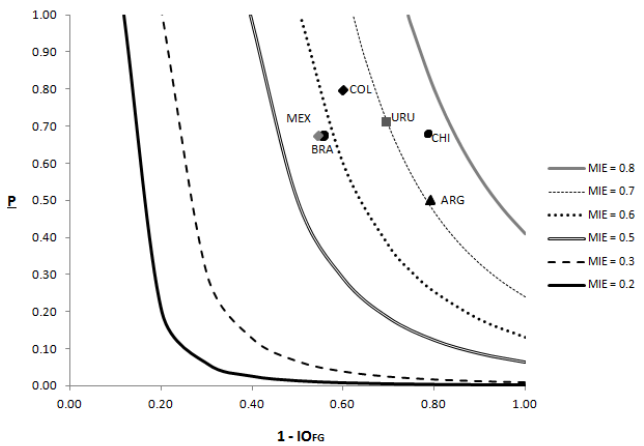
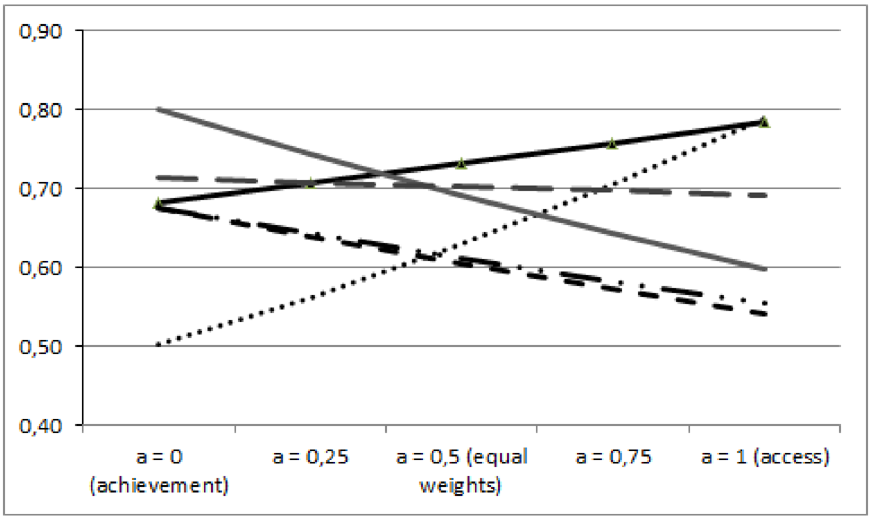


FIGURE 2b.

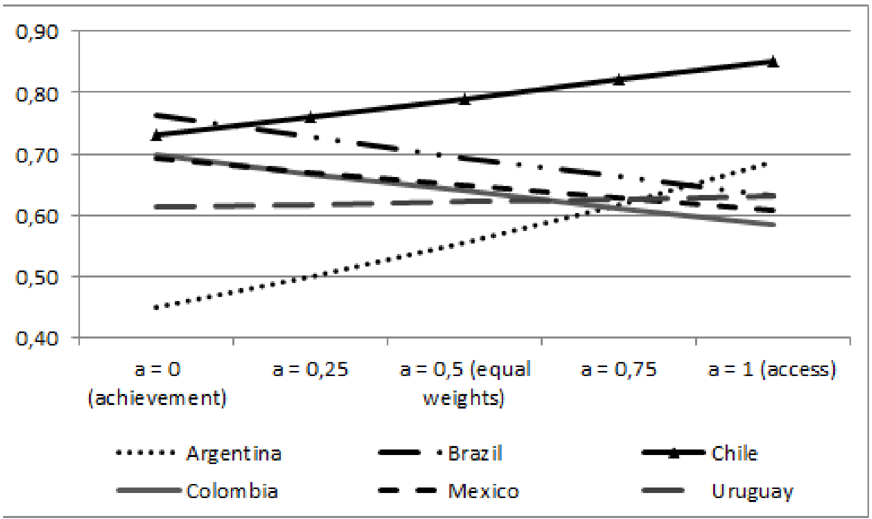
In Figure 2b, we employ different weights for each dimension, privileging the access-EOp dimension (a = ¾; b = 1/4). Uruguay and Argentina, which were in different iso-opportunity curves in Figure 2a, now share the same iso-opportunity curve, since Argentina performs well in access in comparison to other countries. With the new weights, Argentina's iso-opportunity curve is higher than Colombia's.

In Figure 3, we present a wider range of views on the relative importance of access-EOp and achievement-EOp, by ranking our six Latin American countries according to multiple weights.

Figure 3: Equality of educational opportunity in six Latin American countries as measured by BIE1, with varying weights.



PANEL A: PISA 2006



PANEL B: PISA 2009

As expected, as we diminish the relative weight attributed to achievement, we approach rankings based on access alone. Figure 3 also allows a direct visual inspection of strengths and weaknesses of different countries in each dimension of educational EOp. For example, in both years Argentina and Chile increase their level of educational EOp as more emphasis is put on access-EOp (going from left to right in the graph). Brazil, Mexico and Colombia follow an opposite pattern. Uruguay's calculated level of educational EOp is roughly flat across different values for the parameter a.

To some extent Figure 3 summarizes all that has been discussed so far, in three senses. Firstly, it confirms the need to take access into account when dealing with PISA-based analyses encompassing developing countries. Secondly, it shows that different observers might rank differently a given set of countries and thus reach different conclusions regarding educational EOp according to the relative weight attributed to each dimension. Thirdly, it attests that conventional rankings of educational EOp – either based on access alone or on achievement alone – can be viewed as particular cases of *BIE*.

5. Generalization

To employ bidimensional indices defined in previous sections means working with social welfare functions – or, more precisely, "equality-of-opportunity functions" – based upon certain attributes, *BIE* = *f* (access-EOp, achievement-EOp). However, other attributes could be incorporated, or as well as sub-attributes. Although we presented BIE_3 as a bidimensional index in which one dimension is subdivided in two subdimensions, we were already hinting on three dimensions: $\overline{p}$, (1 – *D*), and (1 - IO_{FG}). More generally, the index could be *multidimensional*.

Average scores for example might be a relevant dimension of equality of educational opportunities. Kunz (2012) is concerned with second-generation immigrants in Germany, whose average score in reading in PISA-2009 is 474. What would a Latin-American mother who is considering whether to immigrate to Germany answer if questioned about where her children would have more educational opportunities: in her home country where average scores are lower than those of immigrants in Germany, or in Germany? She might prefer Germany. According to such view, *ceteris paribus*, if country X's average score is higher than country Y's, country X provides more opportunities than country Y.

In Table 2, we show both for 2006 and 2009 the calculated value of multiplying BIE_1 reported in Table 1 by $(\overline{s})$, defined as ratio of the average score in each country to the maximum possible score in PISA (700). Mexico and Brazil exchange positions in this new ranking as compared to Table 1's ranking due to Brazil's extremely low average scores. The same happens in 2009 between Colombia and Uruguay.

Table 2. Multidimensional index: Taking average scores into account.

Country	Panel A: 2006					
	$\overline{p}$	IO_{FG}	BIE_1	Average score	Score as a fraction of the maximum possible score $\overline{s}$ (700)	**MULTIDIMENSIONAL equality of opportunity index**
Argentina	0,790	0,497	0,397	381	0,54	**0,22**
Brazil	0,555	0,325	0,375	370	0,53	**0,20**
Chile	0,784	0,317	0,535	411	0,59	**0,31**
Colombia	0,599	0,199	0,479	370	0,53	**0,25**
Mexico	0,542	0,324	0,367	406	0,58	**0,21**
Uruguay	0,692	0,286	0,494	427	0,61	**0,30**

Country	Panel B: 2009					
	$\overline{p}$	IO_{FG}	BIE_1	Average score	Score as a fraction of the maximum possible score (700) $\overline{s}$	**MULTIDIMENSIONAL equality of opportunity index**
Argentina	0,686	0,550	0,308	388	0,55	**0,17**
Brazil	0,632	0,238	0,481	386	0,55	**0,27**
Chile	0,852	0,268	0,623	421	0,60	**0,37**
Colombia	0,585	0,303	0,408	381	0,54	**0,22**
Mexico	0,607	0,308	0,420	419	0,60	**0,25**
Uruguay	0,631	0,387	0,387	427	0,61	**0,24**

Sources of data: PISA 2006 and 2009.

Those who agree with the view that average scores are a relevant dimension of educational opportunity could advocate the inclusion of four dimensions in a meaningful measure of equality of educational opportunity: achievement, overall access, cross-types dissimilarity with respect to access, and a country's average score, suitably transformed to fit the [0,1] interval. We would then have the following multidimensional index:

$$MIE = (\overline{p})^{\alpha}.(1-D)^{\beta} \cdot (1-IO_{FG})^{\chi}.(\overline{s})^{\delta} \qquad (3.4)$$

where: α, β, γ and δ are (normative) weights, all of them nonnegative and normalized to sum 1.

In Equation 3.4, we have included a country's average score, $(\bar{s})$ as a fourth dimension, but that term can also be interpreted as a subdimension of achievement-EOp. In fact, $(\bar{s})$ is within the achievement dimension the analogue of $(\bar{p})$ within the access dimension, in the sense that both reflect overall educational opportunities available in the country. Similarly the remaining terms, (1 – *D*) and (1 – IO_{FG}), are analogous since both indicate the way the available opportunities are distributed across types.

As argued before, different persons might value differently each dimension of equality of educational opportunity. If someone thinks average scores might have nothing to do with equality of opportunity, it simply requires setting δ = 0. The opposite extreme view is one which would ignore the distributional issues captured by $\bar{p}$, *D* and IO_{FG} and care only about average scores, which would only require setting δ = 1 and α = β = γ = 0. Clearly all those cases can be viewed as particular cases of *MIE*.

6. Final remarks

Measuring educational-EOp has been the focus of recent contributions, which concentrate either on opportunity for access to a given level of studies, or on opportunity in terms of educational achievement. In this paper, we combine both concerns. The method introduced here points out the inappropriateness of EOp indices that overlook participation especially in developing countries where many young individuals still abandon the educational system in early years of their lives, temporarily or for good – in either case, becoming ineligible to PISA and probably facing the prospects of limited life opportunities.

The computation of any multidimensional index requires solving the difficult issue of assigning weights to each dimension in the aggregation step. Our application to Latin-American countries shows how rankings approach unidimensional-EOp orderings as relative weights tend to limiting values (as would be expected). For a sufficiently large weight attributed to the access dimension, countries are ranked very differently than they are by EOp indices concerned only about achievement, confirming the need to take access into account when dealing with PISA-based analyses encompassing developing countries.

It is important to notice that, while motivated by PISA's problem, the index offered here is applicable with due adaptations to analyses that employ other datasets presenting similar problems.

If a unidimensional index can be viewed as a particular case of a bidimensional index, the latter, in turn, can be viewed as a particular case of a multidimensional one. There is no reason why other relevant aspects reflecting educational opportunities should not be taken into account.

In addition to incorporating further dimensions, there are other paths for deepening or extending what has been done here. Regarding the achievement-EOp dimension, the sensitivity of results to different sets of circumstances included in the regression-based estimations could be tested. As for the access-EOp dimension, the results presented here could be compared to those relying on other definitions of access-EOp (enrolment rate, or *HOI* based on enrolment, or *HOI* based on coverage rates). It would also be interesting to study the effect of employing alternative aggregating procedures. Finally it might be useful to decompose the contribution to educational-EOp of different dimensions and subdimensions, and to observe how they evolve over time, possibly connecting the observed evolution to policies or shocks affecting pupils and their families.

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