The Dynamics of Productivity Change: A Review of the Bottom-up Approach

Bert M. Balk (Erasmus University)

Paper Prepared for the IARIW-UNSW Conference
on Productivity: Measurement, Drivers and Trends

Sydney, Australia, November 26-27, 2013

Session 6B: Productivity and Efficiency Analysis

Time: Wednesday, November 27, 11:30-12:30

The Dynamics of Productivity Change: A Review of the Bottom-Up Approach

Bert M. Balk
Rotterdam School of Management
Erasmus University
Rotterdam
E-mail bbalk@rsm.nl

Draft, October 15, 2013

Abstract

This paper considers the relation between (total factor) productivity measures for lower level production units and aggregates thereof such as industries, sectors, or entire economies. In particular, this paper contains a review of the so-called bottom-up approach, which takes an ensemble of individual production units, be it industries or enterprises, as the fundamental frame of reference. At the level of industries the various forms of shift-share analysis are reviewed. At the level of enterprises the additional features that must be taken into account are entry (birth) and exit (death) of production units.

Keywords: Producer; productivity; aggregation; decomposition; shift-share analysis; bottom-up approach; index number theory.

JEL code: C43, O47.

1 Introduction

In a previous article (Balk 2010) I considered the measurement of productivity change for a single, consolidated production unit.[1] The present paper continues by studying an ensemble of such units. The classical form is a so-called sectoral shift-share analysis. The starting point of such an analysis is an ensemble of industries, according to some industrial classification (such as ISIC or NAICE), at some level of detail. An industry is a set of enterprises[2] engaged in the same or similar kind of activities. In the case of productivity analysis the ensemble is usually confined to industries for which independent measurement of input and output is available. Such an ensemble goes by different names: business sector, market sector, commercial sector, or simply measurable sector. Data are published and/or provided by official statistical agencies.

Let us, by way of example, consider labour productivity, in particular value-added based labour productivity. The output of industry k at period t is then measured as real value added RVA^{kt}; that is, nominal value added VA^{kt}(= revenue minus intermediate inputs cost) deflated by a suitable, ideally industry-specific, price index. Real value added is treated as 'quantity' of a single commodity, that may or may not be added across the production units belonging to the ensemble studied, and over time. At the input side there is usually given some simple measure of labour input, such as total number of hours worked L^{kt}; rougher measures being persons employed or full time equivalents employed. Then labour productivity of industry k at period t is defined as RVA^{kt}/L^{kt}.

In the ensemble the industries are of course not equally important, thus some weights reflecting relative importance, θ^{kt}, adding up to 1, are necessary. In the literature there is some discussion as to the precise nature of these weights. Should the weights reflect (nominal) value-added shares $VA^{kt}/\sum_k VA^{kt}$? or real value-added shares $RVA^{kt}/\sum_k RVA^{kt}$? or labour input shares $L^{kt}/\sum_k L^{kt}$? We return to this discussion later on.

Aggregate labour productivity at period t is then defined as a weighted

[1] "Consolidated" means that intra-unit deliveries are netted out. In some parts of the literature this is called "sectoral". At the economy level, "sectoral" output reduces to GDP plus imports, and "sectoral" intermediate input to imports.

[2] There is no unequivocal naming here. So, instead of enterprises one also speaks of firms, establishments, plants, or kind-of-activity units. The minimum requirement is that realistic annual profit/loss accounts can be compiled.

mean, either arithmetic $\sum_k \theta^{kt} RVA^{kt}/L^{kt}$ or geometric $\prod_k (RVA^{kt}/L^{kt})^{\theta^{kt}}$, and the focus of interest is the development of such a mean over time. There are clearly two main factors here, shifting importance and shifting productivity, and their interaction. The usual product of a shift-share analysis is a table which by industry and time periods compared provides detailed decomposition results. Special interest can be directed thereby to industries which are ICT-intensive, at the input and/or the output side; industries which are particularly open to external trade; industries which are (heavily) regulated; *etcetera*.

Things become only slightly more complicated when value-added based *total* factor productivity is considered. At the input side one now needs per industry and time period nominal capital and labour cost as well as one or more suitable deflators. The outcome is real primary input, X_{KL}^{kt}, which can be treated as 'quantity' of another single commodity. Total factor productivity of industry k at period t is then defined as RVA^{kt}/X_{KL}^{kt}. The issue of the precise nature of the weights gets some additional complexity, since we now also could contemplate the use of nominal or real cost shares to measure the importance of the various industries.

More complications arise when one wants to base the analysis on *gross-output* based total factor productivity. For the output side of the industries one then needs nominal revenue as well as suitable, industry-specific deflators. For the input side one needs nominal primary and intermediate inputs cost together with suitable deflators. The question of which weights to use is aggravated by the fact that industries deliver to each other, so that part of one industry's output becomes part of another industry's input. Improper weighting can then easily lead to double-counting of productivity effects. This is where so-called Domar weights enter the picture.

Since the early 1990s an increasing number of statistical agencies made (longitudinal) microdata of enterprises available for research. Economists could now focus their research at production units at the lowest level of aggregation and dispense with the age-old concept of the 'representative firm' that had guided so much theoretical development. At the firm or enterprise level one usually has access to nominal data about output revenue and input cost detailed to various categories, in addition to data about employment and some aspects of financial behaviour. Lowest level quantity data are usually not available, so that industry-level deflators must be used. Also, at the enterprise level the information available is generally insufficient to construct firm-specific capital stock data. Notwithstanding such practical restrictions, microdata research has spawned and is still spawning lots of interesting results. A landmark contribution, including a survey of older results, is Foster, Haltiwanger and Krizan (2001). Good surveys were provided by Bartelsman and Doms (2000) and, more recently, Syverson (2011). Recent examples of research are collected in a special issue on firm dynamics of the journal *Structural Change and Economic Dynamics* 23 (2012), 325-402.

Of course, dynamics at the enterprise level is much more impressive than at the industry level, no matter how fine-grained. Thought-provoking features are the growth, decline, birth, and death of production units. Split-ups as well as mergers and acquisitions occur all over the place. All this is exacerbated by the fact that the annual microdata sets are generally coming from (unbalanced, rotating) samples, which implies that any superficial analysis of given datasets is likely to draw inaccurate conclusions.

This paper contains a review and discussion of the so-called bottom-up approach, which takes an ensemble of individual production units as the fundamental frame of reference. The top-down approach is the subject of two other papers, namely Balk (2009), (2011b). The theory developed here can be applied to a variety of situations, such as 1) a large company consisting of a number of subsidiaries, 2) an industry consisting of a number of enterprises, or 3) an economy or, more precisely, the 'measurable' part of an economy consisting of a number of industries.

2 Accounting identities

We consider an ensemble (or set) $\mathcal{K}^t$ of consolidated production units[3], operating during a certain time period t in a certain country or region. For each unit the KLEMS-Y *ex post* accounting identity in nominal values (or, in current prices) reads

$$C_{KL}^{kt} + C_{EMS}^{kt} + \Pi^{kt} = R^{kt} \ (k \in \mathcal{K}^t), \tag{1}$$

where C_{KL}^{kt} denotes the primary input cost, C_{EMS}^{kt} the intermediate inputs cost, R^{kt} the revenue, and Π^{kt} the profit (defined as remainder). Intermediate inputs cost (on energy, materials, and business services) and revenue concern generally tradeable commodities. It is presupposed that there is some agreed-on commodity classification, such that C_{EMS}^{kt} and R^{kt} can be written as sums

[3]In terms of variables to be defined below, consolidation means that $C_{EMS}^{kkt} = R^{kkt} = 0$.

of quantities times (unit) prices of these commodities. Of course, for any production unit most of these quantities will be zero. It is also presupposed that output prices are available from a market or else can be imputed. Taxes on production are supposed to be allocated to the K and L classes.

The commodities in the capital class K concern owned tangible and intangible assets, organized according to industry, type, and age class. Each production unit uses certain quantities of those assets, and the configuration of assets used is in general unique for the unit. Thus, again, for any production unit most of the asset cells are empty. Prices are defined as unit user costs and, hence, capital input cost C_K^{kt} is a sum of prices times quantities.

Finally, the commodities in the labour class L concern detailed types of labour. Though any production unit employs specific persons with certain capabilities, it is usually their hours of work that count. Corresponding prices are hourly wages. It is presupposed that, wherever necessary, imputations have been made for self-employed workers. Henceforth, labour input cost C_L^{kt} is a sum of prices times quantities. Total primary input cost is the sum of capital and labour input cost, $C_{KL}^{kt} = C_K^{kt} + C_L^{kt}$. Profit Π^{kt} is the balancing item and thus may be positive, negative, or zero.

The KL-VA accounting identity then reads

$$C_{KL}^{kt} + \Pi^{kt} = R^{kt} - C_{EMS}^{kt} \equiv VA^{kt} \; (k \in \mathcal{K}^t), \tag{2}$$

where VA^{kt} denotes value added, defined as revenue minus intermediate inputs cost. In this paper it will always be assumed that $VA^{kt} > 0$.

Adding-up the KLEMS-Y relations over all the units would imply double-counting because of deliveries between units. To see this, it is useful to split intermediate input cost and revenue into two parts, respectively concerning units belonging to the ensemble $\mathcal{K}^t$ and units belonging to the rest of the world. Thus,

$$C_{EMS}^{kt} = \sum_{k' \in \mathcal{K}^t, k' \neq k} C_{EMS}^{k'kt} + C_{EMS}^{ekt}, \tag{3}$$

where $C_{EMS}^{k'kt}$ is the cost of the intermediate inputs purchased by unit k from unit k', and C_{EMS}^{ekt} is the cost of the intermediate inputs purchased by unit k from the world beyond the ensemble $\mathcal{K}$. Similarly,

$$R^{kt} = \sum_{k' \in \mathcal{K}^t, k' \neq k} R^{kk't} + R^{ket}, \tag{4}$$

where $R^{kk't}$ is the revenue obtained by unit k from delivering to unit k', and R^{ket} is the revenue obtained by unit k from delivering to units outside of $\mathcal{K}^t$. Adding up the KLEMS-Y relations (1) then delivers

$$\sum_{k\in\mathcal{K}^t} C_{KL}^{kt} + \sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K},k'\neq k} C_{EMS}^{k'kt} + \sum_{k\in\mathcal{K}^t} C_{EMS}^{ekt} + \sum_{k\in\mathcal{K}^t}\Pi^{kt} =$$

$$\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} R^{kk't} + \sum_{k\in\mathcal{K}^t} R^{ket}. \tag{5}$$

If for all the tradeable commodities output prices are identical to input prices (which is ensured by National Accounting conventions), then the two intra-$\mathcal{K}^t$-trade terms cancel, and the foregoing expression reduces to

$$\sum_{k\in\mathcal{K}^t} C_{KL}^{kt} + \sum_{k\in\mathcal{K}} C_{EMS}^{ekt} + \sum_{k\in\mathcal{K}^t}\Pi^{kt} = \sum_{k\in\mathcal{K}^t} R^{ket}. \tag{6}$$

This is the KLEMS-Y accounting relation for the ensemble $\mathcal{K}^t$, considered as a consolidated production unit. The corresponding KL-VA relation is then

$$\sum_{k\in\mathcal{K}^t} C_{KL}^{kt} + \sum_{k\in\mathcal{K}^t}\Pi^{kt} = \sum_{k\in\mathcal{K}^t} R^{ket} - \sum_{k\in\mathcal{K}^t} C_{EMS}^{ekt}, \tag{7}$$

which can be written as

$$C_{KL}^{\mathcal{K}^t t} + \Pi^{\mathcal{K}^t t} = R^{\mathcal{K}^t t} - C_{EMS}^{\mathcal{K}^t t} \equiv VA^{\mathcal{K}^t t}. \tag{8}$$

where $C_{KL}^{\mathcal{K}^t t} \equiv \sum_{k\in\mathcal{K}^t} C_{KL}^{kt}$, $\Pi^{\mathcal{K}^t t} \equiv \sum_{k\in\mathcal{K}^t}\Pi^{kt}$, $R^{\mathcal{K}^t t} \equiv \sum_{k\in\mathcal{K}^t} R^{ket}$, and $C_{EMS}^{\mathcal{K}^t t} \equiv \sum_{k\in\mathcal{K}^t} C_{EMS}^{ekt}$. One verifies immediately that

$$VA^{\mathcal{K}^t t} = \sum_{k\in\mathcal{K}^t} VA^{kt}. \tag{9}$$

The similarity between expressions (2) and (8), together with the additive relation between all the elements, is the reason why the KL-VA production model is the natural starting point for studying the relation between individual and aggregate measures of productivity change. We will see however that the bottom-up approach basically neglects this framework.

3 Continuing, entering, and exiting production units

As indicated in the previous section the superscript t denotes a time period, the usual unit of measurement being a year. Though data may be available over a longer time span, any comparison is concerned with only two periods: an earlier period 0 (also called base period), and a later period 1 (also called comparison period). These periods may or may not be adjacent. When the production units are industries, then the ensemble $\mathcal{K}^0$ will usually be the same as $\mathcal{K}^1$. But when the production units studied are enterprises, this will in general not hold, and we must distinguish between continuing, exiting, and entering production units. In particular,

$$\mathcal{K}^0 = \mathcal{C}^{01} \cup \mathcal{X}^0 \tag{10}$$

$$\mathcal{K}^1 = \mathcal{C}^{01} \cup \mathcal{N}^1, \tag{11}$$

where $\mathcal{C}^{01}$ denotes the set of continuing units (that is, units active in both periods), $\mathcal{X}^0$ the set of exiting units (active in the base period only), and $\mathcal{N}^1$ the set of entering units (active in the comparison period only). The sets $\mathcal{C}^{01}$ and $\mathcal{X}^0$ are disjunct, as are $\mathcal{C}^{01}$ and $\mathcal{N}^1$.

Of course, when the production units studied form a balanced panel, then the sets $\mathcal{X}^0$ and $\mathcal{N}^1$ are empty. The same holds for the case where the production units are industries. These two situations will in the sequel be considered as specific cases.

The theory developed in the remainder of this paper is cast in the language of intertemporal comparisons. By redefining 0 and 1 as countries or regions, and conditioning on a certain time period, the following can also be applied to cross-sectional comparisons. There is one big difference, however. Apart from mergers, acquisitions and the like, enterprises have a certain perseverance and can be observed through time. But a certain enterprise cannot exist at the same time in two countries or regions. Hence, in cross-sectional comparisons the lowest level production units can only be industries, and 'entering' and 'exiting' units correspond to industries existing in only one of the two countries or regions compared.

4 Productivity indices and levels

As explained in the first part of the previous section, the various components of the accounting identity (1) are values, that is, sums of prices times quantities. We are primarily interested in their development through time, as measured by ratios. It is assumed that all the detailed price and quantity data, underlying the values, are accessible. This is, of course, the ideal situation, which in practice is not likely to materialize. Nevertheless, for conceptual reasons it is good to use this as our starting point. More mundane situations, deviating to a higher or lesser degree from the ideal, will then be considered later.

4.1 Indices

Using index number theory, each value ratio can be decomposed as a product of two components, one capturing the price effect and the other capturing the quantity effect. Thus, let there be price and quantity indices such that for any two periods t and t' the following relations hold:

$$C_{KL}^{kt}/C_{KL}^{kt'} = P_{KL}^{k}(t,t')Q_{KL}^{k}(t,t') \quad (12)$$

$$C_{EMS}^{kt}/C_{EMS}^{kt'} = P_{EMS}^{k}(t,t')Q_{EMS}^{k}(t,t') \quad (13)$$

$$R^{kt}/R^{kt'} = P_{R}^{k}(t,t')Q_{R}^{k}(t,t'). \quad (14)$$

Labour cost is part of primary input cost, thus it can also be assumed that there are functions such that

$$C_{L}^{kt}/C_{L}^{kt'} = P_{L}^{k}(t,t')Q_{L}^{k}(t,t'). \quad (15)$$

We are using here the shorthand notation introduced in the earlier article (Balk 2010). All these price and quantity indices are supposed to be, appropriately dimensioned, functions of the prices and quantities at the two periods that play a role in the value ratios; *e.g.* $P_{L}^{k}(t,t')$ is a labour price index for production unit k, based on all the types of labour distinguished, comparing hourly wages at the two periods t and t', conditional on hours worked at these periods. These functions are supposed to satisfy some basic axioms ensuring proper behaviour, and, dependent on the time span between t and t', may be direct or chained indices (see Balk 2008).

The construction of price and quantity indices for value added was discussed in Balk (2010, Appendix B). Thus there are also functions such that

$$VA^{kt}/VA^{kt'} = P^k_{VA}(t,t')Q^k_{VA}(t,t') \tag{16}$$

Formally, the relations (12), (13), (14), (15) and (16) mean that the Product Test is satisfied. Notice that it is not required that all the functional forms of the price and quantity indices be the same. However, the Product Test in combination with the axioms rules out a number of possibilities.

We recall some definitions. The *value-added based total factor productivity index* for period 1 relative to period 0 was defined by Balk (2010) as

$$ITFPROD^k_{VA}(1,0) \equiv \frac{Q^k_{VA}(1,0)}{Q^k_{KL}(1,0)}. \tag{17}$$

This index measures the 'quantity' change component of value added relative to the quantity change of all the primary inputs. Similarly, the *value-added based labour productivity index* for period 1 relative to period 0 was defined as

$$ILPROD^k_{VA}(1,0) \equiv \frac{Q^k_{VA}(1,0)}{Q^k_{L}(1,0)}. \tag{18}$$

This index measures the 'quantity' change component of value added relative to the quantity change of labour input. Recall that the labour quantity index $Q^k_L(t,t')$ is here defined as an index acting on the prices and quantities of all the types of labour that are being distinguished.

Suppose now that the units of measurement of the various types of labour are in some sense the same; that is, the quantities of all the labour types are measured in hours, or in full-time equivalent jobs, or in some other common unit. Then it makes sense to define the total labour quantity of production unit k at period t as

$$L^{kt} \equiv \sum_{n \in L} x^{kt}_n, \tag{19}$$

and to use the Dutot or simple sum quantity index,

$$Q^{Dk}_L(t,t') \equiv L^{kt}/L^{kt'}. \tag{20}$$

The ratio of a labour quantity index based on types of labour, $Q_L^k(t,t')$, and the simple sum labour quantity index $Q_L^{Dk}(t,t')$ is an index of labour quality (or composition).

The *value-added based simple labour productivity index* for production unit k,

$$ILPROD_{VA}^{Dk}(1,0) \equiv \frac{Q_{VA}^k(1,0)}{L^{kt}/L^{kt'}}, \tag{21}$$

can then be interpreted as an index of real value added per unit of labour.

4.2 Levels

As one sees, some 'level'-language has crept in. The bottom-up approach freely talks about productivity (change) in terms of levels. But what precisely are levels, and what is the relation between levels and indices? Intuitively, indices are just ratios of levels, so that it seems that the difference is merely in the kind of language one prefers. It appears, however, that a closer look is warranted.

For each production unit $k \in \mathcal{K}^t$ *real value added* is (ideally) defined as

$$RVA^k(t,b) \equiv VA^{kt}/P_{VA}^k(t,b); \tag{22}$$

that is, nominal value added at period t divided by (or, as one says, deflated by) a production-unit-k-specific value-added based price index for period t relative to a certain reference period b, where period b may or may not preceed period 0. Notice that this definition tacitly assumes that production unit k, existing in period t, also existed or still exists in period b; otherwise, deflation by a production-unit-k-specific index would be impossible. When production unit k does not exist in period b then for deflation a non-specific index must be used. On the complications thereby we will come back at a later stage.

The foregoing definition implies that

$$RVA^k(b,b) = VA^{kb}/P_{VA}^k(b,b) = VA^{kb}, \tag{23}$$

since, whatever its functional form, for the reference period any price index will deliver the outcome 1. Thus, at the reference period b, real value added equals nominal value added.

For example, one easily checks that when $P^k_{VA}(t,b)$ is a Paasche-type double deflator, then real value added RVA^{kt} is period t value added at prices of period b (recall Balk 2010, Appendix B). The rather intricate form at the left-hand side of expression (22) serves to make clear that unlike VA^{kt}, which is an observable monetary magnitude, $RVA^k(t,b)$ is *the outcome of a function*. Though the outcome is also monetary, its magnitude depends on the reference period and the deflator chosen.

The first kind of dependence becomes clear by considering $RVA^k(t,b')$ for some $b' \neq b$. One immediately checks that $RVA^k(t,b')/RVA^k(t,b) = P^k_{VA}(t,b)/P^k_{VA}(t,b')$, which is a measure of the (k-specific value-added based) price difference between periods b' and b. Put otherwise, real value added depends critically on the price level of the reference period, which is the period for which nominal and real value added coincide.

As to the other dependence, it of course matters whether $P^k_{VA}(t,b)$ is a Paasche-type or a Laspeyres-type or a Fisher-type double deflator. Here the difference in general increases with increasing the time span between the periods t and b.

Like real value added, *real capital-and-labour input*, relative to reference period b, is (ideally) defined as deflated primary input cost,

$$X^k_{KL}(t,b) \equiv C^{kt}_{KL}/P^k_{KL}(t,b), \tag{24}$$

and *real labour input*, relative to reference period b, is (ideally) defined as deflated labour cost,

$$X^k_L(t,b) \equiv C^{kt}_L/P^k_L(t,b), \tag{25}$$

Of course, similar observations as above apply to these two definitions. In particular, it is important to note that at the reference period b real primary input equals nominal input cost, $X^k_{KL}(b,b) = C^{kb}_{KL}$, and real labour input equals nominal labour cost, $X^k_L(b,b) = C^{kb}_L$.

Using the foregoing building blocks, the *value-added based total factor productivity level* of production unit k at period t is defined as real value added divided by real primary input,

$$TFPROD^k_{VA}(t,b) \equiv \frac{RVA^k(t,b)}{X^k_{KL}(t,b)}. \tag{26}$$

Notice that numerator as well as denominator are expressed in the same price level, namely that of period b. Thus $TFPROD^k_{VA}(t,b)$ is a dimensionless variable.

The foregoing definition immediately implies that at the reference period b value-added based total factor productivity equals nominal value added divided by nominal primary input cost, $TFPROD^k_{VA}(b,b) = VA^{kb}/C^{kb}_{KL}$. Now recall the KL-VA accounting identity (2) and assume that profit Π^{kt} is constrained to equal 0 for all production units at all time periods. Then reference period total factor productivity of all production units equals 1, $TFPROD^k_{VA}(b,b) = 1$ $(k \in \mathcal{K}^t)$.

Likewise, the *value-added based labour productivity level* of unit k at period t is defined as real value added divided by real labour input,

$$LPROD^k_{VA}(t,b) \equiv \frac{RVA^k(t,b)}{X^k_L(t,b)}. \quad (27)$$

This is also a dimensionless variable. For the reference period b we obtain

$$LPROD^k_{VA}(b,b) = \frac{VA^{kb}}{C^{kb}_L} = \frac{VA^{kb}}{C^{kb}_{KL}}\frac{C^{kb}_{KL}}{C^{kb}_L}. \quad (28)$$

Hence, when profit $\Pi^{kt} = 0$ for all production units at all time periods then production unit k's labour productivity at reference period b, $LPROD^k_{VA}(b,b)$ equals C^{kb}_{KL}/C^{kb}_L. This is the reciprocal of k's labour cost share at period b.

In case the simple sum quantity index is used for labour, one obtains

$$LPROD^k_{VA}(t,b) = \frac{RVA^k(t,b)}{C^{kt}_L/P^k_L(t,b)} = \frac{RVA^k(t,b)}{C^{kb}_L Q^k_L(t,b)} = \frac{RVA^k(t,b)}{(C^{kb}_L/L^{kb})L^{kt}}, \quad (29)$$

where subsequently expressions (25), (15), and (20) were substituted. The constant in the denominator, C^{kb}_L/L^{kb}, is the mean price of a unit of labour at reference period b. It is not unimportant to notice that real value added per unit of labour, $RVA^k(t,b)/L^{kt}$, is not dimensionless. Its dimension is money-of-period-b per unit of labour.

4.3 Linking levels and indices

We now turn to the relation between levels and indices. One expects that taking the ratio of two levels would deliver an index, but let us have a look. Dividing unit k's total factor or labour productivity level at period 1 by the same at period 0 delivers, using the various definitions and relations (16), (12) and (15),

$$\frac{TFPROD_{VA}^{k}(1,b)}{TFPROD_{VA}^{k}(0,b)} = \frac{Q_{VA}^{k}(1,b)/Q_{VA}^{k}(0,b)}{Q_{KL}^{k}(1,b)/Q_{KL}^{k}(0,b)}, \tag{30}$$

and

$$\frac{LPROD_{VA}^{k}(1,b)}{LPROD_{VA}^{k}(0,b)} = \frac{Q_{VA}^{k}(1,b)/Q_{VA}^{k}(0,b)}{Q_{L}^{k}(1,b)/Q_{L}^{k}(0,b)}, \tag{31}$$

respectively. Surely, if $Q_{VA}^{k}(t,t')$, $Q_{KL}^{k}(t,t')$ and $Q_{L}^{k}(t,t')$ are well-behaving functions then the right-hand sides of expressions (30) and (31) have the form of an output quantity index divided by an input quantity index, both for period 1 relative to period 0. When $b = 0,1$ one easily checks that (30) reduces to $ITFPROD_{VA}^{k}(1,0)$ and that (31) reduces to $ILPROD_{VA}^{k}(1,0)$. But, when $b \neq 0,1$, then

$$TFPROD_{VA}^{k}(1,b)/TFPROD_{VA}^{k}(0,b) = ITFPROD_{VA}^{k}(1,0)$$

if and only if the quantity indices $Q_{VA}^{k}(t,t')$ and $Q_{KL}^{k}(t,t')$ are transitive (that is, satisfy the Circularity Test). Similarly, when $b \neq 0,1$,

$$LPROD_{VA}^{k}(1,b)/LPROD_{VA}^{k}(0,b) = ILPROD_{VA}^{k}(1,0)$$

if and only if the quantity indices $Q_{VA}^{k}(t,t')$ and $Q_{L}^{k}(t,t')$ are transitive. Notice that when the labour quantity index has the Dutot form the last condition is automatically satisfied. Transitive quantity indices, however, are in practice seldom used. Moreover, they would lead to price indices which fail some basic axioms.

4.4 When not all the data are accessible

The word 'ideally' was deliberately inserted in front of definitions (22), (24) and (25). This word reflects the assumption that all the detailed price and quantity data, necessary to compile production-unit-specific price and quantity index numbers, are accessible. In practice, especially in the case of microdata, though the data are *available* at the enterprises — because revenue and cost are sums of quantities produced or used at certain unit prices — they are usually not *accessible* for researchers, due to the excessive cost of obtaining such data, their confidentiality, the response burden experienced by enterprises, or other reasons. In such cases researchers have to fall back at indices which are estimated for a higher aggregation level. This in turn means that real values are contaminated by differential price developments between the production units considered and the higher level aggregate. Let us review a number of situations.

4.4.1 Sectoral studies

The typical sectoral study starts with estimates of nominal value added (that is, value added at current prices) and deflators per industry. These data come from, and are ideally consistent with, the overall National Accounts. Ideally, also the components of value added are given: revenue (or, sales) and cost of intermediate inputs (energy, materials, and services). Using product level Producer Price Indices (PPIs), nominal revenue and the various cost components can be converted into real values, the balance of which is real value added. The ratio of nominal to real value added is then the value-added deflator. Thus, ideally, a value-added deflator turns out to be a so-called *"double deflator"*, which is a technical term saying that a value-added deflator is some function of the revenue and intermediate inputs cost deflators.

The quality of the industry-specific value-added deflators critically depends on the nature of the underlying sample data, their level of detail, and the various compilation, estimation and imputation procedures used; *e.g.* are hedonic methods used to account for quality change? do the procedures take sufficient care of new as well as obsolete products? is sample rotation adequately accounted for?

Sometimes one discovers that there is not enough information to execute a proper deflation of some or all of the cost components, and auxiliary assumptions must be invoked; *e.g.* that the price development of energy is the same as that of (raw) materials, or that the price development of services can be approximated by a Consumer Price Index (CPI).

Sometimes there is no deflator at all for intermediate inputs, so that it turns out that value added is in fact deflated by a revenue-based price index. In such a case one speaks of a *"single deflator"*. An interesting question then arises: what is the best output concept, value added deflated by a single deflator, which means that any differential price change of intermediate inputs relative to gross output contaminates the output measure, or revenue deflated by a specific deflator (that is, gross output is the output measure) and ignoring intermediate inputs entirely? I fear that there is no general answer here, but that it depends on the purpose of the study.[4]

How much harm is caused by the use of an improper deflator for value added?[5] Let our target be to measure simple labour productivity $RVA^k(t,b)/L^{kt}$ for all $k \in \mathcal{K}^t$, where b is some reference period. Suppose that, instead of a value-added based price index $P^k_{VA}(t,b)$, a revenue-based price index $P^k_R(t,b)$ was used to deflate nominal value added VA^{kt}. As one easily checks, this means that, instead of $RVA^k(t,b)$, $RVA^k(t,b)P^k_{VA}(t,b)/P^k_R(t,b)$ is used in the numerator of the labour productivity measure. The bias of the labour productivities is

$$\frac{RVA^k(t,b)}{L^{kt}}\left(\frac{P^k_{VA}(t,b)}{P^k_R(t,b)}-1\right)\ (k \in \mathcal{K}^t), \tag{32}$$

but in general nothing can be said about this distribution. And how large the bias is for any particular industry can only be determined in a case-by-case analysis for which detailed industry data are indispensable.

Is there an aggregate effect? Suppose we are interested in weighted mean simple labour productivity, $\sum_k \theta^k RVA^k(t,b)/L^{kt}$, with certain weights θ^k summing to 1. The use of improper deflators then means that we are looking instead at $\sum_k \theta^k(P^k_{VA}(t,b)/\ P^k_R(t,b))RVA^k(t,b)/L^{kt}$. Put otherwise, the simple labour productivities are not weighted by θ^k but by $\theta^k(P^k_{VA}(t,b)/P^k_R(t,b))$ $(k \in \mathcal{K}^t)$, and these weights don't sum to 1. Now it appears that, using the definition of covariance,

$$\begin{aligned}
&\sum_k \theta^k(P^k_{VA}(t,b)/P^k_R(t,b))RVA^k(t,b)/L^{kt} = \\
&\quad \sum_k \theta^k(P^k_{VA}(t,b)/P^k_R(t,b))\sum_k \theta^k RVA^k(t,b)/L^{kt} \\
&\quad + \mathrm{Covar}\left(P^k_{VA}(t,b)/P^k_R(t,b), RVA^k(t,b)/L^{kt};\theta^k\right),
\end{aligned} \tag{33}$$

[4]See also Timmer *et al.* (2010, 219-221).

[5]A study casting empirical light on this question is Cassing (1996). In terms of our notation she compared a. single-deflated value added, that is $VA^{kt}/P^k_R(t,b)$, where the revenue-based price index is a Törnqvist, b. double-deflated value added, that is $VA^{kt}/P^k_{VA}(t,b)$, where the value-added based price index is a Paasche, and c. double-deflated value added where the price index is a Törnqvist. Using Indonesian manufacturing data over the period 1975-1988 she found the differences in the development of deflated value added more striking the less aggregated the data.

where Covar$(a^k, b^k; \theta^k)$ denotes the (finite) covariance of scalar variables a^k and b^k, weighted by θ^k, where the range of k is a finite set. Assume that this covariance equals 0. Then the actual mean $\sum_k \theta^k (P^k_{VA}(t,b)/P^k_R(t,b))$ $RVA^k(t,b)/L^{kt}$ differs from the target mean $\sum_k \theta^k RVA^k(t,b)/L^{kt}$ by the factor $\sum \theta^k (P^k_{VA}(t,b)/P^k_R(t,b))$. This factor is the mean ratio of the value-added based deflators and the revenue based deflators. The magnitude of such a mean, and especially whether it is greater or less than 1, is an empirical issue. Though in individual cases of little help, the mean ratio can be used to adjust the actually measured labour productivities so that their mean be equal to the target mean.

The message of the preceding two paragraphs is simply that the way value added is deflated influences the distributions of the ensueing productivity levels; and the same holds at the input side. The situation here can roughly be depicted as follows.

Per industry the labour cost of its employees is usually available, together with an estimate of the number of employees or jobs. Such numbers must be converted to hours worked (or paid) by auxiliary survey data. In an interesting recent study, Burda, Hamermesh and Stewart (2012) showed the effect of using different types of surveys for such conversion factors. Next, estimates of hours worked (or paid) must be generated for self-employed workers. Introducing types of labour, *e.g.* according to gender, skill, education, experience, is usually considered a bridge too far. An interesting application with two types of labour, nationals and immigrants, was recently provided by Kangasniemi, Mas, Robinson and Serrano (2012). Consequentially, labour productivity indices, whether value-added or gross-output based, are usually of the simple type; and quality change of labour is neglected.

For the computation of capital input cost one ideally needs detailed estimates of the capital stock, by industry, type and age of the assets, together with sufficient information to generate unit user costs. A general recipe was outlined by Balk (2011a). If this recipe is followed and all the necessary data are available, then one ends up with quantities and prices and, thus, industry-specific quantity and price indices can be compiled. Differing assumptions, with respect to the rate of return, the extent of asset revaluation, and the utilisation rates have impact on the outcomes. So does the scope of the capital concept: are only tangibles included, or also intangibles, land, and subsoil assets?

The NBER Technical Working Paper by Bartelsman and Gray (1996), though pretty old, is still a useful source of information on the many problems that must be solved in the course of constructing a dataset suitable for analyzing industrial productivity change. The paper also contains some interesting tables highlighting the effect of using this or that type of deflator. A more recent survey was provided by Timmer *et al.* (2010, Sections 3.3-7).

4.4.2 Microdata studies

Let us now turn to microdata, and use Foster, Haltiwanger and Krizan's (2001) study of the U. S. manufacturing sector over the 1977 to 1987 period as a first example. This study was based on plant-level data from the Census of Manufactures (CM). The following quote from the Appendix of their article pictures the data situation:

> "The Census of Manufactures (CM) plant-level data includes value of shipments, inventories, book values of equipment and structures, employment of production and nonproduction workers, total hours of production workers, and cost of materials and energy usage. Real gross output is measured as shipments adjusted for inventories, deflated by the four-digit output deflator for the industry in which the plant is classified. All output and materials deflators used are from the four-digit NBER Productivity Database (Bartelsman and Gray, 1996, recently updated by Bartelsman, Becker and Gray). Labor input is measured by total hours for production workers plus an imputed value for the total hours for nonproduction workers. The latter imputation is obtained by multiplying the number of non-production workers at the plant (a collected data item) times the average annual hours per worker for a nonproduction worker from the Current Population Survey. We construct the latter at the 2-digit industry level for each year and match this information to the CM by year and industry. [...]
>
> Materials input is measured as the cost of materials deflated by the 4-digit materials deflator. Capital stocks for equipment and structures are measured from the book values deflated by capital stock deflators (where the latter is measured as the ratio of the current dollar book value to the constant dollar value for the two-digit industry from Bureau of Economic Analysis data). Energy

> input is measured as the cost of energy usage, deflated by the Gray-Bartelsman energy-price deflator. The factor elasticities are measured as the industry average cost shares, averaged over the beginning and ending year of the period of growth. Industry cost shares are generated by combining industry-level data from the NBER Productivity Database with the Bureau of Labor Statistics (BLS) capital rental prices.
>
> The CM does not include data on purchased services (other than that measured through contract work) on a systematic basis (there is increased information on purchased services over time)."

Some features are worth highlighting. First, there are two types of labour (production and nonproduction workers) and two types of capital assets (equipment and structures) distinguished. Second, the imputation for the hours of nonproduction workers at production unit k is based on information from higher-level aggregates, obtained from sources outside k. Third, all the deflators used to obtain k-specific real values are not k-specific, but at best specific for the next-higher aggregate to which k belongs. This implies that all these real values, at the output as well as the input side, contain residual price differences (between k and the higher aggregate). Put otherwise, any productivity measure is contaminated by price differentials. Fourth, there is no information on services input, so only multi-factor productivity (MFP) measures can be compiled.

Cast in our notation, it appears that the following formula for the MFP level was used by Foster, Haltiwanger and Krizan (2001, 318):

$$MFPROD^k_{FHK}(t,b) \equiv \frac{Y^k(t,b)}{X^k_{Ke}(t,b)^{\alpha_{Ke}} X^k_{Ks}(t,b)^{\alpha_{Ks}} \left(L^{kt}_p + L^{kt}_{np}\right)^{\alpha_L} X^k_E(t,b)^{\alpha_E} X^k_M(t,b)^{\alpha_M}}, \quad (34)$$

where $Y^k(t,b)$ is deflated revenue[6], e denotes equipment, s structures, p production worker, and np nonproduction worker. The denominator is easily recognized as a Cobb-Douglas type aggregator, the α's being average *industry* cost shares, adding up to 1. Notice that the two types of labour

[6]Formally, according to expression (14), $Y^k(t,b) \equiv R^{kt}/P^k_R(t,b)$.

are aggregated by a simple sum function. Labour productivity levels were compiled as

$$LPROD^k_{FHK}(t,b) \equiv \frac{Y^k(t,b)}{L^{kt}_p + L^{kt}_{np}}. \tag{35}$$

Seen against the background of the previous sections all this is clearly not ideal, especially the fact that what is presented as productivity change is contaminated by all kinds of price effects. As Foster, Haltiwanger and Krizan (2001, 354) put it in their concluding remarks:

> "Ideally, we would like to measure outputs, inputs, and associated prices of outputs and inputs at the establishment level in a way that permits the analysis of aggregate productivity growth in the manner discussed in this paper. Current practices at statisticsl agencies are far from this ideal with many of the components collected by different surveys with different units of observation (e.g., establishments vs. companies) and indeed by different statistical agencies."

Our second example is the data description provided in a recent article by Dobbelaere and Mairesse (2013):

> "We use an unbalanced panel of French manufacturing firms over the period 1978-2001, based mainly on firm accounting information from EAE (Enquête Annuelle d'Enterprise, Service des Etudes et Statistiques Industrielles (SESSI). We only keep firms for which we have at least 12 years of observations, ending up with an unbalanced panel of 10,646 firms with the number of observations for each firm varying between 12 and 24. We use real current production deflated by the two-digit producer price index of the French industrial classification as a proxy for output (Y). Labor (L) refers to the average number of employees in each firm for each year and material input (M) refers to intermediate consumption deflated by the two-digit intermediate consumption price index. The capital stock (K) is measured by the gross book value of tangible assets as reported in the firm balance sheets at the beginning of the year (or at the end of the previous year),

> adjusted for inflation. The shares of labor (α_L) and material input (α_M) are constructed by dividing, respectively, the firm total labor cost and undeflated intermediate consumption by the firm undeflated production and by taking the average of these ratios over adjacent years."

It is unclear whether energy and services are included in material input or not, but that does not concern us here. In our notation their total factor productivity index (called Solow residual) for adjacent years t and $t-1$ appears to be

$$IMFPROD^k_{DM}(t,t-1;b) \equiv \frac{Y^k(t,b)/Y^k(t-1,b)}{\left(X^k_K(t,b)/X^k_K(t-1,b)\right)^{1-\alpha_L-\alpha_M}(L^{kt}/L^{kt-1})^{\alpha_L}\left(X^k_M(t,b)/X^k_M(t-1,b)\right)^{\alpha_M}}, \quad (36)$$

where $\alpha_L \equiv \left(C^{kt-1}_L/R^{kt-1} + C^{kt}_L/R^{kt}\right)/2$ and $\alpha_M \equiv \left(C^{kt-1}_M/R^{kt-1} + C^{kt}_M/R^{kt}\right)/2$. Notice that the denominator is a Törnqvist quantity index, and that the indices for output, capital, and material input are dependent on the reference year of the deflators (whereby it is here assumed that all those deflators employ the same reference year). Notice also that the definition of the weights $1-\alpha_L-\alpha_M$, α_L and α_M implies that profit $\Pi^{kt}=0$ for all production units at all time periods.

A pervasive feature of microdata studies, of which we just highlighted two examples, is the use of higher-level instead of production-unit specific deflators. Continuing the analysis of the previous subsection we can ask: how bad is this? Let us again assume that our target is to measure simple labour productivity $RVA^k(t,b)/L^{kt}$ for all $k \in \mathcal{K}^t$, where b is some reference period.[7] Suppose that, instead of the k-specific value-added based price indices $P^k_{VA}(t,b)$, a $\mathcal{K}^t$-specific index $P^{\mathcal{K}^t}_{VA}(t,b)$ were used. The bias distribution would be

$$\frac{RVA^k(t,b)}{L^{kt}}\left(\frac{P^k_{VA}(t,b)}{P^{\mathcal{K}^t}_{VA}(t,b)}-1\right) \; (k \in \mathcal{K}^t), \quad (37)$$

[7] It is assumed here that all units $k \in \mathcal{K}^t$ exist(-ed) in reference period b.

about which in general nothing can be said with any certainty. How large the bias is for particular production units can only be determined in a case-by-case analysis, using unit-specific data, the very data we are missing.

Is there an aggregate effect? Again, suppose we are interested in weighted mean simple labour productivity, $\sum_k \theta^k RVA^k(t,b)/L^{kt}$, where θ^k are certain weights adding up to 1. In this case it appears that

$$\begin{aligned}\sum_k \theta^k (P^k_{VA}(t,b)/P^{\mathcal{K}^t}_{VA}(t,b)) RVA^k(t,b)/L^{kt} = \\ \sum_k \theta^k (P^k_{VA}(t,b)/P^{\mathcal{K}^t}_{VA}(t,b)) \sum_k \theta^k RVA^k(t,b)/L^{kt} \\ + \text{Covar}\left(P^k_{VA}(t,b)/P^{\mathcal{K}^t}_{VA}(t,b), RVA^k(t,b)/L^{kt}; \theta^k\right). \quad (38)\end{aligned}$$

Assume that the covariance equals 0. Then the actual mean labour productivity differs from the target mean by the factor $\sum \theta^k (P^k_{VA}(t,b)/P^{\mathcal{K}^t}_{VA}(t,b))$. This factor is the mean of the *relative* k-specific value-added based deflators, and this mean might be expected to be approximately equal to 1. Thus, in the zero-covariance case there is probably no aggregate effect. However, for determining whether this is indeed the case, one needs all the indididual data. Without such data the situation seems a bit hopeless.

There is some literature on the effect of using industry-level deflators instead of enterprise-level deflators on the estimation of production functions and the analysis of productivity change. See the early study of Abbott (1991) and, more recently, Mairesse and Jaumandreu (2005) and Foster, Haltiwanger and Syverson (2008). Of course, for such studies one needs enterprise-level price data, which severely limits the possibilities. In the literature, productivity based on revenue or value added deflated by an industry-level price index is sometimes called 'revenue productivity', to distinguish it from our concept that is then called '(physical) output productivity'. In a recent contribution Smeets and Warzynski (2013) found that physical productivity exhibited more dispersion than revenue productivity. On the failure of revenue productivity measures to identify within-plant efficiency gains from exporting, see Marin and Voigtländer (2013). From the cross-sectional perspective this issue was studied by Van Biesebroeck (2009).

5 Decompositions: arithmetic approach

Let us now assume that productivity levels, real output divided by real input, are somehow observable.[8] We denote the productivity level of unit k at period t by $PROD^{kt}$. Each production unit comes with some measure of relative size (importance) in the form of a weight θ^{kt}. These weights add up to 1 for each period, that is

$$\sum_{k \in \mathcal{K}^0} \theta^{k0} = \sum_{k \in \mathcal{K}^1} \theta^{k1} = 1. \tag{39}$$

We concentrate here on the productivity levels as introduced in the previous section; that is, $PROD^{kt}$ has the form of real value added divided by real primary input or real labour input. Then, ideally, the relative size measure θ^{kt} must be consistent with either of those measures. Though rather vague, this assumption is for the time being sufficient; we will return to this issue in a later section.

The aggregate (or mean) productivity level at period t is quite naturally defined as the weighted arithmetic average of the unit-specific productivity levels, that is $PROD^t \equiv \sum_k \theta^{kt} PROD^{kt}$, where the summation is taken over all production units existing at period t. The weighted geometric average, which is a natural alternative, will be discussed in the next section.

Aggregate productivity change between periods 0 and 1 is then given by

$$PROD^1 - PROD^0 = \sum_{k \in \mathcal{K}^1} \theta^{k1} PROD^{k1} - \sum_{k \in \mathcal{K}^0} \theta^{k0} PROD^{k0}. \tag{40}$$

Given the distinction between continuing, exiting, and entering production units, as defined by expressions (10) and (11), expression (40) can be decomposed as

$$\begin{aligned} PROD^1 - PROD^0 = \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1} PROD^{k1} \\ & + \sum_{k \in \mathcal{C}^{01}} \theta^{k1} PROD^{k1} - \sum_{k \in \mathcal{C}^{01}} \theta^{k0} PROD^{k0} \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0}. \end{aligned} \tag{41}$$

[8]This section updates Balk (2003, Section 6).

The first term at the right-hand side of the equality sign shows the contribution of entering units, the second and third term together show the contribution of continuing units, whereas the last term shows the contribution of exiting units. The contribution of continuing units, $\sum_{k\in\mathcal{C}^{01}} \theta^{k1} PROD^{k1} - \sum_{k\in\mathcal{C}^{01}} \theta^{k0} PROD^{k0}$, is the joint outcome of intra-unit productivity change, $PROD^{k1} - PROD^{k0}$, and inter-unit relative size change, $\theta^{k1} - \theta^{k0}$, for all $k \in \mathcal{C}^{01}$. The problem of decomposing this joint outcome into the contributions of the two factors happens to be structurally similar to the index number (or indicator) problem. Whereas in index number theory we talk about prices, quantities, and commodities, we are here talking about sizes, productivity levels, and (continuing) production units.

It can thus be expected that in reviewing the various decomposition methods familiar names from index number theory, such as Laspeyres, Paasche, and Bennet, will turn up (see Balk 2008 for the nomenclature).

5.1 The first three methods

The first method decomposes the contribution of the continuing units into a Laspeyres-type contribution of intra-unit productivity change and a Paasche-type contribution of relative size change:

$$\begin{aligned} & PROD^1 - PROD^0 = \\ & \quad \sum_{k\in\mathcal{N}^1} \theta^{k1} PROD^{k1} \\ & \quad + \sum_{k\in\mathcal{C}^{01}} \theta^{k0}(PROD^{k1} - PROD^{k0}) + \sum_{k\in\mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) PROD^{k1} \\ & \quad - \sum_{k\in\mathcal{X}^0} \theta^{k0} PROD^{k0}. \end{aligned} \tag{42}$$

The second term at the right-hand side of the equality sign relates to intra-unit productivity change and uses base period weights. It is therefore, using the language of index number theory, called a Laspeyres-type measure. The third term relates to relative size change and is weighted by comparison period productivity levels. It is therefore called a Paasche-type measure. This decomposition was used in the early microdata study of Baily, Hulten and Campbell (1992).

One feature is important to notice. Disregard for a moment entering and exiting production units. Then aggregate productivity change is entirely due to continuing units, and is the sum of two terms. Suppose that all the units experience productivity increase, that is, $PROD^{k1} > PROD^{k0}$ for all $k \in \mathcal{C}^{01}$. Then aggregate productivity change is not necessarily positive, because the relative-size-change term $\sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) PROD^{k1}$ can exert a negative influence. This 'paradox' was extensively discussed by Fox (2012) and we will return to this issue in a later section.

The net effect of exit and entry is given by the sum of the first and the fourth term, $\sum_{k \in \mathcal{N}^1} \theta^{k1} PROD^{k1} - \sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0}$. This cannot be decomposed any further. However, the following consideration sheds some light on the nature of the contributions of exiters and entrants.

Since base period and comparison period weights add up to 1, we can insert an arbitrary scalar a, and obtain

$$\begin{aligned} PROD^1 - PROD^0 = \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1} (PROD^{k1} - a) \\ & + \sum_{k \in \mathcal{C}^{01}} \theta^{k0} (PROD^{k1} - PROD^{k0}) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) (PROD^{k1} - a) \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0} (PROD^{k0} - a). \end{aligned} \quad (43)$$

Thus, entering units contribute positively to aggregate productivity change insofar their comparison period productivity levels exceed a, and exiting units contribute positively insofar their base period productivity levels fall short of a. However, since there are two different periods involved here, it is not entirely clear what value for a one reasonably could use.

The second method uses a Paasche-type measure for intra-unit productivity change and a Laspeyres-type measure for relative size change. This leads to

$$\begin{aligned} PROD^1 - PROD^0 = \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1} (PROD^{k1} - a) \\ & + \sum_{k \in \mathcal{C}^{01}} \theta^{k1} (PROD^{k1} - PROD^{k0}) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) (PROD^{k0} - a) \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0} (PROD^{k0} - a). \end{aligned} \quad (44)$$

It is possible to avoid the choice between the Laspeyres-Paasche-type and the Paasche-Laspeyres-type decomposition. The third method uses for the contribution of both intra-unit productivity change and relative size change Laspeyres-type measures. However, this simplicity is counterbalanced by the necessity to introduce a covariance-type term:

$$
\begin{aligned}
PROD^1 - PROD^0 = \\
& \sum_{k \in \mathcal{N}^1} \theta^{k1}(PROD^{k1} - a) \\
& + \sum_{k \in \mathcal{C}^{01}} \theta^{k0}(PROD^{k1} - PROD^{k0}) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k0} - a) \\
& + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k1} - PROD^{k0}) \\
& - \sum_{k \in \mathcal{X}^0} \theta^{k0}(PROD^{k0} - a). \qquad (45)
\end{aligned}
$$

In view of the overall Laspeyres-type perspective, a natural choice for a now seems to be $PROD^0$, the base period aggregate productivity level. This leads to the decomposition originally proposed by Haltiwanger (1997) and preferred by Foster, Haltiwanger and Krizan (2001) (there called method 1). The method has been employed *inter alia* by Foster, Haltiwanger and Krizan (2006), Foster, Haltiwanger and Syverson (2008), and Collard-Wexler and De Loecker (2013).[9]

5.2 Interlude: The TRAD, CSLS, and GEA decompositions

Let us pause for a while at this expression and consider the case where there is neither exit nor entry; that is $\mathcal{K}^0 = \mathcal{K}^1 = \mathcal{C}^{01}$. Then expression (45) reduces to

$$PROD^1 - PROD^0 =$$

[9] Altomonte and Nicolini (2012) applied the FHK method to aggregate price-cost margin change. For any individual production unit the price-cost margin was defined as nominal cash flow (= value added minus labour cost) divided by nominal revenue, CF^{kt}/R^{kt}. These margins were weighted by market shares $R^{kt}/\sum_{k \in \mathcal{K}^t} R^{kt}$ $(k \in \mathcal{K}^t)$.

$$\sum_{k\in\mathcal{C}^{01}} \theta^{k0}(PROD^{k1} - PROD^{k0})$$
$$+ \sum_{k\in\mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k0} - a)$$
$$+ \sum_{k\in\mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k1} - PROD^{k0}). \quad (46)$$

In order to transform to (forward looking) percentage changes (aka growth rates) both sides of this expression are divided by $PROD^0$, which delivers

$$\frac{PROD^1 - PROD^0}{PROD^0} =$$
$$\sum_{k\in\mathcal{C}^{01}} \theta^{k0}\frac{PROD^{k0}}{PROD^0}\left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}}\right)$$
$$+ \sum_{k\in\mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})\frac{PROD^{k0} - a}{PROD^0}$$
$$+ \sum_{k\in\mathcal{C}^{01}} \frac{PROD^{k0}}{PROD^0}(\theta^{k1} - \theta^{k0})\left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}}\right). \quad (47)$$

Now consider simple labour productivity, that is, real value added per unit of labour; thus $PROD^{kt} \equiv RVA^k(t,b)/L^{kt}$ ($k \in \mathcal{C}^{01}$). Let the relative size of a production unit be given by its labour share; that is, $\theta^{kt} \equiv L^{kt}/\sum_{k\in\mathcal{C}^{01}} L^{kt}$ ($k \in \mathcal{C}^{01}$). It is straightforward to check that then the weights occurring in the first right-hand side term of (47), $\theta^{k0}(PROD^{k0}/PROD^0)$, reduce to real-value-added shares, $RVA^k(0,b)/\sum_{k\in\mathcal{C}^{01}} RVA^k(0,b)$ ($k \in \mathcal{C}^{01}$), so that

$$\frac{PROD^1 - PROD^0}{PROD^0} =$$
$$\sum_{k\in\mathcal{C}^{01}} \frac{RVA^k(0,b)}{\sum_{k\in\mathcal{C}^{01}} RVA^k(0,b)}\left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}}\right)$$
$$+ \sum_{k\in\mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})\frac{PROD^{k0} - a}{PROD^0}$$
$$+ \sum_{k\in\mathcal{C}^{01}} \frac{PROD^{k0}}{PROD^0}(\theta^{k1} - \theta^{k0})\left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}}\right). \quad (48)$$

In view of the fact that $\sum_{k\in\mathcal{C}^{01}}(\theta^{k1} - \theta^{k0}) = 0$, (48) can also be written as

$$\frac{PROD^1 - PROD^0}{PROD^0} = \sum_{k \in \mathcal{C}^{01}} \frac{RVA^k(0,b)}{\sum_{k \in \mathcal{C}^{01}} RVA^k(0,b)} \left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}} \right) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) \frac{PROD^{k0} - a}{PROD^0} + \sum_{k \in \mathcal{C}^{01}} \frac{PROD^{k0}}{PROD^0} (\theta^{k1} - \theta^{k0}) \left(\frac{PROD^{k1} - PROD^{k0} - b}{PROD^{k0}} \right), \quad (49)$$

for another arbitrary scalar b.

Now, chosing $a = 0$ and $b = 0$ yields the TRAD(itional) way of decomposing aggregate labour productivity change into contributions of the various industries, according to three main sources: a within-sector effect, a reallocation level effect, and a reallocation growth effect respectively (see for various other names and their provenance De Avillez 2012). Chosing $a = PROD^0$ and $b = PROD^1 - PROD^0$ yields the CSLS decomposition (which has been developed at the Centre for the Study of Living Standards).

Finally, let the relative size of a production unit be given by its combined labour and relative price share; that is, $\theta^{kt} \equiv (L^{kt}/\sum_{k \in \mathcal{C}^{01}} L^{kt})(P_{VA}^k(t,b)/P_{VA}^{\mathcal{K}}(t,b))$ ($k \in \mathcal{C}^{01}$), where $P_{VA}^{\mathcal{K}}(t,b)$ is some non-k-specific deflator. Notice that these weights do not add up to 1. It is straightforward to check that in this case the weights occurring in the first right-hand side term of (47), $\theta^{k0}(PROD^{k0}/PROD^0)$, reduce to nominal-value-added shares, $VA^{k0}/\sum_{k \in \mathcal{C}^{01}} VA^{k0}$ ($k \in \mathcal{C}^{01}$), so that

$$\frac{PROD^1 - PROD^0}{PROD^0} = \sum_{k \in \mathcal{C}^{01}} \frac{VA^{k0}}{\sum_{k \in \mathcal{C}^{01}} VA^{k0}} \left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}} \right) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) \frac{PROD^{k0} - a}{PROD^0} + \sum_{k \in \mathcal{C}^{01}} \frac{PROD^{k0}}{PROD^0} (\theta^{k1} - \theta^{k0}) \left(\frac{PROD^{k1} - PROD^{k0}}{PROD^{k0}} \right). \quad (50)$$

For $a = 0$ this appears to be the Generalized Exactly Additive Decomposition (GEAD), going back to Tang and Wang (2004) and explored by Dumagan (2013).

De Avillez (2012) provided an interesting empirical comparison of TRAD, CSLS, and GEAD. He found that "despite some similarities, all three decomposition formulas paint very different pictures of which sectors drove labour productivity growth in the Canadian business sector during the 2000-2010 period." The difference between TRAD and CSLS not unexpectedly hinges on the role played by the scalars a and b. Varying a and/or b implies varying magnitudes of the two reallocation effects, not at the aggregate level — because the sums are invariant — but at the level of individual production units (*i.c.* industries).

The difference between TRAD and CSLS on the one hand and GEAD on the other evidently hinges on the absence or presence of relative price levels in the sectoral measures of importance. De Avillez found it "impossible to say which set of estimates provides a more accurate picture of economic reality because the GEAD formula is, ultimately, measuring something very different from the TRAD and CSLS formulas." I concur insofar this conclusion only means that the answer cannot be found within the bottom-up perspective. The top-down perspective is required to obtain a decision.

5.3 The fourth and fifth method

Let us now return to expression (45). Instead of the Laspeyres perspective, one might as well use the Paasche perspective. The covariance-type term accordingly appears with a negative sign. Thus, the fourth decomposition is

$$\begin{aligned} PROD^1 - PROD^0 = & \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1}(PROD^{k1} - a) \\ & + \sum_{k \in \mathcal{C}^{01}} \theta^{k1}(PROD^{k1} - PROD^{k0}) + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k1} - a) \\ & - \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k1} - PROD^{k0}) \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0}(PROD^{k0} - a). \qquad (51) \end{aligned}$$

The natural choice for a would now be $PROD^1$, the comparison period aggregate productivity level. It is left to the reader to explore the analogs to expressions (49) and (50) by using backward looking percentage changes.

The fifth method avoids the Laspeyres-Paasche dichotomy altogether, by using the symmetric Bennet-type method. This amounts to taking the arithmetic average of the first and the second method. The covariance-type term then disappears. Thus,

$$\begin{aligned} PROD^1 - PROD^0 = & \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1}(PROD^{k1} - a) \\ & +(1/2) \sum_{k \in \mathcal{C}^{01}} (\theta^{k0} + \theta^{k1})(PROD^{k1} - PROD^{k0}) \\ & +(1/2) \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k0} + PROD^{k1} - 2a) \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0}(PROD^{k0} - a). \end{aligned} \tag{52}$$

Baldwin and Gu (2003) suggested to set a equal to the base period productivity level of the exiting production units, $\sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0} / \sum_{k \in \mathcal{X}^0} \theta^{k0}$. Then the exiters-term of expression (52) disappears. Put otherwise, entering units are seen as replacing exiting units, contributing positively to aggregate productivity change insofar their productivity level exceeds that of the exiters.

A more natural choice for a is $(PROD^0 + PROD^1)/2$, the average aggregate productivity level. Substituting this in expression (52), we obtain

$$\begin{aligned} PROD^1 - PROD^0 = & \\ & \sum_{k \in \mathcal{N}^1} \theta^{k1} \left(PROD^{k1} - \frac{PROD^0 + PROD^1}{2} \right) \\ & + \sum_{k \in \mathcal{C}^{01}} \frac{\theta^{k0} + \theta^{k1}}{2} \left(PROD^{k1} - PROD^{k0} \right) \\ & + \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) \left(\frac{PROD^{k0} + PROD^{k1}}{2} - \frac{PROD^0 + PROD^1}{2} \right) \\ & - \sum_{k \in \mathcal{X}^0} \theta^{k0} \left(PROD^{k0} - \frac{PROD^0 + PROD^1}{2} \right). \end{aligned} \tag{53}$$

Thus, entering units contribute positively to aggregate productivity change if their productivity level is above average. Similarly, exiting units contribute positively if their productivity level is below average. Continuing units can contribute positively in two ways: if their productivity level increases, or if the units with above (below) average productivity levels increase (decrease) in relative size. This decomposition basically corresponds to the one used by Griliches and Regev (1995). In view of its symmetry it should be the preferred one. Moreover, Foster, Haltiwanger and Krizan (2001) argue that (53) (there called method 2) is presumably less sensitive to (random) measurement errors than (45). The method was employed by Baily, Bartelsman and Haltiwanger (2001) and Foster, Haltiwanger and Syverson (2008).

Balk and Hoogenboom-Spijker (2003) compared the five methods, defined by (43), (44), (45), (51), and (52) respectively, on micolevel data of the Netherlands manufacturing industry over the period 1984-1999. Though there appeared to be appreciable differences between the various decompositions, the pervasive fact was the preponderance of the productivity change of the continuing units (or, the 'within' term).

5.4 Another five methods

A common feature of the five decomposition methods discussed hitherto is that the productivity levels of exiting and entering production units were compared to a single overall benchmark level a, which moreover happens to be completely arbitrary. It seems more natural to compare the productivity levels of exiting units to the mean level of the continuing units at the base period — which is the period of exit, and to compare the productivity levels of entering units to the mean level of the continuing units at the comparison period — which is the period of entrance.

Thus, let the aggregate productivity level of the continuing production units at period t be defined as $PROD^{\mathcal{C}^{01}t} \equiv \sum_{k\in\mathcal{C}^{01}} \theta^{kt} PROD^{kt} / \sum_{k\in\mathcal{C}^{01}} \theta^{kt}$ $(t = 0, 1)$. Since the weights θ^{kt} add up to 1 for both periods (see expression (39)), expression (40) can be decomposed as

$$\begin{aligned} & PROD^1 - PROD^0 = \\ & \quad \sum_{k\in\mathcal{N}^1} \theta^{k1} \left(PROD^{k1} - PROD^{\mathcal{C}^{01}1} \right) \\ & \quad + PROD^{\mathcal{C}^{01}1} - PROD^{\mathcal{C}^{01}0} \end{aligned}$$

$$- \sum_{k \in \mathcal{X}^0} \theta^{k0} \left(PROD^{k0} - PROD^{\mathcal{C}^{01}0} \right). \tag{54}$$

This expression tells us that entering units contribute positively to aggregate productivity change if their productivity level is above that of the continuing units at the entrance period. Similarly, exiting units contribute positively if their productivity level is below that of the continuing units at the exit period. Let the relative size of continuing units be defined by $\tilde{\theta}^{kt} \equiv \theta^{kt} / \sum_{k \in \mathcal{C}^{01}} \theta^{kt}$ ($k \in \mathcal{C}^{01}; t = 0, 1$). The contribution of the continuing units to aggregate productivity change can then be written as

$$PROD^{\mathcal{C}^{01}1} - PROD^{\mathcal{C}^{01}0} = \sum_{k \in \mathcal{C}^{01}} \tilde{\theta}^{k1} PROD^{k1} - \sum_{k \in \mathcal{C}^{01}} \tilde{\theta}^{k0} PROD^{k0}, \tag{55}$$

which has the same structure as the second and third term of expression (41), the difference being that the weights now add up to 1; that is, $\sum_{k \in \mathcal{C}^{01}} \tilde{\theta}^{kt} = 1$ ($t = 0, 1$). Thus the five methods discussed earlier can simply be repeated on the right-hand side of expression (54). The first four, asymmetric, methods are left to the reader. The symmetric Bennet decomposition delivers the following result,

$$\begin{aligned} & PROD^1 - PROD^0 = \\ & \quad \sum_{k \in \mathcal{N}^1} \theta^{k1} \left(PROD^{k1} - PROD^{\mathcal{C}^{01}1} \right) \\ & \quad + \sum_{k \in \mathcal{C}^{01}} \frac{\tilde{\theta}^{k0} + \tilde{\theta}^{k1}}{2} \left(PROD^{k1} - PROD^{k0} \right) \\ & \quad + \sum_{k \in \mathcal{C}^{01}} (\tilde{\theta}^{k1} - \tilde{\theta}^{k0}) \left(\frac{PROD^{k0} + PROD^{k1}}{2} - a \right) \\ & \quad - \sum_{k \in \mathcal{X}^0} \theta^{k0} \left(PROD^{k0} - PROD^{\mathcal{C}^{01}0} \right). \end{aligned} \tag{56}$$

Now the only place where an arbitrary scalar a can be inserted is in the third term, since the relative weights of the continuing production units add up to 1 in both periods. This decomposition was recently developed by Diewert and Fox (2010).

As we see, continuing units may contribute positively in two ways: if their productivity levels on average increase, or if the units with average productivity levels above (or below) the scalar a increase (or decrease) in relative size. Notice that if there are no exiting or entering units, that is, $\mathcal{K}^0 = \mathcal{K}^1 = \mathcal{C}^{01}$, then the Diewert-Fox method (56) reduces to the Griliches-Regev method (53). I am not aware of any application of the Diewert-Fox method.

5.5 Evaluation

The foregoing overview hopefully demonstrates a number of things.

First, there is no unique decomposition of aggregate productivity change as defined by expression (40).[10]

Second, one should be careful with reifying the different components, in particular the covariance-type term, since this term can be considered as a mere artifact arising from the specific (Laspeyres- or Paasche-) perspective chosen.

Third, the undetermined character of the scalar a lends additional arbitrariness to the first set of five decompositions. At the aggregate level it is easily seen that letting a tend to 0 will lead to a larger contribution of the entering units, the exiting units, and the size change of continuing units, at the expense of the intra-unit productivity change. The advantage of the second set of five decompositions, among which the symmetric Diewert-Fox method, is that the distribution of these four parts is kept unchanged. The remaining arbitrariness in (56) is in the size-change term and materializes only at the level of individual continuing production units.

Fourth, what counts as 'entrant' or 'exiter' depends not only on the length of the time span between the periods 0 and 1, but also on observation thresholds employed in sampling.

All in all it can be expected that the outcome of any decomposition exercise depends to some extent on the particular method favoured by the researcher.

A final note. As demonstrated in the previous section, the productivity levels $PROD^{kt}$ depend on the price reference period of the deflators used. In particular this holds for the simple labour productivity levels $RVA^k(t,b)/L^{kt}$. This dependence obviously extends to aggregate produc-

[10] This non-uniqueness should not come as a surprise and finds its parallel in index number theory (see Balk 2008) and in so-called structural decomposition analysis (widely used in input-output analysis; see Dietzenbacher and Los 1998).

tivity change $PROD^1 - PROD^0$. To mitigate its effect, one considers instead the forward looking growth rate of aggregate productivity $(PROD^1 - PROD^0)/PROD^0$ and its decomposition, obtained by dividing each term by $PROD^0$. It would of course be equally justified to consider the backward looking growth rate $(PROD^1 - PROD^0)/PROD^1$. A symmetric growth rate is obtained when the difference $PROD^1 - PROD^0$ is divided by a mean of $PROD^0$ and $PROD^1$. When the logarithmic mean[11] is used, one obtains

$$(PROD^1 - PROD^0)/L(PROD^0, PROD^1) = \ln(PROD^1/PROD^0), \quad (57)$$

which can be interpreted as a percentage change.[12] However, its decomposition still contains differences such as $PROD^{k1} - PROD^{k0}$, which of course can be transformed into logarithmic differences but at the expense of getting pretty complicated weights.

Thus this calls for going geometric right from the start; this is the topic of the next section.

6 Decompositions: geometric approach

In the geometric approach the aggregate productivity level is defined as a weighted *geometric* average of the unit-specific productivity levels, that is $PROD^t \equiv \prod_k (PROD^{kt})^{\theta^{kt}}$. This is equivalent to defining $\ln PROD^t \equiv \sum_k \theta^{kt} \ln PROD^{kt}$, which implies that, by replacing $PROD$ by $\ln PROD$, the entire story of the previous section can be repeated. We will not do this. It is sufficient to mention that the geometric counterpart of the Diewert-Fox decomposition (56) is

$$\ln PROD^1 - \ln PROD^0 = \\ \sum_{k \in \mathcal{N}^1} \theta^{k1} \left(\ln PROD^{k1} - \ln PROD^{\mathcal{C}^{01}1} \right)$$

[11]The logarithmic mean is, for any two strictly positive real numbers a and b, defined by $L(a,b) \equiv (a-b)/\ln(a/b)$ if $a \neq b$ and $L(a,a) \equiv a$. It has the following properties: (1) $\min(a,b) \leq L(a,b) \leq \max(a,b)$; (2) $L(a,b)$ is continuous; (3) $L(\lambda a, \lambda b) = \lambda L(a,b)$ $(\lambda > 0)$; (4) $L(a,b) = L(b,a)$; (5) $(ab)^{1/2} \leq L(a,b) \leq (a+b)/2$; (6) $L(a,1)$ is concave. See Balk (2008) for details.

[12]Since $\ln(a/a') = \ln(1 + (a-a')/a') \approx (a-a')/a'$ when $(a-a')/a'$ is small

$$
\begin{aligned}
&+ \sum_{k \in \mathcal{C}^{01}} \frac{\tilde{\theta}^{k0} + \tilde{\theta}^{k1}}{2} \left(\ln PROD^{k1} - \ln PROD^{k0} \right) \\
&+ \sum_{k \in \mathcal{C}^{01}} (\tilde{\theta}^{k1} - \tilde{\theta}^{k0}) \left(\frac{\ln PROD^{k0} + \ln PROD^{k1}}{2} - a \right) \\
&- \sum_{k \in \mathcal{X}^0} \theta^{k0} \left(\ln PROD^{k0} - \ln PROD^{\mathcal{C}^{01}0} \right), \qquad (58)
\end{aligned}
$$

where now $\ln PROD^{\mathcal{C}^{01}t} \equiv \sum_{k \in \mathcal{C}^{01}} \theta^{kt} \ln PROD^{kt} / \sum_{k \in \mathcal{C}^{01}} \theta^{kt}$.

Now we are having logarithmic differences all over the place, but with the same weights as in expression (56). The term reflecting productivity change of continuing production units is a mean-share weighted sum of logarithmic differences. This term functionally resembles a Törnqvist price or quantity index.

The advantage of expression (58) over (56) is that logarithmic changes can be interpreted immediately as percentage changes. This holds for the left-hand side of (58) as well as for the first, second, and fourth term at the right-hand side. The disadvantage is that, as an aggregate level measure, a geometric mean $\prod_k (PROD^{kt})^{\theta^{kt}}$ is less easy to understand than an arithmetic mean $\sum_k \theta^{kt} PROD^{kt}$. We let the top-down approach here decide.

7 Monotonicity

As already preluded to, both definitions of aggregate productivity change, the arithmetic and the geometric, suffer from what Fox (2012) called the "monotonicity problem" or "paradox".

Again, disregard for a moment entering and exiting production units. Then aggregate productivity change is entirely due to continuing units, and is the sum or (in the geometric case) product of two terms. Suppose that all the units experience productivity increase, that is, $PROD^{k1} > PROD^{k0}$ for all $k \in \mathcal{C}^{01}$. Then the second right-hand side term in the Diewert-Fox decompositions (56) and (58) is positive. However, aggregate productivity change is not necessarily positive, because the relative-size-change terms $\sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k0} + PROD^{k1})/2$ or $\sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0}) \ln(PROD^{k0} PROD^{k1})^{1/2}$, respectively, can exert a counterbalancing influence.[13]

[13] When there are no entering or exiting units then $\tilde{\theta}^{kt} = \theta^{kt}$ ($k \in \mathcal{C}^{01}; t = 0, 1$).

Fox (2012) noticed that the term $\sum_{k \in \mathcal{C}^{01}} (\theta^{k0} + \theta^{k1})(PROD^{k1} - PROD^{k0})/2$ as such has the desired monotonicity property, and proposed to extend this measure to the set $\mathcal{C}^{01} \cup \mathcal{X}^0 \cup \mathcal{N}^1 = \mathcal{K}^0 \cup \mathcal{N}^1 = \mathcal{X}^0 \cup \mathcal{K}^1$. Aggregate productivity change is then defined as

$$\Delta PROD_{Fox}(1,0) \equiv \sum_{k \in \mathcal{C}^{01} \cup \mathcal{X}^0 \cup \mathcal{N}^1} \frac{\theta^{k0} + \theta^{k1}}{2} \left(PROD^{k1} - PROD^{k0} \right). \tag{59}$$

Now, for all exiting production units, $k \in \mathcal{X}^0$, it is evidently the case that in the later period 1 those units have size zero; that is, $\theta^{k1} = 0$. It is then rather natural to set their virtual productivity level also equal to zero; that is, $PROD^{k1} = 0$. Likewise, entering units, $k \in \mathcal{N}^1$, have size zero in the earlier period 0; that is, $\theta^{k0} = 0$. Their virtual productivity level at that period is also set equal to zero; that is, $PROD^{k0} = 0$. Then (59) can be decomposed as

$$\begin{aligned} \Delta PROD_{Fox}(1,0) = {} & (1/2) \sum_{k \in \mathcal{N}^1} \theta^{k1} PROD^{k1} \\ & + \sum_{k \in \mathcal{C}^{01}} \frac{\theta^{k0} + \theta^{k1}}{2} \left(PROD^{k1} - PROD^{k0} \right) \\ & -(1/2) \sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0}. \end{aligned} \tag{60}$$

Unfortunately, there is no geometric analog to expressions (59) and (60), because the logarithm of a zero productivity level is minus infinity. By using logarithmic means, one obtains

$$\begin{aligned} \Delta PROD_{Fox}(1,0) = {} & (1/2) \sum_{k \in \mathcal{N}^1} \theta^{k1} PROD^{k1} \\ & + \sum_{k \in \mathcal{C}^{01}} \frac{\theta^{k0} + \theta^{k1}}{2} L(PROD^{k0}, PROD^{k1}) \ln \left(PROD^{k1} / PROD^{k0} \right) \\ & -(1/2) \sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0}, \end{aligned} \tag{61}$$

which, however, does not provide any advantage vis à vis expression (60).

It is interesting to compare this proposal to the Griliches-Regev decomposition (53) with $a = 0$. It turns out that

$$\begin{aligned} & PROD^1 - PROD^0 = \\ & \quad \Delta PROD_{Fox}(1,0) \\ & \quad +(1/2) \sum_{k \in \mathcal{N}^1} \theta^{k1} PROD^{k1} \\ & \quad +(1/2) \sum_{k \in \mathcal{C}^{01}} (\theta^{k1} - \theta^{k0})(PROD^{k0} + PROD^{k1}) \\ & \quad -(1/2) \sum_{k \in \mathcal{X}^0} \theta^{k0} PROD^{k0}. \end{aligned} \quad (62)$$

Remarkable is that of the entire contribution of entering and exiting production units to $PROD^1 - PROD^0$, half is considered as productivity change and half as non-productivity change. It is difficult to envisage a solid justification for this.

8 The Olley-Pakes decomposition

Though aggregate, or weighted mean, productivity levels are interesting, researchers are also interested in the *distribution* of the unit-specific levels $PROD^{kt}$ ($k \in \mathcal{K}^t$), and the change of such distributions over time, a good example being Bartelsman and Dhrymes (1998). Given the relative size measures θ^{kt} (recall, adding up to 1) a natural question is whether high or low productivity of a unit goes together with high or low size. Are big firms more productive than small firms? Or are the most productive firms to be found among the smallest? Questions multiply when the time dimension is taken into account. Does the ranking of a particular production unit in the productivity distribution sustain through time? Are firms ranked somewhere in a particular period likely to rank higher or lower in the next period? Is there a relation with the age, however determined, of the production units? Do the productivity distributions, and the behaviour of the production units, differ over the industries?

When it comes to size a natural measure to consider is the covariance of weights and productivity levels. Let $\#(\mathcal{K}^t)$ be the number of units in $\mathcal{K}^t$, let

$\overline{PROD}^t \equiv \sum_{k \in \mathcal{K}^t} PROD^{kt}/\#(\mathcal{K}^t)$ be the unweighted mean of the productivity levels, and let $\bar{\theta}^t \equiv \sum_{k \in \mathcal{K}^t} \theta^{kt}/\#(\mathcal{K}^t) = 1/\#(\mathcal{K}^t)$ be the unweighted mean of the weights. One then easily checks that

$$\sum_{k \in \mathcal{K}^t} (\theta^{kt} - \bar{\theta}^t)(PROD^{kt} - \overline{PROD}^t) = PROD^t - \overline{PROD}^t. \qquad (63)$$

This is a particular instance of a general relation derived by Bortkiewicz in 1923/24. Bortkiewicz showed that the difference between two differently weighted means has the form of a covariance. Interesting applications can be found in index number theory (see Balk 2008).

Olley and Pakes (1996, 1290) rearranged this relation to the form

$$PROD^t = \overline{PROD}^t + \sum_{k \in \mathcal{K}^t} (\theta^{kt} - \bar{\theta}^t)(PROD^{kt} - \overline{PROD}^t) \qquad (64)$$

and provided an interpretation which has been repeated, in various forms, by many researchers.[14] The interpretation usually goes like this: There is some event (say, a certain technological innovation or some other shock) that gives rise to a productivity level $\overline{PROD}^t$; but this productivity level is transformed into an aggregate level $PROD^t$ by means of a mechanism called *reallocation*, the extent of which is measured by the covariance term in expression (64). So it seems that the aggregate productivity level $PROD^t$ is 'caused' by two factors, a productivity shock and a reallocation.[15]

I propose to call this the Olley-Pakes fallacy, because there are not at all two factors. Expression (63) is a mathematical identity: reallocation, defined as a covariance, is identically equal to the difference of two means, a weighted and an unweighted one. All that expression (64) does is featuring the unweighted mean rather than the weighted mean as the baseline variable.

I don't dispute the usefulness of studying time-series or cross-sections of covariances such as we see at the left-hand side of expression (63). Additional insight can be obtained when one replaces productivity levels $PROD^{kt}$ by productivity changes, measured as differences $PROD^{k1} - PROD^{k0}$ or percentage changes $\ln(PROD^{k1}/PROD^{k0})$. As a descriptive device this is won-

[14] It is straightforward to generalize the Olley-Pakes decomposition to the case where the ensemble $\mathcal{K}^t$ consists of a number of disjunct groups. The right-hand side of expression (64) then becomes the sum of a between-groups covariance and for each group an unweighted mean productivity level and a within-group covariance. Collard-Wexler and De Loecker (2013) considered a case of two groups.

[15] Foster, Haltiwanger and Krizan (2001) called expression (64) method 3.

derful, especially for comparing ensembles (industries, economies) — see for instance Lin and Huang (2012) where such covariances are regressed on several background variables. In the cross-country study of Bartelsman, Haltiwanger and Scarpetta (2013) within-industry covariances between size and productivity play a key role.

The Olley-Pakes decomposition (64) can of course be used to decompose aggregate productivity change $PROD^1 - PROD^0$ into two terms, the first being $\overline{PROD}^1 - \overline{PROD}^0$, and the second being the difference of two covariance terms. But then we are unable to distinguish between the contributions of exiting, continuing, and entering production units. Thus, it is advisable to restrict the Olley-Pakes decomposition to the continuing units, and substitute into expression (54). Doing this results in the following expression,

$$\begin{aligned}
&PROD^1 - PROD^0 = \\
&\quad \sum_{k \in \mathcal{N}^1} \theta^{k1} \left(PROD^{k1} - PROD^{\mathcal{C}^{01}1} \right) \\
&\quad + \overline{PROD}^{\mathcal{C}^{01}1} - \overline{PROD}^{\mathcal{C}^{01}0} \\
&\quad + \sum_{k \in \mathcal{C}^{01}} (\tilde{\theta}^{k1} - 1/\#(\mathcal{C}^{01}))(PROD^{k1} - \overline{PROD}^{\mathcal{C}^{01}1}) \\
&\quad - \sum_{k \in \mathcal{C}^{01}} (\tilde{\theta}^{k0} - 1/\#(\mathcal{C}^{01}))(PROD^{k0} - \overline{PROD}^{\mathcal{C}^{01}0}) \\
&\quad - \sum_{k \in \mathcal{X}^0} \theta^{k0} \left(PROD^{k0} - PROD^{\mathcal{C}^{01}0} \right), \qquad (65)
\end{aligned}$$

where $PROD^{\mathcal{C}^{01}t}$ is the weighted mean productivity level and $\overline{PROD}^{\mathcal{C}^{01}t} \equiv \sum_{k \in \mathcal{C}^{01}} PROD^{kt}/\#(\mathcal{C}^{01})$ is the unweighted mean productivity level of the continuing units at period t ($t = 0, 1$); $\#(\mathcal{C}^{01})$ is the number of those units.

This then is the decomposition proposed by Melitz and Polanec (2012). Their paper contains an interesting empirical comparison of the Griliches-Regev method (53), the Foster-Haltiwanger-Krizan method (45), and the extended Olley-Pakes method (65).

Wolf (2011, 21-25) used the Olley-Pakes decomposition to enhance the Griliches-Regev decomposition. By substituting expression (64) into expression (53) one obtains

$$PROD^1 - PROD^0 =$$

$$
\begin{aligned}
&\sum_{k\in\mathcal{N}^1}\theta^{k1}\left(PROD^{k1}-\frac{PROD^0+PROD^1}{2}\right)\\
&+\sum_{k\in\mathcal{C}^{01}}\frac{\theta^{k0}+\theta^{k1}}{2}\left(PROD^{k1}-PROD^{k0}\right)\\
&+\sum_{k\in\mathcal{C}^{01}}(\theta^{k1}-\theta^{k0})\left(\frac{PROD^{k0}+PROD^{k1}}{2}-\frac{\overline{PROD}^0+\overline{PROD}^1}{2}\right)\\
&-\sum_{k\in\mathcal{C}^{01}}(\theta^{k1}-\theta^{k0})\Big(\sum_{k\in\mathcal{K}^0}(\theta^{k0}-\bar{\theta}^0)(PROD^{k0}-\overline{PROD}^0)\\
&+\sum_{k\in\mathcal{K}^1}(\theta^{k1}-\bar{\theta}^1)(PROD^{k1}-\overline{PROD}^1)\Big)/2\\
&-\sum_{k\in\mathcal{X}^0}\theta^{k0}\left(PROD^{k0}-\frac{PROD^0+PROD^1}{2}\right). \qquad (66)
\end{aligned}
$$

As one sees, the original Griliches-Regev 'between' term, the third right-hand side term in expression (53), is split into two parts. The first part, which is the third right-hand side term in the last expression, is relatively easy to understand: it is still a covariance between size changes and mean productivity levels. The second part, which is the fourth right-hand side term in the last expression, is far more complex. This part can be rewritten as $(\sum_{k\in\mathcal{N}^1}\theta^{k1}-\sum_{k\in\mathcal{X}^0}\theta^{k0})$ times a mean covariance (of size and productivity level). It is unclear how this could be interpreted.

9 The choice of weights

The question which weights θ^{kt} are appropriate when a choice has been made as to the productivity levels $PROD^{kt}$ ($k\in\mathcal{K}^t$) has received some attention in the literature. Given that somehow $PROD^{kt}$ is output divided by input, should θ^{kt} be output- or input-based? The literature does not provide us with definitive answers.[16] Indeed, as long as one stays in the bottom-up

[16] As Karagiannis (2013) showed, the issue is not unimportant. He considered the Olley-Pakes decomposition (64) on Greek cotton farm data. Output and input shares were used to weight total factor productivity and labour productivity levels. The covariances turned out to be significantly different. An earlier example was provided by Van Beveren (2012), using firm-level data from the Belgian food and beverage industry. De Loecker and Konings (2006) noted that there is no clear consensus on the appropriate weights (shares)

framework it is unlikely that a convincing answer can be obtained. We need the complementary top-down view. The guiding principle here is: aggregate productivity = productivity of the aggregate.

There are a number of options here. We start with the case where $PROD^{kt}$ is value-added based total factor productivity. Next we consider value-added based labour productivity. Finally we turn to gross-output based labour and total factor productivity respectively.

9.1 Value-added based total factor productivity – general

The top-down approach starts with the adding-up relation (9), repeated here as

$$VA^{\mathcal{K}^t t} = \sum_{k \in \mathcal{K}^t} VA^{kt}. \tag{67}$$

This relation tells us that nominal value added of the ensemble $\mathcal{K}^t$ is the sum of nominal value added of the individual production units k making up this ensemble. Next it is important to recall that the KL-VA accounting identities of the individual units, given by expression (2), are structurally identical to the KL-VA accounting identity of the ensemble (8). This means that we can treat the ensemble as a higher level production unit, and that all the definitions of indices and levels, applied up till now to individual units, can be applied to the ensemble as well.

Recall the definition of real value added in expression (22). Rewriting this expression yields

$$VA^{kt} = P_{VA}^{k}(t,b) RVA^{k}(t,b) \; (k \in \mathcal{K}^t). \tag{68}$$

Nominal value added is here as it were decomposed into a price component and a quantity component. For the ensemble we have similarly

$$VA^{\mathcal{K}^t t} = P_{VA}^{\mathcal{K}^t}(t,b) RVA^{\mathcal{K}^t}(t,b), \tag{69}$$

where $P_{VA}^{\mathcal{K}^t}(t,b)$ is a value-added based price index for the ensemble $\mathcal{K}^t$ for period t relative to a certain reference period b. This index is supposed to be

that should be used. In their work they used employment based shares $L^{kt}/\sum_k L^{kt}$ to weight value-added based total factor productivity indices $Q_{VA}^{k}(t,b)/Q_{KL}^{k}(t,b)$.

estimated from a sample of enterprises and products. Notice that all the price indices, in expressions (68) and (69) are using the same reference period.

Substituting expressions (68) and (69) into (67) and rearranging a bit delivers a relation between real value added of the ensemble and real value added of the individual units,

$$RVA^{\mathcal{K}^t}(t,b) = \sum_{k \in \mathcal{K}^t} \frac{P^k_{VA}(t,b)}{P^{\mathcal{K}^t}_{VA}(t,b)} RVA^k(t,b). \tag{70}$$

It is important to observe that, unlike nominal value added, see expression (67), real value added appears in general to be not additive.

For any individual production unit, real primary input has been defined by expression (24). For the ensemble the corresponding definition reads

$$X^{\mathcal{K}^t}_{KL}(t,b) \equiv C^{\mathcal{K}^t t}_{KL} / P^{\mathcal{K}^t}_{KL}(t,b), \tag{71}$$

where $C^{\mathcal{K}^t t}_{KL} \equiv \sum_{k \in \mathcal{K}^t} C^{kt}_{KL}$ and $P^{\mathcal{K}^t}_{KL}(t,b)$ is a suitable deflator for the primary input of the ensemble $\mathcal{K}^t$. Now, dividing both sides of expression (70) by $X^{\mathcal{K}^t}_{KL}(t,b)$ and inserting at the right-hand side $X^k_{KL}(t,b)/X^k_{KL}(t,b) = 1$ ($k \in \mathcal{K}^t$), one obtains

$$\frac{RVA^{\mathcal{K}^t}(t,b)}{X^{\mathcal{K}^t}_{KL}(t,b)} = \sum_{k \in \mathcal{K}^t} \frac{P^k_{VA}(t,b)}{P^{\mathcal{K}^t}_{VA}(t,b)} \frac{X^k_{KL}(t,b)}{X^{\mathcal{K}^t}_{KL}(t,b)} \frac{RVA^k(t,b)}{X^k_{KL}(t,b)}. \tag{72}$$

Employing the definition of value-added based total factor productivity as given by expression (26), expression (72) can be written as

$$TFPROD^{\mathcal{K}^t}_{VA}(t,b) = \sum_{k \in \mathcal{K}^t} \frac{P^k_{VA}(t,b)}{P^{\mathcal{K}^t}_{VA}(t,b)} \frac{X^k_{KL}(t,b)}{X^{\mathcal{K}^t}_{KL}(t,b)} TFPROD^k_{VA}(t,b). \tag{73}$$

This is our desired result. It means that if $PROD^{kt}$ is defined as value-added based total factor productivity $TFPROD^k_{VA}(t,b)$, then the appropriate weights are given by

$$\phi^{kt} \equiv \frac{P^k_{VA}(t,b)}{P^{\mathcal{K}^t}_{VA}(t,b)} \frac{X^k_{KL}(t,b)}{X^{\mathcal{K}^t}_{KL}(t,b)} \; (k \in \mathcal{K}^t). \tag{74}$$

Appropriate in the sense that when these weights are used, then aggregate productivity $\sum_k \phi^{kt} PROD^{kt}$ can be interpreted as the value-added based total factor productivity of the ensemble considered as a higher level production unit. Notice that the weights ϕ^{kt} ($k \in \mathcal{K}^t$) are not necessarily adding up to 1. Thus, though expression (73) is a weighted sum of individual productivities it is not a mean.

There is, however, another way of looking at expression (73). To see this, notice that $(P_{VA}^k(t,b)/P_{VA}^{\mathcal{K}^t}(t,b))TFPROD_{VA}^k(t,b)$ is so-called revenue total factor productivity; that is, the result of deflating VA^{kt} not by its unit-k-specific deflator $P_{VA}^k(t,b)$ but by the ensemble-specific deflator $P_{VA}^{\mathcal{K}^t}(t,b)$. Weighting these revenue TFPs by real input shares $X_{KL}^k(t,b)/X_{KL}^{\mathcal{K}^t}(t,b)$ then delivers aggregate TFP. Notice that these real input shares also not necessarily add up to 1.

Expression (73) as a relation between aggregate and individual productivities is not unique. To see this, consider instead of value-added the adding-up relation for primary input cost,

$$C_{KL}^{\mathcal{K}^t t} = \sum_{k \in \mathcal{K}^t} C_{KL}^{kt}. \tag{75}$$

Employing (24) and (71), expression (75) can be rewritten as

$$X_{KL}^{\mathcal{K}^t}(t,b) = \sum_{k \in \mathcal{K}^t} \frac{P_{KL}^k(t,b)}{P_{KL}^{\mathcal{K}^t}(t,b)} X_{KL}^k(t,b). \tag{76}$$

It is not unimportant to observe that, unlike nominal primary input cost real primary input appears in general to be not additive. Individual and aggregate real value added were defined by (68) and (69) respectively. Now, dividing both sides of expression (76) by $RVA^{\mathcal{K}^t}(t,b)$ and inserting at the right-hand side $RVA^k(t,b)/RVA^k(t,b) = 1$ ($k \in \mathcal{K}^t$), one obtains

$$\frac{X_{KL}^{\mathcal{K}^t}(t,b)}{RVA^{\mathcal{K}^t}(t,b)} = \sum_{k \in \mathcal{K}^t} \frac{P_{KL}^k(t,b)}{P_{KL}^{\mathcal{K}^t}(t,b)} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \frac{X_{KL}^k(t,b)}{RVA^k(t,b)}. \tag{77}$$

Employing the definition of value-added based total factor productivity as given by expression (26), expression (77) can be written as

$$\left(TFPROD_{VA}^{\mathcal{K}^t}(t,b)\right)^{-1} = \sum_{k \in \mathcal{K}^t} \frac{P_{KL}^k(t,b)}{P_{KL}^{\mathcal{K}^t}(t,b)} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \left(TFPROD_{VA}^k(t,b)\right)^{-1}. \tag{78}$$

This is our alternative result. Thus, aggregate total factor productivity can also be obtained as a weighted *harmonic* sum of individual productivities, with weights

$$\psi^{kt} \equiv \frac{P^k_{KL}(t,b)}{P^{\mathcal{K}^t}_{KL}(t,b)} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)}\ (k \in \mathcal{K}^t). \tag{79}$$

Notice that these weights not necessarily add up to 1. It is interesting to compare the structure of the two sets of weights ϕ^{kt} and ψ^{kt}. The former are based on real primary input shares and relative value-added price levels, whereas the latter are based on real output (value added) shares and relative primary input price levels.

9.2 Value-added based total factor productivity – additivity imposed

A sufficient condition for adding up is that deflators for the ensemble, for value added as well as primary inputs, are Paasche-type indices. This can be seen as follows. Adding up of real value added,

$$RVA^{\mathcal{K}^t}(t,b) = \sum_{k \in \mathcal{K}^t} RVA^k(t,b), \tag{80}$$

is, by inserting the definitions of real value added, equivalent to

$$\frac{1}{P^{\mathcal{K}^t}_{VA}(t,b)} = \sum_{k \in \mathcal{K}^t} \frac{VA^{kt}}{VA^{\mathcal{K}^t t}} \frac{1}{P^k_{VA}(t,b)}. \tag{81}$$

But this relation simply expresses that the value-added based deflator for the ensemble is a Paasche index of the deflators for the individual production units (recall that nominal value added is additive). Similarly, if the primary-inputs based deflator for the ensemble is a Paasche index of the unit-specific deflators, then

$$X^{\mathcal{K}^t}_{KL}(t,b) = \sum_{k \in \mathcal{K}^t} X^k_{KL}(t,b). \tag{82}$$

One checks easily that if conditions (80) and (82) are satisfied, then expression (73) reduces to

$$TFPROD_{VA}^{\mathcal{K}^t}(t,b) = \sum_{k\in\mathcal{K}^t} \frac{X_{KL}^k(t,b)}{X_{KL}^{\mathcal{K}^t}(t,b)} TFPROD_{VA}^k(t,b), \tag{83}$$

and expression (78) reduces to

$$TFPROD_{VA}^{\mathcal{K}^t}(t,b) = \left(\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \left(TFPROD_{VA}^k(t,b) \right)^{-1} \right)^{-1}. \tag{84}$$

In both cases the weights now add up to 1. Employing the fact that an harmonic mean is always less than or equal to an arithmetic mean, one obtains the following inequalities:

$$\left(\sum_{k\in\mathcal{K}^t} \frac{X_{KL}^k(t,b)}{X_{KL}^{\mathcal{K}^t}(t,b)} \left(TFPROD_{VA}^k(t,b) \right)^{-1} \right)^{-1} \leq TFPROD_{VA}^{\mathcal{K}^t}(t,b) \tag{85}$$

$$\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} TFPROD_{VA}^k(t,b) \geq TFPROD_{VA}^{\mathcal{K}^t}(t,b). \tag{86}$$

Equality holds only when all the individual productivities $TFPROD_{VA}^k(t,b)$ ($k \in \mathcal{K}^t$) are the same. The first inequality states that taking an harmonic mean of the individual productivities, weighted with real primary *input* shares, understates aggregate productivity. The second inequality states that taking an arithmetic mean, but now weighted with real *output* (value added) shares, overstates aggregate productivity. Interestingly, the left-hand side of expression (86) is the target variable considered by Olley and Pakes (1996).

We also know that a geometric mean is greater than or equal to an harmonic mean, which implies that, using (84),

$$\prod_{k\in\mathcal{K}^t} \left(TFPROD_{VA}^k(t,b) \right)^{RVA^k(t,b)/RVA^{\mathcal{K}^t}(t,b)} \geq TFPROD_{VA}^{\mathcal{K}^t}(t,b). \tag{87}$$

Such a geometric mean was as target variable considered by Melitz and Polanec (2012). It is thus seen to overstate aggregate productivity.

9.3 Value-added based labour productivity

For value-added based labour productivity the setup of the previous subsections can be repeated. The only thing one needs to do is replacing real primary input by real labour input, as defined by expression (25) for individual production units. Of course a similar definition can be supplied for the ensemble. Finally, we recall that nominal labour cost is additive; that is, $C_L^{\mathcal{K}^t t} = \sum_{k \in \mathcal{K}^t} C_L^{kt}$. Labour productivity was defined by expression (27). Starting from the numerator the result appears to be

$$LPROD_{VA}^{\mathcal{K}^t}(t,b) = \sum_{k \in \mathcal{K}^t} \frac{P_{VA}^k(t,b)}{P_{VA}^{\mathcal{K}^t}(t,b)} \frac{X_L^k(t,b)}{X_L^{\mathcal{K}^t}(t,b)} LPROD_{VA}^k(t,b), \tag{88}$$

and starting from the denominator,

$$LPROD_{VA}^{\mathcal{K}^t}(t,b) = \left(\sum_{k \in \mathcal{K}^t} \frac{P_L^k(t,b)}{P_L^{\mathcal{K}^t}(t,b)} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \left(LPROD_{VA}^k(t,b) \right)^{-1} \right)^{-1}. \tag{89}$$

Both expressions relate value-added based labour productivity of the ensemble, considered as a higher level production unit, to the labour productivities of the constituent production units. Notice that the weights not necessarily add up to 1.

Two special cases deserve our attention. First, when for labour the *simple sum quantity index* is used then labour productivity is given by expression (29) and real labour input by $X_L^k(t,b) = (C_L^{kb}/L^{kb})L^{kt}$. Substitution, for the individual production units as well as for the ensemble, into expression (88) delivers the following expression,

$$\frac{RVA^{\mathcal{K}^t}(t,b)}{L^{\mathcal{K}^t t}} = \sum_{k \in \mathcal{K}^t} \frac{P_{VA}^k(t,b)}{P_{VA}^{\mathcal{K}^t}(t,b)} \frac{L^{kt}}{L^{\mathcal{K}^t t}} \frac{RVA^k(t,b)}{L^{kt}}. \tag{90}$$

At the right-hand side of this expression we have a weighted sum of simple value-added based labour productivities, defined as real value added per unit of labour, $RVA^k(t,b)/L^{kt}$ ($k \in \mathcal{K}^t$). At the left-hand side we have similarly $RVA^{\mathcal{K}^t}(t,b)/L^{\mathcal{K}^t t}$, where $L^{\mathcal{K}^t t}$ is labour input of the ensemble considered as a higher level production unit. It is quite natural to assume that $L^{\mathcal{K}^t t} = \sum_{k \in \mathcal{K}^t} L^{kt}$, which implies that the fractions $L^{kt}/L^{\mathcal{K}^t t}$ ($k \in \mathcal{K}^t$) are labour shares adding up to 1. Notice, however, that these labour shares are premultiplied by relative price levels, so that the weights of the labour productivities do not necessarily add up to 1.

There is, however, another way of looking at expression (90). To see this, notice that $(P^k_{VA}(t,b)/P^{\mathcal{K}^t}_{VA}(t,b))(RVA^k(t,b)/L^{kt})$ is so-called revenue labour productivity; that is, the result of deflating VA^{kt} not by its unit-k-specific deflator $P^k_{VA}(t,b)$ but by the ensemble-specific deflator $P^{\mathcal{K}^t}_{VA}(t,b)$. Weighting these revenue labour productivities by labour shares $L^{kt}/L^{\mathcal{K}^t t}$ then delivers aggregate labour productivity.

Finally, we notice that expression (90) is the model underlying the GEAD expression (50). But it now turns out that an alternative decomposition can be developed.

To see this, notice that by employing the simple sum labour quantity index expression (89) reduces to

$$\frac{RVA^{\mathcal{K}^t}(t,b)}{L^{\mathcal{K}^t t}} = \left(\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \frac{C_L^{kt}/L^{kt}}{C_L^{\mathcal{K}^t t}/L^{\mathcal{K}^t t}} \left(\frac{RVA^k(t,b)}{L^{kt}}\right)^{-1}\right)^{-1}. \quad (91)$$

If the unit labour prices are the same across production units, that is, $C_L^{kt}/L^{kt} = \alpha$ $(k \in \mathcal{K}^t)$, then $C_L^{\mathcal{K}^t t}/L^{\mathcal{K}^t t} = \alpha$, so that expression (91) further reduces to

$$\frac{RVA^{\mathcal{K}^t}(t,b)}{L^{\mathcal{K}^t t}} = \left(\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \left(\frac{RVA^k(t,b)}{L^{kt}}\right)^{-1}\right)^{-1}. \quad (92)$$

Interestingly, the same expression results when we start with the adding-up assumption $L^{\mathcal{K}^t t} = \sum_{k\in\mathcal{K}^t} L^{kt}$, divide left- and right-hand side by $RVA^{\mathcal{K}^t}(t,b)$, and insert at the right-hand side $RVA^k(t,b)/RVA^k(t,b) = 1$ $(k \in \mathcal{K}^t)$. Expression (92) can now be used to develop an alternative to the GEAD.

Second, let us assume that *additivity* holds; that is, $RVA^{\mathcal{K}^t}(t,b) = \sum_{k\in\mathcal{K}^t} RVA^k(t,b)$ and $L^{\mathcal{K}^t t} = \sum_{k\in\mathcal{K}^t} L^{kt}$. Then expression (88) reduces to

$$LPROD^{\mathcal{K}^t}_{VA}(t,b) = \sum_{k\in\mathcal{K}^t} \frac{L^{kt}}{L^{\mathcal{K}^t t}} LPROD^k_{VA}(t,b), \quad (93)$$

and expression (89) reduces to

$$LPROD_{VA}^{\mathcal{K}^t}(t,b) = \left(\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} \left(LPROD_{VA}^k(t,b) \right)^{-1} \right)^{-1}. \tag{94}$$

Now in both cases the weights add up to 1, so that the following inequalities hold:

$$\left(\sum_{k\in\mathcal{K}^t} \frac{L^{kt}}{L^{\mathcal{K}^t t}} \left(LPROD_{VA}^k(t,b) \right)^{-1} \right)^{-1} \leq LPROD_{VA}^{\mathcal{K}^t}(t,b) \tag{95}$$

$$\sum_{k\in\mathcal{K}^t} \frac{RVA^k(t,b)}{RVA^{\mathcal{K}^t}(t,b)} LPROD_{VA}^k(t,b) \geq LPROD_{VA}^{\mathcal{K}^t}(t,b). \tag{96}$$

Thus, a labour-share weighted harmonic mean of labour productivities understates aggregate labour productivity while a real-value-added-share weighted arithmetic mean overstates aggregate labour productivity. The second inequality was also obtained by Van Biesebroeck (2008), albeit in a less direct way.

Notice that the right-hand side of expression (93) is the target variable of the TRAD and CSLS decompositions considered in Section 5.2. Thus these decompositions are consistent; that is, aggregate productivity can be interpreted as productivity of the aggregate. However, underlying this result is the assumption of additivity, which is pretty restrictive.

Since a geometric mean is less then or equal to an arithmetic mean, we find that

$$\prod_{k\in\mathcal{K}^t} \left(LPROD_{VA}^k(t,b) \right)^{L^{kt}/L^{\mathcal{K}^t t}} \leq LPROD_{VA}^{\mathcal{K}^t}(t,b). \tag{97}$$

This geometric mean labour productivity features prominently in Melitz and Polanec (2012). In the Appendix of their paper decompositions based on the left-hand side and the right-hand side of expression (97) are empirically compared.

9.4 Gross-output based productivity

There are not so many microdata studies dealing with the concept of gross-output based productivity. For any individual production unit gross-output based total factor productivity is defined as

$$TFPROD_Y^k(t,b) \equiv \frac{Y^k(t,b)}{X_{KLEMS}^k(t,b)} = \frac{R^{kt}/P_R^k(t,b)}{C_{KLEMS}^{kt}/P_{KLEMS}^k(t,b)} \; (k \in \mathcal{K}^t). \quad (98)$$

In the numerator we see real revenue; that is, nominal revenue deflated by a k-specific revenue based price index with reference period b. In the denominator we have real KLEMS input; that is, nominal KLEMS input cost deflated by a k-specific KLEMS input based price index with the same reference period; so that the ratio $TFPROD_Y^k(t,b)$ is a dimensionless variable.

Similarly, gross-output based simple labour productivity is defined as

$$SLPROD_Y^k(t,b) \equiv \frac{Y^k(t,b)}{L^{kt}} \; (k \in \mathcal{K}^t); \quad (99)$$

that is, real revenue per unit of labour. The dimension of this variable is money of reference period b.

Suppose that we have access to production-unit specific data such that either of these measures can be compiled. Which weights would be appropriate?

Let us start with the target variable considered by Baily, Bartelsman and Haltiwanger (2001). This is $SLPROD_Y^k(t,b)$, albeit that instead of unit-specific deflators industry-level deflators were used. The labour unit was an hour worked. These simple labour productivities were weighted by labour shares; that is, by $L^{kt}/L^{\mathcal{K}^t t} = L^{kt}/\sum_{k\in\mathcal{K}^t} L^{kt}$. Thus, aggregate productivity was compiled as

$$LPROD_{BBH}^{\mathcal{K}^t}(t,b) \equiv \sum_{k\in\mathcal{K}^t} \frac{L^{kt}}{\sum_{k\in\mathcal{K}^t} L^{kt}} SLPROD_Y^k(t,b) = \frac{\sum_{k\in\mathcal{K}^t} Y^k(t,b)}{L^{\mathcal{K}^t t}}. \quad (100)$$

But what precisely does this mean? To see this, we must return to the accounting identities discussed in Section 2 and notice that

$$\sum_{k\in\mathcal{K}^t} R^{kt} = \sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} R^{kk't} + R^{\mathcal{K}^t t}. \quad (101)$$

Thus, total revenue is the sum of revenue obtained by internal deliveries (recall that $R^{kk't}$ is the revenue obtained by unit k from delivering to unit k') and aggregate revenue $R^{\mathcal{K}^t t}$, which is the revenue obtained by the ensemble

$\mathcal{K}^t$, considered as a consolidated production unit. Now, imposing additivity, that is, defining the aggregate revenue based price index as a Paasche index of the k-specific revenue based price indices,

$$\frac{1}{P_R^{\mathcal{K}^t}(t,b)} \equiv \sum_{k\in\mathcal{K}^t} \frac{R^{kt}}{\sum_{k\in\mathcal{K}^t} R^{kt}} \frac{1}{P_R^k(t,b)}, \tag{102}$$

implies that expression (101) can be written as

$$P_R^{\mathcal{K}^t}(t,b) \sum_{k\in\mathcal{K}^t} Y^k(t,b) = \sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} R^{kk't} + R^{\mathcal{K}^t t}, \tag{103}$$

or

$$\sum_{k\in\mathcal{K}^t} Y^k(t,b) = \frac{\sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} R^{kk't}}{P_R^{\mathcal{K}^t}(t,b)} + \frac{R^{\mathcal{K}^t t}}{P_R^{\mathcal{K}^t}(t,b)}. \tag{104}$$

If we define real revenue of the ensemble $\mathcal{K}^t$, considered as a consolidated production unit, as $Y^{\mathcal{K}^t}(t,b) \equiv R^{\mathcal{K}^t t}/P_R^{\mathcal{K}^t}(t,b)$, then expression (104) can be simplified to

$$\sum_{k\in\mathcal{K}^t} Y^k(t,b) = \frac{\sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} R^{kk't}}{P_R^{\mathcal{K}^t}(t,b)} + Y^{\mathcal{K}^t}(t,b). \tag{105}$$

Substituting expression (105) into expression (100) and applying definition (99) to the ensemble considered as a production unit delivers the following relation:

$$LPROD_{BBH}^{\mathcal{K}^t}(t,b) = SLPROD_Y^{\mathcal{K}^t}(t,b) \left(1 + \frac{\sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} R^{kk't}}{R^{\mathcal{K}^t t}}\right). \tag{106}$$

Since nominal revenue is non-negative, it appears that aggregate BBH productivity overstates simple labour productivity of the aggregate, and that the magnitude of the bias depends on the relative extent of the intra-ensemble deliveries. The bias vanishes only when there are no intra-ensemble deliveries.

We now turn to $TFPROD_Y^k(t,b)$, a key variable considered by Bartelsman and Dhrymes (1998). They had industry and time effects removed econometrically, but that does not need to concern us here. The individual gross-output based total factor productivities were weighted by real KLEMS input shares $X^k_{KLEMS}(t,b)/\sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b)$, so that aggregate total factor productivity was compiled as

$$\begin{aligned} TFPROD^{\mathcal{K}^t}_{BD}(t,b) &\equiv \sum_{k\in\mathcal{K}^t} \frac{X^k_{KLEMS}(t,b)}{\sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b)} TFPROD^k_Y(t,b) \\ &= \frac{\sum_{k\in\mathcal{K}^t} Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b)}. \end{aligned} \tag{107}$$

Notice that, assuming that additivity at the output side holds, the numerator is given by expression (105). For the denominator a similar expression can be derived. To see this, we again return to the accounting identities in Section 2 and notice that

$$\sum_{k\in\mathcal{K}^t} C^{kt}_{EMS} = \sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} C^{k'kt}_{EMS} + C^{\mathcal{K}^t t}_{EMS}, \tag{108}$$

Adding at both sides $C^{\mathcal{K}^t t}_{KL} = \sum_{k\in\mathcal{K}^t} C^{kt}_{KL}$, we obtain the following accounting relation:

$$\sum_{k\in\mathcal{K}^t} C^{kt}_{KLEMS} = \sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} C^{k'kt}_{EMS} + C^{\mathcal{K}^t t}_{KLEMS}. \tag{109}$$

Thus, total cost is the sum of cost incurred by internal deliveries (recall that $C^{k'kt}_{EMS}$ is the cost incurred by unit k for purchases from unit k') and aggregate cost $C^{\mathcal{K}^t t}_{KLEMS}$, which is the KLEMS input cost of the ensemble $\mathcal{K}^t$, considered as a consolidated production unit. Now, imposing additivity at the input side, that is, defining the aggregate KLEMS input based price index as a Paasche index of the k-specific KLEMS input based price indices,

$$\frac{1}{P^{\mathcal{K}^t}_{KLEMS}(t,b)} \equiv \sum_{k\in\mathcal{K}^t} \frac{C^{kt}_{KLEMS}}{\sum_{k\in\mathcal{K}^t} C^{kt}_{KLEMS}} \frac{1}{P^k_{KLEMS}(t,b)}, \tag{110}$$

implies that expression (109) can be written as

$$P^{\mathcal{K}^t}_{KLEMS}(t,b) \sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b) = \sum_{k\in\mathcal{K}^t} \sum_{k'\in\mathcal{K}^t, k'\neq k} C^{k'kt}_{EMS} + C^{\mathcal{K}^t t}_{KLEMS}, \tag{111}$$

or

$$\sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b) = \frac{\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} C^{k'kt}_{EMS}}{P^{\mathcal{K}^t}_{KLEMS}(t,b)} + \frac{C^{\mathcal{K}^t t}_{KLEMS}}{P^{\mathcal{K}^t}_{KLEMS}(t,b)}. \quad (112)$$

If we define real KLEMS input of the ensemble $\mathcal{K}^t$, considered as a consolidated production unit, as $X^{\mathcal{K}^t}_{KLEMS}(t,b) \equiv C^{\mathcal{K}^t t}_{KLEMS}/P^{\mathcal{K}^t}_{KLEMS}(t,b)$, then expression (112) can be simplified to

$$\sum_{k\in\mathcal{K}^t} X^k_{KLEMS}(t,b) = \frac{\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} C^{k'kt}_{EMS}}{P^{\mathcal{K}^t}_{KLEMS}(t,b)} + X^{\mathcal{K}^t}_{KLEMS}(t,b). \quad (113)$$

Substituting expressions (105) and (113) into expression (107) and applying definition (98) to the ensemble considered as a production unit delivers the following relation:

$$\begin{aligned} &TFPROD^{\mathcal{K}^t}_{BD}(t,b) \\ &\quad = TFPROD^{\mathcal{K}^t}_{Y}(t,b)\frac{1+\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} R^{kk't}/R^{\mathcal{K}^t t}}{1+\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} C^{k'kt}_{EMS}/C^{\mathcal{K}^t t}_{KLEMS}}. \quad (114) \end{aligned}$$

As observed in Section 2, National Accounting conventions imply that revenue and cost of the intra-ensemble transactions are equal, that is

$$\sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} R^{kk't} = \sum_{k\in\mathcal{K}^t}\sum_{k'\in\mathcal{K}^t,k'\neq k} C^{k'kt}_{EMS}.$$

Thus the magnitude of the bias of aggregate BD total factor productivity depends on the magnitude of aggregate revenue $R^{\mathcal{K}^t t}$ relative to aggregate KLEMS input cost $C^{\mathcal{K}^t t}_{KLEMS}$. Put otherwise, the magnitude of the bias depends on aggregate profit $\Pi^{\mathcal{K}^t t}$. If aggregate profit is positive (negative), then aggregate BD total factor productivity understates (overstates) total factor productivity of the aggregate. If aggregate profit equals 0, then the bias vanishes. A sufficient condition for zero aggregate profit is that $\Pi^{kt} = 0$ for each individual production unit $k \in \mathcal{K}^t$. Of course, the bias also vanishes in the trivial case when there are no intra-ensemble deliveries.

It is now interesting to consider a recent paper by Collard-Wexler and De Loecker (2013). These authors also dealt with $TFPROD^k_Y(t,b)$ ($k \in \mathcal{K}^t$), but to obtain aggregate productivity the individual total factor productivities were weighted by nominal revenue shares $R^{kt}/\sum_{k\in\mathcal{K}^t} R^{kt}$. Thus aggregate productivity was defined as

$$TFPROD_{CWL}^{\mathcal{K}^t}(t,b) \equiv \sum_{k\in\mathcal{K}^t} \frac{R^{kt}}{\sum_{k\in\mathcal{K}^t} R^{kt}} TFPROD_Y^k(t,b). \tag{115}$$

To obtain an interpretation for this mean, we first relate it to the alternative where real shares $Y^k(t,b)/\sum_{k\in\mathcal{K}^t} Y^k(t,b)$ are used as weights,

$$\begin{aligned} TFPROD_{CWL}^{\mathcal{K}^t}(t,b) = & \\ & \sum_{k\in\mathcal{K}^t} \frac{Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} Y^k(t,b)} TFPROD_Y^k(t,b) \\ & + \sum_{k\in\mathcal{K}^t} \left(\frac{R^{kt}}{\sum_{k\in\mathcal{K}^t} R^{kt}} - \frac{Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} Y^k(t,b)} \right) TFPROD_Y^k(t,b). \end{aligned} \tag{116}$$

The second term at the right-hand side of this equation has the form of a covariance, but there is in general no compelling reason for this covariance to be positive or negative, large or small.

Next, applying the arithmetic-harmonic mean inequality to the first term at the right-hand side of equation (116), we find that

$$\sum_{k\in\mathcal{K}^t} \frac{Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} Y^k(t,b)} TFPROD_Y^k(t,b) \geq TFPROD_{BD}^{\mathcal{K}^t}(t,b), \tag{117}$$

where definitions (98) and (107) were used. Now expression (114) above tells us that, under additivity at the input and the output side, $TFPROD_{BD}^{\mathcal{K}^t}(t,b)$ is an unbiased measure of total factor productivity of the aggregate if there are no intra-ensemble deliveries, as happens to be the case in the particular industry studied by Collard-Wexler and De Loecker.

Thus, on the assumption that the covariance in equation (116) equals 0, it seems likely that aggregate CWL productivity overstates the productivity of the aggregate.

The study of Foster, Haltiwanger and Krizan (2001) contains interesting comparative results. They examined the effect of using output- or input-based weights on gross-output based total factor or simple labour productivities.

The first measure they considered was total factor productivity weighted by real output shares; that is,[17]

$$TFPROD^{\mathcal{K}^t}_{FHK}(t,b) \equiv \sum_{k\in\mathcal{K}^t} \frac{Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} Y^k(t,b)} TFPROD^k_Y(t,b). \tag{118}$$

We immediately recognize this as the left-hand side of expression (117), so we may conclude that in the cases studied by Foster, Haltiwanger and Krizan, which were four-digit level industries, $TFPROD^{\mathcal{K}^t}_{FHK}(t,b)$ most likely overstates aggregate total factor productivity $TFPROD^{\mathcal{K}^t}_Y(t,b)$.

Second, they considered simple labour productivity weighted by real output shares; that is,

$$LPROD^{\mathcal{K}^t}_{FHK}(t,b) \equiv \sum_{k\in\mathcal{K}^t} \frac{Y^k(t,b)}{\sum_{k\in\mathcal{K}^t} Y^k(t,b)} SLPROD^k_Y(t,b). \tag{119}$$

Applying the arithmetic-harmonic mean inequality, and definitions (99) and (100) respectively, we get

$$LPROD^{\mathcal{K}^t}_{FHK}(t,b) \geq \frac{\sum_{k\in\mathcal{K}^t} Y^k(t,b)}{L^{\mathcal{K}^t t}} = LPROD^{\mathcal{K}^t}_{BBH}(t,b). \tag{120}$$

The right-hand side is familiar from the foregoing. Combining expressions (120) and (106) we may conclude that, though four-digit level industries usually exhibit almost no intra-ensemble trade, $LPROD^{\mathcal{K}^t}_{FHK}(t,b)$ overstates simple labour productivity of the aggregate, $SLPROD^{\mathcal{K}^t}_Y(t,b)$.

The third measure considered by Foster, Haltiwanger and Krizan is simple labour productivity weighted by labour shares.[18] But this is the measure previously defined in expression (100) as $LPROD^{\mathcal{K}^t}_{BBH}(t,b)$. At the four-digit level this may be seen as an unbiased measure of $SLPROD^{\mathcal{K}^t}_Y(t,b)$.

The primary purpose of Foster, Haltiwanger and Krizan was to compare decompositions of intertemporal change of these three aggregate measures. They specifically examined the Foster-Haltiwanger-Krizan (FHK)

[17] Actually, their multi-factor productivity index, discussed in Section 4.4.2, can be seen as a special case of $TFPROD^k_Y(t,b)$.

[18] Actually, two variants were considered, one where the labour unit is an hour worked and one where it is a worker.

method (45) and the Griliches-Regev (GR) method (53). It turned out that, though the levels were of course different, the FHK decompositions of $\Delta TFPROD_{FHK}^{\mathcal{K}^t}(t,b)$ and $\Delta LPROD_{FHK}^{\mathcal{K}^t}(t,b)$ were strikingly similar. Levels as well as FHK decompositions of $\Delta LPROD_{FHK}^{\mathcal{K}^t}(t,b)$ and $\Delta LPROD_{BBH}^{\mathcal{K}^t}(t,b)$ differed remarkably, however. Interestingly, for the three aggregate measures the GR decomposition delivered almost the same results. Overall, the 'within' term appeared dominant.

References

[1] Abbott, T. A., 1991, "Producer Price Dispersion, Real Output, and the Analysis of Production", Journal of Productivity Analysis 2, 179-195.

[2] Altomonte, C. and M. Nicolini, 2012, "Economic Integration and the Dynamics of Firms' Competitive Behavior", Structural Change and Economic Dynamics 23, 383-402.

[3] Baily, M. N., C. Hulten, and D. Campbell, 1992, "Productivity Dynamics in Manufacturing Plants", *Brookings Papers on Economic Activity: Microeconomics* 2, 187-249.

[4] Baily, M. N., E. J. Bartelsman and J. Haltiwanger, 2001, "Labor Productivity: Structural Change and Cyclical Dynamics", The Review of Economics and Statistics 83, 420-433.

[5] Baldwin, J. R. and W. Gu, 2003, Plant Turnover and Productivity Growth in Canadian Manufacturing, Research Paper (Micro Economic Analysis Division, Statistics Canada, Ottawa).

[6] Balk, B. M., 2003, "The Residual: On Monitoring and Benchmarking Firms, Industries, and Economies with Respect to Productivity", Journal of Productivity Analysis 20, 5-47.

[7] Balk, B. M., 2008, Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference (Cambridge University Press, New York).

[8] Balk, B. M., 2009, "Measuring and Relating Aggregate and Subaggregate Productivity Change without Neoclassical Assumptions", Discussion Paper 09026 (Statistics Netherlands, www.cbs.nl).

[9] Balk, B. M., 2010, "An Assumption-free Framework for Measuring Productivity Change", The Review of Income and Wealth 56, Special Issue 1, S224-S256.

[10] Balk, B. M., 2011a, "Measuring and Decomposing Capital Input Cost", The Review of Income and Wealth 57, 490-512.

[11] Balk, B. M., 2011b, Dissecting Aggregate Output and Labour Productivity Change, ERIM Report Series Reference No. ERS-2011-023-MKT. Available at SSRN: http://ssrn.com/abstract=1961469. Revised version to appear in Journal of Productivity Analysis.

[12] Balk, B. M. and E. Hoogenboom-Spijker, 2003, The Measurement and Decomposition of Productivity Change: Exercises on the Netherlands' Manufacturing Industry, Discussion Paper 03001 (Statistics Netherlands, Voorburg/Heerlen).

[13] Bartelsman, E. J. and Ph. J. Dhrymes, 1998, "Productivity Dynamics: US Manufacturing Plants, 1972-1986", Journal of Productivity Analysis 9, 5-34.

[14] Bartelsman, E. J. and M. Doms, 2000, "Understanding Productivity: Lessons from Longitudinal Microdata", *Journal of Economic Literature* XXXVIII, 569-594.

[15] Bartelsman, E. J. and W. Gray, 1996, The NBER Manufacturing Productivity Database, Technical Working Paper 205 (National Bureau of Economic Research, Cambridge MA).

[16] Bartelsman, E., J. Haltiwanger and S. Scarpetta, 2013, "Cross-Country Differences in Productivity: The Role of Allocation and Selection", American Economic Review 103, 305-334.

[17] Beveren, I. van, 2012, "Total Factor Productivity Estimation: A Practical Review", Journal of Economic Surveys 26, 98-128.

[18] Biesebroeck, J. van, 2008, "Aggregating and Decomposing Productivity", Review of Business and Economics LIII, 122-146.

[19] Biesebroeck, J. van, 2009, "Disaggregate Productivity Comparisons: Sectoral Convergence in OECD Countries", Journal of Productivity Analysis 32, 63-79.

[20] Burda, M. C., D. S. Hamermesh and J. Stewart, 2012, Cyclical Variation in Labor Hours and Productivity Using the ATUS, Working Paper 18603 (National Bureau of Economic Research, Cambridge MA).

[21] Cassing, S., 1996, "Correctly Measuring Real Value Added", The Review of Income and Wealth 42, 195-206.

[22] Collard-Wexler, A. and J. de Loecker, 2013, "Reallocation and Technology: Evidence from the U. S. Steel Industry", Working Paper 18739 (National Bureau of Economic Research, Cambridge MA).

[23] De Avillez, R., 2012, "Sectoral Contributions to Labour Productivity Growth in Canada: Does the Choice of Decomposition Formula Matter?", International Productivity Monitor 24, 97-117.

[24] Dietzenbacher, E. and B. Los, 1998, "Structural Decomposition Techniques: Sense and Sensitivity", *Economic Systems Research* 10, 307-323.

[25] Diewert, W. E. and K. J. Fox, 2010, "On Measuring the Contribution of Entering and Exiting Firms to Aggregate Productivity Growth", in Price and Productivity Measurement: Volume 6 – Index Number Theory, edited by W. E. Diewert, B. M. Balk, D. Fixler, K. J. Fox and A. O. Nakamura (Trafford Press, www.vancouvervolumes.com, www.indexmeasures.com). Revised version of Discussion Paper No. 05-02 (Department of Economics, University of British Columbia, Vancouver, 2005).

[26] Dobbelaere, S. and J. Mairesse, 2013, "Panel Data Estimates of the Production Function and Product and Labor Market Imperfections", Journal of Applied Econometrics 28, 1-46.

[27] Dumagan, J. C., 2013, "A Generalized Exactly Additive Decomposition of Aggregate Labor Productivity Growth", The Review of Income and Wealth 59, 157-168.

[28] Foster, L., J. Haltiwanger and C. J. Krizan, 2001, "Aggregate Productivity Growth: Lessons from Microeconomic Evidence", in New Developments in Productivity Analysis, edited by C. R. Hulten, E. R. Dean and M. J. Harper, Studies in Income and Wealth Volume 63 (The University of Chicago Press, Chicago and London).

[29] Foster, L., J. Haltiwanger and C. J. Krizan, 2006, "Market Selection, Reallocation, and Restructuring in the US Retail Trade Sector in the 1990s", The Review of Economics and Statistics 88, 748-758.

[30] Foster, L., J. Haltiwanger and C. Syverson, 2008, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?", The American Economic Review 98, 394-425.

[31] Fox, K. J., 2012, "Problems with (Dis)Aggregating Productivity, and Another Productivity Paradox," Journal of Productivity Analysis 37, 249-259.

[32] Griliches, Z. and H. Regev, 1995, "Firm Productivity in Israeli Industry, 1979-1988", *Journal of Econometrics* 65, 175-203.

[33] Haltiwanger, J., 1997, "Measuring and Analyzing Aggregate Fluctuations: The Importance of Building from Microeconomic Evidence", *Federal Reserve Bank of St. Louis Economic Review* 79, No. 3, 55-77.

[34] Kangasniemi, M., M. Mas, C. Robinson, and L. Serrano, 2012, "The Economic Impact of Migration: Productivity Analysis for Spain and the UK", Journal of Productivity Analysis 38, 333-343.

[35] Karagiannis, G., 2013, Reallocation and Productivity Dynamics: Empirical Evidence on the Role of Aggregation Weights, Mimeo (Department of Economics, University of Macedonia, Thessaloniki).

[36] Lin, Y.-C. and T.-H. Huang, 2012, "Creative Destruction over the Business Cycle: A Stochastic Frontier Analysis", Journal of Productivity Analysis 38, 285-302.

[37] Loecker, J. de, and J. Konings, 2006, "Job Reallocation and Productivity Growth in a Post-Socialist Economy: Evidence from Slovenian Manufacturing", European Journal of Political Economy 22, 388-408.

[38] Mairesse, J. and J. Jaumandreu, 2005, "Panel-data Estimates of the Production Function and the Revenue Function: What Difference Does It Make?", Scandinavian Journal of Economics 107, 651-672.

[39] Marin, A. G. and N. Voigtländer, 2013, Exporting and Plant-level Efficiency Gains: It's in the Measure, Working Paper 19033 (National Bureau of Economic Research, Cambridge MA).

[40] Melitz, M. J. and S. Polanec, 2012, Dynamic Olley-Pakes Productivity Decomposition with Entry and Exit, Working Paper 18182 (National Bureau of Economic Research, Cambridge MA).

[41] Olley, S. and A. Pakes, 1996, “The Dynamics of Productivity in the Telecommunications Equipment Industry”, Econometrica 64, 1263-1297.

[42] Smeets, V. and F. Warzynski, 2013, “Estimating Productivity with Multi-product Firms, Pricing Heterogeneity and the Role of International Trade”, Journal of International Economics 90, 237-244.

[43] Syverson, C., 2011, “What Determines Productivity?”, Journal of Economic Literature 49, 326-365.

[44] Tang, J. and W. Wang, 2004, “Sources of Aggregate Labour Productivity Growth in Canada and the United States”, Canadian Journal of Economics 37, 421-444.

[45] Timmer, M. P., R. Inklaar, M. O’Mahony and B. van Ark, 2010, Economic Growth in Europe: A Comparative Industry Perspective (Cambridge University Press, Cambridge UK).

[46] Wolf, Z., 2011, Aggregate Productivity Growth under the Microscope, Research Series No. 518 (Tinbergen Institute, VU University Amsterdam).