Measuring Chronic Multidimensional Poverty: A Counting Approach

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Abstract

How can indices of multidimensional poverty be adapted to produce measures that quantify both the joint incidence of multiple deprivations and their chronicity? This paper adopts a new approach to the measurement of chronic multidimensional poverty. It relies on the counting approach of Alkire and Foster (2011) for the measurement of multidimensional poverty in each time period; and then on the duration approach of Foster (2011) for the measurement of multidimensional poverty persistence across time. The proposed indices are sensitive both to (i) the share of dimensions in which people are deprived and (ii) the duration of their multidimensional poverty experience. A related set of indices is also proposed to measure transient poverty. The behaviour of the proposed two families is analysed using a relevant set of axioms. An empirical illustration is provided with a Chilean panel dataset spanning the period from 1996 to 2006.

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1. Introduction

Sen (1976) argued that an index of poverty should capture the incidence of poverty, the extent of individual poverty, and inequality among poor people. His seminal contribution inspired numerous proposals of unidimensional indices of poverty based on cross-sections of income or consumption data.[5]

However, the duration of poverty at the individual or household level is a crucial issue for understanding how people experience poverty. Persistent conditions of insufficiency might precipitate detrimental effects on well-being. For instance, an increase in the duration of poverty increases the likelihood of impairment and illness.[6] A person stricken by long-lasting poverty can become socially excluded, and/or lose allegiance to the wider community (Walker, 1995). This, in turn, may lead to social unrest. Furthermore, it is important to know who among the poor are chronically poor and to understand their condition in order to improve policy predictions and responses (Lybbert et al., 2004; Carter and Barrett, 2006). Therefore it often becomes desirable to measure individual poverty *dynamically* using panel data.

An important recent development in poverty measurement research has been the definition of a robust multidimensional framework. The reason for its emergence is that the well-being of a population depends on both monetary and non-monetary dimensions of life (see Kolm, 1977; Streeten, 1981; Sen, 1985, 1987; Anand and Sen, 1997; and Foster and Sen, 1997). Examples of non-income dimensions are housing, schooling, nutrition, etc. A person with sufficiently high income may not always be well-off with respect to some non-monetary dimensions of life. For example she may have an insufficient quantity of a non-club public good. Likewise, a pavement dweller with good nutritional status may have a low income. It may not be possible to trade off income and some non-income dimensions. It also may be necessary to develop policies to address specific deprivations or combinations of deprivations. If so, then the construction of a multidimensional index of poverty and its analysis may be worthwhile.

It is extremely important to combine these two approaches for the study of chronic multidimensional poverty. Hulme et al. (2001) and Hulme and Mckay (2005) argued explicitly that the measurement of chronic poverty should focus on multidimensional situations. 'Chronically poor are commonly multi-dimensionally deprived' (Hulme, 2004, p.3). Furthermore, interesting analyses can be carried out when chronic and transient poverty measures are broken down by dimension. For example, one can perform an analysis to see whether chronic poverty has distinctive components that may comprise 'poverty traps'.

This paper extends the Alkire-Foster multidimensional approach to chronic poverty and, in a related manner, transient poverty, using the Foster (2011) duration approach. The latter is chosen because it is parsimonious and easy to understand and it is based on the same axiomatic foundations as the Alkire-Foster family of multidimensional poverty indices. Moreover, unlike other inter-temporal poverty approaches, Foster's identification criteria explicitly identify the chronically poor; but can easily be adjusted to identify the

[5] For reviews of this literature see Foster and Sen (1997), Zheng (1997), Lambert (2001), Chakravarty (2009), and Foster (2011).
[6] For a discussion of duration issues, see Bane and Ellwood (1986), Gaiha (1989), Gaiha and Deolikar (1993), Morduch (1994), Baluch and Masset (2003), Hulme and Shepherd (2003), Carter and Barrett (2006), and Dercon and Shapiro (2007).

transiently poor. The Alkire-Foster (2011) approach has been chosen for the empirical illustrations because it can be computed with ordinal or ratio-scale data and because it is widely applied.

The next section briefly discusses the most recent literature on inter-temporal poverty measurement including existing proposals to measure chronic multidimensional poverty. That section's purpose is to clarify the concept of chronic poverty and its distinctiveness within the inter-temporal poverty literature. Section 3 presents some notation and definitions. Section 4 presents axioms for a general chronic multidimensional poverty index. Section 5 introduces our class of chronic multidimensional poverty measures. We also introduce a family of transient multidimensional poverty measures and investigate axiom fulfilment for both families. Section 6 compares our proposal and that put forward by Nicholas and Ray (2011) highlighting the main differences between the two. Section 7 presents two empirical illustrations that use ratio scale and, separately, ordinal variables, using the CASEN panel datasets in Chile with observations for 1996, 2001 and 2006. Section 8 concludes.

2. A conceptual clarification on the current state of the literature

The recent literature on poverty measurement that accounts for time, also known as *inter-temporal poverty*, provides normative evaluations that are sensitive to different aspects of people's lifetime poverty experience. This literature does not explicitly distinguish people who are chronically poor from those who are only transiently poor. Instead attention is generally focused on features like the number of consecutive spells in poverty, the number of consecutive spells outside poverty, or the timing of the poverty experience (e.g. whether it is concentrated at the beginning or at the end of a lifetime). For instance, in the individual poverty measures of Bossert et al. (2012a), deprivation gaps belonging to longer spells are assigned greater weight.[8] Other interesting examples of inter-temporal poverty measures include the contributions of Hoy and Zheng (2011), Dutta et al. (2011) and Hojman and Kast (2009). For instance, Dutta et al. (2011) considered a variant of the approach by Bossert et al. (2012a) by discounting the impact of a period in poverty using the number of periods outside poverty directly preceding it. Hojman and Kast (2009) described an inter-temporal poverty measure that trades off poverty levels and changes (gains and losses) over time. Hence this index is an increasing function of absolute levels of poverty and changes in poverty. Bossert et al. (2012b) followed a similar approach.

Finally, Nicholas and Ray (2011) presented a generalization of the Chakravarty-D'Ambrosio (2006) class of multidimensional deprivation measures by explicitly taking into account the duration and persistence of deprivation. Essential to this generalization is the number of dimensions in which a person becomes deprived at different time periods. As this proposal combines a multidimensional framework with time, just like ours, we discuss it more thoroughly below.

None of the recent proposals mentioned above seeks to identify the chronically poor (distinguishing them from the transiently poor), a purpose whose ongoing interest dates from an earlier literature. Several

[8]Gradin et al. (2012), in turn, generalized the proposal of Bossert et al. (2012a).

approaches to the measurement of chronic poverty have been suggested. Jalan and Ravallion (1998) proposed using a person's permanent income in order to identify him or her as chronically poor.[9] According to this approach, a person is regarded as chronically poor if the individual's permanent income falls below a certain poverty line. Because aggregation of incomes over the periods under consideration ignores income variations across periods, Foster and Santos (2012) followed the permanent income approach by explicitly allowing for an imperfect degree of substitutability across periods. They then used a decomposable Clark, Hemming and Ulph (1981) measure in order to compute chronic poverty. Porter and Quinn (2008) suggested a class of chronic poverty indices that incorporates the view that the poorer the individual is, the higher the negative impact of fluctuations in well-being.

Foster (2011) proposed a class of chronic poverty indices that rely on aggregation across time. He defined an individual as chronically poor if his income falls below an exogenously given poverty line for a minimum percentage of time periods. This approach to the measurement of chronic poverty is known as the spell, or duration, approach (see Yaqub 2000a, 2000b, McKay and Lawson 2002, Hoy Thompson and Zheng 2012). The Foster class of indices, which is an extension of the Foster-Greer-Thorbecke (1984) family of poverty indices to the chronic set-up, fulfils a time anonymity condition under which reordering of incomes in the individuals' trajectories does not change chronic poverty. Foster (2011) also suggested an associated index of transient poverty defined to evaluate poverty of a shorter duration.[10]

Our contribution fits into this latter strand of the literature by proposing a way to identify and evaluate the experience of the chronically poor in a multidimensional sense. Like previous contributions, our proposal is guided by a set of relevant axioms.

3. Preliminaries

We have observations on d dimensions or attributes of well-being for a set of N individuals at *T* different time points. Let x_{ij}^t stand for the quantity of attribute j possessed by person i in period t. Let $\mu(v)$ stand for the arithmetic mean of v. It is assumed that $x_{ij}^t \geq 0 \; \forall i, j, t$. Let X^t denote the matrix whose i^{th} row is the row vector $x_{i.}^t = (x_{i1}^t, x_{i2}^t, \ldots, x_{id}^t)$. $X^t$is the $N \times d$ achievement matrix in period t. The distribution of attribute j in period t is represented by the column vector $x_{.j}^t$.

In this multidimensional set-up, a deprivation cut-off z_j is defined for each attribute; these are fixed across periods. These deprivation cut-offs give the minimal quantities of the d attributes necessary to be non-deprived in each attribute. Let $z^t = (z_1^t, \ldots, z_d^t)$ be the vector of deprivation cut-offs in period t and $z_j^t > 0 \; \forall \; j, t$. z^t is an element of the set $Z^t \subset \mathbb{R}_+^d$, a strictly positive part of the *d*-dimensional Euclidean space. Person i is regarded as deprived with respect to dimension j in period t if $x_{ij}^t < z_j^t$. Person i is non-deprived in dimension j in period t if $x_{ij}^t \geq z_j^t$.

[9] See also Rodgers and Rodgers (1993), Calvo and Dercon (2007), Calvo (2008) and Foster (2009).
[10]Chakravarty (2009) investigated properties of subgroup decomposable chronic poverty indices in this framework.

The powered deprivation shortfall of person i in dimension j at period t is: $g_{ij}^t(\alpha) \equiv \left(1 - \frac{x_{ij}^t}{z_j^t}\right)^\alpha$, where $\alpha \geq 0$. Clearly, individuals deprived in j at t have a positive deprivation gap, whereas otherwise $g_{ij}^t(\alpha) = 0$. We also consider the $N \times d$ powered deprivation gap matrix for period t, $G^t(\alpha)$, whose typical element is $g_{ij}^t(\alpha)$.

a. The Alkire-Foster approach to the identification of the multidimensionally poor

Two well-known methods of identification of the multidimensionally poor have been analysed, among others, by Tsui (2002), Atkinson (2003) and Bourguignon and Chakravarty (2003). According to the *union method*, if a person is deprived in any dimension, then he is regarded as poor. On the other extreme, the *intersection method* demands that only persons who are deprived in all dimensions are treated as poor. As Alkire and Foster (2011) argued, a more general alternative to these two criteria is an identification approach which requires a person to be poor if she is deprived in at least k dimensions, where

$$0 < k \leq d$$[11]

Thus k is a poverty cut-off that identifies who is poor. When $k = 1$, the union method is obtained, whereas the intersection method requires $k = d$. This intermediate identification approach assumes that each dimension is assigned equal importance.

However, different dimensions can be assigned different positive weights in order of importance, where $\sum_{j=1}^{d} w_j = d$. In such a case, if $0 < k \leq \min\{w_1, w_2, \ldots, w_d\}$, where w_j is the weight assigned to dimension j, we get the union method. As before, $k = d$ yields the intersection method.[12] In this paper we adopt the Alkire-Foster method for the identification of the multidimensionally poor in each period. The following identification procedure applies equally to any method based on a counting approach in which a weighted sum of deprivations is compared against a poverty cut-off like k.[13]

Identification of the multidimensionally poor in period t proceeds according to the following steps. Having defined a d-dimensional column vector of weights: $W = (w_1, w_2, \ldots, w_d)$ we generate an N-dimensional counting vector, $C^t = G^t(0)W$. A typical element of C^t, e.g. c_i^t, gives the weighted sum of deprivations for person i in period t. Formally, $c_i^t = \sum_{j=1}^{d} w_j g_{ij}^t(0)$.[14] Secondly, we generate an N-dimensional identification (column) vector for period t, $I^t(k)$, such that a typical element, $\rho_i^t(k)$ is defined by: $\rho_i^t(k) = \mathbb{I}(c_i^t \geq k)$.[15] In other words the identification vector elements take two values - 0 and 1; $\rho_i^t(k) = 1$ if and only if individual i is multidimensionally poor, according to deprivation cut-offs z^t, weights W and poverty cut-off k and $\rho_i^t(k) = 0$ otherwise.

b. The Duration Approach

[11] Equivalently, k can be defined as the share of weighted dimensions in which a person must be deprived in order to be identified as poor – $0 < k \leq 1$ – and similarly the weights in this case sum to 1 rather than to d.

[12] See Alkire and Foster (2011) for further discussion.

[13] For further discussions of counting approaches see Atkinson (2003) and Lasso de la Vega (2010).

[14] Note that $g_{ij}^t(0) = 1$ when individual i is deprived in dimension j.

[15] $\mathbb{I}(a)$ is an indicator function whose value is 1 if and only if a is true. Otherwise it is equal to 0.

Now that we have identified the poor in every period, the next step is to identify the chronically poor. As mentioned above we assume that the attribute quantities have been appropriately transformed to take into account variations across time periods (e.g. due to discount factors) and hence for each dimension a common threshold can be used. Let $z = (z_1, z_2, \dots, z_d)$ be the vector of common deprivation cut-offs.

Given the Alkire-Foster method of identification of the multi-dimensionally poor, Foster's (2011) duration approach says that a person is chronically poor if she remains in poverty for at least a certain number of time periods τ.We refer to τ as the duration cut-off. Thus, this duration-based approach involves a third identification step in addition to the two steps implemented above. In the previous subsection, we identified dimensional deprivation in every period (and for every individual) using the deprivation cut-offs (z). Then we identified the multi-dimensionally poor, in each period, using the Alkire-Foster dual cutoff approach. The third step, i.e. the duration approach, identifies the chronically poor among these multi-dimensionally poor persons in different periods using the duration cut-off τ. [16]

We apply the deprivation cutoff across the number[17] of periods in which each individual is multidimensionally poor. First, we count the periods of poverty by constructing a $N \times T$ matrix, $I(k)$ in which each of the t column vector is the identification vector for the tth period, $I^t(k)$. Then we generate the N-dimensional chronic counting vector, $C = I(k)1_T$, where 1_T is a T-dimensional column vector of ones. A typical element of C, $c_i = \sum_{t=1}^{T} \rho_i^t(k)$, gives the number of periods in which person *i* was multidimensionally poor for a given *k* Finally, we apply the cut-off τ to the chronic counting vector, to identify the chronically poor. In particular, we generate an N-dimensional column vector, $P^c(k;\tau)$, for the identification of the chronically poor, such that a typical element, $\rho_i(k;\tau)$ is defined by: $\rho_i(k;\tau) = \mathbb{I}(c_i \geq \tau)$. $\rho_i(k;\tau) = 1$ if and only if individual i is chronically multidimensionally poor, according to deprivation cut-offs z, weights W, poverty cut-off k and duration cut-off τ.[18]

4. Desirable properties

We now define a chronic multidimensional poverty index as a real-valued non-negative function, $\Psi(X; z, W, k, \tau)$, such that Ψ: $\mathbb{R}_+^{N\times d\times T} \rightarrow [0,1]$, where $X = (X^1, X^2, \dots, X^T)$. We assume at the outset that the poverty index is normalized between 0 and 1, and that it is scale invariant, i.e. positive scale transformations of the attribute quantities in all the periods and cut-offs do not change the level of poverty. This property shows that the attributes are measurable on ratio scales.

[16] Note that this order of aggregation could be altered. For instance, people who are chronically deprived in each dimension could be identified using the same duration cut-off τ for each dimension. Then, subsequently, *multidimensional chronic deprivation* could be computed across the chronically deprived. But in this paper we are interested in chronic poverty when poverty is conceived, and measured, as the prevalence of multiple deprivations. Hence the sequence of identification steps used in this paper is the appropriate one.

[17] Tau could also reflect the share of periods in which a person was poor, in which case $0<\tau\leq 1$

[18] The measures presented subsequently could also use different identification strategies, such as the average deprivation level across years $\rho_i(k;\tau) = \mathbb{I}(\sum_{t=1}^{T} c_i^t \geq k)$ or the inclusion of a functional form (or weights) to allow for different valuation across years; however the axioms satisfied by such an approach would change; also, the resulting measures would also not be associated with the set of intuitive partial indices of H^C, A^C, D^C presented below.

The following axioms are multidimensional counterparts to Foster's (2011) single dimensional chronic poverty axioms and/or chronic counterparts to Alkire and Foster's multidimenisonal poverty axioms.

Chronic Poverty Focus (CHF): Suppose person i is not chronically poor in the achievement matrix $X$and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t + \delta$ for a triplet (i,j,t) where $\delta > 0$, and $y_{sq}^l = x_{sq}^l \forall (s,q,l) \neq (i,j,t)$. Then $\Psi(X;z,W,k,\tau) = \Psi(Y;z,W,k,\tau)$.

This axiom says that if a person is not chronically poor, then an increase in the quantity of any of his attributes, in any period, does not affect the value of the poverty index. That is, the poverty index is independent of the achievement levels of non-chronically poor people.

Chronic Monotonicity (CHM): Suppose person i is chronically poor in the achievement matrix X and the matrix Y is obtained from X as follows:$y_{ij}^t = x_{ij}^t - \delta$ for a triplet (i,j,t), where $x_{ij}^t < z_j$, $\delta > 0$ and $y_{sq}^l = x_{sq}^l \forall (s,q,l) \neq (i,j,t)$. Then$\Psi(X;z,W,k,\tau) \leq \Psi(Y;z,W,k,\tau)$.

This property says that if a person who is chronically poor becomes more deprived in an attribute, then poverty does not decrease.

Time Focus (TIF): Suppose person i is chronically poor in the achievement matrix X and the matrix Y is obtained from X as follows: $y_{ij}^t = x_{ij}^t + \delta$ for a triplet (i,j,t) where $x_{ij}^t \geq z_j$, $\delta > 0$ and $y_{sq}^l = x_{sq}^l \forall (s,q,l) \neq (i,j,t)$. Then$\Psi(X;z,W,k,\tau) = \Psi(Y;z,W,k,\tau)$.

This property says that for a chronically poor person who is non-deprived in an attribute in a period, an increase in the quantity of that attribute in the same period leaves poverty unchanged. Thus, if a person is not deprived in an attribute, then giving her more of the attribute does not change the extent of chronic poverty, even if she is deprived in one or more of the other dimensions in that period. Trade-off between two attributes of a person, who is deprived in one but not in the other, is not possible. This does not exclude the possibility of a trade-off if the person is deprived in both attributes.

Time Monotonicity (TIM): Suppose the achievement matrices Y and X are related as follows: for some period t', some attribute j' and a person i' who is chronically poor in Y, $y_{ij}^t < z_j \leq x_{ij}^t$, $(i,j,t) = (i',j',t')$ and $y_{ij}^t = x_{ij}^t \forall (i,j,t) \neq (i',j',t')$. Then $\Psi(X;z,W,k,\tau) < \Psi(Y;z,W,k,\tau)$.

This postulate says that for a chronically poor person an increase in the duration of poverty experienced in a dimension leads to an increase in poverty.

Time Anonymity (TAN): If the sequence $(Y^1, Y^2, \ldots, Y^T)$ in the achievement matrix Y is obtained by a reordering of the sequence $(X^1, X^2, \ldots, X^T)$ in the matrix Y, then $\Psi(X;z,W,k,\tau) = \Psi(Y;z,W,k,\tau)$.

This postulate requires that the time-sequencing of the attributes' distributions does not affect the value of the chronic poverty index. It rules out the possibility that longer poverty spells get higher weights in the aggregation.

None of the axioms stated so far deals with the inequality among the chronically poor. In the case of cross-sectional income poverty, if there is a (progressive) transfer of income from a richer poor to a poorer poor that

does not change their relative positions, then we say that the post-transfer income distribution of the poor is obtained from the pre-transfer one by a 'smoothing of incomes'. This reduces inequality in the income distribution of the poor (Sen, 1976). In the multidimensional set up smoothing requires that poverty should not increase under (progressive) transfers of attribute quantities from richer poor to poorer poor persons, given the relative positions of the donors and the recipients. This is achieved if the post-transfer achievement matrix of the chronically poor in any period can be expressed as the product of a bi-stochastic matrix and the pre-transfer achievement matrix in the period (Kolm, 1977).

A non-negative $N \times N$ matrix $B = (b_{ij})$ is called a bi-stochastic matrix of order N if all its cells are non-negative, and each of its rows and columns sums to one. A bi-stochastic matrix is called a permutation matrix if there is exactly one positive entry in each row and column. For any $t \in (1,2,\dots,T)$, we say that X^t is obtained from Y^t by an averaging or smoothing of achievements among the chronically poor if $X^t = BY^t$ for some non-permutation bi-stochastic matrix B of order N such that $b_{ii} = 1$ for every non-chronically poor person i in Y^t. The condition $b_{ii} = 1$ ensures that the distributions of the attributes among the non-chronically poor remain unaffected, and that smoothing occurs only among the chronically poor (Alkire and Foster, 2011). Hence inequality of the chronically poor in X^t is not higher than that in Y^t. If B is a permutation matrix, then the rows of X^t are rearrangement of the rows of Y^t.

We can now formally state the following:

Chronic Transfer (CHT): If the achievement matrix Y is transformed into the matrix X as follows: For any arbitrary $t \in (1,2,\dots,T)$, X^t is obtained from Y^t by an averaging among the chronically poor and $X^l = Y^l \; \forall \; l \neq t$ Then $\Psi(X; z, W, k, \tau) \leq \Psi(Y; z, W, k, \tau)$. A transfer from a less-chronically poor individual to a more chronically poor individual in a defined period should not increase the poverty index.

These axioms provide an idea of desirable properties in a longitudinal poverty measure based on the existent literature. However, the analysis of multidimensional poverty across time requires the definition of new assumptions on how a poverty measure should behave. The following axioms, which have not been suggested earlier in the literature, also seem appropriate for a duration-based index:

Chronic Normalization (CHN): $\Psi(X; z, W, k, \tau) = 0$ if and only if $\rho_i(k; \tau) = 0 \; \forall i \in (1,2,\dots,N)$.

According to this axiom, if there are no chronically poor people in society then the poverty index takes on the value zero, and vice versa.

Chronic Dimensional Monotonicity (CDM): Suppose the achievement matrices Y and X are related as follows: for some period t', some attribute j' and a person i' who is chronically poor in X, $x_{ij}^t \geq z_j, y_{ij}^t = x_{ij}^t - \delta < z_j, \delta > 0$ for $(i,j,t) = (i',j',t')$, and $y_{ij}^t = x_{ij}^t \; \forall (i,j,t) \neq (i',j',t')$. Then $\Psi(X; z, W, k, \tau) < \Psi(Y; z, W, k, \tau)$.

According to this axiom, if a chronically poor person who is non-deprived in a dimension but poor in a period becomes deprived in the dimension in that period, then chronic poverty increases.

Chronic Monotonicity in Thresholds (CMT): Let the vector of cut-off points z be transformed into the vector z^*, where $z_j^* = z_j + \beta$ for some $j \in (1,2,\dots,d)$, $z_q = z_q^* \; \forall q \neq j$ and $\beta > 0$ is a constant. Then given the achievement matrix X: $\Psi(X; z^*, W, k, \tau) \geq \Psi(X; z, W, k, \tau)$.

This axiom says that an increase in the deprivation threshold of a dimension does not decrease the chronic poverty associated with a given achievement matrix X .

Monotonicity in Multidimensional Poverty Identifier (MMI). Given the achievement matrix X and (z, W, τ), $\delta > 0$, then $\Psi(X; z, W, k, \tau) \geq \Psi(Y; z, W, k + \delta, \tau)$.

Since an increase in the value of k may reduce the number of poor people, although the intensity of their poverty may rise, the poverty index does not increase.

Chronic Duration Monotonicity (CDUM): Given the achievement matrix X and (z, W, τ), $\gamma > 0$, then $\Psi(X; z, W, k, \tau) \geq \Psi(X; z, W, k, \tau + \gamma)$.

Resembling the rationale of MMI, a higher duration cut-off cannot increase the number of people identified as chronically poor.

Following Alkire and Foster (2011) we also propose an inequality axiom related to transfers between pairs of chronically poor people that reduce the degree of association between the dimensions. We say that X^t is obtained from Y^t by an association-decreasing switch among the poor, if for a pair of chronically poor people, i and i', it is the case that: 1) $y_{ij}^t \geq y_{i'j}^t \forall j \in \{1,2,\dots,d\}$, 2) $\exists j | x_{ij}^t \leq x_{i'j}^t$; and 3) $y_{qj}^t = x_{qj}^t \forall q \neq (i, i')$. That is, the vector dominance of i over i' is broken by the association-decreasing switch. The following property describes one way in which a chronic multidimensional poverty measure should react to association-decreasing switches:

Non-increasing Chronic Poverty Under Association Decreasing Switch (NIPA): Suppose X^t is obtained from Y^t by an association-decreasing switch and $Y^l = X^l \; \forall l \neq t$.Then $\Psi(X; z, W, k, \tau) \leq \Psi(Y; z, W, k, \tau)$.

Alternatively, we could also consider a property of non-decreasing chronic poverty under association decreasing switches (NDPA), as well as a property of poverty insensitivity to association-decreasing switches.

The next three postulates are direct chronic counterparts to their single dimensional forms. The first of them is the subgroup decomposability axiom, which ensures coherence between local and global assessments of chronic poverty.

Additive subgroup decomposability (ASD): For an arbitrary subgroup division of the achievement matrix X into m matrices X_1 through X_m, each with respective subgroup populations of N_1 through N_m: $\Psi(X; z, W, k, \tau) = \sum_{q=1}^{m} \frac{\mathrm{N_q}}{\mathrm{N}} \Psi\big(X_q; z, W, k, \tau\big)$.

This axiom says that for any partitioning of the population into m ($\in \mathbb{N}$)subgroups, overall chronic poverty is given by the population-share weighted average of the subgroup chronic poverty levels. Thus, if chronic poverty in one subgroup decreases, while remaining unchanged in other subgroups, then global poverty falls. The next axiom ensures that chronic poverty remains unchanged if individuals trade their places.

Anonymity (ANY): Suppose X is obtained from Y as follows: $X^t = BY^t$, where B is a permutation matrix, and $X^l = Y^l \; \forall l \neq t$. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

This axiom says that, in the measurement of chronic poverty, only people's achievements, in different periods and in different dimensions, matter. The final axiom enables poverty comparisons among societies with different populations.

Population Replication Invariance (PRI): Let Y be the matrix obtained from a q-fold replication of the achievement matrix X, where $q \geq 2$ is a positive integer; that is, in Y the matrix X appears q times. Then $\Psi(X; z, W, k, \tau) = \Psi(Y; z, W, k, \tau)$.

This axiom ensures that chronic poverty is measured in per capita terms.

5. A class of chronic multidimensional poverty measures

Closely following the functional forms proposed by Alkire and Foster (2011) and Foster (2011), we propose the following normalized population average of powered deprivation gaps, in which only the deprivation gaps of the chronically poor are considered:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{NdT}(P^c)' \sum_{t=1}^{T} G^t(\alpha) W \qquad (1)$$

Where $G^t(\alpha)W$ is a N-dimensional column vector whose typical element is $\sum_{j=1}^{d} w_j g_{ij}^t(\alpha)$ and $(P^c)'$ is a N-dimensional row vector whose typical element is $\rho_i(k; \tau) = I(c_i \geq \tau)$ as defined in section 3.b. An alternative notation for M_C^α is:

$$M_C^\alpha(X; z, W, k, \tau) = \frac{1}{NdT} \sum_{i=1}^{N} \rho_i(k; \tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_j g_{ij}^t(\alpha) \qquad (2)$$

$M_C^\alpha$is the population sum of powered censored normalized deprivation gaps divided by the maximum possible value, NdT; which arises if and only if $x_{ij}^t = 0 \; \forall (i, j, t) \in [1, N] \times [1, d] \times [1, T]$, for $\alpha > 0$.[19] If $\alpha = 0$ then the maximum is attained if and only if $x_{ij}^t < z_j \; \forall (i, j, t) \in [1, N] \times [1, d] \times [1, T]$.

$M_C^\alpha$is an an extension of the Alkire-Foster multidimensional poverty index to chronic poverty and is an extension of the Foster index to the multidimensional space. The following theorem describes the behaviour of M_C^α in terms of its fulfillment of the axioms introduced in the previous section:

Theorem 1: $M_C^\alpha(X; z, W, k, \tau)$ satisfies CHF, CHM, TIF, TIM, TAN, CHT, CHN, CDM, CMT, MMI, CDUM, NIPA, ASD, ANY and PRI for all $\alpha \geq 0$. $M_C^\alpha(X; z, W, k, \tau)$ also satisfies a strong version of CHM for $\alpha > 0$, and a strong version of CHT when $\alpha \geq 1$. Proof: Available upon request.

[19]The intervals $[1, N]$, $[1, d]$ and $[1, T]$ are all subsets from the set of natural numbers.

Just like the Alkire-Foster family of measures, M_c^α can be expressed in terms of intuitive partial indices that convey meaningful information on different features of a society's experience of chronic multidimensional poverty:

H^c is the population headcount of chronic poverty. D^c measures the average duration of poverty *among the chronically poor*. Then, by (3), we know that when $\alpha > 0$, $M_c^\alpha$is sensitive to the prevalence of chronic poverty in the population (H^c), the relative duration of chronic poverty (D^c), the relative number of deprivations among the chronically poor (A^c).

a. A class of transient multidimensional poverty measures

Using the same framework we also propose a family of indices of *transient* (multidimensional) poverty, M_{tr}^α. The main difference between the two families is in the identification of the poor. We identify a person as transiently poor if $0 < c_i < \tau$. Hence we use a different N-dimensional column vector, $P^{tr}(k;\tau)$, for the identification of the transiently poor, such that a typical element, $\omega_i(k;\tau)$ is defined by: $\omega_i(k;\tau) = \mathbb{I}(0 < c_i < \tau)$. $\omega_i(k;\tau) = 1$ if and only if individual i is transiently multidimensionally poor, according to deprivation cut-offs z, weights W, multidimensional cut-off k and duration cut-off τ. The family is:

$$M_{tr}^\alpha(X;z,W,k,\tau) = \frac{1}{NdT}(P^{tr})' \sum_{t=1}^{T} G^t(\alpha) W \tag{3}$$

An alternative notation for M_{tr}^α is:

$$M_{tr}^\alpha(X;z,W,k,\tau) = \frac{1}{NdT} \sum_{i=1}^{N} \omega_i(k;\tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_d g_{ij}^t(\alpha) \tag{4}$$

The following theorem describes behaviour of M_{tr}^α in terms of its fulfilment of the axioms introduced in the previous section:

Theorem 2: $M_{tr}^\alpha(X;z,W,k,\tau)$ satisfies TIF, TIM, TAN, CMT, MMI, ASD, ANY and PRI for all $\alpha \geq 0$.

Note that several axioms are not fulfilled by M_{tr}^α. In many cases the reason is that the axioms are stated in terms of chronic poverty. For instance, CHF states that improvements in an attribute of a non-chronically poor person should not affect the poverty measure. However, that person could still be transiently poor, in which case a transient measure, sensitive to the poverty status and intensity of that person, may be affected. Likewise, an averaging of attribute values *among the chronically poor* may not increase chronic poverty, but it could increase transient poverty if some of these previously chronically poor people become transiently poor.

b. However it is straightforward to show that the following axioms can also be fulfilled by M_{tr}^α if they are rephrased in terms of transiently poor people: CHF, CHM, CHT, CHN, CDM and NIPA. In the case of CDUM, an increase in the duration cut-off does not decrease transient poverty (as opposed to not increasing chronic poverty). Proof. Available upon request.

b. The case of ordinal data

Often well-being attributes are measured as binary, ordinal or ordered categorical variables, e.g. sanitation. When data are ordinal, the poverty index must be insensitive to monotonic transformations in the data and associated deprivation cutoff.

The indices M_c^0 and M_{tr}^0 satisfy the requirements of ordinal data and fulfil the above axioms according to theorems 1 and 2. Besides, their respective decompositions have straightforward, meaningful interpretations:

$$M_c^0 = H^c D^c A^c \tag{5}$$

$$M_{tr}^0 = H^{tr} D^{tr} A^{tr} \tag{6}$$

6. Comparison with other approaches: the measures of Nicholas and Ray (2011)

Nicholas and Ray (2011) proposed the first inter-temporal extension of a multidimensional poverty index. Their measures combine the multidimensional approach of Chakravarty and D'Ambrosio (2006) with the inter-temporal poverty approach of Bossert et al. (2012a) and Gradin et al. (2012). Expressed in our notation, their family of indices is the following:

$$\Omega_\beta(X;z) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{1}{dT}\sum_{j=1}^{d}\sum_{t=1}^{T} g_{ij}^t(0)s_{ijt}\right]^\beta \tag{7}$$

Where $\beta \geq 0$, and s_{ijt} is a weight that depends on the length of the deprivation spell to which the deprivation experience of individual i in dimension j and period t belongs.[20] A first fundamental difference between (14) and (1) is that (14) neither identifies the chronically poor, nor distinguishes them from the transiently poor, explicitly. That is, (14) is an index of inter-temporal poverty, but not of chronic poverty. Implicitly, it adopts a union approach to both chronic and multidimensional poverty. By contrast, our indices can adopt several approaches for the identification of the chronically poor, ranging from union to intersection. For that reason $\Omega_\beta = 0 \leftrightarrow g_{ij}^t(0) = 0 \forall (i,j,t)$, whereas that's not the case for M_c^α, unless the union approach is considered for both multidimensional and chronic identification steps. Both (14) and (1) are counting measures, but only (1) uses a counting approach explicitly for chronic multidimensional poverty identification.

Secondly, (1) fulfils Time Anonymity (TAN), which is inconsistent with a property of Durational Persistence Monotonicity (TPM), fulfilled by (14). According to (TPM), a poverty measure should increase with increases in s_{ijt}. Hence the timing of deprivation experiences matters; particularly, an index satisfying (TPM) shows higher poverty when poverty experiences are consecutive rather than scattered. (1) does not fulfil (TPM) but could be extended to do so. Note that (14) uses s_{ijt}, but not W.

[20] For different formulations of s_{ijt} see Bossert et al. (2012a) and Gradin et al. (2012).

Among minor differences, note that (14) can not be broken down by dimensional contributions, unless $\beta = 1$, whereas (1) can be broken down by dimensional contributions, although in a censored way when non-union approaches are used. Also Nicholas and Ray (2011) focus on $g_{ij}^{t}(0)$, although extensions for $g_{ij}^{t}(\alpha)$ should be straightforward.

Finally, it is worth noting that:

$$M_c^{\alpha} = \Omega_{\beta} \leftrightarrow \beta = 1, \alpha = 0, s_{ijt} = 1, W = (1, \dots, 1), 0 < k \leq 1, 0 < \tau \leq 1 \quad (8)$$

That is, (1) and (14) are equivalent if and only if a union approach is adopted for both chronic and multidimensional poverty identification, dimensions and spells are weighted equally, only deprivation counts are considered ($\alpha = 0$), and $\beta = 1$.

7. Empirical illustration

In this section we study chronic multidimensional poverty in Chile with a panel dataset whose data points are 1996, 2001 and 2006. These years relate to three identifiable GDP growth experiences. Firstly, in 1996 Chile underwent one of its most successful decades of GDP growth and income poverty reduction (Contreras, 2003; Contreras et al., 2001). In 2001 the country suffered from the negative impact of the Asian crises (Corbo and Schmidt-Hebbel, 2010); and in 2006 a public policy response to lower growth rates was implemented (Galasso, 2011; Glick and Menon, 2009).

We provide one empirical illustration with continuous variables and another one with ordinal variables. The next subsection discusses the data and the choice of well-being indicators. Then the application with continuous variables is described, followed by the application with ordinal variables. We also show how to compute dimensional and period contributions to overall chronic multidimensional poverty.

c. Data and indicators

The CASEN (National Survey of Economic Characterization) panel follows households in three regions (covering 60% of Chile's population) in three rounds: 1996, 2001 and 2006. The panel survey began in 2001 when the Chilean Government, together with the University of Chile, selected a representative subsample of 5,209 households (20,942 individuals) based on the cross-sectional survey of 1996.

The survey is deemed one of the longest panel datasets for a developing country with longitudinal and cross-sectional representativeness (Dercon and Shapiro, 2007). By design, it tends to overestimate income poverty levels vis-à-vis national ones by approximately 5%. Inflation factors were produced in order to adjust for attrition among young (20-29 years) and elderly people (over 60) in large households, and in rented dwellings (Bendezu et al., 2007). To correct for attrition, sample weights for longitudinal consistency were implemented; consequently results are not comparable with cross-sectional data from 2006.

We provide two illustrations of the chronic poverty indices. Firstly, three continuous variables are used to construct a chronic multidimensional poverty measure sensitive to the deprivation gap of each indicator. With

continuous indicators we can compute M_c^α and M_t^α for any level of α, thus generating information on the breadth and severity of chronic poverty. Secondly, we compute a second measure providing a broader understanding of multidimensional poverty by including discrete variables to the previous set. However, when ordinal, categorical or binary variables are included, we calculate only M_c^0 and M_t^0.

The survey's breadth allows for the computation of multiple well-being indicators. As with most household surveys, the questions elicit information on command over resources and functionings, rather than capabilities. There are examples of several choices of well-being dimensions, and respective indicators, made in the literature. Asselin (2009) presents a summary of commonly used dimensions. Our choices were constrained by the need to guarantee longitudinal comparability (e.g. changes in questionnaires preclude using certain indicators).

We select three dimensions: education, housing and employment/income. For the continuous illustration one indicator is selected in each dimension; for the full illustration three indicators are selected in each dimension. Table 1 presents our chosen set of dimensions and indicators for both illustrations; additionally, deprivation ratios per indicator (raw headcounts) are included.

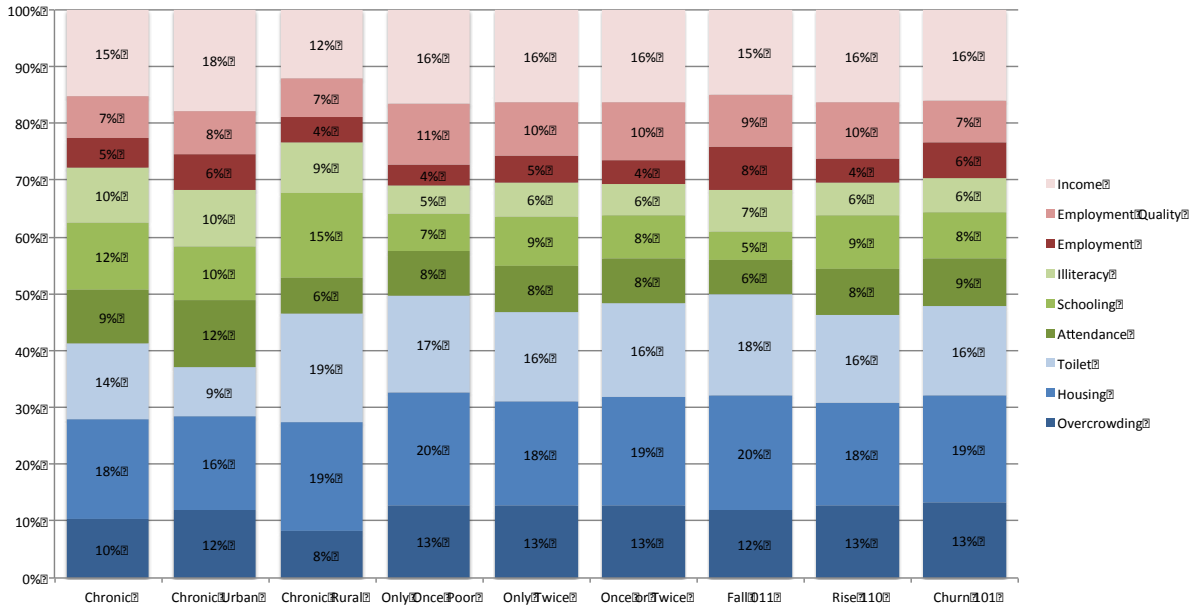
Table 1: Dimensions, Indicators, Weights and Raw Headcounts

			Weights		Raw Headcounts[21]		
Dimension	**Indicator**	**Deprivation Cut-off:** *An individual is deprived if he/she lives in a household with…*	**Continuous illustration**	**Full illustration**	**1996**	**2001**	**2006**
Education	Educational Achievement	no member fulfilling the legal number of compulsory years of education in their born year[22]	1/3	1/9	9% (7%-10%)	6% (5%-8%)	5% (4%-7%)
	School Attendance	at least one individual in school age (6 and 17 years) not attending school or evidences of more than 3 years of educational gap[23]		1/9	12% (10%-13%)	10% (8%-12%)	9% (7%-11%)
	Illiteracy	At least one member older than 17 not able to read or write[24]		1/9	8% (7%-10%)	6% (5%-8%)	4% (3%-5%)
Housing	Overcrowding	more than 2.5 persons per bedroom as defined by the Chilean Ministry of Social Development[25]	1/3	1/9	20% (17%-23%)	13% (11%-15%)	9% (8%-11%)
	Shelter	insufficient housing materials as defined by the Chilean Ministry of Social Development[26] (one or more deprived indicators for walls, floor or roof)		1/9	35% (31%-40%)	28% (24%-31%)	26% (23%-29%)
	Toilet	at least 1 toilet in the household[27]		1/9	19% (15%-23%)	12% (10%-15%)	6% (4%-7%)
Income/Employment	Income	a per capita income lower than the relevant national poverty line defined by the Social Planning Ministry	1/3	1/9	24% (20%-27%)	20% (17%-24%)	11% (9%-11%)
	Unemployment	no member older than 17 is employed[28]		1/9	6% (5%-7%)	10% (8%-12%)	8% (7%-9%)
	Quality of Employment	no member older than 17 has access to the pension system or has signed contract excluding rentiers, pensioners and entrepreneurs as defined by the Chilean Law		1/9	14% (12%-17%)	13% (11%-16%)	10% (8%-12%)

[21] In parentheses: lower and upper 95% confidence intervals.
[22]In 1920, the law 3.654 defines primary education as compulsory. In 1929, the order in council 5.291 extends this regulation to 6 years. Then, in 1965, the Government Decree 27.953 increases the levels of compulsory education to 8 years. Finally, in 2003, the Constitutional Law 19.876 defines 12 as the minimum schooling.
[23]The Constitutional Law 19.876 from 2003 increases from 8 to 12 the years of compulsory education covering from 5 to 17 year old individuals.
[24]The Chilean Government defined a set of policies to promote literacy regardless the age of the individuals (Contigo Aprendo). This indicator differs from schooling because it tries to capture the skill of literacy of each individual in the household. Consequently, if one individual is deprived the entire household is deprived. Conversely, in the schooling indicator if one individual has enough school the household is immediately deprived.
[25] Available at http://www.ministeriodesarrollosocial.gob.cl/casen/definiciones/vivienda.html
[26] Deprived Walls: adobe, wall without interior protection, mud, thatch, artisanal construction, rubbish, cardboard, tin or rubber. Deprived Roof: clinkstone, straw, bulrush, rubbish or cane. Deprived Floor: no protected cement foundation. Definition of acceptable floor changed between 2006 and 2009, stricter standards are preferred.
[27]The Chilean Government defines sanitation standards in using indicators for Access to drinking water and toilet. The former is not available for 2006 and for the latter there is no additional qualitative information regarding the type of toilet.
[28] In Voices of the poor, individuals remark the relevance of employment not only for the pecuniary benefits but also due to social and other outcomes.

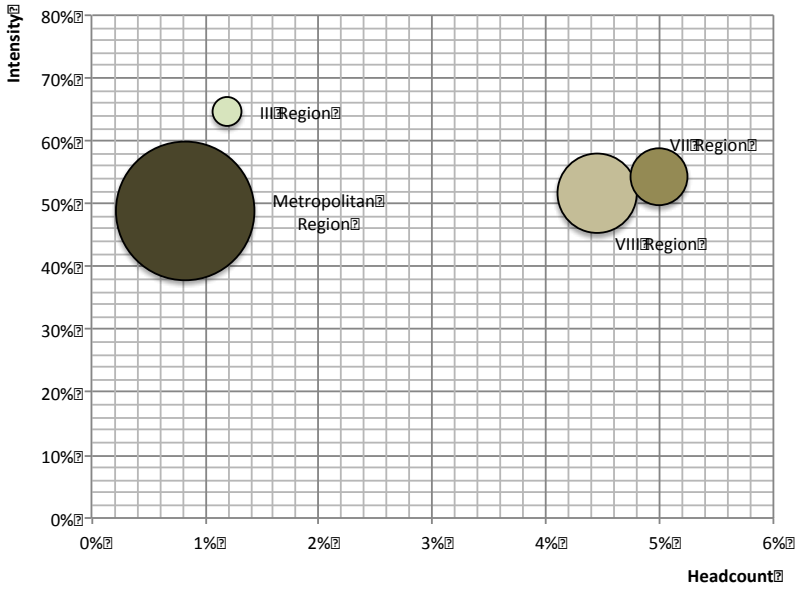
d. Continuous Illustration

The evolution of the three indicators used in the continuous illustration shows similar patterns. The deprivation headcounts of schooling, overcrowding and income fell between 1996 and 2006. The gap and the squared gap measures also declined, as shown in Figure 6: Dimensional breakdown of longitudinal poverty in selected poverty groups (k=3 and τ=3)



Finally, Figure 7 compares the situation of four regions based on the average deprivation share (A^c), or intensity, and the chronic poverty headcount for k=3 and **τ**=3. The bubble's size is corrected by the population share of each region. The metropolitan region presents the lowest level of poverty with the lowest headcount and corrected intensity. Compared to the metropolitan region, the III region has a similar percentage of poverty but with a higher intensity. The VII region has the highest proportion of chronically poor people (nearly 5%), although its average deprivation share of 54.26% is below that of the III region. Meanwhile the VIII region exhibits a similar level of intensity to the VII region, but with a lower headcount.

Figure 7: Chronic Multidimensional Poverty by region (k=3, τ=3)



8. Conclusions

It has been argued explicitly in the literature that poverty should be measured multi-dimensionally in terms of shortfalls of well-being attributes from minimally acceptable levels defined for different individuals in a society. Since, for many people worldwide, poverty is a situation from which it is difficult to escape over time, often it becomes important to track it over multiple periods. This, of course, requires panel data on different dimensions of well-being. Following Foster's (2011) income-based analysis, we have considered the spell, or duration, approach to chronic multidimensional poverty. We have defined multidimensional poverty following Alkire and Foster (2011). In this context two notions of identification are present: Firstly, the identification of the multi-dimensionally poor in each period; secondly, the minimum percentage of time a person has to spend in poverty in order to be identified as chronically poor.

The indices of chronic and transient poverty proposed in this paper represent the most straightforward merger between the Alkire-Foster approach to snapshot multidimensional poverty and the duration approach to chronic poverty. Being both counting approaches to poverty measurement, they blend naturally.As illustrated by the comparison of this paper's proposal with that of Nicholas and Ray (2011), there is scope for further developments on suitable indices of inter-temporal, multidimensional poverty, but these have costs in terms of policy relevance if they do not allow dimensional breakdown (Alkire and Foster 2013). Future research should study the theoretical, empirical and policy implications of combining different approaches to the identification and measurement of multidimensional poverty with different ways of understanding, indentifying and measuring chronic and transient poverty.

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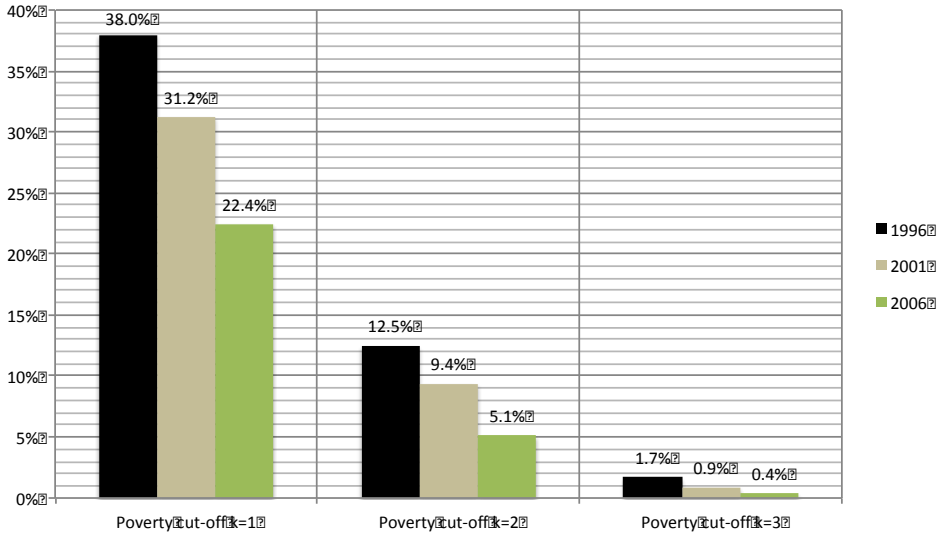
10. Appendix

Figure 8 (see Appendix).

Figure 1 shows that multidimensional poverty reduction is apparent across years and robust to choices of the poverty cut-off and the measure (adjusted headcount ratio, adjusted gap ratio and adjusted squared gap ratio). In terms of decomposition, income is the most important dimension, followed by housing and education (Table 4, in the Appendix). In this example changes in multidimensional poverty are mainly explained by a reduction in the headcount ratio (Table 4, in the Appendix).

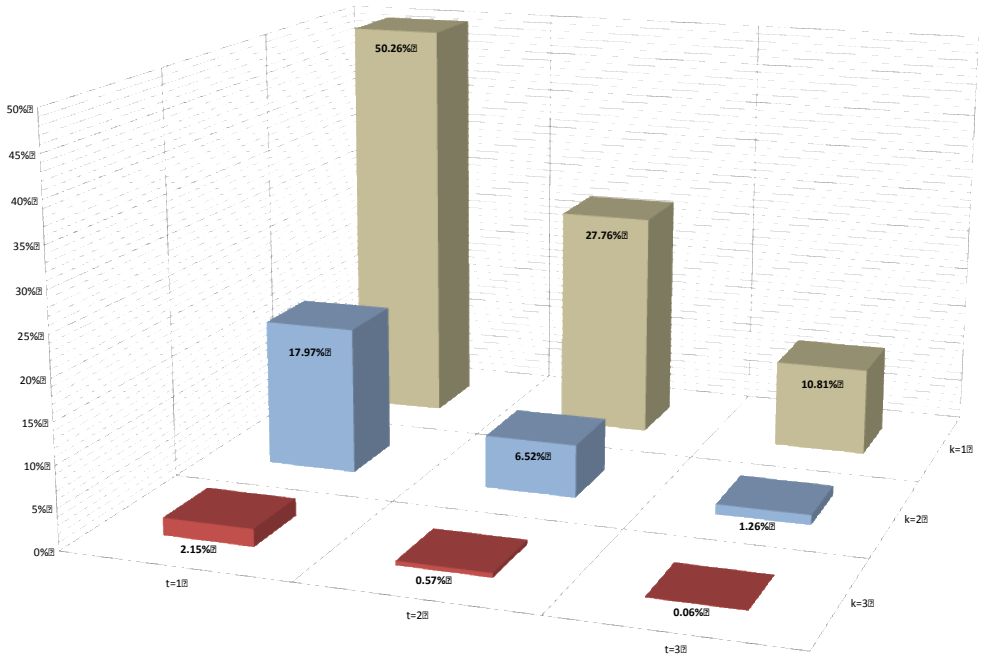
Figure 1: Headcount ratio by poverty cut-off and year



Error! Reference source not found. **shows the headcount ratio for all possible combinations of poverty (k) and time (τ) cut-offs. A double union approach (k=1 and τ=1) identifies 50.26% of the population as chronically poor with an average duration (D^c) of 0.589 periods and an intensity (A^c) of 0.440. On the other extreme, a double intersection approach (k=3 and τ=3) identifies as poor only 0,06% of the population as chronically poor, with an average duration and intensity equal to 1. With an intermediate approach of k=2 and τ=2 6.52% of the population would be identified as poor with an intensity of 0.744. Detailed information can be found in**

Table 5 of the Appendix.

Figure 2: Headcount ratio with all possible poverty (k) and time (τ) cut-offs



	t=1	*t=2*	*t=3*
k=1	*50.26%*	*27.76%*	*10.81%*
k=2	*17.97%*	*6.52%*	*1.26%*
k=3	*2.15%*	*0.57%*	*0.06%*

With a poverty cut-off of two (k=2), the intersections among poor populations in every year are represented in the Venn diagram of Figure 3. 1.26% of the population was poor in 1996, 2001 and 2006. They are the chronically poor for k=2 and τ=3 (as in Figure 2). When τ=3, subtracting that 1.26% from the population in the circles yields the proportion of transiently poor people, i.e. 16.71%. Likewise other headcounts of chronic and transient poverty can be computed with the diagram, which also provides information about the transition of entry into and exit from poverty.

Figure 3: Multidimensional Transitions 1996-2001-2006

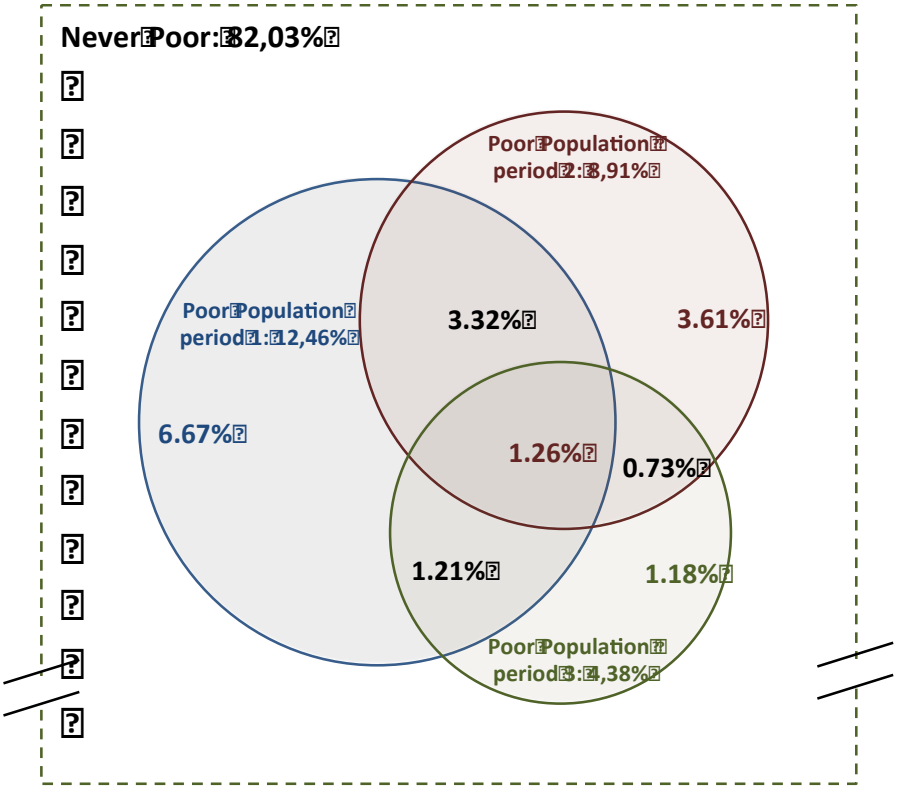
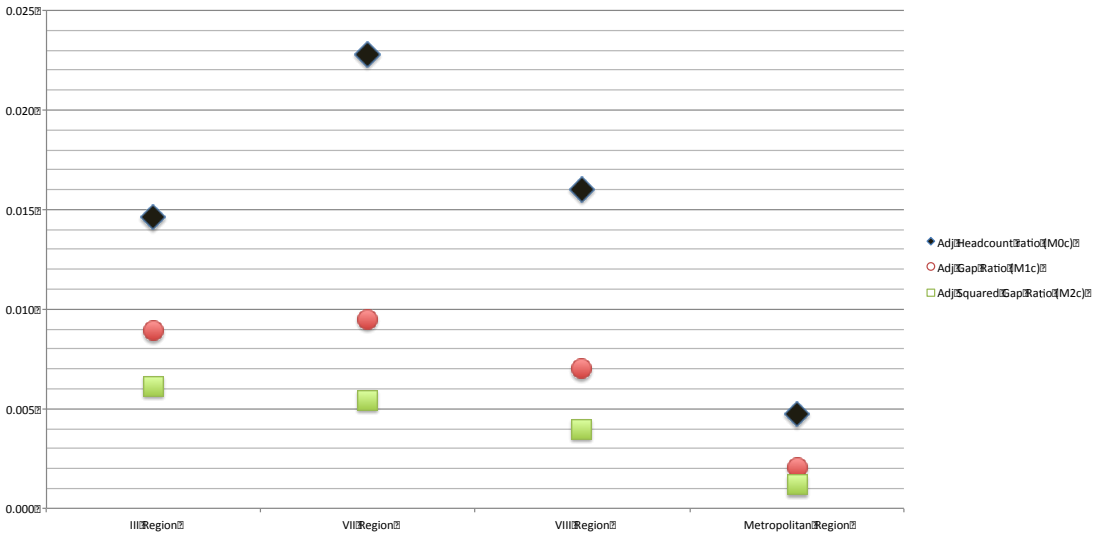


Figure 4 presents the regional breakdown of chronic poverty for $\alpha = 0,1,2$, using k=2 and τ=3. Interestingly, region VII exhibits a higher adjusted headcount ratio and a lower squared gap measure when compared to region III. This is an interesting empirical example showing that higher breadth of deprivations does not necessarily mean higher intensity. Results for the three measures and all natural cut-offs appear in Figure 9 (see Appendix).

Figure 4: Chronic Multidimensional Poverty by Region with k=2, τ=3 for different α



e. Ordinal Illustration

The full illustration provides a more comprehensive picture of multidimensional poverty by including more (ordered categorical) indicators. As Table 1 shows, each dimension is enriched with two additional indicators.

Table 2 shows cross sectional and longitudinal results with k=2. Full results for several other cut-off combinations are available in Table 6 (see Appendix). Similarly to the cardinal illustration results, cross-sectional multidimensional poverty falls from 0.07 to 0.02 between 1996 and 2006. Most of the improvement is due to a lower headcount ratio, whereas the intensity level declines marginally. Clearly, the largest contributors to multidimensional poverty are housing, toilet, overcrowding and income.

The longitudinal results show that under the time union approach (τ=1), 17.81% of the population is chronically poor, experiencing poverty spells during 51.74% of the periods, and in 50.49% of the possible dimensions. The chronic adjusted headcount ratio in this case is 0.05. When τ=3, only 2.38% of the population is chronically poor, and in the 51.81% of their dimensions. The chronic adjusted headcount ratio for intersection approach is 0.01.

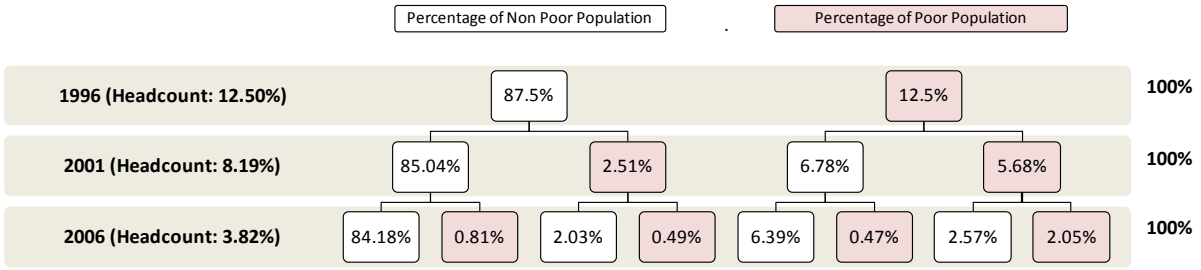
Table 2: Cross sectional and longitudinal poverty measures with k=3

	Cross Sectional Results			***Longitudinal Results***		
	1996-	*2001-*	*2006-*	*t=1*	*t=2*	*t=3*
Headcount Ratio	*12.48%*	*8.19%*	*3.82%*	*15.82%*	*6.59%*	*2.05%*
Duration (Dc)	-	-	-	*51.55%*	*77.04%*	*100.00%*
Intensity	*51.52%*	*49.71%*	*48.78%*	*50.49%*	*51.79%*	*51.84%*
Adjusted Headcount ratio	*0.064*	*0.041*	*0.019*	*0.041*	*0.026*	*0.011*
Censored Headcount						
Overcrowding	*7.08%*	*4.44%*	*1.97%*	*4.50%*	*2.78%*	*0.98%*
Housing	*10.81%*	*6.84%*	*3.10%*	*6.92%*	*4.26%*	*1.69%*
Toilet	*9.82%*	*5.44%*	*2.21%*	*5.82%*	*3.53%*	*1.29%*
Attendance	*4.54%*	*2.96%*	*1.58%*	*3.03%*	*2.00%*	*0.88%*
Schooling	*5.08%*	*3.02%*	*1.69%*	*3.26%*	*2.39%*	*1.15%*
Illiteracy	*3.84%*	*2.34%*	*1.12%*	*2.43%*	*1.75%*	*0.91%*
Employment	*1.82%*	*2.08%*	*1.08%*	*1.66%*	*1.16%*	*0.50%*
Employment Quality	*5.61%*	*3.53%*	*1.32%*	*3.49%*	*2.06%*	*0.70%*
Income	*9.29%*	*5.80%*	*2.73%*	*5.94%*	*3.73%*	*1.46%*
Relative Contribution M0						
Overcrowding	*12.20%*	*11.69%*	*10.65%*	*12.14%*	*11.74%*	*10.28%*
Housing	*18.61%*	*18.00%*	*16.75%*	*18.67%*	*18.00%*	*17.62%*
Toilet	*16.92%*	*14.31%*	*11.92%*	*15.72%*	*14.94%*	*13.52%*
Attendance	*7.82%*	*7.77%*	*8.56%*	*8.17%*	*8.46%*	*9.19%*
Schooling	*8.75%*	*7.95%*	*9.15%*	*8.81%*	*10.11%*	*12.02%*
Illiteracy	*6.61%*	*6.16%*	*6.08%*	*6.57%*	*7.40%*	*9.54%*
Employment	*3.13%*	*5.48%*	*5.83%*	*4.48%*	*4.89%*	*5.25%*
Employment Quality	*9.66%*	*9.29%*	*7.15%*	*9.42%*	*8.69%*	*7.36%*
Income	*15.99%*	*15.26%*	*14.74%*	*16.03%*	*15.78%*	*15.22%*

Figure 5 displays the transitions into and out of poverty spells in a way that highlights the connection between the year-specific poverty headcounts and their chronic counterparts for different choices of τ, similar to the Venn diagram. For instance, with τ=1, the chronic poverty headcount of 15.82% is equal to the headcount of

1996 (12.5%) plus the headcount of 2001 (8.19%) plus the headcount of 2006 (3.75%) minus the proportion of people who were poor in 1996 and 2001 (5.68%) minus the proportion of people who were poor in 2006 and in any earlier period (0.49%+0.47%+2.05%).

Figure 5: Transitions entry and exit from multidimensional poverty (k=3)



Following Figure 5 we can identify and compute different headcounts of chronic and transient poverty using different time cut-offs. For each of these groups of poor people we can also compute complementary measures of incidence, duration and intensity using the methods described in section 5. Table 3 shows the adjusted headcount ratio and its components for different groups of poor people identified by different criteria of chronicity and transiency. Clearly, transient poverty is more prevalent than chronic poverty, although the average intensity of poverty (second-to-last row in Table 3) is lower among the chronically poor in the Chilean case.

Table 3: Chronic and transient poverty for selected groups with k=3 and τ=3

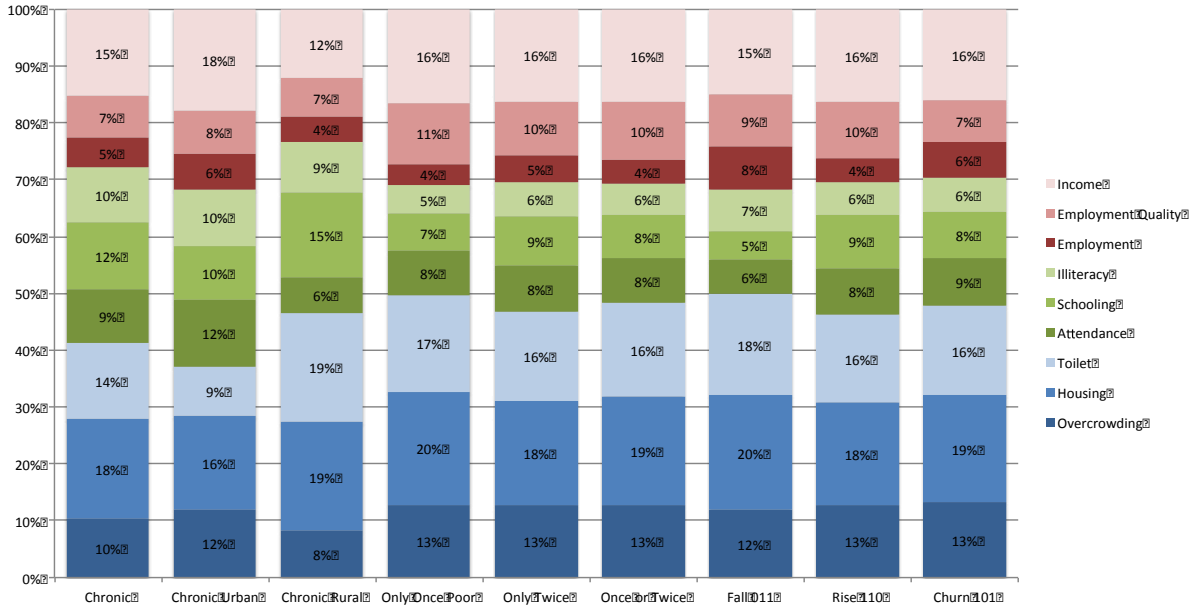
		Transient Measures					
	Chronic	*Only Once Poor*	*Only Twice*	*Once or Twice*	*Fall 011**	*Rise 110***	*Churn 101****
Headcount Ratio(Hc)	*2.052%*	*9.228%*	*4.539%*	*13.767%*	*0.495%*	*3.570%*	*0.474%*
Duration (Dc)	*0.333*	*0.667*	*0.443*	*1.000*	*0.667*	*0.667*	*0.667*
Adj Av Dep Share (Ac)	*0.483*	*0.517*	*0.500*	*0.518*	*0.501*	*0.524*	*0.485*
Adj Headcount ratio (M0c)	*0.015*	*0.016*	*0.031*	*0.011*	*0.002*	*0.012*	*0.002*

*Fall 011: Non-poor in 1996, then poor in the subsequent periods. **Rise 110: Poor in 1996 and 2001, then non-poor in 2006. ***Churn 101: Poor in 1996, non-poor in 2001, poor in 2006.

Additionally, we can assess the contribution of each deprivation to the adjusted headcount ratio of each one of the above poverty groups. The contributions are based on the censored headcounts, i.e. the proportions of people who are poor (e.g. chronically or transiently) *and* deprived in a specific variable. For instance, in Figure 6 the first three leftmost bars decompose poverty by dimensions for the chronically poor at the national level, and by urban and rural areas. The contributions of toilet, housing and schooling are most significant in rural locations, while income and housing are prominent in cities. Similar analyses can be performed for the other

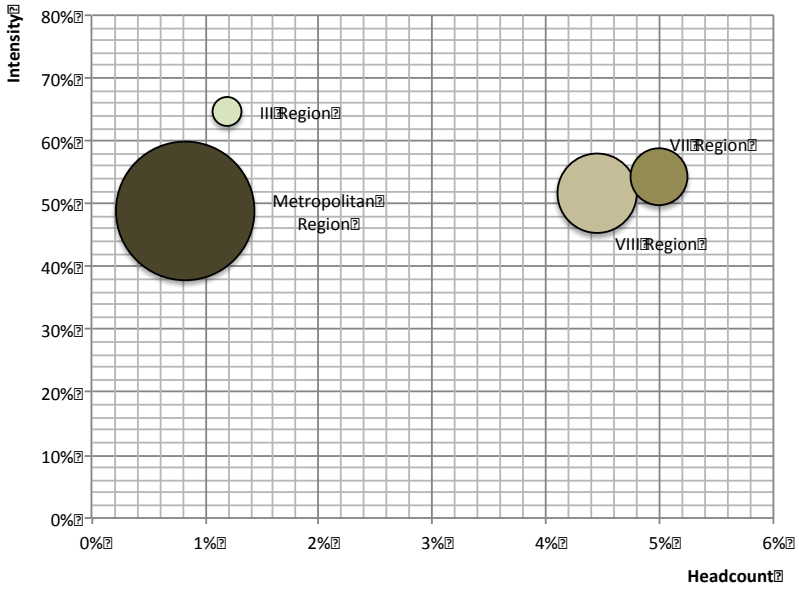
poverty groups (see six rightmost bars in Figure 6). More censored headcounts, and computations of relative contributions, for different choices of k and t are available in Table 6 (in the Appendix).

Figure 6: Dimensional breakdown of longitudinal poverty in selected poverty groups (k=3 and τ=3)



Finally, Figure 7 compares the situation of four regions based on the average deprivation share (A^c), or intensity, and the chronic poverty headcount for k=3 and **τ**=3. The bubble's size is corrected by the population share of each region. The metropolitan region presents the lowest level of poverty with the lowest headcount and corrected intensity. Compared to the metropolitan region, the III region has a similar percentage of poverty but with a higher intensity. The VII region has the highest proportion of chronically poor people (nearly 5%), although its average deprivation share of 54.26% is below that of the III region. Meanwhile the VIII region exhibits a similar level of intensity to the VII region, but with a lower headcount.

Figure 7: Chronic Multidimensional Poverty by region (k=3, τ=3)



8. Conclusions

It has been argued explicitly in the literature that poverty should be measured multi-dimensionally in terms of shortfalls of well-being attributes from minimally acceptable levels defined for different individuals in a society. Since, for many people worldwide, poverty is a situation from which it is difficult to escape over time, often it becomes important to track it over multiple periods. This, of course, requires panel data on different dimensions of well-being. Following Foster's (2011) income-based analysis, we have considered the spell, or duration, approach to chronic multidimensional poverty. We have defined multidimensional poverty following Alkire and Foster (2011). In this context two notions of identification are present: Firstly, the identification of the multi-dimensionally poor in each period; secondly, the minimum percentage of time a person has to spend in poverty in order to be identified as chronically poor.

The indices of chronic and transient poverty proposed in this paper represent the most straightforward merger between the Alkire-Foster approach to snapshot multidimensional poverty and the duration approach to chronic poverty. Being both counting approaches to poverty measurement, they blend naturally.As illustrated by the comparison of this paper's proposal with that of Nicholas and Ray (2011), there is scope for further developments on suitable indices of inter-temporal, multidimensional poverty, but these have costs in terms of policy relevance if they do not allow dimensional breakdown (Alkire and Foster 2013). Future research should study the theoretical, empirical and policy implications of combining different approaches to the identification and measurement of multidimensional poverty with different ways of understanding, indentifying and measuring chronic and transient poverty.

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10. Appendix

Figure 8: Raw Headcount, Gap and Squared Gap ratio per dimension and year

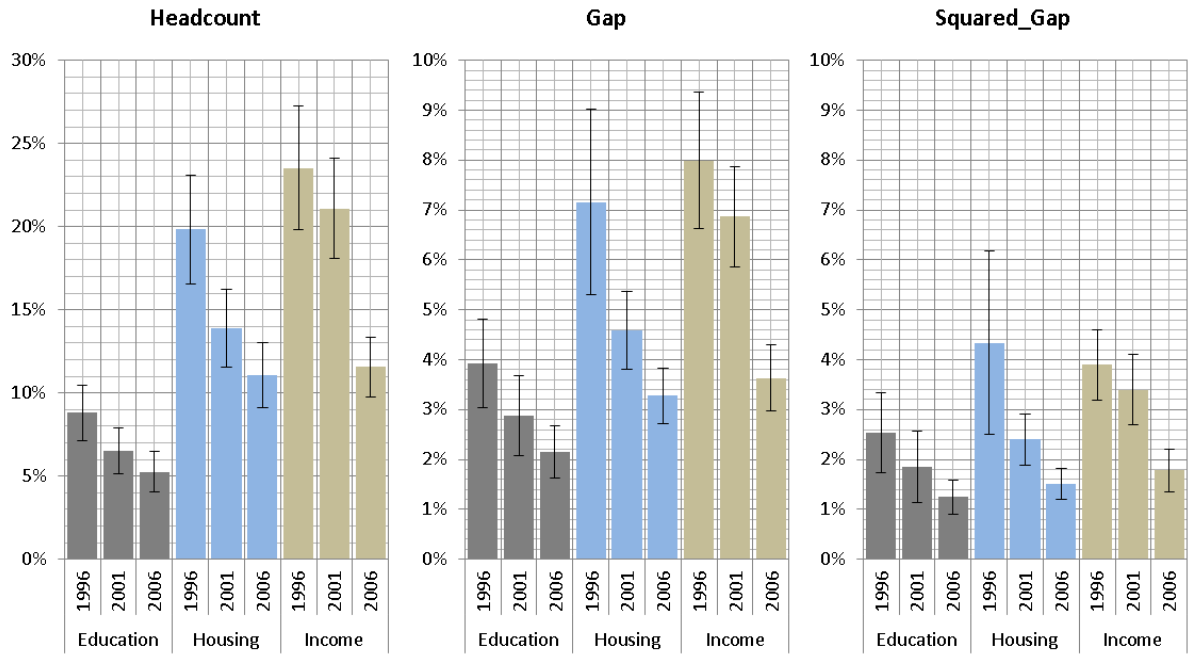

Table 4: Cross-sectional results for cardinal illustration and all possible cut-offs

	Poverty cut-off k=1			***Poverty cut-off k=2***			***Poverty cut-off k=3***		
	1996-	*2001-*	*2006-*	*1996-*	*2001-*	*2006-*	*1996-*	*2001-*	*2006-*
Headcount Ratio	*0.38*	*0.31*	*0.22*	*0.12*	*0.09*	*0.05*	*0.02*	*0.01*	*0.00*
Intensity	*0.46*	*0.44*	*0.42*	*0.71*	*0.70*	*0.69*	*1.00*	*1.00*	*1.00*
Average Gap	*0.35*	*0.34*	*0.32*	*0.40*	*0.35*	*0.35*	*0.44*	*0.35*	*0.31*
Squared Gap	*0.19*	*0.18*	*0.16*	*0.24*	*0.19*	*0.17*	*0.26*	*0.18*	*0.14*
Adjusted Headcount ratio	*0.17*	*0.14*	*0.09*	*0.09*	*0.07*	*0.04*	*0.02*	*0.01*	*0.00*
Adjusted Gap Ratio	*0.06*	*0.05*	*0.03*	*0.04*	*0.02*	*0.01*	*0.01*	*0.00*	*0.00*
Adjusted Squared Gap Ratio	*0.04*	*0.03*	*0.02*	*0.02*	*0.01*	*0.01*	*0.00*	*0.00*	*0.00*
Weighted Contribution M0									
Education	*0.03*	*0.02*	*0.02*	*0.02*	*0.01*	*0.01*	*0.01*	*0.00*	*0.00*
Income	*0.08*	*0.07*	*0.04*	*0.04*	*0.03*	*0.01*	*0.01*	*0.00*	*0.00*
Housing	*0.07*	*0.05*	*0.04*	*0.03*	*0.03*	*0.01*	*0.01*	*0.00*	*0.00*
Relative Contribution M0									
Education	*17%*	*16%*	*19%*	*19%*	*14%*	*19%*	*33%*	*33%*	*33%*
Income	*45%*	*51%*	*41%*	*42%*	*45%*	*42%*	*33%*	*33%*	*33%*
Housing	*38%*	*33%*	*40%*	*39%*	*41%*	*39%*	*33%*	*33%*	*33%*

Table 5: Aggregated results chronic poverty all poverty and temporal cut-offs

	Cut-off k=1			*Cut-off k=2*			*Cut-off k=3*		
	t=1	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*
Headcount Ratio(Hc)	*0.503*	*0.278*	*0.108*	*0.180*	*0.065*	*0.013*	*0.022*	*0.006*	*0.001*
Duration (Dc)	*0.589*	*0.797*	*1.000*	*0.478*	*0.731*	*1.000*	*0.430*	*0.701*	*1.000*
Adj Av Dep Share (Ac)	*0.440*	*0.463*	*0.500*	*0.703*	*0.721*	*0.744*	*1.000*	*1.000*	*1.000*
Adj Av Gap Share	*0.154*	*0.168*	*0.198*	*0.263*	*0.278*	*0.326*	*0.395*	*0.399*	*0.422*

Adj Av Squared Gap Share	*0.084*	*0.092*	*0.111*	*0.147*	*0.148*	*0.188*	*0.221*	*0.231*	*0.240*
Adj Headcount ratio (M0c)	*0.130*	*0.102*	*0.054*	*0.060*	*0.034*	*0.009*	*0.009*	*0.004*	*0.001*
Adj Gap Ratio (M1c)	*0.046*	*0.037*	*0.021*	*0.023*	*0.013*	*0.004*	*0.004*	*0.002*	*0.000*
Adj Squared Gap Ratio (M2c)	*0.025*	*0.020*	*0.012*	*0.013*	*0.007*	*0.002*	*0.002*	*0.001*	*0.000*

Figure 9: Chronic Multidimensional Poverty with $\alpha = 0$, $\alpha = 1$ and $\alpha = 2$

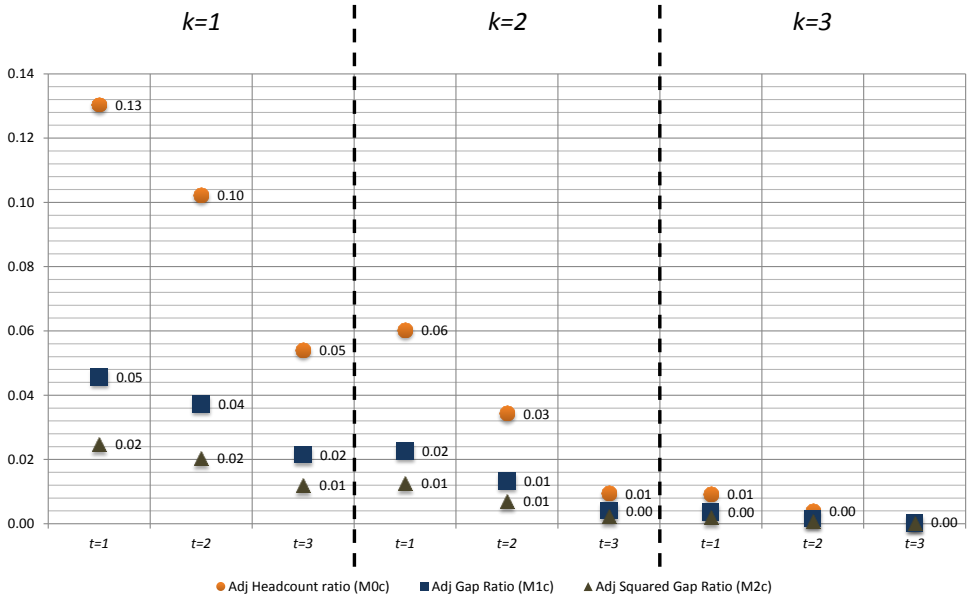

Table 6: Multidimensional Poverty Full Model

	Cut-off k=20%			*Cut-off k=40%*			*Cut-off k=60%*			*Cut-off k=80%*			*Cut-off k=100%*		
	t=1	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*	*t=1*	*t=2*	*t=3*
Headcount Ratio(Hc)	*49.67%*	*29.23%*	*13.71%*	*15.82%*	*6.59%*	*2.05%*	*2.46%*	*0.58%*	*0.02%*	*0.04%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Duration (Dc)	*0.62*	*0.82*	*1.00*	*0.52*	*0.77*	*1.00*	*0.41*	*0.68*	*1.00*	*0.33*	.	.	.	.	.
Adj Av Dep Share (Ac)	*0.33*	*0.35*	*0.37*	*0.50*	*0.52*	*0.52*	*0.69*	*0.70*	*0.73*	*0.89*	.	.	.	.	.
Adj Headcount ratio (M0c)	*0.10*	*0.08*	*0.05*	*0.04*	*0.03*	*0.01*	*0.01*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*	*0.00*
Censored Headcount															
Overcrowding	*11.51%*	*8.98%*	*5.20%*	*4.50%*	*2.78%*	*0.98%*	*0.82%*	*0.32%*	*0.02%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Housing	*19.97%*	*16.69%*	*9.91%*	*6.92%*	*4.26%*	*1.69%*	*0.98%*	*0.39%*	*0.02%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Toilet	*11.57%*	*10.16%*	*5.85%*	*5.82%*	*3.53%*	*1.29%*	*0.91%*	*0.38%*	*0.01%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Attendance	*8.06%*	*6.62%*	*4.13%*	*3.03%*	*2.00%*	*0.88%*	*0.58%*	*0.21%*	*0.01%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Schooling	*6.05%*	*5.61%*	*3.97%*	*3.26%*	*2.39%*	*1.15%*	*0.78%*	*0.31%*	*0.02%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Illiteracy	*5.20%*	*4.72%*	*3.40%*	*2.43%*	*1.75%*	*0.91%*	*0.52%*	*0.22%*	*0.02%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Employment	*4.97%*	*3.77%*	*2.64%*	*1.66%*	*1.16%*	*0.50%*	*0.26%*	*0.14%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Empl_qual	*8.31%*	*6.55%*	*3.65%*	*3.49%*	*2.06%*	*0.70%*	*0.62%*	*0.17%*	*0.00%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Income	*14.79%*	*11.80%*	*6.76%*	*5.94%*	*3.73%*	*1.46%*	*0.85%*	*0.32%*	*0.01%*	*0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
Relative Contribution M0															
Overcrowding	*12.73%*	*11.99%*	*11.44%*	*12.14%*	*11.74%*	*10.28%*	*13.00%*	*12.87%*	*15.30%*	*12.50%*	.	.	.	.	.
Housing	*22.09%*	*22.28%*	*21.77%*	*18.67%*	*18.00%*	*17.62%*	*15.47%*	*15.86%*	*15.30%*	*12.50%*	.	.	.	.	.
Toilet	*12.79%*	*13.57%*	*12.85%*	*15.72%*	*14.94%*	*13.52%*	*14.35%*	*15.40%*	*13.33%*	*12.50%*	.	.	.	.	.
Attendance	*8.91%*	*8.84%*	*9.07%*	*8.17%*	*8.46%*	*9.19%*	*9.14%*	*8.41%*	*6.26%*	*12.50%*	.	.	.	.	.
Schooling	*6.69%*	*7.50%*	*8.72%*	*8.81%*	*10.11%*	*12.02%*	*12.37%*	*12.80%*	*15.30%*	*12.50%*	.	.	.	.	.
Illiteracy	*5.75%*	*6.30%*	*7.46%*	*6.57%*	*7.40%*	*9.54%*	*8.25%*	*8.87%*	*15.30%*	*12.50%*	.	.	.	.	.
Employment	*5.50%*	*5.03%*	*5.81%*	*4.48%*	*4.89%*	*5.25%*	*4.09%*	*5.71%*	*3.94%*	*0.00%*	.	.	.	.	.
Empl_qual	*9.19%*	*8.74%*	*8.02%*	*9.42%*	*8.69%*	*7.36%*	*9.82%*	*7.03%*	*1.97%*	*12.50%*	.	.	.	.	.
Income	*16.35%*	*15.76%*	*14.86%*	*16.03%*	*15.78%*	*15.22%*	*13.53%*	*13.04%*	*13.33%*	*12.50%*	.	.	.	.	.