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Reassessing China's Productive Performance Using Tight Bounds of "True" Index Numbers

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REASSESSING CHINA'S PRODUCTIVE PERFORMANCE USING TIGHT BOUNDS OF "TRUE" INDEX NUMBERS

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"We know that all economic data are a boring form of science fiction. But the Chinese are more fictional than most. What's happening in China─God knows!"

Paul Krugman (2012, p. 25).

1. Introduction

The global economic downturn that has been caused by the current financial crisis has revealed certain important characteristics of the strength and weakness of the economic growth of China and other fast-growing emerging countries. The strength derives from increasing openness of the national production system to foreign direct investments and industrial integration with "global supply chains" taking advantage of the low cost of labour. The weakness of this model becomes apparent as that of the typical emerging export-oriented economy that is highly vulnerable during recession periods. Moreover, as in other fast-growing Asian countries, the economic expansion is still capital intensive rather than technology driven. This increases the exposure of China to bubble risks.

The most dynamic Chinese provinces were particularly hit during the economic downturn of 2008. More than 50,000 SME's went bankrupt in Guangdong province with the net exports' contribution to economic growth becoming negative during that year[1]. During the recession it became evident that China's status of "world's factory" makes domestic economy highly vulnerable at least in certain industrialized regions. It is widely recognized that innovation in products, technology, and organization is the key factor also in China. From the domestic demand side, increase in the overall level of consumption and change in tastes and typology of consumed commodities and services will entail increase in personal income. This could not be achieved without strong productivity growth.

The need of shifting the Chinese economy to the more active role of one of the "locomotives" of the world can be met only under significant structural changes that can result to be even stronger and more profound that those that have been taking places during the last two decades. These entail the surge in domestic demand, which, in turn, could be achieved only with an increase in real per-capita incomes, and, in other terms, an improvement in the standard of living of the population via total factor productivity (*TFP*) growth.

How far is China from attaining this stage? In a joint research paper, one of us has analysed China's experience following that of Japan in the 60s-70s - learning and absorbing by copying the received pattern of production (see Harry Wu *et al*.. 2011). But, apparently, Japan emerged from that stage and entered a stage of true innovation more quickly than China is now doing. China seems to have been active in the investment of new imported technologies, but less successful with reference to the available best-practice techniques.

We may note that, notwithstanding the primary position of the Chinese economy among the exporting countries, Chinese brands still lack the main components for global success. China has recently acquired several worldwide recognized brands through mergers and acquisitions, as for example Thomson Electrics acquired by *TCL*, *IBM*'s PC business acquired by *Lenovo*, and *Rover'*s assets acquired by *Nanjing Automotive*. However, native globally recognized brands like *Sony* and *Toyota* in Japan, and *Samsung* in Korea, will become possible in China only through autonomous innovation in products and organization which will build competitiveness on high quality and new technologies rather than on low costs and prices[2]. To be sure, there are positive signs on the markets for strong incentives to innovation and the establishment of local Chinese brands in the future. On the side of consumers, as the standard of living improves, demand for quality and branded products increases while demand for counterfeit products is fading away. A survey study by McKinsey (cited by *The Economist*, 14th January 2012, p. 64) revealed, for example, that the proportion of Chinese consumers declaring to be willing to buy fake jewellery dropped from 32% in 2008 to 12% in 2010.

Innovation and high productivity growth are therefore of primary importance for a sustainable economic growth. As Schumpeter (1934) strongly emphasized, innovation may take the form of the introduction of new and improved processes (*process innovation* with new and better management and technologies) and products (*product innovation*). With limitation in data, where it is not possible to distinguish these two components, innovation can be subsumed in terms of change in the production function relating output to inputs of production, which in shorted terminology can be called "technological change" or more

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[1] See, for example, Wang et al. (2011, p. 17).

[2] See Fan (2005)(2006)(2008) and Floyd, Ardley, and McManus (2011) on the absence of new Chinese brands and Anholt (2007) on competitive identity based on new brand management.

simply “technology”. Economic performance and, more strictly, “total factor productivity” have a wider meaning including components such as (non-constant) economies of scale, externalities and efficiency.

In order to assess the direction and sustainability of economic development of a country, we need to decompose, both at macro and sectoral level, changes in TFP giving account of the relative importance and implications of “technology” and “scale” factors. While the former is of primary importance for the sustainability of economic growth in the long run, the latter are less reliable since they could soon encounter limits and exhaust their contribution.

The proposed methodology is based on index number approach where certain restricted hypotheses generally used in previous studies are generalized in order to take into account of the characteristics of an emerging economy benefiting of increasing returns to scale. The economic performance of China will be measured using a new data set including, in particular, the sectoral *Wu Chinese Economy Database* referring to the period 1949-2009. It will be also assessed at what degree our calculations are consistent with the official estimation of the real GDP and its components.

Previous empirical findings

The study of productivity growth in China has been one of the economic subject matters that have been closely scrutinized in recent times along these lines. One of the reasons for this lies in the need to understand China’s growth mechanism and sustainability of prolonged periods of high growth rates in the future. The debate on the experience of other countries in the comparison between USA and Europe and, more significantly, in the analysis of the type of growth in other Asian countries has pointed to the importance of the relative contribution to output of changes in factor inputs and total factor productivity. The famous discussion by Krugman (1994), Young (1994) and Kim and Lau (1994) raised serious doubts on the sustainability of growth in East Asian countries, which they found more inputs intensive and returns to scale rather than based on technological progress. This would lead to reduce the economic expansion of those countries as the limits in the availability of factor inputs are inevitably reached if productivity contribution were not boosted. Their estimations were, however, put into question by other authors (Kawai 1994, Oshima 1995, Sarel, 1995) leaving the debate to further insights.

The debate on China’s economic growth is similarly open today. Many studies based on growth accounting and other methods have not reached unanimous conclusions, although they seem with some variations to recognize a minor role of TFP growth. In a recent review of the literature on this subject, Yanrui Wu (2011) of Business School at University of Western Australia, has taken into account 151 empirical studies of TFP contribution to China’s growth and constructed statistical averaging indicators of the results obtained by them. The estimated mean indicates that about one-third of China’s growth can be attributed to TFP growth. Wu concludes that, although such a measure is not as high as that found in the most advanced economies, it indicates that further growth is in some extent sustainable. However, substantial variation can be noted in the results among the studies under review. In particular, the results seem to be sensitive to the choice of techniques of analysis, types of models, and types of indicators used for productivity assessment. Many of these studies exhibit the limitations that we shall try to bypass in the present studies.

Among the latest studies, that are not taken into account in Yanrui Wu's, 2011 survey, Li and Liu (2009)(2011) found that the major contributor to economic growth in China is input growth, with human capital still remaining inadequate. Productivity growth was of minor importance and was due mainly to technical progress, whereas scale effects had become visible only in recent years. Özyurt (2009) and Cao et al. (2009) distinguished quantity and quality components of capital and labour inputs in their computation of TFP growth at aggregate and disaggregate industry level. Özyurt et al (2011) found that, in some Chinese provinces, scale effects were even negative and inefficiency was decreased only in casesere technical progress took place.

The data used in previous analysis have been put into question by a number of studies. Harry Wu (1993, 2000, 2002a, 2002b, 2008, 2011), Wu and Shea, 2001, Shiu and Wu, 2007, Wu and Ximing, 2010, Wu et al, 2011), Maddison (1998), Holz (2004)(2006), Sun and Ren (2007), Wang and Szirmai (2012) have reconstructed their own economic accounts of China. The analytical results on productivity growth seem to lead to substantial differences with respect to those based on official statistics. The main conclusion of these studies is that productivity growth has contributed very little to China's economic development except during the recent years, during which however remained still below the inputs' contribution to growth. The amount of productivity growth to be attributed to technical progress (imported or spurred by domestic innovation) remains, however, to be systematically explored.

2. Technical progress as a factor of growth

Schumpeter (1934) constructed his theory of economic development on three basic elements:

1) *Innovation* as the essential function of the entrepreneur;
2) *Credit mechanism*;
3) *Profit maximization* as the main objective driving the entrepreneur's behaviour.

Schumpeter (1939, Vol. I, p. 84) defined innovation explicitly in terms of a change in the form of the *production function*:

> "We will now define innovation more rigorously by means of the production function. […] This function describes the way in which quantity of products varies if quantity of factors vary. If, instead of quantities of factors we vary the form of the function, we have an innovation. […] *[W]e will simply define innovation as the setting up of new production function. This covers the case of a new commodity as well as those of a new form of organization or a merger, or the opening up of new markets*." (Italics added.)

As noted by early commentators in the 1950s, Schumpeter's definition of innovation based on the change in the production function resembles the definition of technological change used by students of *productivity* and *technical progress*. Brozen (1951, p.238) started his article as follows:

> “Investigation of the role of technological change in economic growth is made easier if we examine it at three different levels: at the level of invention, of innovation, and of imitation. We are led to this approach quite naturally through the circumstance that movement in technology has been defined as a change in the production function and that this may have any one of three different meanings”.

More explicitly, Ruttan (1959, p. 598) noted that the above quotation from Schumpeter appears remarkably close to the following definition of *technical change* given by Solow (1957, p. 312):

> “If Q represents output and K and L represent capital and labor in “physical” units, then the aggregate production function can be written as:
>
> $$Q = F(K, L; t)$$
>
> The variable t […] appears in F to allow for technical change. I am using the phrase “technical change” as a shorthand expression for *any kind of a shift* in the production function”. (Italics in the original.)

(For a similar definition, see Ruttan, 1956.) Fellner (1956a, 1956b) discussed the same concept under the heading of technological-organization change. Ruttan (1959, p. 599) also noted:

> “It seems fairly clear that current interest in technological change and growth in total [factor] productivity is focused on the same problem which Schumpeter treated under the heading of innovation. That problem is the effect of technological and organizational change, operating through the production function, on economic growth. Schumpeter was primarily interested in changes in the production functions of the technological leaders — the innovating firms — because of the growth forces which adoption of the new methods of production set in motion. Recent students of growth and productivity have, on the other hand, given major attention to the production function set in motion. Recent students of growth and productivity have, on the other hand, given major attention to the production function which describes the average performance of the economy or industry”.

Summing up, Paul R. Schweitzer (1961, p. 153) claimed that [Ruttan’s] “term ‘technological change’ and Schumpeter’s term ‘innovation’ as well as Fellner’s ‘technological-organizational change’ and Solow’s ‘technical change’ all refer to the same phenomenon, namely, a shift in the production function". However, he noted that the techniques in production may change also because the level of output changes (implying that the returns to scale are not constant). We note that the most appropriate term that will encompass the Schumpeterian notion of innovation within the production function is "technical progress", which together other components such as inefficiencies, non-constant returns to scale, externalities, make up what we call total factor productivity (see also Domar, 1961), that is

$$T\dot{F}P = \dot{R}S + \dot{T}C$$

where $T\dot{F}P$ is total factor productivity change, $\dot{R}S$ is the component due to scale economies and externalities, and $\dot{T}C$ is technological change component due to innovation and efficiency gains.

As Schumpeter himself noted (followed by many others, among whom Domar, 1961 and Johnston, 1966), innovation may take the form of the introduction of new and improved processes (*process innovation* with new and better management and technologies) and products (*product innovation*). Both can be captured by the shift in the production function, which is part of "The Residual" in the growth accounting literature.

3. Growth accounting methodology: Further refinement

Our methodology of productivity measurement is built upon a modification of the Solow-Jorgenson-Griliches growth accounting method by following the Schumpeterian interpretation and developing the economic theory of "true" index numbers consistently with this interpretation. In his historical account of this method, Zvi Griliches (1996) did not mentioned Schumpeter in reporting on the discovery of the "Residual" (the measure of technical progress), but he mentioned Ruttan (1956) among the precursors of Solow (1957). He attributed to Solow not the method of calculation "which by then was being taught to most graduate students", but the "explicit integration of economic theory into such calculations" (p. 1328). Griliches cited Morris Copeland (1937) as the first mention of an output-to-input index but, in a footnote, he conceded that "more thorough research may unearth even earlier references" (p. 1934). Schumpeter was certainly one major contributor at the centre of economic theory of technical change, but Griliches pointed Solow for having explicitly "clarified the meaning of what were heretofore relatively arcane index number calculations" by bringing them in direct relation with the theory of economic growth.

The fundamental index number problem, which is essentially that of aggregation, have never completely solved. Under the influence of Marshall (1887), who doubted that a unique and true measure of the price index (needed also to compute the real aggregate output) could ever be founded, in his famous paper on index numbers winning the Adam Smith Prize, Keynes (1909) reached the following conclusion with reference to Walsh (1901):

> "If there was a perfect measure of general exchange value, Mr. Walsh would certainly have found it; but the method of exhaustion is barren, if the object of search has no real existence"(p. 135).

If individual preferences are not of the same kind, tastes change over time or tastes differ across space, then aggregation problems may arise because the object of measure (the aggregate price index) does not exist. This conclusion was confirmed in Keynes' (1930) theory of limits for the aggregate price index by stating that such an index can be computed only under homothetic conditions. Hicks (1940) had exposed a similar index number problem that would invalid any valuation of social income in presence of non-constant returns to scale and imperfect competition. As he stated,

> "Returns are not always constant; prices (ex tax) are not always equal to marginal cost; once these assumptions are dropped the whole argument loses its validity. [...] The collapse is much worse when we allow for increasing returns and imperfect competition. Prices (before taxation) cease to represent marginal costs; we have no reliable information about the convexity or concavity of the substitution curves. Some people may be tempted to rush in with the suggestion that a constant degree of market imperfection and constant marginal costs would mend the situation, and is not too bad a hypothesis; but there seem to be crushing objections against this. While there may be some sense in which it is normal for marginal costs to be constant under imperfect competition, that sense can hardly be relevant here, where we are thinking about a whole economy, not a single firm, so that the specificity of factors is of first-rate importance to us. Further, unless we have some way of measuring marginal costs directly, we need to assume not only a constant degree of market imperfection, but the same degree of market imperfection in all industries─and that assumption is hardly tolerable. It does not seem as if the collapse can be prevented." (p. 121).

Samuelson (1950), in explicit reference to Hicks (1940), reinforced this presentation and pointed to inconsistent comparisons when the consumer has changed tastes or is not in equilibrium. Non-neutral changing tastes produce distortive effects in consumption similarly to those produced by non-neutral technical progress in production. In two other memorable articles, Hicks (1956)(1958) reiterated the discussion of the index number problem on consumer demand and real income, respectively where non-neutral effects on the composition of the bundle of goods may devoid the resulting index numbers of any analytical value. We note, in passim, that the non-neutral income effect addressed to by Hicks is equivalent to what Samuelson (1974)(1984) called "Engel-Gerschenkron effect" in the comparisons of standard of living and to the effect of non-homothetic returns to scale in the production activities.

This happens when the expected inequality between Laspeyres and Paasche indexes (the so-called L-P inequality) turns out to be with the "wrong" sign. However, the non-neutral income effects may still be present in the case of the right sign of this inequality, but at such level that it does not offset the price-induced substitution effects completely. Indeed, in this last situation, the apparently "well-behaved" L-P inequality could be used as a boundary interval of possible values of constructed "true" indexes which rationalize the data.

The Hicksian theory on which our own price index theory is built indicates that a positive *LP* difference is a necessary and sufficient condition for using the observed data on prices and quantities to reconstruct "true" index numbers based on *hypothetical* homothetic preferences. These, however, do not necessarily coincide with the actual criteria governing the observed behaviour. Rather, they can be seen as index numbers that are "exact" for certain supporting functions (the utility or production functions) that may rationalize the observed data. In other words, the *LP* inequality might be the result of the concomitant "non-proportional" effects of real income changes as well as substitution effects under non-homothetic preferences (if any), but *the observed data could always be rationalized by a hypothetical homothetic preference field if* $L - P > 0$ *in the consumer case, or* $L - P < 0$ *in the producer case*. Under this condition we could always reconstruct "true" price and quantity index numbers that are consistent with that homothetic preference field and, as such, always respect all Fisher's requirement, including transitivity. This is, in fact, (as Keynes, 1930, among others, had recalled) the only condition under which it is possible to make such construction. It also corresponds to the Antonelli's (1886) integrability condition under which the data on the observed behavioural choices can be used in order to compare economic welfare and productivity.

In order to mitigate the difficulties arising from possible non-homothetic scale economies, our measurement can be done by using the most general accounting framework as possible, where output-input separability is not imposed. This can be conveniently obtained by analysing the formation of net (normalized) profits which discounts the effects of scale economies and externalities. The required homotheticity condition for aggregation, if not met because of scale effects at the level of the outputs or inputs considered separately, could be more easily met at the higher level of outputs and inputs pooled together in the process of the formation of net profits.

This approach would allow the decomposion of total factor productivity growth into effects from technical change, returns to scale and possible inefficiency and to assess the impact of these components on real profits and factor rewards using all possible "true" measures while taking into account market imperfections. It is will be carried out with the additional qualification that the restrictive hypothesis of well behaved (smooth) technology is released by introducing the hypothesis of non-smooth contour of the alternative techniques with a given technology.

Using the profit function approach, we generilize other contributions as, for example, Kumbhakar (2002), Diewert and Fox (2010), and Fernald and Neiman (2011), who have defined a decomposition of the "residual" productivity indexes into effects from scale economies, technical progress and imperfect competition[3]. In order to implement this decomposition, these authors consider an exogenous estimation of marginal costs and construct the markup over average total costs as a simultaneous function of supply and demand conditions.

[3] Precursors of this line of refinement of growth accounting are the pioneering methodological papers by Lau (1972), Caves, Christensen and Diewert (1982), Chambers (1988).

In the case of Chinese economy, we can instead assume that, in general, private and public firms, even state-owned enterprises, are price takers in both input and output markets even when these markets are fragmented and non-competitive[4]. Consequently, we can assume that the firms' output is determined at the level where producer prices are equal to marginal costs and the ad-valorem markups over average costs signal directly the degree of scale economies. This fact, allow us to simplify the decomposition of productivity growth since scale effects can be measured with index numbers constructed using the same data on output and inputs prices and quantities without requiring additional exogenous information or econometric estimates.

The relaxation of the assumption that the technological frontier is smoothly shaped (as in the traditional index number approach), is made in favour of the hypothesis of a "piece-wise linear" contour[5] of the technical frontier. We consider the index numbers that can be constructed using the Afriat's approach in the version revised by one of us (Milana, 2010). This last approach consists in defining chain-consistent (transitive) tight bounds of the numerical interval of all possible "true" measures of productivity and technology change. In the presence of changes in allocative inefficiency, the "true" measures are obtained by correction for this distortion.

We start from the accounting of nominal profits as a residual between gross revenues and total costs

$$\Pi^t = p^t y^t - \sum_i w_i^t x_i^t$$

where Π^t is the total nominal profit at period t, p^t and y^t are, respectively, the output price and quantity, and w_i^t and x_i^t are, respectively, the ith input price and quantity. The index number defined as ratio of its numerical values at two observation points, may be decomposed into price and quantity components[6]:

$$\Pi^1 / \Pi^0 = P_\Pi^{0,1} \cdot Q_\Pi^{0,1}$$

The indexes of *TC* and *TFP* between any pair of observation points can be obtained from the absolute change in normalized real profits $\Delta TC^{0,1}$

[4] In the case of increasing returns to scale, the firm might incur losses as the exogenously given output prices equal marginal costs at a lower level of average total costs. These losses are usually covered with public subsidies in the case of state-owned enterprises.

[5] This means that the derived output supply and input demand quantities can be multi-valued functions of prices whereas certain other output-input combinations can be associated with multiple levels of relative prices.

[6] We note that both price and quantity components $P_\Pi^{0,1}$ and $Q_\Pi^{0,1}$ can be seen as ratios of aggregate levels or aggregation of ratios between pairs of elementary prices or quantities.

$$\Delta TC^{0,1} \equiv \frac{\Pi^0}{p^0 y^0}\left(\underbrace{\frac{\Pi^1}{\Pi^0} / P_{\Pi}^{0,1}}_{\substack{\text{implicit real}\\ \text{profit index}\\ \text{number}}} - 1\right) = \frac{\Pi^0}{p^0 y^0} \cdot \left(\underbrace{Q_{\Pi}^{0,1}}_{\substack{\text{real profit}\\ \text{index number}}} - 1\right)$$

where $P_{\Pi}^{0,1}$ and $Q_{\Pi}^{0,1}$ are index numbers of price and quantity components of nominal profit index numbers $(\Pi^1 / \Pi^0 = P_{\Pi}^{0,1} \cdot Q_{\Pi}^{0,1})$. Hence, the index number of technical change is obtained as

$$TC^{0,1} = \frac{y^1 / y^0}{y^1 / y^0 - \Delta TC_{A-M}^{0,1}} \qquad \text{(index number of } TC\text{)}$$

In the case of input-output separability and aggregability of the input quantity changes in the form of $X^{0,1}$, the complete accounting of output growth is obtained as

$$\frac{y^1}{y^0} = \underbrace{TC^{0,1} \cdot RS^{0,1}}_{TFP^{0,1}} \cdot X^{0,1} \quad \text{with} \quad \overbrace{RS^{0,1} \cdot X^{0,1}}^{y_x^1 / y^0}.$$

where y_x^1 / y^0 is the index number of the contribution of the quantity change of inputs to output; the index number of *TFP* measures the distance between the actual output quantity and the input quantities, whereas the index number of *TC* measures the distance between that output quantity and the level that, *ceteris paribus*, it would have had with no technological change. The index numbers of *TC* and (under the aggregability conditions) *TFP* and *X* can be computed using formulas or algorithms. In particular, to compute these index numbers, we propose the chain-consistent upper and lower bounds of "true" index numbers of the price and quantity components of nominal profit changes. These indexes recently were proposed by one of us (Milana, 2010) as a further solution within the well-known Afriat's approach (see also Afriat and Milana, 2009). For comparison, we also complement these indexes with those obtained using the traditional bilateral Laspeyres, Paasche, Fisher, and Törnqvist indexes.

4. The Required Data

Our data construction is based on a series of data work by Wu and his associates that applies the standard production function approach covering industry-level output and labor and capital input measures (e.g. Wu, 2002a, 2002b, 2007, 2008, 2011a and 2011b; Maddison and Wu, 2008; Wu and Yue, 2010). In this study, we further revise and update his data series. The new efforts include an adjustment to the official industrial output data, a standardization of the numbers employed based on our estimates of hours worked, and revising and updating the estimates of net capital stock.

Coverage and Classification

This study covers all industrial enterprises in China for the period 1987-2009. In the official industry statistics the coverage of data has changed over time without a clear and transparent explanation, which has caused confusions and difficulties to empirical research at industry level. One of the major difficulties to researchers is that the official criterion for industrial enterprises to be covered has been changed from ownership to the level of administration and then to the value of annual sales.

For most of the planning period, the available industry data can only cover the state-owned enterprises (SOEs). From 1980, the coverage was enlarged to include enterprises as independent accounting units at or above the (rural) township administrative level regardless of ownership type. However, the coverage was changed in 1998 again by a "designated size" approach under which all SOEs plus non-state enterprises with total annual sales of five million yuan or more were included.[7] The differences between these criteria cannot be coherently or logically reconciled. Moreover, the sum of the total outputs in value added by any of these criteria is not consistent with the sector or national aggregates in the national accounts. Worse still, from 2005 onwards the sum of value added by enterprises at or above the "designated size" has become illogically larger than the industrial GDP in the national accounts (Wu, 2011a).

In the present study we focus on the total industrial economy for the period 1987-2009. Our question is how to ensure a complete coverage for major inputs (capital and labor) and outputs and a consistent industrial classification that matches all input and output variables over time. We introduce a "formal sector" concept to ensure a "conceptually-consistent" coverage of industrial enterprises over time. Industrial enterprises in the "formal sector" refer to those legally registered with the authorities as complete business entities with independent accounting status regardless of their ownership type, administrative level or "size". By using this "formal sector" umbrella, we can to a large extent "bypass" the inconsistent coverage problem in the official industry statistics. We will discuss how this coverage is defined and maintained in measuring input and output in the following sub-sections.

The official industry statistics are available at two-digit level but based on different Chinese standards of industrial classification (CSIC) introduced at different time (i.e. CSIC/1972, CSIC/1985, CSIC/1994 and CSIC/2002). To make it consistent over time, the CSIC/2002 is used as a standard to re-classify all the historical data as well as to adjust the coverage. We finally adopt a classification system used in Wu and Yue (2010) that regroup (inconsistent) Chinese industries into 24 sectors out of 39 industries as in CSIC/2002, basically reconcilable with the EU-KLEMS system of classification (Timmer et al., 2007).

[7] Note that in 2007 the "designated size of 5 million yuan" was changed from the annual sales of *all* production or business to the annual sales by *major* activities only. Since 2011, the value of annual sales by major activities has been increased from 5 to 20 million yuan (NBS, 2011), creating further difficulties in maintaining data consistency.

Value Added and Gross Output[8]

Studies have shown that conceptual and methodological problems and institutional deficiencies in the Chinese statistical system have tended to exaggerate the growth of GDP while underestimating the level of GDP (Maddison, 1998; Keidel, 1992). Official industrial statistics is one of the areas that have most suffered (Wu, 2000, pp.479-484). There have been a number of important empirical studies attempting to provide alternative estimates using various approaches such as commodity-based physical output index (Wu, 2002a), alternative price indices (Wu, 2000; Woo, 1998; Ren, 1997; Jefferson et al., 1996), and energy consumption approximation (Adams and Chen, 1996). Despite their different estimates, all appear to strongly support the upward bias hypothesis about the official growth estimates. Wu's work on output index based on commodity data is perhaps the most systematic and independent studies of the official estimates (Wu, 2002a and 2011b).

However, Wu's approach is more appropriate for assessing the real output (value added) growth rate of total industry rather than individual industries. Because Chinese industry statistics are based on *enterprise* rather than *establishment* (for narrowly defined activities or single product production), commodity-based estimates may not closely match labor and capital statistics used for multi-activity enterprises that may contain several establishments engaged in different industries. For this reason, we adopt Wu's recent gross value added (GVA) estimate for total industry as the "control total" in nominal terms, which has been adjusted for the significant inconsistency found in GVA between the sum of the "designated size" enterprises and the national accounts aggregate (Wu, 2011a).

Our main data work for the construction of the nominal GVA and GVO series by industry follows a novel "ownership approach": 1) more systematic and easily available SOE data are used as the "hard core" for the entire period 1987-2009, 2) non-SOE data for enterprises at or above the "township level" prior to 1998 and the "designated size" since 1998 are used to define the main industrial activities that have been closely monitored and controlled by the planning authorities, and 3) less systematic data for enterprises at the "village level" (below the township level) prior to 1998 and below the "designated size" since 1998 are used to define the border of the "formal sector" and hence to construct the output for the outer layer of the economy. We argue that since this "ownership approach" is applied at industry level, it gives a more plausible estimate of the industrial structure.

Regarding data source, the basic GVA and GVO data are from *China Industrial Economic Statistics Yearbook* (DIS, 2009 and earlier issues). However, before China shifted to the System of National Accounts (SNA) in 1992, there were no statistics on value added but net value of output (NVO) complied under the Material Product System (MPS). We adjust NVO to the concept of GVA by adding back an estimated capital consumption component. We also make intensive use of the census data from China's 1985 and 1995 industrial censuses and statistics for rural township and village enterprises in the construction of the outer layer of the "formal sector". The output value of the "informal sector" is simply estimated as the difference (residual) between the national account "control totals" and the constructed GVA and GVO for the "formal sector".

[8] Although China in principle switched to the System of National Accounts (SNA) in 1992 and has since continuously improved its national accounts through surveys and censuses, some of the concepts and practices used by the National Bureau of Statistics (NBS) are to some extent still influenced by the old Material Product System (MPS) (for details see Xu, 1999 and 2009).

Finally, the constructed industry GVA in nominal terms is deflated by our adjusted industry-specific producer price index (PPI) (see NBS, 2009, Table 8-11 and 8-12, and earlier issues for historical data). We choose to use PPI because it suggests much higher changes of output prices than the traditional "comparable price index" (CPPI) under MPS (Wu, 2000; Woo, 1998; Ren, 1997; Jefferson *et al.*, 1996).[9] However, due to data limitation we are unable to construct input prices for each industry. This means that we have to assume that changes in input prices are the same as changes in output prices.

Numbers Employed

Moreover, following China's 1990 population census, official statistics exhibit a big jump in employment by 17 percent or 94.2 million, creating thereafter a huge discrepancy between the total employment and the sum of sectoral employment (Maddison and Wu, 2008; Wu, 2011a). Direct usage of the officially reported numbers employed would be very misleading.

In our data construction, we first adopt Wu's (2011a) adjusted total numbers employed for the industrial sector as a new "control total". His adjustment is based on a careful examination of the relationship between annual employment statistics and population census for 1982, 1987 and 1990. He showed that the structural break could have appeared in 1982 if the 1982 census results were incorporated into the national totals without altering the annual employment estimates. This break was caused by the fact that the official annual estimates did not take into account the activities emerged outside the labor planning and administration system as a result of policy change in the early 1970s that encouraged small, collective enterprises to employ surplus labor especially in rural areas. His adjustment to China's total employment series is therefore for the period 1970-1990 using a trend-deviation approach with 1982 as the mid-point to "anchor" the series (Wu, 2011a).[10]

Given the "control total" for industry as a whole, the allocation within the industrial sector are based on weights given by the structure of labor-intensive, small-sized enterprises (village-level or below the "designated size"). With the new control totals for individual industries, the rest of the adjustment adopts the approach used in Wu and Yue (2010) which contains several steps. First, in line with what we do for the output, we ensure the consistency of the coverage by the "formal sector" at industry level. Second, we convert the numbers employed to hours worked based on a) institutional working hours, b) industry-specific standard working hours according to the nature of each industry and hence different shift arrangement, and c) assumptions for extra hours especially in labor-intensive (should standardize this throughout the paper) industries.[11]

[9] The practice of CPPI was stopped after 2002, ending with CPPI's last or 1990 benchmark (see Wu 2011b).

[10] The additional workers uncounted in the annual statistics are allocated by weights into agriculture, industry, construction and services, excluding the so-called "non-material/non-market services" (banking, business services, government services etc.) because these workers were most likely engaged in labor-intensive manufacturing, construction and services (Wu, 2011a).

[11] As a long tradition under central planning, non-industrial staff and workers working in child care centers, educational and medical units, commercial outlets, and social and political organizations are inherent in the official industrial statistics. The separation or commercialization of these auxiliary services began in the late 1990s, but has not yet been completed in some SOEs. Before 1998, unemployed workers remained on the payroll in all enterprises. Strictly speaking these service employees should be re-allocated to service industries. This has not yet been done.

Measuring Labor Input

We follow the same procedures in Wu and Yue (2010) but use new source of data for the 2005 benchmark, that is, a large sample data from the one-percent population survey in 2005. Details will be followed…

…We first construct marginal employment and compensation matrices for benchmarks 1987, 1990, 1995, 2000 and 2005. With population censuses and sample surveys for these benchmarks, we can have more information than regular time series of numbers and total wage bills at industry level.

……

Net Capital Stock

As discussed in Wu (2008), a significant mistake often made in constructing capital stock is the direct use of official statistics on "total investment in fixed assets" (TIFA) as the investment variable in the perpetual inventory method (PIM) equation.[12] By the official definition, it refers to the *workload* of investment activity in money terms including construction and purchase of fixed assets whether or not the investment projects are completed and actually transferred to investors or users (NBS, 2001, p.220). As commented by Xu (1999, pp. 62-63), this is different from the gross fixed capital formation (GFCF) concept in the SNA that capital formation only takes place when a contract-based ownership transaction of capital goods from a producer or constructor to a user (investor) is completed (CEC et al., 1993).[13] This is regarded as the key difference between SNA and the Chinese system in measuring fixed asset investment (Xu, 1999, pp.62-63). The problem is, as critically noted in Chow (1993, p.816), the work performed as recorded TIFA may not produce results that meet standards for fixed assets in the *current* period. In fact, some of the work (investment projects) may take many years to become qualified for production use and some may never meet the standards, hence completely wasted. Even if there is no wasteful investment, TIFA still tends to exaggerate investment while underestimate inventory, which will, more importantly, distort the growth statistics of real investment.

To bypass the problem, following Wu's earlier work (2002b) and his later revision (Wu, 2008), we opt for constructing a new investment series by using official industry statistics on year-end "original value of fixed assets" (OVFA). However, OVFA is a well-known "dirty indicator" that mixes structures with equipment, assets purchased in different periods, i.e. in historical costs, and residential and non-industrial structures in one measure by value.

The first step is to derive an annual flow of investment by taking the first difference of the OVFA adjusted for scrapings.[14] Compared with Wu's earlier work, we have allowed earlier and shorter scraping process along with the marketization of the economy. Next, based

[12] For example, see Young (2000), Huang et al. (2002), Hu and Khan (1997) and Li *at el.* (1992).

[13] The general SNA principles governing the time of recording and valuation of gross fixed capital formation is "when the ownership of the fixed assets is transferred to the institutional unit that intends to use them in production" (CEC, 1993, p.223).

[14] Earlier studies by Chen et al. (1988a and 1988b) conducted a similar exercise to derive an annual investment flow from OVFA but ignored the effect of scrapings, which underestimate the investment. However, some studies (e.g. Wang and Szirmai, 2011) argue that the scraping effect is likely minor.

on the information on type of fixed assets in investment as surveyed by the Ministry of Finance (MoF), we have identified and removed non-industrial assets and residential structures from the so-derived investment flow. Third, we construct deflators for individual industries based on the MoF detailed (6-digit) asset evaluation data for the period 1984-2000 (MoF *et al.*, 2002) with an extension back to 1952 and updated to 2008 by PPIs for investment goods (building materials and machinery industries).

In the PIM exercise, we follow Hulten and Wykoff (1981a and 1981b) assuming a geometric function of depreciation that reflects changes in economic efficiency of different types of fixed assets. As depreciation (δ) of an asset is equal to its declining-balance rate (R) divided by its service-live (T), we need to estimate proper R and T for equipment and structures of each industry. We adopt the BEA (Bureau of Economic Analysis, Washington D.C.) estimates of the declining-balance rates for major industrial equipment and structures as given in Kaze and Herman (1997, pp.72-3) based on the seminal empirical work by Hulten and Wykoff (1981a and 1981b). To gauge the service lives of assets in China's manufacturing, we rely on scattered information from official documents.[15]

Measuring Capital Input

…

5. Empirical results

The application of the methodology developed above allows us to unveil interesting features of China's industrial growth. During the overall period 1987-2009, TFP has increased in all industries by about 1% per year, whereas technology has contributed to output growth by about 2.2%. (Figure 1). A slightly different picture is obtained using the traditional growth accounting approach based on Törnqvist index numbers. In general, however, the contribution of TFP and TC to output growth has been less important than factor input growth as demonstrated by the relatively attenuated dynamics of these two indexes (Figures 2 and 3 and Table 1). The TFP index shows an average 2.2% increase per year during the whole examined period whereas the output growth in total industry has grown at an average rate of 12.3% per year. The major input contribution appears to be from intermediate input quantities with an average 8.1% increase per year and only a marginal contribution from an average increase in capital by 1.7% and labour by 0.1% per year. This is consistent with some earlier empirical studies which have drawn the analyst's attention to inadequate accumulation of capital.

During the period 1987-2009, we can distinguish sub-periods with a different TFP dynamics. Although gross output in total industry has grown almost steadily with an average of annual growth rates in the range 10-15%, we can observe a wide variation in the speed of growth of TFP. Due to an apparent inefficient reallocation of factor inputs and some data

[15] There are three sources of information: a) official depreciation rates (by the straight-line approach) used by MoF since 1963, b) a detailed list of the standard service lives for fixed assets issued by the State Council in 1985 (No. 63 Circular), and c) a new regulation on service lives by MoF in 1992 (No. 574).

problems in certain years (for example 1993), we can also note significant differences even in the algebraic sign between the Tornqvist-based measure and "True" index of TFP growth rates in the examined sub-periods. The former measure points to a substantial fall of an average 18.6% per year in 1991-1995 and an increase of 17.4% in the subsequent period of 1996-2000, followed by a slower but more stable TFP growth in the following periods up to 2009. The latter index signals a different pattern with a sustained 9.0% annual increase of TFP in 1991-1995 followed by a fall of -5,3% in 1996-2000 and virtually no growth of TFP during more recent periods.

Particularly striking is the weak dynamics of fixed capital inputs, which have move in many cases in opposite direction with respect to the variable capital inputs (the so-called intermediate inputs). The difficulty in keeping the fixed capital and labour inputs at the pace of output growth have in general brought about decreasing returns to scale, which in turn partially offset the positive contribution of technical change on output growth (Figure 4). The inadequacy of investments to maintain the pace of output growth has been pointed out recently as a cause of concern regarding both the attenuation of the overall impact of technology on economic welfare and through inflationary effects of increasing marginal costs[16].

A second important message that we can derive from our computations is that misallocation of factor inputs and technical inefficiency seem to have been widespread in China during the examined period. The frequent violation of the Laspeyres-Paasche inequality condition is the evidence that allocative inefficiency of factor inputs maybe the main cause of this outcome rather than non-homothetic changes in the technology frontier. From the methodological point of view, this is a serious problem which invalidates the application of all the traditional index number formulas. In such conditions, the resulting measures of TFP changes loose meaning and produce erratic results.

A comparison between the results obtained using the "true" index numbersafter correction for allocative inefficiencies have a much less erratic behaviour than the traditional Tornqvist and Fisher index numbers. The dynamics of the "true" indexes of TFP and TC closely track the effects of the economic developments in China and other Asian countries. Particularly relevant is the depressing effect of the Asian financial crises on TFP and TC during 1997 followed by a new period of sustained growth up to the first half of the last decade (Figure 1). However, the persistent insufficient accumulation of capital in domestic activities has been the cause of the worrisome stagnation of TFP during the financial crisis after 2008. Technical change has continued at a slower pace than before.

At industry level, notable differences have been registered in both growth rates of productivity and technology. The most dynamic industry was the ICT-producing sector, as expected, which has experienced a seven-fold increase in productivity and an almost ten-fold improvement in technology. However, some industries even decreased productivity as, for

[16] For a survey of the literature on investments and economic growth in China, se for example, Milana and Wu (2012) and the studies mentioned therein.

example, Coal, Gas, and Oil products. All the other industries have experienced an increase of TFP and an even higher TC.

6. Conclusion

This paper has presented a new way to account for total factor productivity changes in the context of the methodology of growth accounting. Using index numbers, changes in TFP have been decomposed in technical change and effects of returns to scale The conditions of a heavily administered economic regime like that of China present misallocation problems that complicate the theoretical approach, but on the other hand simplify the picture of how prices are determined on the markets. Even large enterprises behave here as price-takers and their supply does not affect prices. In this context, the assumption of the equality between sale prices and marginal costs allows us to consider the ratio of total costs to total revenues as an index of the returns to scale. Appropriate “true” index numbers where therefore constructed using a procedure that takes also into account allocative inefficiencies. The results obtained from China are astoundingly suggestive and turn out to be much more credible than the analyses using the traditional methods for an interpretation of the specific reality such as that of China.

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APPENDIX

A reformulation of TFP growth accounting

The traditional growth accounting is based on the following hypotheses:

*H*1: Firms operate in free and competitive markets in all sectors of the economy.

*H*2: The technology of production is characterized by constant returns to scale.

*H*3: The firms are technically and allocation efficient.

*H*4: Pure profits are always equal to zero (so that the ex-post and ex-ante user costs of capital coincide implying that 1) the internal rate of return is equal to the market rate of return of competing investments, 2) the average total cost of production is equal to production price prevailing on the market.

The extended growth accounting applied here is based on the following more general hypotheses:

*H*1*: Firms operate in some sectors in free and competitive markets, whereas many others operate in sectors where the markets are subjected directly or indirectly to the State control.

*H*2*: The technology of production is locally characterized by non-constant returns to scale in many sectors of the economy.

*H*3*: Technical and allocative inefficiency may occur in production units..

*H*4*: Non-zero pure profits are registered in the economic accounts at industry level.

A general formulation of *TFP* growth accounting

Let us consider that one output y is produced using the technology at period t by N inputs x_1, $x_{2,\ ...,}\ x_N$. Given the respective input prices $w_1\ w_2\ ...\ w_N$, the production price is $p = \mu \cdot c$, where μ is the markup of the output producer price over the average total cost c. Therefore (omitting, for the moment, the time superscript),

(1) $$p \cdot y = \underbrace{\mu \cdot c}_{p} \cdot y = \mu \underbrace{\sum_i w_i \cdot x_i}_{c \cdot y}$$

If input quantities and prices can be aggregated as functions of *only* quantities and prices, respectively, then

(2) $$p \cdot y = \mu \cdot \underbrace{W(\mathbf{w}) \cdot X(\mathbf{x})}_{\sum_i w_i \cdot x_i}$$

As shown by Shephard (1953), the functions W and X are conjugate in the sense that their functional forms are related to each other so that equality (2) holds over the relevant domain of **w** and **x.**

Defining the ratio $y/X(\mathbf{x})$ as total factor productivity (*TFP*), we can rewrite (A2) as

(A3)
$$p \cdot y = \underbrace{\mu \underbrace{\frac{1}{TFP} W(\mathbf{w})}_{c}}_{p} \cdot \underbrace{TFP \cdot X(\mathbf{x})}_{y}$$

Thus, from (A3), we have the equivalence between the so-called primal and dual measures of *TFP*

(A4)
$$TFP \equiv \underset{\substack{\text{Primal}\\ \text{measure}}}{\frac{y}{X(\mathbf{x})}} = \underbrace{\mu \cdot \frac{W(\mathbf{x})}{p}}_{\substack{\text{Dual}\\ \text{measure}}}$$

Accounting for TFP changes

Absolute changes in output quantity (dy) can be decomposed into two elements: (i) technical change and (ii) changes in input quantities, that is

$$\mathrm{d}y = \underbrace{\frac{\partial y}{\partial T} \cdot \mathrm{d}T}_{\substack{\text{Change}\\ \text{due to } TC}} + \underbrace{\sum_i \frac{\partial y}{\partial x_i} \cdot \mathrm{d}x_i}_{\substack{\text{Change due to}\\ \text{changes in inputs}}}$$

Dividing through by y, the foregoing equation becomes

$$\dot{y} = \underbrace{\frac{1}{y} \cdot \frac{\partial y}{\partial T} \cdot \mathrm{d}T}_{\substack{\text{Relative change}\\ \text{of } y \text{ due to } TC\\ (TC)}} + \underbrace{\sum_i \frac{\partial y}{\partial x_i} \cdot \frac{x_i}{y} \cdot \dot{x}_i}_{\substack{\text{Relative change in } y\\ \text{due to changes in inputs}}}$$

And, assuming that the producer optimizes its factor demand, the real factor rewards are equal to the respective factor marginal productivities (that is for each ith factor $\partial y / \partial x_i = w_i / p$), then

$$\varepsilon \equiv \frac{1}{\mu} = \sum_i \frac{\partial y}{\partial x_i} \cdot \frac{x_i}{y} = \frac{\sum_i w_i x_i}{py} = \frac{C}{R}$$

Equation ( ) becomes

$$\dot{y} \quad = TC + \underbrace{\frac{\sum_j w_j x_j}{py}}_{\text{Inverse mark-up}} \cdot \underbrace{\sum_i \frac{w_i x_i}{\sum_j w_j x_j} \cdot \dot{x}_i}_{\substack{\text{Weighted average of} \\ \text{relative changes in inputs} \\ \dot{X}}}$$

$$= TC + \varepsilon \cdot \dot{X}$$

Where $\dot{X} \equiv \sum_i \frac{w_i x_i}{\sum_j w_j x_j} \cdot \dot{x}_i$, which has a meaning of pure aggregate of input quantity changes if it is path-independent from relative input price changes.

Since, from (A4)

$$\dot{y} = T\dot{F}P + \dot{X}$$

and setting $\dot{S} \equiv (\varepsilon - 1)\dot{X}$, total factor productivity change can be decomposed into technological change and scale effects , that is

$$T\dot{F}P = \dot{T}C + \dot{S}$$

The real income distribution of gains from technology and productivity growth

The dual measures of $\dot{T}C$ can be also derivable using the profit function defined as $\Pi(p, \mathbf{w}, t) \equiv \max_{y,\mathbf{x}} \{ p \cdot y - \mathbf{w} \cdot \mathbf{x} : f(\mathbf{x}) \geq y; \mathbf{x} \geq \mathbf{0}_N \}$. Let us start from its total differentiation

$$\mathrm{d}\Pi(p, \mathbf{w}, t) = \frac{\partial \Pi}{\partial p} \cdot \mathrm{d}p - \sum_n \frac{\partial \Pi}{\partial w_n} \cdot \mathrm{d}w_n + \underbrace{\frac{\partial \Pi}{\partial t} \mathrm{d}t}_{p \frac{\partial f}{\partial t} \mathrm{d}t}$$

Using Hotelling-Shephard's lemma, this becomes

$$\mathrm{d}\Pi = y \cdot \mathrm{d}p - \sum_n x_n \cdot \mathrm{d}w_n + \underbrace{p \cdot \frac{\partial y}{\partial t} \mathrm{d}t - \sum_n w_n \cdot \frac{\partial x_n}{\partial t} \mathrm{d}t}_{p \frac{\partial f}{\partial t} \mathrm{d}t}$$

Rearranging terms, after a simple algebraic manipulation, yields

$$\varPi\cdot \mathrm{d}\varPi\cdot\frac{1}{\varPi}-py\cdot \mathrm{d}p\frac{1}{p}+\sum\nolimits_n(w_n\cdot x_n)\cdot(\frac{\mathrm{d}w_n}{w_n})=\underbrace{py\cdot \mathrm{d}y\frac{1}{y}-\sum\nolimits_n(w_n\cdot x_n)\cdot(\frac{\mathrm{d}x_n}{x_n})}_{p\frac{\partial f}{\partial t}\mathrm{d}t}$$

The instantaneous relative rate of change of production due to technical change and the distribution of the gains are obtained by dividing the foregoing equation through by $p\cdot y$ and rearranging, thus obtaining

$$\underbrace{\frac{\varPi}{py}\left[\dot{\varPi}-\underbrace{\left(\frac{py}{\varPi}\dot{p}-\frac{WX}{\varPi}\cdot\dot{W}\right)}_{\substack{\text{Price component}\\ \text{of unit profit change}}}\right]}_{\text{Dual measure of }\dot{TC}}=\underbrace{\frac{\varPi}{py}\underbrace{\left[\frac{py}{\varPi}\dot{y}-\frac{WX}{\varPi}\dot{X}\right]}_{\substack{\text{Quantity component}\\ \text{of unit profit change}}}}_{\text{Primal measure of }\dot{TC}}$$

$$=\underbrace{\frac{\varPi}{R}(\dot{\varPi}-\dot{p})+\frac{C}{R}(\dot{W}-\dot{p})}_{\text{Dual measure of }\dot{TC}}\quad=\quad\underbrace{\dot{y}-k\cdot\dot{X}}_{\text{Primal measure of }\dot{TC}}$$

where $\dot{W}\equiv\sum_n\frac{w_n\cdot x_n}{\sum_i w_i\cdot x_i}\cdot\frac{\mathrm{d}w_n}{w_i}$, which has a meaning of a pure aggregate of input price changes if it is path-independent from relative input quantity changes.

The primal measure of technological change ($\dot{TC}$) component of $T\dot{F}P$ is represented in the right-hand side and the dual measure of $\dot{TC}$ in the left-hand side of the foregoing equation. We can interpret the left-hand side of the foregoing equation as the rate of change of output attributable to technological change only.

Since $C\equiv W\cdot X$, dividing through this identity by y and taking the instantaneous rate of changes of all these variables yields

$$\underbrace{\dot{y}-\dot{X}}_{T\dot{F}P}\quad=\quad\dot{W}-\dot{c}\quad=\quad\dot{W}-\dot{p}-\dot{C}+\dot{R}$$

where $c\equiv\frac{C}{y}=k\cdot p$ with $k\equiv\frac{C}{R}$.

Therefore

$$
\begin{aligned}
RS &= T\dot{F}P - TC = (1-k)\dot{X} \\
&= \underbrace{(\dot{W} - \dot{p} - \dot{C} + \dot{R})}_{T\dot{F}P} - \underbrace{\left(k\dot{W} + \frac{\mathrm{d}\Pi / \mathrm{d}t}{R} - \dot{p} \right)}_{\dot{T}C} \\
&= (1-k)\dot{W} + \frac{\mathrm{d}C}{R} - \dot{C}
\end{aligned}
$$

In equilibrium, in a perfectly competitive economic environment and with constant returns to scale, $k = 1$ since $R = C$ $(\Pi = 0)$, $c = p$, and both additive elements of the right-hand side of the foregoing equation are null. Under these conditions, it is immediate to see that the general derivation of $\dot{T}C$ given above collapses the following equality traditional used in *TFP* growth accounting:

$$
\underset{\substack{\text{Dual measure} \\ \text{of } T\dot{F}P=\dot{T}C}}{\dot{W} - \dot{p}} \quad = \quad \underset{\substack{\text{Primal measure} \\ \text{of } T\dot{F}P=\dot{T}C}}{\underbrace{\dot{y} - \dot{X}}}.
$$

Table A1. Formulas of relative *TFP* changes in instant time and its components (*TC* and *RS*)

	Variable	Primal approach	Dual approach
(1) = (2) + (3)	$\dot{TFP}$ $= \dot{TC} + \dot{RS}$	$\dot{y} - \dot{X}$ $= \sum_n s_n (\dot{y} - \dot{x}_n)$	$\dot{W} - \dot{c}$ $= \dot{R} - \dot{C} + \dot{W} - \dot{p}$ $= \dot{R} - \dot{C} + \sum_n s_n (\dot{w}_n - \dot{p})$
(2) = (1) - (3)	$\dot{TC}$ $= \dot{TFP} - \dot{RS}$	$\dot{y} - k \cdot \dot{X}$	$\frac{\mathrm{d}\Pi}{R} - \dot{p} + \frac{C}{R}\dot{W}$
(3) = (1)-(2)	$\dot{RS}$ $= \dot{TFP} - \dot{TC}$	$(k-1) \cdot \dot{X}$	$\frac{\mathrm{d}C}{R} - \dot{C} + \left(1 - \frac{C}{R}\right) \cdot \dot{W}$

Legenda:

$$\dot{W} \equiv \sum_n s_n^t \dot{w}_n \,,\ \dot{X} \equiv \sum_n s_n^t \dot{x}_n \,,\ s_n^t \equiv \frac{w_n^t x_n^t}{\sum_n w_n^t x_n^t} = \frac{w_n^t x_n^t}{W \cdot X}$$

List of variables:

$C \equiv W \cdot X$: Nominal total costs of production

W : Aggregate factor price level

X : Aggregate factor input quantity level

$R \equiv p \cdot y$: Nominal total revenues

p : producer price level

y : output quantity level

$\Pi \equiv R - C$: Nominal pure profits

$$k \equiv \frac{W \cdot X}{p \cdot y} = \frac{C}{R}$$

$$c \equiv \frac{W \cdot X}{y} = p \cdot k$$

Traditional bilateral index number index numbers

Laspeyres-type index numbers of

The Laspeyres measure of incremental output due to *TC* is the following:

$$\Delta TC_L^{0,1} \equiv \frac{\Pi^0}{p^0y^0}\left(\frac{\Pi^1}{\Pi^0} / \underbrace{\frac{\Pi^1}{p^0y^1-\sum_i w_i^0 x_i^1}}_{\substack{\text{Paasche price component of}\\ \Pi^1/\Pi^0}} -1\right) = \frac{\Pi^0}{p^0y^0}\left(\underbrace{\frac{p^0y^1-\sum_i w_i^0 x_i^1}{p^0y^0-\sum_i w_i^0 x_i^0}}_{\substack{\text{Laspeyres (implicit Paasche)}\\ \text{quantity component of } \Pi^1/\Pi^0}} -1\right)$$

$$= \frac{p^0y^1-\sum_i w_i^0 \cdot x_i^1}{p^0y^0} - (1-\varepsilon^0) = \left(\frac{y^1}{y^0}-1\right) - \varepsilon^0 \cdot \left(\frac{\sum_i w_i^0 x_i^1}{\sum_i w_i^0 x_i^0}-1\right)$$

where $\varepsilon^0 \equiv \frac{\sum_i w_i^0 x_i^0}{p^0y^0}$ and $(1-\varepsilon^0) = \frac{\Pi^0}{p^0y^0}$.

Hence, the Laspeyres-based index number of technical change is

$$TC_L^{0,1} = \frac{y^1/y^0}{y^1/y^0 - \Delta TC_L^{0,1}} = \frac{y^1/y^0}{1+\varepsilon^0\cdot\left(\frac{\sum_i w_i^0 x_i^1}{\sum_j w_j^0 x_j^0}-1\right)}$$

where $TC_L^{0,1} = TFP_L^{0,1} \equiv \frac{y^1/y^0}{\frac{\sum_i w_i^0 x_i^1}{\sum_i w_i^0 x_i^0}}$ if $\varepsilon^0 = 1$.

It is immediate to note that, irrespective of the value of ε^0, the denominator of the foregoing $TC_L^{0,1}$ formula is equal to unity if the input volume does not change $\left(\text{if } \frac{\sum_i w_i^0 x_i^1}{\sum_j w_j^0 x_j^0} = 1\right)$. In such case, the entire change in the output quantity is attributed to technical change, that is $TC_L^{0,1} = y^1/y^0$.

Finally, the Laspeyers-based index number of scale effects is given by

$$RS_L^{0,1} = TFP_L^{0,1}/TC_L^{0,1} = (1-\varepsilon^0)\frac{\sum_i w_i^0 x^0}{\sum_i w_i^0 x_i^1} + \varepsilon^0$$

Paasche-type index numbers

The Paasche measure of the incremental output due to *TC* can be derived with the following procedure.

$$\Delta TC_K^{0,1} \equiv \frac{p^1y^0-\sum_i w_i^1\cdot x_i^0}{p^1y^0}\left(\frac{\Pi^1}{\Pi^0}\Big/\underbrace{\frac{p^1y^0-\sum_i w_i^1\cdot x_i^0}{p^0y^0-\sum_i w_i^0\cdot x_i^0}}_{\text{Laspeyres price component of } \Pi^1/\Pi^0}-1\right)=\frac{p^1y^0-\sum_i w_i^1\cdot x_i^0}{p^1y^0}\left(\underbrace{\frac{p^1y^1-\sum_i w_i^1\cdot x_i^1}{p^1y^0-\sum_i w_i^1\cdot x_i^0}}_{\text{Paasche (implicit Laspeyres) quantity component of } \Pi^1/\Pi^0}-1\right)$$

$$=\frac{p^1y^0-\sum_i w_i^1\cdot x_i^0}{p^1y^1}-(1-\varepsilon^1)=\left(\frac{y^0}{y^1}-1\right)-\varepsilon^1\cdot\left(\frac{\sum_i w_i^1x_i^0}{\sum_i w_i^1x_i^1}-1\right)$$

where $\varepsilon^1 \equiv \frac{\sum_i w_i^1x_i^1}{p^1y^1}$ and $(1-\varepsilon^1)=\frac{\Pi^1}{p^1y^1}$.

Hence, a simple algebraic manipulation yields the TC index number

Hence, the Paasche-based index number of technical change is

$$TC_K^{0,1}=\frac{y^1/y^0}{[y^0/y^1-\Delta TC_L^{1,0}]^{-1}}=\frac{y^1/y^0}{\left[1+\varepsilon^1\cdot\left(\frac{\sum_i w_i^1x_i^0}{\sum_j w_j^1x_j^1}-1\right)\right]^{-1}}$$

where $TC_K^{0.1}=TFP_K^{0,1}\equiv\frac{y^1/y^0}{\frac{\sum_i w_i^1x_i^1}{\sum_i w_i^1x_i^1}}$ if $\varepsilon^1=1$.

It is immediate to note that, irrespective of the value of ε^1, the denominator of the foregoing $TC_K^{0,1}$ formula is equal to unity if the input volume does not change $\left(\text{if } \frac{\sum_i w_i^1x_i^0}{\sum_j w_j^1x_j^1}=1\right)$. In such case, the entire change in the output quantity is attributed to technical change, that is $TC_K^{0,1}=y^1/y^0$.

Finally, the Paasche-based index number of scale effects is given by

$$RS_K^{0,1} = TFP_K^{0,1} / TC_K^{0,1} = \left[(1-\varepsilon^1) \frac{\sum_i w_i^1 x^1}{\sum_i w_i^1 x_i^0} + \varepsilon^1 \right]^{-1}$$

Fisher-type index numbers

Taking the geometric mean of the Laspeyres- and Paasche-type index numbers yields Fisher'type index numbers of *TFP* and *TC*, that is

$$TC_F^{0,1} = (TC_L^{0,1} \cdot TC_K^{0,1})^{\frac{1}{2}}$$

$$= \frac{y^1 / y^0}{\left[1 + \varepsilon^0 \cdot \left(\frac{\sum_i w_i^0 x_i^1}{\sum_j w_j^0 x_j^0} - 1 \right) \right] \cdot \left[1 + \varepsilon^1 \cdot \left(\frac{\sum_i w_i^1 x_i^0}{\sum_j w_j^1 x_j^1} - 1 \right) \right]^{-1}}$$

$$TFP_F^{0,1} = (TFP_L^{0,1} \cdot TFP_K^{0,1})^{\frac{1}{2}}$$
$$= \frac{y^1 / y^0}{\left(\frac{\sum_i w_i^0 x_i^1}{\sum_i w_i^0 x_i^0} \frac{\sum_i w_i^1 x_i^1}{\sum_i w_i^1 x_i^0} \right)^{1/2}}$$

$$RS_F^{0,1} = (RS_L^{0,1} \cdot RS_K^{0,1})^{\frac{1}{2}}$$

$$= \left\{ \left[(1-\varepsilon^0) \frac{\sum_i w_i^0 x^0}{\sum_i w_i^0 x_i^1} + \varepsilon^0 \right] \cdot \left[(1-\varepsilon^1) \frac{\sum_i w_i^1 x^1}{\sum_i w_i^1 x_i^0} + \varepsilon^1 \right]^{-1} \right\}^{\frac{1}{2}}$$

Törnqvist-type index numbers

The Törnqvist measure of the incremental output due to *TC* could be obtained by computing the following index numbers of price and quantity components of nominal profit changes, respectively given by

$$P_T^{0,1} \equiv \exp\left[\frac{1}{2}(\frac{p^0 y^0}{\Pi^0} + \frac{p^1 y^1}{\Pi^1})(\ln p^1 - \ln p^0) - \frac{1}{2}\sum_i (\frac{w_i^0 x_i^0}{\Pi^0} + \frac{w_i^1 x_i^1}{\Pi^1})(\ln w_i^1 - \ln w_i^0)\right]$$

$$Q_T^{0,1} \equiv \exp\left[\frac{1}{2}(\frac{p^0 y^0}{\Pi^0} + \frac{p^1 y^1}{\Pi^1})(\ln y^1 - \ln y^0) - \frac{1}{2}\sum_i (\frac{w_i^0 x_i^0}{\Pi^0} + \frac{w_i^1 x_i^1}{\Pi^1})(\ln x_i^1 - \ln x_i^0)\right]$$

and, since the Tornqvist price and quantity index numbers are not dual conjugate, the primal and dual Tornqvist-type index numbers of *TC* do not coincide. We therefore have

$$\ln TC_{PT}^{t-1,t} \equiv (\ln y^1 - \ln y^0) - \sum_i \frac{1}{2}\left(\varepsilon_i^0 \frac{w_i^0 x_i^0}{\sum_j w_j^0 x_j^0} + \varepsilon_i^1 \frac{w_1^1 x_i^1}{\sum_i w_i^1 x_i^1}\right)\cdot\left(\ln x_i^1 - \ln x_i^0\right)$$

$$\ln TFP_{PT}^{t-1,t} \equiv (\ln y^1 - \ln y^0) - \sum_i \frac{1}{2}\left(\frac{w_i^0 x_i^0}{\sum_j w_j^0 x_j^0} + \frac{w_1^1 x_i^1}{\sum_i w_i^1 x_i^1}\right)\cdot\left(\ln x_i^1 - \ln x_i^0\right)$$

An important remark is that the primal and dual measures should be mutually consistent in the sense that the Laspeyres-type primal (dual) measure is conjugate (and equal to) the Paasche-type dual (primal) measure. We can also take advantage of the use of both primal and dual measures of technical change as they give us complementary information of the *TFP* growth accounting exercise. While the primal measure accounts for the *sources* of productivity growth on the side of factor quantities, the dual measure allows us to detect the *distribution* of the productivity gains between real profits and real factor rewards.

Index numbers in the multilateral or intertemporal comparisons: Afriat's approach

It is well known that all index number formulas devised so far in the literature fail to satisfy at least one of the economic requirements in the context of multilateral comparisons. The approach due to Sydney Afriat is based on the rejection of the use of one single formula. It relies, instead, on a computational method. This can be described as follows.

Let us start with the matrices of bilateral Laspeyres (**L**) and Paasche (**K**) index numbers comparing aggregate prices at the point of observation *i* relative to those at point *j*, for *i,j* =1, 2, …, *N*. They are respectively

$$\mathbf{L} \equiv \begin{bmatrix} L_{11} & L_{12} & ... & L_{1N} \\ L_{21} & L_{22} & ... & L_{2N} \\ ... & ... & ... & ... \\ L_{N1} & L_{N2} & ... & L_{NN} \end{bmatrix} \quad \text{and} \quad \mathbf{K} \equiv \begin{bmatrix} K_{11} & K_{12} & ... & K_{1N} \\ K_{21} & K_{22} & ... & K_{2N} \\ ... & ... & ... & ... \\ K_{N1} & K_{N2} & ... & K_{NN} \end{bmatrix}$$

where $L_{ij} \equiv \frac{\mathbf{p}^i\mathbf{q}^j}{\mathbf{p}^j\mathbf{q}^j}$, and $K_{ij} \equiv \frac{\mathbf{p}^i\mathbf{q}^i}{\mathbf{p}^j\mathbf{q}^i} = \frac{1}{L_{ji}}$. Obviously, $K_{ij} = \frac{1}{L_{ji}}$ and $L_{ii} = K_{ii} = 1$.

The Laspeyres and Paasche index numbers are usually considered as two alternative measures of the unknown "true" index number P_{ij} which can be seen as an aggregation of the elementary price ratios p_r^i / p_r^j or, alternatively, as a ratio of aggregate price levels, *i.e.* $P_{ij} \equiv P_i / P_j$, where P_i and P_j are "true" aggregate price levels at the *i*th and *j*th points of observation. The price level ratio, always respects, by construction, the "base reversal" test, that is $P_{ij} = 1/P_{ji}$, and the "circularity" test, that is $P_{it} \cdot P_{tj} = P_{ij}$. By contrast, in the general case where the elementary price ratios *and* the relative quantity weights change, the Laspeyres and Paasche indices fail to be "base-" and "chain-consistent", that is $L_{ij} \neq 1/L_{ji} = K_{ij}$, $L_{it} \cdot L_{tj} \neq L_{ij}$ and $K_{it} \cdot K_{tj} \neq K_{ij}$. Even more unacceptable is well-known failure of chained indexes to return on the previous levels if all elementary prices go back to their older levels (the so-called "drift effect"): $L_{it} \cdot L_{ti} \neq L_{ii} = 1.$ and $K_{it} \cdot K_{ti} \neq K_{ii} = 1.$ These failures make the two index number formulas, like all the other alternative formulas, unsuitable to represent a price index. Nevertheless, as we shall see below, they are useful for testing the existence of the "true" price index and constructing its consistent bounds.

The so-called *LP-inequality* condition is that $L_{ij} \geq K_{ij}$ on the purchaser's side ($L_{ij} \leq K_{ij}$ on the supplier's side) is necessary and sufficient for the existence of a "true" price index number P_{ij} with a numerical value falling between the Laspeyres and Paasche indices. If this condition is not satisfied for all pairs of observation, then a correction of the data for possible inefficiency can be devised and/or an alternative more general model using a wider or different set of variables could be considered.

If the *LP*-inequality condition is satisfied for all pairs of points of observation, let us define, in the purchaser's case (following Afriat, 1981, 1984, p. 47, 2005, p. 167, 2008),

(11.1) $$M_{ij} \equiv \min_{kl...m} L_{ik} L_{kl} ... L_{mj}$$ (minimum chained Laspeyres price index number)

(11.2) $$H_{ij} \equiv \max_{kl...m} K_{ik} K_{kl} ... K_{mj} = \frac{1}{M_{ji}}$$ (maximum chained Paasche price index number)

so that we have tighter bounds with $L_{ij} \geq M_{ij} \geq P_{ij} \geq H_{ij} \geq K_{ij}$ for $i \neq j$ and $L_{ii} = M_{ii} = P_{ii} = H_{ii} = K_{ii} = 1$. In the case of supplier, the inequality signs and the "min/max" problems are reversed.

The efficient computation procedure is based on the application of Edmunds' (1973) minimum path and Bainbridge's (1978) power algorithm to the Laspeyres matrix L for all compared years as adapted by Afriat (1979)(1980b)(1981)(1982) for the identification of the optimized chained indexes. It consists in raising the Laspeyres matrix to powers N times, with N being the number of the compared observation points (6 years in the case of Fisher's data), in a modified arithmetic where + means *min.* In this special arithmetic, the resulting matrix M (corrected for inefficiency) remains unchanged if multiplied further by L, that is $M \equiv L^N = L^{N+1} = M \cdot L$.

If the LP-inequality condition is not satisfied for some or all pairs of points of observation, then we could "correct" the data for inefficiency. Diagonal elements $M_{ii} < 1$ and $H_{ii} < 1$ tell the inconsistency of the system.

A critical efficiency parameter e^* can be found for correction of the L matrix. For any element $M_{ii} < 1$, let d_i represent the number of nodes in the path $i...i$, then

(11.3) $$e_i = (M_{ii})^{\frac{1}{d_i}}$$

If $M_{ii} \geq 1$, let e_i take the value of 1 and then the critical efficiency parameter is determined as

(11.4) $$e^* = \min_i e_i$$

The adjusted Laspeyres matrix is obtained as

(11.5) $$L^*_{ij} = L_{ij} / e^* \qquad \text{for} \quad i \neq j$$

and the procedure goes on as before with L^* in place of the original L.

However, the optimized chained Laspeyres and Paasche indexes (the elements of the matrices L and M, respectively) are still intransitive – like any other chained index – since they exhibit the *triangle inequalities* $M_{it}M_{tj} \geq M_{ij}$ and $H_{it}H_{tj} \leq H_{ij}$. The matrix of the geometric mean elements $(M_{ij} \cdot H_{ij})^{1/2}$ proposed by Afriat (2008) and used by Afriat and Milana (2009) in practical illustrations may turn out to be only approximately transitive.

Proposed solution of the index number problem in the multilateral context

The chain-consistent (transitive) tight bounds are "true" index numbers themselves. They can be derived by adopting the following new procedure. Let us assume, without loss of generality, that all prices are normalized with an arbitrary aggregate price level, say for example P_1, and define the maximum and minimum price levels

(12.1) $\widehat{p}_i = (\max_t\ M_{it} / M_{(i-1)t}) \cdot \widehat{p}_{i-1} = (\max_t\ M_{it} \cdot H_{t(i-1)}) \cdot \widehat{p}_{i-1}$ for i = 2, 3, ..., N; t = 1, 2, ..., N

(12.2) $\breve{p}_i = (\min_t H_{it} / H_{(i-1)t}) \cdot \breve{p}_i = (\min_t H_{it} \cdot M_{t(i-1)}) \cdot \breve{p}_i$ for i = 2, 3, ..., N; t = 1, 2, ..., N

with $\widehat{P}_1$ and $\breve{P}_1$ being equal to 1.

The chain-consistent bounds of the "true" index numbers are therefore obtained as

(12.3) $$\widehat{P}_{ij} = \widehat{p}_i / \widehat{p}_j \quad \text{and} \quad \breve{P}_{ij} = \breve{p}_i / \breve{p}_j$$

With only to observation points $(N = 2)$, the index-number problem of a consumer is solved by finding the following bounds:

(12.4) $$\widehat{\mathbf{P}} \equiv \left[\widehat{P}_{ij}\right] = \begin{bmatrix} 1 & K_{12} \\ L_{21} & 1 \end{bmatrix} \quad \text{and} \quad \breve{\mathbf{P}} \equiv \left[\breve{P}_{ij}\right] = \begin{bmatrix} 1 & L_{12} \\ K_{21} & 1 \end{bmatrix}$$

With 4 observation points, after reordering their sequence of comparison conveniently, we might obtain

(12.5) $$\widehat{\mathbf{P}} = \begin{bmatrix} 1 & K_{12} & K_{12}K_{23} & K_{12}K_{23}K_{34} \\ L_{21} & 1 & K_{23} & K_{23}K_{34} \\ L_{32}L_{21} & L_{32} & 1 & K_{34} \\ L_{43}L_{32}L_{21} & L_{43}L_{32} & L_{43} & 1 \end{bmatrix}$$

and

$$\breve{\mathbf{P}} = \begin{bmatrix} 1 & L_{12} & L_{12}L_{23} & L_{12}L_{23}L_{34} \\ K_{21} & 1 & L_{23} & L_{23}L_{34} \\ K_{32}K_{21} & K_{32} & 1 & L_{34} \\ K_{43}K_{32}K_{21} & K_{43}K_{32} & K_{43} & 1 \end{bmatrix} \qquad (12.6)$$

Chain-consistent bounds of *quantity indices* can be obtained by using a similar procedure directly or implicitly by deflating the nominal total expenditure by means of the respective consistent bounds $\hat{P}_{ij}$ and $\breve{P}_{ij}$.

In fact, it is well known (see, for example, Prasada Rao and Banerjee, 1986) that, if price and quantity index numbers are constructed as ratios between levels of aggregate prices and quantities respectively, they satisfy *all Fisher's tests* including transitivity.

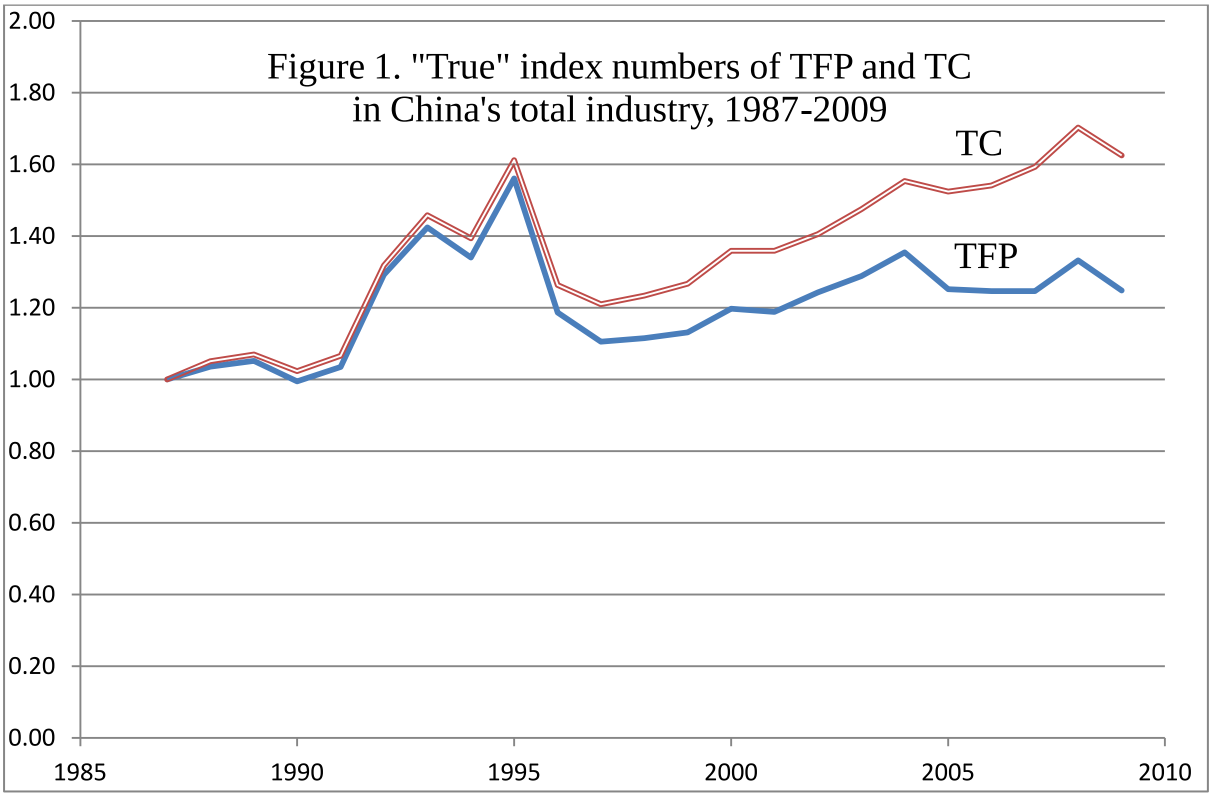


Figure 1. "True" index numbers of TFP and TC
in China's total industry, 1987-2009
TC
TFP
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1985
1990
1995
2000
2005
2010

Figure 2. "True" index numbers of *TFP* in Chinese individual industries, (1987=1.0)

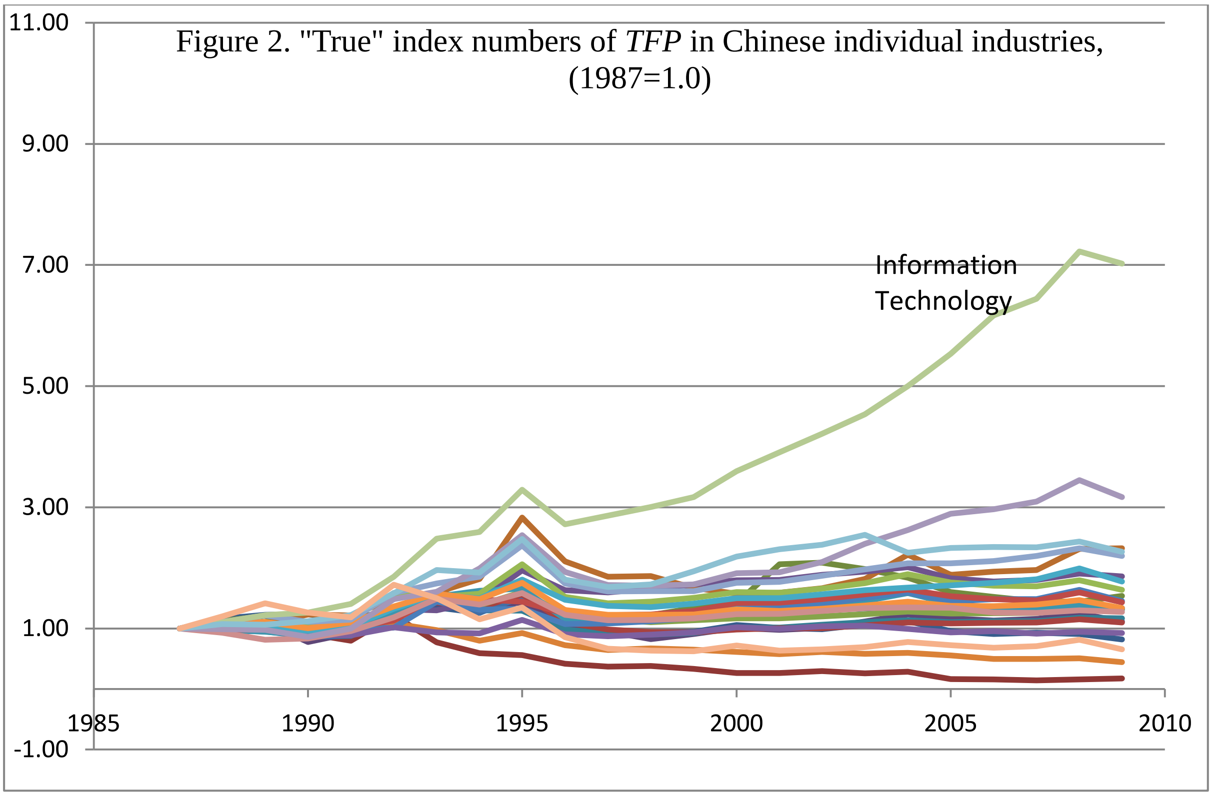
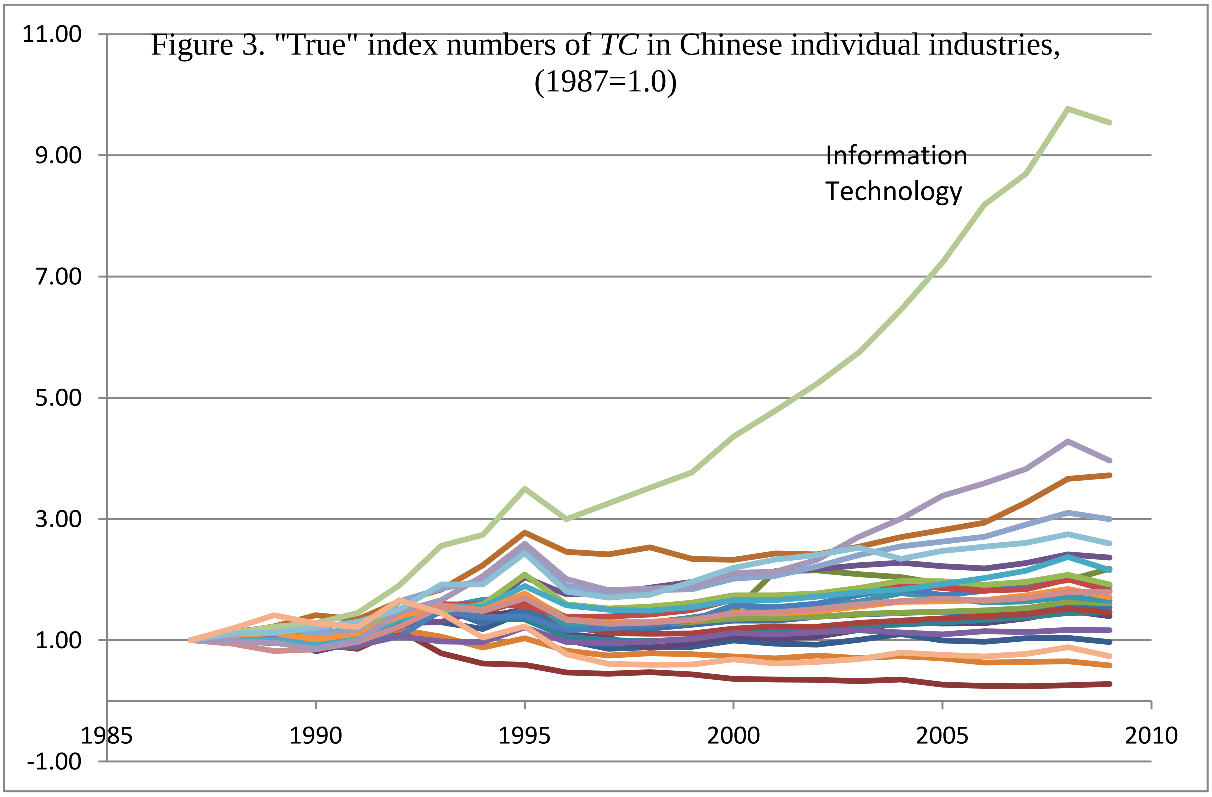


Figure 3. "True" index numbers of *TC* in Chinese individual industries, (1987=1.0)



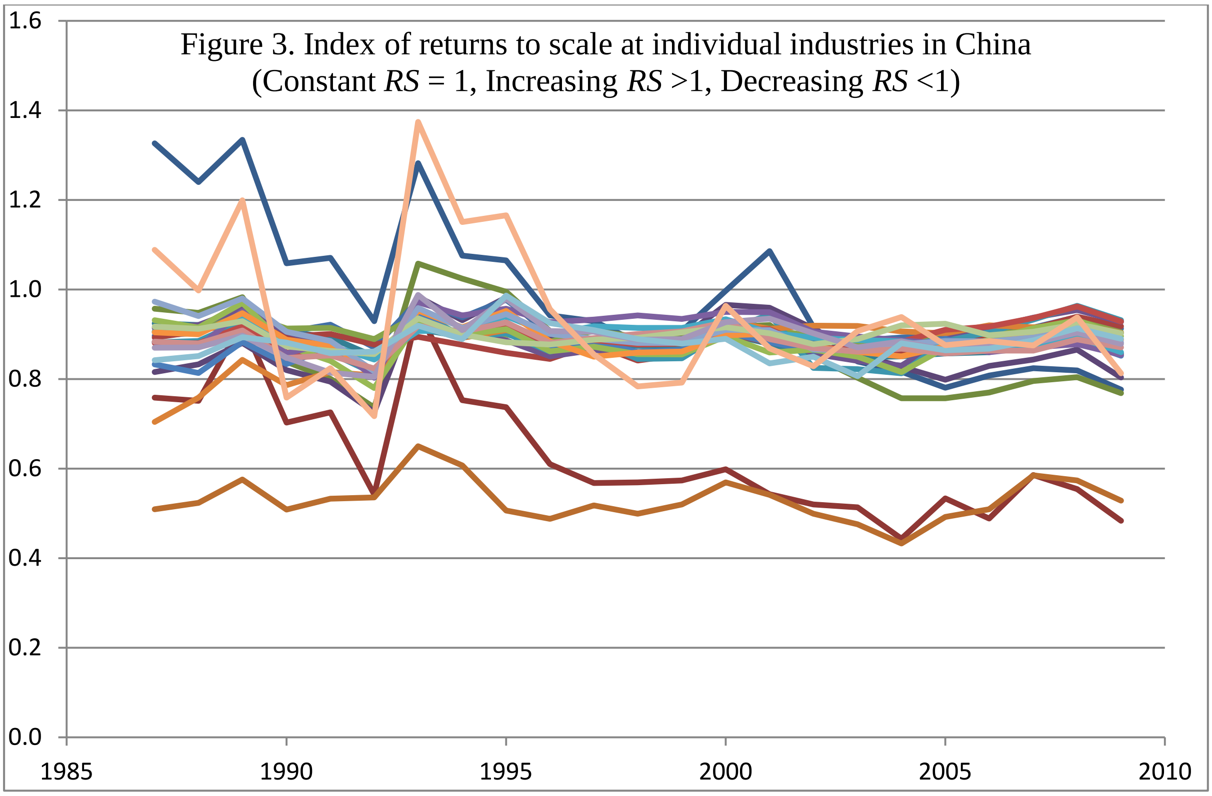


Figure 3. Index of returns to scale at individual industries in China
(Constant RS = 1, Increasing RS >1, Decreasing RS <1)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1985
1990
1995
2000
2005
2010