

Session Number: 4D Issues Related to the National Accounts

Time: Tuesday, August 7, Afternoon

*Paper Prepared for the 32nd General Conference of
The International Association for Research in Income and Wealth*

Boston, USA, August 5-11, 2012

Foundations of Balance Sheet Economics

Kazusuke Tsujimura and Masako Tsujimura

For additional information please contact:

Kazusuke Tsujimura
Faculty of Economics, Keio University

Masako Tsujimura
Keio Economic Observatory, Keio University

Email address: mizosita@sanken.keio.ac.jp

This paper is posted on the following website: <http://www.iariw.org>

Foundations of Balance Sheet Economics

Kazusuke Tsujimura Masako Tsujimura

Keio University, Tokyo, Japan

Paper Prepared for the 32nd General Conference of
The International Association for Research in Income and Wealth

Boston, USA, August 5 - 11, 2012

Abstract

The recent financial crisis and global recession reminded us of the importance of a database that enables us to review the interdependence between the real and financial economy. The objective of this paper is threefold; (i) to reconstruct a flow-of-funds based national accounting system; (ii) to clarify the meaning of economic bubbles in the national accounting perspective; (iii) to propose a from-whom-to-whom payer-payee matrix format of national accounting presentation. In historical cost accounting, the book value of an item in the balance sheet is the amount of funds that have changed hands in the last transaction. Thus the changes in the balance sheets of the concerning parties are directly correlated with the flow of funds from the payer to the payee. The payer-payee matrix that records the flow of funds between the all economic entities can be a useful device for national accounting.

JEL: C82, E01

Keywords: Flow of Funds, From Whom to Whom, Economic Bubble

1. Introduction

In April 2009, in the midst of the financial crisis and global recession, the Group of Twenty (G-20) finance ministers and central bank governors called for exploration of information gaps and provide appropriate proposals for strengthening data collection. As has been true of previous international financial crises, these gaps are highlighted when a lack of timely, accurate information hinders the ability of policy makers and market participants to develop effective responses. Indeed, the recent crisis has reaffirmed an old lesson—good data and good analysis are the lifeblood of effective surveillance and policy responses at both the national and international levels. In response to the G-20 initiative, International Monetary Fund (IMF) and Financial Stability Board (FSB) recommended to develop a strategy to promote the compilation and dissemination of the balance sheet approach, flow of funds, and sectoral data more generally¹. Although the term “flow of funds” implies the use of a time dimension associated with movement, spending, and consummation of transactions, it is probably more clearly discussed in the framework of “balance sheet economics” as Klein (1977) suggests. Tsujimura and Tsujimura (2011)² delineated the collapsing mechanism of the U.S. subprime mortgage bubble in Klein’s framework³ and clarified the theoretical relationship between the conventional flow of funds accounts of the balance sheet format and the asset-liability matrix of the from-whom-to-whom format⁴. However, the present-day flow of funds accounts covers only the lender-borrower, or more generally,

¹ Financial Stability Board and International Monetary Fund (2009, 2011), Recommendation 15. From-whom-to-whom framework of presentation is recommended in both Bank for International Settlement (2010) and Shrestha, Mink and Fassler (2012).

² The paper uses both U.S. Flow of Funds Accounts and IMF Coordinated Portfolio Investment Survey data.

³ For details, see Klein (1983), pp. 36-56. His idea has much resemblance to Stone (1966).

⁴ For the details of the asset-liability matrix, see Tsujimura and Mizoshita (2003) and Tsujimura and Tsujimura (2010a).

creditor-debtor relationship so that it is insufficient to extend the analysis into the interdependence between the real and financial economy.

As Vanoli (2002) points out, Irving Fisher in his theoretical works⁵, has formerly evoked the extension of the accounting treatment of individuals and businesses to society as a whole and the possibility, in principle, of obtaining the capital and income of society as a combination of balances of businesses and income accounts of individuals⁶. Frisch (1943), Stone (1947)⁷, Aukrust (1949), Gaathon (1953) among many other authors tried to combine business and national accounting into one common framework. It is well known that it was Morris Copeland who systematically drew the ground design of the money-flows accounts, or flow of funds accounts as we now call it. However, one of Morris Copeland's significant but less well-known roles is his contribution to the development of national accounting, which means the general application of business accounting methods to aggregative economic measurement⁸. In the basic conception of the money-flows accounts, Copeland (1947, 1949, 1952) conceives of the economy as composed of groups of institutional units or institutional sectors. Units in each of these groups make and receive payments to other units in the same and the other sectors. The basic unit of analysis is a transaction between two parties — payer and payee. Each transaction involves a money outflow for one party and an equal inflow for another. Each sector's inflows and outflows are shown for each

⁵ Fisher (1906, 1928).

⁶ Quoted from the English translation (2005), p. 16; it is equivalent to p. 35 of the original French version.

⁷ The prototype of the system of national accounts (SNA) in Stone (1947) is completely different from his later edition of SNA 1968 that is incorporated with the input-output table originated by Leontief (1941).

⁸ Dawson (1991), p.93. For further details, see Tsujimura and Tsujimura (2010b).

of the typical types of transaction. These money-flows together constitute the “main money circuit” that is measured⁹.

The objective of this paper is threefold; (i) to reconstruct a flow-of-funds based national accounting system; (ii) to clarify the meaning of economic bubbles in the national accounting perspective; (iii) to propose a from-whom-to-whom payer-payee matrix format of national accounting presentation. Section 2 gives the fundamental concepts of national accounting that is based on the balance sheets, which is in turn based on the Roman law concepts of *jus in rem* and *jus in personam*. The claim (i.e. financial asset) and obligation (i.e. liability) that relates to the same *jus in personam* are recorded as a pair in the balance sheets of two different entities; that is the basis for the flow-of-funds based national accounting. The classification of financial instruments and institutional/economic sectors will be considered in Subsections 2-2 and 2-3 while more general topics are discussed in 2-1.

Section 3 gives the framework of the flow-of-funds based national accounting. The valuation of assets and liabilities are examined in Subsection 3-1. Among four alternatives, historical cost accounting has the most relevance to the flow-of-funds based national accounting because the book value of the assets and liabilities are the amount of funds that have changed hands in the acquisition process. In Subsection 3-2, we will discuss the operations of funds in detail. Transmission of funds from a bank to its customers is an operation in which there are equal increases of deposit in both depository institution’s liability and customer’s asset; it is the creation of funds. Transmission of funds from its customer to a bank is an operation in which there are equal decrease of deposit in both depository institution’s liability and customer’s asset;

⁹ Rutherford (2002).

it is the repayment of funds. Transfer of funds is a process in which payer's deposit account is debited and payee's account is credited the same amount as a result of a series of transmission of funds through the banking system.

Journalizing is the main topic of Subsections 3.3 through 3.5. As a general rule, we record the economic event whenever transfer of funds takes place. Some economic events involve not only the transfer of funds (i.e. monetary assets) but also that of non-monetary financial assets or non-financial assets. In this respect, the economic events are classified into 9 categories including one non-pecuniary category (i.e. disposal of non-financial asset) as detailed in Subsection 3.4. This subsection also gives the fundamental accounting rules of historical cost accounting. All the economic events are recorded in the ledger matrix for the final preparation of the balance sheets. The mathematical process to obtain the flow and stock variables of assets and liabilities are detailed in Subsections 3.5 and 3.6. The fundamental properties of historical cost accounting will be examined for each category of economic events in Subsection 3.7.

In Section 4, we will examine if the transactions involving the existing assets alter the composition of the macroeconomic net worth. Some authors including Wicksell (1898) and Keynes (1930) suggest that the existing asset transactions may change the course of the economy. Furthermore, the above literature of Keynes suggests that the divergence between saving and investment may create economic bubbles. It is intriguing because Keynes (1930) uses unconventional definition of monetary income, which is an equivalent of the amount of received funds¹⁰. We will examine the argument in detail in Subsection 4.2.

In historical cost accounting, the book value of an item in the balance sheet is the

¹⁰ For further discussion on saving, investment and bubble, see Haberler (1938).

amount of funds that have changed hands in the last onerous transaction involving that item. That means the changes in the balance sheets have one-to-one relationship with flow of funds from the payer to the payee. Therefore, the payer-payee matrix that records all the transference of funds on the from-whom-to-whom basis can be a useful device for national accounting. In Section 5, we will propose a flow-of-funds based national accounting presentation format that has the from-whom-to-whom payer-payee matrix as its core. Some concluding remarks are in Section 6.

2. Basic Concepts

2.1 National Balance Sheet Accounts

National balance sheet accounts, which is an essential part of the system of national accounts, consist of a coherent set of articulated balance sheets of various economic or institutional sectors. A balance sheet is a list of the claims and obligations that relate to *jus in personam* as well as the claims that relates to *jus in rem* which are measured in the unit of account¹¹. The concepts of *jus in rem* and *jus in personam* are the foundations of the Roman law that is the most widely used platform of civil law in the contemporary world¹². Roman law classifies the social phenomena into as few categories as possible and to give no more than basic principles. This process of abstraction is important because by combining two or more principles, it is possible to create new principles that can apply to new phenomena¹³. *Jus in rem* is the exclusive dominion of a person over a thing. *Dominium* or ownership is the exclusive rights to use, obtain profit from, and dispose of a particular *Res*. *Dominium* is the most

¹¹ For the relationship between law and accounting, see Dohr (1928).

¹² Although the U.S. is not usually considered to be a Roman law country, the U.S. uniform commercial code is one of the best specimens of its kind.

¹³ Nicholas (1962), p. 1.

comprehensive *Jus in Rem*. However, if *locatio* or leasehold is established on the property, *dominium* is divided into two independent rights: the leasehold and the ownership of the leased property. They are also referred to as *Jus in Rem*. *Res* can be either *res corporales* or *res incorporales*. *Res corporales* are physical objects such as automobile, building, land, etc. *Res incorporales* are abstract things such as method of manufacture, writing, music, etc. Although *jus in rem* is considered to be an implicit contract among people en masse, the duties that correlate with it are always negative — they merely are duties to forbear or abstain¹⁴.

In contrast to this, *jus in personam* or *obligatio* is an explicit agreement between specific parties in which one party is obliged to do or to perform some specific duties on behalf of the other. Most of *jus in personam* arise from contract (*ex contractu*) ; however some others arise from tort (*ex delicto*). Many of the duties categorized as *jus in personam* are to fulfill pecuniary obligations. A specific claim relating to *jus in personam* that serves as unit of account or measurement of value is referred to as monetary asset. A specific duty corresponding to monetary asset is referred to as monetary liability. Other specific claims and duties relating to *jus in personam*, which are measurable in terms of the exchange value to monetary assets, are categorized as non-monetary financial assets and liabilities respectively. A specific claim relating to *jus in rem*, which are measurable in terms of the exchange value to monetary assets, is referred to as non-financial asset. Financial assets (liabilities) consist both of monetary assets (liabilities) and non-monetary financial assets (liabilities); however we will refer to the latter simply as financial assets (liabilities) in this paper. The financial asset and liability relating to the same *jus in personam* are recorded in the balance sheets of

¹⁴ Austin (1879), pp. 381-382.

different economic entities as a pair. The simultaneous recording of the asset and liability relating to the same *jus in personam* in two different entities' balance sheets as a pair is often referred to as horizontal double entry.

2.2 Financial Instruments

A financial instrument is a *jus in personam* that gives rise to a monetary/financial asset of one entity, and a liability of another. Financial instruments are classified into several categories: (1) loans and deposits; (2) debt securities; (3) assurance/insurance technical reserves; and (4) equities.

When a depository financial institution such as a bank grants credit to its customer, a set of asset and liability is created on its balance sheet. In this particular case, the asset and liability are referred to as loan and deposit respectively. Alternatively, a deposit is created when a financial institution purchases IOU note from its customer. Although a deposit account belongs to one specific customer and not transferable, the value stored in it is transferable between deposit accounts. The value in this sense is referred to as funds. Bank deposit is considered to be monetary asset for the creditor and monetary liability for the issuing bank because they serve as unit of account. A bank deposit agreement is an agreement in which a financial institution promises to make a specified amount of funds available to its customer for transfer at a specified or unspecified future date. When the date is specified, it is referred to as time deposit. When the date is not specified so that the fund is freely transferable at any time, it is referred to as current deposit. A bank loan agreement is a contract whereby a financial institution credit certain amount of funds to its customer's account and the customer agrees that he/she keeps the same amount in his/her deposit account at a specific future date; the customer

also agrees that the funds to be forfeited by the financial institution at that time. As a consequence, both loan and deposit disappear from the balance sheets of the financial institution and of its customer at the time of repayment. The loan agreement between parties other than financial institutions is an agreement in which the lender transfer a specific amount of funds to the borrower's deposit account and the borrower promises to transfer the same amount of funds to the lender's account at a specific future date. All the deposit and loans that are evidenced by negotiable instruments are excluded from the category because they are classified as debt security mentioned below.

Currency is a tool that is exclusively used as a medium of exchange in an economic territory. Currency includes specie coins, banknotes and subsidiary coins. A specie coin is a fixed-quantity minted piece of metal such as gold whose face value is always equivalent to its exchange value. A banknote or a subsidiary coin is an order directing a reserve bank to transfer designated amount of funds to the bearer's account, and thus serves as a medium of exchange. A designated amount of funds is moved from the customer's account to an inscribed deposit pool upon the issuance of a banknote; then it is moved from the deposit pool to the bearer's account upon presentation. Therefore, banknotes and subsidiary coins are considered to represent the claim to the funds stored at the reserve bank. Specie coins are categorized as non-financial assets while banknotes and subsidiary coins are merely instruction notes so that currency does not constitute an independent class of asset.

Debt securities are negotiable instruments serving as evidence of a debt. In most cases, debt security is an indenture of loan or deposit agreement. They include but are not limited to bills, notes, bonds, negotiable certificates of deposit, commercial paper, debentures, and similar instruments normally traded in the financial markets. While

“freely negotiable” means transferable by delivery only, “negotiable” usually means an endorsement from previous owner is required to verify the transfer.

Assurance is a special type of deposit agreement between a large number of policy owners and an underwriter, in which the underwriter agrees to repay the principal to a designated beneficiary upon the occurrence of some specified event, such as insured individual’s retirement or death, which is certain to happen at some unknown future time. So called permanent life insurance and endowment insurance belong to this category. Some annuity schemes also belong to the category. Outstanding claims reserves in assurance are referred to as assurance/pension technical reserve and appear on the liability side of the balance sheet of an insurer; but it also appears on the asset side of the balance sheets of policy holders. Meanwhile insurance is an onerous contract between a large number of policy owners and an underwriter, in which the underwriter agrees to pay a designated beneficiary the value of the damage or loss upon the occurrence of some specified event, such as fire or traffic accident, that happens to a certain portion of policy owners at random during the term of contract. Temporary term life insurance, pure endowment insurance and life annuity alongside with non-life insurance schemes belong to this category. Accrual adjustments in insurance are referred to as insurance technical reserves and recorded on the liability side of the balance sheet of an insurer. Insurance technical reserves include both unearned premiums and claims due. The former is recorded on the asset side of the balance sheet of policy holders while the latter is recorded on that of beneficiaries.

Equity securities are negotiable or non-negotiable instruments that give the holder a residual claim on the assets of the issuer. Joint stock corporation, a popular type of business entity involving one or more persons, issues certificates representing a residual

claim (shares or stocks) in return for each financial contribution, and the shareholders are free to transfer their ownership interest at any time by selling their stockholding to others. Although investment fund shares also belong to equity securities, they have a specialized role in financial intermediation as a kind of collective investment in other assets, so they are identified separately whenever it is possible.

2.3 Economic and Institutional Sectors

An economic sector is a group of economic entities, categorized on the basis of their principal functions, behavior and objectives. An economic entity is a group of natural persons that makes economic decisions collectively. Although a household that is considered to be an economic entity does not have a legal personality, many married couples have joint checking accounts or joint mortgages so that they share not only the decision making but also the responsibility. Meanwhile an institutional sector is a group of institutional unit, categorized on the basis of their principal functions, behavior and objectives. An institutional unit is an entity, which is capable of entering into contracts, of suing and of being sued, such as natural person, legal person or the government¹⁵. A legal person is a group of natural persons that is regarded by law to have the status of personhood. Each economic entity (institutional unit) in the economy must belong to one and only one economic (institutional) sector. For example, a non-financial business corporation is an institutional unit while its establishments such as factory or branch office are economic entities. However, both belong to non-financial corporate sector. In such a case, non-financial corporations as a group can be either an economic sector or institutional sector.

¹⁵ This definition of institutional unit is different from that of SNA 2008. In the SNA, economic entity and institutional unit are synonyms. See Paragraph 4.2, *ibid*.

In most cases, economic sector and institutional sector are synonymous. However, a clear distinction must be observed between these two concepts when resident/non-resident status is concerned. The economic entities as well as institutional units are divided into two classes: residents and non-residents. An economic territory is a single currency area, which is under the effective economic control of a single government. The economic territory includes areas of land, water and airspace. An institutional unit is said to be resident within an economic territory when a natural person has his/her domicile or residence in the economic territory or a legal entity has its principal office in the economic territory. All other institutional units are treated as non-residents. An economic entity is said to be resident within an economic territory when it maintains a center of predominant economic activity in that territory. For example, an establishment such as factory or branch office that resides in economic territory 'A' and belongs to a business corporation whose principal office resides in economic territory 'B' is a resident of territory 'A' when categorized by economic sector but is a resident of territory 'B' when categorized by institutional sector.

The total economy consists of all the economic entities (institutional units) which are resident in the economic territory. The rest of the world account covers transactions between resident and non-resident economic entities (institutional units) and the related stocks of assets and liabilities where relevant. As already mentioned, the economic entities (institutional units) are grouped together to form economic sectors (institutional sectors), on the basis of their principal functions, behavior and objectives: (1) non-financial corporations; (2) financial institutions; (3) households; (4) nonprofit institutions; and (5) the general government. They are further divided into several sub-sectors.

A non-financial corporation is a legal person whose principal activity is the production of market goods or non-financial services. All the resident non-financial corporations are grouped into the non-financial corporation institutional sector. A quasi-corporation is an unincorporated enterprise owned by a resident institutional unit that has sufficient information to compile a complete set of accounts and is operated as if it were a separate corporation and whose de facto relationship to its owner is that of a corporation to its shareholders¹⁶. All the resident business establishments belonging to non-financial corporations inclusive of quasi corporations are grouped into the non-financial corporation economic sector.

Financial corporations consist of all legal persons that are principally engaged in providing financial services, including insurance and pension funding services, to other institutional units. All the resident financial corporations are grouped into the financial corporation institutional sector. All the resident business establishments belonging to financial corporations inclusive of quasi corporations are grouped into the financial institution economic sector. The sector is divided into several sub-sectors: reserve bank, other depository financial institutions, other financial intermediaries, financial auxiliaries and insurance corporations.

Depository financial institutions or simply banks are financial intermediaries that have liabilities in the form of deposits. The reserve banks, also known as central bank in some countries, are the national depository financial institutions that issue currency and provide financial and banking services for its country's government and commercial banking system. Other financial intermediaries are engaged in providing financial services by incurring liabilities, in forms other than deposits on their own account for

¹⁶ SNA 2008, Paragraph 4.42.

the purpose of acquiring financial assets by engaging in financial transactions on the market. Financial auxiliaries consist of financial institutions that provide financial services but do not take ownership of the financial assets and liabilities being transacted such as a brokerage. Insurance corporations provide either assurance or insurance to individual institutional units or groups of units or reinsurance services to other insurance corporations.

A household is a collective consumption unit consisting of natural persons living together. A natural person who is living alone is also treated as a household. All the resident households are grouped into the household sector. The household sector is an economic sector as well as an institutional sector. A household that actively engages in market production, which is referred to as an unincorporated enterprise, is sometimes included in the household sector but preferably categorized as a separate sector under the name of non-corporate business.

A non-profit institution is an economic entity other than government, which provides goods and services free or at prices significantly below the cost of production. They are financed either by regular membership subscriptions, by contributions, or by earnings on their holdings of financial or non-financial assets. A non-profit institution that provides goods and services predominantly to households or that consists mainly of individuals is referred to as a non-profit institution serving households. Some non-profit institutions have legal personality, but many others do not have any legal status. Therefore resident non-profit institution as a group is an economic sector but not necessarily an institutional sector.

Government units are institutional units established by political processes that have legislative, judicial and/or administrative authority over other economic/

institutional units within a given area. The government, as an economic entity, collects taxes and in return provides public services. Such economic activities of the national as well as local governments are referred to as general government in the national accounting. The general government as a group is an economic sector but also treated as an institutional sector, although their legal status is unclear in many cases. The economic activities of a government to supply marketable goods and services are classified either as non-financial corporation or as financial corporation. They are referred to as public corporations and are treated as separate sectors whenever it is possible.

3. Flow-of-Funds Based National Accounting

3.1 Valuation of Assets and Liabilities

The issue of asset and liability valuation has long been a contentious issue for accounting standard setters and for national accountants alike. In original cost accounting, the book value of an item in the balance sheet is the amount of funds that have changed hands at the first onerous transaction involving that item; the book value of the item does not change even if it is exchanged for funds thereafter. In historical cost accounting, the book value of an item in the balance sheet is the amount of funds that have changed hands in the last onerous transaction involving that item. In current cost accounting, every item in the balance sheet should be valued as if it were being acquired on the date to which the balance sheet relates¹⁷. In buy-back cost accounting, every asset in the balance sheet should be valued as if it were being acquired on the date to which the balance sheet relates; the book value of a liability is equivalent to the book value of

¹⁷ Since liabilities seldom change hands, their reference value tends to be their original value.

the asset that relates to the same *jus in personam*. Therefore, both in historical cost accounting and current cost accounting, there can be a difference between the book value of the asset and the book value of the liability that relates to the same *jus in personam*; there is no difference between them in original cost accounting and buy-back cost accounting.

Paragraph 2.60 of SNA 2008 stipulates that assets and liabilities are recorded at current values at the time to which the balance sheet relates, not at their original valuation. Paragraph 2.58 also states that a financial asset and its liability counterpart have to be recorded for the same amount in the creditor and the debtor accounts. It is widely considered that SNA 2008 is based on buy-back cost accounting. Traditionally national accountants favored either current or buy-back cost accounting because productivity analysts assert that non-financial assets should be measured by their replacement cost rather than by the acquisition cost. However, if the ultimate purpose of national accounting is the depiction of flow of funds rather than the flow of products, historical cost accounting is a more suitable option because the book value of an item in the balance sheet is the amount of funds that have changed hands.

3.2 Operations of Funds

Transmission of funds from a bank to its customer is an operation in which the bank credits the customer's account with deposit and incurs the liability; thus the asset of the customer and the liability of the bank increase the same amount. Transmission of funds from its customer to a bank is an operation in which the bank debits the customer's deposit account and discharges its liability; thus the asset of the customer and the liability of the bank decrease the same amount. (See Figure 1.) As we have

mentioned in Section 2.2, creation of funds is an operation in which the bank credits the customer's account with deposit and the customer promises to repay it at a specified future date; thus the asset of the customer and the liability of the bank increase the same amount with the deposit but also the asset of the bank and the liability of the customer increase the same amount with the loan. Repayment of funds is an operation in which the bank debits the customer's deposit account and discharges the customer's loan liability, thus the deposit of the customer and the corresponding liability of the bank decrease and, at the same time, the loan liability of the customer and the corresponding asset of the bank decrease the same amount. Alternatively, funds are created when a financial institution purchases IOU note inclusive of any type of negotiable instrument from its customer and his/her account is credited as a consequence; when it is repaid, the issuer's account is debited. (See Figure 2.) Therefore a creation of funds increases and repayment decreases the amount of outstanding funds in total.

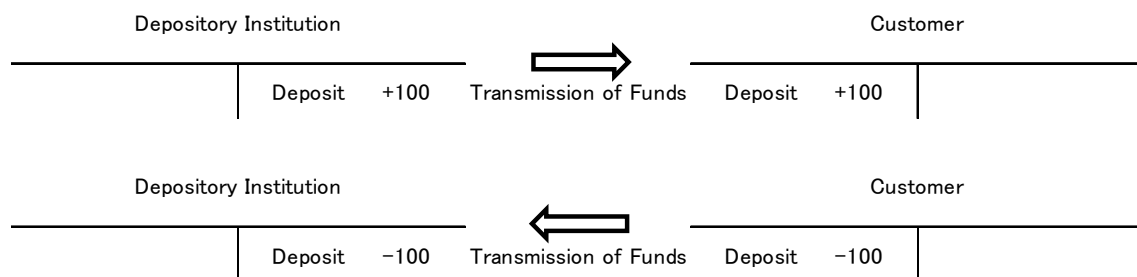


Figure 1. Transmission of Funds between Depository Institution and its Customer

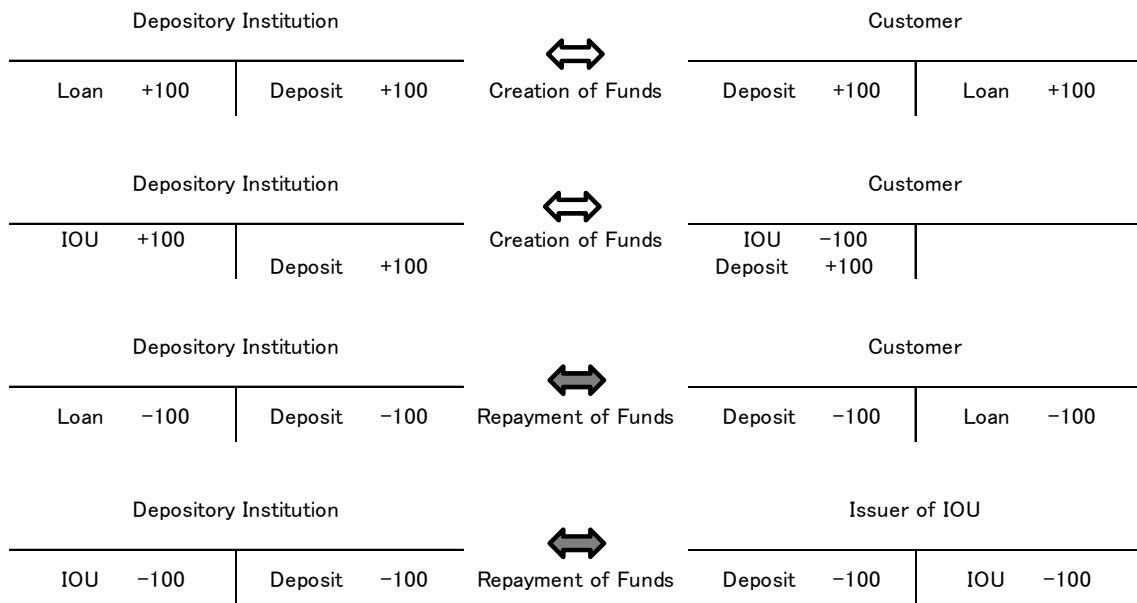


Figure 2. Creation and Repayment of Funds

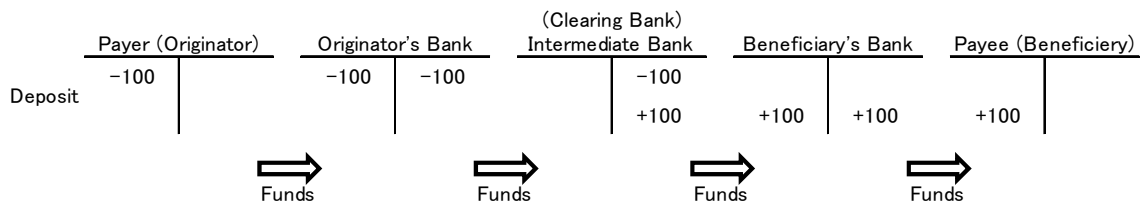


Figure 3. Transfer of Funds between Non-bank Economic Entities

From \ To	Payer (Originator)	Originator's Bank	Intermediate Bank	Beneficiary's Bank	Payee (Beneficiary)
Payer (Originator)					100
Originator's Bank					
Intermediate Bank					
Beneficiary's Bank					
Payee (Beneficiary)					

Figure 4. Transfer-of-Funds Matrix

From \ To	Payer (Originator)	Originator's Bank	Intermediate Bank	Beneficiary's Bank	Payee (Beneficiary)
Payer (Originator)		100 ①			
Originator's Bank			100 ②		
Intermediate Bank				100 ③	
Beneficiary's Bank					100 ④
Payee (Beneficiary)					

Figure 5. Transmission-of-Funds Matrix

In the modern economy, most economic transactions are settled through bank deposit account operations. Transfer of funds is a process in which a payer's deposit account is debited and payee's account is credited for the same amount. In this case, we say that the amount of funds is transferred from payer's (originator's) account to payee's (beneficiary's) account. Transfer of funds in this sense involves at least one, but typically three depository institutions: the originator's bank, the intermediary bank or clearing bank, and the beneficiary's bank. Transfer of funds between non-bank economic entities is a sequence of the same number and amount of transmission of funds in the opposite direction between a bank and its customer. (See Figure 3.) Therefore a transfer between non-bank economic entities neither increases nor decreases the amount of outstanding funds in total. If we put this into matrix form, the transfer of funds is simply recorded in the cell at the intersection of payer's row and the payee's column as in Figure 4 while the transmission of funds behind it is a four-step operation as depicted in Figure 5. Note that there is no entry into the intersection cell of the payer's row and payee's column in the latter matrix.

Bank checks are written orders to a depository financial institution to transfer a

stated sum of funds from the payer's account to the payee's account. As we have already mentioned, a banknote or a subsidiary coin is an order directing a reserve bank to transfer designated amount of funds. The funds are moved from the original owner's account to an inscribed deposit pool upon the issuance of a banknote; they are then moved from the deposit pool to the bearer's account upon presentation of a banknote or a subsidiary coin. Therefore, banknotes or subsidiary coins are considered to represent the claim to the funds stored at the reserve bank just as bank checks represent the claim to the funds stored at a commercial bank.

3.3 Journalizing

We record the economic event whenever transfer of funds (inclusive of creation and repayment) takes place. An economic event that involves transfer of funds is referred to as a pecuniary event. Since all pecuniary events involve changes in the balance sheet, the changes are recorded in a T-shaped pecuniary event journal. As a general rule, if an event involves transfer of *jus in rem* or existing *jus in personam*, it is recorded on the left hand side of the pecuniary event journal of the new owner as an increment of non-financial or financial asset in the amount of funds that changed hands. If the asset was already on the balance sheet of the previous owner, it is erased at the moment of the transaction so that the reduction of asset is recorded as a negative value on the left hand side of the journal at the previous book value. If an economic event creates *jus in personam*, an increment of financial asset is recorded on the left hand side of the journal of the creditor in the amount of funds that changed hands. It is also recorded on the right hand side of the journal of the debtor as an increment of liability at the same value. When a debt is repaid, both financial asset and liability are erased from

the balance sheets of the parties so that they are recorded as negative values in their journal at the previous book values. The left hand side of the journal is referred to as increment of asset while the right hand side is referred to as increment of liability.

When there is a change in the asset without any transfer of funds, the change will be recorded in the non-pecuniary event journal. For example, disposal of machinery due to obsolescence is a non-pecuniary event. We should treat it separately because it is a unilateral rather than a bilateral event. The left hand side of the non-pecuniary event journal records a reduction of assets. An increment in non-financial asset is recorded in the pecuniary event journal when the asset changes hands for the first time in such a transaction. Therefore no increment will be recorded in the non-pecuniary event journal. Cancellation of debt such as forgiveness is regarded as a special case of redemption, in which no funds change hands, so that it will be recorded in the pecuniary event journal rather than in the non-pecuniary event journal.

3.4 Economic Events

Let M_{ut} and D_{ut} be the monetary assets and liabilities respectively; F_{ut} and L_{ut} be the financial assets and liabilities; and N_{ut} be the non-financial assets; where t refers to any point of time, and u could be any member of the economic entity or institutional unit population. The assets are recorded on the left-hand side while the liabilities are listed on the right-hand side of the balance sheet, which is a T-shaped account, at their acquisition cost as mentioned above (i.e. the amount of funds changed hands at the time of last transaction). We define net worth as the difference between the total assets and liabilities:

$$(3-1) \quad W_{ut} \equiv (M_{ut} + F_{ut} + N_{ut}) - (D_{ut} + L_{ut}).$$

We define economic event e ($e = 1, \dots, \mathcal{E}, \dots, \mathcal{E}$) which includes both pecuniary and non-pecuniary events. Typically a pecuniary event involves transfer of funds from payer a to payee b . In addition to that, up to three financial intermediaries are involved: the originator's bank, the intermediary bank and the beneficiary's bank. Any economic event causes at least some changes in the balance sheets of at least one participant. There are ten patterns of changes in any of the balance sheet: δM^+ , δM^- , δD^+ , δD^- , δF^+ , δF^- , δL^+ , δL^- , δN^+ and δN^- . The superscripts $+$ and $-$ indicate the increasing and decreasing factor of net worth respectively. For example, δL^+ refers to a decrease in liability while δL^- means an increase in it as a result of an economic event.

Economic events are broadly divided into nine categories:

Category 1: Creation of funds;

Category 2: Repayment of funds;

Category 3: Transfer of funds with no non-monetary asset involved;

Category 4: Transfer of funds that create new financial asset/liability;

Category 5: Transfer of funds in exchange for existing financial asset/liability;

Category 6: Elimination of existing financial asset/liability;

Category 7: Transfer of funds that create new non-financial asset;

Category 8: Transfer of funds in exchange for existing non-financial asset;

Category 9: Disposal of non-financial asset.

As we have already mentioned, Category 1 and 2 are arrangements directly between a financial intermediary and its customer. All other category of pecuniary event involves

two participants and up to three financial intermediaries. While Category 1 and 2 are related to bank loans, Category 4 and 6 are corresponding to loan arrangements that involve non-bank lenders. Category 6 events are typically redemption but they also include such cases in which no funds change hands (e.g. forgiveness of debt). Although creation of non-financial asset can be treated as a special case of Category 8 where the previous book value of the item was zero, they are treated separately as Category 7. Likewise, disposal of non-financial asset is a special case of Category 8 where the disposal value is zero. However, since it is a unilateral event, we treat it separately as Category 9. When a Category 4 event takes place, a pair of financial asset and liability is created. In such a case, we will give an identification number f ($f = 1, \dots, \varphi, \dots, \Phi$) to the newly created pair of financial asset and liability. In the same manner, when a Category 7 event takes place, we give an identification number g ($g = 1, \dots, \gamma, \dots, \Gamma$) to the newly created non-financial asset.

Category of Economic Events	Payer a (Originator)	Payee b (Beneficiary)	Originator's Bank	Intermediary Bank	Beneficiary's Bank
Category 1	$\delta D^- + \delta F^+$	$\delta M^+ + \delta L^-$	na	na	na
Category 2	$\delta M^- + \delta L^+$	$\delta D^+ + \delta F^-$	na	na	na
Category 3	δM^-	δM^+	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 4	$\delta M^- + \delta F^+$	$\delta M^+ + \delta L^-$	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 5	$\delta M^- + \delta F^+$	$\delta M^+ + \delta F^-$	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 6	$\delta M^- + \delta L^+$	$\delta M^+ + \delta F^-$	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 7	$\delta M^- + \delta N^+$	δM^+	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 8	$\delta M^- + \delta N^+$	$\delta M^+ + \delta N^-$	$\delta M^- + \delta D^+$	$\delta D^+ + \delta D^-$	$\delta M^+ + \delta D^-$
Category 9	na	$+\delta N^-$	na	na	na

na: not applicable

Table 1. Changes in Assets and Liabilities for each Category of Economic Events

Category of Economic Events	Payer a (Originator)	Payee b (Beneficiary)	Macroeconomic Net Worth	Applicable Rules
Category 1	0	0	0	I
Category 2	0	0	0	I and IV
Category 3	δM^{a-}	δM^{b+}	0	V
Category 4	0	0	0	I
Category 5	0	$\delta M^{b+} + \delta F^{b-}$	$\delta M^{b+} + \delta F^{b-}$	I and II
Category 6	$\delta M^{a-} + \delta L^{a+}$	$\delta M^{b+} + \delta F^{b-}$	$\delta L^{a+} + \delta F^{b-}$	III and V
Category 7	0	δM^{b+}	δM^{b+}	I
Category 8	0	$\delta M^{b+} + \delta N^{b-}$	$\delta M^{b+} + \delta N^{b-}$	I and II
Category 9	na	δN^{b-}	δN^{b-}	na

na: not applicable

Table 2. Changes in Net Worth for each Category of Events

	Monetary Asset	Monetary Liability	Financial Asset	Financial Liability	Non-financial Asset	Macroeconomic Net Worth	Applicable Rules
Category 1	δM^{b+}	δD^{a-}	δF^{a+}	δL^{b-}	na	0	I and V
Category 2	δM^{a-}	δD^{b+}	δF^{b-}	δL^{a+}	na	0	I and IV
Category 3	0	0	na	na	na	0	V
Category 4	0	0	δF^{a+}	δL^{b-}	na	0	I and V
Category 5	0	0	$\delta F^{a+} + \delta F^{b-}$	na	na	$\delta F^{a+} + \delta F^{b-}$	I, II and V
Category 6	0	0	δF^{b-}	δL^{a+}	na	$\delta L^{a+} + \delta F^{b-}$	III and V
Category 7	0	0	na	na	δN^{a+}	δN^{a+}	V
Category 8	0	0	na	na	$\delta N^{a+} + \delta N^{b-}$	$\delta N^{a+} + \delta N^{b-}$	I, II and V
Category 9	na	na	na	na	δN^{b-}	δN^{b-}	na

na: not applicable

Table 3. Net Changes in the Book Value of Assets/Liabilities for each Category of Economic Events

The changes in assets and liabilities of the participants and intermediaries for each category of economic events are listed in Table 1. The following historical cost accounting rules will be applied to each category of economic events.

[Rule I] When a pecuniary event involves transfer or creation of financial asset/liability or non-financial asset, it is recorded in the account of the concerning party as an increment of asset/liability in the amount of funds changed hands in that particular event.

[Rule II] When a pecuniary event involves transfer of financial or non-financial assets, and in case that the asset has already been on the account of the old owner, it is simply erased so that the reduction of asset is recorded at the previous book value.

[Rule III] If the financial asset and liability that relates to the same *jus in personam* are owned and incurred by the same economic entity (institutional unit) as a result of an economic event, it is referred to as redemption and the asset and liability are eliminated from the balance sheets of the concerned parties.

[Rule IV] As a special case, when a bank loan is repaid, that is the same amount of funds that created the monetary asset is paid from the customer to the financial institution, both monetary and financial asset and liability are erased from the account of the concerned parties.

[Rule V] The book values of the monetary asset and liability relating to the same *jus in personam* are always equal because they serve as the unit of account. By the same reason, when a monetary asset changes hands, its book value does not change.

When we apply these rules to each category of economic events, the changes in net worth for each category of events are summarized in Table 2. The columns of intermediary banks are omitted from the table, because there are no changes in the net worth according to Rules IV and V above.

In categories 1 and 2, which correspond to bank loan and redemption, there is an increase and decrease in the total amount of funds; however it does not change the net worth of either the lender or the borrower because the lending is measured as the amount of deposit granted to the customer. In contrast to this, redemption gain or loss is possible when non-bank lending is securitized and becomes subject to sale. Also, whenever either financial or non-financial assets are subject to sale, changes in net worth are inevitable. In category 3, which is a pecuniary transaction involving no non-monetary asset, the payer incurs a net worth loss while the payee accrues a net worth gain of the same amount; however there is no change in the macroeconomic net worth because it is simply a transfer of funds.

In table 3, the changes in the book value of assets/liabilities for each category of economic events are listed. It is obvious that the changes in the book value occur only when either financial or non-financial asset becomes subject to sale. The corresponding line in Table 2 and 3 refer to the same event; the former records the economic event from the viewpoint of participating economic entities or institutional units while the latter observes it from the viewpoint of assets and/or liabilities involved in the transaction. Since the entries in both tables are changes in the net worth of the same event, the column total of both tables is identical to the change in the macroeconomic net worth.

3.5 Posting in the Ledger Matrix

Let $e[t_e, a_e, b_e, f_e, g_e, h_e, k_e] \in \mathbb{E}$ be any of economic event that is characterized by seven attributes: time of occurrence $t_e \in \mathbb{T}$; two participants $a_e, b_e \in \mathbb{U}$ ($a_e \neq b_e$) where \mathbb{U} is the economic entity (institutional unit) population; financial asset/liability identification number f_e ; non-financial asset identification number g_e ; the transaction identity number for each asset h_e (we set $h_e = 1$ when the asset is created and add one whenever it changes hands); and event category k_e as explained earlier. Let t_0 be the origin of the time axis \mathbb{T} , and Δt be the duration of an accounting period so that we can define $\mathfrak{t}_\tau = \{t \in \mathbb{T} \mid t_0 + (\tau - 1)\Delta t < t \leq t_0 + \tau\Delta t\}$ as accounting period τ . Thus $\mathfrak{t}_\tau = \{t \in \mathbb{T} \mid t_0 < t \leq t_0 + \tau\Delta t\}$ means any time before the end of accounting period τ .

We define subsets of economic events as follows:

$$(3-2-1) \quad \mathbb{T}_\tau = \{e \in \mathbb{E} \mid t_e \in \mathfrak{t}_\tau\} ;$$

$$(3-2-2) \quad \mathbb{T}_\tau = \{e \in \mathbb{E} \mid t_e \in \mathfrak{t}_\tau\} ;$$

$$(3-2-3) \quad \mathbb{P}_{av} = \{e \in \mathbb{E} \mid a_e = v\} ;$$

$$(3-2-4) \quad \mathbb{P}_{bv} = \{e \in \mathbb{E} \mid b_e = v\} ;$$

$$(3-2-5) \quad \mathbb{F}_\varphi = \{e \in \mathbb{E} \mid f_e = \varphi\} ;$$

$$(3-2-6) \quad \mathbb{G}_\gamma = \{e \in \mathbb{E} \mid g_e = \gamma\} ;$$

$$(3-2-7) \quad \mathbb{H}_\eta = \{e \in \mathbb{E} \mid h_e = \eta\} ;$$

$$(3-2-8) \quad \mathbb{K}_\kappa = \{e \in \mathbb{E} \mid k_e = \kappa\} .$$

For example, in equation (3-2-1), T_τ is the set of economic events that have taken place during time period τ . Likewise, in equation (3-2-3), P_{av} means the set of events in which v 'th member of the economic entity population plays the role of payer a .

Ledger Matrix Λ consists of all the economic events that have taken place, with all the necessary information relating to them.

$$(3-3) \quad \Lambda = \begin{bmatrix} \lambda_{1a} & \lambda_{1b} \\ \vdots & \vdots \\ \lambda_{\varepsilon a} & \lambda_{\varepsilon b} \\ \vdots & \vdots \\ \lambda_{\mathcal{E}a} & \lambda_{\mathcal{E}b} \end{bmatrix} ;$$

where

$$\lambda_{ea} = (\delta M_e^{a-} \quad \delta D_e^{a-} \quad \delta F_e^{a+} \quad \delta L_e^{a+} \quad \delta N_e^{a+}) ;$$

and

$$\lambda_{eb} = (\delta M_e^{b+} \quad \delta D_e^{b+} \quad \delta F_e^{b-} \quad \delta L_e^{b-} \quad \delta N_e^{b-}) .$$

We further define auxiliary vectors \mathbf{v}_l ($l=1, \dots, 5$). \mathbf{v}_l is a 5×1 column vector with 1 as the l 'th element and 0 elsewhere. For example, the change in net worth for a participant that assumes the role of payer a in an event is obtainable using the following formula:

$$(3-4) \quad \delta W_e^a = \sum_{l=1}^5 \lambda_{ea} \mathbf{v}_l = \lambda_{ea} \mathbf{i} ;$$

where \mathbf{i} is a 5×1 unit vector whose all elements are 1.

3.6 Stock and Flow Variables

3.6.1 Stocks

We can calculate the book value of the outstanding assets and liabilities owned by economic entity (institutional unit) \mathfrak{U} at the end of period τ in the following manner.

$$(3-5-1) \quad M_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{v}_1 + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{v}_1 \ .$$

$$(3-5-2) \quad D_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{v}_2 + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{v}_2 \ .$$

$$(3-5-3) \quad F_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{v}_3 + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{v}_3 \ .$$

$$(3-5-4) \quad L_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{v}_4 + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{v}_4 \ .$$

$$(3-5-5) \quad N_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{v}_5 + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{v}_5 \ .$$

The net worth of economic entity (institutional unit) \mathfrak{U} at the end of period τ is obtainable in the following formula:

$$(3-6) \quad W_{\mathfrak{U}\tau} = \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{i} + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{i} \ .$$

Likewise, we can calculate the macroeconomic book value of the outstanding assets and liabilities at the end of period τ .

$$(3-7-1) \quad M_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{v}_1 + \lambda_{eb} \mathbf{v}_1) \ .$$

$$(3-7-2) \quad D_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{v}_2 + \lambda_{eb} \mathbf{v}_2) \ .$$

$$(3-7-3) \quad F_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{v}_3 + \lambda_{eb} \mathbf{v}_3) \ .$$

$$(3-7-4) \quad L_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{v}_4 + \lambda_{eb} \mathbf{v}_4) \ .$$

$$(3-7-5) \quad N_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{v}_5 + \lambda_{eb} \mathbf{v}_5) \ .$$

The macroeconomic net worth at the end of period τ is obtainable in the following

formula:

$$(3-8) \quad W_{\tau} = \sum_{e \in \mathbb{T}_{\tau}} (\lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i}) .$$

3.6.2 Flows

The flow variables corresponding to each category of asset and liability for time period τ for each economic entity (institutional unit) \mathbf{u} are obtainable using following formulae. As far as flow variables are concerned, the sign of the variables has much importance so that we sum up the positive and negative numbers separately.

$$(3-9-1-1) \quad \Delta M_{\mathbf{u}\tau}^+ = \sum_{e \in \mathbb{P}_{b\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{eb} \mathbf{v}_1 ;$$

$$(3-9-1-2) \quad \Delta M_{\mathbf{u}\tau}^- = \sum_{e \in \mathbb{P}_{a\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{ea} \mathbf{v}_1 .$$

$$(3-9-2-1) \quad \Delta D_{\mathbf{u}\tau}^+ = \sum_{e \in \mathbb{P}_{b\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{eb} \mathbf{v}_2 ;$$

$$(3-9-2-2) \quad \Delta D_{\mathbf{u}\tau}^- = \sum_{e \in \mathbb{P}_{a\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{ea} \mathbf{v}_2 .$$

$$(3-9-3-1) \quad \Delta F_{\mathbf{u}\tau}^+ = \sum_{e \in \mathbb{P}_{a\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{ea} \mathbf{v}_3 ;$$

$$(3-9-3-2) \quad \Delta F_{\mathbf{u}\tau}^- = \sum_{e \in \mathbb{P}_{b\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{eb} \mathbf{v}_3 .$$

$$(3-9-4-1) \quad \Delta L_{\mathbf{u}\tau}^+ = \sum_{e \in \mathbb{P}_{a\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{ea} \mathbf{v}_4 ;$$

$$(3-9-4-2) \quad \Delta L_{\mathbf{u}\tau}^- = \sum_{e \in \mathbb{P}_{b\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{eb} \mathbf{v}_4 .$$

$$(3-9-5-1) \quad \Delta N_{\mathbf{u}\tau}^+ = \sum_{e \in \mathbb{P}_{a\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{ea} \mathbf{v}_5 ;$$

$$(3-9-5-2) \quad \Delta N_{\mathbf{u}\tau}^- = \sum_{e \in \mathbb{P}_{b\mathbf{u}} \cap \mathbb{T}_{\tau}} \lambda_{eb} \mathbf{v}_5 .$$

The change in the net worth of economic entity (institutional unit) during period τ is calculated likewise:

$$(3-10) \quad \Delta W_{v\tau} = \sum_{e \in P_{av} \cap T_\tau} \lambda_{ea} \mathbf{i} + \sum_{e \in P_{bv} \cap T_\tau} \lambda_{eb} \mathbf{i} ;$$

When non-pecuniary events must be excluded, the set operation $e \in P_{\pi v} \cap T_\tau$ is replaced by $e \in P_{\pi v} \cap T_\tau \cap K_9^c$, where K_9^c is the complementary set of K_9 .

The corresponding macroeconomic values can be calculated in the following manner.

$$(3-11-1-1) \quad \Delta M_\tau^+ = \sum_{e \in T_\tau} \lambda_{eb} \mathbf{v}_1 ;$$

$$(3-11-1-2) \quad \Delta M_\tau^- = \sum_{e \in T_\tau} \lambda_{ea} \mathbf{v}_1 .$$

$$(3-11-2-1) \quad \Delta D_\tau^+ = \sum_{e \in T_\tau} \lambda_{eb} \mathbf{v}_2 ;$$

$$(3-11-2-2) \quad \Delta D_\tau^- = \sum_{e \in T_\tau} \lambda_{ea} \mathbf{v}_2 .$$

$$(3-11-3-1) \quad \Delta F_\tau^+ = \sum_{e \in T_\tau} \lambda_{ea} \mathbf{v}_3 ;$$

$$(3-11-3-2) \quad \Delta F_\tau^- = \sum_{e \in T_\tau} \lambda_{eb} \mathbf{v}_3 .$$

$$(3-11-4-1) \quad \Delta L_\tau^+ = \sum_{e \in T_\tau} \lambda_{ea} \mathbf{v}_4 ;$$

$$(3-11-4-2) \quad \Delta L_\tau^- = \sum_{e \in T_\tau} \lambda_{eb} \mathbf{v}_4 .$$

$$(3-11-5-1) \quad \Delta N_\tau^+ = \sum_{e \in T_\tau} \lambda_{ea} \mathbf{v}_5 ;$$

$$(3-11-5-2) \quad \Delta N_\tau^- = \sum_{e \in T_\tau} \lambda_{eb} \mathbf{v}_5 .$$

The changes in the macroeconomic net worth during period $t = \tau$ is calculated likewise:

$$(3-12) \quad \Delta W_\tau = \sum_{e \in T_\tau} (\lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i}) .$$

As in the previous case, when non-pecuniary events must be excluded, the set operation

$e \in T_\tau$ is replaced by $e \in T_\tau \cap K_9^c$.

3.7 Fundamental Properties of historical cost accounting

We can derive the following relations from Table 2, which is based on the fundamental rules of historical cost accounting. For economic event categories 1, 2 and 4, that is $e \in K_1 \cup K_2 \cup K_4$:

$$(3-13-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = 0 \quad (\text{for } e \in K_1 \cup K_2 \cup K_4) ;$$

$$(3-13-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = 0 \quad (\text{for } e \in K_1 \cup K_2 \cup K_4) ;$$

$$(3-13-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = 0 \quad (\text{for } e \in K_1 \cup K_2 \cup K_4) ;$$

where δW_e^a , δW_e^b and δW_e are changes in the net worth of payer a , payee b and macroeconomy as the result of an economic event. For event category 3:

$$(3-14-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = \delta M_e^{a-} \quad (\text{for } e \in K_3) ;$$

$$(3-14-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta M_e^{b+} \quad (\text{for } e \in K_3) ;$$

$$(3-14-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = 0 \quad (\text{for } e \in K_3) .$$

For event category 5:

$$(3-15-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = 0 \quad (\text{for } e \in K_5) ;$$

$$(3-15-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta M_e^{b+} + \delta F_e^{b-} \quad (\text{for } e \in K_5) ;$$

$$(3-15-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = \delta M_e^{b+} + \delta F_e^{b-} \quad (\text{for } e \in K_5) .$$

For event category 6:

$$(3-16-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = \delta M_e^{a-} + \delta L_e^{a+} \quad (\text{for } e \in K_6) ;$$

$$(3-16-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta M_e^{b+} + \delta F_e^{b-} \quad (\text{for } e \in K_6) ;$$

$$(3-16-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = \delta L_e^{a+} + \delta F_e^{b-} \quad (\text{for } e \in K_6) .$$

For event category 7:

$$(3-17-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = 0 \quad (\text{for } e \in K_7) ;$$

$$(3-17-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta M_e^{b+} \quad (\text{for } e \in K_7) ;$$

$$(3-17-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = \delta M_e^{b+} \quad (\text{for } e \in K_7) .$$

For event category 8:

$$(3-18-1) \quad \delta W_e^a = \lambda_{ea} \mathbf{i} = 0 \quad (\text{for } e \in K_8) ;$$

$$(3-18-2) \quad \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta M_e^{b+} + \delta N_e^{b-} \quad (\text{for } e \in K_8) ;$$

$$(3-18-3) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = \delta M_e^{b+} + \delta N_e^{b-} \quad (\text{for } e \in K_8) .$$

For event category 9, which is a non-pecuniary event:

$$(3-19) \quad \delta W_e = \delta W_e^b = \lambda_{eb} \mathbf{i} = \delta N_e^{b-} \quad (\text{for } e \in K_9) .$$

In addition to the above relations, we have the following relations from the second column to the last of Table 3, which is the change in the macroeconomic net worth as the result of an economic event. For event categories 1, 2, 3 and 4:

$$(3-20) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = (\lambda_{ea} + \lambda_{eb}) \mathbf{i} = 0 \quad (\text{for } e \in K_1 \cup K_2 \cup K_3 \cup K_4) .$$

For event category 5:

$$(3-21) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = (\lambda_{ea} + \lambda_{eb}) \mathbf{i} = \delta F_e^{a+} + \delta F_e^{b-} \quad (\text{for } e \in K_5) .$$

For event category 6:

$$(3-22) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = (\lambda_{ea} + \lambda_{eb}) \mathbf{i} = \delta L_e^{a+} + \delta F_e^{b-} \quad (\text{for } e \in K_6) .$$

For event category 7:

$$(3-23) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = (\lambda_{ea} + \lambda_{eb}) \mathbf{i} = \delta N_e^{a+} \quad (\text{for } e \in K_7) .$$

For event category 8:

$$(3-24) \quad \delta W_e = \lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i} = (\lambda_{ea} + \lambda_{eb}) \mathbf{i} = \delta N_e^{a+} + \delta N_e^{b-} \quad (\text{for } e \in K_8) .$$

For event category 9:

$$(3-25) \quad \delta W_e = \lambda_{eb} \mathbf{i} = \delta N_e^{b-} \quad (\text{for } e \in K_9) .$$

Also from Table 3, we can calculate the outstanding assets and liabilities on the macroeconomic balance sheet at the end of accounting period τ :

$$(3-26-1) \quad M_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} + \lambda_{eb}) \mathbf{v}_1 = \sum_{e \in \mathbb{T}_\tau \cap K_1} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_\tau \cap K_2} \delta M_e^{a-} ;$$

$$(3-26-2) \quad D_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} + \lambda_{eb}) \mathbf{v}_2 = \sum_{e \in \mathbb{T}_\tau \cap K_1} \delta D_e^{a-} + \sum_{e \in \mathbb{T}_\tau \cap K_2} \delta D_e^{b+} ;$$

$$(3-26-3) \quad F_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} + \lambda_{eb}) \mathbf{v}_3 = \sum_{e \in \mathbb{T}_\tau \cap K_1} \delta F_e^{a+} + \sum_{e \in \mathbb{T}_\tau \cap K_2} \delta F_e^{b-} \\ + \sum_{e \in \mathbb{T}_\tau \cap K_4} \delta F_e^{a+} + \sum_{e \in \mathbb{T}_\tau \cap K_5} (\delta F_e^{a+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap K_6} \delta F_e^{b-} ;$$

$$(3-26-4) \quad L_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} + \lambda_{eb}) \mathbf{v}_4 = \sum_{e \in \mathbb{T}_\tau \cap K_1} \delta L_e^{b-} + \sum_{e \in \mathbb{T}_\tau \cap K_2} \delta L_e^{a+} \\ + \sum_{e \in \mathbb{T}_\tau \cap K_4} \delta L_e^{b-} + \sum_{e \in \mathbb{T}_\tau \cap K_6} \delta L_e^{a+} ;$$

$$(3-26-5) \quad N_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} + \lambda_{eb}) \mathbf{v}_5 = \sum_{e \in \mathbb{T}_\tau \cap K_7} \delta N_e^{a+} \\ + \sum_{e \in \mathbb{T}_\tau \cap K_8} (\delta N_e^{a+} + \delta N_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap K_9} \delta N_e^{b-} .$$

Finally, Rules I, II, III, IV and V give the following relations between economic events that involve same *jus in rem* or *personam*. For financial assets:

$$(3-27-1) \quad -\delta F_{\{F_\varphi \cap H_\eta\}}^{b-} = \delta F_{\{F_\varphi \cap H_{\eta-1}\}}^{a+} = -\delta M_{\{F_\varphi \cap H_{\eta-1}\}}^{a-} = \delta M_{\{F_\varphi \cap H_{\eta-1}\}}^{b+} \quad (\text{for } \eta \geq 2) ;$$

specifically

$$(3-27-2) \quad -\delta F_{\{F_\varphi \cap H_2\}}^{b-} = \delta F_{\{F_\varphi \cap H_1\}}^{a+} = -\delta M_{\{F_\varphi \cap H_1\}}^{a-} \\ = \delta M_{\{F_\varphi \cap H_1\}}^{b+} = -\delta L_{\{F_\varphi \cap H_1\}}^{b-} \quad (\text{for } \eta = 2) .$$

Note that all the subscript sets $\{\}$ in the above and following equations are singletons.

The economic event relating to H_1 undoubtedly belongs to category 4. For non-financial assets:

$$(3-27-3) \quad -\delta N_{\{G_\gamma \cap H_\eta\}}^{b-} = \delta N_{\{G_\gamma \cap H_{\eta-1}\}}^{a+} = -\delta M_{\{G_\gamma \cap H_{\eta-1}\}}^{a-} = \delta M_{\{G_\gamma \cap H_{\eta-1}\}}^{b+} \quad (\text{for } \eta \geq 2) .$$

The above equations mean that the book value of the asset removed from the payee's balance sheet as a result of category 5, 6, 8 or 9 event is the amount of funds that changed hands in the previous event (category 4, 5, 7 or 8) relating to the particular asset. Furthermore according to Rule III,

$$(3-27-4) \quad \delta L_{\{F_\varphi \cap K_6\}}^{a+} = -\delta L_{\{F_\varphi \cap K_4\}}^{b-} = \delta M_{\{F_\varphi \cap K_4\}}^{b+} = -\delta M_{\{F_\varphi \cap K_4\}}^{a-} = \delta F_{\{F_\varphi \cap K_4\}}^{a+} .$$

4. Economic Meanings

4.1 Composition of the Net Worth

In the previous section, we have examined the fundamental properties of historical cost accounting using mathematical expressions. In this section, we will reconsider the above results from the macroeconomic perspectives. Specifically we will examine if the transactions involving the existing assets alter the composition of the macroeconomic

net worth. Some authors including Wicksell (1898; p. 135) and Keynes (1930; pp. 121-126) suggest that the existing asset transactions may change the course of the economy. Furthermore, the above literature of Keynes suggests that the divergence between saving and investment may create economic bubbles. It is interesting because the definition of income in Keynes (1930) is not the conventional one of income as produce, but of income as earnings¹⁸, which is equivalent to monetary income or the amount of received funds¹⁹. We will examine the argument in detail in the next subsection.

Equation (3-6), which describes the net worth of economic entity (or institutional unit) \mathfrak{U} at the end of period τ , can be rewritten as follows using equations (3-13-1), (3-13-2), (3-14-1), (3-14-2), (3-15-1), (3-15-2), (3-16-1), (3-16-2), (3-17-1), (3-17-2), (3-18-1), (3-18-2) and (3-19):

$$\begin{aligned}
(4-1) \quad W_{\mathfrak{U}\tau} &= \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{ea} \mathbf{i} + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau} \lambda_{eb} \mathbf{i} \\
&= \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_3} \delta M_e^{a-} + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_3} \delta M_e^{b+} + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_5} (\delta M_e^{b+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{P}_{a\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_6} (\delta M_e^{a-} + \delta L_e^{a+}) + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_6} (\delta M_e^{b+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_7} \delta M_e^{b+} + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_8} (\delta M_e^{b+} + \delta N_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{P}_{b\mathfrak{U}} \cap \mathbb{T}_\tau \cap \mathbb{K}_9} \delta N_e^{b-} .
\end{aligned}$$

Likewise equation (3-8), which is the macroeconomic net worth, is rewritten as below using equations (3-13-3), (3-14-3), (3-15-3), (3-16-3), (3-17-3), (3-18-3) and (3-19):

$$(4-2) \quad W_\tau = \sum_{e \in \mathbb{T}_\tau} (\lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i})$$

¹⁸ In this paper, we use Lindahl's classification of concept of income: income as consumption, income as interest, income as earnings, and income as produce. For further details, see Lindahl (1919).

¹⁹ For further discussion on saving, investment and bubble, see Haberler (1938).

$$\begin{aligned}
&= \sum_{e \in \mathbb{T}_T \cap K_5} (\delta M_e^{b+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_T \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{T}_T \cap K_7} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_T \cap K_8} (\delta M_e^{b+} + \delta N_e^{b-}) + \sum_{e \in \mathbb{T}_T \cap K_9} \delta N_e^{b-} .
\end{aligned}$$

It should be noted that the first and second terms of the right hand side of equation (4-1) offset against each other in the aggregation process. The above equation is also expressed in the following way because equations (3-15-3), (3-16-3), (3-17-3) , (3-18-3) and (3-19) are equivalent to equations (3-21) through (3-25):

$$\begin{aligned}
(4-3) \quad W_\tau &= \sum_{e \in \mathbb{T}_T} (\lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i}) \\
&= \sum_{e \in \mathbb{T}_T} (\lambda_{ea} + \lambda_{eb}) \mathbf{i} \\
&= \sum_{e \in \mathbb{T}_T \cap K_5} (\delta F_e^{a+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_T \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{T}_T \cap K_7} \delta N_e^{a+} + \sum_{e \in \mathbb{T}_T \cap K_8} (\delta N_e^{a+} + \delta N_e^{b-}) + \sum_{e \in \mathbb{T}_T \cap K_9} \delta N_e^{b-} .
\end{aligned}$$

The terms of the right hand side of equations (4-2) and (4-3) correspond to each other. While the former sees the net worth from the economic entity's viewpoint, the latter sees it from the asset and liability perspective. In the latter view, net worth consists not only of the accumulated investment goods but also of the accumulation of asset value changes.

Now let us examine each term of the right hand side of equation (4-3). If category 4 and 6 events take place consecutively before the end of time period τ , equations (3-27-2) and (3-27-4) give the following relations:

$$\begin{aligned}
(4-4) \quad \sum_{e \in \mathbb{T}_T \cap \mathbb{F}_\varphi \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) &= \delta L_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap K_6\}}^{a+} - \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap K_4\}}^{a+} \\
&= -\delta M_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap K_4\}}^{a-} - \delta M_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap K_4\}}^{b+} = 0 .
\end{aligned}$$

That is to say if a pair of financial asset and liability is created and redeemed before the

end of time period τ , the period-end net worth relating to it is zero. In case if category 4, 5 and 6 events take place consecutively before the end of time period τ , we have the following relations in which $\eta(\tau)$ means the last transaction relating to the particular set of financial asset and liability that took place during time period τ :

$$\begin{aligned}
(4-5) \quad & \sum_{e \in \mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5} (\delta F_e^{a+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
& = \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{a+} + \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{b-} \right) \\
& + \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}}^{a+} + \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}}^{b-} \right) \\
& \quad \vdots \\
& + \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{a+} + \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{b-} \right) \\
& + \left(\delta L_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_6 \cap \mathbb{H}_{\eta(\tau)}\}}^{a+} + \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_6 \cap \mathbb{H}_{\eta(\tau)}\}}^{b-} \right)
\end{aligned}$$

(from equations (3-27-1) and (3-27-2))

$$\begin{aligned}
& = \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{a+} - \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_4 \cap \mathbb{H}_1\}}^{a+} \right) \\
& + \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}}^{a+} - \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{a+} \right) \\
& \quad \vdots \\
& + \left(\delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{a+} - \delta F_{\{\mathbb{T}_\tau \cap \mathbb{F}_\phi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-2}\}}^{a+} \right)
\end{aligned}$$

$$\begin{aligned}
& + \left(\delta L_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_6 \cap \mathbb{H}_{\eta(\tau)}\}}^{a+} - \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{a+} \right) \\
& = \delta L_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_6\}}^{a+} - \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4\}}^{a+}
\end{aligned}$$

(from equations (3-27-2) and (3-27-4))

$$\begin{aligned}
& = - \delta M_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4\}}^{a-} - \delta M_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4\}}^{b+} \\
& = 0 .
\end{aligned}$$

In other words, if a pair of financial asset and liability is created and redeemed before the end of time period τ , the period-end net worth relating to it is zero, no matter how many times it is traded in the existing instrument market. Else if category 4 and 5 events occur consecutively but category 6 event does not take place before the end of time period τ , we have the following relations:

$$\begin{aligned}
(4-6) \quad & \sum_{e \in \mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5} (\delta F_e^{a+} + \delta F_e^{b-}) \\
& = \left(\delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{a+} + \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}}^{b-} \right) \\
& + \left(\delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}}^{a+} + \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}}^{b-} \right) \\
& \quad \vdots \\
& + \left(\delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{a+} + \delta F_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{b-} \right)
\end{aligned}$$

$$+ \left(\delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)}\}} + \delta F^{b-}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)}\}} \right)$$

(from equations (3-27-1) and (3-27-2))

$$\begin{aligned} &= \left(\delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}} - \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4 \cap \mathbb{H}_1\}} \right) \\ &+ \left(\delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_3\}} - \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_2\}} \right) \\ &\quad \vdots \\ &+ \left(\delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}} - \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-2}\}} \right) \\ &+ \left(\delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)}\}} - \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)-1}\}} \right) \\ &= \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)}\}} - \delta F^{a+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4 \cap \mathbb{H}_1\}} \end{aligned}$$

(from equations (3-27-1) and (3-27-2))

$$= \delta M^{b+}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_5 \cap \mathbb{H}_{\eta(\tau)}\}} + \delta M^{a-}_{\{\mathbb{T}_T \cap \mathbb{F}_\varphi \cap \mathbb{K}_4 \cap \mathbb{H}_1\}}$$

$\neq 0$ (except for by accident) .

That is to say if a pair of financial asset and liability is created and traded in the existing instrument market but not redeemed before the end of time period τ , the period-end net worth relating to it is the difference between the issuing value and the acquisition cost of the last owner so that it is non-zero except for by accident.

If only category 7 but not category 8 or 9 events take place before the end of time period τ , we have the following relations:

$$(4-7) \quad \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7} \delta N_e^{a+} = \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7\}}^{a+} \cdot \\ = -\delta M_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7\}}^{a-} \cdot$$

It means that new creation of non-financial asset results in an increase in net worth equivalent to the amount of funds to have paid to obtain it. If category 7 and 9 events relating to the same non-financial asset take place consecutively before the end of time period τ , equation (3-27-3) gives the following relations:

$$(4-8) \quad \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7} \delta N_e^{a+} + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_9} \delta N_e^{b-} \\ = \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7\}}^{a+} + \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_9\}}^{b-} \\ = \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7\}}^{a+} - \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7\}}^{a+} \\ = 0 \cdot$$

In this case, no increment in net worth remains on the balance sheet because the sum of positive and negative entries before the end of the period is equal. If category 7, 8 and 9 events take place consecutively before the end of time period τ , we have the following relations:

$$(4-9) \quad \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7} \delta N_e^{a+} + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_8} (\delta N_e^{a+} + \delta N_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_9} \delta N_e^{b-} \\ = \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_7 \cap \mathbb{H}_1\}}^{a+} \\ + \left(\delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{a+} + \delta N_{\{\mathbb{T}_\tau \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{b-} \right)$$

$$\begin{aligned}
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_3\}} + \delta N^{b-}_{\{T_T \cap G_T \cap K_8 \cap H_3\}} \right) \\
& \quad \vdots \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-2}\}} + \delta N^{b-}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-2}\}} \right) \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-1}\}} + \delta N^{b-}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-1}\}} \right) \\
& + \delta N^{b-}_{\{T_T \cap G_T \cap K_9 \cap H_{\eta(\tau)}\}} \\
& \text{(from equation (3-27-3))} \\
= & \delta N^{a+}_{\{T_T \cap G_T \cap K_7 \cap H_1\}} \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_2\}} - \delta N^{a+}_{\{T_T \cap G_T \cap K_7 \cap H_1\}} \right) \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_3\}} - \delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_2\}} \right) \\
& \quad \vdots \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-2}\}} - \delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-3}\}} \right) \\
& + \left(\delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-1}\}} - \delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-2}\}} \right) \\
& - \delta N^{a+}_{\{T_T \cap G_T \cap K_8 \cap H_{\eta(\tau)-1}\}} \\
= & \mathbf{0} .
\end{aligned}$$

In other words, if a non-financial asset is created and disposed before the end of time

period τ , the period-end net worth relating to it is zero, no matter how many times it is traded in the market. If category 7 and 8 events occur consecutively but a category 9 event does not take place before the end of time period τ , we have the following relations:

$$\begin{aligned}
(4-10) \quad & \sum_{e \in \mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_7} \delta N_e^{a+} + \sum_{e \in \mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8} (\delta N_e^{a+} + \delta N_e^{b-}) \\
&= \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_7 \cap \mathbb{H}_1\}}^{a+} \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{a+} + \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{b-} \right) \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_3\}}^{a+} + \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_3\}}^{b-} \right) \\
&\quad \vdots \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{a+} + \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_{\eta(\tau)-1}\}}^{b-} \right) \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_{\eta(\tau)}\}}^{a+} + \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_{\eta(\tau)}\}}^{b-} \right)
\end{aligned}$$

(from equation (3-27-3))

$$\begin{aligned}
&= \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_7 \cap \mathbb{H}_1\}}^{a+} \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{a+} - \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_7 \cap \mathbb{H}_1\}}^{a+} \right) \\
&+ \left(\delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_3\}}^{a+} - \delta N_{\{\mathbb{T}_T \cap \mathbb{G}_\gamma \cap \mathbb{K}_8 \cap \mathbb{H}_2\}}^{a+} \right)
\end{aligned}$$

$$\begin{aligned}
& \vdots \\
& + \left(\delta N^{a+}_{\left\{T_T \cap G_\gamma \cap K_8 \cap H_{\eta(\tau)-1}\right\}} - \delta N^{a+}_{\left\{T_T \cap G_\gamma \cap K_8 \cap H_{\eta(\tau)-2}\right\}} \right) \\
& + \left(\delta N^{a+}_{\left\{T_T \cap G_\gamma \cap K_8 \cap H_{\eta(\tau)}\right\}} - \delta N^{a+}_{\left\{T_T \cap G_\gamma \cap K_8 \cap H_{\eta(\tau)-1}\right\}} \right) \\
& = \delta N^{a+}_{\left\{T_T \cap G_\gamma \cap K_8 \cap H_{\eta(\tau)}\right\}} \\
& \neq 0 .
\end{aligned}$$

That is to say, if a non-financial asset is created and traded in the market but not demolished before the end of time period τ , the period-end net worth relating to it is the acquisition cost of the last owner so that it is always non-zero.

Therefore, from equations (4-4) through (4-10), equation (4-3) can be rewritten as below:

$$\begin{aligned}
(4-11) \quad W_\tau &= \sum_{g \in \mathbb{V}_{g7}} \delta N^{a+}_{\left\{T_T \cap G_g \cap K_7\right\}} \\
& + \sum_{g \in \mathbb{V}_{g8}} \left(\delta N^{a+}_{\left\{T_T \cap G_g \cap K_8 \cap H_{\eta(\tau)}\right\}} - \delta N^{a+}_{\left\{T_T \cap G_g \cap K_7 \cap H_1\right\}} \right) \\
& + \sum_{f \in \mathbb{V}_f} \left(\delta F^{a+}_{\left\{T_T \cap F_f \cap K_5 \cap H_{\eta(\tau)}\right\}} - \delta F^{a+}_{\left\{T_T \cap F_f \cap K_4 \cap H_1\right\}} \right) \\
& = \sum_{g \in \mathbb{V}_{g7}} \delta N^{a+}_{\left\{T_T \cap G_g \cap K_7\right\}}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{g \in V_{g8}} \left(\delta N_{\{T_t \cap G_g \cap K_8 \cap H_{\eta(t)}\}}^{a+} - \delta N_{\{T_t \cap G_g \cap K_7 \cap H_1\}}^{a+} \right) \\
& + \sum_{f \in V_f} \left(\delta F_{\{T_t \cap F_f \cap K_5 \cap H_{\eta(t)}\}}^{a+} + \delta L_{\{T_t \cap F_f \cap K_4 \cap H_1\}}^{b-} \right),
\end{aligned}$$

where $V_f = \{f \mid T_t \cap F_f \cap K_5 \cap H_{\eta(t)} \neq \emptyset\}$, $V_{g7} = \{g \mid T_t \cap G_g \cap K_9 \cap H_{\eta(t)} = \emptyset\}$ and $V_{g8} = \{g \mid T_t \cap G_g \cap K_8 \cap H_{\eta(t)} \neq \emptyset\}$. That is, if we assume liabilities are never bought or sold, the macroeconomic net worth is the sum of the original acquisition value (i.e. installment cost) of the existing non-financial assets, the difference between the book value of existing non-financial assets and their original value, and the difference between the book value of existing financial assets and corresponding liabilities. Note that, unlike equation (4-3), the redeemed financial assets and disposed non-financial assets do not appear in equation (4-11) because they are already removed from the balance sheet.

4.2 Macroeconomic Savings

In modern economics, income is defined as produce after Keynes' *General Theory* (1936; p. 53). However, in his *Treatise on Money* (1930; p.121), Keynes was using an alternative definition: money income or income as earnings. One of the advantages of the latter definition is that it is free from the production boundary problems. Along with the above discussions, macroeconomic income as earnings can be defined in the following manner:

$$(4-12) \quad Y_t^{gross} = \sum_{e \in T_t \cap K_3} \delta M_e^{b+} + \sum_{e \in T_t \cap K_7} \delta M_e^{b+}$$

$$\begin{aligned}
& + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_5} (\delta M_e^{b+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
& + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_8} (\delta M_e^{b+} + \delta N_e^{b-}) .
\end{aligned}$$

Category 9 events are deliberately omitted in the above equation because income as earnings is usually defined in gross terms rather than in net terms. Roughly speaking, the right hand side of equation (4-12) is itemized as follows:

- [1] Revenue from the sale of consumption goods and services; Transfer receipts;
- [2] Revenue from the sale of investment goods;
- [3] Capital gain/loss from the sale of financial assets;
- [4] Capital gain/loss from the redemption of financial assets;
- [5] Capital gain/loss from the sale of non-financial assets.

As Lindahl (1919) asserts, income as earnings is calculated as the sum of consumption and the increase of capital value which has taken place during a period²⁰. For now, let us simply take

$$(4-13) \quad C_\tau = \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_3} \delta M_e^{a-} = \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_3} \delta M_e^{b+}$$

as macroeconomic consumption. In such a case, the macroeconomic gross saving can be written as follows:

$$\begin{aligned}
(4-14) \quad S_\tau^{gross} &= Y_\tau^{gross} - C_\tau \\
&= \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_7} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_5} (\delta M_e^{b+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_6} (\delta L_e^{a+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_\tau \cap \mathbb{K}_8} (\delta M_e^{b+} + \delta N_e^{b-}) .
\end{aligned}$$

We will call it gross saving out of earnings just for convenience.

Technically speaking, it is also possible to rewrite (4-12) and (4-14) in net terms,

²⁰ For detailed discussion on the definition of income, see Fisher (1937), Blough and Hewett (1938), etc.

which include category 9 events as well:

$$\begin{aligned}
(4-15) \quad Y_{\tau}^{net} &= \sum_{e \in \mathbb{T}_{\tau} \cap K_3} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta M_e^{b+} \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_5} (\delta M_e^{b+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_{\tau} \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_8} (\delta M_e^{b+} + \delta N_e^{b-}) + \sum_{e \in \mathbb{T}_{\tau} \cap K_9} \delta N_e^{b-} ;
\end{aligned}$$

$$\begin{aligned}
(4-16) \quad S_{\tau}^{net} &= Y_{\tau}^{net} - C_{\tau} \\
&= \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_{\tau} \cap K_5} (\delta M_e^{b+} + \delta F_e^{b-}) \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_{\tau} \cap K_8} (\delta M_e^{b+} + \delta N_e^{b-}) \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_9} \delta N_e^{b-} .
\end{aligned}$$

Likewise equation (3-12), which is the increase in macroeconomic net worth during time period τ , is rewritten as below using equations (3-13-3), (3-14-3), (3-15-3), (3-16-3), (3-17-3), (3-18-3) and (3-19):

$$\begin{aligned}
(4-17) \quad \Delta W_{\tau} &= \sum_{e \in \mathbb{T}_{\tau}} (\lambda_{ea} \mathbf{i} + \lambda_{eb} \mathbf{i}) \\
&= \sum_{e \in \mathbb{T}_{\tau} \cap K_5} (\delta M_e^{b+} + \delta F_e^{b-}) + \sum_{e \in \mathbb{T}_{\tau} \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta M_e^{b+} + \sum_{e \in \mathbb{T}_{\tau} \cap K_8} (\delta M_e^{b+} + \delta N_e^{b-}) \\
&+ \sum_{e \in \mathbb{T}_{\tau} \cap K_9} \delta N_e^{b-} .
\end{aligned}$$

It means $S_{\tau}^{net} = \Delta W_{\tau}$ so that net saving is equivalent to the changes in net worth.

On the other hand, macroeconomic gross and net investment can be written as follows:

$$(4-18) \quad I_{\tau}^{gross} = \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta N_e^{a+} = - \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta M_e^{a-} = \sum_{e \in \mathbb{T}_{\tau} \cap K_7} \delta M_e^{b+} ;$$

$$\begin{aligned}
(4-19) \quad I_{\tau}^{net} &= \sum_{e \in T_{\tau} \cap K_7} \delta N_e^{a+} + \sum_{e \in T_{\tau} \cap K_9} \delta N_e^{b-} \\
&= - \sum_{e \in T_{\tau} \cap K_7} \delta M_e^{a-} + \sum_{e \in T_{\tau} \cap K_9} \delta N_e^{b-} \\
&= \sum_{e \in T_{\tau} \cap K_7} \delta M_e^{b+} + \sum_{e \in T_{\tau} \cap K_9} \delta N_e^{b-}
\end{aligned}$$

In either case, it is apparent that the identity between macroeconomic saving and investment does not hold. Actually, the difference between savings out of earnings and investments is as follows in either case:

$$\begin{aligned}
(4-20) \quad S_{\tau} - I_{\tau} &= \sum_{e \in T_{\tau} \cap K_5} (\delta M_e^{b+} + \delta F_e^{b-}) + \sum_{e \in T_{\tau} \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in T_{\tau} \cap K_8} (\delta M_e^{b+} + \delta N_e^{b-}) .
\end{aligned}$$

We can rewrite the above equation as follows because equations (3-21) is equivalent to (3-15-3); (3-22) is to (3-16-3); and (3-24) is to (3-18-3) respectively.

$$\begin{aligned}
(4-21) \quad S_{\tau} - I_{\tau} &= \sum_{e \in T_{\tau} \cap K_5} (\delta F_e^{a+} + \delta F_e^{b-}) + \sum_{e \in T_{\tau} \cap K_6} (\delta L_e^{a+} + \delta F_e^{b-}) \\
&\quad + \sum_{e \in T_{\tau} \cap K_8} (\delta N_e^{a+} + \delta N_e^{b-}) .
\end{aligned}$$

Therefore, the difference between savings out of earnings and investments is equivalent to the sum of realized capital and redemption gain in financial as well as non-financial assets.

5. Flow of Funds Table as a National Accounting Device

In historical cost accounting, the book value of an item in the balance sheet is the amount of funds that have changed hands in the last onerous transaction involving that item. In other words, the changes in the balance sheets of the parties involved in a pecuniary event have a one-to-one relationship with the flow of funds from the payer to the payee. Therefore, the transfer of funds or payer-payee matrix that records all the

transference of funds as described in Subsection 3.2 can be a useful device for national accounting. Since, in national accounting, all the economic entities (institutional units) are aggregated into economic (institutional) sectors, the detailed depiction of transmission of funds within the financial sector is not necessary for most purposes.

Taking the classification of economic events into consideration, there must be several basic flow of funds tables. The prototype of the tables is shown in the appendix. Figure A-1 is the flow of funds table for economic event category 3 in which no non-monetary assets change hands. Final as well as intermediate consumption, exports and imports, salary and wages, interest and dividend, tax and subsidiary, and other current payments inclusive of transfers are recorded here in the sector \times sector payer-payee matrix. For example, household consumption expenditure is recorded at the intersection of the row of households and column of non-financial corporations. Intermediate consumptions are mainly recorded at the intersection of the row and column of non-financial corporations. If we divide the sector into industries it looks like transposed input-output table. Since no non-monetary assets change hands in the category 3 events, the receipt and payment of funds (i.e. increase or decrease of monetary asset) are directly reflected in the changes in the net worth of the sectors.

The flow of funds table depicted in Figure A-2 is that for economic event category 7, which relates to creation of non-financial assets. Frankly speaking, all the capital formation is recorded in this table. For example, purchase of a newly built house is recorded at the intersection of the row of households and column of non-financial corporations. Likewise, public investment in the infrastructure such as highway or irrigation system is written in the cell on the row of general government and the column of non-financial corporations. For the payer of funds, an increment of the non-financial

asset is recorded in the last column. The change in the net worth is the sum of the net increase in monetary and non-financial assets.

Figure A-3 is the flow of funds table for economic event category 8 that corresponds to the transactions involving existing non-financial assets including both produced and non-produced assets such as land, natural resources, etc. For example, secondary market transactions of private residential property are recorded at the crosspoint of household's row and column. However, if inventory is sold to others it is also recorded in this table. As in the previous table, an increment of the non-financial asset is recorded in the rightmost cell of the payer's row. In this table, a decrement of non-financial asset is also recorded in the third row from the bottom. While, in the payer's case, the increase in the non-financial asset is always equals to the decrease in the monetary asset, in the payee's case, the decrease in the non-financial asset is not necessarily equals to the increase in the monetary asset. The change in the net worth is the sum of the net increase in monetary and non-financial assets.

Since, Figures A-1 through A-3 are all relating to non-financial transactions, we can consolidate them into one table. The flow of funds table depicted in Figure A-4 includes not only category 3, 7 and 8 economic events but also category 9 events that do not involve payment of funds. With this table, we can overview the creation, disposal, and secondary market transaction of the non-financial assets and their effects on the net worth. Not only that, we can grasp all the payments of non-financial nature at a glance. When the cells of the payer-payee matrix are filled with greater numbers, the economy is booming. If the numbers are getting smaller, we are facing recession. We can even tell which part of the economy is malfunctioning.

While Figures A-1 through A-4 portray the non-financial events, Figures A-5

through A-8 exhibit economic events of financial nature. Figure A-5 is the flow of funds table for economic event category 1 and 4 that are relating to creation of financial assets. Since bank lending is classified as category 1 rather than 4, it is recorded in the row of creation of funds in the payer's section of the table. All other lending is classified as category 4 and recorded in the payer-payee matrix of the table. For example, if an individual purchases Treasury bond, the transaction is recorded at the intersection of the row of households and the column of general government. However, if the bond is purchased by a bank, it is recorded in the row of creation of funds at the intersection with the column of general government. The payer-payee matrix that is marked as **Z₁** represents the creation of new lender-borrower relationship²¹. While an increase in the financial asset is recorded in the last column for the payers; that in liability is recorded in the row marked as category 4. A category 1 economic event creates financial asset as well as monetary liability for the depository financial corporation, and financial liability for the sector to which the customer belongs. The change in the net worth is the sum of net acquisition of monetary/financial assets and incurrence of the corresponding liabilities. As we have already mentioned, category 1 and 4 events cause no changes in the net worth of either lending or borrowing sectors.

Figure A-6 is the flow of funds table for economic event category 5 that corresponds to the transactions involving existing financial assets. For example, secondary market transactions of Treasury securities are recorded at the intersection of the seller's row and the buyer's column. Since category 5 events cause changes to the existing lender-borrower relations, they are recorded in the matrices marked as **Z₂** and **Z₃**. The increments of financial assets for payers are recorded by issuers of the financial

²¹ In matrix **Z₁**, the rows of depository and non-depository financial institutions are aggregated into one row that represents all the financial institutions as in the other tables.

asset in Z_2 . The decrements of financial assets for payees are also recorded by issuers of the financial asset in Z_3 . The two matrices are more or less symmetric because a sale of an instrument is recorded for payer (purchaser) as well as for payee (seller). While, in the payer's case, the increase in the financial asset is always equals to the decrease in the monetary asset, in the payee's case, the decrease in the financial asset is not necessarily equals to the increase in the monetary asset. The change in the net worth is the sum of the net increase in monetary and financial assets, which is equivalent to the accrued capital gains or losses.

The flow of funds table depicted in Figure A-7 is that for economic event categories 2 and 6, which relate to the redemption of financial assets. As in the case of creation of financial assets, repayment of bank loan is classified as category 2 rather than 6, it is recorded in the column of repayment of funds in the payee's section of the table. All other redemptions are classified as category 6 and recorded in the payer-payee matrix of the table. For example, if a Treasury security is redeemed to an individual, it is recorded at the intersection of the row of general government and the column of households. However, if it is redeemed to a bank, it is recorded in the column of repayment of funds at the cell of general government. The payer-payee matrix that is marked as Z_4 represents the termination of lender-borrower relationship²². It should be noted that, as far as lender-borrower relationship are concerned, matrices Z_1 and Z_4 are diagonal symmetric. Redemption of a financial asset eliminates both payer's liability and payee's asset simultaneously; they are recorded outside of the payer-payee matrix as in the previous cases. The change in the net worth can be non-zero because of redemption gains or losses.

²² In matrix Z_4 , the columns of depository and non-depository financial institutions are aggregated into one column that represents all the financial institutions.

Figures A-5 through A-7 are integrated into Figure A-8 that is referred to as financial flow of funds table. This table covers all the financial transactions, creation, redemption as well as secondary market transactions of financial assets. This table will help us to answer questions like “who is responsible for the creation of funds?” or “who is the winner in the secondary market transactions?”. To answer the first question, we should refer to the column of creation of funds, which is just below the payer-payee matrix. To answer the second, we should refer to the very last row at the bottom because all the changes in net worth in this table relate to capital (redemption) gains or losses.

An integration of Figures A-4 and A-8 produce Figure A-9 so that it is referred to as integrated flow of funds table. This table is useful because we can overview all the transactions between sectors in terms of flow of funds. As a result, some sectors have net increase in monetary asset while others have net decreases in it. The row of ΔM in this table shows the winners and losers in this respect. This table also shows the gross increase and decrease as well as the net increase in all classes of asset and liability. It also allows us to browse the changes in net worth sector by sector. Of course we can make period-end balance sheet of each sector using these figures in addition to the balance sheet of previous period. We can also make asset-liability matrix using matrices appeared in the previous tables. Since matrix $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 + (\mathbf{Z}_3)' - (\mathbf{Z}_4)'$ is the increment matrix, we can construct the asset-liability matrix just by summing up. Asset-liability matrix is a sector \times sector matrix so that it is an ideal tool to browse the lender-borrower relationship. However, we should note two points. First of all, this matrix does not include monetary asset-liability relationship — bank deposits are counted neither as the asset of the customer nor the liability of the financial institution. Second of all, this matrix shows the current book value of the creditor rather than that of

the debtor. To know who owes how much to whom, we should rewrite it with the debtor's book value.

6. Concluding Remarks

In Section 4, we examined the meaning of economic bubbles in the historical-cost national accounting perspective. As shown in equation (4-11), the macroeconomic net worth is the sum of (a) the original acquisition value of the existing non-financial assets (i.e. their value at the date when it appeared on the balance sheet for the first time); (b) the difference between the current book value and the original acquisition value of the existing non-financial assets; and (c) the difference between the current book value of the existing financial assets and the corresponding liabilities. It is apparent that the sum of (a) and (b) are the current book value of the existing non-financial assets so that, in an economy where financial assets and liabilities are never bought or sold, the macroeconomic net worth is equivalent to the current book value of the existing non-financial assets. However, in an economy where financial assets are subject to sale, the macroeconomic net worth also includes (c) the difference between the current book value of existing financial assets and the corresponding liabilities. It is noteworthy that both (b) and (c) are the direct result of the changes in asset valuation. In this context, an economic bubble is defined as a situation in which the growth rate of (b) and (c) is unusually high. If (b) is the dominant factor, it is a commodity or real estate bubble; if (c) is the foremost factor, it is a financial bubble. The advantage of this definition of bubble is that it is readily measurable in the framework of the national accounting. We should remember that the conventional definition of bubble, which is the divergence of

the market price of an asset from its fundamental price²³, is not easily applicable to the macro economy when measurement is the final goal.

According to equations (4-16) and (4-17), the increment of macroeconomic net worth is equivalent to the net saving out of macroeconomic monetary income or income as earnings as Lindahl (1919) put it. The macroeconomic net saving consists of three parts, (i) acquisition and disposal of non-financial assets; (ii) capital gains or losses in the non-financial assets; and (iii) capital (and redemption) gains or losses in the financial assets. It is obvious that components (i), (ii) and (iii) correspond to (a), (b) and (c) respectively. Since (i) means net investment, we can conclude that the disparity between macroeconomic saving and investment, i.e. (ii) and (iii), creates economic bubbles.

In historical cost accounting, the book value of an item in the balance sheet is the amount of funds that have changed hands in the last onerous transaction involving that item. In other words, the changes in the balance sheets of the parties involved in a pecuniary event have a one-to-one relation with flow of funds from the payer to the payee. Therefore, the transfer-of-funds or payer-payee matrix that records all the transference of funds between the economic entities can be a useful device for national accounting as we have described in Section 5. The funds in the macro economy is comparable to the circulatory system in the human body. In order to stay healthy it is very important to have good circulation. Just as physicians detect diseases by examining the circulatory system, we can diagnose the economy by observing the flow of funds. By comparing the current payer-payee matrix with that of the past, we must be able to tell which part of the system is malfunctioning. The detailed information will certainly

²³ Flood and Garber (1980).

help the analysts thoroughly examine the whole economy; however an integrated flow of funds table such as that shown in Figure A-9 will be enough for the policy makers to have an overview. In an age of electronic banking, most transactions are settled through computer based funds transfer system of one sort or another so that it is not too difficult to collect from-whom-to-whom payment data; the only thing we have to do is earmark the bank customers to identify which sector they belong to. Once they are earmarked, we can automatically collect the real-time data without any additional cost. Although payer-payee matrix is no substitute for the product-flow based SNA, it is a handy and convenient tool for the policy makers and the private analysts alike.

References

- Aukrust, Odd (1949) "On the Theory of Social Accounting," *Review of Economic Studies*, 16(3), 170-188.
- Austin, John (1879) *Lectures on Jurisprudence, or The Philosophy of Positive Law*, two volumes, 4th ed., Robert Campbell (ed.), London: John Murray.
- Bank for International Settlements (2010) *Handbook on Securities Statistics, Part 2, Debt Securities Holdings*.
- Blough, Roy and W. W. Hewett (1938) "Capital Gains in Income Theory and Taxation Policy," *Studies in Income and Wealth*, vol.2, 191-239.
- Copeland, Morris A. (1947) "Tracing Money Flows through the United States Economy," *American Economic Review*, 37(2), pp. 31-49.
- Copeland, Morris A. (1949) "Social Accounting for Moneyflows," *Accounting Review*, 24(3), pp. 254-264.
- Copeland, Morris A. (1952) *A Study of Moneyflows in the United States*, NBER.

- Dawson, John C. (1991) "Copeland on Social Accounting," prepared for the annual meeting of American Statistical Association. Reprinted as chap.5 of Dawson ed. (1996).
- Dawson, John C. ed. (1996) *Flow-of-Funds Analysis: A Handbook for Practitioners*, Armonk: M.E. Sharpe.
- Dohr, James Lewis (1928) "A Legal Analysis of the Balance Sheet," *The Accounting Review*, 3(2), pp. 117-123.
- European Commission, International Monetary Fund, Organisation for Economic Co-operation and Development, United Nations and World Bank (2009) *System of National Accounts 2008*.
- Financial Stability Board and International Monetary Fund (2009) *The Financial Crisis and Information Gaps, Report to the G-20 Finance Ministers and Central Bank Governors*.
- Financial Stability Board and International Monetary Fund (2011) *The Financial Crisis and Information Gaps, Implementation Progress Report*.
- Fisher, Irving (1906) *The Nature of Capital and Income*, New York: Macmillan.
- Fisher, Irving (1928) "The Income Concept in the Light of Experience," Reprinted in English from the German original (1927) *Die Wirtschaftstheorie der Gegenwart*, Vienna: Wieser Festschrift, vol. III.
- Fisher, Irving (1937) "Income in Theory and Income Taxation in Practice," *Econometrica*, 5(1), pp. 1-55.
- Flood, Robert P. and Peter M. Garber (1980) "Market Fundamentals versus Price-Level Bubbles: The First Test," *Journal of Political Economy*, 88(4), pp. 745-770.
- Frisch, Ragnar (1943) "Økosirk-Systemet: det Økonomiske Sirkulasjonssystem," *Økonomisk Tidsskrift*, 45, pp. 106-121.
- Frisch, Ragnar (1964) "A Generalized Form of the REFI Interflow Table," in Kowalik, T. ed. (1964) *Problems of economic dynamics and planning: Essays in Honour of Michał Kalecki*, Warszawa: Państwowe Wydawnictwo Naukowe, pp. 133-156.
- Gaathon (Gruenbaum), Ludwig A. (1953) "A Real-Financial System of Social Accounting," *Review of Economics and Statistics*, 35(1), pp. 1-19.
- Haberler, Gottfried (1938) "National Income, Saving, and Investment," in Conference

- on Research in National Income and Wealth (1938) *Studies in Income and Wealth*, Volume 2, NBER, pp. 139-166.
- Keynes, John Maynard (1930) *A Treatise on Money*, two volumes, Reprinted in 1971, London: Macmillan.
- Keynes, John Maynard (1936) *The General Theory of Employment, Interest and Money*, Reprinted in 1973, London: Mcmillan.
- Klein, Laurence R. (1977) "Building Economic Models that Work", *Society*, 14(3), pp. 30-34.
- Klein, Lawrence R. (1983) *Lectures in Econometrics*, Amsterdam: North-Holland.
- Leontief, Wassily W. (1941) *The Structure of American Economy, 1919-1939: An Empirical Application of Equilibrium Analysis*, Cambridge, Mass.: Harvard University Press.
- Lindahl, Erik R. (1919) "The Concept of Income," in *Economic Essays in Honour of Gustav Cassel*, reprinted in 1967, New York: Augustus M. Kelley, pp. 399-407.
- Nicholas, Barry (1962) *An Introduction to Roman Law*, Oxford: Clarendon Press.
- Rutherford, Malcolm (2002) "Morris A. Copeland: A Case Study in the History of Institutional Economics," *Journal of the History of Economic Thought*, 24(3), pp. 261-290.
- Shrestha, Manik, Reimund Mink and Segismundo Fassler (2012) "An Integrated Framework for Financial Positions and Flows on a From-Whom-to-Whom Basis: Concepts, Status, and Prospects," IMF Working Paper WP/12/57.
- Stone, Richard (1947) "Definition and Measurement of the National Income and Related Totals," in United Nations (1947) *Measurement of National Income and the Construction of Social Accounts*.
- Stone, Richard (1966) "The Social Accounts from a Consumer's Point of View," *Review of Income and Wealth*, 12(1), pp. 1-33.
- Tsujimura, Kazusuke and Masako Mizoshita (2003) "Asset-Liability-Matrix Analysis Derived from Flow-of-Funds Accounts: the Bank of Japan's Quantitative Monetary Policy Examined," *Economic Systems Research*, 15(1), pp. 51-67.
- Tsujimura, Kazusuke and Masako Tsujimura (2010a) "A Flow-of-Funds Analysis of Quantitative Monetary Policy," in Shinichi Ichimura and Laurence R. Klein

- (eds.) *Macroeconometric Analysis of Japan*, London: World Scientific, ch. 7, pp. 173-193.
- Tsujimura, Kazusuke and Masako Tsujimura (2010b) “Copeland’s Money-Flows Accounts and the Presentation Format of National Accounts,” presented at the International Association for Research in Income and Wealth, 31st General Conference, St-Gallen, Switzerland, 22-28 August, 2010.
- Tsujimura, Masako and Kazusuke Tsujimura (2011) “Balance Sheet Economics of the Subprime Mortgage Crisis,” *Economic Systems Research*, 23(1), pp. 1-25.
- United Nations Statistical Office (1968) *A System of National Accounts*, Studies in Methods, series F, no.2, rev.3, New York: United Nations.
- Vanoli, André (2002) *Une Histoire de la Comptabilité Nationale*, Paris: La Découverte. Translated by Marion Pinot Libreros and Gayle H. Partmann in 2005 as *A History of National Accounting*, Amsterdam: IOS Press.
- Wicksell, Knut (1898) *Geldzins und Güterpreise*, Jena: Gustav Fischer, Translated by Richard F. Kahn in 1936 as *Interest and Prices*, Reprinted in 1965, New York: Augustus M. Kelley.

APPENDIX

Figure A-1. Flow of Funds Table for Economic Event Category 3

		Payee						ΔM^-
		Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	Total Payment of Funds
Payer	Financial Institutions							
	Non-financial Corporations							
	Households							
	Nonprofit Institutions							
	General Government							
	ROW							
ΔM^+	Total Receipt of Funds							
$\Delta M = \Delta M^+ + \Delta M^-$								
$\Delta W = \Delta M$								

Figure A-2. Flow of Funds Table for Economic Event Category 7

		Payee						ΔM^-	ΔN^+
		Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	Total Payment of Funds	Category 7
Payer	Financial Institutions								
	Non-financial Corporations								
	Households								
	Nonprofit Institutions								
	General Government								
	ROW								
ΔM^+	Total Receipt of Funds								
$\Delta M = \Delta M^+ + \Delta M^-$									
$\Delta N = \Delta N^+$									
$\Delta W = \Delta M + \Delta N$									

Figure A-3. Flow of Funds Table for Economic Event Category 8

		Payee						ΔM^-	ΔN^+
		Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	Total Payment of Funds	Category 8
Payer	Financial Institutions								
	Non-financial Corporations								
	Households								
	Nonprofit Institutions								
	General Government								
	ROW								
ΔM^+	Total Receipt of Funds								
$\Delta M = \Delta M^+ + \Delta M^-$									
ΔN^-	Category 8								
$\Delta N = \Delta N^+ + \Delta N^-$									
$\Delta W = \Delta M + \Delta N$									

Figure A-5. Flow of Funds Table for Economic Event Categories 1 and 4

		Payee						ΔM^-	ΔF^+
		Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	Total Payment of Funds	Category 4
Creation of Funds	Category 1 *								
Payer	Financial Institutions **			Z₁					
	Non-financial Corporations								
	Households								
	Nonprofit Institutions								
	General Government								
	ROW								
ΔM^+	Total Receipt of Funds								
$\Delta M = \Delta M^+ + \Delta M^-$									
ΔL^-	Category 1								
	Category 4								
ΔF^+	Category 1								
ΔD^-	Category 1								
$\Delta F = \Delta F^+$									
$\Delta L = \Delta L^-$									
$\Delta D = \Delta D^-$									
$\Delta W = \Delta M + \Delta F + \Delta L + \Delta D$		0	0	0	0	0	0	0	

* Depository Financial Institutions

** Non-depository Financial Institutions

Note: In matrix Z1, the rows of depository and non-depository financial institutions are aggregated into one row that represents all the financial institutions as in the other tables.

Figure A-6. Flow of Funds Table for Economic Event Category 5

		Payee						ΔM^-	ΔF^+						
									Category 5						
		(Issuers of Asset)							Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	
		Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	Total Payment of Funds	Financial Institutions	Non-financial Corporations	Households	Nonprofit Institutions	General Government	ROW	
Payer	Financial Institutions														
	Non-financial Corporations														
	Households														
	Nonprofit Institutions														
	General Government														
	ROW														
ΔM^+	Total Receipt of Funds														
$\Delta M = \Delta M^+ + \Delta M^-$															
ΔF^-	Category 5 (Issuers of Asset)	Financial Institutions													
		Non-financial Corporations													
		Households													
		Nonprofit Institutions													
		General Government													
		ROW													
$\Delta F = \Delta F^+ + \Delta F^-$															
$\Delta W = \Delta M + \Delta F$															

Z_2

Z_3

Figure A-7. Flow of Funds Table for Economic Event Categories 2 and 6

		Repayment of Funds	Payee						ΔM^-	ΔL^+		ΔF^-	ΔD^+
			Category 2 *	Financial Institutions **	Non-financial Corporations	Households	Nonprofit Institutions	General Government		ROW	Total Payment of Funds		
Payer	Financial Institutions												
	Non-financial Corporations												
	Households												
	Nonprofit Institutions												
	General Government												
	ROW												
ΔM^+	Total Receipt of Funds												
$\Delta M = \Delta M^+ + \Delta M^-$													
ΔF^-	Category 6												
$\Delta F = \Delta F^-$													
$\Delta L = \Delta L^+$													
$\Delta D = \Delta D^+$													
$\Delta W = \Delta M + \Delta F + \Delta L + \Delta D$													

Z₄

* Depository Financial Institutions

** Non-depository Financial Institutions

Note: In matrix Z₄, the columns of depository and non-depository financial institutions are aggregated into one column that represents all the financial institutions.

