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Abstract

A crucial feature of multidimensional poverty lies in the interactions that exist across the dimensions of wellbeing. This has been recognized in the recent literature on multidimensional poverty measurement, which, in addition to the levels of dimensional poverty, has stressed that multidimensional poverty should also reflect the joint distribution of the dimensions. To our knowledge, the literature has failed, however, in following up on this important insight to address the important issue of designing policy to optimally reduce multidimensional poverty. From an empirical perspective, the different dimensions of poverty are often interconnected and can mutually reinforce each other, especially in cases of severe poverty and deprivation over a relatively long period. From a normative perspective, it is reasonable to argue that a greater correlation of deprivation increases, *ceteris paribus*, multidimensional poverty. From a policy perspective, improving one dimension of well-being produces a triple effect on a person's multidimensional poverty: a direct effect on the targeted dimension, an effect on the joint deprivation, and an indirect and spill-over effect on the other dimensions. For these reasons, optimal multidimensional-poverty policy can then differ from optimal unidimensional-poverty reducing policy. This paper assesses formally the optimal design of targeting under multidimensional approach; the theoretical results are then applied to data from Vietnam (1992-1993 and 1997-1998) and South Africa (1993).

Keywords: Targeting; Multidimensional poverty; Optimal policy; Vietnam; South Africa.

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1 Introduction

It is increasingly well understood that, to assess poverty in a multidimensional framework, it is important to take into account the interactions and the correlation between the relevant dimensions. This matters both for identifying the multidimensional poor (see for instance Alkire 2010) and for measuring the magnitude of their poverty. The interactions across the dimensions can occur in various ways, and can feed each other in the short or in the longer term. For instance, one person's well-being attributes might be jointly determined with the attributes of his household or community (such as when a child's chances of survival depend on parental and community characteristics), external attributes that may also affect the individual's well-being. In many situations, the attributes are positively correlated (*i.e.*, a deterioration in income worsens nutrition, and worse nutrition diminishes household productivity); however, negative correlations also occur, as when a fall in child labor decreases household income, at least in the short term.

The interconnections between dimensions of well-being can be especially strong in cases of severe deprivation over a long period, where the poor can be victims of vicious circles (Makdissi and Wodon (2004)). Formulating policy taking into account these interactions can be particularly important; in such instances, even transitory shocks can affect permanently the future level of well-being by generating "continuing multi-dimensional poverty traps" (Thorbecke 2005). In the absence of appropriate policy, it may well be that the prevalence of multiple forms of deprivation and greater correlations across dimensions of well-being will increase over time, at the cost of greater multidimensional poverty.

The importance of properly taking into account multidimensional poverty objectives when designing policy has been recognized both in the policy arena and in the scientific literature. This is implicit for instance in the well-known Latin American conditional cash transfers (CCT) programs. Cash transfers are mostly given to families with poor children with the joint objective of improving both income and other dimensions of well-being, such as education and health. Although the rationale for this is not always made perfectly clear and explicit, the usual motivation is to design policy in such a way as to take advantage of the spill-over effects across dimensions, effects that take place both in the short and in the longer term, as well as to target more specifically those that are deprived in more than one dimension of well-being. Conditionality rules are imposed to leverage as much as possible the cross-dimension effects of the cash transfers.

It is important to note that means and objectives are sometimes confused. For instance, although the ultimate policy objective is often to reduce uni-dimensional monetary poverty, it is regularly the case that multiple means and proxies are used to reduce that uni-dimensional indicator of deprivation. Although the policy tools may be multiple, the ultimate objective in such cases is still the reduction of uni-dimensional poverty. In many Latin American countries, the CCT targeting tools are based on multidimensional poverty indices, with the primary objective of finding proxies for permanent income, without truly being interested in dimensional deprivations other than income or consumption poverty. As pointed out by Azevedo and Robles (2010), this implementation strategy runs the risk of leading to a sub-optimal fall in multidimensional poverty.[1]

Using multidimensional poverty indices to design policies to reduce uni-dimensional poverty can also be non-optimal. This is a point clearly made by Ravallion (2011), who argues for instance that, to reduce income poverty, it is better to target the income poor, and that to reduce deprivation in access to public services, it is analogous better to target those that are deprived of such services. Using a poverty index that mixes up the two dimensions and produces a multidimensional index of poverty (MIP), would lead to a sub-optimal reduction of unidimensional income and public services poverty:

> "The total impact on (multidimensional) poverty would be lower if one based the allocation on the MIP [multidimensional index of poverty] rather than the separate poverty measures — one for incomes and one for access to services. It is not the aggregate index that we need for this purpose but its components." (Ravallion 2011, (p. 240))

Unlike Ravallion (2011), we take it as given in the context of this paper that we are interested in reducing multidimensional poverty. It has been well known for some time, however, that the optimal policy rules to reduce poverty are not necessarily based on the poverty indices themselves — see for instance Kanbur (1987) and Besley and Kanbur (1988) in the unidimensional poverty case.[2] We are not aware of previous work that derives optimal policy rules in order to reduce multidimensional poverty. The main goal of

[1] One exception is the *Chile Solidario* program, which has the explicit objective of reducing multidimensional poverty (Fiszbein and Schady 2009).

[2] Referring to their MIP, Alkire and Santos (2010) suggest that it "could be used to target the poorest, track the Millennium Development Goals, and design policies that directly address the interlocking deprivations poor people experience." (p. 1). Although the intention is clear (to reduce a MIP), it is unclear how the MIP itself can be of direct policy use. Rather, it would seem that explicit policy rules need to be derived in order to reduce optimally the MIP.

this paper is to do this, by formally setting the social objective function in terms of multidimensional poverty reduction, and by taking into account both the empirical, the normative and the spill-over informational importance of the joint distribution of well-being dimensions. This work thus stresses the importance of considering the interdependencies among multiple deprivations, rather than looking separately at each dimension, as advocated in the 2009 Report of the Commission on the Measurement of Economic Performance and Social Progress (see Stiglitz, Sen, and Fitoussi 2009):

> "[T]he consequences for quality of life of having multiple disadvantages far exceed the sum of their individual effects. Developing measures of these cumulative effects requires information on the 'joint distribution' of the most salient features of quality of life across everyone in a country through dedicated surveys. (...) When designing policies in specific fields, impacts on indicators pertaining to different quality-of-life dimensions should be considered jointly, to address the interactions between dimensions and the needs of people who are disadvantaged in several domains." (pp. 15-6)

The paper proceeds as follows. Section 2 presents the bi-dimensional poverty index followed in this study. Section 2.1 discusses the theoretical results of the impact of targeting one dimension on the bi-dimensional poverty, due to an additive and a multiplicative transfer. Section 2.2 derives the conditions determining which population subgroup should be targeted first in order to get the largest reduction in population poverty following an additive and a multiplicative transfer. Section 2.3 enriches the previous results by adding the inter-dimensional spill-over effect. Such theoretical results and their robustness are then tested with data from Vietnam (1992-1993 and 1997-1998) and South Africa (1993) in section 3. Interesting insights emerge from this application; it is found, for instance, that rules to decentralize geographical targeting funds may differ according to whether it is unidimensional or multidimensional poverty that is set to be reduced by the national authorities.

2 Theoretical framework

It is one thing to concur that poverty is multidimensional; it is another to agree on a specific procedure to measure it. The literature has been building up a stock of various multidimensional indices; see for instance Chakravarty, Mukherjee, and Ranade (1998), Tsui (2002), Bourguignon and Chakraverty (2003), Alkire and Foster (2011). All such indices have the potential to order differently the extent of poverty across distributions. This also means that they will provide different policy guidelines, especially when it comes to informing the design of targeting schemes. This is problematic, as conflicting policy guidance will potentially flow from separate ways of measuring multidimensional poverty.

One way to circumvent this problem is to seek unanimity of policy guidance across classes of poverty measurement procedures. To do this, we follow Duclos, Sahn, and Younger (2006), which we briefly summarize below. They start by defining well-being (measured, for expositional simplicity, over two dimensions of well-being, x and y) as a function $\lambda(x, y)$ that increases in both x and y. An unknown poverty frontier $\lambda(x, y) = 0$ is then supposed to exist that separates the poor from the rich, a frontier at which overall well-being of an individual is precisely equal to a "poverty level" of well-being, and below which individuals are in poverty. The set of the poor is then given by $\Lambda(\lambda) = \{(x, y) \,|(\lambda(x, y) \leq 0\}$. Multidimensional additive poverty indices can then be represented by

$$P(\lambda) = \int\int_{\Lambda(\lambda)} \pi(x, y; \lambda)\, dF(x, y), \tag{1}$$

where $\pi(x, y; \lambda)$ is the contribution to poverty of an individual with well-being indicators x and y and where $F(x, y)$ is the joint distribution of x and y.

Duclos, Sahn, and Younger (2006) then defined a first-order class of bidimensional poverty indices. The indices that belong to that class must consider as potentially poor only those individuals that belong to the largest reasonable poverty set, defined by $\Lambda(\lambda^*)$. The indices must also be continuous along the poverty frontier, be weakly decreasing in x and in y, and be such that the marginal poverty benefit of an increase in either x or y decreases with the value of the other variable. Atkinson and Bourguignon (1982) refer to this as a property of non-decreasing poverty under a “correlation-increasing switch”; this implies that, *ceteris paribus*, the greater the correlation of deprivation and the greater the incidence of multiple deprivation, the higher the level of multidimensional poverty. Higher-order classes of poverty indices are obtained by imposing further assumptions on the derivatives of $\pi(x, y; \lambda)$.

To test for whether the poverty ranking of two distributions is robust across all members of one of the above classes of poverty indices, Duclos, Sahn, and Younger (2006) introduce the following bi-dimensional poverty indices (for $\alpha_x, \alpha_y \geq 0$):

$$P(\alpha_x, \alpha_y, z_x, z_y) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_x - x}{z_x} \right)^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} dF(x, y) \tag{2}$$

where z_x and z_y are poverty lines in dimensions x and y respectively, and where $\left(\frac{z_x - x}{z_x}\right)$ and $\left(\frac{z_y - y}{z_y}\right)$ are called normalized "poverty gaps" in the poverty literature. Tracing (2) over areas of values of z_x and z_y draws a "dominance surface".

They then show that if $P_A(\alpha_x, \alpha_y, z_x, z_y)$ for some distribution A is greater than $P_B(\alpha_x, \alpha_y, z_x, z_y)$ for some distribution B over all choices of (z_x, z_y) within $\Lambda(\lambda^*)$, then poverty will be unambiguously higher in A than in B for all of the poverty indices that are members of the class of multidimensional poverty measures of order (α_x, α_y) and for all poverty frontiers that lie within $\Lambda(\lambda) \subset \Lambda(\lambda^*)$. Note that these classes of measures include intersection, union, and intermediate poverty measures, as long as these fit within $\Lambda(\lambda^*)$, although the index in (2) is an intersection index. The converse is also true: only if $P_A(\alpha_x, \alpha_y, z_x, z_y)$ is larger than $P_B(\alpha_x, \alpha_y, z_x, z_y)$ can we be certain that poverty is unambiguously larger in A.

It cannot be argued convincingly that the intersection index in (2) is necessarily better than all other possible multidimensional poverty indices. The superiority of one index over another is essentially a matter of a value judgment. The precise form of that index is also debatable: there are many other forms of intersection indices. There are, however, important advantages in focusing on the form of (2). (2) it is a natural generalization of the popular uni-dimensional FGT indices — see Foster, Greer, and Thorbecke (1984). Through its intersection nature, (2) also focuses on the poorest of the poor, that is, on those that suffer from multiple deprivation. Perhaps most importantly, if we find that (2) is consistently lower after some policy than before for a wider range of intersection poverty lines, then, by the result above, we also know that a large class of other poverty indices with different poverty frontiers will also generate the same pre-and post-policy poverty ranking. Such a result is unfortunately not available with the use of other indices. The use of (2) can therefore help establish robustness of policy guidance and poverty rankings in a transparent manner.

Much of the paper relies on the derivation of (2) with respect to changes in dimensional values, through targeting policies or shocks to well-being indicators. Because of this, it is useful to extend (2) to cases in which α_x or α_y may be equal to minus one. Let then

$$P(\alpha_x = -1, \alpha_y, z_x, z_y) = f\left(x = z_x\right) \int_0^{z_y} \left(\frac{z_y - y}{z_y}\right)^{\alpha_y} f\left(y \middle|\, x = z_x\right) dy \tag{3}$$

and, similarly,

$$P(\alpha_x, \alpha_y = -1, z_x, z_y) = f\left(y = z_y\right) \int_0^{z_x} \left(\frac{z_x - x}{z_x}\right)^{\alpha_x} f\left(x \middle|\, y = z_y\right) dx. \tag{4}$$

It is also useful to rewrite $P(\alpha_x, \alpha_y, z_x, z_y)$ in a way that shows explicitly the role of attribute correlation in the valuation of multidimensional poverty. Knowing that

$$\begin{aligned} &\operatorname{cov}\left[\left(\tfrac{z_x-x}{z_x}\right)_+^{\alpha_x}, \left(\tfrac{z_y-y}{z_y}\right)_+^{\alpha_y}\right] \\ &= E\left[\left(\tfrac{z_x-x}{z_x}\right)_+^{\alpha_x} \left(\tfrac{z_y-y}{z_y}\right)_+^{\alpha_y}\right] - E\left[\left(\tfrac{z_x-x}{z_x}\right)_+^{\alpha_x}\right] E\left[\left(\tfrac{z_y-y}{z_y}\right)_+^{\alpha_y}\right] \end{aligned} \tag{5}$$

where $f_+ = \max(f, 0)$, we can rewrite (2) as:

$$\begin{aligned} &P(\alpha_x, \alpha_y, z_x, z_y) \\ &= P(\alpha_x, z_x) P(\alpha_y, z_y) + \operatorname{cov}\left[\left(\tfrac{z_x-x}{z_x}\right)_+^{\alpha_x}, \left(\tfrac{z_y-y}{z_y}\right)_+^{\alpha_y}\right]. \end{aligned} \tag{6}$$

where $P(\alpha_x, z_x)$ and $P(\alpha_y, z_y)$ are the usual unidimensional FGT indices. Thus, the dominance surface over areas of z_x and z_y has a height that is determined by the product of the two unidimensional poverty curves plus the covariance between the poverty gaps in the two attributes. This latter term captures the importance of the "correlation" between the two dimensions.

Duclos, Sahn, and Younger (2006) illustrates how this covariance term can have a crucial influence on the dominance surfaces. It can for instance happen that urban areas unidimensionally dominate rural areas both in income and in nutrition, but not bidimensionally. Unidimensional comparisons may be ambiguous, but the ambiguity can be resolved by the joint distribution information.

2.1 The effect of targeting *x*

2.1.1 Additive transfer

Assume that an additive transfer γ is granted to everyone in a population. This is a simplifying framework; we will enrich it later on in the paper. We can then re-write (2) (dropping z_x and z_y from the P's for expositional simplicity) as

$$P(\alpha_x, \alpha_y, \gamma) = \int_0^{z_x}\int_0^{z_y} \left(\frac{z_x - x - \gamma}{z_x}\right)^{\alpha_x} \left(\frac{z_y - y}{z_y}\right)^{\alpha_y} dF(x, y). \tag{7}$$

For $\alpha_x > 0$, a marginal change in γ will change the bi-dimensional poverty index (2) by

$$\begin{aligned} \frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma} &= P(\alpha_x - 1, \alpha_y, \gamma) \\ &= -\frac{\alpha_x}{z_x} P(\alpha_x - 1) P(\alpha_y) - \text{cov}\left[\left(\frac{z_x - x - \gamma}{z_x}\right)_+^{\alpha_x - 1}, \left(\frac{z_y - y}{z_y}\right)_+^{\alpha_y}\right] \end{aligned} \tag{8}$$

The first term in (8) is the targeting impact on unidimensional poverty identified in Kanbur (1985). It also corresponds to the well known result that the sensitivity of FGT poverty to changes in well-being is related to the same FGT index, but with a value α equal to $\alpha - 1$. For multidimensional poverty, this effect must be multiplied by the level of uni-dimensional poverty in the other dimensions — the term $P(\alpha_y)$ in (8) — although these other dimensions are not targeted by the transfer. The multidimensional poverty impact also incorporates the covariance between the poverty gaps in the dimensions, to the power $\alpha_x - 1$ and α_y.

If $\alpha_x = 0$, the change in bidimensional poverty is given by:

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma)}{\partial \gamma} = -P(\alpha_x = -1, \alpha_y, \gamma) \tag{9}$$

Considering (9), the impact is directly proportional to the density of individuals around z_x and to the (conditional) unidimensional FGT index of order α in the other dimension (y), for those around $x = z_x$). The impact of targeting on the intersection bidimensional headcount is therefore quite different from the value of the headcount itself.

2.1.2 Multiplicative transfer

A commonly-modeled form of transfer increases pre-transfer indicators by some proportion λ. Algebraically, post-transfer poverty can be written as

$$P(\alpha_x, \alpha_y, \lambda) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_x - x(1+\lambda)}{z_x} \right)^{\alpha_x} \left(\frac{z_y - y}{z_y} \right)^{\alpha_y} dF(x, y) \tag{10}$$

When $\alpha_x > 0$, the derivative of (10) with respect to λ is given by

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} = -\frac{\alpha_x}{(1+\lambda)} [P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{11}$$

The *per capita* cost, R, of such a multiplicative transfer is

$$R = (1+\lambda)\,\overline{x}, \tag{12}$$

where $\overline{x}$ is the average of x. The change in aggregate poverty per dollar spent *per capita* is then:

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \bigg/ \frac{\partial R}{\partial \lambda} = -\frac{\alpha_x}{\overline{x}\,(1+\lambda)} [P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]. \tag{13}$$

If $\alpha_x = 0$, the change in the bidimensional headcount per dollar spent is

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda)}{\partial \lambda} \bigg/ \frac{\partial R}{\partial \lambda} = -\frac{z}{\overline{x}\,(1+\lambda)} P(\alpha_x = -1, \alpha_y, \lambda). \tag{14}$$

Comparing (8) to (13), and (9) to (14), it is not possible to say *a priori* whether for every *per capita* dollar spent, an additive transfer contributes more than a multiplicative transfer to multidimensional poverty reduction. As for bidimensional poverty index of order $\alpha > 0$, two components play a crucial role: while we know that $[P(\alpha_x - 1, \alpha_y) - P(\alpha_x, \alpha_y)]$ (which appears in the multiplicative case) is smaller then $[P(\alpha_x - 1, \alpha_y)]$ (which appears in the additive case), we do not know whether $\frac{\alpha_x}{\overline{x}(1+\lambda)}$ (in the multiplicative case) is larger or smaller than $\frac{\alpha_x}{z_x}$. In particular, the latter component in the multiplicative case should be larger enough with respect to the additive case to be able to more than compensate the first component (which, as discussed, under the multiplicative case results to be lower). In poor societies where $\overline{x}$ is particularly low and far from the poverty line, we may then prefer the multiplicative prototype. As for bidimensional poverty index of order $\alpha = 0$, we need to compare z_x with $\overline{x}$. If z_x is greater than $\overline{x}$, then the multiplicative transfer scheme may be preferred. This is more likely to be the case for poor societies with a fairly equal distribution in x.

2.2 Socio-economic targeting of dimension x

2.2.1 An additive transfer

Developing the framework above, we can provide insights into which population subgroup should be targeted in other three used population poverty that most learned dollar spent. For simplicity, assume that we can we divide the total population into two exclusive groups, A and B (such as urban and rural areas). Bidimensional poverty is then given by

$$P(\alpha_x, \alpha_y, \gamma^A, \gamma^B) = \omega^A P^A(\alpha_x, \alpha_y, \gamma^A) + \omega^B P^B(\alpha_x, \alpha_y, \gamma^B) \tag{15}$$

where ω^A and ω^B are the population shares of groups A and B, respectively, and where γ^A and γ^B are transfer targeted specifically to members of the groups A and B respectively.

To assess whether an additive transfer is better targeted towards group A or group B, we need to check whether

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \Big/ \frac{\partial R}{\partial \gamma^A} \lesseqgtr \frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \Big/ \frac{\partial R}{\partial \gamma^B}. \tag{16}$$

The *per capita* cost of an additive transfer is given by

$$R = \omega^A \gamma^A + \omega^B \gamma^B. \tag{17}$$

We start with the case of $\alpha_x > 0$. We then have:

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^A)}{\partial \gamma^A} \Big/ \frac{\partial R}{\partial \gamma^A} = -\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y, \gamma^A) \tag{18}$$

and, similarly,

$$\frac{\partial P(\alpha_x, \alpha_y, \gamma^B)}{\partial \gamma^B} \Big/ \frac{\partial R}{\partial \gamma^B} = -\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y, \gamma^B). \tag{19}$$

The largest aggregate poverty reduction per dollar spent is then obtained by targeting that group that has the highest $P(\alpha_x - 1, \alpha_y, \gamma^B)$ index. Looking back to (8), we note that this will be the case for the group which displays the highest $P(\alpha_x - 1)$ index, the largest $P(\alpha_y)$ index, and the highest covariance between $P(\alpha_x - 1)$ and $P(\alpha_y)$ uni-dimensional gaps. It is clear that choosing the group to target on the basis of the $P(\alpha_x)$ indices will generally not lead to an optimal multidimensional poverty reduction strategy.

For $\alpha_x = 0$, $\frac{\alpha_x}{z_x} P^A(\alpha_x - 1, \alpha_y, \gamma^A)$ and $\frac{\alpha_x}{z_x} P^B(\alpha_x - 1, \alpha_y, \gamma^B)$ in (18) and (19) above are replaced respectively by $P^A(\alpha_x = -1, \alpha_y, \gamma^A)$ and $P^B(\alpha_x = -1, \alpha_y, \gamma^B)$. Again, the multidimensional poverty index itself (in this case, the usual FGT index for dimension y but counting only those that are also poor in dimension x) is not the right guide to selecting the optimal group to target in order to reduce national poverty the most *per capita* dollar spent. Instead, the optimal targeting rule uses the y-dimension FGT index of those that are around the x poverty line, multiplied by how many of the group's individuals are close to the x-dimension poverty line.

2.2.2 Multiplicative transfer

Let us now consider the optimal group selection rule under a multiplicative targeting scheme. The *per capita* cost of such a scheme is given by

$$R = \omega^A (1 + \lambda^A) \overline{x}^A + \omega^B (1 + \lambda^B) \overline{x}^B \tag{20}$$

and, when $\alpha_x > 0$, changes in poverty due to a transfer λ in groups A and B respectively are

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda^A)}{\partial \lambda^A} \Big/ \frac{\partial R}{\partial \lambda^A} = -\frac{\alpha_x}{\overline{x}^A (1 + \lambda^A)} [P^A(\alpha_x - 1, \alpha_y) - P^A(\alpha_x, \alpha_y)] \tag{21}$$

and

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda^B)}{\partial \lambda^B} \Big/ \frac{\partial R}{\partial \lambda^B} = -\frac{\alpha_x}{\overline{x}^B (1 + \lambda^B)} [P^A(\alpha_x - 1, \alpha_y) - P^B(\alpha_x, \alpha_y)]. \tag{22}$$

For $\alpha_x = 0$, the expressions above become

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda^A)}{\partial \lambda^A} \Big/ \frac{\partial R}{\partial \lambda^A} = -\frac{z_x}{\overline{x}^A (1 + \lambda^A)} P^A(\alpha_x = -1, \alpha_y) \tag{23}$$

and

$$\frac{\partial P(\alpha_x, \alpha_y, \lambda^B)}{\partial \lambda^B} \bigg/ \frac{\partial R}{\partial \lambda^B} = -\frac{z_x}{\overline{x}^B (1+\lambda^B)} P^B(\alpha_x = -1, \alpha_y). \tag{24}$$

The case in which the public transfer is a proportion of dimension x is less straightforward to analyze then the case of an additive transfer. As is clear from (13), the reduction in the multidimensional poverty index is the largest for those populations with the lowest average income and, at the same time, the distance between the poverty gaps in dimension x at the power $\alpha - 1$ and α, weighted by the poverty gap in dimension y, summed to the intercorrelation of the two dimensions is the highest. People living in extreme poverty in a dimension, and thus showing a lower average value of that dimension, are usually those for which poverty gaps decrease faster to an increase in α; the difference in poverty gaps of order $\alpha - 1$ and α is likely to be larger for this group. In addition, as discussed in the introduction, these people are also likely to show higher poverty gap in other dimension and, as a consequence, a higher intercorrelation between poverty gaps in different dimensions. However, for this group, changes in the average value of x due to a proportional transfer are the lowest.

Similarly to what we already discussed for (13), for α higher than 0 the policy advice is less straightforward and some ambiguity may arise. Similarly, no clearcut conclusions emerge when α equals 0. The group the Government should target depends on different factors: the population weighted density for dimension x around its poverty line, the average value for dimension x and the poverty gap of order α in dimension y (at $x = z_x$).

In this case, for every *per capita* dollar spent, the reduction in poverty is highest for populations with the largest density of dimension x around the dimension's poverty line and showing the lowest average value for dimension x and the highest poverty gap of order α in dimension y (at $x = z_x$).

2.3 Targeting with inter-dimensional spill-over effects

2.3.1 Transfers to dimension x

Now suppose that dimension y is also indirectly affected by additive transfers γ made to dimension x. We suppose that this spill-over, indirect, effect on y is captured by a function $y(\gamma)$, which is equal to y in the absence of spill-over effects. We may re-write (7) as

$$P(\alpha_x, \alpha_y, \gamma) = \int_0^{z_x} \int_0^{z_y} \left(\frac{z_x - x - \gamma}{z_x} \right)^{\alpha_x} \left(\frac{z_y - y(\gamma)}{z_y} \right)^{\alpha_y} dF(x, y). \tag{25}$$

For expositional simplicity, let us think of x and y as income and nutrition, respectively, two dimensions in which welfare analysts are supposed to be jointly interested. (25) shows that a policy that targets income explicitly (for instance, through a cash transfer) affects multidimensional poverty directly through its impact on the poverty gap in dimension x, through its multiplying effect on the gap in the other dimension y, and through its spill-over effect on that other dimension, captured in (25) by the function $y(\gamma)$.

For $\alpha_y > 0$, the marginal *spill-over effect* on bi-dimensional poverty of a change in γ is then given by

$$\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spillover effect}} = -\frac{\alpha_y}{z_y}P(\alpha_x,\gamma)\int_0^{z_y}\frac{\partial y(\gamma)}{\partial\gamma}\left(\frac{z_y-y(\gamma)}{z_y}\right)^{\alpha_y-1}dF(y) \quad (26)$$
$$+\frac{\alpha_y}{z_y}\,\text{cov}\left[\left(\frac{z_x-x-\gamma}{z_x}\right)_+^{\alpha_x},\frac{\partial y(\gamma)}{\partial\gamma}\left(\frac{z_y-y(\gamma)}{z_y}\right)_+^{\alpha_y-1}\right],$$

and for $\alpha_y = 0$, it equals

$$\left.\frac{\partial P(\alpha_x,\alpha_y,\gamma)}{\partial\gamma}\right|_{\text{spillover effect}} = \quad (27)$$
$$-\left.\frac{\partial y(\gamma)}{\partial\gamma}\right|_{y(\gamma)=z_y} f(y(\gamma)=z_y)\int_0^{z_x}\left(\frac{z_x-x}{z_x}\right)^{\alpha_x}dF\left(x|\,y(\gamma)=z_y\right).$$

This spill-over effect adds to the other effects described above, either through the impact of an additive or of a multiplicative transfer on dimension x. For instance, the net multidimensional poverty effect of an additive transfer to dimension x would be the sum of (8) (or (9) for $\alpha_x = 0$) and either (26) or (27). For a multiplicative transfer, expression (8) is replaced by (11), and analogously for $\alpha_x = 0$.

The formulation of $y(\gamma)$ is sufficiently general to allow for several types of spill-over effects on the second dimension. Special cases include additive spillover effects, when $y(\gamma) = y + \gamma$, or multiplicative ones, when when $y(\gamma) = (1+\gamma)y$. In all cases, the spill-over effect is given by the mean of the product of the y poverty gaps to the power $\alpha - 1$ and the marginal change in $y(\gamma)$, weighted by the x poverty gaps to the power α_x . Whether this indirect effect favors targeting the more severely poor depends on whether the severely poor's well-being indicator y is more sensitive to changes in x. That may or may not be the case.

These spillover effects can then be normalized by the *per capita* cost of targeting dimension x. This is done in the same way as in section 2.2. Doing so makes it possible to assess which population subgroup should be targeted first in order to reduce multidimensional poverty as quickly as possible, subject to resource constraints. If a *per capita* cost can be assessed for each of the the two dimensions, x and y, then such a normalization also allows establishing which dimension should be preferably targeted by public expenditures.

3 Applications

3.1 Correlations and inter-dimensional spill-overs

As discussed above, the correlation — and more generally, the joint distribution — of dimensions is important both for measurement and for policy purposes. Much of this correlation usually reflects a "natural" distribution of dimensions. An example is the relation between maternal nutrition during pregnancy and child weight at birth. Another is the correlation between child nutrition and schooling performance (and adult labor outcomes): child malnutrition (especially if experienced during the first two years of life) is usually associated with lower school and labor performance (see, for example, Glewwe and King 2001, Heckman 2008 and Alderman, Hoddinott, and Kinsey 2006 for some evidence and discussion).

Some of that joint distribution between indicators of well-being can also be driven (at least partly) by policy. Public investments in perinatal care (for instance, through pre-natal health visits and nutritional programs for pregnant women) can improve the health status of newborn children. Important direct and indirect educational costs (including opportunity costs) can limit the school attendance of the monetarily poor children, leading to class repetition and late or no enrollment. In the absence of affordable and good-quality public health services, those that are monetarily poor will also be more likely to experience bad health conditions. This is true in the short term, although the effects may be reinforced over time through the existence of multidimensional poverty traps.

Policy can influence the multidimensional distribution of such indicators in a number of different ways. The subsidized or free provision of social services such as education, health and housing may be one way to alleviate poverty in its multiple dimensions; it may also serve to reduce the prevalence of multiple deprivations, or, equivalently, to reduce the correlation across deprivations. Some of the former Socialist countries provide a good example in that regard. Countries such as Kyrgyzstan and Uzbekistan show relatively high child poverty rates in the income dimension, but reasonably good outcomes in terms of schooling. UNICEF (2011) argues this is partly due to public commitments towards social service delivery and to the effects of the inherited socialist system, which produced relatively high educational outcomes.

Policies are often designed in a way that (at least in appearance) tries to address the multidimensionality of poverty. The popularized conditional cash transfer (CCT) programs intend for instance to break down the multidimensional (and multi-generational) poverty traps both by alleviating monetary poverty and by increasing human capital and health status. A key mechanism that is employed is the conditionality of the transfers. The effect on multidimensional poverty of a cash transfer conditioned on family investments in child nutrition is likely to be higher than one without conditionality; the effect on monetary policy may, however, be reduced by conditionality, if, for instance, some of the transfers cannot then be used for purely income production purposes. The cross-dimension effects of transfer conditionality have been most extensively demonstrated in the context of Latin American countries. For example, Fiszbein and Schady (2009) show plenty of cross-country evidence of CCT's positive impacts on child labor, various health indicators and access to health services, school enrolment and attendance, and, most prominently because of the nature of the program, on income poverty.

The "natural" correlation across attributes of well-being also depends on the quality of markets and of social services. When markets do not exist or are highly imperfect, social programs maybe relatively ineffective at producing positive spill-over effects on dimensions other than the targeted one. For example, in remote areas where appropriate schooling infrastructure is missing or is of poor quality, social cash transfers for children may have meagre effects on school outcomes (see for instance Kakwani, Soares, and Son 2006 and Cockburn, Fofana, and Tiberti 2010).

It is not possible to take into account all of the possible interactions that may exist between policy and the multiple dimensions of poverty. It is nevertheless feasible and, we believe, valuable to use the analytical framework developed above to illustrate how these interactions should feed into policy design and policy evaluation. We do this in two different ways. We first assess the poverty impact and the optimality of simple targeting rules established on the basis of socioeconomic characteristics, following the strong targeting tradition of the unidimensional poverty literature. We then go beyond the simple rules by assessing the impact of more realistic policies, policies that can have spill-over effects beyond the dimensions that are targeted.

3.2 Data and estimation procedures

We apply the analytical approach presented above to three separate datasets from Vietnam and South Africa. These are the Vietnam Living Standard Survey (VLSS) 1992-1993, the VLSS 1997-1998 and the South Africa Integrated Household Survey (SAIHS) 1993. These three data sets include information on household consumption and anthropometric measures, which is a major reason for which we are using them. This information leads to the construction of *per capita* household consumption (deflated by appropriate spatial and temporal price deflators) and height-for-age z scores (*HAZ*), standardized by the growth standards released by WHO (2006). These indicators of monetary well-being and of health are used for income poverty and health poverty respectively. The analysis focuses on children under five years old. It is supposed that policy can target *per capita* expenditure (dimension x), but that policy depends on the joint distribution of expenditure and *HAZ* (which we take as dimension y).

The spill-over effect of targeting expenditures on health is obtained by estimating the following simple linear regression model:

$$y_i = \alpha + \beta_x x_i + \sum_k \beta_k z_{k,i} + \epsilon_i \tag{28}$$

where y_i is the z-score variable for individual i, x_i is *per capita* consumption, β_k is the coefficient associated to *per capita* consumption, z_k is a set of k determinants, β_k are their associated coefficients, and ϵ_i is the error term. The econometric model finally retained to estimate the spill-over coefficient is that proposed by Wagstaff, van Doorslaer, and Watanabe (2003), which uses an OLS estimation with community-level fixed effects at the level of child's commune and where the standard errors are corrected both for heteroscedasticity and for the effects of geographic commune-level clustering. Note that the model is intended to provide a simple, reduced-form, representation of potentially complex mechanisms linking consumption with children's health. These mechanisms will generally depend on household composition and intra-household allocation rules, anyway that is non observable to the analyst. The cash transfers can be distributed across household members, with a reduced effect on the targeted children. Nutritional transfers could in principle be potentially better targetable to children, but again there exist substitution strategies that parents can use in order to substitute away from children some of the additional resources intended for them.

Weighted average values of *HAZ*'s determinants as well as their estimated coefficients are shown in Table 1. Most of coefficients take the expected sign in all three surveys: per capita real expenditure are positively associated to child health condition; child's health is negatively (convexly) linked to his or her age; in South Africa being male is associated to worse nutrition while having access to improved sanitation facilities improves nutrition only in Vietnam 1992-93. Surprisingly, access to safe water sources as well as mother's schooling are not statistically significant. The spill-over parameters that are produced are 0.0171 percent for VLSS 1992-1993, 0.0097 percent for VLSS 1997-1998 and 0.1766 percent for SAIHS 1993. These parameters are obtained as ratio between $ln(pc\_consumption)$'s coefficients in Table 1 and the weighted values of the exponential of $ln(pc\_consumption)$. It is then calculated as $0.2470/\exp(7.2705)$ for VLSS 1992-1993; $0.1885/\exp(7.5709)$ for VLSS 1997-1998 and $0.2842/\exp(5.0808)$ for SAIHS 1993.

Table 1: HAZ regression and descriptive statistics

explanatory variables	**VLSS92-93**		**VLSS97-98**		**SAIHS93**	
	coeff.	***mean***	***coeff.***	***mean***	***coeff.***	***mean***
ln(pc_consumption)	0.2470	7.27	0.1885	7.57	0.2842	5.08
	(3.61)		-2.24		-6.47	
age_months	-0.0764	32.02	-0.0652	33.43	-0.0567	31.45
	(-12.55)		(-9.6)		(-9.8)	
age_months2	0.0010	1328.86	0.0007	1433.78	0.0008	1269.50
	(10.88)		(7.38)		(8.53)	
gender	0.0262	0.50	-0.0368	0.51	-0.1232	0.50
	(0.54)		(-0.67)		(-2.71)	
safe_water	0.0543	0.79	0.0945	0.73	-0.1752	0.83
	(0.5)		(1)		(-1.72)	
safe_sanitation	0.2405	0.14	0.1007	0.20	0.1404	0.35
	(2.68)		(1.12)		(0.95)	
schooling_mother	0.0167	6.51	0.0043	2.73	0.0135	5.56
	(1.61)		(0.11)		(1.74)	
_cons	-3.0117		-2.1653		-1.7532	
	(-6.27)		(-3.42)		(-7.14)	
Adj R2	0.1551		0.2013		0.1696	
observations	2754		2195		3858	

Note: t-stat are reported in parenthesis. Explanatory variables are not necessarily comparable across surveys since their definition can differ

Source: authors' analysis based on VLSS 1992-1993, VLSS 1997-1998 and SAIHS 1993

3.3 Discussion of results

We proceed by separating the population into separate sub population geographical groups — see their definition in 2. As suggested in WHO (2006), out-of-range values (<-5 and >3) for the z-scores are dropped. For ease of exposition, a value of 10 is added to the *HAZ* variable and to the poverty line in the health dimension; such a transformation does not affect any of the substantive results since we are interested in absolute multidimensional poverty, not relative multidimensional poverty or inequality.

A annual monetary poverty line of 1790 thousands Dong (in 1998 prices) is used for the two surveys of Vietnam, while a monetary poverty line of 1970.4 Rand is used for South Africa. These values correspond to around 385 and 902 international dollars (in 2005 prices) respectively. For health, a poverty line of -2 standard deviations is used for each of the three countries — this threshold is often used to identify moderate-to-severe stunting (following the transformation of the *HAZ* variable, the poverty line then changed to 8). For dominance purposes, different poverty lines are also used. Specifically, ten different poverty lines (equal or lower than the official poverty line) for each of the two dimensions were estimated, then giving 100 possible combinations.

Table 2: Geographical units

(a) *Vietnam 1992-1993*

region	milieu: urban	milieu: rural
RedRiverDelta	8	6
Northeast	3	3
Northwest	4	4
NorthCentralCoast	7	7
SouthCentralCoast	5	10
CentralHighlands	1	1
Southeast	5	2
MekongRiverDelta	9	2

(b) *Vietnam 1997-1998*

region	milieu: urban	milieu: rural
RedRiverDelta	5	1
Northeast	10	10
Northwest	3	3
NorthCentralCoast	10	10
SouthCentralCoast	7	2
CentralHighlands	9	9
Southeast	4	6
MekongRiverDelta	8	2

(c) *South Africa1993*

province	milieu: metro	milieu: urban	milieu: rural
1	1	1	1
2	2	2	
3	3	4	5
4	6	7	8
5	9	10	
6	11	12	
7	13	14	
8	15	16	
9	17	17	18

We focus on policy impact on bi-dimensional poverty with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$, normalized by the *per capita* cost of the policy. The geographical units are ordered according to the importance of the marginal poverty reduction following a marginal increase of a transfer in the monetary dimension. An important lesson is that those rankings change significantly once we move away from uni-dimensional towards multidimensional poverty alleviation.

We start with Vietnam 1992, using $\alpha_x = \alpha_y = 0$. Focusing first on unidimensional poverty, a significantly larger reduction in poverty headcount per dollar spent is obtained by targeting unit 1 in comparison with units 6, 5, 3 and 7. As for equations (9) and (3), group 1 shows the largest density around z_x. The method followed to produce the standard errors as well as the statistical test to evaluate the difference in poverty change between units is described in Appendix A. The results are presented in Table 3 (Panel A). The second-best unit to be targeted is unit 2, whose uni-dimensional poverty impact per dollar spent is significantly larger than 3 and 7. A statistical ranking cannot be established with respect to the other geographical units.

Let us now add the nutritional component as shown in Table 3 (Panel A). When this term is added (the poverty headcount ratio in the second dimension), a significant re-ranking across the geographical units is obtained. Units 1 and 2 continue to be prioritized by the targeting policy but in comparison to different units: differently from the previous step, unit 1 is now preferred to units 9 and 8 and no more to units 6 and 3. The latter units shows indeed the largest headcount poverty ratio in nutrition — see Table 4. Targeting unit 2 is now statistically preferable compared to units 9, 8 and 5. The next units to be prioritized are now units 3, 10 and 6, which give larger poverty reduction compared to unit 5.

Adding the covariance term, which enables to take into account the impact of targeting geographical units on the joint deprivation of individuals in the total population, a few changes in the targeting ranking are observable. Units 1 and 2 are both the first units to be prioritized by the policy-maker; these units are now statistically preferred to units 5, 3, 8, 6, 4 and 7. Compared to the previous step, targeting unit 1 now becomes statistically better than units 3, 6 and 4, but loses its dominance on unit 4; similarly, poverty change in unit 2 is now statistically larger than in units 3, 6, 4 and 7, but not anymore with respect to unit 9. Unit 10 follows; its poverty reduction is statistically larger than that in unit 7. Finally, unit 9 is preferred to units 6 and 7 as, around the monetary poverty line, the covariance between the its density function and the poverty headcount ratio in nutrition is sufficiently large.

When also the spill-over component is added as indicated in equation (27), units 1 and 2 both lose their dominance with respect to unit 8. In addition, unit 10 is now also preferred to unit 6 while targeting unit 5 allows a statistically larger reduction in bi-dimensional population poverty headcount ratio in comparison to unit 7.

Comparing the policy guidance obtained under unidimensional poverty to that generated by a consideration of multidimensional poverty, a few interesting cases emerge. As an example, from a uni-dimensional perspective (in both dimensions), there is no reason to prefer targeting unit 1 relative to unit 4. A preference for targeting unit 1 instead of unit 4 becomes, however, statistically significant under multidimensional poverty. Another interesting case concerns the comparison between units 9 and 7. While targeting the monetary dimension, one cannot establish any statistical preference under the unidimensional perspective, conversely unit 7 is statistically preferable to unit 9 if the nutrition dimension is targeted. However, under a multidimensional approach, targeting the monetary results in a statistical preference for unit 9 as opposed to unit 7.

With $\alpha_x = \alpha_y = 1$ — see Table 3 (Panel B) — a large re-ranking across units is found again when we move away from uni-dimensional towards multidimensional poverty alleviation. As with $\alpha_x = \alpha_y = 0$, the policy guidance can change substantially under unidimensional poverty as compared to a multidimensional approach. Under a uni-dimensional perspective (in both dimensions), targeting unit 5 does not dominate any other geographical units. A preference for targeting unit 5 compared to unit 8 becomes, however, statistically significant from a multidimensional perspective; similarly, a preference for targeting unit 7 as opposed to units 4 and 6 cannot be established on the basis of unidimensional poverty, but it does become statistically significant under multidimensional poverty. On the contrary, targeting units 2 and 7 as opposed to units 10 and 1 can be rationalized under the objective of reducing unidimensional poverty, but not under the objective of alleviating multidimensional poverty.

As well-know from the poverty literature, the use of different poverty indices can affect substantially the ranking across groups. This is what we observe in Table 3 when Panel A is compared to Panel B. In particular, if we look at the last group of results (*total impact with spill-over*) we learn that units 1 and 2 are by far the first ones to be prioritized under $\alpha_x = \alpha_y = 0$ while they are statistically preferred only to units 5, 9 and 8 with $\alpha_x = \alpha_y = 1$. Interestingly, there is no reason to prefer unit 9 under the poverty gap, while it does become statistically preferable to units 6 and 7 when the bi-dimensional headcount is considered.

Table 3: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1992-1993

Panel (A): $\alpha_x = \alpha_y = 0$

	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
Ranking	group	Population poverty change	Groups dominated	Group	population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
1	1	-0.00060	6:5:3:7	1	-0.00036	7:9:8:5	1	-0.00036	5:3:8:6:4:7	1	-0.00038	5:3:6:7:4
2	2	-0.00045	3:7	2	-0.00025	9:8:5	2	-0.00025	5:3:8:6:4:7	2	-0.00027	5:3:6:7:4
3	10	-0.00039		3	-0.00023	5	10	-0.00020	7	9	-0.00023	6:7
4	9	-0.00035		10	-0.00022	5	9	-0.00020	6:7	10	-0.00022	6:7
5	4	-0.00035		6	-0.00022	5	5	-0.00015		8	-0.00020	
6	6	-0.00033		4	-0.00022		3	-0.00013		5	-0.00018	7
7	5	-0.00032		7	-0.00019		8	-0.00013		3	-0.00014	
8	3	-0.00032		9	-0.00016		6	-0.00011		6	-0.00012	
9	8	-0.00031		8	-0.00014		4	-0.00010		7	-0.00011	
10	7	-0.00028		5	-0.00013		7	-0.00010		4	-0.00011	

Panel (B): $\alpha_x = \alpha_y = 1$

	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
Ranking	group	Population poverty change	Groups dominated	Group	population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
1	4	-0.00052	7:6:10:1:2:9:5:8	3	-0.00005	6:4:10:2:9:5:8	3	-0.00005	6:4:10:2:5:9:8	3	-0.00006	6:4:10:2:5:9:8
2	3	-0.00050	6:10:1:2:9:5:8	7	-0.00005	6:4:10:2:9:5:8	7	-0.00005	6:4:10:2:5:9:8	7	-0.00005	6:4:10:2:5:9:8
3	7	-0.00048	10:1:2:9:5:8	6	-0.00004	10:2:9:5:8	1	-0.00004	2:5:9:8	1	-0.00005	5:9:8
4	6	-0.00046	10:2:9:5:8	1	-0.00004	9:5:8	6	-0.00004	2:5:9:8	6	-0.00004	2:5:9:8
5	10	-0.00039	9:5:8	4	-0.00004	2:9:5:8	4	-0.00004	5:9:8	4	-0.00004	5:9:8
6	1	-0.00039	9:5:8	10	-0.00003	9:5:8	10	-0.00003	5:9:8	10	-0.00004	5:9:8
7	2	-0.00037	9:5:8	2	-0.00003	9:5:8	2	-0.00003	5:9:8	2	-0.00003	5:9:8
8	9	-0.00023		9	-0.00001	8	5	-0.00001	8	5	-0.00001	8
9	5	-0.00017		5	-0.00001		9	-0.00001		9	-0.00001	
10	8	-0.00012		8	-0.00000		8	-0.00000		8	-0.00000	

Source: authors' analysis based on data from the VLSS 1992-1993. Note: Difference in poverty change between groups significant at 5 percent

Table 4: Population share and poverty gaps in the monetary and nutritional dimensions: Vietnam 1992-1993

Group	**Population share**	**Monetary**		**Nutrition**	
		P0	P1	P0	P1
1	0.034	0.695	0.249	0.600	0.096
2	0.260	0.669	0.211	0.551	0.073
3	0.160	0.893	0.321	0.710	0.100
4	0.036	0.930	0.336	0.620	0.071
5	0.070	0.304	0.077	0.397	0.041
6	0.172	0.829	0.277	0.678	0.083
7	0.145	0.862	0.307	0.686	0.101
8	0.021	0.207	0.031	0.431	0.031
9	0.032	0.404	0.119	0.449	0.038
10	0.068	0.706	0.261	0.572	0.074
Population	1	0.729	0.247	0.607	0.080

Source: authors' analysis based on data from the VLSS 1992-1993

When we move to Vietnam 1997-1998, a similar large re-ranking across units is observed when we move from unidimensional to multidimensional poverty index under both $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$. Results are shown in Appendix (Figure 7 Panel A and Panel B; Figure 8).The only notable exception concerns the spillover component. With $\alpha_x = \alpha_y = 0$, moving from the index including the joint deprivation towards the complete definition of the multidimensional poverty adopted in this paper (i.e. with the spill-over effect among the dimensions) does not statistically affect the socio-economic ranking of targeting.

Let us now turn to regional targeting in South Africa. The results are shown in Table 5. Let's concentrate here on the results with $\alpha_x = \alpha_y = 1$ (Panel (B)) and discuss some particular cases. A few interesting links between unidimensional and multidimensional poverty also emerge when pondering which unit should be targeted first. Unit 17 shows, for instance, the largest level of health headcount poverty among almost all geographical units (see Table 6) as well as an above the average health poverty gap, but its level of monetary poverty is not statistically larger than any of the other units. Considering multidimensional poverty, a statistically significant policy preference for targeting unit 17 can be established only with respect to unit 7. Targeting unit 11 is better than targeting unit 12 in terms of unidimensional poverty in both the monetary and the health dimensions, but this is nevertheless not the case when the impact of such targeting on multidimensional poverty is taken into account.

When we compare the results for the total multidimensional poverty index with $\alpha_x = \alpha_y = 0$ and $\alpha_x = \alpha_y = 1$ we learn that the policy guidance changes dramatically. As an example, unit 3 is dominated by most of other geographical units when $\alpha_x = \alpha_y = 0$, while with $\alpha_x = \alpha_y = 1$ we found that it dominates 16 units out of 17 possible (unit 5 is the only one not statistically preferred by unit 3). While unit 13 shows extraordinary large headcount monetary ratio and nutritional poverty gap (which explains its superiority under $\alpha = 1$) (Figure 1 panel (A)), nearly nobody lies around the monetary poverty line (which broadly justifies the small bi-dimensional impact when $\alpha = 0$) (panel (B)). This explain the big reversal when we move away from $\alpha_x = \alpha_y = 0$ towards $\alpha_x = \alpha_y = 1$.

Table 5: Impact of targeting monetary dimension on bi-dimensional poverty: South Africa 1993

Panel (A): $\alpha_x = \alpha_y = 0$

	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
ranking	group	population poverty change	groups dominated	group	population poverty change	groups dominated	group	population poverty change	groups dominated	group	population poverty change	groups dominated
1	4	-0.00458	18:3	17	-0.00133	2:12:16:1:3:14:18:7	4	-0.00307	3	4	-0.00315	3
2	6	-0.00385	12:1:7:2:16:18:3	5	-0.00125	18:7	15	-0.00295	13:1:17:18:16:2:3	15	-0.00308	13:1:17:18:16:2:3
3	8	-0.00383	1:2:18:3	6	-0.00125	2:12:16:1:3:14:18:7	6	-0.00253	1:17:18:16:2:3	6	-0.00268	1:17:18:16:2:3
4	15	-0.00365	1:7:2:16:18:3	4	-0.00119		9	-0.00221		9	-0.00233	
5	5	-0.00348	1:2:18:3	8	-0.00109	7	8	-0.00221	18:3	8	-0.00231	3
6	17	-0.00323	18:3	15	-0.00105	12:1:3:14:18:7	10	-0.00208	3	12	-0.00226	3
7	10	-0.00319	18:3	13	-0.00100	2:12:1:3:14:18:7	11	-0.00207	18:3	10	-0.00225	3
8	11	-0.00304	18:3	10	-0.00098	12:1:3:14:18:7	5	-0.00201	3	11	-0.00224	3
9	13	-0.00288	18:3	11	-0.00085	1:3:18:7	12	-0.00200	3	5	-0.00215	3
10	9	-0.00259		9	-0.00073		13	-0.00193	3	14	-0.00210	3
11	12	-0.00258	3	2	-0.00067	18:7	14	-0.00193		13	-0.00203	3
12	14	-0.00235		12	-0.00050		7	-0.00184	3	7	-0.00203	3
13	1	-0.00225	3	16	-0.00050		1	-0.00154	3	1	-0.00179	3
14	7	-0.00215		1	-0.00047		17	-0.00143	3	17	-0.00178	3
15	2	-0.00189		3	-0.00045		18	-0.00135	3	18	-0.00159	3
16	16	-0.00185		14	-0.00041		16	-0.00134		16	-0.00158	
17	18	-0.00179		18	-0.00035		2	-0.00126		2	-0.00155	
18	3	-0.00117		7	-0.00025		3	-0.00075		3	-0.00081	

Panel (B): $\alpha_x = \alpha_y = 1$

	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
ranking	group	population poverty change	groups dominated	group	population poverty change	groups dominated	group	population poverty change	groups dominated	group	population poverty change	groups dominated
1	3	-0.00544	13:6:15:11:4:10:2:5: 12:16:17:14:8:7:1:18	3	-0.00026	13:6:4:2:15:10:9:11: 8:17:16:12:1:18:7:14	3	-0.00027	13:6:10:15:4:8:2:9: 11:16:12:17:18:14:1:7	3	-0.00031	13:6:10:15:9:4:8:2: 11:16:12:17:14:18:1:7
2	9	-0.00478	11:2:5:12:16:17:14:8: 7:1:18	13	-0.00018	9:11:8:17:16:12:1:18: 7:14	13	-0.00020	9:11:12:17:18:14:1:7	13	-0.00024	9:2:11:12:17:14:18:1: 7
3	13	-0.00460	6:15:11:2:5:12:16:17: 14:8:7:1:18	6	-0.00016	11:8:17:16:12:1:18:7: 14	5	-0.00020		5	-0.00022	
4	6	-0.00396	5:12:16:17:14:8:7:1: 18	5	-0.00013		6	-0.00017	12:17:18:14:1:7	6	-0.00018	12:17:14:18:1:7
5	15	-0.00384	12:16:17:14:8:7:1:18	4	-0.00013	12:1:18:7:14	10	-0.00016	12:17:18:14:1:7	10	-0.00018	12:17:14:18:1:7
6	11	-0.00371	12:17:14:8:7:1:18	2	-0.00012	12:1:18:7:14	15	-0.00015	12:17:18:14:1:7	15	-0.00018	12:17:14:18:1:7
7	4	-0.00361	12:17:14:8:7:1:18	15	-0.00012	12:1:18:7:14	4	-0.00014	17:18:14:1:7	9	-0.00016	12:17:14:18:1:7
8	10	-0.00360	12:17:14:8:7:1:18	10	-0.00012	12:1:18:7:14	8	-0.00014	7	4	-0.00015	14:18:1:7
9	2	-0.00314	17:8:7:1:18	9	-0.00011	12:1:18:7:14	2	-0.00013	18:14:1:7	8	-0.00015	18:1:14
10	5	-0.00233		11	-0.00010	12:1:18:7:14	9	-0.00013	17:18:14:1:7	2	-0.00015	18:1:14
11	12	-0.00203	18	8	-0.00008		11	-0.00012	17:18:14:1:7	11	-0.00014	17:14:18:1:7
12	16	-0.00194		17	-0.00007	18:7:14	16	-0.00010		16	-0.00011	
13	17	-0.00179		16	-0.00005		12	-0.00007	7	12	-0.00008	7
14	14	-0.00174		12	-0.00004		17	-0.00006	7	17	-0.00008	7
15	8	-0.00167		1	-0.00003		18	-0.00005	7	14	-0.00006	
16	7	-0.00165		18	-0.00002		14	-0.00005		18	-0.00005	7
17	1	-0.00153		7	-0.00002		1	-0.00004		1	-0.00005	
18	18	-0.00119		14	-0.00002		7	-0.00002		7	-0.00002	

Source: authors' analysis based on data from the SAIHS 1993. Note: Difference in poverty change between groups significant at 5 percent

Table 6: Population share and poverty gaps in the monetary and nutritional dimensions: South Africa 1993

group	population share	monetary		nutrition	
		P0	P1	P0	P1
1	0.068	0.251	0.078	0.210	0.021
2	0.013	0.515	0.207	0.356	0.040
3	0.146	0.894	0.473	0.384	0.048
4	0.016	0.593	0.209	0.261	0.036
5	0.015	0.383	0.125	0.361	0.057
6	0.140	0.649	0.232	0.324	0.041
7	0.038	0.271	0.085	0.115	0.014
8	0.022	0.274	0.097	0.286	0.046
9	0.024	0.785	0.399	0.284	0.024
10	0.034	0.591	0.250	0.306	0.033
11	0.063	0.610	0.252	0.279	0.028
12	0.025	0.333	0.096	0.196	0.018
13	0.151	0.755	0.355	0.346	0.039
14	0.011	0.285	0.118	0.171	0.012
15	0.057	0.631	0.274	0.287	0.031
16	0.012	0.318	0.125	0.273	0.026
17	0.029	0.294	0.123	0.412	0.038
18	0.137	0.196	0.063	0.193	0.021
National	1	0.554	0.241	0.292	0.034

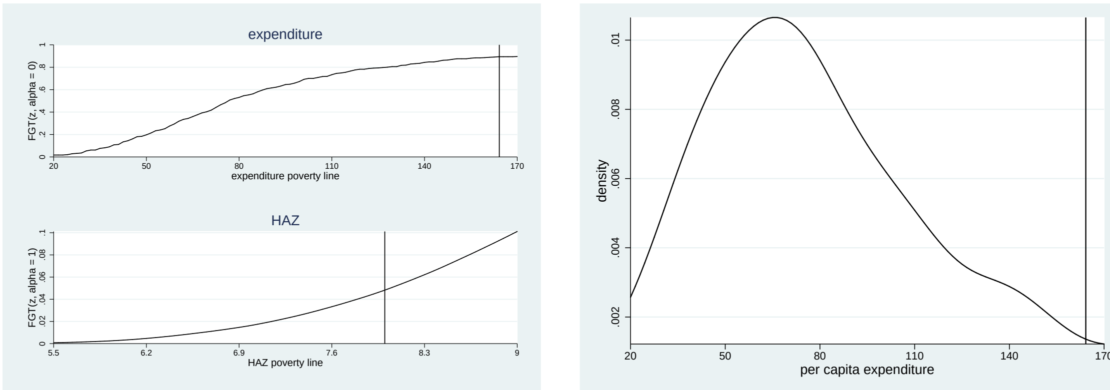
Source: authors' analysis based on data from the SAIHS 1993

3.3.1 Bivariate and Univariate Poverty Comparison

This section presents the results of bivariate and univariate dominance tests. Recall that the condition for distribution *A* to dominates distribution *B* is that the difference between multidimensional poverty in *A* is lower than multidimensional poverty in *B* over a sufficiently large range of poverty lines. In other terms, over a stochastic dominance surface delimited by different combination of poverty lines, after a social policy the reduction in multidimensional poverty in *A* is larger than the reduction in multidimensional poverty in *B*.

Let us start with multidimensional poverty dominance tests (for simplicity, only dominance tests for $\alpha_x = \alpha_y = 1$ are presented here). Recall that we used 10 different poverty

Figure 1: Expenditure density and FGT measures in South Africa in unit 3



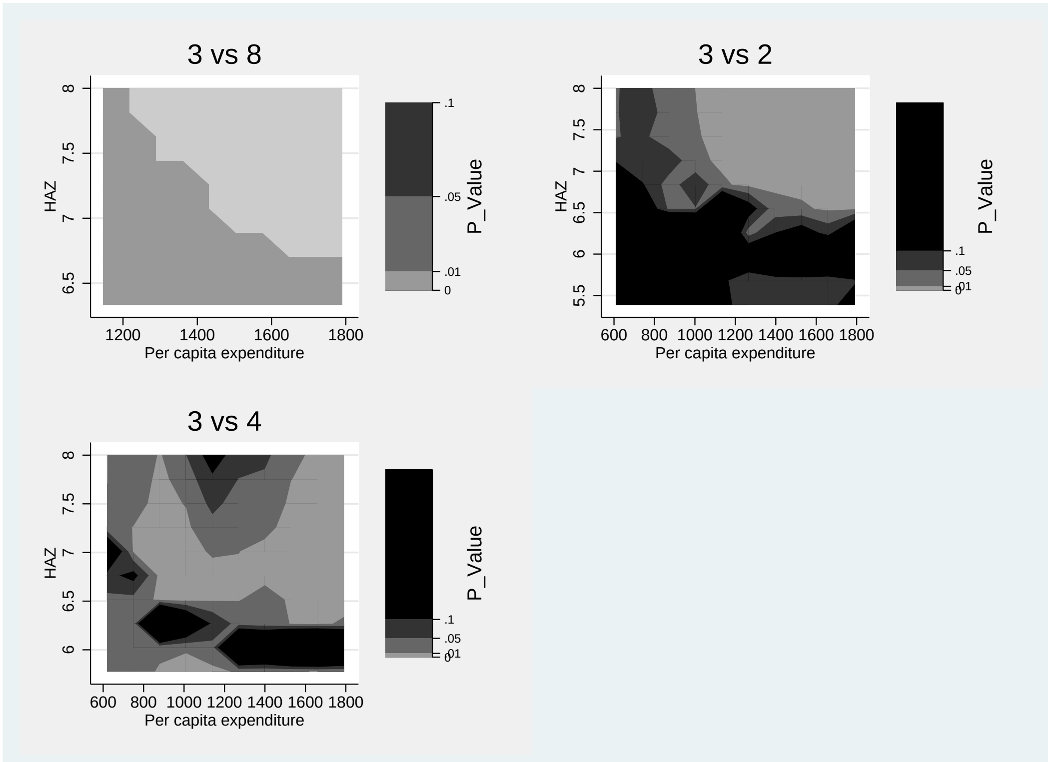
(a) FGT measures (b) Expenditure density

Source: authors' analysis based on data from SAIHS 1993

lines for the two dimensions, giving 100 possible combinations. We estimated the deciles of the segment given by the difference between the official poverty line and the minimum value in a given distribution; then this segment is added to that minimum value. This is where the 10 poverty lines for each dimension where set. The upper limit of the resulting region (highest right corner) corresponds to the official poverty lines, while the poorest people are in the lower left corner. Let limit the discussion to some interesting cases and start with Vietnam 1992-1993. As reported in Figure 2, when unit 3 is compared to unit 8, its dominance is verified over the whole region plot constructed for this test. If compared to unit 2, unit 3 is dominant only for upper nutritional poverty lines. While the dominance is confirmed also for lower poverty lines with respect to the monetary dimension, for *HAZ* values equal or below (around) 6.5 nutritional poverty gaps in the two units are not statistically different. Finally, unit 3 dominates unit 4 for most of the subregions traced by the poverty lines. However, it is interesting to note that a statistically robust result cannot be established when the monetary poverty lines is set between 1200 and 1800, and the nutritional poverty line below 6. Indeed, in this subregion plot there are no children.

Let turn to South Africa — Figure 3. As already discussed, for $\alpha_x = \alpha_y = 1$, unit 3 should be the group to be targeted first. Some specific comparisons are worth to be presented more in detail. Unit 3 dominates unit 14 over the whole region of poverty lines. As compared to unit 13, unit 3 dominates only limited to the region identified by monetary poverty lines above around 70 Rand and nutritional poverty lines above around 7.5. While the multiplication of the monetary and nutritional poverty alone does allow a robust ranking

Figure 2: Testing the dominance of targeting unit 3 versus other units



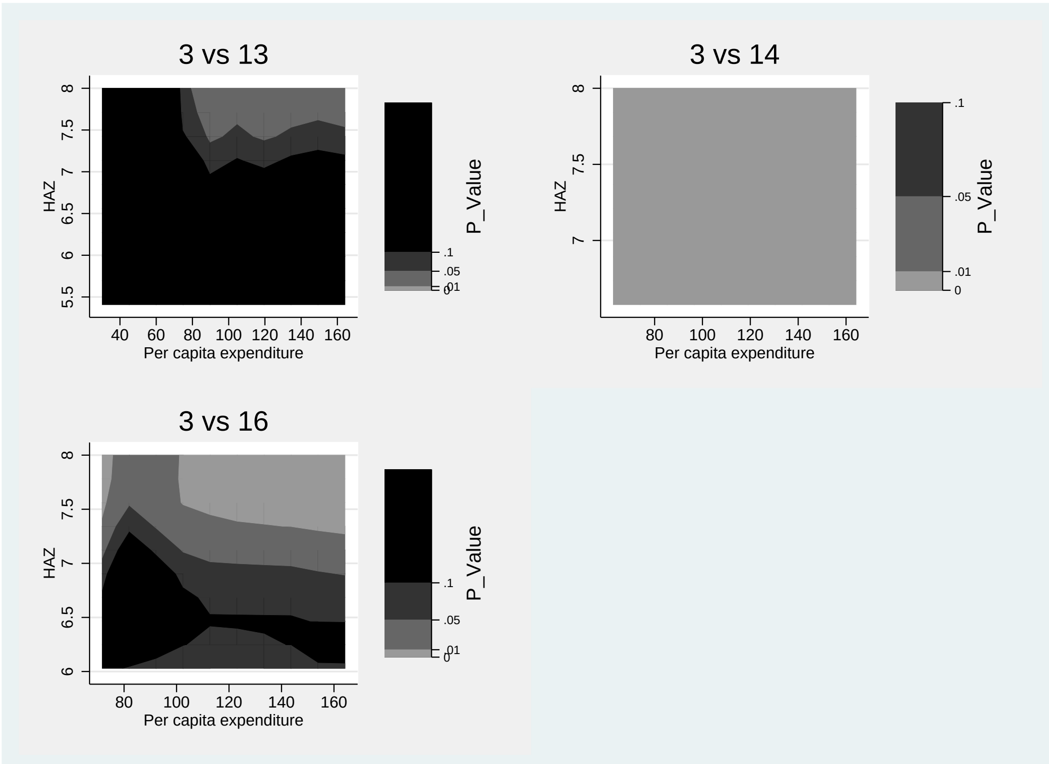
Note: the graphs show the P-value of the difference in poverty change;
Source: authors' analysis based on data from the VLSS 1992-1993

even for lower poverty lines, this is not anymore the case when the correlation component is added in. With regard to unit 16, under the univariate perspective unit 3 dominates in the case of the monetary dimension over the whole range of poverty lines; this is not the case of the nutritional component. For lower monetary and nutritional poverty lines, unit 3 stops to dominate unit 16 when the nutritional component is added (that is, multiplied by the monetary poverty).

As discussed above, focusing on the reduction of univariate or of multidimensional poverty does not necessarily lead to the same policy agenda. It is theoretically clear, and empirically observed, that some socio-economic targeting scheme may be efficient at reducing univariate poverty, but may be sub optimal at alleviating multidimensional poverty; the reverse is also true.

Consider the results shown in Figure 4 as an example. The Figure displays the poverty impact differences of targeting unit 6 with respect to units 10, with respect to a relatively large range of poverty lines. From the perspective of a deviating uni-dimensional univariate

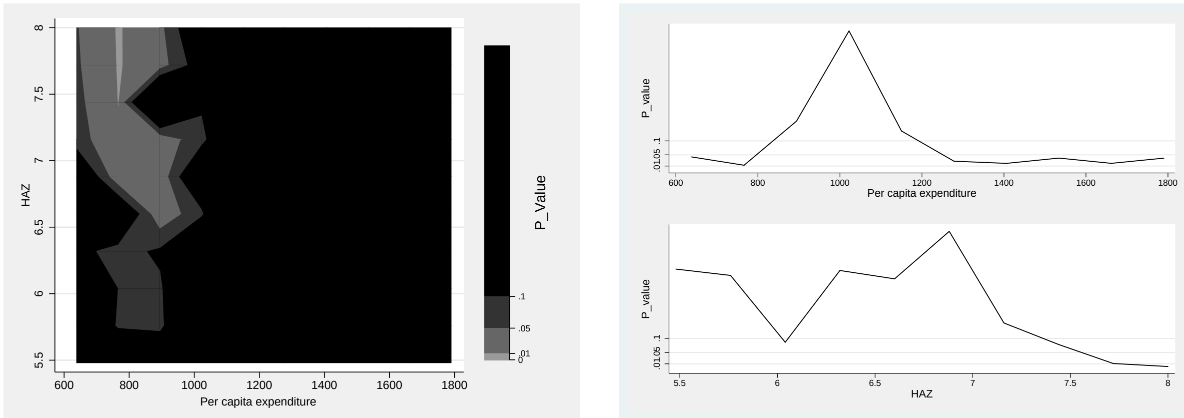
Figure 3: Testing the dominance of targeting unit 3 versus other units



Note: the graphs show the P-value of the difference in poverty change;
Source: authors' analysis based on data from the SAIHS 1993

poverty in either dimension (Panel (b)), it is possible to favor unit 6 with respect to unit 10 over a relative wide region (though results for nutritional dimension are robust only for poverty lines equal or larger than 7.5). The preference becomes, however, not statistically significant when assessing multidimensional poverty over the largest part of the considered region of poverty lines (Panel (a)).

Figure 4: Testing the dominance of targeting of group 6 versus 10

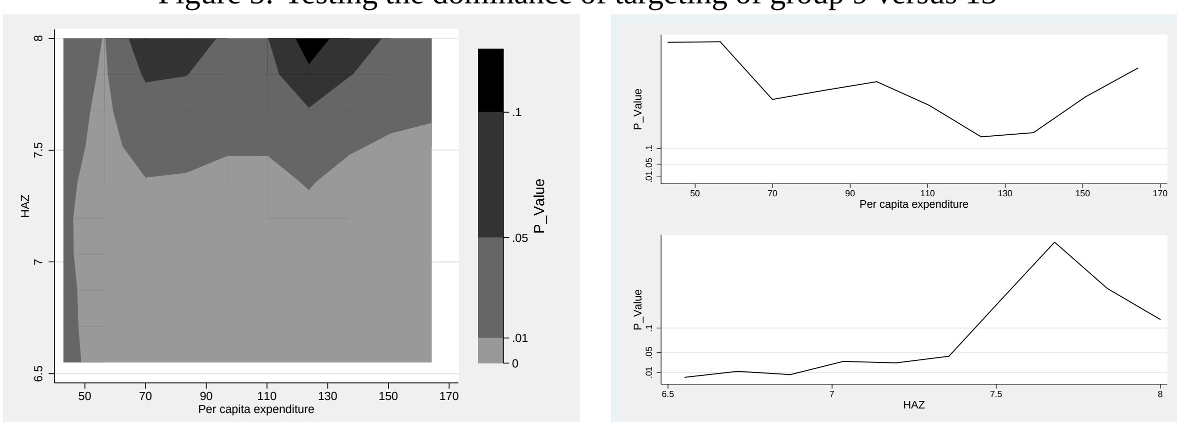


(a) multidimensional (b) unidimensional

Note: the graphs show the P-value of the difference in poverty change;
Source: authors' analysis based on data from the VLSS 1992-1993

Conversely, using the South African survey, although in most cases neither univariate poverty reduction is conclusive, for a large range of combinations of poverty lines presented here we can conclude that reduction in bivariate poverty in unit 9 is statistically dominant that in unit 13 — see Figure 5.

Figure 5: Testing the dominance of targeting of group 9 versus 13



(a) multidimensional (b) unidimensional

Note: the graphs show the P-value of the difference in poverty change;
Source: authors' analysis based on data from the SAIHS 1993

4 Conclusion

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A Appendix 1

Following Theorem ### , one can state formally that targeting group A leads to a greater decrease in (bi or uni-dimensional) poverty, and then a greater increase in social welfare than targeting group B at order s if and only if:

$$\Delta^s(z_1, z_2) = \Delta P_A(z_1, z_2; s) - \Delta P_B(z_1, z_2; s) < 0 \, \forall \, z_1 \in [0, z_1^+] \text{ and } \forall \, z_2 \in [0, z_2^+]. \quad \text{(A.1)}$$

For statistical tests of dominance of targeting a given group over another, a natural formulation of a null hypothesis is thus that of a union of null hypotheses:

$$H_0 : \Delta^s(z_1, z_2) < 0 \text{ for some } \forall \, z_1 \in [0, z_1^+] \text{ and } \forall \, z_2 \in [0, z_2^+] \quad \text{(A.2)}$$

to be tested against an alternative hypothesis that is an intersection of alternative hypotheses

$$H_1 : \Delta^s(z_1, z_2 \geq 0 \text{ for all } \forall \, z_1 \in [0, z_1^+] \text{ and } \forall \, z_2 \in [0, z_2^+]. \quad \text{(A.3)}$$

The decision rule we adopt is then to reject the union set of null hypotheses (non-dominance) in favor of the intersection set of alternative hypotheses (dominance) only if we can reject each of the individual hypotheses in the null set at a $100 \cdot \theta\%$ significance level.

To see this in greater details, denote by $\hat{\Delta}^s(z)$ the sample estimator of $\Delta^s(z)$, by $\Delta_0^s(z)$ its sample value, and by $\sigma^2_{\hat{\Delta}^s(z)}$ the sampling variance of $\hat{\Delta}^s(z)$. It is worth noting here that the first order Taylor approximation approach was used in order to estimate the standard errors. [3] Given that by the law of large numbers and the central limit theorem, all of the estimators used in this paper can be shown to be consistent and asymptotically normally distributed. Among the statistical test values, that one can use in order to take the statistical judgement on the test, is the *P-value*. This value is equal to the smallest significance level for which the null hypothesis would be rejected in favor of the alternative hypothesis. Let $\hat{\upsilon}(z_1, z_2)$ denotes the estimated *P-value* of the test for a given combination of poverty lines Let (z_1, z_2) . Our decision rule is then to reject the set of null hypotheses (A.2) in favor of

[3] See Duclos and Araar (2006), chapter 17 and Rao (1973).

(A.3) if and only if:

$$\hat{v}(z_1, z_2) < \theta \; \forall \; z_1 \in [0, z_1^+] \text{ and } \forall \; z_2 \in [0, z_2^+]. \tag{A.4}$$

B Appendix 2

Table 7: Impact of targeting monetary dimension on bi-dimensional poverty: Vietnam 1997-1998

Panel (A): $\alpha_x = \alpha_y = 0$

Ranking	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
1	1	-0.00063	6:8:7:5:4	1	-0.00028	6:8:7:5:4	1	-0.00031	9:7:5:8:4	1	-0.00034	9:7:5:8:4
2	2	-0.00057	6:8:7:5:4	10	-0.00028	6:8:7:5:4	2	-0.00026	5:8:4	2	-0.00027	5:8:4
3	3	-0.00053	4	2	-0.00027	6:8:7:5:4	10	-0.00023	8:4	10	-0.00024	8:4
4	10	-0.00052	6:7:5:4	9	-0.00024	7:5:4	6	-0.00020	4	6	-0.00022	4
5	9	-0.00043	5:4	3	-0.00023	5:4	9	-0.00016		9	-0.00018	
6	6	-0.00033	5:4	6	-0.00011	5:4	3	-0.00016		3	-0.00017	
7	8	-0.00029		8	-0.00010		7	-0.00015		7	-0.00017	
8	7	-0.00026	4	7	-0.00009	5:4	5	-0.00014		5	-0.00016	
9	5	-0.00019		5	-0.00004		8	-0.00012		8	-0.00015	
10	4	-0.00011		4	-0.00002		4	-0.00006		4	-0.00009	

Panel (B): $\alpha_x = \alpha_y = 1$

Ranking	$-(\alpha_x/z_x)[P_x(\alpha_x-1)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)]$			$-(\alpha_x/z_x)[P_x(\alpha_x-1)P_y(\alpha_y)+Cov(.)]$			Total impact with spill-over		
	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated	Group	Population poverty change	Groups dominated
1	3	-0.00043	1:2:6:8:7:5:4	9	-0.00003	1:6:8:7:4:5	9	-0.00003	3:1:8:6:7:4:5	9	-0.00003	3:1:8:6:7:4:5
2	10	-0.00036	1:2:6:8:7:5:4	10	-0.00002	1:6:8:7:4:5	10	-0.00002	1:8:6:7:4:5	10	-0.00002	1:8:6:7:4:5
3	9	-0.00035	6:8:7:5:4	2	-0.00001	6:8:7:4:5	2	-0.00002	6:7:4:5	2	-0.00002	8:6:7:4:5
4	1	-0.00028	6:8:7:5:4	3	-0.00001	4:5	3	-0.00001	4:5	3	-0.00001	4:5
5	2	-0.00028	6:8:7:5:4	1	-0.00001	6:4:5	1	-0.00001	4:5	1	-0.00001	4:5
6	6	-0.00016	5:4	6	-0.00001	4:5	8	-0.00001	4:5	8	-0.00001	4:5
7	8	-0.00014		8	-0.00001		6	-0.00001	4:5	6	-0.00001	4:5
8	7	-0.00011		7	-0.00000		7	-0.00001	5	7	-0.00001	5
9	5	-0.00006		4	-0.00000		4	-0.00000		4	-0.00000	
10	4	-0.00006		5	-0.00000		5	-0.00000		5	-0.00000	

Source: authors' analysis based on data from the VLSS 1997-1998. Note: Difference in poverty change between groups significant at 5 percent

Table 8: Population share and poverty gaps in the monetary and nutritional dimensions: Vietnam 1997-1998

Group	**Population share**	**Monetary**		**Nutrition**	
		P0	P1	P0	P1
1	0.123	0.495	0.113	0.448	0.039
2	0.223	0.495	0.122	0.481	0.053
3	0.028	0.773	0.255	0.440	0.028
4	0.059	0.105	0.022	0.198	0.018
5	0.031	0.116	0.015	0.202	0.013
6	0.103	0.288	0.087	0.339	0.032
7	0.018	0.197	0.063	0.350	0.034
8	0.030	0.252	0.048	0.363	0.036
9	0.049	0.627	0.232	0.560	0.073
10	0.337	0.649	0.188	0.546	0.057
Population	1	0.493	0.136	0.456	0.047

Source: authors' analysis based on data from the VLSS 1997-1998