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New Estimates of Canadian TFP Growth and the Contribution of Changes in Real Export and Import Prices to Real Income Growth

W. Erwin Diewert and Emily Yu

For additional information please contact:

Name: W. Erwin Diewert
Affiliation: University of British Columbia, Canada

Email Address: diewert@econ.ubc.ca

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W. Erwin Diewert[1] and Emily Yu
Department of Economics
University of British Columbia
Vancouver, Canada, V6T 1Z1
Email: diewert@econ.ubc.ca

Abstract

Using new data from Statistics Canada, the paper shows that the productivity performance of the business sector of the Canadian economy has been reasonably satisfactory over the past 47 years. In particular, traditional gross income Total Factor Productivity (TFP) growth averaged 1.01 percentage points per year over the period 1961-2007 and when a net income framework was used, TFP growth averaged 1.04 percentage points per year. The focus of the study is on the real income generated by the business sector of the Canadian economy. Two concepts of income are used: a gross concept that includes depreciation as a part of income and a more appropriate net concept where depreciation is excluded from income. In both the gross and net income frameworks, the growth of quality adjusted labour input growth was the main driver of growth in real income followed by TFP growth, followed by growth in capital input and then by falling real import prices. However, in recent years, the contribution of falling real import prices turned out to be more than twice as important as capital deepening. A feature of this study is that we look at the contribution of changes in real export and import prices by commodity class. We find that increases in real energy export prices contributed an average of 0.18 percentage points per year to real (net) income growth while falls in the real export prices of machinery and equipment and automotive products subtracted an average of 0.17 and 0.13 percentage points per year from real (net) income growth. Somewhat surprisingly, the effects on real income growth of changing import prices by commodity class were fairly small with two exceptions: falling real prices of machinery and equipment imports (excluding autos) contributed an average of 0.49 percentage points per year to (net) real income growth while rising real prices of energy imports subtracted 0.13 percentage points per year. The falling real prices of imports of other consumer goods only added an average of 0.06 percentage points per year to real income growth. The study encountered many data problems which should be addressed in future work on Canadian business sector productivity performance.

Journal of Economic Literature Classification Numbers

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Keywords

Total factor productivity, real income, terms of trade effects, measurement of capital, measurement of inventory change, user costs, real interest rates.

1. Introduction

Many observers have noted that an improvement in a country's terms of trade has effects that are similar to an improvement in a country's productivity growth. However, it is not straightforward to work out the exact magnitude of each source of gain. Diewert (1983), Diewert and Morrison (1986), Diewert, Mizobuchi and Nomura (2005), Diewert and Lawrence (2006), Morrison and Diewert (1990) and Kohli (1990) (1991) (2003) (2004a) (2004b) (2006) (2007) developed production theory methodologies which enable one to obtain index number estimates of the contribution of each type of gain. In Appendix 1 below, we outline the Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) methodology and in sections 2-4 of the main text, we apply this methodology to the business sector of the Canadian economy over the years 1961-2007.

Appendix 2 below describes how the Canadian business sector data were developed from Statistics Canada sources.[2] Section 2 of the main text aggregates up the data from Appendix 2 and develops conventional measures of Canadian business sector Total Factor Productivity for the years 1961-2007.

Productivity growth, while perhaps the most important source of growth in living standards, is not the entire story. If a country's export prices increase more rapidly than its import prices, then it is well known that this has an effect that is similar to a productivity improvement.[3] Thus in section 3, we measure the relative contributions of productivity improvements, changes in real export and import prices and the growth of labour and capital input to the growth of (gross) real income generated by the business sector in Canada using the methodology explained in sections 1-4 of Appendix 1.

However, this is not the end of the story. GDP is an (imperfect) measure of productive potential, not welfare.[4] For welfare measurement purposes, it is generally conceded that Net Domestic Product (NDP) is a better measure of output, since investment that just meets depreciation means that society is not made any better off from the viewpoint of

[2] This paper is similar to Diewert (2008) but there are some new developments in the present paper: (i) Statistics Canada has substantially revised their published KLEMS data base and we use these revised data as of October 2008 in the present paper; (ii) Statistics Canada made available to the authors some unpublished data from their KLEMS data base on land, inventories and ICT capital which improved our earlier estimates and narrowed the differences between our estimates of TFP growth in Canada as compared to the official Statistics Canada estimates and (iii) we provide a more detailed breakdown of exports and imports in the present paper so that the effects of changes in real export and import prices by commodity category on real income can be determined.

[3] See for example Diewert and Morrison (1986).

[4] For a more extensive discussion of the merits of GDP versus net income, see Diewert (2006a).

sustainable final consumption possibilities. Hence, in the second part of the paper, we subtract depreciation off from gross investment and use consumption plus sales to the nonbusiness sector plus *net investment* plus the trade balance as our business sector output concept. Thus depreciation will be treated as an intermediate input in this model of production. Section 5 of Appendix 1 explains this real net product approach and adapts a translog model of production based on the work of Diewert and Morrison (1986) and Kohli (1990) to this new model of market sector real net income generation.[5] This approach is implemented for the Canadian business sector in section 4 of the main text. The main determinants of growth in real net income generated by the business or market sector of the economy are:

- Technical progress or improvements in Total Factor Productivity;
- Growth in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- Growth in primary inputs.

It turns out that productivity growth becomes a more important factor for explaining real *net* income growth compared to explaining real *gross* income growth. Also the importance of capital deepening is greatly reduced in the net income framework compared to the gross income framework. Somewhat surprisingly, for the years 2000-2007, improvements in the terms of trade made almost the same contribution to real income growth as capital deepening in the gross income framework and in the net income framework, the effects of falling real import prices contributed substantially more to real income growth than capital deepening over the period 2000-2007.

Section 5 of the main text looks at changes in real export and import prices by commodity class and their effects on real income growth in both the gross income and the net income frameworks. These new results are somewhat surprising.

Appendix 3 compares our methodology for determining the effects of improvements in the terms of trade on real income growth with the methodology worked out by Kohli (2006).

Appendix 4 compares our estimates of TFP growth with the business sector Multifactor Productivity Growth estimates recently developed by the Statistics Canada KLEMS program; see Baldwin, Gu and Yan (2007) for a description of the methodology used in the KLEMS program. It should be noted that the Statistics Canada KLEMS program uses detailed industry data in order to construct TFP estimates by industry and then these industry estimates are aggregated up to give aggregate business sector estimates of TFP growth, whereas our approach uses aggregate estimates for the outputs produced and inputs used for the entire business sector. Thus our estimates may suffer from some aggregation bias.

[5] For previous implementations of this model of real net income to Japan and Australia, see Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

Section 6 concludes.[6]

2. Output and Input Aggregates and Conventional Productivity Growth for Canada

In Appendix 2, we constructed price and quantity series for 23 net outputs and 9 primary inputs for the business sector of the Canadian economy for the years 1961-2007. The 23 net outputs are:

- Q_1: Domestic consumption (excluding market residential rents and the services of owner occupied housing);
- Q_2: Real sales of goods and services by the business sector to the nonmarket sector less real sales of goods and services from the nonmarket sector to the business sector;
- Q_3: Government investment;
- Q_4: Business sector investment in residential structures;
- Q_5: Business sector investment in ICT machinery and equipment;
- Q_6: Business sector investment in non ICT machinery and equipment;
- Q_7: Business sector investment in nonresidential structures;
- Q_8: Inventory change;
- Q_9: Exports of agricultural and fish products;
- Q_{10}: Exports of energy products;
- Q_{11}: Exports of forest products;
- Q_{12}: Exports of industrial goods and materials (excluding energy and forest product exports);
- Q_{13}: Exports of machinery and equipment (excluding automotive products);
- Q_{14}: Exports of automotive products;
- Q_{15}: Exports of other consumer goods (excluding automotive products);
- Q_{16}: Exports of services;
- Q_{17}: Imports of agricultural and fish products;
- Q_{18}: Imports of energy products;
- Q_{19}: Imports of industrial goods and materials (including imports of forest products but excluding imports of energy products);
- Q_{20}: Imports of machinery and equipment (excluding automotive products);
- Q_{21}: Imports of automotive products;
- Q_{22}: Imports of other consumer goods and
- Q_{23}: Imports of services.

[6] The final section of Appendix 1 has some new material on how the real net income model used in the present paper can be extended from a single production sector to the case of many industries. Also the final section of Appendix 4 has some suggestions for improving the measurement of productivity by Statistics Canada but these recommendations may be useful for other official productivity programs. The next revision of the *System of National Accounts 1993* will introduce capital services into the accounts so that price and quantity decompositions of primary inputs will be possible, which will greatly facilitate the measurement of productivity by sector. However, the introduction of capital services into the main production accounts will not be easy and it will be necessary for statistical agencies to do a considerable amount of preparatory work.

The nine primary inputs into the business sector are:

- Q_{24}: The labour services of workers with primary or secondary education
- Q_{25}: The labour services of workers with some or completed post secondary certificate or diploma;
- Q_{26}: The labour services of workers with a university degree or above;[7]
- Q_{27}: The stock of ICT machinery and equipment available to the business sector at the start of each year;
- Q_{28}: The stock of non ICT machinery and equipment available to the business sector at the start of each year;
- Q_{29}: The starting stock of business sector nonresidential structures;
- Q_{30}: The starting stocks of inventories used by the business sector;
- Q_{31}: The stock of agricultural land used by the business sector;
- Q_{32}: The stock of nonagricultural, nonresidential land used by the business sector.

As is explained in Appendix 2, user cost prices for the last six primary inputs were constructed, using balancing or endogenous real rates of return that made the value of net output produced by the business sector equal to the value of primary inputs used by the business sector. All of the price and quantity series for the above 32 outputs produced and inputs used by the Canadian business sector are listed in Appendix 2.

In this section, we will aggregate the above net outputs and primary inputs into D, domestic output, equal to an aggregate of the first eight net outputs listed above; X, exports equal to an aggregate of the eight types of exports of goods and services; M, imports equal to an aggregate of seven types of imports of goods and services; L, labour services equal to an aggregate of the three types of labour and K, capital services equal to an aggregate of the six types of capital services. Once these aggregates have been constructed, we further aggregate the three net outputs, D+X−M, into real gross domestic product Y and aggregate the two inputs, L and K, into domestic input Z and finally construct a conventional measure of productivity Y/Z. The aggregations were performed using chained Törnqvist price indexes.[8] The results are listed in Tables 1 and 2 below.

Table 1: Prices of Canadian Business Sector Output and Input Aggregates

Year t	P_C^t	P_D^t	P_X^t	P_M^t	P_L^t	P_K^t	P_Y^t	P_Z^t
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.00538	1.00538	1.02992	1.05787	1.03625	1.01691	0.99683	1.03034
1963	1.02055	1.02264	1.04054	1.09648	1.06604	1.15494	1.00691	1.09305
1964	1.02437	1.02959	1.05692	1.10526	1.11049	1.23317	1.01577	1.14776
1965	1.03690	1.05092	1.07980	1.10214	1.18272	1.25857	1.04400	1.20580
1966	1.07553	1.08861	1.12451	1.11930	1.25929	1.34193	1.08930	1.28444
1967	1.11050	1.12289	1.15421	1.14426	1.33342	1.23578	1.12505	1.30269

[7] These three types of labour input are taken directly from Statistics Canada recent KLEMS program; see Baldwin, Gu and Yan (2007; 26-27) for a description of these data.
[8] More specifically, the chained Divisia option in Shazam was used to do the aggregations.

1968	1.15168	1.15976	1.20119	1.16726	1.41620	1.30595	1.16924	1.38147
1969	1.18980	1.19995	1.23088	1.19648	1.52197	1.34691	1.20982	1.46616
1970	1.22208	1.23858	1.2671	1.21965	1.61278	1.40223	1.25289	1.54532
1971	1.24828	1.28233	1.28552	1.24798	1.72528	1.39719	1.29339	1.61828
1972	1.29847	1.34008	1.33325	1.27498	1.86392	1.49709	1.35869	1.74411
1973	1.38744	1.44587	1.51489	1.35954	2.03837	2.06511	1.49788	2.05198
1974	1.58382	1.65206	1.91283	1.64641	2.35044	2.47332	1.73334	2.39718
1975	1.82198	1.87090	2.16694	1.89029	2.70332	2.21008	1.95274	2.53719
1981	1.90726	1.97372	2.29702	1.92853	3.10646	2.39955	2.09001	2.86510
1977	2.03175	2.09967	2.50231	2.17241	3.38889	2.74058	2.19373	3.17107
1978	2.19264	2.26119	2.73837	2.41667	3.53495	3.09969	2.34654	3.39783
1979	2.40645	2.4806	3.20786	2.73027	3.78520	3.80986	2.60728	3.82051
1980	2.69497	2.75772	3.73464	2.97957	4.11781	3.97428	2.97313	4.09308
1981	2.95335	3.03059	3.99821	3.26618	4.59295	4.05499	3.23724	4.42379
1982	3.22860	3.28484	4.08926	3.43918	5.02021	3.61928	3.46983	4.51822
1983	3.46323	3.46344	4.15051	3.42324	5.22085	4.44415	3.68247	4.97537
1984	3.61506	3.60128	4.29656	3.58225	5.48101	4.94318	3.81283	5.33580
1985	3.72257	3.71138	4.38071	3.67711	5.75670	5.17976	3.91833	5.59935
1986	3.80422	3.80294	4.37060	3.74437	5.90250	5.0834	3.98140	5.65108
1987	3.89726	3.90844	4.45792	3.69150	6.11054	5.74526	4.14995	6.03889
1988	4.00205	4.01307	4.47080	3.60108	6.51242	5.76285	4.30933	6.29679
1989	4.11690	4.12618	4.56005	3.59364	6.78864	5.58998	4.47092	6.40227
1990	4.35206	4.28654	4.52868	3.64367	7.04667	5.31935	4.60964	6.45499
1991	4.59099	4.42296	4.37107	3.57992	7.34245	4.50786	4.72731	6.30363
1992	4.65258	4.47108	4.49573	3.72567	7.48023	4.98747	4.75269	6.58795
1993	4.74252	4.55596	4.69389	3.9246	7.45961	5.15125	4.82121	6.64462
1994	4.77089	4.62372	4.97322	4.16089	7.41314	6.03823	4.89058	6.98418
1995	4.79147	4.65787	5.29132	4.27688	7.53622	6.56286	5.02353	7.27300
1996	4.88952	4.71581	5.32097	4.22185	7.63205	6.99768	5.13502	7.50733
1997	4.96547	4.77523	5.32718	4.23677	7.91076	7.00934	5.19090	7.68381
1998	5.03224	4.84195	5.31558	4.37960	8.13918	6.93415	5.15347	7.79482
1999	5.12045	4.91274	5.37870	4.36044	8.33890	7.36196	5.27960	8.08916
2000	5.25425	5.02564	5.71039	4.44227	8.74780	8.33839	5.54730	8.73198
2001	5.40970	5.14618	5.80311	4.58416	8.97770	8.04237	5.62691	8.75403
2002	5.47743	5.22044	5.68895	4.61494	9.09489	8.53655	5.61217	9.02599
2003	5.61543	5.29558	5.64768	4.31493	9.26253	8.32732	5.87497	9.04388
2004	5.69551	5.37836	5.78629	4.20643	9.48779	9.25929	6.12630	9.55745
2005	5.81654	5.49862	5.95274	4.15511	9.84265	9.80219	6.39760	9.99376
2006	5.92386	5.63197	5.97000	4.12736	10.29074	9.81133	6.58063	10.27222
2007	6.02712	5.75706	6.02590	4.02177	10.66121	10.14055	6.84476	10.63228

Note that we have also listed the price of our household consumption aggregate, P_C^t, in Table 1, which will play a role in subsequent sections. The *productivity level* in year t of the Canadian business sector T^t can be defined as the aggregate year t output, Q_Y^t divided by aggregate year t input, Q_Z^t:[9]

[9] This is also known as Multifactor Productivity or Total Factor Productivity.

(1) $T^t \equiv Q_Y{}^t/Q_Z{}^t$; t = 1961, ..., 2007.

Productivity growth for year t, τ^t, is defined as the productivity level in year t divided by the previous year's productivity level:

(2) $\tau^t \equiv T^t/T^{t-1}$; t = 1962, ..., 2007.

Table 2 lists the quantities that match up to the prices in Table 1 and it also lists productivity levels and growth rates.

Table 2: Quantities of Canadian Business Sector Output and Input Aggregates, TFP Levels and TFP Growth Rates

Year t	$Q_D{}^t$	$Q_X{}^t$	$Q_M{}^t$	$Q_L{}^t$	$Q_K{}^t$	$Q_Y{}^t$	$Q_Z{}^t$	T^t	τ^t
1961	28752	6867	-7897	19202	8520	27722	27722	1.00000	
1962	30578	7195	-8033	20042	8739	29750	28782	1.03361	1.03361
1963	32261	7832	-8031	20574	9007	32113	29582	1.08555	1.05024
1964	34602	9105	-8989	21446	9324	34766	30768	1.12995	1.04090
1965	37904	9418	-10180	22416	9751	37149	32164	1.15499	1.02216
1966	40831	10696	-11579	23550	10328	39949	33880	1.17914	1.02091
1967	41117	11827	-12306	24056	11057	40656	35112	1.15790	0.98198
1968	42835	12910	-13527	24158	11627	42246	35756	1.18152	1.02040
1969	46062	13802	-15377	24718	12059	44521	36737	1.21189	1.02571
1970	46017	15211	-15293	24798	12569	45988	37285	1.23341	1.01776
1971	48326	15929	-16480	25333	13007	47844	38238	1.25120	1.01442
1972	52098	17257	-18892	26101	13416	50590	39410	1.28367	1.02595
1973	59241	19008	-21754	27591	13866	56663	41362	1.36992	1.06719
1974	65163	18347	-23977	28558	14615	59579	43080	1.38298	1.00953
1975	63453	16951	-23228	28530	15571	57119	43961	1.29930	0.93949
1981	67488	18390	-24774	28499	16309	61084	44559	1.37086	1.05508
1977	70950	19678	-24836	28805	17018	65759	45492	1.44552	1.05446
1978	73085	21544	-26197	30018	17690	68588	47367	1.44801	1.00173
1979	78467	22467	-28092	31737	18344	72879	49736	1.46532	1.01195
1980	77309	22548	-28715	32833	19285	71254	51757	1.37669	0.93951
1981	80810	23012	-30716	33723	20148	73083	53480	1.36653	0.99262
1982	70930	22882	-25710	32059	21330	68632	52707	1.30214	0.95288
1983	75524	24326	-28444	32283	21742	72008	53296	1.35109	1.03759
1984	80477	28444	-33270	33497	22102	76807	54884	1.39943	1.03578
1985	85491	29938	-35548	34871	22585	81087	56743	1.42902	1.02114
1986	88667	31456	-37965	36384	23167	83519	58843	1.41937	0.99325
1987	94688	32933	-39889	38166	23747	89072	61210	1.45517	1.02522
1988	101144	35371	-45163	39912	24550	93146	63746	1.46120	1.00414
1989	104645	35434	-47820	40981	25638	94280	65839	1.43198	0.98000
1990	102201	37556	-48551	41030	26721	93557	66811	1.40032	0.97789
1991	96067	38167	-49281	39707	27455	87853	65884	1.33345	0.95224
1992	98558	40921	-51473	39311	27831	91077	65705	1.38615	1.03952
1993	98528	45382	-55461	40184	28050	92144	66858	1.37821	0.99427
1994	103227	51076	-60606	41745	28099	97970	68602	1.42809	1.03619

1995	105681	55452	-64385	42958	28426	101581	70163	1.44779	1.01380
1996	108585	58646	-67340	44212	28923	105126	71906	1.46199	1.00981
1997	116675	63457	-77378	45636	29440	109300	73839	1.48025	1.01249
1998	120947	69086	-81755	47078	30519	115417	76307	1.51254	1.02182
1999	124963	76337	-88261	48781	31631	121154	79074	1.53216	1.01297
2000	132088	83350	-95661	50512	32736	128862	81864	1.57409	1.02737
2001	132099	80654	-90649	51183	33919	130142	83653	1.55574	0.98834
2002	140327	81599	-92347	52290	34561	137310	85376	1.60829	1.03377
2003	142198	79268	-96341	53129	35172	133617	86799	1.53939	0.95716
2004	150705	83281	-104558	55049	35675	139173	89210	1.56007	1.01343
2005	158450	84730	-112492	55875	36549	141962	90878	1.56211	1.00131
2006	165871	85022	-117772	56905	37720	145225	93035	1.56098	0.99928
2007	173734	86002	-124556	58347	38998	148654	95699	1.55335	0.99511

The average rate of TFP growth over the 46 years 1962-2007 is 1.01% per year,[10] which is much higher than the 0.5 to 0.7% per year range that Diewert and Lawrence (2000) found over the period 1962-1996. The present 1.01% average rate of TFP growth can also be compared with Statistics Canada's recent KLEMS program average Multifactor Productivity Growth over the same years of 0.38% per year,[11] which is a rather substantial difference! In Appendix 4 below, we attempt to determine why our results are so different from the official Statistics Canada results.[12]

Over the golden years 1962-1973, TFP growth averaged 2.68% per year; over the dismal years, 1974-1991, TFP growth was essentially 0 (−0.086 per year) but over the remainder of the nineties, 1992-1999, TFP growth nicely recovered to average a very respectable 1.76% per year. During the naughts, 2000-2007, TFP growth again fell off to average only 0.20% per year. There were two recent poor productivity growth years, 2001 and 2003, where drops of 1.17% and 4.28% occurred and if these years are excluded, the average TFP growth rate during the remaining years of the naughts is about 1.4% per year. Hopefully the Canadian productivity recovery since the recession of 1991 is not a statistical mirage and it will continue into the future.

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[10] This rate of TFP growth is reasonably close to the average rate of productivity growth for Australia obtained by Diewert and Lawrence (2006) using a similar methodology and over a similar period. The Diewert and Lawrence market sector average rate of TFP growth for Australia over the period 1961-2004 was 1.49% per year. However, there is an upward bias in the Diewert and Lawrence results due to the fact that they essentially used hours worked as their measure of labour input instead of a quality adjusted measure of labour input for Australia (which was not available).

[11] See CANSIM II series V41712881, Canada, Multifactor Productivity, Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Comparing levels of TFP with the starting level being 1 in 1961, our TFP ended up at 1.553 in 2007 whereas KLEMS Multifactor Productivity ended up at 1.184 in 2007. This is a very substantial difference.

[12] Our measures of business sector output and capital input were different from the KLEMS measures because we excluded rental housing from our measure of value added and we excluded the land and residential structures inputs from our measure of capital services, whereas the KLEMS measures included rental housing in their output and capital input measures. Our measures of labour input were identical and it turned out that the average rate of growth of our business sector value added measure was very close to the corresponding KLEMS average growth rate. The capital services growth rates differed substantially.

However, productivity growth is not necessarily the entire story behind the growth in living standards: if the price of Canadian exports increases more rapidly than the price of Canadian imports, then the real income generated by the business sector should increase. This terms of trade effect is not taken into account in the above productivity computations. Thus in the following section, we implement the translog real income methodology explained in sections 1- 4 of Appendix 1 below and this approach will enable us to assess the contribution to Canadian living standards of improvements in Canada's terms of trade.

3. Explaining Real Income Growth Generated by the Canadian Business Sector: the Gross Output Approach

The basic methodology used in this section can easily be explained in nontechnical terms. The business sector faces (exogenous) domestic and international prices for the net outputs it produces: domestic outputs, exports and (minus) imports. The business sector also utilizes inputs of labour and capital in order to produce its outputs. The value of outputs produced by the business sector less the value of imports used (value added) must eventually flow back to the labour and capital primary inputs that were used to produce value added. This is the (gross) income generated by the business sector. We divide this gross income in year t by the price of consumption in year t, P_C^t, in order to turn this nominal income into *real income* ρ^t; this real income is the number of consumption bundles that *could* be purchased by the owners of the labour and capital inputs that were used in year t by the Canadian business sector. We also divide each of the prices P_D^t, P_X^t, P_M^t, P_L^t and P_K^t by the price of consumption, P_C^t, in order to form the corresponding real output and input prices facing the Canadian business sector in each year. Our measure of the (gross) real income generated by the business sector ρ^t and the corresponding real output and input prices are listed in Table 3.

Table 3: Gross Real Income Generated by the Canadian Business Sector and Real Output and Input Prices

Year t	ρ^t	P_D^t/P_C^t	P_X^t/P_C^t	P_M^t/P_C^t	P_L^t/P_C^t	P_K^t/P_C^t
1961	27722	1.00000	1.00000	1.00000	1.00000	1.00000
1962	29497	1.00000	1.02441	1.05221	1.03070	1.01147
1963	31684	1.00205	1.01959	1.07440	1.04457	1.13169
1964	34474	1.00509	1.03178	1.07897	1.08407	1.20383
1965	37404	1.01352	1.04137	1.06292	1.14063	1.21379
1966	40460	1.01216	1.04554	1.0407	1.17085	1.24769
1967	41188	1.01115	1.03936	1.03040	1.20074	1.11281
1968	42890	1.00701	1.04299	1.01353	1.22968	1.13395
1969	45270	1.00853	1.03453	1.00561	1.27918	1.13205
1970	47147	1.01350	1.03684	0.99801	1.31970	1.14742
1971	49573	1.02727	1.02983	0.99976	1.38213	1.11929
1972	52936	1.03205	1.02679	0.98191	1.43547	1.15296
1973	61173	1.04211	1.09186	0.97989	1.46916	1.48843
1974	65204	1.04308	1.20773	1.03952	1.48403	1.56162

1975	61218	1.02685	1.18933	1.03749	1.48373	1.21301
1981	66937	1.03485	1.20436	1.01115	1.62875	1.25811
1977	71002	1.03343	1.23160	1.06923	1.66796	1.34888
1978	73403	1.03126	1.24889	1.10217	1.61219	1.41368
1979	78961	1.03081	1.33303	1.13456	1.57294	1.58319
1980	78608	1.02328	1.38578	1.10560	1.52796	1.47470
1981	80108	1.02615	1.35379	1.10592	1.55517	1.37301
1982	73760	1.01742	1.26657	1.06522	1.55492	1.12101
1983	76566	1.00006	1.19845	0.98845	1.50751	1.28324
1984	81009	0.99619	1.18852	0.99092	1.51616	1.36739
1985	85351	0.99699	1.17680	0.98779	1.54643	1.39145
1986	87409	0.99966	1.14888	0.98427	1.55157	1.33625
1987	94847	1.00287	1.14386	0.94720	1.56791	1.47418
1988	100298	1.00275	1.11713	0.89981	1.62727	1.43997
1989	102387	1.00225	1.10764	0.87290	1.64897	1.35781
1990	99095	0.98495	1.04058	0.83723	1.61916	1.22226
1991	90462	0.96340	0.95210	0.77977	1.59932	0.98189
1992	93036	0.96099	0.96629	0.80078	1.60776	1.07198
1993	93673	0.96066	0.98975	0.82753	1.57292	1.08618
1994	100428	0.96915	1.04241	0.87214	1.55383	1.26564
1995	106501	0.97212	1.10432	0.89260	1.57284	1.36970
1996	110404	0.96447	1.08824	0.86345	1.56090	1.43116
1997	114262	0.96169	1.07285	0.85325	1.59315	1.41162
1998	118198	0.96219	1.05630	0.87031	1.61741	1.37794
1999	124920	0.95943	1.05044	0.85157	1.62855	1.43776
2000	136049	0.95649	1.08681	0.84546	1.66490	1.58698
2001	135368	0.95129	1.07272	0.84740	1.65956	1.48666
2002	140688	0.95308	1.03862	0.84254	1.66043	1.55850
2003	139793	0.94304	1.00574	0.76841	1.64948	1.48294
2004	149700	0.94432	1.01594	0.73855	1.66584	1.62572
2005	156144	0.94534	1.02342	0.71436	1.69218	1.68523
2006	161326	0.95073	1.00779	0.69673	1.73717	1.65624
2007	168821	0.95519	0.99980	0.66728	1.76887	1.68249

Thus the gross real income generated by the Canadian business sector has grown from $27,722 million dollars worth of 1961 consumption bundles in 1961 to $168,821 million in 2007, a 6.09 fold increase. Looking at the change in real input and output prices, the real price of domestic output has fallen to .955 times the starting level (due to the fact that machinery and equipment prices have risen less rapidly than the price of consumption) and the real price of exports has fallen slightly to .9998 times the starting level. However, the real price of imports has fallen substantially to .667 times the starting level. The quality adjusted real wages of business sector workers have risen to 1.77 times their initial 1961 levels. The real price of capital services has risen 1.68 fold, reflecting rapidly rising prices of agricultural land and nonagricultural business land as well as upward trends in machinery and equipment depreciation rates and in real rates of return; see Appendix 2 for details.[13]

There are six quantitative factors that can be used to explain the real income ρ^t generated by the business sector in year t:

- The price of domestic production (an aggregate of C+I+G) relative to the price of consumption in year t, P_D^t/P_C^t;
- The price of exports relative to the price of consumption in year t, P_X^t/P_C^t;
- The price of imports relative to the price of consumption in year t, P_M^t/P_C^t;
- The quantity of labour used by the business sector in year t, Q_L^t;
- The quantity of capital used by the business sector in year t, Q_K^t and
- The level of technology of the business sector in year t.

The formal model outlined in Appendix 1, based on the work of Diewert and Morrison (1986) and Kohli (1990), allows us to decompose the growth of real income from year t−1 to t, ρ^t/ρ^{t-1}, into multiplicative year to year contribution factors α_D^t, α_X^t, α_M^t, β_L^t, β_K^t and τ^t that describe the effects of changes in the six explanatory variables listed above going from year t−1 to t. The model outlined in Appendix 1 leads to the following equation which decomposes the year to year growth in real income generated by the business sector, ρ^t/ρ^{t-1}, into a product of six year to year explanatory contribution factors:[14]

$$(3)\ \rho^t/\rho^{t-1} = \tau^t\, \alpha_D^t\, \alpha_X^t\, \alpha_M^t\, \beta_L^t\, \beta_K^t\ ; \qquad t = 1962, 1963,...,2007.$$

Thus if α_D^t is greater than one, this means that the domestic price of output grew faster than the price of consumption going from year t−1 to t and α_D^t measures the contribution of rising real domestic output prices to the growth in real income. Similarly, if α_X^t is greater than one, this means that Canadian export prices grew faster than the price of consumption going from year t−1 to t and α_X^t measures the contribution of rising real export prices to the growth in real income generated by the Canadian business sector. However, if if α_M^t is greater than one, this means that Canadian import prices did not increase as quickly as the price of consumption going from year t−1 to t and α_M^t measures the contribution of falling real import prices to the growth in real income generated by the Canadian business sector. If β_L^t is greater than one, then business sector labour input increased going from year t−1 to t and β_L^t measures the contribution of the increase in labour input to the growth in real income generated by the Canadian business sector. Similarly, if β_K^t is greater than one, then business sector capital services input

[13] The volatility of the real price of capital services reflects the fact that we have used balancing real rates of return in our user costs and these real rates are subject to a considerable amount of measurement error. One would expect the aggregate real price of capital services to decline, reflecting the decline in the real price of machinery and equipment, but this decline is offset by a large increase in the real price of land services.

[14] See equations (42), (51) and (56) in Appendix 1 in order to derive this equation. All of the variables in equation (3) can be identified using the data in Appendix 2.

increased going from year t−1 to t and β_K^t measures the contribution of the increase in capital input to the growth in real income generated by the Canadian business sector. Finally, if τ^t is greater than one, then the efficiency of the Canadian business sector increased from year t−1 to t and τ^t measures the contribution of the efficiency increase to the growth in real income generated by the Canadian business sector. These year to year contribution factors are listed in Table 4 along with the averages of these contribution factors in the last two rows of the Table.[15]

Table 4: Business Sector Year to Year Growth in Real Income and Year to Year Contribution Factors

Year t	ρ^t/ρ^{t-1}	τ^t	α_D^t	α_X^t	α_M^t	β_L^t	β_K^t	α_{XM}^t
1962	1.06402	1.03361	1.00000	1.00602	0.98556	1.03028	1.00773	0.9915
1963	1.07414	1.05024	1.00210	0.99882	0.99418	1.01820	1.00943	0.99301
1964	1.08806	1.04090	1.00308	1.00312	0.99883	1.02848	1.01127	1.00194
1965	1.08499	1.02216	1.00854	1.00248	1.00428	1.03048	1.01448	1.00677
1966	1.08172	1.02091	0.99862	1.00108	1.00622	1.03428	1.01841	1.00730
1967	1.01799	0.98199	0.99899	0.9983	1.00302	1.01478	1.02126	1.00131
1968	1.04133	1.02040	0.99587	1.00107	1.00519	1.00296	1.01535	1.00627
1969	1.05548	1.02571	1.00153	0.99744	1.00259	1.01608	1.01117	1.00003
1970	1.04146	1.01776	1.00497	1.00073	1.00253	1.00223	1.01266	1.00326
1971	1.05145	1.01443	1.01352	0.99775	0.99943	1.01508	1.01033	0.99717
1972	1.06785	1.02595	1.00468	0.99901	1.00617	1.02134	1.00911	1.00518
1973	1.15561	1.06713	1.00988	1.02092	1.00072	1.03876	1.01043	1.02166
1974	1.06589	1.00953	1.00096	1.03484	0.97865	1.02287	1.01826	1.01275
1975	0.93887	0.93953	0.98361	0.99488	1.00076	0.99933	1.02109	0.99563
1981	1.09342	1.05508	1.00821	1.00415	1.00992	0.99926	1.01435	1.01412
1977	1.06072	1.05446	0.99858	1.00755	0.97932	1.00735	1.01349	0.98671
1978	1.03382	1.00172	0.99784	1.00495	0.98842	1.02793	1.01294	0.99331
1979	1.07573	1.01194	0.99955	1.02461	0.98852	1.03661	1.01293	1.01285
1980	0.99553	0.93951	0.99259	1.01519	1.01049	1.02181	1.01843	1.02584
1981	1.01907	0.99262	1.00286	0.99086	0.99988	1.01743	1.01559	0.99074
1982	0.92076	0.95289	0.99143	0.97430	1.01502	0.96690	1.01927	0.98894
1983	1.03805	1.03756	0.98324	0.97884	1.02800	1.00458	1.00660	1.00625
1984	1.05802	1.03578	0.99618	0.99668	0.99903	1.02358	1.00609	0.99572
1985	1.05360	1.02114	1.00080	0.99590	1.00130	1.02563	1.00803	0.99719
1986	1.02411	0.99325	1.00270	0.99013	1.00150	1.02750	1.00923	0.99162
1987	1.08509	1.02522	1.00323	0.99823	1.01598	1.03099	1.00898	1.01417
1988	1.05747	1.00414	0.99988	0.99069	1.02084	1.02900	1.01207	1.01133
1989	1.02083	0.98000	0.99949	0.99669	1.01242	1.01743	1.01513	1.00907

[15] The fifth row from the bottom gives the average over the years 1962-2007 and the remaining rows give the averages over the years 1962-1973, 1974-1991, 1992-1999 and 2000-2007. The careful reader will notice that the productivity growth rates τ^t listed in Table 4 do not quite agree with those listed in Table 2. The reason for these small differences is that when calculating τ^t in Table 4, the input aggregate is a direct Törnqvist quantity index whereas in Table 2, the input aggregate was an implicit quantity index; i.e., the value of inputs was deflated by the Törnqvist input price index.

1990	0.96784	0.97789	0.98239	0.97601	1.01721	1.0008	1.01395	0.9928
1991	0.91289	0.95226	0.9777	0.96525	1.03013	0.97776	1.00854	0.99433
1992	1.02845	1.03952	0.99745	1.00613	0.98853	0.9931	1.00421	0.9946
1993	1.00684	0.99427	0.99965	1.01091	0.98478	1.01498	1.00253	0.99552
1994	1.07211	1.03618	1.00887	1.02652	0.97368	1.0255	1.00059	0.99949
1995	1.06047	1.01379	1.003	1.03239	0.98772	1.0185	1.00418	1.01971
1996	1.03665	1.00981	0.99248	0.99158	1.01786	1.01829	1.00644	1.00929
1997	1.03495	1.01249	0.99721	0.99167	1.00659	1.02019	1.00656	0.99800
1998	1.03444	1.02182	1.00051	0.99062	0.98839	1.02012	1.01304	0.97912
1999	1.05687	1.01297	0.99722	0.99650	1.01318	1.02301	1.01296	1.00963
2000	1.08909	1.02737	0.99710	1.02251	1.00432	1.02211	1.01289	1.02693
2001	0.99499	0.98834	0.99495	0.99152	0.99867	1.00825	1.01348	0.99021
2002	1.03930	1.03377	1.00177	0.98014	1.00323	1.01340	1.00711	0.98331
2003	0.99364	0.95716	0.98994	0.98132	1.05112	1.00995	1.00664	1.03148
2004	1.07087	1.01343	1.00129	1.00574	1.02093	1.02224	1.00542	1.02679
2005	1.04305	1.00131	1.00104	1.00412	1.01731	1.00912	1.00950	1.02100
2006	1.03319	0.99928	1.00552	0.99168	1.01286	1.01119	1.01240	1.00443
2007	1.04646	0.99511	1.00461	0.99587	1.02185	1.01543	1.01301	1.01763
A62-07	1.0410	1.0101	0.99904	0.99969	1.0043	1.0160	1.0113	1.0038
A62-73	1.0687	1.0268	1.0035	1.0022	1.0007	1.0211	1.0126	1.0029
A74-91	1.0234	0.99914	0.99562	0.99665	1.0054	1.0132	1.0131	1.0019
A92-99	1.0413	1.0176	0.99955	1.0058	0.99509	1.0167	1.0063	1.0007
A00-07	1.0388	1.0020	0.99953	0.99661	1.0163	1.0140	1.0101	1.0128

Looking at the sample averages listed in the fifth last row of Table 4, it can be seen that the (gross) real income generated by the Canadian business sector over the entire sample period grew at 4.10 percent per year over the 47 years 1961-2007. The biggest contributor to this growth was the growth of quality adjusted labour input at 1.6 percentage points per year. Next was Total Factor Productivity growth, τ^t, which contributed on average 1.01 percentage points per year, followed by capital services growth (1.13 percentage points per year) and declines in real import prices (0.43 percentage points per year). Declines in real domestic output prices and real export prices gave rise to negative average contribution factors, − 0.096 and −0.03 percentage points per year respectively. The last column in Table 4 gives the product of the real export and real import price contribution factors, $\alpha_{XM}{}^t$, defined as:

$$(4)\ \alpha_{XM}{}^t \equiv \alpha_X{}^t \alpha_M{}^t .$$

Roughly speaking, $\alpha_{XM}{}^t$ is a *terms of trade contribution factor*; it gives the contribution to real income growth of the combined effects of real changes in the international prices facing the Canadian business sector.[16] It can be seen that the effects of changing real

[16] Ulrich Kohli has pointed out that this is a slight abuse of terminology. Strictly speaking, the terms of trade is the price of exports over the price of imports and hence involves only two prices. Our definition of $\alpha_{XM}{}^t$ involves three prices: the price of exports, the price of imports and the price of domestic consumption. Our terms of trade contribution factor is the rate of change counterpart to Kohli's (2006; 50) *trading gains factor*. See Appendix 3 for a discussion of the differences between our approach and that of Kohli.

international prices are not negligible for Canada: on average, changing real export and import prices contributed 0.38 percentage points per year to real income growth over the entire sample period.[17] However, for shorter periods, the effects of changing real international prices can be far more important in explaining changes in the real income generated by the market sector of an economy. Thus if we restrict our attention to the period 2000-2007, it can be seen by looking at the last row of Table 4 that the effects of improvements in Canada's terms of trade become almost as important as the effects of capital deepening; i.e., during this period, the average annual growth in the real income generated by the Canadian business sector was 3.88 percent per year and the following factors explained this growth rate: decreases in the real price of imports (1.63), increases in quality adjusted labour input (1.40), increases in capital services input, 1.01) and improvements in TFP (0.20). There were small negative contributors to market sector real income growth during the naughts: decreases in the real price of domestically produced goods and services (−0.05) and decreases in the real price of exports (−0.34). Thus decreases in the real price of imports proved to be the most important factor in explaining the growth in real income generated by the market sector during this period. Overall, the joint effects of changes in real export and import prices contributed about 1.28 percentage points per year on average to the growth of market sector real income during the naughts, which was greater than the contribution of capital deepening over this period (which was 1.01 percentage points per year on average).[18]

The last four rows of Table 4 present the various growth factors for 4 subperiods:

- The 12 golden years for the Canadian economy, 1962-1973, when the real income generated by the business sector grew by 6.87% per year and TFP growth was a stellar 2.68% per year;
- The 18 dismal years for the Canadian economy, 1974-1991, characterized by stagflation, oil shocks and rapidly increasing tax rates when the real income generated by the business sector grew by 2.34% per year and TFP growth was essentially 0;
- The 8 years in the nineties after the recession of 1991, 1992-1999, when real income growth recovered to 4.13% per year and TFP growth also recovered to 1.76% per year and
- The 8 years in the present century, 2000-2007, when TFP growth dropped off to 0.20% per year but real income growth was still strong at 3.88% per year due to the very strong contribution made by falling real import prices during this period, which contributed on average 1.63% per year to real income growth.

The annual change information in Table 4 can be converted into levels using equations (46) in Appendix 1 (with obvious extensions to multiple inputs and outputs). Thus let T^t, A_D^t, A_X^t, A_M^t, B_L^t, B_K^t and A_{XM}^t be the cumulated products of the annual link factors τ^t, α_D^t, α_X^t, α_M^t, β_L^t, β_K^t and α_{XM}^t respectively. Using these definitions and cumulating

[17] Thus the contribution of falling real import prices outweighs the effects of falling real export prices.

[18] These results are very similar to the results obtained for Australia using a similar framework by Diewert and Lawrence (2006); i.e., both Australia and Canada have had very favourable changes in their terms of trade in recent years which contributed greatly to real income growth during the naughts.

equations (3) leads to the following equation, which explains the cumulative growth in real gross income generated by the Canadian business sector relative to the base year 1961:

(5) $\rho^t/\rho^{1961} = T^t\, A_D{}^t\, A_X{}^t\, A_M{}^t\, B_L{}^t\, B_K{}^t$; t = 1962, 1963, ... , 2007.

The cumulated variables that appear in (5) above are reported in Table 5 below along with the cumulated terms of trade contribution factor, $A_{XM}{}^t$ defined to be the product of the two cumulated international price factors, $A_X{}^t$ and $A_M{}^t$.

Table 5: Business Sector Cumulated Growth in Real Income and Cumulated Contribution Factors

Year t	ρ^t/ρ^{1961}	T^t	$A_D{}^t$	$A_X{}^t$	$A_M{}^t$	$B_L{}^t$	$B_K{}^t$	$A_{XM}{}^t$
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.06402	1.03361	1.00000	1.00602	0.98556	1.03028	1.00773	0.99150
1963	1.14291	1.08554	1.00210	1.00483	0.97983	1.04903	1.01723	0.98456
1964	1.24355	1.12994	1.00519	1.00796	0.97868	1.07891	1.02869	0.98648
1965	1.34925	1.15498	1.01378	1.01046	0.98288	1.11179	1.04359	0.99316
1966	1.45950	1.17913	1.01238	1.01155	0.98899	1.14991	1.06280	1.00042
1967	1.48576	1.15790	1.01136	1.00983	0.99197	1.16691	1.08540	1.00172
1968	1.54716	1.18151	1.00719	1.01091	0.99713	1.17037	1.10206	1.00800
1969	1.63300	1.21189	1.00873	1.00832	0.99971	1.18918	1.11437	1.00803
1970	1.70070	1.23340	1.01374	1.00905	1.00224	1.19184	1.12848	1.01131
1971	1.78820	1.25120	1.02745	1.00678	1.00167	1.20981	1.14014	1.00845
1972	1.90953	1.28367	1.03226	1.00578	1.00785	1.23563	1.15052	1.01367
1973	2.20668	1.36984	1.04246	1.02683	1.00857	1.28352	1.16252	1.03563
1974	2.35207	1.38289	1.04346	1.06260	0.98704	1.31287	1.18375	1.04883
1975	2.20829	1.29927	1.02636	1.05716	0.98778	1.31200	1.20871	1.04425
1981	2.41459	1.37083	1.03478	1.06155	0.99759	1.31103	1.22605	1.05899
1977	2.56120	1.44548	1.03331	1.06956	0.97696	1.32066	1.24259	1.04492
1978	2.64781	1.44798	1.03108	1.07485	0.96565	1.35754	1.25867	1.03793
1979	2.84833	1.46527	1.03062	1.10130	0.95457	1.40724	1.27495	1.05127
1980	2.83559	1.37664	1.02298	1.11803	0.96458	1.43794	1.29844	1.07844
1981	2.88968	1.36648	1.02590	1.10781	0.96447	1.46300	1.31869	1.06845
1982	2.66070	1.30211	1.01711	1.07935	0.97896	1.41457	1.34410	1.05663
1983	2.76193	1.35101	1.00006	1.05651	1.00637	1.42105	1.35297	1.06324
1984	2.92218	1.39934	0.99624	1.05300	1.00540	1.45455	1.36120	1.05869
1985	3.07883	1.42893	0.99704	1.04868	1.00671	1.49183	1.37214	1.05571
1986	3.15306	1.41928	0.99972	1.03833	1.00822	1.53286	1.38481	1.04686
1987	3.42136	1.45508	1.00295	1.03649	1.02432	1.58036	1.39724	1.06170
1988	3.61799	1.46111	1.00284	1.02684	1.04567	1.62620	1.41411	1.07373
1989	3.69336	1.43190	1.00233	1.02344	1.05865	1.65455	1.43550	1.08347
1990	3.57459	1.40024	0.98467	0.99889	1.07687	1.65588	1.45553	1.07567
1991	3.26319	1.33340	0.96272	0.96417	1.10931	1.61905	1.46796	1.06957
1992	3.35604	1.38609	0.96026	0.97009	1.09659	1.60787	1.47414	1.06379

1993	3.37900	1.37814	0.95993	0.98067	1.07990	1.63196	1.47788	1.05903
1994	3.62268	1.42800	0.96844	1.00667	1.05148	1.67357	1.47875	1.05849
1995	3.84175	1.44770	0.97135	1.03928	1.03856	1.70452	1.48493	1.07936
1996	3.98254	1.46190	0.96404	1.03053	1.05711	1.73569	1.49449	1.08938
1997	4.12172	1.48016	0.96135	1.02195	1.06407	1.77074	1.50430	1.08743
1998	4.26368	1.51245	0.96184	1.01236	1.05172	1.80636	1.52391	1.06472
1999	4.50616	1.53206	0.95917	1.00882	1.06558	1.84792	1.54366	1.07497
2000	4.90761	1.57400	0.95639	1.03152	1.07018	1.88878	1.56356	1.10392
2001	4.88304	1.55565	0.95156	1.02278	1.06876	1.90437	1.58464	1.09311
2002	5.07494	1.60819	0.95324	1.00247	1.07221	1.92989	1.59591	1.07486
2003	5.04267	1.53930	0.94365	0.98374	1.12702	1.94909	1.60651	1.10869
2004	5.40004	1.55997	0.94487	0.98939	1.15061	1.99244	1.61521	1.13840
2005	5.63249	1.56201	0.94585	0.99346	1.17052	2.01060	1.63056	1.16287
2006	5.81943	1.56088	0.95107	0.98519	1.18558	2.03310	1.65078	1.16802
2007	6.08978	1.55325	0.95545	0.98112	1.21149	2.06447	1.67226	1.18862

Looking at the last row of Table 5, it can be seen that the gross real income generated by the business sector grew 6.09 fold over the years 1961-2007. The main factors explaining this growth are growth of quality adjusted labour input (cumulative growth factor 2.06), productivity increases (cumulative growth factor 1.55), growth of capital services (cumulative growth factor 1.67) and lower real import prices (cumulative growth factor 1.21). There were negative contributions from declining real domestic output prices (cumulative growth factor 0.955) and declining real export prices (cumulative growth factor .98). In recent years, the real prices of Canada's raw materials exports have increased dramatically. However, these increases do not show up in the A_X^t column of Table 5; i.e., the overall real price of Canadian exports has remained relatively constant in recent years. This apparent contradiction can be explained by falling real prices for Canadian exports of manufactured goods as we shall see in section 5 below. As already noted above, the effects of falling real import prices in recent years have been substantial.

As is noted in section 5 of Appendix 1, the income concept used in this section is biased upwards. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence the measure of real income ρ^t that is used in this section is overstated. In the following section, we implement the *net real income model* that is described in more detail in section 5 of Appendix 1.

4. Explaining Real Income Growth Generated by the Canadian Business Sector: the Net Output Approach

The overstatement of income problem that is implicit in the approach used in the previous section can readily be remedied: all we need to do is to take the user cost formula for an asset that has investment price P_I^t in year t and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and

- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

The depreciation part $\delta P_I^t K^t$, will be removed from the user cost and treated as an intermediate input as an offset to gross investment. We now explain this rather simple idea in more detail below.

In our empirical work thus far (described in detail in Appendix 2), our user costs took the following form:

(6) $U^t = (r^t + \delta^t + \tau_B^t)P_I^t$

where r^t is the balancing period t real rate of interest, δ^t is a geometric depreciation rate for period t, τ_B^t is an appropriate business taxation rate on the asset (including property taxes if applicable) and P_I^t is the period t investment price for the asset. However, in the net product approach to the measurement of income,[19] we split up each (gross product) user cost times the beginning of the period stock K^t into the depreciation component $\delta^t P_I^t K^t$ and the remaining term $(r^t+\tau_B^t)P_I^t K^t$ and we regard the second term as a genuine income component but we treat the first term as an intermediate input cost for the business sector and treat it as an offset to gross investment made by the business sector during the year under consideration. Thus in the present section, our new aggregate for domestic output will aggregate the same C+I+G components as before, but now we add the depreciation series for business structures, ICT and for machinery and equipment as negative outputs of the business sector. As noted above, the ICT, machinery and equipment and nonresidential structures user costs are also changed since the depreciation terms are now omitted. Thus the new investment aggregate I is a *net investment aggregate* (gross investment components were indexed with a positive sign in the aggregate and depreciation components were indexed with a negative sign in the aggregate) and the new capital services aggregate is now a “reward for waiting” capital services aggregate.[20]

The above changes mean that our aggregate data series have changed somewhat. The new net product counterparts to the old gross product Tables 1-3 are presented below as Tables 6-8.[21]

Table 6: Prices of Canadian Business Sector (Net) Output and Input Aggregates

[19] See Diewert (2006a) for a more detailed discussion of the net income approach to income measurement.
[20] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: “Second, one can consider the ‘waiting’ or ‘abstinence’ associated with the net returns to capital as the nonlabour primary input.” T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.
[21] The TFP growth rates τ^t in Tables 7 and 9 differ slightly because when calculating τ^t in Table 9, the input aggregate is a direct Törnqvist quantity index whereas in Table 7, the input aggregate was an implicit quantity index; i.e., the value of inputs was deflated by the Törnqvist input price index. Both the direct and implicit Törnqvist indexes are superlative and hence will generally approximate each other very closely; see Diewert (1978).

Year t	P_C^t	P_D^t	P_X^t	P_M^t	P_L^t	P_K^t	P_Y^t	P_Z^t
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.00538	1.00477	1.02992	1.05787	1.03625	1.01517	0.99506	1.03177
1963	1.02055	1.01951	1.04054	1.09648	1.06604	1.21066	1.00167	1.09654
1964	1.02437	1.02570	1.05692	1.10526	1.11049	1.32368	1.01000	1.15544
1965	1.03690	1.04437	1.07980	1.10214	1.18272	1.32976	1.03635	1.21382
1966	1.07553	1.08188	1.12451	1.11930	1.25929	1.43384	1.08239	1.29617
1967	1.11050	1.11723	1.15421	1.14426	1.33342	1.23143	1.11942	1.31158
1968	1.15168	1.15726	1.20119	1.16726	1.41620	1.33236	1.16779	1.39834
1969	1.18980	1.19642	1.23088	1.19648	1.52197	1.35774	1.20736	1.48633
1970	1.22208	1.23208	1.26710	1.21965	1.61278	1.39712	1.24801	1.56563
1971	1.24828	1.27432	1.28552	1.24798	1.72528	1.33648	1.28652	1.63857
1972	1.29847	1.33283	1.33325	1.27498	1.86392	1.45722	1.35367	1.77330
1973	1.38744	1.44088	1.51489	1.35954	2.03837	2.36282	1.49982	2.11395
1974	1.58382	1.64631	1.91283	1.64641	2.35044	2.88611	1.73846	2.47391
1975	1.82198	1.86583	2.16694	1.89029	2.70332	2.22381	1.95864	2.59155
1976	1.90726	1.96938	2.29702	1.92853	3.10646	2.45812	2.10208	2.95469
1977	2.03175	2.09474	2.50231	2.17241	3.38889	2.93446	2.20157	3.28528
1978	2.19264	2.25525	2.73837	2.41667	3.53495	3.40699	2.35179	3.51206
1979	2.40645	2.47513	3.20786	2.73027	3.78520	4.47127	2.61939	3.96092
1980	2.69497	2.75871	3.73464	2.97957	4.11781	4.51719	3.00661	4.22499
1981	2.95335	3.03348	3.99821	3.26618	4.59295	4.37549	3.27072	4.55078
1982	3.22860	3.29615	4.08926	3.43918	5.02021	3.31517	3.50723	4.60204
1983	3.46323	3.51354	4.15051	3.42324	5.22085	4.86150	3.76691	5.15888
1984	3.61506	3.65788	4.29656	3.58225	5.48101	5.66773	3.90201	5.56258
1985	3.72257	3.76949	4.38071	3.67711	5.75670	5.97348	4.00777	5.84765
1986	3.80422	3.86708	4.37060	3.74437	5.90250	5.68664	4.07122	5.88540
1987	3.89726	3.98569	4.45792	3.69150	6.11054	6.89128	4.26595	6.34107
1988	4.00205	4.09782	4.47080	3.60108	6.51242	6.81320	4.44386	6.62912
1989	4.11690	4.21841	4.56005	3.59364	6.78864	6.34569	4.62309	6.72753
1990	4.35206	4.39764	4.52868	3.64367	7.04667	5.68557	4.77719	6.76258
1991	4.59099	4.59362	4.37107	3.57992	7.34245	4.19549	4.95539	6.60905
1992	4.65258	4.64682	4.49573	3.72567	7.48023	5.09947	4.98029	6.94680
1993	4.74252	4.73209	4.69389	3.92460	7.45961	5.28267	5.04452	6.97845
1994	4.77089	4.78795	4.97322	4.16089	7.41314	6.90899	5.10110	7.35256
1995	4.79147	4.81456	5.29132	4.27688	7.53622	7.86053	5.24941	7.68006
1996	4.88952	4.87348	5.32097	4.22185	7.63205	8.64495	5.37425	7.94364
1997	4.96547	4.93146	5.32718	4.23677	7.91076	8.52931	5.42748	8.13030
1998	5.03224	4.99337	5.31558	4.37960	8.13918	8.19333	5.36027	8.22664
1999	5.12045	5.08140	5.37870	4.36044	8.33890	9.06262	5.51701	8.58639
2000	5.25425	5.20795	5.71039	4.44227	8.74780	11.02286	5.83458	9.36276
2001	5.40970	5.34378	5.80311	4.58416	8.97770	10.23916	5.91943	9.35715
2002	5.47743	5.42904	5.68895	4.61494	9.09489	11.21972	5.89394	9.67799
2003	5.61543	5.56047	5.64768	4.31493	9.26253	10.98273	6.26151	9.75266
2004	5.69551	5.66394	5.78629	4.20643	9.48779	12.99255	6.57696	10.39319
2005	5.81654	5.80418	5.95274	4.15511	9.84265	14.07954	6.90744	10.91980

2006	5.92386	5.95701	5.97000	4.12736	10.29074	13.95161	7.12388	11.23987
2007	6.02712	6.10280	6.02590	4.02177	10.66121	14.56298	7.44707	11.66971

Comparing Table 6 with Table 1, we see that the 2007 price of domestic absorption, P_D, has increased to 6.10 from its earlier 2007 gross approach level of 5.76. This is due to the fact that net investment is considerably smaller than gross investment and so the relatively low inflation prices of ICT equipment and other machinery and equipment get much smaller weights in net domestic absorption compared to their weights in gross domestic absorption. The other striking difference between Tables 1 and 6 is that the price of waiting services, P_K^t, in Table 6 grew 14.6 fold over the sample period whereas the price of traditional capital services, P_K^t, in Table 1, grew only 10.1 fold. This difference in growth rates is explained by the fact that the prices of ICT equipment and machinery and equipment services get much lower weights in the Table 6 capital services aggregate compared to their weights in the Table 1 capital services aggregate because the corresponding user costs for the net concept of capital services now excludes the very large depreciation terms in the net user costs for these two capital inputs. Thus the price of agricultural land and business nonagricultural land get much higher weights in the net user cost compared to the gross concept user cost.[22] However, even though the land components now get a much higher weight in weighting services compared to machinery and equipment, the overall price increase in input prices has only increased to 11.7 fold (compared to the gross output model 10.6 fold increase in input prices) over the sample period due to the fact that the importance of capital services dramatically shrinks relative to labour services in the net output framework.

Table 7: Quantities of Canadian Business Sector Net Output and Input Aggregates, TFP Levels and TFP Growth Rates

Year t	Q_D^t	Q_X^t	Q_M^t	Q_L^t	Q_K^t	Q_Y^t	Q_Z^t	T^t	τ^t
1961	25452	6867	-7897	19202	5220	24422	24422	1.00000	______
1962	27156	7195	-8033	20042	5348	26328	25391	1.03690	1.03690
1963	28698	7832	-8031	20574	5509	28554	26083	1.09471	1.05576
1964	30880	9105	-8989	21446	5701	31052	27143	1.14400	1.04503
1965	33935	9418	-10180	22416	5925	33184	28332	1.17124	1.02381
1966	36532	10696	-11579	23550	6230	35653	29773	1.19750	1.02242
1967	36362	11827	-12306	24056	6592	35906	30646	1.17166	0.97842
1968	37696	12910	-13527	24158	6852	37114	30995	1.19743	1.02199
1969	40646	13802	-15377	24718	7070	39110	31769	1.23106	1.02808
1970	40280	15211	-15293	24798	7342	40264	32096	1.25450	1.01904
1971	42286	15929	-16480	25333	7549	41816	32831	1.27365	1.01526
1972	45763	17257	-18892	26101	7730	44262	33788	1.30999	1.02854
1973	52582	19008	-21754	27591	7933	49995	35471	1.40947	1.07594
1974	58017	18347	-23977	28558	8319	52422	36838	1.42305	1.00963
1975	55709	16951	-23228	28530	8833	49405	37340	1.32313	0.92979
1976	59148	18390	-24774	28499	9121	52781	37551	1.40561	1.06233

[22] From Table 13 in Appendix 2, we estimate that the price of agricultural land increased 19.3 fold and the price of business nonagricultural land increased 52.4 fold over the period 1961-2007. For comparison purposes, the price of residential land increased 88.9 fold over this period.

1977	62034	19678	-24836	28805	9410	56883	38119	1.49224	1.06164
1978	63659	21544	-26197	30018	9728	59212	39650	1.49336	1.00075
1979	68536	22467	-28092	31737	10037	62995	41659	1.51215	1.01259
1980	66718	22548	-28715	32833	10516	60768	43244	1.40523	0.92929
1981	69487	23012	-30716	33723	10875	61904	44491	1.39137	0.99013
1982	58666	22882	-25710	32059	11336	56604	43138	1.31216	0.94307
1983	62713	24326	-28444	32283	11395	59449	43409	1.36953	1.04372
1984	67214	28444	-33270	33497	11520	63785	44743	1.42557	1.04092
1985	71770	29938	-35548	34871	11757	67611	46338	1.45908	1.02351
1986	74416	31456	-37965	36384	12018	69537	48102	1.44561	0.99077
1987	79795	32933	-39889	38166	12246	74450	50086	1.48644	1.02824
1988	85403	35371	-45163	39912	12556	77740	52113	1.49175	1.00357
1989	87899	35434	-47820	40981	12974	77984	53590	1.45520	0.97550
1990	84568	37556	-48551	41030	13358	76421	53985	1.41560	0.97278
1991	77953	38167	-49281	39707	13573	70326	52730	1.33371	0.94215
1992	79864	40921	-51473	39311	13581	72949	52299	1.39486	1.04585
1993	79507	45382	-55461	40184	13599	73662	53248	1.38337	0.99177
1994	83827	51076	-60606	41745	13567	79042	54838	1.44137	1.04192
1995	85817	55452	-64385	42958	13673	82146	56148	1.46303	1.01503
1996	88197	58646	-67340	44212	13898	85143	57603	1.47809	1.01029
1997	95522	63457	-77378	45636	14100	88674	59196	1.49799	1.01346
1998	98627	69086	-81755	47078	14461	93588	60980	1.53474	1.02454
1999	101444	76337	-88261	48781	14834	98099	63031	1.55635	1.01408
2000	107179	83350	-95661	50512	15180	104411	65066	1.60470	1.03107
2001	106033	80654	-90649	51183	15588	104590	66165	1.58075	0.98507
2002	113127	81599	-92347	52290	15743	110657	67391	1.64202	1.03876
2003	114374	79268	-96341	53129	16011	106675	68489	1.55756	0.94856
2004	121796	83281	-104558	55049	16135	111286	70423	1.58024	1.01457
2005	128251	84730	-112492	55875	16435	113118	71554	1.58088	1.00040
2006	134178	85022	-117772	56905	16858	115217	73025	1.57777	0.99804
2007	140386	86002	-124556	58347	17304	117368	74899	1.56702	0.99318

Thus the level of business sector Total Factor Productivity using the net approach increased 1.57 fold over the period 1961-2007 and the average rate of net TFP growth was 1.04 percent per year. Recall that using the gross approach, the level of business sector Total Factor Productivity increased 1.55 fold over the period 1961-2007 and the average rate of gross product TFP growth was 1.01 percent per year. Thus switching to the more appropriate net approach does not substantially increase Canadian business sector TFP growth on average. For a more detailed breakdown of net TFP growth, see the last rows of Table 9 below, which is the net product counterpart to Table 4 above.

The net counterpart to Table 3 above is Table 8 below; ρ^t now represents the *net real income* generated by the Canadian business sector in year t.

Table 8: Net Real Income Generated by the Canadian Business Sector and Real Output and Input Prices

Year t	ρ^t	P_D^t/P_C^t	P_X^t/P_C^t	P_M^t/P_C^t	P_L^t/P_C^t	P_K^t/P_C^t
1961	24422	1.00000	1.00000	1.00000	1.00000	1.00000

1962	26058	0.99939	1.02441	1.05221	1.03070	1.00974
1963	28025	0.99899	1.01959	1.07440	1.04457	1.18628
1964	30616	1.00130	1.03178	1.07897	1.08407	1.29219
1965	33166	1.00720	1.04137	1.06292	1.14063	1.28244
1966	35880	1.00591	1.04554	1.04070	1.17085	1.33315
1967	36195	1.00606	1.03936	1.03040	1.20074	1.10890
1968	37633	1.00484	1.04299	1.01353	1.22968	1.15688
1969	39687	1.00556	1.03453	1.00561	1.27918	1.14115
1970	41119	1.00818	1.03684	0.99801	1.31970	1.14323
1971	43096	1.02086	1.02983	0.99976	1.38213	1.07066
1972	46143	1.02647	1.02679	0.98191	1.43547	1.12226
1973	54045	1.03851	1.09186	0.97989	1.46916	1.70300
1974	57540	1.03945	1.20773	1.03952	1.48403	1.82225
1975	53111	1.02407	1.18933	1.03749	1.48373	1.22055
1976	58173	1.03257	1.20436	1.01115	1.62875	1.28882
1977	61637	1.03101	1.23160	1.06923	1.66796	1.44430
1978	63509	1.02855	1.24889	1.10217	1.61219	1.55383
1979	68569	1.02854	1.33303	1.13456	1.57294	1.85804
1980	67795	1.02365	1.38578	1.10560	1.52796	1.67616
1981	68556	1.02713	1.35379	1.10592	1.55517	1.48153
1982	61489	1.02092	1.26657	1.06522	1.55492	1.02681
1983	64662	1.01453	1.19845	0.98845	1.50751	1.40375
1984	68848	1.01184	1.18852	0.99092	1.51616	1.56781
1985	72791	1.01260	1.17680	0.98779	1.54643	1.60467
1986	74417	1.01652	1.14888	0.98427	1.55157	1.49482
1987	81494	1.02269	1.14386	0.94720	1.56791	1.76824
1988	86322	1.02393	1.11713	0.89981	1.62727	1.70243
1989	87573	1.02466	1.10764	0.87290	1.64897	1.54138
1990	83886	1.01047	1.04058	0.83723	1.61916	1.30641
1991	75908	1.00057	0.95210	0.77977	1.59932	0.91385
1992	78088	0.99876	0.96629	0.80078	1.60776	1.09605
1993	78353	0.99780	0.98975	0.82753	1.57292	1.11390
1994	84512	1.00358	1.04241	0.87214	1.55383	1.44816
1995	89997	1.00482	1.10432	0.89260	1.57284	1.64053
1996	93584	0.99672	1.08824	0.86345	1.56090	1.76806
1997	96925	0.99315	1.07285	0.85325	1.59315	1.71772
1998	99689	0.99228	1.05630	0.87031	1.61741	1.62817
1999	105696	0.99237	1.05044	0.85157	1.62855	1.76989
2000	115943	0.99119	1.08681	0.84546	1.66490	2.09789
2001	114445	0.98781	1.07272	0.84740	1.65956	1.89274
2002	119071	0.99117	1.03862	0.84254	1.66043	2.04835
2003	118949	0.99021	1.00574	0.76841	1.64948	1.95581
2004	128508	0.99446	1.01594	0.73855	1.66584	2.28119
2005	134333	0.99788	1.02342	0.71436	1.69218	2.42060
2006	138557	1.00560	1.00779	0.69673	1.73717	2.35515
2007	145019	1.01256	0.99980	0.66728	1.76887	2.41624

Note that from Table 8, the starting level of net real income in 1961, $24,422 million, is less than the corresponding starting level of gross real income in 1961 from Table 3, which was $27,722 million. This makes sense since we now subtract depreciation from the previous estimates of gross income. Net real income generated by the Canadian business sector grew 5.94 fold over the period 1961-2007, which is 2.5 percent less than the 6.09 fold growth of gross real income. The real price of waiting capital services grew 2.42 fold, which is more rapid than the previous 1.68 fold increase in the real price of gross capital services from Table 3. This difference is due to the fact that depreciation gave the prices of ICT equipment and other machinery and equipment (which decrease in real terms) a larger role in the price of gross capital services but when depreciation is regarded as an intermediate input, the price of land (which increases in real terms) gets a much bigger weight in the price of waiting capital services.

The same translog contributions methodology explained in Appendix 1 can be applied to the net output model used in the present section. Thus equation (3) in the previous section is applicable to our new measure of real income generated by the Canadian business sector and Table 9 below is the net income counterpart to Table 4 in the previous section.

Table 9: Business Sector Year to Year Growth in Net Real Income and Net Year to Year Contribution Factors

Year t	ρ^t/ρ^{t-1}	τ^t	$\alpha_D{}^t$	$\alpha_X{}^t$	$\alpha_M{}^t$	$\beta_L{}^t$	$\beta_K{}^t$	$\alpha_{XM}{}^t$
1962	1.06695	1.03690	0.99936	1.00682	0.98365	1.03439	1.00509	0.99037
1963	1.07552	1.05573	0.99958	0.99866	0.99342	1.02062	1.00654	0.99209
1964	1.09244	1.04503	1.00235	1.00352	0.99868	1.03219	1.00817	1.00219
1965	1.08330	1.02381	1.00602	1.00279	1.00483	1.03441	1.00908	1.00764
1966	1.08183	1.02242	0.99868	1.00121	1.00702	1.03875	1.01165	1.00824
1967	1.00877	0.97846	1.00016	0.99807	1.00342	1.01676	1.01232	1.00148
1968	1.03974	1.02199	0.99878	1.00122	1.00592	1.00337	1.00799	1.00714
1969	1.05457	1.02808	1.00073	0.99708	1.00296	1.01835	1.00651	1.00003
1970	1.03608	1.01904	1.00263	1.00083	1.00289	1.00256	1.00771	1.00372
1971	1.04810	1.01527	1.0125	0.99741	0.99934	1.01734	1.00547	0.99676
1972	1.07070	1.02854	1.00555	0.99887	1.00709	1.02456	1.00446	1.00595
1973	1.17124	1.07552	1.01191	1.02388	1.00082	1.04429	1.00568	1.02472
1974	1.06467	1.00963	1.00093	1.03955	0.97585	1.02594	1.01228	1.01445
1975	0.92302	0.93008	0.98430	0.99415	1.00087	0.99924	1.01408	0.99501
1976	1.09531	1.06233	1.00882	1.00478	1.01144	0.99915	1.00651	1.01627
1977	1.05956	1.06162	0.99842	1.00869	0.97623	1.00846	1.00663	0.98472
1978	1.03037	1.00074	0.99754	1.00571	0.98666	1.03229	1.00764	0.99229
1979	1.07966	1.01254	0.99999	1.02844	0.98677	1.04236	1.00801	1.01484
1980	0.98872	0.92929	0.99516	1.01757	1.01214	1.02525	1.01249	1.02992
1981	1.01123	0.99015	1.00348	0.98937	0.99986	1.02031	1.00834	0.98923
1982	0.89691	0.94322	0.99391	0.96965	1.01780	0.96092	1.00885	0.98690
1983	1.05161	1.04339	0.99387	0.97483	1.03347	1.00546	1.00113	1.00745

1984	1.06473	1.04092	0.99739	0.99609	0.99886	1.02789	1.00279	0.99495
1985	1.05728	1.02351	1.00075	0.99518	1.00152	1.03017	1.00531	0.99670
1986	1.02234	0.99078	1.00390	0.98843	1.00176	1.03236	1.00552	0.99017
1987	1.09509	1.02822	1.00612	0.99793	1.01871	1.03633	1.00477	1.01659
1988	1.05925	1.00358	1.00122	0.98918	1.02427	1.03381	1.00644	1.01319
1989	1.01449	0.97551	1.00072	0.99614	1.01449	1.02035	1.00782	1.01058
1990	0.95790	0.97280	0.98583	0.97186	1.02025	1.00094	1.00640	0.99155
1991	0.90490	0.94235	0.98998	0.95890	1.03585	0.97366	1.00297	0.99327
1992	1.02871	1.04579	0.99815	1.00731	0.98635	0.99178	1.0001	0.99356
1993	1.00340	0.99177	0.99902	1.01303	0.98186	1.01790	1.00025	0.99466
1994	1.07861	1.04182	1.00581	1.03168	0.96871	1.03047	0.99950	0.99940
1995	1.06490	1.01503	1.00121	1.03853	0.98545	1.02197	1.00188	1.02342
1996	1.03985	1.01029	0.99235	0.99006	1.02114	1.02164	1.00419	1.01099
1997	1.03570	1.01346	0.99657	0.99019	1.00777	1.02386	1.00370	0.99788
1998	1.02852	1.02454	0.99914	0.98892	0.98629	1.02383	1.00616	0.97536
1999	1.06026	1.01408	1.00009	0.99586	1.01562	1.02730	1.00618	1.01141
2000	1.09695	1.03105	0.99888	1.02656	1.00509	1.02609	1.00605	1.03178
2001	0.98708	0.98508	0.99688	0.99002	0.99844	1.00973	1.00708	0.98847
2002	1.04043	1.03876	1.00315	0.97657	1.00382	1.01586	1.00263	0.98029
2003	0.99897	0.94856	0.99909	0.97802	1.06051	1.01173	1.00451	1.03720
2004	1.08037	1.01455	1.00406	1.00672	1.02453	1.02607	1.00212	1.03142
2005	1.04532	1.00040	1.00326	1.00479	1.02017	1.01062	1.00538	1.02506
2006	1.03145	0.99804	1.00745	0.99032	1.01498	1.01303	1.00744	1.00516
2007	1.04664	0.99318	1.00676	0.99519	1.02549	1.01799	1.00754	1.02056
A62-07	1.0407	1.0104	1.0003	0.99958	1.0051	1.0185	1.0062	1.0045
A62-73	1.0691	1.0292	1.0032	1.0025	1.0008	1.0240	1.0076	1.0034
A74-91	1.0209	0.99781	0.99791	0.99591	1.0065	1.0153	1.0071	1.0021
A92-99	1.0425	1.0196	0.99904	1.0069	0.99415	1.0198	1.0027	1.0008
A00-07	1.0409	1.0012	0.0024	0.99602	1.0191	1.0164	1.0053	1.0150

The net real income generated by the Canadian business sector grew at an annual rate of 4.07 percent on average over the 47 year period 1961-2006, which is a bit less than the gross real income growth rate of 4.10 percent. Real domestic output prices averaged a tiny positive contribution to the growth in real net income of 0.03 per year and falling real export prices made a tiny negative contribution of −0.04 per year. Positive average contributions to the growth of real net income were due to productivity improvements (1.04 per year compared to 1.01 in the gross income framework), growth of labour input (1.85 per year compared to the previous gross income 1.60), growth of capital input (0.62 per year compared to the previous 1.13) and falls in real import prices (0.51 per year compared to the previous 0.43). Comparing these average contribution growth rates in the gross and net real income frameworks leads to the following important observations:

- The role of productivity improvements is *magnified* in the net income framework compared to the gross income framework;[23]

[23] This phenomenon is reasonably well known and is explained in Schreyer (2001): as the input denominator in a total factor productivity measure shrinks (by treating inputs as negative outputs and

- The role of increases in labour input is also *magnified* in the net income framework;
- The role of increases in capital input (capital deepening) is greatly *diminished* in the net income framework and
- The role of falling real import prices is also *magnified* in the net income framework.

During the naughts, the average contribution factor for changes in real export and import prices together was 1.28 percentage points per year in the gross framework and 1.50 percentage points per year in the net framework. The corresponding contribution factor for capital growth during the naughts was 1.01 percentage points in the gross framework and 0.53 percentage points in the net framework. Looking at the contribution of falling import prices alone in the net income framework, during the entire sample period, falling import prices contributed about 0.51 percentage points per year to real income growth whereas the effects of net capital accumulation contributed about 0.62 percentage points per year. During the years of the present decade, falling import prices contributed a very large 1.91 percentage points per year to real income growth whereas the effects of net capital accumulation contributed only 0.53 percentage points per year and TFP improvements contributed only 0.12 percentage points per year. Thus for short periods, changes in the real export or import prices that a country faces can have substantially larger effects on living standards than the effects of (net) capital accumulation or improvements in TFP.

The average annual rate of TFP growth in the net income framework was a satisfactory 1.04 percentage points per year. As usual, there are considerable variations in this average rate over different periods. During the golden years, 1962-1973, TFP growth averaged a spectacular 2.92 percentage points per year. During the dismal years 1974-1991, TFP growth averaged −.22 percentage points per year but over the period, 1992-1999, TFP growth recovered to average a very respectable 1.96 percentage points per year. However, during the years 2000-2006, net TFP growth fell to 0.12 percentage points per year, but all of this decline is explained by two poor performance years (2001 and 2003).

As in the previous section, the year over year growth factors listed in Table 9 above can be cumulated and the decomposition given by equation (5) in the previous section will apply to our new net data. The cumulated variables that appear in the new version of equation (5) are reported below in Table 10.

Table 10: Business Sector Cumulated Growth in Net Real Income and Cumulated Contribution Factors

placing them in the net output numerator), the resulting measure of TFP will increase. This magnification effect shows up most clearly during periods of large productivity growth; i.e., during the period 1962-1973, the average net TFP growth rate was 2.92% per year compared to the average gross rate of 2.68% and during the period 1992-1999, the average net TFP growth rate was 1.96% per year compared to the average gross rate of 1.76 %.

Year t	ρ^t/ρ^{1961}	T^t	$A_D{}^t$	$A_X{}^t$	$A_M{}^t$	$B_L{}^t$	$B_K{}^t$	$A_{XM}{}^t$
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.06695	1.03690	0.99936	1.00682	0.98365	1.03439	1.00509	0.99037
1963	1.14753	1.09468	0.99895	1.00548	0.97718	1.05572	1.01166	0.98254
1964	1.25360	1.14397	1.00130	1.00902	0.97589	1.08970	1.01993	0.98469
1965	1.35802	1.17121	1.00732	1.01184	0.98060	1.12720	1.02920	0.99221
1966	1.46915	1.19747	1.00599	1.01306	0.98749	1.17088	1.04118	1.00039
1967	1.48203	1.17168	1.00615	1.01111	0.99086	1.19050	1.05401	1.00187
1968	1.54093	1.19745	1.00492	1.01234	0.99672	1.19452	1.06243	1.00902
1969	1.62502	1.23108	1.00565	1.00939	0.99967	1.21644	1.06935	1.00906
1970	1.68365	1.25452	1.00830	1.01022	1.00256	1.21955	1.07760	1.01281
1971	1.76462	1.27368	1.02091	1.00761	1.00190	1.24070	1.08349	1.00953
1972	1.88938	1.31002	1.02657	1.00647	1.00901	1.27116	1.08832	1.01553
1973	2.21292	1.40896	1.03879	1.03050	1.00983	1.32747	1.09450	1.04063
1974	2.35603	1.42253	1.03975	1.07126	0.98545	1.36190	1.10794	1.05567
1975	2.17468	1.32306	1.02343	1.06499	0.98630	1.36086	1.12354	1.05040
1976	2.38194	1.40553	1.03246	1.07008	0.99758	1.35971	1.13085	1.06749
1977	2.52380	1.49215	1.03083	1.07939	0.97387	1.37121	1.13835	1.05118
1978	2.60045	1.49325	1.02829	1.08555	0.96087	1.41549	1.14704	1.04308
1979	2.80761	1.51198	1.02828	1.11643	0.94816	1.47544	1.15623	1.05856
1980	2.77594	1.40507	1.02330	1.13605	0.95967	1.51269	1.17067	1.09023
1981	2.80710	1.39124	1.02686	1.12397	0.95954	1.54342	1.18044	1.07849
1982	2.51771	1.31225	1.02061	1.08985	0.97661	1.48310	1.19088	1.06436
1983	2.64765	1.36919	1.01435	1.06242	1.00930	1.49120	1.19222	1.07230
1984	2.81903	1.42521	1.01171	1.05826	1.00815	1.53279	1.19555	1.06689
1985	2.98050	1.45871	1.01246	1.05316	1.00969	1.57904	1.20190	1.06336
1986	3.04707	1.44526	1.01641	1.04097	1.01146	1.63013	1.20853	1.05291
1987	3.33683	1.48605	1.02263	1.03882	1.03038	1.68935	1.21430	1.07038
1988	3.53455	1.49137	1.02388	1.02758	1.05539	1.74646	1.22212	1.08450
1989	3.58575	1.45484	1.02462	1.02362	1.07068	1.78199	1.23167	1.09597
1990	3.43478	1.41527	1.01011	0.99481	1.09237	1.78368	1.23956	1.08670
1991	3.10813	1.33368	0.99998	0.95392	1.13153	1.73670	1.24324	1.07939
1992	3.19738	1.39475	0.99813	0.96090	1.11609	1.72243	1.24336	1.07244
1993	3.20824	1.38326	0.99715	0.97342	1.09584	1.75326	1.24367	1.06672
1994	3.46043	1.44111	1.00295	1.00426	1.06155	1.80668	1.24305	1.06607
1995	3.68503	1.46276	1.00416	1.04295	1.04611	1.84638	1.24539	1.09104
1996	3.83187	1.47781	0.99648	1.03258	1.06822	1.88633	1.25061	1.10302
1997	3.96868	1.49771	0.99306	1.02245	1.07652	1.93133	1.25523	1.10069
1998	4.08186	1.53446	0.99220	1.01112	1.06176	1.97735	1.26297	1.07356
1999	4.32783	1.55606	0.99229	1.00693	1.07835	2.03132	1.27078	1.08582
2000	4.74740	1.60437	0.99119	1.03367	1.08383	2.08432	1.27846	1.12032
2001	4.68605	1.58043	0.98809	1.02335	1.08214	2.10460	1.28752	1.10741
2002	4.87549	1.64169	0.99121	0.99937	1.08627	2.13798	1.29090	1.08559
2003	4.87047	1.55724	0.9903	0.97741	1.15200	2.16307	1.29673	1.12597
2004	5.26189	1.57990	0.99433	0.98398	1.18026	2.21947	1.29948	1.16135
2005	5.50038	1.58054	0.99756	0.98870	1.20406	2.24303	1.30647	1.19045

2006	5.67335	1.57743	1.00500	0.97913	1.22210	2.27225	1.31619	1.19659
2007	5.93796	1.56668	1.01179	0.97442	1.25325	2.31313	1.32611	1.22119

The net real income generated by the business sector grew 5.94 fold over the years 1961-2007. The main factors explaining this growth are growth of labour input (cumulative growth factor 2.31), productivity increases (cumulative growth factor 1.57), growth of waiting capital services (cumulative growth factor 1.33), lower real import prices (cumulative growth factor 1.25)[24] and higher real domestic output prices (cumulative growth factor 1.01). There was a small negative contribution from declining real export prices (cumulative growth factor .97).

In the following section, we will use our disaggregated data on exports and imports and our net income methodology to determine the effects of changing disaggregated real export and import prices on the growth of real income generated by the production sector.

5. The Effects of Changing Real Export and Import Prices on Real Income Growth

The methodology explained in Appendix 1 can be used to work out the contribution of each change in the real export price for our 8 classes of exports to the business sector real income growth. These commodity classes are as follows:

- X_1: Exports of agricultural and fish products;
- X_2: Exports of energy products;
- X_3: Exports of forest products;
- X_4: Exports of industrial goods and materials (excluding energy and forest product exports);
- X_5: Exports of machinery and equipment (excluding automotive products);
- X_6: Exports of automotive products;
- X_7: Exports of other consumer goods (excluding automotive products);
- X_8: Exports of services;

The following Table can be viewed as a decomposition of the aggregate export contribution factor α_X^t which appeared in Table 9 of the previous section into 8 commodity specific factors, α_{X1}^t to α_{X8}^t, which multiply together to yield the overall export contribution factor α_X^t.

Table 11: Year to Year Contribution Factors for Eight Export Classes (Net Income Model)

Year t	α_{X1}^t	α_{X2}^t	α_{X3}^t	α_{X4}^t	α_{X5}^t	α_{X6}^t	α_{X7}^t	α_{X8}^t
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.00682	1.00279	0.99976	1.00198	1.00119	1.00053	1.00001	0.99998
1963	0.99866	0.99897	1.00014	0.99982	0.99939	0.99992	1.00000	0.99998
1964	1.00352	1.00027	0.99979	1.00087	1.00096	1.00032	1.00003	1.00005

[24] Note that most of this growth took place over the years 1999-2007.

1965	1.00279	1.00006	1.00010	1.00028	1.00093	1.00031	0.99989	0.99998
1966	1.00121	1.00119	0.99960	0.99899	1.00100	0.99994	0.99957	0.99994
1967	0.99807	0.99900	0.99878	0.99929	0.99949	1.00028	0.99940	0.99994
1968	1.00122	0.99800	1.00005	0.99939	1.00233	1.00091	0.99916	0.99995
1969	0.99708	0.99744	1.00023	1.00072	0.99977	0.99966	0.99843	1.00003
1970	1.00083	0.99770	0.99975	0.99814	1.00278	1.00139	0.99954	0.99983
1971	0.99741	1.00020	1.00067	1.00074	0.99453	0.99939	1.00040	0.99996
1972	0.99887	1.00072	0.99971	1.00214	0.99755	0.99974	0.99842	0.99993
1973	1.02388	1.01333	1.00258	1.00708	1.00584	0.99881	0.99581	0.99997
1974	1.03955	1.01322	1.01566	1.00429	1.01077	0.99941	0.99617	0.99994
1975	0.99415	0.99265	1.01001	1.00097	0.99525	0.99922	0.99696	0.99979
1976	1.00478	0.99481	1.00755	0.99928	1.00078	0.99909	1.00093	1.00008
1977	1.00869	0.99473	1.00566	1.00175	1.00440	0.99995	1.00148	0.99998
1978	1.00571	1.00129	1.00116	1.00160	1.00190	0.99862	1.00187	0.99993
1979	1.02844	1.00450	1.00548	1.00617	1.01346	0.99898	1.00042	0.99981
1980	1.01757	1.00069	1.01179	0.99718	1.01202	0.99699	0.99926	1.00003
1981	0.98937	0.99959	0.99896	0.99792	0.99258	0.99793	1.00073	1.00009
1982	0.96965	0.99373	0.9970	0.99302	0.98655	0.99842	0.99984	0.99985
1983	0.97483	0.99523	0.99518	0.99578	0.99467	0.99654	0.99722	0.99985
1984	0.99609	0.99984	0.99636	1.00365	0.99833	0.99623	1.00174	0.99990
1985	0.99518	0.99857	0.99624	0.99913	0.99663	0.99793	1.00542	1.00008
1986	0.98843	0.99829	0.98267	1.00468	1.00171	1.00557	0.99313	1.00056
1987	0.99793	0.99782	0.99802	1.00391	1.00042	0.99932	0.99774	1.00000
1988	0.98918	1.00086	0.99231	1.00024	1.00358	0.99877	0.99355	1.00005
1989	0.99614	1.00126	1.00187	1.00086	0.99698	0.99871	0.99586	1.00020
1990	0.97186	0.99568	1.00295	0.99309	0.98997	0.99710	0.99509	0.99959
1991	0.95890	0.99434	0.99241	0.99113	0.98961	0.99449	0.99781	0.99971
1992	1.00731	1.00359	1.00095	1.00094	0.99920	0.99860	1.00466	0.99986
1993	1.01303	1.00193	1.00163	1.00497	0.99940	0.99927	1.00509	0.99996
1994	1.03168	1.00260	0.99828	1.00917	1.01170	1.00192	1.00593	1.00035
1995	1.03853	1.00489	0.99885	1.01349	1.01202	1.00125	1.00479	1.00037
1996	0.99006	1.00232	1.00993	0.99206	0.98943	0.99581	1.00054	0.99994
1997	0.99019	0.99638	1.00064	0.99857	0.99735	0.99522	1.00123	0.99981
1998	0.98892	0.99844	0.98881	1.00164	0.99493	0.99837	1.00633	1.00003
1999	0.99586	0.99843	1.01101	0.99976	0.99579	0.99493	0.99629	0.99989
2000	1.02656	0.99940	1.03446	0.99851	1.00477	0.99451	0.99523	0.99961
2001	0.99002	1.00128	0.99988	0.99931	0.99531	0.99622	1.00093	0.99970
2002	0.97657	0.99948	0.98571	0.99511	0.99761	0.99878	0.99952	0.99982
2003	0.97802	0.99785	1.01620	0.99463	0.99701	0.99068	0.98397	0.99940
2004	1.00672	0.99880	1.00776	1.00317	1.00888	0.99559	0.99231	0.99972
2005	1.00479	0.99616	1.02318	0.99610	1.00351	0.99557	0.99067	0.99965
2006	0.99032	0.99885	0.99539	0.99668	1.01090	0.99568	0.99340	0.99974
A62-07	0.99977	1.0018	1.0001	1.0004	0.99834	0.99871	0.99993	1.0004

The arithmetic average of the contribution factors for the years 1962-2007 are listed in the last row of Table 11. Thus we see that changes in the real export prices of agricultural products contributed on average −0.02 percentage points per year to real income growth. The average contribution factors for the 8 classes of exports are as follows:[25]

- −0.02: Exports of agricultural and fish products;
- 0.18: Exports of energy products;
- 0.01: Exports of forest products;
- 0.04: Exports of industrial goods and materials (excluding energy and forest product exports);
- −0.17: Exports of machinery and equipment (excluding automotive products);
- −0.13: Exports of automotive products;
- −0.007: Exports of other consumer goods (excluding automotive products);
- 0.04: Exports of services.

Thus increasing real prices of energy exports over the sample period contributed 0.18 percentage points per year on average to the growth of real income generated by the Canadian business sector. This increase was more than offset by decreasing real export prices of exports of machinery and equipment including automotive products. The remaining 5 classes of exports did not contribute much to real income growth over the sample period.

The Appendix 1 methodology can be used to work out the contribution to business sector real income growth of changes in real import prices for our 7 classes of imports. These commodity classes are as follows:

- M_1: Imports of agricultural and fish products;
- M_2: Imports of energy products;
- M_3: Imports of industrial goods and materials (including imports of forest products but excluding imports of energy products);
- M_4: Imports of machinery and equipment (excluding automotive products);
- M_5: Imports of automotive products;
- M_6: Imports of other consumer goods and
- M_7: Imports of services.

The following Table can be viewed as a decomposition of the aggregate import contribution factor α_M^t which appeared in Table 9 of the previous section into 7 commodity specific factors, α_{M1}^t to α_{M7}^t, which multiply together to yield the overall export contribution factor α_M^t.

Table 12: Year to Year Contribution Factors for Seven Import Classes (Net Income Model)

[25] The cumulative contribution factors for the 8 classes of exports in 2007 are as follows: 0.98905, 1.08622, 1.00484, 1.01928, 0.92633, 0.94215, 0.99662 and 1.01834.

Year t	α_{M1}^{t}	α_{M2}^{t}	α_{M3}^{t}	α_{M4}^{t}	α_{M5}^{t}	α_{M6}^{t}	α_{M7}^{t}
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	0.99858	0.99956	0.99626	0.99337	0.99879	0.99978	0.99722
1963	0.99295	1.00063	0.99897	1.00113	0.99996	1.00040	0.99941
1964	1.00075	1.00016	0.99888	1.00030	0.99947	0.99986	0.99925
1965	1.00604	0.99986	0.99958	0.99928	1.00094	1.00040	0.99875
1966	1.00154	1.00014	1.00259	1.00083	1.00087	1.00062	1.00039
1967	1.00162	1.00152	1.00036	1.00009	1.00023	1.00042	0.99916
1968	1.00016	0.99967	1.00353	1.00236	1.00024	1.00069	0.99926
1969	1.00068	1.00118	1.00053	1.00135	1.00061	1.00043	0.99817
1970	0.99902	1.00027	1.00095	1.00200	1.00116	1.00060	0.99889
1971	0.99980	0.99880	1.00337	0.99956	0.99923	1.00045	0.99815
1972	0.99913	0.99939	1.00338	1.00291	1.00124	1.00016	1.00086
1973	0.99587	0.99804	0.99767	1.00393	1.00367	1.00119	1.00048
1974	0.99743	0.97463	0.98980	1.00400	1.00397	1.00162	1.00394
1975	1.00300	0.99554	1.00325	0.99771	0.99916	1.00109	1.00112
1976	1.00363	1.00038	1.00241	1.00414	0.99987	0.99988	1.00108
1977	0.99578	0.99763	0.99622	0.99880	0.99511	0.99773	0.99472
1978	0.99900	0.99898	0.99479	1.00614	0.99467	0.99783	0.99519
1979	0.99932	0.99457	0.99174	1.00386	0.99929	0.99983	0.99810
1980	1.00078	0.98766	0.99669	1.02697	1.00055	0.99930	1.00028
1981	1.00000	0.99563	1.00075	1.01400	0.99317	0.99826	0.99816
1982	1.00359	1.00424	1.00560	1.00046	1.00090	1.00169	1.00121
1983	1.00292	1.00432	1.00586	1.01228	1.00334	1.00278	1.00156
1984	0.99950	1.00068	1.00083	1.00230	0.99899	0.99876	0.99782
1985	1.00159	0.99997	1.00384	1.00180	0.99760	0.99984	0.99691
1986	0.99876	1.00937	0.99975	1.00128	0.99757	0.99789	0.99716
1987	1.00132	0.99965	1.00205	1.00892	1.00383	1.00087	1.00194
1988	1.00024	1.00353	0.99990	1.00844	1.00562	1.00144	1.00490
1989	1.00104	0.99942	1.00236	1.00690	1.00140	1.00107	1.00222
1990	1.00141	0.99697	1.00581	1.00862	1.00320	1.00187	1.00222
1991	1.00163	1.00441	1.00750	1.00998	1.00539	1.00249	1.00394
1992	1.00041	1.00030	0.99949	0.99733	0.99553	0.99780	0.99542
1993	1.00045	1.00094	0.99913	0.99635	0.99561	0.99761	0.99168
1994	0.99780	1.00012	0.99398	0.99410	0.99343	0.99668	0.99219
1995	0.99826	0.99949	0.99110	1.00443	0.99668	0.99852	0.99691
1996	1.00103	0.99736	1.00662	1.01249	1.00140	1.00162	1.00048
1997	0.99955	1.00051	1.00191	1.00686	1.00040	1.00068	0.99787
1998	1.00067	1.00407	0.99834	0.99761	0.99557	0.99660	0.99341
1999	1.00165	0.99741	1.00466	1.00883	1.00229	1.00124	0.99948
2000	1.00091	0.99101	0.99879	1.00954	1.00338	1.00145	0.99997
2001	0.99972	1.00195	0.99896	1.00149	1.00038	0.99876	0.99719
2002	0.99992	0.99954	1.00260	1.00215	1.00014	1.00062	0.99884
2003	1.00205	0.99792	1.00927	1.02232	1.01032	1.00874	1.00854
2004	1.00128	0.99604	0.99772	1.01434	1.00536	1.00609	1.00356
2005	1.00133	0.99177	1.00051	1.01253	1.00643	1.00413	1.00336

2006	1.00135	0.99560	0.99768	1.01012	1.00489	1.00403	1.00129
A62-07	1.0003	0.99871	1.0004	1.0049	1.0006	1.0006	0.99945

The arithmetic average of the contribution factors for the years 1962-2007 are listed in the last row of Table 12. Thus we see that changes in the real import prices of agricultural products contributed on average 0.03 percentage points per year to real income growth. The average contribution factors for the 7 classes of imports are as follows:[26]

- 0.03: Imports of agricultural and fish products;
- –0.13: Imports of energy products;
- 0.05: Imports of industrial goods and materials (including forest products but excluding energy products);
- 0.49: Imports of machinery and equipment (excluding automotive products);
- 0.06: Imports of automotive products;
- 0.06: Imports of other consumer goods (excluding automotive products);
- –0.06: Imports of services.

Thus lower real import prices for machinery and equipment explained most of the overall favourable effects on real income growth of changes in import prices; this factor contributed on average 0.49 percentage points per year. There were negative contributions to real income growth of higher energy prices (–0.13 percentage points per year) and from higher prices for imported services (–0.06 percentage points per year. There were small but significant positive contributions to real income growth from lower prices for agricultural products (+.03%), materials (+.05%), automotive products (+.06%) and other consumer goods (+.06%). What was surprising to us was the relatively small effect of lower import prices for finally demanded consumer goods.

The methodology explained in this section could be applied to industrial sectors with suitable data on outputs and intermediate inputs; see section 6 of Appendix 1.

6. Conclusion

There are four major conclusions that we can draw from the above results.

First, using new data from Statistics Canada, we have shown that the productivity performance of the business sector of the Canadian economy has been reasonably satisfactory over the past 47 years. In particular, traditional gross income Total Factor Productivity growth averaged 1.01 percent per year over the period 1962-2007[27] and when the net income framework was used, TFP growth averaged 1.04 percent per year.

[26] The cumulative contribution factors for the 7 classes of imports in 2007 are as follows: 1.01313, 0.94191, 1.01973, 1.24849, 1.02793,1.02824 and 0.97493.

[27] The corresponding Statistics Canada average Multifactor Productivity growth rate over this period was only 0.43 percent per year. We attempt to explain this puzzling discrepancy in Appendix 4 below but without complete success.

However, there was a long period (1974-1991) where the productivity performance of the Canadian business sector was decidedly unsatisfactory.

Second, we have shown that the role of explanatory factors for growth in the real income generated by the business sector of the Canadian economy changes substantially when we shift from the standard gross product growth accounting model to a theoretically more appropriate net product growth accounting framework. In general, the main positive drivers of real income growth (growth in labour input, TFP growth and declining real import prices) are *magnified* but the effects of capital services input growth are greatly *diminished* when we switch to the net framework as compared to the gross product framework.[28] An important implication of this result is that improvements in TFP probably become the most important factor for explaining improvements in per capita living standards in the long run and the favourable effects of capital deepening are not as big as they appear to be in the traditional gross income growth accounting methodology.

Third, the results presented here show that over short periods of time, changes in the external price environment facing an economy can have substantial effects on living standards. Thus during the years of the present decade, the real (net) income generated by the Canadian business sector grew at an average rate of 4.09 percent per year and declines in real import prices contributed 1.91 percentage points to this increase, which was greater than the effects of quality adjusted labour input growth (1.64 percentage points per year), increases in waiting services (0.53 percentage points per year).[29]

Finally, the study uncovered many data problems which should be addressed in future work on Canadian productivity performance. A discussion of these data problems can be found in Appendix 4 below. More generally, it is evident that statistical agencies are able to provide reasonably accurate data on the prices and quantities of the outputs produced and intermediate inputs used by the various industries in the economy. This is in large part due to the fact that the *System of National Accounts 1993* used by most statistical agencies has developed an adequate methodology for the treatment of gross outputs and intermediate inputs. However, the corresponding methodology for the treatment of primary inputs was not well developed.[30] In particular, the treatment of capital services was absent the *System of National Accounts 1993* and will only be introduced in the next international version of the System of National Accounts. This absence of a standard methodology for the treatment of capital services means that national statistical agencies have not been able to deliver a generally accepted treatment of capital services in their productivity accounts. Thus detailed data on capital stocks and flows by industry is either not available from national statistical agencies or is not provided due to the lack of information on capital inputs. Given the importance of accurate information on productivity growth, it is important that international agencies provide guidance on

[28] Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) found similar results for Japan and Australia using a similar net income framework.

[29] The Canadian experience with improvements in the terms of trade during the past decade is similar to the Australian experience; see Diewert and Lawrence (2006).

[30] The *System of National Accounts 1993* has a good chapter on wage indexes but does not provide a standard methodology for the treatment of self employment labour input. The recent Manual on the measurement of capital by Schreyer (2009) fills in an important methodological gap in the existing SNA.

acceptable methods for measuring primary input prices and volumes and that national statistical agencies provide more details on how they construct their estimates of primary inputs in their productivity accounts. National departments that have an interest in better productivity measurement (e.g., central banks, departments of finance and industry departments) should support initiatives that will improve the measurement of primary input growth.

Appendix 1: Explaining Real Income Growth: The Translog Approach

1. The Production Theory Framework

In this section, we present the production theory framework which will be used in the main text of the paper.[31] The main reference is Diewert and Morrison (1986)[32] but we also draw on the theory of the output price index, which was developed by Fisher and Shell (1972) and Archibald (1977). This theory is the producer theory counterpart to the theory of the cost of living index for a single consumer (or household) that was first developed by the Russian economist, A. A. Konüs (1924). These economic approaches to price indexes rely on the assumption of (competitive) *optimizing behavior* on the part of economic agents (consumers or producers). Thus we consider only the market sector of the economy in what follows; i.e., that part of the economy that is motivated by profit maximizing behavior. In our empirical work, we define the market sector to be the Canadian business sector of the economy less the rental and owner occupied housing sectors.[33]

Initially, we assume that the market sector of the economy produces quantities of M (net)[34] outputs, $y \equiv [y_1,...,y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$. We further assume that the market sector of the economy uses positive quantities of N primary inputs, $x \equiv [x_1,...,x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is

[31] With the exception of the last section of this Appendix, this material is drawn from Diewert (2005b), Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).
[32] The theory also draws on Samuelson (1953), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Fox and Kohli (1998), Kohli (1978) (1990) (1991) (2003) (2004a) (2004b) (2006) (2007), Morrison and Diewert (1990), Samuelson (1953) and Sato (1976).
[33] The Canadian business sector excludes all of the general government sectors such as schools, hospitals, universities, defence and public administration where no independent measures of output can be obtained. For owner occupied housing, output is equal to input and hence no productivity improvements can be generated by this sector according to SNA conventions. Due to the difficulties involved in splitting up the residential housing stock into the rental and owner occupied portions, we omit the entire residential housing stock and the consumption of residential housing services in our data. However, we do include investment in residential housing, since that investment is part of the output of the market production sector.
[34] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity y_m is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

utilized by the market sector of the economy; denote this period t production possibilities set by S^t. We assume that S^t is a closed convex cone that exhibits a free disposal property.[35]

Given a vector of output prices P and a vector of available primary inputs x, we define *the period t market sector GDP function*, $g^t(P,x)$, as follows:[36]

$$(1)\ g^t(P,x) \equiv \max_y \{P\cdot y : (y,x) \text{ belongs to } S^t\} ; \qquad t = 0,1,2, \ldots .$$

Thus market sector GDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on x, the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the market sector during period t and if the GDP function is differentiable with respect to the components of P at the point P^t,x^t, then the period t vector of market sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:[37]

$$(2)\ y^t = \nabla_P\, g^t(P^t,x^t) ; \qquad t = 0,1,2, \ldots .$$

Thus the period t market sector supply vector y^t can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t output price vector P^t.

If the GDP function is differentiable with respect to the components of x at the point P^t,x^t, then the period t vector of input prices W^t will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of x; i.e., we will have the following equations for each period t:[38]

$$(3)\ W^t = \nabla_x\, g^t(P^t,x^t) ; \qquad t = 0,1,2, \ldots .$$

[35] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974).

[36] The function $g^t(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1973) (1974; 136). Notation: $P\cdot y \equiv \sum_{m=1}^{M} P_m y_m$.

[37] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P\, g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial P_1, \ldots,\partial g^t(P^t,x^t)/\partial P_M]$.

[38] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla x\ g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial x_1, \ldots,\partial g^t(P^t,x^t)/\partial x_N]$.

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t input quantity vector x^t.

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t; i.e., we have the following relationships:

(4) $g^t(P^t,x^t) = P^t \cdot y^t = W^t \cdot x^t$; $\quad t = 0,1,2, \ldots$.

The above material will be useful in what follows but of course, our focus is not on GDP; instead our focus is on the income generated by the market sector or more precisely, on *the real income generated by the market sector*. However, since market sector GDP (the value of market sector production) is distributed to the factors of production used by the market sector, nominal market sector GDP will be equal to nominal market sector income; i.e., from (4), we have $g^t(P^t,x^t) = P^t \cdot y^t = W^t \cdot x^t$. As an approximate welfare measure that can be associated with market sector production,[39] we will choose to measure the *real income generated by the market sector in period t*, r^t, in terms of the number of consumption bundles that the nominal income could purchase in period t; i.e., define ρ^t as follows:

(5) $\rho^t \equiv W^t \cdot x^t / P_C^t$; $\quad t = 0,1,2, \ldots$
$= w^t \cdot x^t$
$= p^t \cdot y^t$
$= g^t(p^t,x^t)$

where $P_C^t > 0$ is the *period t consumption expenditures deflator* and the market sector period t *real output price* p^t and *real input price* w^t vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index; i.e., we have the following definitions:[40]

(6) $p^t \equiv P^t/P_C^t$; $w^t \equiv W^t/P_C^t$; $\quad t = 0,1,2, \ldots$.

The first and last equality in (5) imply that period t real income, ρ^t, is equal to the period t GDP function, evaluated at the period t real output price vector p^t and the period t input vector x^t, $g^t(p^t,x^t)$. Thus *the growth in real income over time can be explained by three*

[39] Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

[40] Our approach is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

main factors: t (Technical Progress or Total Factor Productivity growth), growth in real output prices and the growth of primary inputs. We will shortly give formal definitions for these three growth factors.

Using the linear homogeneity properties of the GDP functions $g^t(P,x)$ in P and x separately, we can show that the following counterparts to the relations (2) and (3) hold using the deflated prices p and w:[41]

(7) $y^t = \nabla_p\, g^t(p^t,x^t)$; t = 0,1,2, ...
(8) $w^t = \nabla_x\, g^t(p^t,x^t)$; t = 0,1,2, ...

Now we are ready to define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$:[42]

(9) $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$; t = 1,2, ...

Thus $\tau(p,x,t)$ measures the proportional change in the real income produced by the market sector at the reference real output prices p and reference input quantities used by the market sector x where the numerator in (9) uses the period t technology and the denominator in (9) uses the period t−1 technology. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t−1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (9): a *Laspeyres type measure* τ_L^t that chooses the period t−1 reference vectors p^{t-1} and x^{t-1} and a *Paasche type measure* τ_P^t that chooses the period t reference vectors p^t and x^t:

(10) $\tau_L^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$; t = 1,2, ... ;
(11) $\tau_P^t \equiv \tau(p^t,x^t,t) = g^t(p^t,x^t)/g^{t-1}(p^t,x^t)$; t = 1,2, ...

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from index number theory[43] (so that the estimate going backwards is equal to the reciprocal of

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[41] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,x)$, which uses the original output prices P, then they will also solve the profit maximization problem that uses the normalized output prices $p \equiv P/P_C$; i.e., they will also solve the problem defined by $g^t(p,x)$.
[42] This measure of technical progress is due to Diewert and Morrison (1986; 662). A special case of it was defined earlier by Diewert (1983; 1063).
[43] See Fisher (1922; 64).

the estimate going forwards), then the geometric mean will be the best simple average to take.[44] Thus we define the geometric mean of (10) and (11) as follows:[45]

$$(12)\ \tau^t \equiv [\tau_L{}^t\, \tau_P{}^t]^{1/2}\ ; \qquad\qquad t = 1,2, \ldots .$$

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (10)-(12). One obvious way would be to assume a functional form for the GDP function $g^t(p,x)$, collect data on output and input prices and quantities for the market sector for a number of years (and for the consumption expenditures deflator), add error terms to equations (7) and (8) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[46] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus we will suggest methods for implementing measures like (12) in this Appendix that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t real output price growth factors* $\alpha(p^{t-1},p^t,x,s)$:[47]

$$(13)\ \alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x)/g^s(p^{t-1},x)\ ; \qquad\qquad s = 1,2, \ldots .$$

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in real output prices going from period t−1 to t, using the technology that is available during period s and using the reference input quantities x. Thus each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (13): a *Laspeyres type measure* $\alpha_L{}^t$ that chooses the period t−1 reference technology and

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[44] See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.

[45] The theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and Morrison (1986; 662-663) in the nominal GDP context. See Diewert (1993) for properties of symmetric means.

[46] "The estimation of GDP functions such as (19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).

[47] This measure of real output price change was essentially defined by Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89). Readers who are familiar with the theory of the true cost of living index will note that the real output price index defined by (13) is analogous to the Konüs (1924) *true cost of living index* which is a ratio of cost functions, say $C(u,p^t)/C(u,p^{t-1})$ where u is a reference utility level: g^s replaces C and the reference utility level u is replaced by the vector of reference variables x.

reference input vector x^{t-1} and a *Paasche type measure* α_P^t that chooses the period t reference technology and reference input vector x^t:

$$(14)\ \alpha_L^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, ... ;$$
$$(15)\ \alpha_P^t \equiv \alpha(p^{t-1},p^t,x^t,t) = g^t(p^t,x^t)/g^t(p^{t-1},x^t)\ ; \qquad t = 1,2, ...$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:[48]

$$(16)\ \alpha^t \equiv [\alpha_L^t\, \alpha_P^t]^{1/2}\ ; \qquad t = 1,2, ...$$

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in input quantities. Define a family of *period t real input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:[49]

$$(17)\ \beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t)/g^s(p,x^{t-1})\ ; \qquad s = 1,2, ...$$

Thus $\beta(x^{t-1},x^t,p,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in input quantities used by the market sector going from period t−1 to t, using the technology that is available during period s and using the reference real output prices p. Thus each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (17): a *Laspeyres type measure* β_L^t that chooses the period t−1 reference technology and reference real output price vector p^{t-1} and a *Paasche type measure* β_P^t that chooses the period t reference technology and reference real output price vector p^t:

$$(18)\ \beta_L^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad t = 1,2, ... ;$$
$$(19)\ \beta_P^t \equiv \beta(x^{t-1},x^t,p^t,t) = g^t(p^t,x^t)/g^t(p^t,x^{t-1})\ ; \qquad t = 1,2, ...$$

Since both measures of real input growth are equally valid, it is natural to average them to obtain an overall measure of the effects of input growth on real income:[50]

$$(20)\ \beta^t \equiv [\beta_L^t\, \beta_P^t]^{1/2}\ ; \qquad t = 1,2, ...$$

[48] The indexes defined by (13)-(16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.
[49] This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).
[50] The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

Recall that market sector real income for period t was defined by (5) as ρ^t equal to nominal period t factor payments $W^t \cdot x^t$ deflated by the household consumption price deflator P_C^t. It is convenient to define γ^t as the *period t chain rate of growth factor for real income*:

$$(21)\ \gamma^t \equiv \rho^t/\rho^{t-1}\ ; \qquad t = 1,2, \ldots .$$

It turns out that the definitions for γ^t and the technology, output price and input quantity growth factors $\tau(p,x,t)$, $\alpha(p^{t-1},p^t,x,s)$, $\beta(x^{t-1},x^t,p,s)$ defined by (9), (13) and (17) respectively satisfy some interesting identities, which we will now develop. We have:

$$\begin{aligned}(22)\ \gamma^t &\equiv \rho^t/\rho^{t-1}\ ; && t = 1,2, \ldots .\\ &= g^t(p^t,x^t)/g^{t-1}(p^{t-1},x^{t-1}) && \text{using definitions (4) and (5)}\\ &= [g^t(p^t,x^t)/g^{t-1}(p^t,x^t)][g^{t-1}(p^t,x^t)/g^{t-1}(p^{t-1},x^t)][g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})] &&\\ &= \tau_P{}^t\ \alpha(p^{t-1},p^t,x^t,t-1)\ \beta_L{}^t && \text{using definitions (11), (13) and (18).}\end{aligned}$$

In a similar fashion, we can establish the following companion identity:

$$(23)\ \gamma^t \equiv \tau_L{}^t\ \alpha(p^{t-1},p^t,x^{t-1},t)\ \beta_P{}^t \qquad \text{using definitions (10), (13) and (19).}$$

Thus multiplying (22) and (23) together and taking positive square roots of both sides of the resulting identity and using definitions (12) and (20), we obtain the following identity:

$$(24)\ \gamma^t \equiv \tau^t\ [\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}\ \beta^t\ ; \qquad t = 1,2, \ldots .$$

In a similar fashion, we can derive the following alternative decomposition for γ^t into growth factors:

$$(25)\ \gamma^t \equiv \tau^t\ \alpha^t\ [\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}\ ; \qquad t = 1,2, \ldots .$$

It is quite likely that the real output price growth factor $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}$ is fairly close to α^t defined by (16) and it is quite likely that the input growth factor $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$ is quite close to β^t defined by (20); i.e., we have the following approximate equalities:

$$(26)\ [\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2} \approx \alpha^t\ ; \qquad t = 1,2, \ldots ;$$
$$(27)\ [\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2} \approx \beta^t\ ; \qquad t = 1,2, \ldots .$$

Substituting (26) and (27) into (24) and (25) respectively leads to the following approximate decompositions for the growth of real income into explanatory factors:

$$(28)\ \gamma^t \approx \tau^t\ \alpha^t\ \beta^t\ ; \qquad t = 1,2, \ldots$$

where τ^t is a technology growth factor, α^t is a growth in real output prices factor and β^t is a growth in primary inputs factor.

Rather than look at explanatory factors for the growth in real market sector income, it is sometimes convenient to express the level of real income in period t in terms of an *index of the technology level* or of Total Factor Productivity in period t, T^t, of the *level of real output prices* in period t, A^t, and of the *level of primary input quantities* in period t, B^t.[51] Thus we use the growth factors τ^t, α^t and β^t as follows to define the levels T^t, A^t and B^t:

(29) $T^0 \equiv 1$; $T^t \equiv T^{t-1}\,\tau^t$; t = 1,2, ... ;
(30) $A^0 \equiv 1$; $A^t \equiv A^{t-1}\alpha^t$; t = 1,2, ... ;
(31) $B^0 \equiv 1$; $B^t \equiv B^{t-1}\beta^t$; t = 1,2, ...

Using the approximate equalities (28) for the chain links that appear in (29)-(31), we can establish the following approximate relationship for the level of real income in period t, ρ^t, and the period t levels for technology, real output prices and input quantities:

(32) $\rho^t/\rho^0 \approx T^t\,A^t\,B^t$; t = 0,1,2, ...

In the following section, we note a set of assumptions on the technology sets that will ensure that the approximate real income growth decompositions (28) and (32) hold as exact equalities.

3. The Translog GDP Function Approach

We now follow the example of Diewert and Morrison (1986; 663) and assume that the log of the period t (deflated) GDP function, $g^t(p,x)$, has the following translog functional form:[52]

$$(33)\ \ln g^t(p,x) \equiv a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + (1/2)\sum_{m=1}^M\sum_{k=1}^M a_{mk} \ln p_m^t \ln p_k^t + \sum_{n=1}^N b_n^t \ln x_n^t + (1/2)\sum_{n=1}^N\sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M\sum_{n=1}^M c_{mn} \ln p_m^t \ln x_n^t ;\quad t = 0,1,2, \ldots .$$

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients must satisfy the following restrictions in order for g^t to satisfy the linear homogeneity properties that we have assumed in section 2 above:[53]

[51] This type of levels presentation of the data is quite instructive when presented in graphical form. It was suggested by Kohli (1990) and used extensively by him; see Kohli (1991), (2003) (2004a) (2004b) and Fox and Kohli (1998).
[52] This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.
[53] There are additional restrictions on the parameters which are necessary to ensure that $g^t(p,x)$ is convex in p and concave in x. Note that when we divide the original prices by one of the prices, then one of the scaled prices will be identically equal to one and hence its logarithm will be identically equal to zero.

(34) $\sum_{m=1}^{M} a_m^{\,t} = 1$ for t = 0,1,2, ...;
(35) $\sum_{n=1}^{N} b_n^{\,t} = 1$ for t = 0,1,2, ...;
(36) $a_{mk} = a_{km}$ for all k,m ;
(37) $b_{nj} = b_{jn}$ for all n,j ;
(38) $\sum_{k=1}^{M} a_{mk} = 0$ for m = 1,...,M ;
(39) $\sum_{j=1}^{N} b_{nj} = 0$ for n = 1,...,N ;
(40) $\sum_{n=1}^{N} c_{mn} = 0$ for m = 1,...,M ;
(41) $\sum_{m=1}^{M} c_{mn} = 0$ for n = 1,...,N .

Recall the approximate decomposition of real income growth going from period t−1 to t given by (28) above, $\gamma^t \approx \tau^t\, \alpha^t\, \beta^t$. Diewert and Morrison (1986; 663) showed that[54] if g^{t-1} and g^t are defined by (33)-(41) above and there is competitive profit maximizing behavior on the part of all market sector producers for all periods t, then (28) holds as an exact equality; i.e., we have

$$(42)\ \gamma^t = \tau^t\, \alpha^t\, \beta^t\,; \qquad t = 1,2, \ldots\,.$$

In addition, Diewert and Morrison (1986; 663-665) showed that τ^t, α^t and β^t could be calculated using empirically observable price and quantity data for periods t−1 and t as follows:

$$(43)\ \ln\alpha^t = \sum_{m=1}^{M} (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^{t}y_m^{t}/p^{t}\cdot y^{t})]\ \ln(p_m^{t}/p_m^{t-1})$$
$$= \ln P_T(p^{t-1},p^t,y^{t-1},y^t);$$
$$(44)\ \ln\beta^t = \sum_{n=1}^{N} (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^{t}x_n^{t}/w^{t}\cdot x^{t})]\ \ln(x_n^{t}/x_n^{t-1})$$
$$= \ln Q_T(w^{t-1},w^t,x^{t-1},x^t);$$
$$(45)\quad \tau^t = \gamma^t/\alpha^t\, \beta^t$$

where $P_T(p^{t-1},p^t,y^{t-1},y^t)$ is the Törnqvist (1936) and Törnqvist and Törnqvist (1937) output price index and $Q_T(w^{t-1},w^t,x^{t-1},x^t)$ is the Törnqvist input quantity index.

Since equations (42) now hold as exact identities under our present assumptions, equations (32), the cumulated counterparts to equations (28), will also hold as exact decompositions; i.e., under our present assumptions, we have

$$(46)\ \rho^t/\rho^0 = T^t\, A^t\, B^t\,; \qquad t = 1,2, \ldots\,.$$

We will implement the real income decompositions (42) and (46) in the main text.

4. The Translog GDP Function Approach and Changes in the Terms of Trade

For some purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices α^t into separate effects for each change

[54] Diewert and Morrison established their proof using the nominal GDP function $g^t(P,x)$. However, it is easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) = g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P.

in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all market sector primary input quantities β^t into separate effects for each change in each input quantity. In this section, we indicate how this can be done, making the same assumptions on the technology that were made in the previous section.

We first model the effects of a change in a single (deflated) output price, say p_m, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type price indexes defined by (14) and (15) above for changes in all (deflated) output prices are the following *Laspeyres type measure* α_{Lm}^t that chooses the period t−1 reference technology and holds constant other output prices at their period t−1 levels and holds inputs constant at their period t−1 levels x^{t-1} and a *Paasche type measure* α_{Pm}^t that chooses the period t reference technology and reference input vector x^t and holds constant other output prices at their period t levels:

$$(47)\ \alpha_{Lm}^t \equiv g^{t-1}(p_1^{t-1},...,p_{m-1}^{t-1},p_m^t,p_{m+1}^{t-1},..., p_M^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad m = 1,...,M;\ t = 1,2, ... ;$$

$$(48)\ \alpha_{Pm}^t \equiv g^t(p^t,x^t)/g^t(p_1^t,...,p_{m-1}^t,p_m^{t-1},p_{m+1}^t,..., p_M^t,x^t)\ ; \qquad m = 1,...,M;\ t = 1,2, ...$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on real income of the change in the real price of output m*:[55]

$$(49)\ \alpha_m^t \equiv [\alpha_{Lm}^t\, \alpha_{Pm}^t]^{1/2}\ ; \qquad m = 1,...,M\ ;\ t = 1,2, ...$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 666) can be adapted to give us the following result:

$$(50)\ \ln\alpha_m^t = (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)]\ \ln(p_m^t/p_m^{t-1})\ ; \qquad m = 1,...,M\ ;\ t = 1,2, ...$$

Note that $\ln\alpha_m^t$ is equal to the mth term in the summation of the terms on the right hand side of (43). This observation means that we have the following exact decomposition of the period t aggregate real output price contribution factor α^t into a product of separate price contribution factors; i.e., we have under present assumptions:

$$(51)\ \alpha^t = \alpha_1^t\alpha_2^t ...\ \alpha_M^t\ ; \qquad t = 1,2, ...$$

The above decomposition is useful for analyzing how real changes in the price of exports (i.e., a change in the price of exports relative to the price of domestic consumption) and in the price of imports impact on the real income generated by the market sector. In the

[55] The indexes defined by (47)-(49) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

empirical illustration which follows later, we let M equal three. The three net outputs are:

- Domestic sales (C+I+G);
- Exports (X) and
- Imports (M).

Since commodities 1 and 2 are outputs, y_1 and y_2 will be positive but since commodity 3 is an input into the market sector, y_3 will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (50) for m = 2 (where $y_m^t > 0$) and for m = 3 (where $y_m^t < 0$).

As mentioned above, it is also useful to have a decomposition of the aggregate contribution of input growth to the growth of real income into separate contributions for each important class of primary input that is used by the market sector. We now model the effects of a change in a single input quantity, say x_n, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type quantity indexes defined by (18) and (19) above for changes in input n are the following *Laspeyres type measure* β_{Ln}^t that chooses the period t−1 reference technology and holds constant other input quantities at their period t−1 levels and holds real output prices at their period t−1 levels p^{t-1} and a *Paasche type measure* β_{Pn}^t that chooses the period t reference technology and reference real output price vector p^t and holds constant other input quantities at their period t levels:

$$(52)\ \beta_{Ln}^t \equiv g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad n = 1,...,N;\ t = 1,2, ...\ ;$$

$$(53)\ \beta_{Pn}^t \equiv g^t(p^t,x^t)/g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t)\ ; \qquad n = 1,...,N;\ t = 1,2, ...\ .$$

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income of the change in the quantity of input n*:[56]

$$(54)\ \beta_n^t \equiv [\beta_{Pn}^t\,\beta_{Pn}^t]^{1/2}\ ; \qquad n = 1,...,N\ ;\ \ t = 1,2, ...\ .$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 667) can be adapted to give us the following result:

$$(55)\ \ln\beta_n^t = (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t\,x_n^t/w^t\cdot x^t)]\ \ln(x_n^t/x_n^{t-1})\ ; \qquad n = 1,...,N\ ;\ t = 1,2, ...\ .$$

[56] The indexes defined by (52)-(54) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

Note that $\ln\beta_n^t$ is equal to the nth term in the summation of the terms on the right hand side of (44). This observation means that we have the following exact decomposition of the period t aggregate input growth contribution factor β^t into a product of separate input quantity contribution factors; i.e., we have under present assumptions:

(56) $\beta^t = \beta_1^t\beta_2^t ... \beta_N^t$; $\qquad t = 1,2, ...$

5. The Deflated NDP Translog Approach

There is a severe flaw with all of the analysis presented in the previous sections of this Appendix.. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence our real income measure defined by (5) above is overstated.

To see why Gross Domestic Product overstates income, consider the model of production that is described by the following quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).[57]

All of the "flow" inputs that are purchased during the period and all of the "flow" outputs that are sold during the period are the inputs and outputs that appear in the usual definition of cash flow. These are the flow inputs and outputs that are very familiar to

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[57] For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980). The usual user cost of capital can be derived from this framework if depreciation is independent of use.

national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred due to its sales of outputs and its purchases of inputs during the period. As well, *it will have a stock of assets that it can use when it starts production in the following period*. Just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm's productive activities. Hence, to get a complete picture of the firm's production activities over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period.

We illustrate the above theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in period t, Y^t and the other output is an investment good, I^t. One of the inputs is the flow of noncapital primary input X^t and the other input is K^t, capital services. Suppose that the average prices during period t of a unit of Y^t, X^t and I^t are $P_Y{}^t$, $P_X{}^t$ and $P_I{}^t$ respectively. Suppose further that the interest rate prevailing at the beginning of period t is r^t. The value of the beginning of period t capital stock is assumed to be $P_I{}^t$, the investment price for period t. In order to induce households to let the business sector use the initial stock of capital, firms have to pay households interest equal to $r^t P_I{}^t K^t$. Then neglecting balance sheet items, the market sector's period t *cash flow* is:[58]

(57) $CF^t \equiv P_Y{}^t Y^t + P_I{}^t I^t - P_X{}^t X^t - r^t P_I{}^t K^t$.

K^t is interpreted as the firm's beginning of period t stock of capital it has at its disposal and its end of period stock of capital is defined to be K^{t+1}. These capital stocks are valued at the balance sheet prices prevailing at the beginning and end of period t, $P_I{}^t$ and $P_I{}^{t+1}$ respectively.

The market sector period t *pure profit* is defined as its cash flow plus the value of its end of period t capital stock less the value of its beginning of period t capital stock:

(58) $\prod^t \equiv CF^t + P_I{}^{t+1} K^{t+1} - P_I{}^t K^t$.

Now the end of period depreciated stock of capital is related to the beginning of the period stock by the following equation:

(59) $K^{t+1} = (1 - \delta)K^t$

where $0 < \delta < 1$ denotes the depreciation rate.

Now substitute (57) and (59) into the definition of pure profits (58) and we obtain the following expression:

[58] For equity financed firms, we need to include an imputed return for equity capital.

(60) $\Pi^t \equiv P_Y{}^t\, Y^t + P_I{}^t\, I^t - P_X{}^t\, X^t - r^t\, P_I{}^t\, K^t + P_I{}^{t+1}(1-\delta)K^t - P_I{}^t\, K^t$
$= P_Y{}^t\, Y^t + P_I{}^t\, I^t - P_X{}^t\, X^t - \{r^t\, P_I{}^t + \delta P_I{}^{t+1} - (P_I{}^{t+1} - P_I{}^t)\}K^t.$

The expression that precedes the capital stock K^t, $\{r^t\, P_I{}^t + \delta P_I{}^{t+1} - (P_I{}^{t+1} - P_I{}^t)\}$, can be recognized as the *user cost of capital*;[59] it is the gross rental price that must be paid to a capitalist in order to induce him or her to loan the services of a unit of the capital stock to the production sector.

Some simplifications for (60) occur if we make two additional assumptions:

- Assume that producers and households expect price level stability so that the end of the period price for a new unit of capital $P_I{}^{t+1}$ is expected to be equal to the beginning of the period price for a new unit of capital $P_I{}^t$; in this case, we can interpret r^t as the period t real interest rate;[60]
- Assume that pure profits are zero so that Π^t equals zero.

Substituting these two assumptions into equation (60) leads to the following expression:

(61) $\Pi^t = P_Y{}^t\, Y^t + P_I{}^t\, I^t - P_X{}^t\, X^t - \{r^t\, P_I{}^t + \delta P_I{}^t\}K^t = 0.$

Equation (61) can be rearranged to yield the following value of output equals value of input equation:

(62) $P_Y{}^t\, Y^t + P_I{}^t\, I^t = P_X{}^t\, X^t + \{r^t\, P_I{}^t + \delta P_I{}^t\}K^t.$

Equation (62) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (4), $P^t \cdot y^t = W^t \cdot x^t$, that we have been using thus far in this Appendix. We now come to the point of this rather long digression: *the (gross) payments to primary inputs that is defined by the right hand side of (62) is not income,* in the sense of Hicks.[61] The owner of a unit of capital cannot spend the entire period t gross rental income $\{r^t\, P_I{}^t + \delta P_I{}^t\}$ on consumption during period t because the depreciation portion of the rental, $\delta P_I{}^t$, is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $\{r^t\, P_I{}^t + \delta P_I{}^t\}$ at the end of the period plus the depreciated unit of the initial capital stock, which is worth only $(1-\delta)P_I{}^t$. Thus $\delta P_I{}^t$ of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means

[59] See Christensen and Jorgenson (1969) for a derivation in continuous time and Diewert (1980; 471) for a derivation in discrete time.
[60] This assumption can be relaxed somewhat and we can still end up with much the same model; see Diewert (2006a).
[61] We will use Hicks' third concept of income here: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1946; 174).

that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_X^t X^t + \{r^t P_I^t + \delta P_I^t\}K^t$; instead it is the value of payments to labour $P_X^t X^t$ plus the reward for waiting, $r^t P_I^t K^t$. Using this definition of market sector (net) Hicksian income, we can rearrange equation (62) as follows:

(63) Hicksian market sector income $\equiv P_X^t X^t + r^t P_I^t K^t$
$= P_Y^t Y^t + P_I^t I^t - \delta P_I^t K^t$
= Value of consumption + value of gross investment − value of depreciation.

Thus in this Hicksian net income framework, our new output concept is equal to our old output concept less the value of depreciation. We take the price of depreciation to be the corresponding investment price P_I^t and the quantity of depreciation is taken to be the depreciation rate times the beginning of the period stock, δK^t.

Hence the overstatement of income problem that is implicit in the approaches used in previous sections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

In our empirical work, our user costs in the gross product approach took the following form:

(64) $u^t = (r^t + \delta^t + \tau^t)P_I^t$

where r^t is the balancing period t real rate of interest, δ^t is a geometric depreciation rate for period t, τ^t is an average capital taxation rate on the asset and P_I^t is the period t investment price for the asset. However, when we used the net product approach, we split up each (gross product) user cost times the beginning of the period stock K^t into the depreciation component $\delta^t P_I^t K^t$ and the remaining term $(r^t+\tau^t)P_I^t K^t$ and we regarded the second term as a genuine income component but the first term was treated as an intermediate input cost for the market sector and was an offset to gross investment made by the market sector during the period under consideration. In the main text, when the net approach was used, the investment aggregate I was a *net investment aggregate* (gross investment components were indexed with a positive sign in the aggregate and depreciation components were indexed with a negative sign in the aggregate). The capital services aggregate in the net approach was a "reward for waiting" capital services aggregate rather than the gross return aggregate that was used in the gross product approach.[62]

[62] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.

6. Sectoral Contributions to Real Income Growth

The above theory applied to the market sector as a whole. However, it is of considerable interest to determine which separate industries contributed the most to the overall growth of real income generated by the market sector of the economy. Hence, in this section, we outline how this can be done if industry data on outputs, inputs and the corresponding prices are available.[63] However, at the outset, it should be noted that in general, we will not be able to single out the effects of changes in real international prices as we were able to do when the entire business sector is treated as a single industry.[64]

We assume that there are I industries in the market sector of the economy. As in section 2, we assume that there is a common list of M (net) outputs which each industry produces or uses as intermediate inputs. The net output vector for industry i in period t is $y^{it} \equiv [y_1^{it},...,y_M^{it}]$, which are sold at the positive producer prices for industry i in period t, $P^{it} \equiv [P_1^{it},...,P_M^{it}]$, for i = 1,...,I. There is also a common list of N primary inputs used by each industry. In period t, we assume that industry i uses nonnegative quantities of N primary inputs, $x^{it} \equiv [x_1^{it},...,x_N^{it}]$ which are purchased at the positive primary input prices $W^{it} \equiv [W_1^{it},...,W_N^{it}]$ for i = 1,...,I. In each period t, we assume that there is a feasible set of net output vectors y^i that can be produced by industry i if the vector of primary inputs x^i is utilised by that industry; denote this period t production possibilities set by S^{it}. We assume that S^{it} is a closed convex cone that exhibits a free disposal property. We shall take the net product point of view developed in the previous section for each industry in what follows.

Given a vector of industry i net output prices P^{it} and a vector of available primary inputs x^{it} for that industry, we define *the industry i period t net product function*, $g^{it}(P^{it},x^{it})$, as follows:

$$(65)\ g^{it}(P^{it},x^{it}) \equiv \max_y \{P^{it}\cdot y : (y,x^{it}) \text{ belongs to } S^{it}\} = P^{it}\cdot y^{it} ; \qquad i = 1,...,I ; t = 0,1,2, ...$$

Since we have assumed constant returns to scale for each industry, it is natural to assume that the income generated by industry i in period t, $W^{it}\cdot x^{it}$, is equal to the corresponding value of net product, $P^{it}\cdot y^{it}$; i.e., we have:

$$(66)\ P^{it}\cdot y^{it} = W^{it}\cdot x^{it} ; \qquad i = 1,...,I ; t = 0,1,2, ...$$

[63] In Canada, such data are available from the Input-Output and Productivity Divisions of Statistics Canada. However, these data for the past 5 years are not available at present.

[64] The problem is not methodological; it is a data problem. In order to determine the effects of changing real import and export prices on the real income generated by an industry, we require information on the value and price of exports produced by the industry and on the value and price of imports used by the industry. However, the *System of National Accounts 1993* does not set up the production accounts so that the exports produced and imports used by an industry are recorded in the recommended system of production accounts. In theory, this problem can be remedied simply by distinguishing industry outputs as being either exported or delivered to domestic users and by distinguishing industry inputs as being either imports or supplied by a domestic producer; see Diewert (2007b) (2007c) for the details of the resulting modified industry accounts. In practice, it will be extremely difficult to collect the required information. For further discussion of these issues, see section 1 of Appendix 2 below.

Define the *period t, industry i real input and output price vectors*, w^{it} and p^{it} respectively, as follows:

$$(67)\ w^{it} \equiv W^{it}/P_C^{\,t}\ ;\ p^{it} \equiv P^{it}/P_C^{\,t}\ ; \qquad i = 1,...,I\ ;\ t = 0,1,2, ...\ .$$

As in section 2 of this Appendix, we can define the *real income generated by industry i in period t*, ρ^{it}, as the nominal income generated by industry i in period t, $W^{it}\cdot x^{it}$, divided by the consumption price deflator for period t, $P_C^{\,t}$. Using (65)-(67), we have:

$$\begin{aligned}(68)\ \rho^{it} &\equiv W^{it}\cdot x^{it}/P_C^{\,t} \qquad i = 1,...,I\ ;\ t = 0,1,2, ...\ .\\ &= w^{it}\cdot x^{it}\\ &= P^{it}\cdot y^{it}/P_C^{\,t}\\ &= p^{it}\cdot y^{it}\\ &= g^{it}(p^{it},x^{it})\end{aligned}$$

where the last equality follows using (65)-(67) and the linear homogeneity of $g^{it}(P^{it},x^{it})$ in P^{it}.

We now rework the theoretical analysis presented in sections 2-4 above, except we apply it at the industry level instead of the economy wide market sector level. Thus define γ^{it} as the *period t chain link rate of growth factor for the real income generated by industry i*:

$$(69)\ \gamma^{it} \equiv \rho^{it}/\rho^{it-1}\ ; \qquad i = 1,...,I\ ;\ t = 1,2,...\ .$$

Now assume that the industry i, period t (deflated) GDP function, $g^{it}(p,x)$, has a translog functional form analogous to that defined above by (33)-(41). Repeat the analysis at the national level that led up to equation (42), except now apply it at the industry level. We can derive the following industry counterparts to the national equation (42):

$$(70)\ p^{it}\cdot y^{it}/p^{it-1}\cdot y^{it-1} = \rho^{it}/\rho^{it-1} = \gamma^{it} = \tau^{it}\ \alpha^{it}\ \beta^{it}\ ; \qquad i = 1,...,I\ ;\ t = 0,1,2, ...$$

where the *period t, industry i chain link technical progress growth rate* τ^{it}, *output price growth rate* α^{it} and *input quantity growth rate* β^{it} can be calculated using the period t and t−1 price and quantity data for industry i as follows, for i = 1,...,I ; t = 0,1,2, ... :

$$\begin{aligned}(71)\ \ln\alpha^{it} &\equiv \textstyle\sum_{m=1}^{M} (1/2)[(p_m^{\,it-1}y_m^{\,it-1}/p^{it-1}\cdot y^{it-1}) + (p_m^{\,it}y_m^{\,it}/p^{it}\cdot y^{it})]\ \ln(p_m^{\,it}/p_m^{\,it-1})\\ &= \ln P_T(p^{it-1},p^{it},y^{it-1},y^{it});\\ (72)\ \ln\beta^{it} &\equiv \textstyle\sum_{n=1}^{N} (1/2)[(w_n^{\,it-1}x_n^{\,it-1}/w^{it-1}\cdot x^{it-1}) + (w_n^{\,it}x_n^{\,it}/w^{it}\cdot x^{it})]\ \ln(x_n^{\,it}/x_n^{\,it-1})\\ &= \ln Q_T(w^{it-1},w^{it},x^{it-1},x^{it});\\ (73)\qquad \tau^{it} &\equiv \gamma^{it}/\alpha^{it}\ \beta^{it}\end{aligned}$$

where $P_T(p^{it-1},p^{it},y^{it-1},y^{it})$ is the period t, industry i Törnqvist output price index and $Q_T(w^{it-1},w^{it},x^{it-1},x^{it})$ is the period t, industry i Törnqvist input quantity index.

Recall that in section 2, we defined cumulated counterparts to the chain link equations (42). We can do the same type of operation for the industry data. Thus define the industry i *level of total factor productivity* in period t relative to period 0 as T^{it}, the industry i *level of real output prices* in period t relative to period 0 as A^{it} and the industry i level of *primary input* in period t relative to period 0 as B^{it}. These industry levels can be defined in terms of the corresponding industry chain link factors, τ^{it},α^{it} and β^{it} as follows:

(74) $T^{i0} \equiv 1$; $T^{it} \equiv T^{it-1} \tau^{it}$; $t = 1,2, ...$;
(75) $A^{i0} \equiv 1$; $A^{it} \equiv A^{it-1}\alpha^{it}$; $t = 1,2, ...$;
(76) $B^{i0} \equiv 1$; $B^{it} \equiv B^{it-1}\beta^{it}$; $t = 1,2, ...$.

Since equations (70) hold as exact identities under our present assumptions, the following cumulated counterparts to these equations will also hold as exact decompositions:

(77) $p^{it}\cdot y^{it}/p^{i0}\cdot y^{i0} = \rho^{it}/\rho^{i0} = T^{it} A^{it} B^{it}$; $\qquad i = 1,...,I$; $t = 1,2, ...$.

Thus three factors contribute to the period t level of real income generated by industry i relative to the period 0 level: the level of period t total factor productivity of industry i in period t (relative to period 0), T^{it}, the growth in real output prices for industry i going from period 0 to t, A^{it}, and the growth in primary inputs utilized by industry i going from period 0 to t, B^{it}.

The nominal value of market sector output in period t is the corresponding sum of industry nominal values, $\sum_{i=1}^{I} P^{it}\cdot y^{it}$, which can be converted into the *period t real income generated by the market sector*, ρ^t, by dividing this sum by the period t consumption price deflator, P_C^t:

(78) $\rho^t \equiv \sum_{i=1}^{I} P^{it}\cdot y^{it}/P_C^t = \sum_{i=1}^{I} p^{it}\cdot y^{it} = \sum_{i=1}^{I} \rho^{it}$; $\qquad t = 0,1,...$

where the last equality follows using (68). Define industry i's share of market sector nominal (or real) net output in period 0 as

(79) $s_i^0 \equiv \rho^{i0}/\rho^0$; $\qquad i = 1,...,I.$

Using the above definitions, we can decompose the growth in market sector real income, going from period 0 to t, as follows:

(80) $\rho^t/\rho^0 = [\sum_{i=1}^{I} \rho^{it}]/\rho^0$ $\qquad$ using (78)
$= \sum_{i=1}^{I} [\rho^{it}/\rho^{i0}][\rho^{i0}/\rho^0]$
$= \sum_{i=1}^{I} s_i^0 [\rho^{it}/\rho^{i0}]$ $\qquad$ using (79)
$= \sum_{i=1}^{I} s_i^0 T^{it} A^{it} B^{it}$ $\qquad$ using (77).

Equation (80) shows the factors that determine the evolution of market sector real income growth over time. There are four sets of factors at work:

- The industrial structure of net product in the base period; i.e., the base period industry shares of market sector net output, s_i^0;
- The total factor productivity performance of industry i cumulated from the base period to the current period; i.e., the industry productivity factors, T^{it};
- The growth in industry output prices (deflated by the price of the consumption aggregate) going from period 0 to t; i.e., the industry real output price factors, A^{it} and
- The growth in primary inputs utilized by industry i going from period 0 to t; i.e., the industry primary input growth factors, B^{it}.

Note that if an industry i experiences growth in its (net) output prices relative to the price of consumption, then the corresponding real output price factor A^{it} will be greater than one and this effect will contribute to overall real income growth. It is this type of factor that is missing in traditional Total Factor Productivity decompositions; i.e., the traditional analysis ignores favourable (or unfavourable) output price effects.[65]

Appendix 2: Business Sector Data on Outputs and Inputs for Canada 1961-2007

1. Introduction

The basic approach to measuring productivity growth in this paper is to use recently released information on business sector outputs and inputs from Statistics Canada's KLEMS data base along with information on aggregate final demand expenditures in order to construct "top down" measures of the productivity performance of the Canadian business sector.[66] We also make extensive use of Statistics Canada's National Balance Sheet estimates for information on various capital inputs used by the business sector. Thus the present approach to productivity measurement is an aggregate "top down" approach as opposed to the usual industry "bottom up" approach which makes use of detailed data on inputs used and outputs produced by industrial sectors and aggregates up sectoral productivity growth rates in order to obtain national business sector estimates.[67] With reliable data, the two approaches should give very similar answers.[68] Unfortunately, data on industry inputs and outputs are not likely to be as reliable as the corresponding national data for a variety of reasons[69] so it is useful to provide a check on the industry approach to productivity measurement by using the national aggregate approach.

[65] Improvements in the country's terms of trade are also ignored by the traditional methodology. This does not mean that the traditional emphasis on pure efficiency improvements is "wrong"; it just does not answer the question that we are focusing on, which is: what is the rate of growth in consumption equivalents that the market sector of the economy is generating?
[66] The present data base was constructed in October, 2008.
[67] The bottom up approach is used by the Statistics Canada KLEMS program; see Baldwin, Wu and Yan (2007) for an overview and Baldwin and Yu (2007) for additional information on the construction of the Statistics Canada KLEMS capital services aggregates.
[68] In fact, if indirect tax effects could be ignored and if nominal and real input output tables were perfectly consistent, the two approaches should give exactly the same answer; see Diewert (2006b) (2007c).
[69] For a detailed discussion of these reasons, see Diewert (2001).

There is another reason for undertaking a productivity study using final demand data and this reason is that the effects of changes in a country's terms of trade can be measured in this framework whereas these effects cannot be measured in the industry accounts framework using the existing System of National Accounts 1993 (SNA 1993); see Chapter 15 in Eurostat, IMF, OECD, UN and World Bank (1993). In particular, the Input Output accounts as outlined in Table 15.1 in the SNA 1993 do not show the role of international trade in goods and services by industry. Exports and imports enter the main supply and use tables (Table 15.1) as additions (or subtractions) to total net supply or to total domestic final demand in the familiar C+I+G+X−M setup. This means that Table 15.1 in the main production accounts of *SNA 1993* does not elaborate on which industries are actually using the imports or on which industries are actually doing the exporting by commodity.[70] Thus at present, data difficulties prevent us from looking at the effects of changes in the terms of trade using the bottom up industry aggregation approach.

Diewert and Lawrence (2000) undertook a study of Canada's business sector productivity using the national approach for the years 1962-1996 and Diewert (2002) extended their data to cover the years 1962-1998. Diewert (2008) updated these early studies that used the national approach, noting that there were some differences in his study compared to the earlier studies:

- Statistics Canada provided new data on national expenditure aggregates back to 1961 using annual *chained* index numbers and so it was no longer necessary to work with the old fixed base data on the most disaggregated level possible and then use chain indexes to aggregate up these data.
- Statistics Canada also provided new data on the outputs produced and inputs used by the Canadian business sector back to 1961 using chained Fisher indexes as part of their KLEMS productivity measurement program. In particular, Statistics Canada provided new estimates of labour input, which were a big improvement over the estimates of labour input used by Diewert and Lawrence.
- Diewert and Lawrence (2000) worked with a rather narrow definition of the government sector; their definition included only the public administration industry. In Diewert (2008), the Statistics Canada definition of the nonbusiness sector was used (except that Diewert also added the residential rental housing industry to the nonbusiness sector). The Statistics Canada definition of the nonbusiness sector includes the general government sector and the publicly funded defence, hospital and education sectors in the nonbusiness sector.[71] Since output in the nonbusiness sector is measured by input, the use of the

[70] It should be noted that *SNA 1993* does have a recommended optional Table 15.5 which is exactly suited to our present needs; i.e., this table provides the detail for imports by commodity and by industry. However, *SNA 1993* does not provide a recommendation for a corresponding commodity by industry table for exports.

[71] The nonbusiness sector consists of the following industries: (1) Government funding of hospitals; (2) Government funding of residential care; (3) Government funding of universities; (4) Government funding of other education; (5) Defence services; (6) Other municipal government services; (7) Other provincial government services and (8) Other federal government services.

broader definition of the government sector should lead to higher estimates of productivity growth in the business sector compared to the estimates tabled in Diewert and Lawrence (2000) and Diewert (2002).
- Statistics Canada reorganized its information on indirect taxes (less subsidies) into two categories: taxes that fall primarily on outputs and taxes that fall primarily on inputs. This new information was very useful in making adjustments to output prices for indirect tax effects.[72]

However, since Diewert (2008) appeared, there has been a substantial revision of the Statistics Canada data used in his study. Thus the present paper uses more recent published Statistics Canada data[73] as well as some unpublished disaggregated capital data made available to us from the Statistics Canada KLEMS database. Note that the business sector used here differs from the Statistics Canada business sector in that we have excluded all residential housing services (Owner Occupied Housing services plus Rental Housing services) from our business aggregate whereas Statistics Canada includes the services of rental housing in its business aggregate.[74]

The main conceptual changes in our present data base from the data tabled in Diewert (2008) are as follows:

- The trade data were disaggregated;
- Machinery and equipment investment in Diewert (2008) has been disaggregated into ICT machinery and equipment and non ICT machinery and equipment;[75]
- We used the Statistics Canada KLEMS data on the price of inventory stocks in the present study whereas before, we used another Statistics Canada price series to value inventory stocks;
- The depreciation rates for nonresidential structures, ICT machinery and equipment and non ICT machinery and equipment were reestimated using balance sheet and KLEMS program information along with revised investment information.

In section 2 of this Appendix, we will list the basic final demand expenditure series that were used in this study. Section 3 simply lists the three published business sector measures of quality adjusted labour input for the Canadian business sector that are available on CANSIM as part of the Statistics Canada KLEMS program. Section 4

[72] In early studies of the Total Factor Productivity of an economy like those done by Solow (1957) and Jorgenson and Griliches (1967), outputs were priced at final demand prices, which include indirect taxes. However, Jorgenson and Griliches (1972; 85) noted that this treatment was not consistent with competitive price taking behavior on the part of producers, since producers do not derive any benefit from indirect taxes that fall on their outputs and thus these taxes should be removed.
[73] These data were downloaded in October, 2008.
[74] Our reason for excluding the services of rental housing from our business sector aggregate is due to the lack of accurate data on residential structures investment on rental housing and the lack of information on the quantity and value of land that is occupied by rental housing. Our measure of business sector labour input is exactly the same as that used by the Statistics Canada KLEMS program so only our output measure and capital services input measures differ from the corresponding KLEMS estimates.
[75] Our thanks to Wulong Gu for making the disaggregated data available to us.

studies the problems associated with forming estimates for capital inputs. Section 5 concludes by forming estimates of tax rates on primary inputs. This information is used to calculate estimates of balancing after tax real rates of return. Then this information is used along with the information developed in previous sections in order to calculate user costs for five classes of capital input: machinery and equipment, nonresidential structures, agricultural land, nonagricultural and nonresidential business land and inventories. Section 6 concludes with some observations on the weak points in the data and recommendations for further work on developing a set of productivity accounts for Canada.

2. Estimates of Canadian Final Demand Expenditures

Much of the information tabled in this section is updated information that can be found in the *Canadian Economic Observer*, Statistics Canada (2005), Table 1: Gross Domestic Product (GDP) by Income and Expenditure (millions of dollars and in chained 2002 dollars). The October 2008 version of these data were used, using the Statistics Canada online data service CANSIM II, which were listed as quarterly data. If the quarterly data were seasonally adjusted, then the data for a year were summed and divided by 4 in order to obtain annual data. If the quarterly data were not seasonally adjusted, then they were simply summed in order to obtain annual data. In what follows, we will use the CANSIM individual series label to identify the exact series used.

The first two series are Personal Expenditures on Goods and Services in current and constant chained 2002 dollars, CANSIM II series V498087 and V1992044 respectively. Dividing the current dollar series V_{CT} by the constant dollar series Q_{CT} gives us an implicit price series P_{CT} for personal consumption.

We would like to exclude the imputed expenditures on Owner Occupied Housing (OOH) from the above series since there is no possibility of productivity gains occurring in this sector. However, if we exclude imputed rent from the business sector output series, we also need to exclude the services of the owner occupied housing capital stock as an input into the business sector. Unfortunately, we are not able to construct a reliable measure of the Owner Occupied Housing capital stock from available data; we can only construct a more reliable residential housing capital stock which includes the housing capital stock that is rented. We also were not able to split residential land input into reliable owner occupied and rental components.[76] Hence we excluded both imputed and paid rents from our list of business sector outputs and we excluded the entire residential housing stock

[76] The determination of the structures and land inputs into the production of rented residential housing is a difficult task since the investment data on residential housing is not decomposed into owned and rented investments. This lack of information was also a problem for the Statistics Canada KLEMS program: "Data on investment in rental residential buildings are not available. For the annual MFP programs, we divide the total investment in residential building into rental building and owner-occupied dwelling using paid rents for rental buildings and imputed rents for owner occupied dwelling as the split ratios. The investment in residential buildings and paid and imputed rents are available from the Income and Expenditure Accounts. On average, we find that about 30% of total rents are paid rents and the remaining 70% are imputed rents." Baldwin, Wu and Yan (2007; 43).

and the associated land as an inputs into the business sector.[77] Information on current dollar expenditures on imputed rents and paid rents (this is the series V_{PR} in Table 1 below) for the years 1961-2007 is available from CANSIM II series V498532 and V498533 respectively. The corresponding information on chained 1997 constant dollar expenditures on imputed rents and paid rents (Q_{PR}) is available from CANSIM II series V1992078 and V1992079 only for the years 1981-2007. We divide V_{PR} by Q_{PR} in order to form a price index for paid rents, P_{PR}. We could follow the same strategy to form a price index for imputed rents for the years 1981-2006.[78] However, an alternative series on the imputed value of OOH services for the years 1961-2004 is available from the industry accounts. This series is CANSIM II series V3859926, Business Sector: Owner Occupied Dwellings, from Table 370023: Gross Domestic Product (GDP) at Basic Prices in Current Dollars, System of National Accounts, Benchmark Values, by North American Industry Classification System (NAICS). This series is listed as V_{IMR} in Table 1 below.[79] The final demand value series for imputed rents (not listed) is about 13% higher than its industry counterpart, V_{IMR}. We use the industry series for imputed rents rather than the final demand series because we want our business sector value added to closely approximate the Statistics Canada KLEMS program business sector value added, except that our aggregate will not include paid residential rents.[80]

We now describe how we estimated a price index for the paid rents series for the years 1961-1981 and how we formed a price index for the industry value added series for the imputed rents for OOH for the years 1961-2007. An old series for the industry value added generated by OOH, CANSIM II series V334072, Canada: Current Prices; Business Sector; Owner Occupied Dwellings, from Table 3790001, Gross Domestic Product (GDP) at Factor Cost, System of National Accounts Benchmark Values, by Industry, is available for the years 1961-1997. The corresponding series in constant 1992 dollars is available for the years 1961-2000 as CANSIM II Series V328857 in Table 3790004. We use these two series to form a price index for imputed rent for the years 1961-1997, P_{IMR}^t in Table 1 below. A constant dollar industry series for the services of OOH for the years 1997-2007 can be obtained from CANSIM II Series V14183160, Canada; Seasonally Adjusted at Annual Rates; Chained 1997 Dollars; Owner Occupied Dwellings in Table 3790018, Gross Domestic Product (GDP) at Basic Prices by NAICS.[81] Dividing V_{IMR}^t by

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[77] This means our productivity estimates will be biased downward slightly since the inputs that are used in the rental housing market are included in our estimates but the corresponding outputs are not included.

[78] We did construct the corresponding expenditure based price series for imputed rents for the period 1981-2006 and compared this price index with the corresponding industry based price index for imputed rents described below for the years 1981-2004 and found that the movements were similar. We used the expenditure based price index for the years 2004-2007 to extend the industry based price index from 2004 to 2007.

[79] We explain below how this industry based value series for imputed rents was extended from 2004 to 2007.

[80] The KLEMS business sector value added aggregate excludes imputed rents whereas our business sector value added aggregate will exclude both imputed and paid rents. Our treatment of inventory change is also different.

[81] Somewhat mysteriously, this constant dollar series extends all the way to 2006 whereas the corresponding current dollar series ends at 2004. As noted above, we extended the industry price index for imputed rents from 2004 to 2006 using the movements in the corresponding expenditure based price index for imputed rents over the years 2004-2006. Given this extended price index plus the industry based

this constant dollar series will give us a price index for imputed rents running from 1997 to 2007 and we link this series to the earlier P_{IMR}^t series that ran from 1961 to 1997. We then normalized the price series to equal 1 in 1961 and formed the quantity series Q_{IMR}^t as V_{IMR}^t divided by P_{IMR}^t. V_{IMR}^t, Q_{IMR}^t and P_{IMR}^t are listed in Table 1 below. Recall that we have a value series for paid rents, V_{PR}^t, that covers the years 1961-2007 but the corresponding price index series, P_{PR}^t, covers only the years 1981-2007. We extend P_{PR}^t back to 1961 using the movements in P_{IMR}^t. The resulting price series is normalized to equal 1 in 1961 and a quantity series for paid rents, Q_{PR}^t, is obtained by dividing V_{PR}^t by P_{PR}^t. These three series are also listed in Table 1 below.

Table 1: Housing Value, Quantity and Price Series for Imputed and Paid Rents[82]

Year t	V_{IMR}^t	Q_{IMR}^t	V_{PR}^t	Q_{PR}^t	P_{IMR}^t	P_{PR}^t
1961	2292	2292	1107	1107	1.00000	1.00000
1962	2436	2380	1176	1149	1.02350	1.02350
1963	2660	2412	1290	1170	1.10275	1.10275
1964	2832	2477	1396	1221	1.14316	1.14316
1965	2976	2531	1503	1278	1.17565	1.17565
1966	3249	2620	1658	1337	1.23992	1.23992
1967	3585	2678	1860	1390	1.33856	1.33856
1968	3985	2707	2091	1420	1.47212	1.47212
1969	4416	2784	2342	1476	1.58633	1.58633
1970	4897	2833	2645	1530	1.72855	1.72855
1971	5388	2864	2918	1551	1.88118	1.88118
1972	5757	2866	3183	1584	2.00889	2.00889
1973	6307	2862	3451	1566	2.20366	2.20366
1974	7107	2923	3787	1558	2.43126	2.43126
1975	8313	2992	4290	1544	2.77854	2.77854
1976	10038	3072	4842	1482	3.26746	3.26746
1977	12126	3084	5443	1384	3.93199	3.93199
1978	14090	3051	6106	1322	4.61807	4.61807
1979	15797	2996	6829	1295	5.27283	5.27283
1980	17869	3053	7686	1313	5.85278	5.85278
1981	20512	3159	8822	1359	6.49322	6.49322
1982	23489	3213	10082	1410	7.31046	7.15154
1983	26285	3256	11295	1444	8.07270	7.82159
1984	28446	3294	12181	1471	8.63567	8.28079
1985	30694	3360	12967	1500	9.13517	8.64482
1986	33386	3463	13955	1539	9.64089	9.06928
1987	36117	3573	15090	1599	10.10837	9.43653
1988	39587	3801	16419	1662	10.41493	9.87670
1989	44078	4011	18201	1726	10.98935	10.54481

constant dollar series for imputed rents, the industry based value series for imputed rents was extended to 2006.

[82] The units for all value and quantity series in this Appendix are millions of current dollars for the V series and millions of 1961 dollars for the Q series.

1990	48016	4221	19786	1798	11.37552	11.00446
1991	51779	4469	21133	1853	11.58636	11.40566
1992	54872	4627	22269	1899	11.85900	11.72872
1993	57263	4770	23108	1943	12.00486	11.89235
1994	60557	4887	24056	1982	12.39142	12.13540
1995	63613	5001	24869	2016	12.72013	12.33820
1996	65418	5116	25632	2049	12.78691	12.51068
1997	67405	5245	26425	2097	12.85127	12.59838
1998	69835	5389	27223	2139	12.95872	12.72809
1999	72144	5557	28173	2187	12.98263	12.87911
2000	74582	5704	29059	2231	13.07545	13.02515
2001	77093	5843	30092	2279	13.19410	13.20509
2002	80895	6074	31491	2341	13.31831	13.44940
2003	83916	6250	32829	2413	13.42651	13.60407
2004	87614	6482	34133	2487	13.51648	13.72279
2005	91546	6730	35435	2560	13.60266	13.83987
2006	96714	6985	37137	2638	13.84590	14.07851
2007	103152	7246	39262	2714	14.23568	14.46634

Recall the price and quantity series for a consumption aggregate (which includes all rents, paid and imputed), P_{CT} and Q_{CT}, along with the two price and quantity series for imputed and paid rents in Table 1 above. We changed the sign of the rent quantity series from plus to minus and then calculated a chained Fisher net consumption aggregate by aggregating all consumption (plus sign on the quantities) and rents (negative sign on the quantities). The resulting price and quantity series should closely approximate the price and quantity of consumption excluding housing services. However, the price series includes indirect taxes (less subsidies) on outputs but for productivity measurement purposes, as mentioned earlier, these tax wedges should be excluded. Statistics Canada has a series for indirect taxes less subsidies on products V_{IT}^t, CANSIM II series V1997473, for the years 1961-2007. We subtracted two other tax series from this indirect tax series because these other tax series will be taken into account separately in the price of exports of goods (this is the Oil Export Tax series, CANSIM series V499746) and in the price of imports of goods (this is the Customs Import Duties series, CANSIM series V499741). The resulting indirect taxes less subsidies on products (less trade taxes) series was used to remove the tax wedges on the price of consumption series. The resulting price and quantity of consumption series, P_C^t and Q_C^t, are listed in Tables 2 and 3 below.[83]

We turn our attention to the investment components of final demand. Current dollar government gross fixed capital formation is available as CANSIM II series V498093 for the years 1961-2007. The corresponding chained 2002 dollar series is CANSIM II series

[83] We renormalize all price and quantity series so that the normalized price is 1 in 1961. The units for quantity and value series in this Appendix are in millions of current and 1961 dollars respectively.

V1992050 and we use these two series to form price and quantity series for general government sector investment, P_{IG}^t and Q_{IG}^t, which are listed in Tables 1 and 2 below.[84]

The current and constant chained dollar series for the years 1961-2007 for residential structures investment can be obtained as CANSIM II series V498096 and V1992053 respectively, the current and constant chained dollar series for nonresidential structures investment can be obtained as CANSIM II series V498098 and V1992053 respectively and the resulting price and quantity series are denoted by P_{IR}^t, P_{INR}^t, Q_{IR}^t, Q_{INR}^t and are listed in Tables 2 and 3 below. Statistics Canada provided us with unpublished series on the price and value of ICT investment and for non ICT machinery and equipment for the years 1961-2006. The resulting price and quantity series for ICT investment and non ICT machinery and equipment are denoted by P_{IICT}^t, Q_{IICT}^t, P_{IME}^t and Q_{IME}^t respectively[85]; see Tables 2 and 3 below. These Tables also include the price and quantity of inventory change, P_{II}^t and Q_{II}^t change but the description of how they were constructed is deferred until we discuss how we formed estimates of the beginning of the year stocks of inventories.

Table 2: Price Indexes for Business Sector Outputs: Consumption and Investment

Year t	P_C^t	P_{IG}^t	P_{IR}^t	P_{INR}^t	P_{IME}^t	P_{IICT}^t	P_{II}^t
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.00538	1.00855	1.00504	1.00592	1.01477	0.99939	1.00240
1963	1.02055	1.03939	1.02769	1.03251	1.06598	0.99650	1.01346
1964	1.02437	1.06231	1.07312	1.06158	1.06773	1.00502	1.03169
1965	1.03690	1.13926	1.13368	1.12281	1.10198	1.02393	1.05486
1966	1.07553	1.21007	1.20765	1.19323	1.12251	1.02944	1.07231
1967	1.11050	1.23160	1.28518	1.24188	1.12792	1.07473	1.08449
1968	1.15168	1.23844	1.31431	1.25227	1.13973	1.10902	1.10971
1969	1.1898	1.28464	1.38118	1.32495	1.17197	1.14444	1.13490
1970	1.22208	1.33877	1.42615	1.39058	1.23029	1.19472	1.15046
1971	1.24828	1.40873	1.53179	1.46812	1.27216	1.22722	1.20264
1972	1.29847	1.48528	1.67349	1.55098	1.30444	1.26956	1.31611
1973	1.38744	1.64838	1.97123	1.71873	1.34535	1.30782	1.42683
1974	1.58382	2.01078	2.36134	2.03419	1.51430	1.35698	1.54691
1975	1.82198	2.23421	2.56072	2.27337	1.73112	1.45867	1.65112

[84] The price series for investment should be adjusted for indirect taxes that fall on investment outputs. Since these taxes are relatively small and it is difficult to collect consistent information on these taxes over our sample period, we neglect these indirect tax wedges on investment components of final expenditure.

[85] We used the rates of increase in the price and quantity of all investment in machinery and equipment going from 2006 to 2007 from the published Statistics Canada expenditure accounts (see the current and constant chained dollar series for machinery and equipment investment, CANSIM II series V498099 and V1992056 respectively) to extend the ICT and non ICT M&E price and quantity series to 2007. When we aggregated up the two unpublished ICT and other M&E investment series using chained Fisher indexes and compared the resulting aggregate KLEMS based M&E investment series with the corresponding national accounts based series, we found that the value series were very close but the KLEMS based price series grew 2.32 fold over the years 1961-2006 whereas the national accounts based price series grew 2.60 fold, which is 12% higher. We chose to use the KLEMS based data series for investment in machinery and equipment.

1976	1.90726	2.34499	2.76853	2.40093	1.82918	1.45190	1.77007
1977	2.03175	2.48990	2.87768	2.52980	1.98862	1.44467	1.92666
1978	2.19264	2.66206	3.04069	2.71145	2.19453	1.40748	2.13561
1979	2.40645	2.88522	3.28046	2.96312	2.44348	1.38366	2.34120
1980	2.69497	3.19858	3.55455	3.3252	2.69736	1.21888	2.55883
1981	2.95335	3.68313	3.99273	3.68676	2.98813	1.12880	2.75849
1982	3.22860	3.92658	4.08226	3.96113	3.19883	1.16812	2.91040
1983	3.46323	4.01498	4.25350	3.93090	3.26756	1.02222	3.04428
1984	3.61506	4.17063	4.41785	4.08142	3.40812	0.96342	3.12590
1985	3.72257	4.20827	4.55564	4.21351	3.57237	0.89167	3.15838
1986	3.80422	4.20267	4.90827	4.27520	3.70845	0.82129	3.20668
1987	3.89726	4.22375	5.40819	4.47320	3.69804	0.75277	3.26271
1988	4.00205	4.33769	5.78293	4.72840	3.69075	0.71401	3.31521
1989	4.1169	4.43728	6.13195	4.92520	3.79390	0.64691	3.36284
1990	4.35206	4.53066	6.11231	5.08853	3.87130	0.61403	3.36458
1991	4.59099	4.31837	6.32257	5.00311	3.74170	0.54586	3.51938
1992	4.65258	4.31896	6.39710	4.97541	3.88288	0.51043	3.52557
1993	4.74252	4.34342	6.58445	5.03758	4.04523	0.50164	3.70024
1994	4.77089	4.42033	6.76485	5.20497	4.24754	0.48119	3.83041
1995	4.79147	4.51572	6.76717	5.27332	4.44740	0.44755	3.91857
1996	4.88952	4.53812	6.75581	5.43035	4.57050	0.41150	3.78206
1997	4.96547	4.57906	6.87512	5.56694	4.69221	0.39399	3.80677
1998	5.03224	4.59706	6.95993	5.71450	4.90374	0.36919	3.86651
1999	5.12045	4.57201	7.13210	5.82995	4.94358	0.33873	3.95640
2000	5.25425	4.68967	7.29782	6.02775	5.01831	0.32384	4.04039
2001	5.40970	4.68012	7.48766	6.07934	5.17017	0.31733	4.10861
2002	5.47743	4.72977	7.81242	6.18175	5.25354	0.30560	3.87128
2003	5.61543	4.72659	8.21290	6.30506	4.94953	0.27567	3.91048
2004	5.69551	4.79882	8.71618	6.70389	4.85253	0.24913	3.95736
2005	5.81654	4.91308	9.11452	7.12403	4.79667	0.23077	4.02157
2006	5.92386	5.09878	9.78750	7.64002	4.71044	0.21439	4.14724
2007	6.02712	5.28795	10.49192	8.04719	4.58255	0.20857	4.15442

Table 3: Quantity Indexes for Business Sector Outputs: Consumption and Investment

Year t	$Q_C{}^t$	$Q_{IG}{}^t$	$Q_{IR}{}^t$	$Q_{INR}{}^t$	$Q_{IME}{}^t$	$Q_{IICT}{}^t$	$Q_{II}{}^t$
1961	20265	1887	2211	2618	1925	312	1
1962	21331	2094	2271	2545	2067	358	560
1963	22290	2101	2354	2637	2199	399	936
1964	23529	2141	2715	3050	2744	412	532
1965	24974	2426	2825	3320	3257	472	1228
1966	26240	2668	2699	3802	3899	580	1313
1967	27228	2718	2754	3613	3866	617	454
1968	28525	2758	3132	3593	3537	592	619
1969	29923	2700	3551	3592	3817	673	1580

1970	30450	2645	3254	3946	3868	677	180
1971	32321	2985	3728	4089	3924	744	-232
1972	34891	2938	4066	4074	4338	806	148
1973	37676	2781	4371	4396	5336	932	2542
1974	39789	2845	4464	4675	5925	1167	4839
1975	41468	2962	4386	5286	6089	1310	-334
1976	43911	2855	5172	5168	6219	1568	250
1977	45480	2916	5242	5479	6057	1647	1319
1978	47255	2875	5291	5626	6202	1895	1300
1979	48694	2803	5251	6337	6988	2225	3691
1980	49521	2869	4977	7055	7165	3074	167
1981	49914	2967	5279	7620	8211	4423	891
1982	48210	3095	4340	6929	6867	4205	-4603
1983	49567	2991	5079	6361	6414	5184	-1049
1984	51955	3124	5131	6288	6397	6297	1744
1985	54842	3497	5578	6590	6909	7438	1467
1986	56973	3485	6267	6210	7454	8984	1029
1987	59433	3621	7190	6454	8084	12586	1701
1988	61835	3789	7340	7110	9704	14111	2078
1989	63760	4184	7640	7345	10096	17019	1406
1990	64001	4463	6835	7346	9331	18392	-583
1991	62103	4692	5824	7075	8856	19836	-3899
1992	62821	4621	6238	5960	7995	24332	-551
1993	63783	4560	6024	5993	7449	26001	-878
1994	65822	4894	6271	6533	8001	30044	709
1995	67161	4740	5340	6574	8373	33319	3164
1996	68967	4536	5852	6696	8500	39474	2417
1997	72495	4390	6330	7881	10434	49575	1780
1998	74536	4361	6106	7906	10647	60330	2329
1999	77521	5039	6324	8101	11106	72466	1249
2000	80901	5229	6656	8266	11270	85220	2915
2001	82720	5830	7363	8712	10798	84440	-1856
2002	85721	6044	8403	8195	10625	83843	3367
2003	88311	6370	8854	8651	11417	90145	-1722
2004	91157	6773	9519	9257	12185	105113	1058
2005	94522	7521	9845	10184	13473	115589	1981
2006	98708	8021	10061	11093	14066	130883	1104
2007	103366	8644	10363	11047	15069	140217	1757

All of the outputs described above can be regarded as outputs produced by the business sector and sold to final demanders. However, the business sector also sells goods and services to the nonbusiness sector and it also purchases smaller amounts of goods and services from the nonbusiness sector. We now describe how we formed price and quantity estimates for the net sales of the business sector to the nonbusiness sector.

For the years 1961-2007 from the National Income and Expenditure Accounts, CANSIM II series V498092; Government Current Expenditure on Goods and Services, Table 3800002, we have estimates of total government gross current expenditures on goods and services (less sales of goods and services to the business sector) in current dollars. From the same Table and for the same years, CANSIM II series V1992049; Government Current Expenditure on Goods and Services, Table 3800002, we have estimates of total government gross current expenditure on goods and services (less sales of goods and services to the business sector) in chained 2002 dollars. We use these two series to form price and quantity series for final demand government sector net expenditures on goods and services, P_G^t and Q_G^t, which are listed in Table 4.

Recall that the Statistics Canada KLEMS productivity program business sector value added aggregate *includes* rental residential housing but *excludes* the services of owned residential housing (whereas our business sector value added aggregate *excludes* all forms of residential rents). The Industry Division of Statistics Canada produces yet another business sector estimate of nominal and real value added (at factor cost) which *includes* all residential rents, both imputed and paid. We will denote this value added aggregate by V_B^t in year t. Statistics Canada also produces a companion nonbusiness sector value added aggregate (at factor cost) which we will denote by V_N^t in year t. If the value of indirect taxes less subsidies on products for year t, V_{IT}^t, is added to the sum of these two industry value added aggregates, we get an estimate of the value of GDP at final demand prices in year t; i.e., we have the following identity:

(1) $V_B^t + V_N^t + V_{IT}^t = V_{GDP}^t$.

We will now describe how we formed estimates for V_B^t and V_N^t along with the corresponding price and quantity decompositions. From Table 3790024, Gross Domestic Product (GDP) at Basic Prices in Current Dollars, SNA, Benchmark Values, Special Industry Aggregations Based on the North American Industry Classification System (NAICS), we can obtain the V_B^t series (title is Canada: Business Sector Industries) for the years 1961-2004 from CANSIM II Series V3860037. From the same Table 3790024, we can obtain the V_N^t series (title is Canada: Non-Business Sector Industries) for the years 1961-2004 from CANSIM II Series V3860040. We can obtain price indexes P_B^t, P_N^t and quantity indexes Q_B^t,Q_N^t for V_B^t and V_N^t for the years 1961-1997 by using the series V334562, V335071, V334565 and V335074 from CANSIM Table 3790002, Gross Domestic Product (GDP) at Factor Cost, System of National Accounts Benchmark Values by Industry (Special Aggregations). These series give business and nonbusiness sector value added at basic prices in current dollars and in constant 1992 dollars. Using CANSIM Table 3790020, we can find estimates for Q_B^t (Series V14182646) and for Q_N^t (Series V14182651) in chained 1997 dollars for the years 1997-2007. Hence using these series in conjunction with our earlier value series V_B^t and V_N^t which run from 1961 to 2004, we can obtain price series for business and nonbusiness sector value added at basic prices for the years 1997-2004. These price series can be linked to our earlier price series P_B^t and P_N^t which extended to 1997 so that the resulting price series will run from 1961 to 2004. However, we still do not have price or value series for the B and N sectors for 2004-2007, although we do have quantity series for these years. We extend the price series P_N^t from 2004 using an implicit price index for government goods and services, which was constructed using CANSIM Table 3800002, series V498092 in current dollars and series V1992049 for chained 2002 dollars. It turns out that the total of V_B^t and V_N^t is available in another CANSIM II V3860274. Canada, Gross Domestic Product (GDP) at Basic Prices in Table 3800030: GDP and GNP at Market Prices and Net National Income at Basic Prices. Thus we have enough information to deduce the price P_B^t and the value of business sector output V_B^t for the years 2004-2007. The business and nonbusiness sector price and quantity series, P_B^t, P_N^t and Q_B^t, Q_N^t are listed in Table 4 below.

Table 4: Business Sector, Nonbusiness Sector, Government Final Demand and KLEMS Business Sector Price and Quantity Aggregates

Year t	Q_B^t	Q_N^t	Q_G^t	Q_{BKLEMS}^t	P_B^t	P_N^t	P_G^t	P_{BKLEMS}^t
1961	33097	5204	6624	30805	1.00000	1.00000	1.00000	1.00000
1962	35338	5480	6928	33059	1.00919	1.03863	1.02916	1.00509
1963	37217	5713	7164	35013	1.02992	1.08205	1.05990	1.01881
1964	39810	5952	7542	37567	1.04877	1.14227	1.09761	1.03600
1965	42658	6120	7883	40122	1.07554	1.21527	1.15160	1.06938
1966	45529	6409	8581	42827	1.12248	1.34490	1.24333	1.11745
1967	46616	6870	9334	43728	1.16053	1.45671	1.32836	1.15516
1968	49335	7263	9944	46133	1.19304	1.54780	1.41575	1.18948
1969	51965	7585	10376	48537	1.23452	1.71317	1.53846	1.23074
1970	52968	7962	11287	49889	1.29437	1.84146	1.64279	1.27609
1971	55844	8255	11631	51843	1.33615	1.95964	1.75921	1.33535
1972	59086	8549	11995	54998	1.40300	2.12810	1.89268	1.40260
1973	63467	8887	12559	59206	1.54872	2.31234	2.04912	1.55366
1974	65346	9295	13357	61310	1.79635	2.65779	2.33927	1.79872
1975	65545	9790	14251	62061	2.03754	3.05325	2.65962	2.01795
1976	70082	10097	14525	66118	2.17572	3.45337	2.99471	2.15434
1977	72425	10348	15205	68823	2.32657	3.73913	3.24567	2.27212
1978	74875	10644	15473	71979	2.51505	3.96850	3.45708	2.42047
1979	77878	10805	15635	75134	2.80145	4.29236	3.77635	2.69350
1980	79169	11138	16169	76938	3.12982	4.66836	4.14895	2.98837
1981	81847	11496	16441	80244	3.40152	5.22919	4.64970	3.21385
1982	78970	11693	16767	77088	3.65996	5.83630	5.18504	3.44460
1983	81077	11952	17045	79192	3.89493	6.12278	5.48059	3.65571
1984	86041	12198	17222	84752	4.03944	6.37296	5.69544	3.76522
1985	90944	12471	17959	89260	4.13895	6.57967	5.90593	3.87314
1986	93580	12708	18283	91514	4.19906	6.81314	6.09362	3.92907
1987	97824	12840	18525	96022	4.38354	7.17180	6.36273	4.08969
1988	102723	13057	19370	100981	4.58086	7.53056	6.60377	4.26785
1989	105427	13224	19903	103685	4.75619	8.03351	6.95695	4.41096
1990	106128	13541	20605	103235	4.85194	8.60166	7.34874	4.52286
1991	104194	13849	21208	99027	4.90597	9.01919	7.64969	4.63908
1992	105171	14045	21414	99628	4.92162	9.36188	7.88200	4.64471
1993	108151	14150	21422	102483	4.97722	9.50865	7.98997	4.69377

1994	113766	14218	21156	108795	5.08269	9.55910	8.11082	4.75837
1995	117124	14279	21034	112401	5.23638	9.61980	8.19895	4.89045
1996	119744	14025	20786	114956	5.33957	9.72826	8.23447	4.99301
1997	125797	13787	20579	121417	5.40168	9.95401	8.34626	5.04144
1998	131475	13890	21240	127127	5.37429	10.0751	8.44225	5.00882
1999	139515	14320	21687	135542	5.47529	10.18237	8.57899	5.10349
2000	147808	14614	22356	144108	5.71191	10.65177	8.94989	5.34163
2001	149733	14926	23229	146512	5.80825	10.88586	9.11375	5.41622
2002	153895	15241	23802	150269	5.82601	11.29659	9.42889	5.42824
2003	156933	15608	24551	153124	6.02555	11.73665	9.71085	5.62745
2004	162130	15921	25044	158534	6.23215	11.96951	9.87845	5.82088
2005	167081	16154	25429	163342	6.45631	12.39824	10.23229	6.04364
2006	171718	16519	26385	168301	6.65413	12.80917	10.57143	6.21454
2007	175972	16945	27359	172358	6.89723	13.13041	10.83655	6.44335

It is also of some interest to compare the price and quantity of the above Industry Division business sector prices and quantities P_B^t and Q_B^t with the corresponding business sector prices and quantities P_{BKLEMS}^t and Q_{BKLEMS}^t that originate with the Statistics Canada productivity program.[86] These series are also listed in Table 4. The source for Q_{BKLEMS}^t for the years 1961-2007 is CANSIM II series V41712932: Canada, Real Gross Domestic Product (GDP), Business Sector from Table 3830021: Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Subsectors by the North American Industry Classification (NAICS). The corresponding nominal value added series V_{BKLEMS}^t is available in the same table for the years 1961-2004 as CANSIM II series V41713153: Canada: Gross Domestic Product (GDP), Business Sector. The values V_{BKLEMS}^t for the missing years 2005-2007 can be obtained by adding the value of imputed rents, V_{IMR}^t, to the Industry Division value added for the Business Sector, V_B^t. Finally, P_{BKLEMS}^t can be obtained by dividing V_{BKLEMS}^t by Q_{BKLEMS}^t. As usual, we normalized the resulting price and quantity series so that P_{BKLEMS}^t equals 1 when t equals 1961. The resulting P_{BKLEMS}^t and Q_{BKLEMS}^t are listed in Table 4.

Recall the GDP identity defined by (1) above, which expressed the nominal value of GPD, V_{GDP}^t, at final demand prices as being equal to the value added of the Industry Division business sector value added at basic prices, V_B^t, plus nonbusiness sector value added, V_N^t, plus the value of indirect taxes less subsidies on products, V_{IT}^t. We can also express the value of GDP at final demand prices as the familiar sum of final demand values; i.e., as the following sum of final demand expenditures on consumption plus investment plus government expenditures on goods and services plus exports less imports:

(2) $V_{GDP}^t = V_{CT}^t + V_I^t + V_G^t + V_X^t - V_M^t$.

[86] Recall that the Productivity Program business sector value added aggregate V_{KLEMS}^t should be equal to the Industry Division value added aggregate V_B^t less the value of imputed rents from the Industry Division, V_{IMR}^t.

Define a new consumption aggregate at basic prices V_{CN}^t as the value of consumption at final demand prices, V_{CT}^t, less indirect taxes less subsidies on products, V_{IT}^t:

$$(3)\ V_{CN}^t \equiv V_{CT}^t - V_{IT}^t\ .$$

Now equate the two expressions for the value of GDP given by (1) and (2) and use the resulting equation to express business sector value added V_B^t in terms of final demand components and the value of nonbusiness sector value added V_N^t. Making use of (3), the resulting equation is the following one:[87]

$$(4)\ V_B^t = V_{CN}^t + V_I^t + V_X^t - V_M^t + (V_G^t - V_N^t).$$

Conceptually, the aggregate $V_G^t - V_N^t$ should be equal to the sales of the business sector of goods and services to the nonbusiness sector less the purchases of intermediate inputs of the business sector from the nonbusiness sector. Put another way, the business sector's net sales of goods and services should equal its net deliveries to final demand sectors ($V_C^t + V_I^t + V_X^t - V_M^t$) plus its net deliveries to the nonbusiness sector ($V_G^t - V_N^t$).

Recall that we did not use the Industry Division's concept of Business Sector value added; we subtracted the value of imputed and paid residential rent from our business sector aggregate. Let V_R^t be equal to the sum of imputed residential rent V_{IMR}^t and paid residential rent V_{IMR}^t (see Table 1 for these series). Conceptually, if we subtract rents V_R^t from V_{CN}^t, we should get V_C^t, the consumption aggregate whose price and quantity is listed in Tables 2 and 3 above. Thus subtracting V_R^t from both sides of (4) leads to the following identity:

$$(5)\ V_B^t - V_R^t = V_C^t + V_I^t + V_X^t - V_M^t + (V_G^t - V_N^t).$$

Thus our business sector value added aggregate can be formed using either the left or right hand sides of the identity (5). We will use the right hand side of (5) to form our value measure of business sector net output since we want to focus on the effects of changing international prices on the performance of the business sector.

How should the corresponding real quantities that correspond to the value aggregates on either side of (5) be calculated? Obviously, each cell in the supply and use tables that correspond to the value aggregate on the left hand side of (5) could be aggregated up using a chained superlative index number formula provided that an appropriate price deflator were available for each cell.[88] On the other hand, the value cells that are components on the right hand side of (5) that correspond to final demand components (at basic prices) could be aggregated up using a chained superlative index number formula.

[87] The identity (4) is not quite consistent with our treatment of indirect taxes less subsidies since we also made some indirect tax adjustments to the prices of exports and imports as explained above; i.e., since we used a slight modification of (3) to adjust final demand consumption prices for indirect tax wedges, we used a corresponding slight modification of the identity (4).

[88] Quantities in the Make matrix would have a positive sign while quantities in the Use matrix would have a negative sign.

We can then ask: under what conditions would the corresponding quantity aggregates be equal? This question is addressed by Moyer, Reinsdorf and Yuskavage (2006) and in more detail by Diewert (2006b) (2007b) (2007c). The answer to this question is that if the detailed data are constructed in an appropriate manner and the Fisher formula is used, then the direct industry aggregation and the aggregation of final demand component approaches are perfectly consistent.[89] In addition, if two stage aggregation procedures are used and a superlative index number formula is used at each stage of aggregation, then the theoretical and empirical results in Diewert (1978) show that the commonly used single stage superlative indexes will approximate their two or more stage counterparts to a high degree of approximation if the chain principle is used.[90]

Using the above results, we will construct our measure of business sector real value added by aggregating up the value components on the right hand side of (5). Rather than work with both V_G^t and V_N^t as final demand components, we will aggregate over these two components to form the value aggregate V_{GN}^t equal to $(V_G^t - V_N^t)$, and conceptually, this value aggregate should be equal to the net deliveries of goods and services of our business sector to the nonbusiness sector less the purchases of intermediate inputs by our business sector from the nonbusiness sector. The year t price and quantity aggregates, P_{GN}^t and Q_{GN}^t, that correspond to these value aggregates V_{GN}^t are calculated using chained Fisher indexes with Q_N^t getting a negative weight in the index number formula. P_{GN}^t and Q_{GN}^t are listed in Table 5 below.

Table 5: Price and Quantity Indexes for the Net Sales of the Business Sector to the Nonbusiness Sector

Year t	P_{GN}^t	Q_{GN}^t
1961	1.00000	1420
1962	0.99388	1447
1963	0.97566	1446
1964	0.92683	1596
1965	0.91239	1798
1966	0.88328	2320
1967	0.89087	2684
1968	0.96139	2950
1969	0.96769	3068
1970	1.00582	3858
1971	1.10291	3885
1972	1.14412	3942
1973	1.22092	4247
1974	1.35752	4819
1975	1.48446	5397
1976	1.64265	5254
1977	1.78697	5963

[89] See Diewert (2006b) (2007b) and the numerical examples in Diewert (2007c) in particular.
[90] The results of Hill (2006) show that these approximation results will not necessarily hold for mean of order r superlative indexes if r is large in magnitude.

1978	1.92861	5833
1979	2.18540	5796
1980	2.48896	6062
1981	2.79431	5845
1982	3.10614	6019
1983	3.37412	5998
1984	3.48567	5838
1985	3.67204	6539
1986	3.74218	6635
1987	3.79747	6789
1988	3.78607	7814
1989	3.82606	8423
1990	3.86395	9043
1991	3.92576	9508
1992	3.94383	9458
1993	3.96988	9223
1994	4.17941	8537
1995	4.29853	8165
1996	4.20205	8263
1997	4.10280	8414
1998	4.13974	9511
1999	4.29033	9380
2000	4.43225	10022
2001	4.45850	11040
2002	4.56662	11443
2003	4.56905	12088
2004	4.60878	12331
2005	4.77387	12551
2006	4.93210	13651
2007	5.05578	14632

We now turn our attention to the export and import components of final demand. Current dollar exports of goods are available as CANSIM II series V498104 for the years 1961-2007. The corresponding chained 2002 dollar series is CANSIM II series V1992061 and these two series could be used to form price and quantity series for the exports of goods. However, in this study, we will form series for more detailed components of the exports and imports of goods. Current dollar exports of services are available as CANSIM II series V498105 for the years 1961-2007. The corresponding chained 2002 dollar series is CANSIM II series V1992062 and we use these two series to form price and quantity series for the exports of services, P_{16}^t and Q_{16}^t, which are listed in Tables 6 and 7 below.

Our starting point for obtaining disaggregated data on the exports and imports of goods is CANSIM Table 3800012, Exports and Imports of Goods and Services, Canada, Current Prices. It is possible to obtain disaggregated information on the value of exports for the following 7 classes for the years 1971-2007:

- Q_9: Exports of agricultural and fish products;
- Q_{10}: Exports of energy products;
- Q_{11}: Exports of forest products;
- Q_{12}: Exports of industrial goods and materials (excluding energy and forest product exports);
- Q_{13}: Exports of machinery and equipment (excluding automotive products);
- Q_{14}: Exports of automotive products;
- Q_{15}: Exports of other consumer goods (excluding automotive products);

The CANSIM series numbers for the first 7 classes of exports are V498730-V498736. It is also possible to find corresponding constant dollar series in 1992 constant dollars over the period 1971-1997 in CANSIM Table 3800012 and the CANSIM series numbers are V498767-V498773. Finally, constant dollar chained estimates for these export categories (in 1997 chained dollars) can be found for the years 1981-2007 in CANSIM Table 3800012 and the series numbers are V1992162- V1992168. We used these series to form chained price and quantity series for these 7 export categories for the years 1981-2007. The constant dollar price series were linked to each chained price series at the year 1981 in order to extend the chained series back to 1971.[91]

There remains the problem of obtaining price series for the above 7 classes of exports to cover the years 1961-1971. From Leacy (1983), Series G415-428 Foreign trade, domestic exports, excluding coin and bullion, by main commodity sections, current values, we can obtain value series covering exports for the years 1946-1975 for the following 5 commodity classes:

- Live animals (G415);
- Food, feed, beverages and tobacco (G417);
- Crude materials (inedible) (G419);
- Fabricated materials (inedible) (G421);
- End products (inedible) (G423).

From the same source, price indexes for each of the above 5 classes of exports are available as Series K57-K61 in the Table with the title: Export price indexes, trade of Canada commodity classification, 1926-1975. Thus we can find price and quantity series for these 5 classes of exports that cover the years 1961-1971. Unfortunately, these price indexes are of the fixed base variety with a base year of 1948 so they are likely to differ substantially from the corresponding chain indexes (which are not publicly available). However, Leacy (1983) also lists as part of export price Series K57-K61 (Panel A) for the above 5 classes of exports some indexes that have a 1971 base year but these price indexes cover only the years 1968-1975. We use these latter price indexes to construct export price indexes for the years 1968-1971 and then we use the 1948 based indexes to further extend these 5 series back to 1961.

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[91] There were two other categories in the export and import classifications: Special transactions and Other balance of payments adjustments. These categories were small and were omitted in our analysis.

The above operations give us 5 disaggregated export price and quantity series for the period 1961-1971 but we have 7 classes of exports of goods for the years 1971-2007. We generated Fisher chained price and quantity indexes for exports of Live animals and for exports of Food, feed, beverages and tobacco for the years 1961-1971 and linked these series to our earlier series, P_9 and Q_9, exports of agricultural and fish products. But we need some additional series so that we can match the export and import series for the 1960s to the series that cover the post 1971 period. We will create separate export series for energy, forest products, automotive products and other consumer goods. Our sources for these extra series are the input output tables for the Canadian economy that cover the years 1961-1981; see Statistics Canada (1987a) (1987b).

In order to create a price and quantity series for aggregate Energy exports for the years 1961-1971, we aggregated data for 6 classes of energy exports using the M level of aggregation: Coal, Crude mineral oils, Natural gas, Gasoline and fuel oil, Other petroleum and coal products and Electric power. These components were aggregated using Fisher (1922) chained indexes. The resulting price and quantity series were linked to our earlier price and quantity series, P_{10} and Q_{10}, for energy at the year 1971.

In order to create aggregate Forestry exports for the years 1961-1971, we aggregated data for 7 classes of forest product exports using the M level of aggregation: Lumber and timber, Veneer and plywood, Other wood fabricated materials, Furniture and fixtures, Pulp, Newsprint and other paper stock, and Paper products. These components were aggregated using Fisher (1922) chained indexes. The resulting price and quantity series were linked to our earlier price and quantity series, P_{11} and Q_{11}, for forest product exports at the year 1971.

We aggregated the input output data for 2 classes of automotive product exports using the M level of aggregation: Motor vehicles and Motor vehicle parts. These components were aggregated using Fisher (1922) chained indexes. The resulting price and quantity series were linked to our earlier price and quantity series, P_{14} and Q_{14}, for automotive product exports at the year 1971.

In order to create an aggregate for Exports of other consumer goods (excluding automotive products) for the years 1961-1971, we aggregated data for 8 classes of consumer goods type exports using the M level of aggregation: Leather and leather products, Other textile products, Hosiery and knitted wear, clothing and accessories, appliances and receivers (households), Pharmaceuticals, Other chemical products and Other manufactured products. These components were aggregated using Fisher (1922) chained indexes. The resulting price and quantity series were linked to our earlier price and quantity series, P_{15} and Q_{15}, for exports of other consumer goods at the year 1971.

We generate price and quantity series over the years 1961-1971 for Exports of industrial goods and materials (excluding energy and forest product exports), P_{12} and Q_{12}, as a chained Fisher aggregate of our price and quantity series for Crude materials (inedible) (G419) and Fabricated materials (inedible) (G421) less our series for Exports of energy products P_{10} and Q_{10}) and Exports of forest products (P_{11} and Q_{11}).[92] The resulting export price and quantity series for the years 1961-1971 are linked to our earlier series for P_{12} and Q_{12} at the year 1971.

Finally, we generate price and quantity series over the years 1961-1971 for Exports of machinery and equipment (excluding automotive products), P_{13} and Q_{13}, as a chained Fisher aggregate of our price and quantity series for Exports of end products (inedible) (G423) less our series for Exports of automotive products P_{14} and Q_{14}) and less Exports of other consumer goods (P_{15} and Q_{15}).[93] The resulting export price and quantity series for the years 1961-1971 are linked to our earlier series for P_{13} and Q_{13} at the year 1971.

There is one additional adjustment which affects the price of energy exports. During the years 1974-1985, Canada imposed an export tax on its energy exports, which is included in the price of exports. However, producers do not receive this export tax revenue and so it must be subtracted from the export price. This adjustment of the export price index for exports of goods can be accomplished using the Oil Export Tax series, CANSIM series V499746 from the National Income and Expenditure Accounts. After making this adjustment, the resulting price and quantity series are $P_{10}{}^t$ and $Q_{10}{}^t$, which are listed in Tables 6 and 7 below along with the other price and quantity series for the 8 classes of exports.

Table 6: Price Indexes for Eight Commodity Classes of Exports, 1961-2007

Year t	$P_9{}^t$	$P_{10}{}^t$	$P_{11}{}^t$	$P_{12}{}^t$	$P_{13}{}^t$	$P_{14}{}^t$	$P_{15}{}^t$	$P_{16}{}^t$
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.05659	0.98534	1.03706	1.01977	1.03488	1.01167	0.99880	1.01858
1963	1.05292	1.01104	1.04975	1.02746	1.04642	1.02781	1.00853	1.04402
1964	1.06146	0.99854	1.06857	1.04333	1.06452	1.03785	1.02909	1.07736
1965	1.07552	1.01886	1.08666	1.06777	1.09035	1.03617	1.03692	1.12159
1966	1.13905	1.02441	1.10752	1.12360	1.12856	1.05032	1.05855	1.18364
1967	1.15369	0.96541	1.12890	1.15308	1.17596	1.06607	1.07630	1.26762
1968	1.14873	1.00454	1.15804	1.22731	1.25191	1.08830	1.10360	1.34554
1969	1.11043	1.05284	1.21130	1.26468	1.28144	1.09898	1.14735	1.41327
1970	1.07460	1.06679	1.20448	1.33958	1.36481	1.12160	1.14015	1.50120
1971	1.10272	1.12301	1.24675	1.28936	1.37181	1.15193	1.15590	1.57903
1972	1.16574	1.15550	1.34600	1.30236	1.41712	1.17423	1.18547	1.66505
1973	1.65562	1.34434	1.61655	1.49521	1.46833	1.18656	1.25988	1.79018
1974	2.47603	2.37211	1.97968	1.94140	1.65047	1.27902	1.42184	2.02338
1975	2.44172	3.47049	2.31660	2.10541	1.86186	1.40313	1.56638	2.29526
1976	2.27544	4.36340	2.39386	2.22651	1.90587	1.48840	1.66746	2.51896
1977	2.13627	5.37242	2.62455	2.50906	2.02758	1.61582	1.76846	2.72893

[92] All four prices are entered as positive numbers in the index number formula while the first two quantities are entered positively and the last two quantities are entered negatively.
[93] All three prices are entered as positive numbers in the index number formula while the first quantity is indexed with a positive sign and the last two quantities are indexed with negative signs.

1978	2.37658	5.96944	2.90165	2.77108	2.11566	1.78223	1.88273	2.90258
1979	2.88982	7.40098	3.47704	3.53655	2.27395	1.96622	2.00068	3.14587
1980	3.28365	10.39436	3.73797	4.45064	2.41362	2.17714	2.24946	3.49696
1981	3.56932	11.17124	3.96546	4.55036	2.55522	2.41373	2.49659	3.94719
1982	3.46707	11.69844	3.85284	4.31903	2.72280	2.63304	2.66992	4.34103
1983	3.39310	11.53824	3.83642	4.35230	2.75291	2.73815	2.80369	4.66353
1984	3.53005	11.27486	4.26336	4.45480	2.69323	2.90539	2.88654	4.87299
1985	3.51503	10.86479	4.32498	4.41174	2.68327	3.12972	3.00305	5.13320
1986	3.43645	7.75428	4.78008	4.59745	2.97757	3.01503	3.28669	5.40392
1987	3.32350	7.55588	5.20316	4.73208	3.02031	3.02347	3.40733	5.59379
1988	3.49449	6.35623	5.36339	5.05742	3.04258	2.91102	3.52049	5.72724
1989	3.73084	6.87199	5.59564	5.03255	3.06798	2.86952	3.71759	5.93246
1990	3.47655	7.84685	5.24427	4.74911	3.11615	2.87959	3.73982	6.08151
1991	3.14681	6.82156	4.69849	4.45427	3.07176	2.96769	3.82601	6.29162
1992	3.48817	7.07296	4.84492	4.47367	3.06279	3.15213	3.82959	6.33223
1993	3.71935	7.4753	5.37177	4.53039	3.09777	3.35449	3.89234	6.51684
1994	3.96389	7.25372	6.18573	5.11913	3.17302	3.52359	3.99910	6.66734
1995	4.40988	7.11026	7.36498	5.72144	3.21973	3.65787	4.09757	6.88429
1996	4.71556	8.77046	6.80562	5.32837	3.18323	3.74681	4.16933	7.03680
1997	4.46079	9.0083	6.77946	5.28875	3.12343	3.83816	4.19780	7.22032
1998	4.38396	7.35245	7.02909	5.13339	3.13173	4.05650	4.25986	7.34563
1999	4.31935	9.26009	7.12821	5.03383	3.08837	4.03743	4.31663	7.46192
2000	4.37477	15.28916	7.16500	5.39152	3.06944	4.02821	4.36260	7.72519
2001	4.62634	15.71996	7.30172	5.31976	3.0918	4.17233	4.43899	7.74816
2002	4.63446	13.37888	6.82281	5.26906	3.10623	4.21091	4.46478	7.87210
2003	4.53195	16.64222	6.33572	5.24802	2.97965	3.84484	4.47389	7.93621
2004	4.46913	18.36274	6.82304	5.79875	2.92001	3.66998	4.48713	8.09962
2005	4.14794	23.46718	6.44732	6.11917	2.87531	3.46196	4.51188	8.29947
2006	4.09923	22.90367	6.08392	6.87211	2.82144	3.31328	4.54101	8.42538
2007	4.38863	23.0415	5.71715	7.39951	2.78882	3.16568	4.54902	8.58781

Table 7: Quantity Indexes for Eight Commodity Classes of Exports, 1961-2007

Year t	Q_9^t	Q_{10}^t	Q_{11}^t	Q_{12}^t	Q_{13}^t	Q_{14}^t	Q_{15}^t	Q_{16}^t
1961	1432.1	252.3	1573.6	2057.2	402.1	50.1	63.7	1036.0
1962	1328.6	363.9	1593.1	2130.2	511.5	62.5	77.7	1132.0
1963	1571.9	364.1	1684.6	2242.7	581.5	93.7	90.9	1204.0
1964	1963.5	408.9	1834.5	2531.7	792.8	187.7	102.2	1285.6
1965	1798.9	425.8	1869.5	2721.2	791.3	347.5	112.1	1359.7
1966	1954.0	484.2	1964.9	2809.4	910.9	985.6	135.7	1492.0
1967	1613.2	578.7	1927.6	2997.6	1147.4	1620.4	146.5	1864.9
1968	1589.7	657.2	2127.4	3282.7	1239.5	2533.5	183.3	1499.0
1969	1492.5	757.0	2290.6	3091.0	1378.2	3181.6	214.8	1657.1
1970	1967.9	951.2	2339.2	3675.8	1437.1	3137.5	236.2	1767.3
1971	2168.3	1154.0	2374.2	3611.1	1436.8	3613.9	243.1	1747.3
1972	2268.9	1479.9	2662.0	3707.9	1662.5	4001.8	270.8	1720.7

1973	2217.2	1789.7	2825.7	4085.7	1950.5	4539.2	319.8	1893.1
1974	1778.2	1508.4	2809.5	3986.3	2088.5	4430.7	323.6	2094.5
1975	1879.8	1216.3	2185.5	3513.3	2165.0	4554.8	289.8	1997.6
1976	2047.5	977.6	2718.2	3824.4	2325.5	5499.2	311.3	2048.5
1977	2445.0	919.8	3001.2	3955.3	2345.1	6388.1	338.2	2052.1
1978	2524.7	937.6	3313.3	4281.0	2914.9	6954.2	404.7	2272.8
1979	2538.2	1101.5	3369.6	4292.6	3880.9	6004.4	506.8	2556.4
1980	2785.9	951.8	3286.8	4633.9	4469.2	5002.0	570.8	2658.0
1981	2923.8	950.1	3126.5	4529.3	4810.1	5585.9	548.3	2738.1
1982	3142.7	1009.7	2962.3	4133.5	4586.9	6387.7	525.1	2508.2
1983	3256.3	1079.1	3284.3	4169.5	4413.1	7748.3	546.8	2534.8
1984	3299.9	1210.3	3501.3	4965.2	5761.5	10115.0	652.0	2685.2
1985	2979.2	1461.0	3544.7	5178.9	6360.5	10539.3	666.0	2839.4
1986	3178.0	1416.6	3708.8	5606.6	6826.0	10494.1	767.4	3254.1
1987	3565.5	1701.0	4031.4	5790.7	6884.4	10540.5	775.7	3294.5
1988	3527.3	2009.2	4025.0	6316.1	7120.6	11928.4	798.8	3546.0
1989	3101.7	1997.4	3836.1	6412.9	7810.3	11838.6	709.3	3704.0
1990	3830.8	1779.1	3877.8	6765.1	9259.5	12042.3	895.2	3857.1
1991	4169.0	2068.3	3958.3	7016.2	9536.5	10949.5	908.0	3892.6
1992	4397.4	2184.7	4131.5	7237.9	10413.1	12087.4	1167.0	4156.5
1993	4342.7	2374.6	4352.4	7773.7	11895.0	14490.7	1440.8	4519.2
1994	4746.4	2646.9	4708.9	8301.8	14402.7	16349.5	1775.9	5093.3
1995	4754.3	2868.3	4989.3	8896.4	17402.0	17200.2	2029.5	5395.8
1996	4913.1	2970.5	5073.4	9821.5	19457.0	16912.7	2278.7	5850.5
1997	5553.7	3016.9	5178.1	10708.6	22070.0	18099.8	2555.4	6263.6
1998	5711.7	3238.6	5042.1	11525.7	25770.1	19342.2	2949.8	7084.9
1999	5929.8	3226.3	5623.2	11889.3	28713.2	24097.5	3239.8	7400.4
2000	6309.4	3476.8	5969.8	12608.7	35853.8	24300.2	3484.0	7936.8
2001	6717.8	3547.7	5517.3	12744.0	33169.5	22176.3	3673.6	7967.1
2002	6661.8	3687.1	5458.9	13317.5	31256.9	22958.2	3959.7	8276.2
2003	6450.4	3636.6	5448.2	12730.0	29760.9	22727.9	3841.6	7982.5
2004	6863.6	3708.9	5777.2	13443.3	31200.7	24629.3	3848.1	8268.8
2005	7256.1	3705.8	5653.2	13721.1	32346.3	25417.7	3800.7	8284.5
2006	7613.6	3789.4	5478.7	13664.5	33058.2	24838.7	3922.2	8185.4
2007	7831.6	3977.5	5118.7	14111.7	33500.8	24419.5	4118.9	8042.4

We now turn our attention to imports.

Current dollar information on imports of services can be found as CANSIM II series V498108 for the years 1961-2007 and the corresponding constant 2002 chained dollar series is CANSIM II series V1992065. We use these two series to form price and quantity series for the imports of services, P_{23}^t and Q_{23}^t, which are listed in Tables 8 and 9. Note that since imported goods and services are inputs into the business sector, when we form a value added aggregate, we need to append a minus sign to any quantity series pertaining to imports.

As was the case for our treatment of exports, the starting point for obtaining disaggregated data on imports of goods is CANSIM Table 3800012, Exports and Imports of Goods and Services, Canada, Current Prices. Using this Table, it is possible to obtain disaggregated information on the value of imports for the same 7 classes of imported good that was used for exports for the years 1971-2007. However, imports of forest products was small throughout the sample period and so this import component was aggregated with imports of industrial goods and materials (excluding forest and energy imports).[94] Thus we used CANSIM Table 38000012 in order to generate prices and quantities for the following 7 classes of imports for the years 1971-2007:[95]

- Q_{17}: Imports of agricultural and fish products;
- Q_{18}: Imports of energy products;
- Q_{19}: Imports of industrial goods and materials (including imports of forest products but excluding imports of energy products);
- Q_{20}: Imports of machinery and equipment (excluding automotive products);
- Q_{21}: Imports of automotive products;
- Q_{22}: Imports of other consumer goods and
- Q_{23}: Imports of services.

There remains the problem of obtaining price series for the above 6 classes of imports to cover the years 1961-1971. From Leacy (1983), Series G429-442: Foreign trade, imports, excluding coin and bullion, by main commodity sections, current values, 1946-1975, millions of dollars (all countries), we can obtain value series covering imports for the years 1946-1975 for the following 5 commodity classes:

- Live animals (G429);
- Food, feed, beverages and tobacco (G431);
- Crude materials (inedible) (G433);
- Fabricated materials (inedible) (G435);
- End products (inedible) (G437).

From the same source, price indexes for each of the above 5 classes of imports are available as Series K62-K67 in the Table with the title: Import price indexes, trade of Canada commodity classification, 1926-1975. Thus we can find price and quantity series for these 5 classes of exports that cover the years 1961-1971. Unfortunately, these price indexes are of the fixed base variety with a base year of 1948 so they are likely to differ substantially from the corresponding chain indexes. However, as was the case for export price indexes, Leacy (1983) also lists as part of import price Series K62-K67 (Panel A) for the above 5 classes of imports counterpart indexes that have a 1971 base year but these price indexes cover only the years 1968-1975. We use these latter price indexes to construct import price indexes for the years 1968-1971 and then we use the 1948 based indexes to further extend these 5 series back to 1961.

[94] Chained Fisher indexes were used in order to do the aggregation.
[95] As in the case of export indexes, we used chained indexes whenever they were available.

The above operations give us 5 disaggregated export price and quantity series for the period 1961-1971 but we have 6 classes of imports of goods for the years 1971-2007. We generated Fisher chained price and quantity indexes for imports of Live animals and for exports of Food, feed, beverages and tobacco for the years 1961-1971 and linked these series to our earlier series, P_{17} and Q_{17}, imports of agricultural and fish products. As was the case for extending our export series back to the 1960s, we need some additional series so that we can match the import series for the 1960s to the series that cover the post 1971 period. We will create separate import series for energy, automotive products and other consumer goods using the input output tables for the Canadian economy that cover the years 1961-1981; see Statistics Canada (1987a) (1987b). The rest of our import series computations parallel our export series computations, except that we did not generate a separate series for forest product imports due to their small size throughout the sample period.

The price of imports does not include import duties that are added to the international cost of these imported goods. Hence we must add these import duties to the price of imports. We assumed that energy, automotive and service imports were exempt from import duties and we assumed a uniform rate for the remaining import categories.[96] The series on customs import duties is CANSIM II series V499741 and after adjusting the price of imports using this series, the resulting price and quantity series for the imports of goods and services are listed in Tables 8 and 9 below.

Table 8: Price Indexes for Seven Commodity Classes of Imports, 1961-2007

Year t	P_{17}^t	P_{18}^t	P_{19}^t	P_{20}^t	P_{21}^t	P_{22}^t	P_{23}^t
1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	1.04883	1.02856	1.05578	1.09816	1.05195	1.01253	1.05228
1963	1.30508	1.00981	1.08637	1.09743	1.06958	1.01429	1.07921
1964	1.28197	1.00471	1.10651	1.09682	1.09414	1.02308	1.09777
1965	1.06938	1.02524	1.12598	1.12082	1.07648	1.02077	1.13619
1966	1.05080	1.05430	1.13068	1.15070	1.09270	1.03586	1.17024
1967	1.02344	0.98958	1.16189	1.18684	1.12302	1.05402	1.22610
1968	1.05513	1.04752	1.14800	1.19862	1.16016	1.06806	1.28761
1969	1.06398	1.00624	1.17757	1.21963	1.18860	1.08873	1.36902
1970	1.13052	1.01555	1.19414	1.22480	1.20108	1.09806	1.42900
1971	1.16283	1.11732	1.16445	1.25747	1.24031	1.10649	1.49900
1972	1.24651	1.20329	1.15700	1.26668	1.26989	1.14591	1.53952
1973	1.52015	1.43514	1.27659	1.29789	1.29640	1.18635	1.63282
1974	1.87300	4.22172	1.65984	1.42128	1.40659	1.29665	1.75102
1975	1.97450	5.43394	1.83511	1.67222	1.63529	1.44968	1.98130
1976	1.85375	5.63358	1.85932	1.67883	1.71458	1.52194	2.04366
1977	2.24663	6.42113	2.09118	1.81185	1.93225	1.71419	2.34119
1978	2.49806	7.14772	2.42204	1.83462	2.20965	1.95044	2.69834
1979	2.79922	9.22856	2.93293	1.94108	2.44387	2.14927	3.04229

[96] This is only a very rough approximation to the truth.

1980	3.05858	14.04487	3.40659	1.71574	2.71841	2.44883	3.39335
1981	3.35205	16.89476	3.70305	1.67637	3.26363	2.79785	3.81611
1982	3.27980	16.65354	3.79588	1.82550	3.52543	2.94081	4.10352
1983	3.19245	15.35401	3.78529	1.74893	3.62370	2.95617	4.30819
1984	3.39093	15.57686	3.91220	1.78888	3.82410	3.17305	4.63802
1985	3.29961	16.05971	3.85316	1.81429	4.02632	3.27951	4.98855
1986	3.52718	10.45556	3.94881	1.83440	4.20229	3.51229	5.29276
1987	3.44257	10.91648	3.94832	1.74520	4.15738	3.53064	5.28864
1988	3.50154	9.09050	4.05930	1.67407	4.03524	3.50861	5.09254
1989	3.45272	9.69611	4.06260	1.63403	4.08857	3.52386	5.09401
1990	3.45595	11.94481	4.01098	1.61746	4.16440	3.5793	5.24914
1991	3.43419	10.19284	3.86391	1.58265	4.12731	3.59254	5.31731
1992	3.43112	10.16216	3.93946	1.63533	4.39452	3.79083	5.63381
1993	3.44632	9.84044	4.05394	1.70897	4.68083	4.02323	6.19826
1994	3.70890	9.83576	4.32449	1.78245	4.99133	4.27046	6.68231
1995	3.92677	10.17736	4.70222	1.74664	5.15679	4.39186	6.90224
1996	3.88275	11.94335	4.52874	1.66405	5.19805	4.36419	7.01253
1997	3.99625	11.84732	4.52516	1.62932	5.26105	4.38392	7.26074
1998	3.97357	9.76539	4.64624	1.67064	5.51748	4.66991	7.80221
1999	3.85703	11.43514	4.55850	1.62996	5.52100	4.67035	7.97505
2000	3.84946	16.95310	4.72363	1.59815	5.52428	4.69368	8.18543
2001	3.99707	16.32263	4.90529	1.63313	5.67016	4.91888	8.64576
2002	4.05634	16.80830	4.85791	1.63385	5.73431	4.93873	8.84642
2003	3.91718	18.58307	4.58281	1.46138	5.39577	4.49645	8.39644
2004	3.81803	21.35868	4.74735	1.34910	5.21353	4.18264	8.24087
2005	3.72978	27.03564	4.82559	1.26664	5.00314	4.01666	8.14870
2006	3.62982	30.53146	5.01981	1.20443	4.84982	3.84873	8.19437
2007	3.73318	31.10128	4.92307	1.14851	4.63116	3.67501	8.15599

Table 9: Quantity Indexes for Seven Commodity Classes of Imports, 1961-2007

Year t	Q_{17}^t	Q_{18}^t	Q_{19}^t	Q_{20}^t	Q_{21}^t	Q_{22}^t	Q_{23}^t
1961	824.1	478.0	1850.2	1840.7	615.1	754.0	1535.0
1962	840.3	480.7	1926.5	1797.9	699.1	811.6	1479.6
1963	786.0	541.8	1952.1	1788.1	707.1	799.6	1466.8
1964	810.8	547.7	2235.8	2092.0	857.4	842.2	1618.7
1965	934.5	583.4	2482.1	2400.7	1157.2	948.7	1692.5
1966	1026.7	592.8	2666.5	2826.7	1573.4	1066.8	1850.0
1967	1086.6	638.9	2528.6	3077.0	1963.9	1109.3	1934.6
1968	1127.9	685.5	2726.8	3135.8	2649.7	1240.3	2013.8
1969	1290.9	737.9	3052.2	3522.5	3030.9	1461.0	2336.7
1970	1279.6	766.3	3041.7	3548.7	2758.7	1459.4	2471.7
1971	1293.5	817.1	3359.3	3712.9	3249.2	1650.7	2476.3
1972	1424.2	895.9	3855.8	4450.5	3819.2	2010.7	2585.9
1973	1625.4	928.1	4182.5	5370.1	4619.0	2325.5	2888.3
1974	1674.4	788.6	4742.9	6357.6	4931.1	2642.7	3277.5

1975	1675.3	766.1	4071.1	6031.1	4954.5	2583.3	3550.2
1976	1940.0	719.6	4177.8	6189.4	5417.1	2998.7	3971.3
1977	1846.2	654.2	4160.4	6312.2	5864.7	2943.3	3883.1
1978	1872.6	625.7	4499.4	7651.1	5918.6	2988.3	3824.2
1979	1855.1	624.7	5082.4	9325.4	6096.9	3148.4	3637.1
1980	1921.5	598.8	4917.6	12269.7	4900.3	3016.1	3760.0
1981	1994.6	573.7	5163.8	15342.7	4799.6	3113.5	3860.0
1982	1914.8	404.5	4146.0	11976.5	4128.6	2890.3	3594.5
1983	1985.9	336.2	4779.6	13552.5	5142.3	3228.1	3689.7
1984	2169.3	393.7	5469.4	16671.9	6668.0	3613.1	3774.9
1985	2190.2	395.2	6052.7	17490.1	7684.4	3557.1	3904.9
1986	2287.9	486.3	6369.6	19038.1	7845.2	3806.2	4267.7
1987	2388.1	541.7	6424.7	21231.5	7844.3	3991.8	4536.3
1988	2381.6	569.4	7298.0	26838.5	8225.8	4266.7	5184.6
1989	2625.7	641.6	7645.9	29100.6	7812.7	4681.5	5792.5
1990	2779.6	686.3	7577.4	29167.1	7319.2	4868.6	6405.6
1991	2871.4	650.4	7342.9	29676.0	7501.5	5065.0	6664.1
1992	3100.3	637.4	7950.1	31183.3	7664.1	5459.7	6738.8
1993	3437.0	708.1	8947.7	33412.5	8533.4	5711.6	6865.2
1994	3617.1	707.6	10111.7	39325.5	9583.4	5854.9	6755.0
1995	3597.8	711.1	10694.6	45780.5	9712.7	6144.3	6763.0
1996	3839.3	804.2	11268.5	48400.6	9832.0	6242.8	7110.7
1997	4101.5	897.0	13177.8	58701.8	11561.6	7109.8	7374.5
1998	4531.5	884.1	14104.5	63169.9	12105.0	7726.6	7366.6
1999	4760.9	936.4	14810.3	69069.7	13753.7	8239.7	7683.6
2000	4998.2	1053.1	15872.4	79742.9	14016.8	8861.7	8114.0
2001	5301.6	1087.2	15123.8	71310.1	12799.3	9072.2	7954.1
2002	5594.2	985.7	15449.5	67567.7	14207.5	9805.4	8090.6
2003	5735.7	1066.2	15563.8	70541.0	14176.3	10757.9	8830.5
2004	5842.3	1160.3	16838.7	80434.0	14839.8	11893.8	9363.0
2005	6159.0	1245.4	17648.1	91268.2	15667.0	12840.8	9823.2
2006	6731.2	1134.1	18076.1	99174.3	16464.5	14081.0	10104.3
2007	7118.8	1175.8	18659.2	105849.0	17274.7	15541.1	10697.4

We turn our attention to forming estimates of business sector labour input.

3. Business Sector Labour Input Estimates

Quality adjusted measures of the quantity of three types of labour for the years 1961-2007 are available from the Statistics Canada KLEMS productivity program; see CANSIM Table 3830021 which has the title: Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors, by the North American Industry Classification System (NAICS). The three series are V41713000 (the title is Canada: Labour Input of Workers with Primary or Secondary Education; Business Sector), V41713017 (Labour Input of workers with Some or Completed Post-Secondary Certificate or Diploma; Business Sector) and V41713034

(Labour Input of Workers with University Degree or Above, Business Sector). The corresponding value of labour input or labour compensation series are found in the same Table and their CANSIM series numbers are V41713187, V41713204 and V41713221. These value series only cover the years 1961-2004.[97] These KLEMS labour series allowed us to construct the three business sector labour input series Q_{L1}^t, Q_{L2}^t and Q_{L3}^t for the years 1961-2007 (see Table 10 below for a listing of these data) and the corresponding wage index series P_{L1}^t, P_{L2}^t and P_{L3}^t for the years 1961-2004 (see Table 10).

The Statistics Canada productivity program aggregate labour input measure is described as follows:

"The labour input is an aggregate of the hours worked of all persons classified by their education, work experience and class of employment (paid versus self-employed workers). This aggregate labour input measure is constructed by aggregating hours at work data for each of 56 types of workers classified by their educational attainment (4), work experience (7) and class of workers (2) using an annual chained-Fisher index. The effect of Fisher aggregation is to produce a measure of labour input that reflects both changes in total hours of work and changes in the composition of workers." John R. Baldwin, Wulong Gu and Beiling Yan (2007; 37).

Baldwin, Gu and Yan (2007; 26) describe their more disaggregated measures of labour input as follows:

"Labour input for MFP measures reflects the compositional shifts of workers by education, experience and class of workers (paid versus self-employed). The growth of labour input (labour services) is an aggregate of the growth of hours worked by different classes of workers, weighted by the hourly wages of each class."

Thus each of the three types of labour classified by educational attainment Q_{L1}^t, Q_{L2}^t and Q_{L3}^t is a Fisher quantity aggregate over the other characteristics, holding constant the relevant educational levels. Baldwin, Gu and Yan (2007; 26) also comment on the difficulties associated with breaking up the net operating surplus generated by the self employed into labour and capital compensation components:

"We have modified the assumptions about the share of labour going to the self-employed workers to reflect changes that occurred during the 1990s. In the past, it had been assumed that the self employed essentially earned incomes similar to the employed. The Census of Population up to 1990 showed that this was a reasonable assumption; however, during the 1990s, self-employed income fell behind that of production workers. The new measure of self-employed for calculating labour input assumes that the hourly earning of self-employed workers is proportional to that of paid workers with the same level of education and experience. The proportional or scaling factor for each level of education and experience is based on the relative hourly earnings of paid versus self-employed workers derived from the Census of Population."

Overall, we believe that Statistics Canada has done an excellent job in constructing their new measures of labour input and we will use these measures in the present study.[98] The

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[97] This is very puzzling: the quantity series run from 1961 to 2007; why stop the corresponding value series at 2004?

[98] The labour input that is used in the residential rental of housing industry should be deducted from our measure of labour input (since we exclude all residential housing outputs from our definition of the

effect of using the Statistics Canada measures of quality adjusted labour input is to increase the growth of labour input by about 37% over the sample period compared to using hours worked as the measure of labour input.[99] Basically, there was a big shift in labour inputs from less skilled and less educated workers to more educated workers over this period which served to greatly increase quality adjusted labour input compared to unweighted hours worked by all types of labour.

As noted above, the KLEMS estimates of real labour input for the three types of labour run from 1961-2007 but the corresponding value series stop at 2004. Hence we need to estimate either wages or values for the three types of labour for the years 2004-2007. In order to accomplish this task, we formed our own estimates of the total value of labour input over the years 1961-2007. Estimates of wages, salaries and supplementary labour income for the business sector are available from CANSIM II series V498167 for the years 1961-2007. However, this measure of business sector payments for labour services neglects the labour input of the self employed (and unpaid family workers); i.e., it includes only the gross wages of employees. The value of the labour services rendered by the self employed are part of the gross operating surplus of the household sector, which includes also the returns to the capital and land used by the self employed. An upper bound to the value of self employed labour services is the sum of unincorporated business net income which is available for 1961-2007 as CANSIM II series V498170. We assume that two thirds of unincorporated net income is a return to labour and one third is the return to capital. We added this imputed labour income of the self employed to the labour income of employees in the business sector and compared this measure of total business sector labour compensation to the corresponding total labour compensation from the KLEMS data base[100] and found that these two series were very close until about 1995 and then they gradually diverged to end up about 4% apart in 2003. We used the rates of growth of our imperfect measure of business sector labour income growth to extend the official KLEMS business sector labour compensation series from 2004 to 2007. We then divided this extended measure of total labour compensation by the KLEMS business sector measure of aggregate labour input[101] in order to obtain an implicit wage rate for aggregate business sector labour for the years 2004-2007. We used the movements in this implicit wage rate to extend the KLEMS wage indexes $P_{L1}{}^t$, $P_{L2}{}^t$ and $P_{L3}{}^t$ from 2004 to 2007; see Table 10 below for the results of these manipulations.

Table 10: Price and Quantity Indexes for Three Types of Business Sector Labour

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business sector while the KLEMS program business sector excludes only the services of Owner Occupied Housing). However, the KLEMS data base that is available in CANSIM does not include information on the three types of labour input that is used in the residential housing rental industry so we were not able to deduct these labour inputs from total business sector labour input. Thus our productivity estimates will have a tiny downward bias due to this factor.

[99] Estimates of total hours worked in the KLEMS business sector for the years 1961-2007 are available from CANSIM II series V41712966, (Canada, Hours Worked, Business Sector) in Table 3830021 (Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub Sectors, by North American Industry Classification System (NAICS)).

[100] See the CANSIM II series V41713170, Canada, Labour Compensation, Business Sector, in Table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors, by NAICS.

[101] See the CANSIM II series V41712949 with the title Canada, Labour Input, Business Sector.

Year t	P_{L1}^{t}	P_{L2}^{t}	P_{L3}^{t}	Q_{L1}^{t}	Q_{L2}^{t}	Q_{L3}^{t}
1961	1.00000	1.00000	1.00000	17122	710	1370
1962	1.02980	1.18632	1.01126	17345	1216	1448
1963	1.05959	1.21347	1.04509	17259	1723	1487
1964	1.10555	1.2456	1.09234	17482	2230	1566
1965	1.18267	1.29309	1.15814	17774	2762	1657
1966	1.26544	1.33357	1.24479	18152	3345	1788
1967	1.34988	1.36529	1.31145	18066	3852	1853
1968	1.44254	1.40528	1.40547	17740	4282	1853
1969	1.55946	1.46444	1.53399	17740	4789	1918
1970	1.66484	1.50846	1.62547	17379	5195	1983
1971	1.77957	1.67298	1.59351	17190	5777	2166
1972	1.93443	1.81847	1.60097	17190	6411	2336
1973	2.13905	1.97009	1.63324	17705	7247	2505
1974	2.49530	2.23902	1.78277	17826	7931	2740
1975	2.90404	2.52097	1.99402	17328	8362	2870
1976	3.39630	2.82284	2.16028	16916	8793	2897
1977	3.75369	3.03206	2.22664	16675	9300	3066
1978	3.90502	3.15557	2.39954	16967	10110	3314
1979	4.17384	3.37618	2.61573	17482	11149	3640
1980	4.51590	3.71514	2.83118	17654	11960	3901
1981	5.00927	4.09363	3.43078	17843	12492	4188
1982	5.50893	4.47191	3.61769	16761	11985	4149
1983	5.58062	4.79842	3.92881	16727	12188	4254
1984	5.95405	4.90205	4.14250	17053	12771	4697
1985	6.20453	5.19867	4.38987	17499	13455	5062
1986	6.33749	5.30440	4.65051	17946	14241	5480
1987	6.64512	5.43184	4.69809	18530	15153	5911
1988	7.12085	5.79003	4.88684	19045	16040	6432
1989	7.26900	5.96671	5.70897	19268	16648	6798
1990	7.26005	6.49660	5.97755	18908	16927	7007
1991	7.42364	6.73848	6.66739	17843	16547	7137
1992	7.57474	6.90164	6.68101	17242	16471	7450
1993	7.69212	6.83616	6.46124	16864	17079	8246
1994	7.69473	6.84046	6.22327	16795	18371	8768
1995	7.81823	7.01644	6.20187	16692	19537	9055
1996	7.93652	6.96834	6.54102	16812	20322	9538
1997	8.12992	7.18146	7.03193	16332	21918	10073
1998	8.34882	7.36378	7.31313	16383	22679	10856
1999	8.51971	7.52497	7.58425	16984	23414	11325
2000	8.92053	7.90809	7.95222	17311	24199	12082
2001	9.06503	8.10605	8.30514	16967	24706	12695
2002	9.14718	8.18965	8.50473	17173	25339	13048
2003	9.33522	8.37832	8.56340	16778	26226	13517
2004	9.56758	8.56262	8.80245	17156	27113	14313

2005	9.92027	8.90341	9.10008	17122	27062	15292
2006	10.37190	9.30873	9.51436	17396	27189	15983
2007	10.74529	9.64385	9.85688	17242	28101	16818

We now turn our attention to the problems associated with the estimation of beginning of the year capital stocks for the business sector.

4. Business Sector Capital Stock Estimates

Our general strategy in this section will be to use estimates from the National Balance Sheets to obtain estimates of inventory and land stocks used by the business sector; see Statistics Canada (1997). This balance sheet information is also used to calibrate estimates of depreciation for reproducible capital stocks used by the business sector. However, Statistics Canada made some unpublished information on components of the capital stock that are inputs into the Canadian business sector available to the authors and we will use some of this information on machinery and equipment investments and capital stocks in the present study.

For the years 1962-2008, beginning of the year estimates of various national wealth components can be obtained from the CANSIM II data base. National totals for the value of various assets can be obtained from CANSIM table 3780004 (National Balance Sheet Accounts, by Sectors) for residential structures (see series V34675), nonresidential structures (V34676), machinery and equipment (V34677), inventories (V34679) and land (V34680). The same table has the corresponding asset values for the persons and unincorporated business sector; for residential structures (see series V33464), nonresidential structures (V33465), machinery and equipment (V33466), inventories (V33468) and land (V33469). Table 3780004 also has the corresponding asset values for corporations and government business enterprises; for residential structures (see series V31693), nonresidential structures (V31694), machinery and equipment (V31695), inventories (V31696) and land (V31697). Finally, table 3780004 has the corresponding asset values for the government sector; for residential structures (see series V32575), nonresidential structures (V32576), machinery and equipment (V32577), inventories (V32578) and land (V32579). We subtract the government sector value of nonresidential structures, machinery and equipment and inventories from the corresponding total economy asset values in order to obtain business sector estimates of the value of beginning of the year t business sector nonresidential structure stocks VK_{NR}^t, business machinery and equipment stocks VK_{ME}^t, and business inventory stocks VK_{BI}^t; see Table 11 below for a listing of these business sector stock values. Although residential structures are not part of our domain of definition for business sector output, it will prove useful to have some information on the value of residential structures and residential land for comparison purposes. Thus the total value of residential structures from the national balance sheets for Canada, VK_{RS}^t, is also listed in Table 11.

We ended up not using the balance sheet information on the value of business sector machinery and equipment. Instead, we used the investment series on ICT machinery and equipment, P_{IICT}^t and Q_{IICT}^t, and on Non ICT machinery and equipment, P_{IME}^t and Q_{IME}^t, listed in Tables 2 and 3 above that were provided to us by Statistics Canada from their KLEMS data base. Statistics Canada also provided us with the companion business sector price and quantity series for the beginning of the year capital stocks for ICT and Non ICT machinery and equipment. This allowed us to compare the two price series for ICT. The KLEMS ICT investment price decreased from 1 in 1961 to 0.214 in 2006 whereas the KLEMS ICT capital stock price decreased from 1 in 1961 to 0.462 in 2006, which is a considerable difference.[102] If there were no asset heterogeneity in the class of ICT investments (which there certainly is), then using the geometric model of depreciation (which is used by the KLEMS program), we would expect these two price series to be very close to each other. The KLEMS Non ICT M&E investment price increased from 1 in 1961 to 4.71 in 2006 whereas the KLEMS Non ICT capital stock price increased from 1 in 1961 to 5.23 in 2006, which again indicates some asset heterogeneity, but the divergence between these two series is not nearly as large as the divergence in the two ICT investment and capital stock price series. The question now arises: which of these two price series should we use for a geometric model of depreciation?

Using the geometric or declining balance depreciation model of depreciation, the starting capital stock of a generic asset in period t+1, QK^{t+1}, is equal to one minus the depreciation rate in period t, δ^t, times the previous period's starting stock, QK^t, plus the new investment in the previous period, Q_I^t; i.e., we have:

(6) $QK^{t+1} = (1-\delta^t)QK^t + Q_I^t$.

Given information on beginning of the year capital stocks and investment during each year, the above equation can be solved for a balancing depreciation rate, δ^t, that reconciles the investment information with the balance sheet information:

(7) $\delta^t = [QK^t - QK^{t+1} + Q_I^t]/QK^t$.

Using the Statistics Canada KLEMS data base for ICT investment and capital, we tried deflating the value data by either the ICT investment price deflator or the ICT capital stock price deflator. Using the latter deflator, the implied ICT depreciation rate trended up from 0.238 in 1961 to 0.837 in 2005. These depreciation rates seem to be too large. However, when we implemented equation (7) using the ICT investment price deflator, the implied ICT depreciation rate trended up from 0.239 in 1961 to 0.329 in 2005. These depreciation rates seem to be very reasonable so we decided to use the ICT investment price deflator as the price of both ICT investments and capital stocks. Since the implied depreciation rates for ICT capital using equation (7) had a pronounced upward trend, we regressed these rates on a constant and a time trend. The estimated constant was 0.19909 with a standard error of 0.01644 and the estimated trend parameter was 0.00263 with a standard error of 0.0006437 so that both estimated parameters were significant. Thus we decided to assume a starting depreciation rate of 0.200 in 1961 and we increased this depreciation rate by 0.00263 each subsequent year. We then used equation (6) above

[102] There are even bigger differences in the rates of growth of the KLEMS ICT capital stocks versus the corresponding KLEMS measures of ICT services growth: the ICT stock grew 110 fold over the period 1961-2006 while the flow of ICT services grew 591 fold over the same period. This seems implausible.

along with the Statistics Canada 1961 value of the ICT capital stock as our starting value for the 1961 quantity of ICT capital in order to generate a new series for the business sector beginning of the year ICT capital stock, QK_{ICT}^t (see Table 12 below). An entirely analogous procedure was used to generate a new series for the business sector beginning of the year Non ICT machinery and equipment capital stock, QK_{ME}^t. Again, we used the KLEMS investment price deflator for Non ICT M&E, P_{IME}^t listed above in Table 2 as the deflator for the value of the Non ICT M&E capital stocks and the associated values of investments from the KLEMS data base and we calculated the implied depreciation rates using equation (7). The resulting implied depreciation rates had a small upward trend, starting at 0.160 in 1961 and ending up at 0.175 in 2005. Since the implied depreciation rates for ICT capital using equation (7) had an upward trend, we regressed these rates on a constant and a time trend. The estimated constant was 0.15034 with a standard error of 0.005459 and the estimated trend parameter was 0.00067351 with a standard error of 0.0002137 so that both estimated parameters were significant. Thus we decided to assume a starting depreciation rate of 0.150 in 1961 and we increased this depreciation rate by 0.00067 each subsequent year. We then used equation (6) above along with the Statistics Canada KLEMS program 1961 value of the Non ICT M&E capital stock as our starting value for the 1961 quantity of Non ICT M&E capital in order to generate a new series for the business sector beginning of the year ICT capital stock, QK_{ME}^t (see Table 12 below).[103]

Determining the value of business sector land is difficult. The problem is that the household sector owns a considerable amount of land that is used for business purposes; i.e., unincorporated persons own farm land and rental business properties and the land used in these enterprises should appear as inputs into the business sector. The corporate business sector also owns some land associated with residential rental properties and we are trying to exclude these inputs from our measure of business sector input. We will make some rough approximations in an attempt to solve these difficulties.

We first find estimates for the price and quantity of agricultural land, P_{AL}^t and QK_{AL}^t. Estimates of the area of agricultural land are available for the Census years 1981,1986, 1991, 1996, 2001 and 2006 from CANSIM II series V32166910 and we interpolated the quantity of land in use in agriculture between these years using constant rates of growth (geometric interpolation). From Leacy (1983), series M-23, Area of Land in Farm Holdings, Census Data in thousands of acres, we can obtain estimates of the area of farm land for 1961 and 1971. After converting from acres to hectares, these data can be appended to the previous data and again geometric interpolation between the various census years can be used to complete our estimates for QK_{AL}^t; see Table 14 for a listing.[104] CANSIM table 20020 (Balance Sheet of the Agricultural Sector at December 31) has asset value data for the end of the year for 1981-2007, which is beginning of the year values for the years 1982-2008. The two series that are of interest to us from this table are V157698 (the value of farm real estate) and V157699 (the value of farm land), which we denote by VK_{AL}^t for year t. Thus for the years 1982-2008, the price of

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[103] We used the rate of price inflation in Machinery and Equipment investment (using national accounts data) over the years 2006-2007 in order to extend P_{IICT}^t and P_{IME}^t to 2007.

[104] As usual, the listed data are normalized so that the corresponding price is unity in 1961.

agricultural land, P_{AL}^t, can be obtained by dividing VK_{AL}^t by QK_{AL}^t. For the years 1961-1980, we link P_{AL}^t to CANSIM series V381831 (the title is Canada, Value per Acre) in Table 20003, Value per Acre of Farm Land and Buildings. This last series runs from 1961 to 2006 and we found that it was quite close to P_{AL}^t for the overlap years 1981-2006. With estimates for the price and quantity of agricultural land for the years 1961-1980, we can form estimates for the corresponding values, VK_{AL}^t; see Table 11 below. The price and quantity of agricultural land, P_{AL}^t and QK_{AL}^t, are listed in Tables 13 and 14 below.

We assume that agricultural land is an input into our business sector. We also assume that the value of residential land, VK_{RL}^t, is equal to the total value of household and unincorporated business land less the value of agricultural land. Finally, we assume that the value of nonagricultural business land is equal to equal to the value of corporate enterprise land, VK_{BL}^t; see Table 11 below.

Table 11: Beginning of Year Asset Values for Residential Structures and Land and Six Business Sector Capital Stocks

Year t	VK_{ICT}^t	VK_{ME}^t	VK_{NR}^t	VK_{BI}^t	VK_{AL}^t	VK_{BL}^t	VK_{RL}^t	VK_{RS}^t
1961	1236	11994	27850	13594	5954	6376	10680	28710
1962	1297	12291	29388	13698	6203	6820	11423	29923
1963	1384	13161	31414	14292	6573	7281	12083	31707
1964	1506	13526	33599	15398	7303	7840	12548	34310
1965	1633	14853	37288	16224	8156	8537	13682	37875
1966	1778	16465	41694	17884	9132	9621	14974	42144
1967	2079	18394	46012	19588	10281	10971	16365	46525
1968	2361	20118	48638	20303	11298	12138	17719	49296
1969	2575	21618	53654	21462	11476	13161	19830	54058
1970	2892	23849	58457	23742	11534	14717	22130	58649
1971	3129	25718	64343	24275	11709	16421	24498	65459
1972	3440	27339	70782	25097	12542	18614	28056	74892
1973	3777	29576	81305	27660	15088	21533	32837	92703
1974	4266	36087	100032	33614	19533	26117	40030	116929
1975	5202	44936	116373	43928	24583	32759	47559	133160
1976	5841	51020	128858	46336	28876	38954	53411	150350
1977	6670	58919	141312	50117	33354	43763	59187	164910
1978	7226	67818	157802	57091	39666	49318	66442	183222
1979	7968	78428	179306	66060	49189	55774	73636	207120
1980	7976	91337	210699	81062	62176	64557	83608	234113
1981	8990	106059	245999	89024	70108	75224	101065	272325
1982	12096	121176	278898	98428	70197	86636	118825	288844
1983	12154	125826	287481	90451	68838	93532	117239	307383
1984	13465	131389	306422	91417	65974	96021	127037	328929
1985	14798	137700	323505	99318	62107	101438	139331	348994
1986	16116	144732	336131	104983	57733	105796	142788	388346
1987	17570	147727	357496	109889	54270	113337	165711	444678

1988	21135	152490	384524	117358	53316	122547	195773	498048
1989	23038	167116	409950	126135	56789	134213	223325	551991
1990	26275	180720	433558	132675	61388	145366	262598	574912
1991	26883	179871	435196	130781	63028	155133	258677	614114
1992	28186	189167	439422	123077	61853	159092	287087	633754
1993	32035	195633	445261	121352	62227	162090	307046	667294
1994	34428	201705	460244	124117	64707	169691	330460	698905
1995	36200	210289	468982	131198	69745	179044	352720	713616
1996	37274	216893	485422	146615	75539	185095	343616	719997
1997	40724	223783	500321	150648	81546	193888	352946	743640
1998	45118	244159	522564	158409	86376	202313	374636	766757
1999	49415	255591	541413	169901	89497	211188	394371	797843
2000	56416	269491	568533	178794	92140	221506	425256	829875
2001	65453	286828	582195	194366	95020	234213	452800	867239
2002	69434	296458	602645	190023	97613	243534	502343	926184
2003	66302	282138	621034	192080	99905	254448	573841	1003732
2004	63616	282551	669186	187291	102197	270107	633362	1099801
2005	64580	287617	723667	193723	104627	293163	733545	1190816
2006	65678	295028	795080	204832	107323	313706	834713	1323948
2007	70684	299576	862641	215812	111010	334379	949764	1468026

We assume that the quantity of residential land QK_{RL}^t and the quantity of business nonagricultural land QK_{BL}^t are constant over the sample period and hence the corresponding price series P_{RL}^t and P_{BL}^t are proportional to the corresponding value series VK_{RL}^t and VK_{BL}^t for the years 1962-2007. We extend these two price series back to 1961 using the movement from 1961 to 1962 in another land price series; namely series S319 in Leacy (1983): Average Land Cost per Dwelling Unit, NHA, Single Detached. These land price series, P_{RL}^t and P_{BL}^t, are listed in Table 14 below and the corresponding quantity series, QK_{RL}^t and QK_{BL}^t, are listed in Table 13.[105]

From Table 2 above, we have price deflators for nonresidential structures for year t, P_{INR}^t, and we use these deflators to divide VK_{NR}^t by P_{INR}^t in order to obtain preliminary beginning of the year capital stock quantity series, QK_{NR}^t.[106] Recall that a series for the annual quantity of investment in nonresidential structures, Q_{INR}^t, is available from Table 3 above. Now we are in a position to apply equation (7) again and generate a series of

[105] The Statistics Canada KLEMS program made available to us their aggregate price and quantity series for land used in the business sector. These series cover our agricultural land and our nonagricultural and nonresidential land series and also cover the part of residential land that applies to rental housing. The KLEMS capital stock of land grew 2.01 fold over the period 1961-2006 and the corresponding price series grew 18.7 fold. On the other hand, our estimate of the growth in the quantity of agricultural land and nonresidential and nonagricultural business land is essentially zero, with corresponding 18.6 fold and 48.5 fold increases in the price of these two business land components. We estimate even higher growth rates in the price of residential land but more research in this area is needed.

[106] The use of these prices (which are average prices over the year) for stock deflation purposes is not quite appropriate because conceptually, we should be using the prices that prevail for these stock components at the beginning of the year rather than the average prices in the year which follows. However, for our purposes, the errors made here will not be material.

geometric depreciation rates δ_{NR}^t for the nonresidential stock of structures for the years 1962-2006. The mean of these depreciation rates turned out to be 0.0616 with a standard deviation of 0.03253, so there was a considerable amount of variability in these rates. There appeared to be a slight upward trend in these depreciation rates so we regressed them on a time trend. The estimated constant was 0.045439 with a standard error of 0.009565 and the estimated trend parameter was 0.0007034 with a standard error of 0.0003621. Thus we decided to assume a starting depreciation rate of 0.045 in 1961 and we increased this depreciation rate by 0.0007 each subsequent year. We then used equation (6) above along with the Statistics Canada balance sheet value of the stock of nonresidential structures in 1962as our starting value in order to generate a new series for the business sector beginning of the year nonresidential capital stock, QK_{NR}^t (see Table 12 below).

From Table 2 above, we have price deflators for residential structures for year t, P_{IR}^t, and we use these deflators to divide VK_{RS}^t by P_{IR}^t in order to obtain preliminary beginning of the year capital stock quantity series, QK_{RS}^t.[107] Recall that a series for the annual quantity of investment in residential structures, Q_{IR}^t, is available from Table 3 above. Now we are in a position to apply equation (7) again and generate a series of geometric depreciation rates δ_{RS}^t for the residential structures capital stock for the years 1962-2006. The mean of these depreciation rates turned out to be 0.040239 with a standard deviation of 0.01795. There appeared to be no trends in these depreciation rates so we decided to assume a constant geometric depreciation rate of 0.04 for each year. We then used equation (6) above along with the Statistics Canada balance sheet value of the stock of residential structures in 1962 as our starting value in order to generate a new series for the business sector beginning of the year nonresidential capital stock, QK_{RS}^t (see Table 14 below).

The smoothed geometric depreciation rates δ_{ICT}^t, δ_{ME}^t, δ_{NR}^t and δ_{RS}^t are listed in Table 12 below.

Table 12: Smoothed Geometric Depreciation Rates for ICT, Non ICT Machinery and Equipment, Nonresidential Structures and Residential Structures Capital Stocks Implied by the Balance Sheets and Investment Flow Data

Year t	δ_{ICT}^t	δ_{ME}^t	δ_{NR}^t	δ_{RS}^t
1961	0.20000	0.15000	0.0450	0.04
1962	0.20263	0.15067	0.0457	0.04
1963	0.20526	0.15134	0.0464	0.04
1964	0.20789	0.15201	0.0471	0.04
1965	0.21052	0.15268	0.0478	0.04
1966	0.21315	0.15335	0.0485	0.04

[107] The use of these prices (which are average prices over the year) for stock deflation purposes is not quite appropriate because conceptually, we should be using the prices that prevail for these stock components at the beginning of the year rather than the average prices in the year which follows. However, for our purposes, the errors made here will not be material.

1967	0.21578	0.15402	0.0492	0.04
1968	0.21841	0.15469	0.0499	0.04
1969	0.22104	0.15536	0.0506	0.04
1970	0.22367	0.15603	0.0513	0.04
1971	0.22630	0.15670	0.0500	0.04
1972	0.22893	0.15737	0.0527	0.04
1973	0.23156	0.15804	0.0534	0.04
1974	0.23419	0.15871	0.0541	0.04
1975	0.23682	0.15938	0.0548	0.04
1976	0.23945	0.16005	0.0555	0.04
1977	0.24208	0.16072	0.0562	0.04
1978	0.24471	0.16139	0.0569	0.04
1979	0.24734	0.16206	0.0576	0.04
1980	0.24997	0.16273	0.0583	0.04
1981	0.25260	0.16340	0.0590	0.04
1982	0.25523	0.16407	0.0597	0.04
1983	0.25786	0.16474	0.0604	0.04
1984	0.26049	0.16541	0.0611	0.04
1985	0.26312	0.16608	0.0618	0.04
1986	0.26575	0.16675	0.0625	0.04
1987	0.26838	0.16742	0.0632	0.04
1988	0.27101	0.16809	0.0639	0.04
1989	0.27364	0.16876	0.0646	0.04
1990	0.27627	0.16943	0.0653	0.04
1991	0.27890	0.17010	0.0660	0.04
1992	0.28153	0.17077	0.0667	0.04
1993	0.28416	0.17144	0.0674	0.04
1994	0.28679	0.17211	0.0681	0.04
1995	0.28942	0.17278	0.0688	0.04
1996	0.29205	0.17345	0.0695	0.04
1997	0.29468	0.17412	0.0702	0.04
1998	0.29731	0.17479	0.0709	0.04
1999	0.29994	0.17546	0.0716	0.04
2000	0.30257	0.17613	0.0723	0.04
2001	0.30520	0.17680	0.0730	0.04
2002	0.30783	0.17747	0.0737	0.04
2003	0.31046	0.17814	0.0744	0.04
2004	0.31309	0.17881	0.0751	0.04
2005	0.31572	0.17948	0.0758	0.04
2006	0.31835	0.18015	0.0765	0.04
2007	0.32098	0.18082	0.0772	0.04

Table 13: Prices for Residential Structures and Land and Six Business Sector Capital Stocks

Year t	PK_{ICT}^t	PK_{ME}^t	PK_{NR}^t	PK_{BI}^t	PK_{AL}^t	PK_{BL}^t	PK_{RL}^t	PK_{RS}^t

1961	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1962	0.99939	1.01477	1.00592	1.00758	1.04000	1.06956	1.06956	1.00504
1963	0.99650	1.06598	1.03251	1.01000	1.10000	1.14186	1.13138	1.02769
1964	1.00502	1.06773	1.06158	1.02114	1.22000	1.22953	1.17489	1.07312
1965	1.02393	1.10198	1.12281	1.03951	1.36000	1.33883	1.28108	1.13368
1966	1.02944	1.12251	1.19323	1.06286	1.52000	1.50884	1.40204	1.20765
1967	1.07473	1.12792	1.24188	1.08044	1.72000	1.72055	1.53236	1.28518
1968	1.10902	1.13973	1.25227	1.09271	1.90000	1.90357	1.65913	1.31431
1969	1.14444	1.17197	1.32495	1.11812	1.94000	2.06400	1.85677	1.38118
1970	1.19472	1.23029	1.39058	1.14350	1.96000	2.30803	2.07210	1.42615
1971	1.22722	1.27216	1.46812	1.15918	2.00000	2.57526	2.29387	1.53179
1972	1.26956	1.30444	1.55098	1.21176	2.16000	2.91918	2.62704	1.67349
1973	1.30782	1.34535	1.71873	1.32609	2.62000	3.37696	3.07471	1.97123
1974	1.35698	1.51430	2.03419	1.43765	3.42000	4.09586	3.74822	2.36134
1975	1.45867	1.73112	2.27337	1.55864	4.34000	5.13750	4.45312	2.56072
1976	1.45190	1.82918	2.40093	1.66364	5.14000	6.10905	5.00114	2.76853
1977	1.44467	1.98862	2.52980	1.78349	5.92000	6.86323	5.54193	2.87768
1978	1.40748	2.19453	2.71145	1.94126	7.02000	7.73441	6.22122	3.04069
1979	1.38366	2.44348	2.96312	2.15180	8.68000	8.74688	6.89491	3.28046
1980	1.21888	2.69736	3.32520	2.35895	10.94000	10.12430	7.82863	3.55455
1981	1.12880	2.98813	3.68676	2.57823	12.30000	11.79717	9.46319	3.99273
1982	1.16812	3.19883	3.96113	2.77940	12.28000	13.58689	11.12613	4.08226
1983	1.02222	3.26756	3.93090	2.93246	12.00746	14.66837	10.97760	4.25350
1984	0.96342	3.40812	4.08142	3.06736	11.47452	15.05871	11.89508	4.41785
1985	0.89167	3.57237	4.21351	3.14959	10.77083	15.90824	13.04615	4.55564
1986	0.82129	3.70845	4.27520	3.18232	9.98330	16.59100	13.36985	4.90827
1987	0.75277	3.69804	4.47320	3.23099	9.38642	17.77433	15.51626	5.40819
1988	0.71401	3.69075	4.72840	3.28744	9.22342	19.21871	18.33108	5.78293
1989	0.64691	3.79390	4.92520	3.34034	9.82631	21.04826	20.91090	6.13195
1990	0.61403	3.87130	5.08853	3.38833	10.62426	22.79735	24.58825	6.11231
1991	0.54586	3.74170	5.00311	3.39008	10.91042	24.32908	24.22111	6.32257
1992	0.51043	3.88288	4.97541	3.54606	10.69755	24.94996	26.88125	6.39710
1993	0.50164	4.04523	5.03758	3.55229	10.75269	25.42013	28.75011	6.58445
1994	0.48119	4.24754	5.20497	3.72829	11.17139	26.61217	30.94243	6.76485
1995	0.44755	4.44740	5.27332	3.85944	12.03048	28.07898	33.02674	6.76717
1996	0.41150	4.57050	5.43035	3.94827	13.01832	29.02794	32.17431	6.75581
1997	0.39399	4.69221	5.56694	3.81073	14.07653	30.40692	33.04791	6.87512
1998	0.36919	4.90374	5.71450	3.83563	14.93456	31.72819	35.07889	6.95993
1999	0.33873	4.94358	5.82995	3.89582	15.49944	33.12003	36.92678	7.13210
2000	0.32384	5.01831	6.02775	3.98639	15.98327	34.73818	39.81866	7.29782
2001	0.31733	5.17017	6.07934	4.07102	16.50975	36.73098	42.39774	7.48766
2002	0.30560	5.25354	6.18175	4.13975	16.95600	38.19277	47.03668	7.81242
2003	0.27567	4.94953	6.30506	3.90062	17.34990	39.90438	53.73129	8.21290
2004	0.24913	4.85253	6.70389	3.94012	17.74345	42.36014	59.30452	8.71618
2005	0.23077	4.79667	7.12403	3.98736	18.16070	45.97595	68.68515	9.11452
2006	0.21439	4.71044	7.64020	4.05205	18.62402	49.19765	78.15794	9.78750

2007	0.20857	4.58255	8.04719	4.17868	19.25890	52.43974	88.93074	10.49192

Table 14: Quantities of Residential Structures and Land and Six Business Sector Capital Stocks

Year t	QK_{ICT}^t	QK_{ME}^t	QK_{NR}^t	QK_{BI}^t	QK_{AL}^t	QK_{BL}^t	QK_{RL}^t	QK_{RS}^t
1961	1236	11994	27850	13594	5954	6376	10680	28710
1962	1298	12112	29215	13595	5965	6376	10680	29773
1963	1389	12346	30425	14150	5976	6376	10680	30853
1964	1499	12668	31650	15079	5986	6376	10680	31972
1965	1595	13478	33210	15607	5997	6376	10680	33409
1966	1727	14668	34942	16826	6008	6376	10680	34898
1967	1934	16308	37050	18130	5977	6376	10680	36201
1968	2129	17651	38840	18580	5946	6376	10680	37507
1969	2250	18446	40495	19195	5915	6376	10680	39139
1970	2420	19385	42038	20763	5885	6376	10680	41124
1971	2550	20216	43827	20942	5854	6376	10680	42733
1972	2710	20958	45637	20711	5806	6376	10680	44752
1973	2888	21984	47305	20858	5759	6376	10680	47028
1974	3144	23831	49175	23381	5711	6376	10680	49518
1975	3566	25958	51190	28184	5664	6376	10680	52001
1976	4023	27892	53670	27852	5618	6376	10680	54307
1977	4617	29628	55859	28101	5634	6376	10680	57307
1978	5134	30903	58198	29409	5650	6376	10680	60257
1979	5759	32097	60513	30700	5667	6376	10680	63137
1980	6544	33862	63364	34364	5683	6376	10680	65863
1981	7965	35493	66725	34529	5700	6376	10680	68205
1982	10355	37881	70409	35413	5716	6376	10680	70756
1983	11890	38508	73134	30845	5733	6376	10680	72266
1984	13976	38552	75077	29803	5750	6376	10680	74455
1985	16596	38546	76778	31534	5766	6376	10680	76607
1986	19623	39028	78623	32989	5783	6376	10680	79121
1987	23340	39947	79920	34011	5782	6376	10680	82223
1988	29601	41317	81322	35699	5781	6376	10680	86124
1989	35612	44049	83235	37761	5779	6376	10680	90019
1990	42792	46682	85203	39156	5778	6376	10680	94058
1991	49249	48072	86985	38578	5777	6376	10680	97130
1992	55220	48718	88319	34708	5782	6376	10680	99069
1993	63861	48362	88388	34162	5787	6376	10680	101344
1994	71547	47487	88424	33291	5792	6376	10680	103314
1995	80884	47284	88935	33994	5797	6376	10680	105453
1996	90581	47455	89391	37134	5803	6376	10680	106574
1997	103363	47692	89874	39533	5793	6376	10680	108164
1998	122207	49790	91445	41299	5784	6376	10680	110167
1999	145882	51702	92868	43611	5774	6376	10680	111867
2000	174209	53702	94319	44851	5765	6376	10680	113715

2001	206260	55477	95766	47744	5755	6376	10680	115823
2002	227207	56430	97488	45902	5757	6376	10680	118553
2003	240511	57003	98498	49243	5758	6376	10680	122214
2004	255354	58228	99821	47534	5760	6376	10680	126179
2005	279847	59962	101581	48584	5761	6376	10680	130651
2006	306347	62633	104065	50550	5763	6376	10680	135269
2007	338899	65373	107198	51646	5764	6376	10680	139920

End of the year current market value starting stocks of inventories for the entire economy and for the government sector are available from the National Balance Sheet Accounts; see CANSIM series V34679 and V32578 (Table 3780004) for the years 1961-2007. Subtracting the government inventory stocks from the total inventory stocks will give us estimates for the value of the business sector beginning of the year inventory stocks for the years 1962-2008, VK_{BI}^t. We can subtract the value of inventory change for 1961 (see CANSIM II series V498100; table 3800002; Canada, Current Prices, Business Investment in Inventories) from the starting stock of inventories in 1962 in order to extend the value of inventory stock series back to 1961. Statistics Canada provided us with a price index for business sector inventory stocks for the years 1961-2005, PK_{BI}^t. We extended this price series to the years 2006-2007 by using the January indexes for the Industrial Product Price Index for Canada and for All Commodities, CANSIM II series V1574377, Table 3290039. The inventory value series VK_{BI}^t can be divided by the inventory stock price series P_{BI}^t, in order to obtain a real beginning of the year business sector stock of inventories, QK_{BI}^t. The resulting price and quantity series (after normalization so that the price is unity in 1961) are listed in Table 13 for P_{BI}^t and Table 14 for QK_{BI}^t.

It is possible to generate an alternative value of inventory stock series by cumulating information on the value of inventory change from the System of National Accounts. Thus the CANSIM II series V498100 estimates the current value of business investment in inventories, which conceptually, should equal the value of inventory change over the year. Using the balance sheet estimates of the starting stock of inventories for 1962 (which was $13,698 million) and the above series, we can cumulate inventory changes and obtain an alternative SNA based estimated value of inventory change, which ended up at $91,315 million at the start of 2007. However, using the balance sheet estimates for the beginning of 2007 value of business inventories, we obtain the estimate $215,812 million, which is 2.35 times as big as the implied SNA estimate. Thus the SNA based estimates basically give us an inventory to output ratio that is implausibly low at the end of the sample period. It is true that inventory to output ratios have been falling due to just in time delivery and other inventory management techniques but the number of goods that are being produced has also been growing, which implies an increasing need for inventories. In any case, we will take the balance sheet estimates of inventory stocks as the "truth".[108]

[108] This choice will lead to an increase in measured Total Factor Productivity compared to estimates that rely on the SNA estimates of inventory change. See Diewert and Smith (1994) for a detailed accounting

Recalling Table 2 and 3 in this Appendix, a preliminary price series for inventory change P_{II}^{t} in year t is set equal to P_{BI}^{t+1} listed in Table 13.[109] A preliminary series for the quantity of inventory change in year t listed, Q_{II}^{t}, is set equal to the stock at the beginning of year t+1, QK_{BI}^{t+1}, less the stock at the beginning of year t, QK_{BI}^{t}. These preliminary series, P_{II}^{t} and Q_{II}^{t} are then renormalized so that P_{II}^{t} equals unity in 1961 and these renormalized series are the series which appear in Tables 2 and 3 above.

5. Primary Input Tax Rates, Balancing Real Rates of Return and User Costs

Nonresidential structures (office buildings, factories, etc.) and business land have to pay property taxes on these inputs whereas machinery and equipment and inventory stocks are generally exempt from paying these taxes. Thus it is necessary to take into account property taxes when constructing user costs of capital for business nonresidential structures and business land. Information on property taxes for the years 1961-2007 is available from Statistics Canada; see CANSIM II series V499942, table 3800035 (Real Property Taxes of Local Governments) and CANSIM II series V499841, Table 3800033 (Real Property Taxes of Provincial Governments). We approximate the asset base on which these taxes fall as the total beginning of the year national value of land, residential structures and nonresidential structures. Data on these values are available for the years 1962-2007 from the National Balance Sheets and these data were described at the beginning of section 4. These series were summed and the sum was used as the tax base for the sum of the two property tax series, V499942 plus V499841. The resulting property tax rates are reported as the series τ_{P}^{t} in Table 15 below[110] and it will be used in the construction of the user costs of business sector land and nonresidential structures.[111]

It is of some interest to calculate the average business tax rate for taxes that apply to the use of financial capital in the business sector so we provide estimates for this tax rate by year. These business taxes that fall on the return to capital are defined to be the sum of the following taxes:

- Taxes less subsidies on factors of production (CANSIM II series V1992216, table 3800001) less local government and Provincial government property taxes;
- Total government taxes on income from corporations and government business enterprises (CANSIM II series V499131, table 3800007) and
- Total government taxes on income from nonresidents (CANSIM II series V499132, table 3800007).

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framework for inventories that is consistent with the Hicks (1961) and Edwards and Bell (1961) model of production and Diewert (2005b) for a critical review of SNA conventions for measuring inventory change.

[109] Diewert (2005b) showed that in order to obtain a user cost of inventories that is consistent with other user costs and the measurement of output, inventory changes should be valued at end of year prices.

[110] The tax rate for 1961 was set equal to the corresponding rate for 1962.

[111] This is a very rough approximation to the actual property tax rates on business sector land and nonresidential structures since actual property tax rates are different across different sectors and assets. For example, business sector property assets are generally taxed more heavily than household property assets.

The sum of the above three sources of general business taxes that fall on capital stock components was divided by the corresponding sum of the beginning of the year value of assets for our six types of business sector asset; i.e., the above sum of taxes for year t was divided by $PK_{ICT}^{t} \times QK_{ICT}^{t}$ (year t starting value of ICT machinery and equipment) plus $PK_{ME}^{t} \times QK_{ME}^{t}$ (year t starting value of non ICT machinery and equipment) plus $PK_{NR}^{t} \times QK_{NR}^{t}$ (year t starting value of nonresidential structures) plus $PK_{BL}^{t} \times QK_{BL}^{t}$ (year t starting value of business sector land) plus $PK_{AL}^{t} \times QK_{AL}^{t}$ (year t starting value of agricultural land) plus $PK_{BI}^{t} \times QK_{BI}^{t}$ (year t value of starting stocks of inventories) and the resulting year t general business tax rate is denoted as τ_B^{t}, which is listed in Table 15 below.

Using the property tax rates τ_P^{t}, the general business tax rates τ_B^{t}, the ICT depreciation rates δ_{ICT}^{t}, the non ICT machinery and equipment depreciation rates δ_{ME}^{t} and the nonresidential structures depreciation rates δ_{NR}^{t}, the *user costs* of ICT stocks, machinery and equipment, nonresidential structures, business inventories, agricultural land and nonagricultural, nonresidential business land, U_{ICT}^{t}, U_{ME}^{t}, U_{NR}^{t}, U_{BI}^{t}, U_{AL}^{t} and U_{BL}^{t} respectively, can be defined as follows:[112]

(8) $U_{ICT}^{t} \equiv [r^{t} + \tau_B^{t} + \delta_{ICT}^{t}]\, PK_{ICT}^{t}$
(9) $U_{ME}^{t} \equiv [r^{t} + \tau_B^{t} + \delta_{ME}^{t}]\, PK_{ME}^{t}$;
(10) $U_{NR}^{t} \equiv [r^{t} + \tau_B^{t} + \tau_P^{t} + \delta_{NR}^{t}]\, PK_{NR}^{t}$;
(11) $U_{BI}^{t} \equiv [r^{t} + \tau_B^{t}]\, PK_{BI}^{t}$;
(12) $U_{AL}^{t} \equiv [r^{t} + \tau_B^{t} + \tau_P^{t}]\, PK_{AL}^{t}$;
(13) $U_{BL}^{t} \equiv [r^{t} + \tau_B^{t} + \tau_P^{t}]\, PK_{BL}^{t}$

where r^{t} is suitable real rate of return that applies to the business sector in year t. In the present study, we will follow national income accounting conventions and will take r^{t} to be the *balancing real rate of return*;[113] i.e., it is the rate of return that is consistent with the year t value of business sector net output being equal to the value of primary inputs

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[112] For additional material on user costs and many historical references, see Jorgenson (1989) (1996a) (1996b) and Diewert (2005a) and (2006a).

[113] For most purposes, it is probably preferable to use an exogenous real rate of return in the user costs (8)-(13) since the resulting prices will probably approximate market rental prices better. For discussion of this topic, see Diewert (2006a). However, in the present study, there was little difference in the empirical results if the sample average real rate of return (4.95 %) was used in place of the balancing real rate; i.e., in the gross income model, average TFP growth changed from 1.14 % to 1.13 % per year and in the net income model, average TFP growth changed from 1.26 % to 1.25 % per year This is similar to results obtained by Diewert and Lawrence (2005) (2006) for Australia. Their first study used the sample average balancing real rate for Australia whereas their second study used the year by year balancing real rates of return. However, Baldwin and Gu (2007; 27) found substantial differences for the Canadian business sector in their TFP growth rates for the period 1961-1981 where their estimated average TFP growth rates increased from the 0.90 to 1.01 % per year range using balancing or endogenous interest rates to the 1.18 to 1.26 % range using an exogenous interest rate. The differences that Baldwin and Gu (2007; 28) found for the 1981-2001 period were not nearly as large: an increase from the 0.30-0.38 % range to the 0.32-0.43 % range. Baldwin and Gu (2007; 18) mention that they used a constant real rate of interest equal to 5.1 % in their exogenous interest rate models, which is very close to the 4.95 % real rate that we used in our exogenous real rate computations.

used by the business sector in year t, where the user costs (8)-(12) are used as prices for the beginning of the year capital inputs. Thus r^t can be determined as the solution to the following linear in r^t equation:

$$\begin{aligned}(14)\ & P_C{}^tQ_C{}^t + P_{IG}{}^tQ_{IG}{}^t + P_{IR}{}^tQ_{IR}{}^t + P_{INR}{}^tQ_{INR}{}^t + P_{IME}{}^tQ_{IME}{}^t + P_{II}{}^tQ_{II}{}^t + P_{GN}{}^tQ_{GN}{}^t \\ & \quad + P_{XG}{}^tQ_{XG}{}^t + P_{XS}{}^tQ_{XS}{}^t + P_{MG}{}^tQ_{MG}{}^t + P_{MS}{}^tQ_{MS}{}^t \\ &= P_{L1}{}^tQ_{L1}{}^t + P_{L2}{}^tQ_{L2}{}^t + P_{L3}{}^tQ_{L3}{}^t \\ & \quad + [r^t + \tau_B{}^t + \delta_{ICT}{}^t]\, PK_{ICT}{}^tQK_{ICT}{}^t + [r^t + \tau_B{}^t + \delta_{ME}{}^t]\, PK_{ME}{}^tQK_{ME}{}^t \\ & \quad + [r^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t]\, PK_{NR}{}^tQK_{NR}{}^t + [r^t + \tau_B{}^t + \tau_P{}^t]\, PK_{BL}{}^tQK_{BL}{}^t \\ & \quad + [r^t + \tau_B{}^t + \tau_P{}^t]\, PK_{AL}{}^tQK_{AL}{}^t + [r^t + \tau_B{}^t]\, PK_{BI}{}^tQK_{BI}{}^t\end{aligned}$$

where the various price and quantity series are defined in the above Appendix 2 tables.[114] The resulting series of balancing real rates of return is listed in Table 15 below. Once r^t has been determined, then the six series of user costs defined by (8)-(13) can also be calculated; these series are also listed in Table 15. Note that r^t is a *real after tax rate of return* because we do not include a capital gains term in our user costs and all user costs are evaluated at the average prices for the corresponding investment good for year t.

Table 15: Business Sector Tax Rates, Balancing Real Rates of Return and User Costs

Year t	$\tau_P{}^t$	$\tau_B{}^t$	r^t	$U_{ICT}{}^t$	$U_{ME}{}^t$	$U_{NR}{}^t$	$U_{BI}{}^t$	$U_{AL}{}^t$	$U_{BL}{}^t$
1961	0.01528	0.03225	0.03650	0.26875	0.21875	0.12903	0.06875	0.08403	0.08403
1962	0.01528	0.03281	0.03578	0.27106	0.22250	0.13034	0.06912	0.08723	0.08971
1963	0.01525	0.03318	0.04750	0.28494	0.24733	0.14696	0.08149	0.10552	0.10954
1964	0.01534	0.03468	0.05119	0.29524	0.25400	0.15745	0.08769	0.12348	0.12445
1965	0.01536	0.03353	0.04779	0.29882	0.25786	0.16222	0.08453	0.13148	0.12944
1966	0.01533	0.03249	0.05042	0.30478	0.26520	0.17510	0.08812	0.14933	0.14823
1967	0.01524	0.03054	0.03555	0.30294	0.24827	0.16210	0.07141	0.13989	0.13993
1968	0.01590	0.03299	0.03652	0.3193	0.25552	0.16944	0.07595	0.16227	0.16258
1969	0.01624	0.03428	0.03291	0.32986	0.26082	0.17758	0.07513	0.16185	0.17220
1970	0.01607	0.03153	0.03428	0.34584	0.27292	0.18519	0.07525	0.16047	0.18897
1971	0.01554	0.03206	0.02737	0.35065	0.27495	0.18640	0.06889	0.14993	0.19306
1972	0.01506	0.03382	0.02783	0.36891	0.28570	0.20072	0.07471	0.16570	0.22394
1973	0.01380	0.03757	0.05809	0.42794	0.34131	0.27990	0.12685	0.28677	0.36963
1974	0.01255	0.04094	0.06004	0.45483	0.39326	0.34100	0.14518	0.38829	0.46503
1975	0.01217	0.03639	0.02907	0.44093	0.38922	0.30106	0.10203	0.33691	0.39882
1976	0.01283	0.03314	0.03320	0.44398	0.41412	0.32335	0.11038	0.40696	0.48369
1977	0.01333	0.03120	0.04273	0.45653	0.46663	0.36292	0.13185	0.51658	0.59888
1978	0.01317	0.03105	0.04783	0.45546	0.52730	0.40389	0.15314	0.64625	0.71202
1979	0.01205	0.03217	0.06274	0.47357	0.62792	0.48763	0.20424	0.92848	0.93563
1980	0.01198	0.03248	0.05146	0.40699	0.66535	0.51280	0.19800	1.04933	0.97109
1981	0.01207	0.03143	0.04056	0.36639	0.70337	0.52742	0.18560	1.03391	0.99164

[114] $P_{XG}{}^t$ and $Q_{XG}{}^t$ are chained Fisher aggregates of our 7 classes of exports of goods, $P_{XS}{}^t$ and $Q_{XS}{}^t$ are the price and quantity of exports of services, $P_{MG}{}^t$ and $Q_{MG}{}^t$ are chained Fisher aggregates of our 6 classes of imports of goods and $P_{MS}{}^t$ and $Q_{MS}{}^t$ are the price and quantity of imports of services.

1982	0.01218	0.02773	0.02063	0.35463	0.67952	0.47628	0.13440	0.74340	0.82251
1983	0.01273	0.02880	0.04443	0.33845	0.77758	0.57532	0.21474	1.03215	1.26088
1984	0.01254	0.03198	0.05234	0.33220	0.85111	0.64470	0.25864	1.11142	1.45859
1985	0.01260	0.03173	0.05501	0.31196	0.90318	0.67898	0.27321	1.07001	1.58038
1986	0.01276	0.03070	0.05025	0.28474	0.91858	0.66783	0.25761	0.93553	1.55481
1987	0.01262	0.03305	0.06412	0.27517	0.97846	0.77381	0.31395	1.03052	1.95141
1988	0.01253	0.03318	0.05905	0.25936	0.96079	0.79750	0.30321	0.96627	2.01340
1989	0.01267	0.03323	0.04822	0.22971	0.94929	0.78175	0.27209	0.92490	1.98116
1990	0.01303	0.03249	0.03650	0.21200	0.92299	0.74964	0.23376	0.87139	1.86982
1991	0.01360	0.03066	0.01785	0.17872	0.81795	0.64092	0.16443	0.67758	1.51093
1992	0.01418	0.03054	0.02931	0.17425	0.89545	0.70016	0.21221	0.79188	1.84691
1993	0.01418	0.03271	0.02850	0.17325	0.94110	0.71929	0.21742	0.81059	1.91629
1994	0.01392	0.03528	0.04441	0.17635	1.06954	0.84170	0.29711	1.04577	2.49121
1995	0.01369	0.03729	0.05185	0.16943	1.16489	0.90509	0.34405	1.23716	2.88751
1996	0.01371	0.04085	0.05500	0.15962	1.23083	0.97235	0.37844	1.42627	3.18027
1997	0.01366	0.04436	0.04773	0.15238	1.24910	0.97949	0.35092	1.48857	3.21548
1998	0.01386	0.04021	0.04498	0.14121	1.27485	0.97115	0.32674	1.47920	3.14252
1999	0.01395	0.04567	0.04749	0.13315	1.32792	1.04185	0.36292	1.66008	3.54736
2000	0.01320	0.05092	0.06134	0.13434	1.44719	1.19200	0.44748	2.00514	4.35800
2001	0.01296	0.04095	0.06042	0.12901	1.43815	1.13880	0.41265	1.88744	4.19919
2002	0.01255	0.03963	0.07025	0.12765	1.50961	1.21244	0.45488	2.07595	4.67601
2003	0.01239	0.04193	0.06610	0.11536	1.41639	1.22833	0.42137	2.08922	4.80516
2004	0.01212	0.04536	0.07973	0.10916	1.47468	1.42330	0.49287	2.43457	5.81222
2005	0.01167	0.04517	0.08577	0.10307	1.48894	1.55589	0.52207	2.58973	6.55621
2006	0.01110	0.04621	0.07823	0.09493	1.43474	1.62001	0.50423	2.52426	6.66814
2007	0.01065	0.04605	0.07965	0.09316	1.40462	1.71843	0.52524	2.62584	7.14985
Average	0.01351	0.03569	0.04827	0.26874	0.78978	0.64956	0.23351	0.95212	1.91810

Note that the sample average of the balancing after tax real rates of return r^t was a rather large 4.827% per year.[115] The average property tax rate $\tau_P{}^t$ was 1.351% while the average business tax rate on assets was 3.569%. Thus the before business tax real rate of return averaged 8.396%. Thus it appears that governments are taking about 42.5% of the before tax return to capital assets on average.[116] However, it must be kept in mind that

[115] The corresponding balancing real rate of return for Australia averaged around 3 percent; see Diewert and Lawrence (2006). Normally, after tax real rates of return are in the 1 to 3 percent rate whereas our average rate is close to 5 percent. This suggests that our estimates of the value of output may be too high or that the value of labour input are too low or that our estimated asset values for business sector capital inputs are too small. We think that the last possibility is the most probable one. Using the data tabled in this appendix, we calculated a business sector nominal and real value of business sector output and we also calculated the corresponding business sector nominal and real capital stock inputs where the real measures were calculated using chained Fisher indexes. We found that the nominal business sector capital output ratio fell from 2.417 in 1961 to 1.861 in 2007 while the real capital output ratio fell from 2.417 in 1961 to 1.538 in 2007. These falls in the capital output ratio seem unlikely. See Diewert and Fox (2001) for a discussion of output mismeasurement problems.

[116] This relatively high rate of business taxation has two negative effects: (i) it raises the user cost of capital and hence lessens the beneficial effects of capital deepening and (ii) the high rates lead to a relatively large loss of productive efficiency; i.e., the deadweight losses of such large tax rates are likely to be large. See

these balancing rates of return may not be very reliable; they contain the net effect of all the measurement errors that were made in constructing this data set. The volatility in the above real rates of return is a source of concern since it is likely that a considerable proportion of the volatility is caused by various measurement errors. The volatility in the real rates of return also causes volatility in the user costs and possible volatility in productivity growth rates. However, we repeated our productivity calculations using a constant after tax real rate of return (equal to the sample average real rate of 4.83%) and found no material difference in our productivity growth rates. Hence the volatility in the productivity growth rates is mainly due to volatility in our output measures.

6. Sources of Error

There are many problems with the data constructed in this Appendix. Some of the more important possible sources of error are listed as follows:

- Our adjustments for converting final demand prices (those facing the final demanders of the goods and services produced by the business sector) into basic prices (prices facing the producers of the goods and services) were rather crude and some aggregation error will be associated with our procedures. In particular, only crude adjustments for the effects of indirect taxes on the components of consumption were made. Also our method for estimating the net supplies of the business sector to the nonbusiness sector are rather indirect and subject to some error.[117]
- Our tax adjustments for the price of imports and exports were also not completely satisfactory due to various aggregation errors; i.e., we were not able to assign taxes accurately to the various components of imports and exports.
- Our measure of labour input relies on the Statistics Canada KLEMS program estimates for quality adjusted labour and their may be some amount of error in these estimates. In particular, it is very difficult to account for the hours of work and labour compensation for the self employed.
- It proved to be difficult to reconcile balance sheet information with investment information. Our treatment of investment and capital services was highly aggregated and hence contains some aggregation errors. We also relied heavily on the Statistics Canada Balance Sheet estimates and these estimates are highly aggregated; in particular, there is not enough detail on the allocation of land. Moreover, the Balance Sheet stocks appear to give asset values that are too small.[118]

Diewert and Lawrence (2002) for a methodology for estimating the deadweight losses due to capital taxation.

[117] In particular, we did not have access to *chained* price indexes for the nonbusiness sector for the years prior to 1997 and this will lead to some aggregation errors.

[118] Evidence of this possible undercounting of asset values in the Balance Sheet accounts are the declining capital output ratios that are implied by our data. Moreover, the assessed value of real property (land and structures) in British Columbia for 2007 was just over one trillion dollars. If we add up the value of land and structures in the National Balance Sheets for the beginning of 2007, we get a value of about 4 trillion dollars. If we multiply the British Columbia value by a factor of 8, it seems that the national value of real property should be equal to about 8 trillion instead of the 4 trillion in the accounts.

- Our treatment of property taxes is very approximate.
- Our user costs of capital were constructed using a particular set of assumptions (no capital gains and endogenous real rates of return) and these assumptions are not universally accepted.
- The roles of infrastructure capital and R&D investments were not taken into account.
- The role of resource depletion was also not taken into account.

The next international version of the System of National Accounts will recognize capital services in the production accounts. This will be a big step forward since it will allow inputs in the SNA production accounts to be decomposed into price and quantity components and hence the revised SNA will facilitate the development of productivity accounts for each country that implements the revised SNA. However, just introducing capital services into the SNA will not be sufficient in order to develop accurate sectoral productivity accounts. The revised SNA also needs to consider the following problems:

- More attention needs to be given to the development of basic prices by industry and by commodity; i.e., we need accurate information on the exact location of indirect taxes (and commodity subsidies) by commodity and industry on both outputs and intermediate inputs.
- In order to deal adequately with the complications introduced by international trade, the existing Input Output production accounts need to be reworked so that the role of traded goods and services can be tracked by industry.
- The treatment of inventory change in the present SNA seems inadequate for the needs of productivity accounts. Inventory change should be integrated with the balance sheet accounts and the user cost accounts.
- The investment accounts need to be integrated with the corresponding balance sheet accounts, both in nominal and real terms.
- The treatment of land in the balance sheets requires additional work; i.e., there are problems in obtaining information on the quantity of land used by each industry and sector and valuing the land appropriately.[119]
- Difficult decisions must be made on the exact form of the user cost formula to be used when measuring capital services; i.e., the revised SNA should make specific recommendations on how user costs should be constructed so that some measure of international comparability can be achieved in the accounts.
- The problems involved in making imputations for the labour input of the self employed (and unpaid family workers) should also be addressed.

The introduction of capital services into the SNA will provide challenges for statistical agencies. However, as national statistical agencies make productivity accounts a part of their regular production of the national accounts, there will be benefits to the statistical system as a whole since a natural output of the new system of accounts will be balancing real rates of return by sector or industry. These balancing real rates of return will provide

[119] There are some difficult conceptual and practical problems involved in separating structure value from land value; see Diewert (2007) for a discussion of some of these problems.

a check on the accuracy of the sectoral data: if the rates are erratic or very large or very small, this can indicate measurement error in the sectoral data and hence will give the statistical agency an early indication of problems with the data.

Statistics Canada already has an extensive productivity program. It is to be hoped that as the program evolves in the future, the data will be presented to the public in some detail and hopefully, at some level of aggregation, revised series will be made available back to 1961.[120]

Appendix 3: Kohli's Treatment of the Gains from Trade

Ulrich Kohli, the chief economist for the Swiss National Bank, has long had an interest in adjusting income measures for changes in a country's terms of trade using production theory; see Kohli (1990) (2003) (2004a) (2004b) (2008) and Fox and Kohli (1998). His latest methodology is conveniently laid out in Kohli (2006) and this paper also has an application to Canada so it should be possible to compare his empirical results with the results presented here.

For our purposes, there are four main differences in Kohli's methodology for determining the welfare effects of changes in a country's terms of trade:

- Kohli's production sector is the entire economy whereas our production sector is just the business sector;
- Kohli uses final demand prices to value outputs whereas we use the prices that producers face; i.e., our methodology follows Jorgenson and Griliches (1972; 85) and adjusts prices for indirect tax wedges;
- Kohli divides the nominal income produced by his production sector by the price of domestic absorption (the price of C + G + I) in order to obtain his real income concept whereas we divide the nominal income produced by the market sector of the economy by the price of consumption (the price of C) in order to obtain our real income concept and finally
- Kohli's methodology requires information only on the prices and volumes (quantities) of the components of final demand whereas our methodology seemingly requires information on the prices and volumes of primary inputs as well, which is a strike against the use of our methodology, since information on the prices and quantities of primary inputs used by the economy is much harder to obtain than the comparable information on outputs produced by the economy.

The first methodological difference will only be important empirically if the nonbusiness sector grows faster or slower than the business sector and the second difference will only be important if the ratio of indirect taxes on products to GDP is changing. The second factor is not likely to be important in the case of Canada but the first factor is important,

[120] It is important to have data back to the early 1960's since the 1950's and 1960's were decades of very high productivity growth. Hence if we want to explain the productivity slowdown that took place in the 1970's, it is important to have comparable data for the 1960's.

particularly since our definition of the business sector totally excludes residential housing from outputs and inputs.

The effects of the third factor can readily be determined. Our basic methodology explained in Appendix 1 is not affected by the choice of deflator; i.e., instead of dividing by the price of consumption P_C, we can just as easily divide by our domestic price P_D (or any other price that we think is relevant for welfare evaluation purposes) and our basic production theory methodology is not affected. Below, we will divide by P_D instead of P_C and we will find that using our definition of the business sector, it does not make a lot of difference whether we divide by the price of domestic consumption or by the price of domestic absorption. However, the question raised by Kohli's methodology is: which deflator is the "right" one? We would argue that the price of consumption has a closer connection with welfare than the price of absorption and is easier to understand but we concede that "reasonable" economists might well opt for Kohli's alternative.[121]

The fourth factor is the most interesting one from a methodological point of view. At first glance, it would appear that Kohli's methodology for determining the effects of changes in the terms of trade on real income growth has a clear advantage over our methodology, since our methodology evidently requires price and quantity information on primary inputs, whereas his methodology requires information only on the prices and quantities of final demand components. However, this apparent methodological difference is illusory; we will show below that our methodology is actually equivalent to that of Kohli, except that he divides his nominal income by P_D (the price of C + G + I) whereas in section 2 of the main text, we divided our nominal income by P_C (the price of C).

We now rework our (gross) real income methodology, explained in sections 2 and 3 of the main text, but in this Appendix, we will substitute the price of business sector domestic value added $P_D{}^t$ for the price of consumption produced by the business sector $P_C{}^t$ as our deflator for the nominal income generated by the Canadian business sector. Referring back to section 2 in the main text, recall that the year t price of consumption was $P_C{}^t$, the price of domestic sales was $P_D{}^t$, the price of exports was $P_X{}^t$, the price of imports was $P_M{}^t$, the price of labour services was $P_L{}^t$, the price of capital services was $P_K{}^t$, the price of business sector value added was $P_Y{}^t$ and the price of business sector primary input was $P_Z{}^t$. The corresponding quantity aggregates were defined as $Q_C{}^t$, $Q_D{}^t$, $Q_X{}^t$, $Q_M{}^t$, $Q_L{}^t$, $Q_K{}^t$, $Q_Y{}^t$ and $Q_Z{}^t$ respectively. We use the same definitions here but in order to apply the translog methodology developed in Appendix 1, we need to define $P_Y{}^t$ as the Törnqvist price index for the components of business sector value added (D, X and −M) and $Q_Y{}^t$ as the corresponding implicit output quantity index. Using these definitions and the material in Appendix 1, the year t TFP growth for the business sector can be defined as (one plus) output growth divided by (one plus) input growth as follows:[122]

[121] Kohli (2006; 49) presents some additional arguments justifying his preference for the price of absorption as a deflator over other alternatives.
[122] It is traditional to use X rather than Z to denote an input aggregate but unfortunately, we have already used X to denote aggregate exports.

(1) $\tau^t \equiv [Q_Y^t/Q_Y^{t-1}]/[Q_Z^t/Q_Z^{t-1}]$;   t = 1962, 1963,..., 2007.

Year t nominal income generated by the business sector can still be defined either as $P_Y^tQ_Y^t$ (output side definition) or $P_Z^tQ_Z^t$ (input side definition). Recall that in section 3 of the main text, we divided year t nominal income by P_C^t, the year t price of consumption. Now we will follow Kohli's example and divide nominal income by the domestic price index P_D^t. Thus define the year t *Kohli type real income*, ρ^t, as follows:

(2) $\rho^t \equiv P_Y^tQ_Y^t/P_D^t$;   t = 1961, ..., 2007.

The formal model outlined in Appendix 1, based on the work of Diewert and Morrison (1986) and Kohli (1990), again allows us to decompose the growth of Kohli type real income from year t−1 to t, ρ^t/ρ^{t-1}, into multiplicative year to year contribution factors α_D^t, α_X^t, α_M^t, β_L^t, β_K^t and τ^t that describe the effects of changes in these six explanatory variables going from year t−1 to t. Thus the model outlined in Appendix 1 leads to the following equation which decomposes the year to year growth in Kohli type real income generated by the business sector, ρ^t/ρ^{t-1}, into a product of six year to year explanatory contribution factors:[123]

(3) $\rho^t/\rho^{t-1} = \tau^t\,\alpha_D^t\,\alpha_X^t\,\alpha_M^t\,\beta_L^t\,\beta_K^t$;   t = 1962, 1963,...,2007.

- It should be noted that the TFP growth factor τ^t which appears in (3) is equal to the same τ^t which appears in equation (1) above and hence can be estimated empirically if data on output and input prices and quantities are available.
- The contribution factors for TFP growth, labour input growth and capital input growth are exactly the same in both frameworks, except that Kohli deflates nominal values by P_D rather than P_C.[124]

The product of the real export and real import price contribution factors, α_{XM}^t, is defined in the usual way as follows:

(4) $\alpha_{XM}^t \equiv \alpha_X^t\,\alpha_M^t$.

As noted in section 3 of the main text, α_{XM}^t is our *terms of trade contribution factor*; it gives the contribution to real income growth of the combined effects of real changes in the international prices facing the Canadian business sector.

There is one additional task left to do in this Appendix and that is to reconcile Kohli's trading gains factor with our terms of trade contribution factor, α_{XM}^t defined above by (4).

Kohli (2006; 50) defines Gross Domestic Income in year t as nominal income in year t, $P_Y^tQ_Y^t$, divided by the nontraded goods price index for year t which is P_D^t using our

[123] See equations (42), (51) and (56) in Appendix 1 in order to derive this equation.
[124] It can be shown that this will always happen.

notation; i.e., he defines real income in year t, ρ^t, by (2) above, which we rewrite in a slightly different way as follows:

$$(5)\ \rho^t \equiv P_Y{}^t Q_Y{}^t / P_D{}^t \qquad t = 1961, ..., 2007$$
$$= [P_Y{}^t / P_D{}^t] Q_Y{}^t .$$

Thus real income decomposes into a price factor, $[P_Y{}^t/P_D{}^t]$, times real output, $Q_Y{}^t$. Kohli (2006; 50) defines this price factor as his *trading gains factor*; i.e., we have

$$(6)\ TGF^{\,t} \equiv P_Y{}^t / P_D{}^t ; \qquad t = 1961, ..., 2007.$$

We will show that the rate of growth of Kohli's trading gains factor is equal to our terms of trade contribution factor, which is already expressed as a rate of growth. We will require two additional results in order to do this. We have already noted that when we choose to define real income by deflating nominal income by the domestic price index $P_D{}^t$, then using the translog methodology explained in Appendix 1, we will find that the domestic price contribution factor, $\alpha_D{}^t$, will be identically unity; i.e., we have

$$(7)\ \alpha_D{}^t = 1 ; \qquad t = 1962, ..., 2007.$$

Again using the translog methodology explained in Appendix 1, it can be shown that the product of the labour and capital input growth factors is equal to the rate of growth of aggregate input; i.e., we have

$$(8)\ \beta_L{}^t\, \beta_K{}^t = Q_Z{}^t / Q_Z{}^{t-1} ; \qquad t = 1962, ..., 2007.$$

Now start with the translog identity given by (3):

$$(9)\ \rho^t/\rho^{t-1} = \alpha_D{}^t\, \alpha_X{}^t\, \alpha_M{}^t\, \beta_L{}^t\, \beta_K{}^t\, \tau^t ; \qquad t = 1962, ...,2007$$
$$= \alpha_X{}^t\, \alpha_M{}^t\, [Q_Z{}^t/Q_Z{}^{t-1}]\, \tau^t \qquad \text{using (7) and (8)}$$
$$= \alpha_X{}^t\, \alpha_M{}^t\, [Q_Z{}^t/Q_Z{}^{t-1}]\, [Q_Y{}^t/Q_Y{}^{t-1}]/[Q_Z{}^t/Q_Z{}^{t-1}] \qquad \text{using (1)}$$
$$= \alpha_X{}^t\, \alpha_M{}^t\, [Q_Y{}^t/Q_Y{}^{t-1}] \qquad \text{canceling terms}$$
$$= \alpha_{XM}{}^t\, [Q_Y{}^t/Q_Y{}^{t-1}] \qquad \text{using definition (4).}$$

Using (5), (6) and (9), it can be seen that we have the following equality between $\alpha_{XM}{}^t$ and the rate of growth of Kohli's Trading Gain Factor:

$$(10)\ TGF^{\,t}/TGF^{\,t-1} = \alpha_{XM}{}^t ; \qquad t = 1962, ...,2007.$$

Thus Kohli's basic methodology is equivalent to our methodology, provided that we divide nominal income by the relevant domestic deliveries price deflator, $P_D{}^t$, in place of the consumption price deflator, $P_C{}^t$. The equivalence of the two approaches should not be a big surprise since both approaches rely on the same translog decomposition analysis originally developed by Diewert and Morrison (1986) and independently by Kohli (1990).

A final implication of the analysis in this Appendix is that we can use the decomposition of real income growth given by (9) above to rewrite our basic decomposition of real income growth given by (3) in the following alternative form:

$$(11)\ \rho^t/\rho^{t-1} = \alpha_D{}^t\, \alpha_X{}^t\, \alpha_M{}^t\, [Q_Y{}^t/Q_Y{}^{t-1}].$$

Thus if we go back to deflating year t nominal income by the price of consumption, $P_C{}^t$, instead of by $P_D{}^t$, then the domestic real price change term $\alpha_D{}^t$ again makes its appearance in (11).

Appendix 4: The Statistics Canada KLEMS Estimates of Business Sector Multifactor Productivity Growth

1. Introduction

As was mentioned in the main text, the Statistics Canada KLEMS program has recently provided estimates of the multifactor productivity growth for the Canadian business sector; see Baldwin, Gu and Yan (2007) and Baldwin and Gu (2007) for a description of the methods used in this program. In section 2 of the main text, we explained that our level of business sector Total Factor Productivity using our user cost framework ended up at 1.553 in 2007 from its starting value of 1 in 1961 whereas the KLEMS Multifactor business sector productivity ended up at 1.184 in 2007.[125] In this Appendix, we will try to determine why our estimates are so different from the corresponding KLEMS program estimates.

Since our measure of business sector labour input is identical to the measure used by the KLEMS program, the differences between the two sets of results must be due to either differences in the outputs or in the capital services inputs. We will address each source of potential difference in turn.

2. Differences in the Output Concepts

As was mentioned in section 2 of the main text, our business sector output concept differs from the corresponding KLEMS concept in two ways:

- We exclude the services of both owned and rented residential housing from our output concept whereas the KLEMS program excludes only owned residential housing services and
- We measure real inventory change as a difference in real inventory stocks whereas the KLEMS program follows national income accounting conventions and measures inventory change in a rather different manner.

[125] See CANSIM II series V41712881, Canada, Multifactor Productivity, Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors.

We will now attempt to adjust our output measure to make it closer to the corresponding KLEMS measure. Before making any adjustments, our average rate of business sector real output growth over the years 1962-2007 was 3.78% per year compared to the corresponding KLEMS average rate of 3.85%[126] which is not a large difference.

From Table 1 in Appendix 2, we have listed series for the price and quantity of paid residential rents in Canada, P_{PR}^t and Q_{PR}^t respectively. From Tables 6 and 7 in the main text, we have listed the price and quantity of business sector output, P_Y^t and Q_Y^t respectively. We can construct an adjusted measure of Canadian business sector output that will be closer to the KLEMS measure by taking a chained Fisher index of these two series. The average rate of growth of the resulting quantity aggregate turned out to be 3.71% per year which compares to the KLEMS average growth rate of 3.85% per year. Thus our neglect of rents in our output aggregate does not explain the large differences in our estimated productivity growth rates as compared to the corresponding official KLEMS estimates.

Our treatment of inventory change is different from the KLEMS treatment of inventory change and is likely to lead to a slightly larger average output growth rate as compared to the KLEMS treatment but more research on this topic is required.

Adjusting our output aggregate for our different treatment of paid residential rents and inventory change appears to make very little overall difference on average to our rate of growth of business sector output: making these adjustments does not change our overall average growth rate very much and thus our average business sector output growth rate is comparable to the corresponding KLEMS average growth rate.[127]

3. Differences in Labour Input Concepts

Since we used the KLEMS program estimates for the price and quantity of the three types of labour that are available on CANSIM, there are no differences in our estimates of TFP growth for the Canadian business sector and the KLEMS Multifactor growth rates due to differing measures of labour input.[128]

4. Differences in Capital Input Concepts

The Statistics Canada KLEMS program has developed new estimates of capital services input over the period 1961-2006 for the Canadian business sector, Q_{KO}^t; see CANSIM II

[126] The source for the KLEMS real output series is CANSIM II series V41712932, Canada; Real Gross Domestic Product (GDP); Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors.

[127] The remaining differences between the average growth rates can be explained by (i) index number aggregation errors; (ii) the fact that our treatment of indirect tax wedges is only approximately correct and (iii) we have not adjusted our output measure for any intermediate inputs that may be used by the rental of residential housing industry.

[128] However, since our output measure excludes the provision of residential rental services, we should also exclude the labour input associated with these services. We did not do this because it is difficult to find a breakdown of the rental of structures industry into residential, commercial and industrial components.

series V41713051, Canada; Capital Input; Business Sector, in table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Using this series, we can compute the KLEMS average growth rate for capital *services* for the years 1962-2007 and this average growth rate turns out to be 5.04% per year. This rate can be compared to the average growth rate for our estimates for the quantity of capital *services* implied by the Q_K^t series listed in Table 2 of the main text, which turned out to be 3.37% per year. This is an enormous difference in average growth rates and explains why our estimates of business sector TFP growth are so much larger than the corresponding estimates from the KLEMS program. Our series of capital services (one plus) growth rates, Q_K^t/Q_K^{t-1}, are listed in Table 1 below along with the corresponding official KLEMS series, Q_{KO}^t/Q_{KO}^{t-1}.

The Statistics Canada KLEMS program has also developed new estimates of the stock of capital used by the Canadian business sector over the period 1961-2007, QK_O^t; see CANSIM II series V41713068, Canada; Capital Stock; Business Sector, in Table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Using this series, we can compute the KLEMS average growth rate for the business sector capital *stock* for the years 1962-2007 and this average growth rate turns out to be 3.29% per year. This rate can be compared to the average growth rate for our estimates for the capital *stock* used by our Canadian business sector which turned out to be 2.71% per year. Our capital stock aggregate for year t, QK^t (with corresponding stock price PK^t) is defined as a chained Fisher aggregate of:

- The smoothed data quantity series for ICT stocks QK_{ICT}^t, machinery and equipment stocks QK_{ME}^t and nonresidential structures QK_{NR}^t and
- The quantity series for the stock of inventories, QK_{BI}^t, the stock of business land, QK_{BL}^t, and the stock of agricultural land, QK_{AL}^t. These quantity series and the corresponding stock prices are listed in Tables 13 and 14 of Appendix 2.

Our series of aggregate capital stock (one plus) growth rates, QK^t/QK^{t-1}, is listed in Table 2 below along with the counterpart official KLEMS capital stock growth rate series, QK_O^t/QK_O^{t-1}.

Table 1: Unofficial and KLEMS Capital Services and Capital Stock Growth Rates

Year	Q_K^t/Q_K^{t-1}	Q_{KO}^t/Q_{KO}^{t-1}	QK^t/QK^{t-1}	QK_O^t/QK_O^{t-1}
1962	1.02570	1.03817	1.02312	1.02672
1963	1.03066	1.04412	1.03021	1.02974
1964	1.03523	1.07042	1.03575	1.05415
1965	1.04580	1.08553	1.03985	1.05822
1966	1.05915	1.10909	1.05340	1.06796
1967	1.07054	1.07104	1.06044	1.05758
1968	1.05154	1.05102	1.04042	1.04011
1969	1.03716	1.06311	1.03242	1.04683
1970	1.04229	1.05479	1.04046	1.04474

1971	1.03488	1.05195	1.02817	1.04030
1972	1.03144	1.06584	1.02345	1.04600
1973	1.03352	1.08108	1.02645	1.05093
1974	1.05405	1.07500	1.05074	1.06167
1975	1.06543	1.06977	1.06543	1.05187
1976	1.04741	1.06522	1.03281	1.05325
1977	1.04346	1.05248	1.03236	1.03933
1978	1.03949	1.05263	1.03448	1.04144
1979	1.03694	1.06316	1.03240	1.04152
1980	1.05134	1.07426	1.04936	1.04319
1981	1.04472	1.07143	1.03420	1.04459
1982	1.05868	1.02366	1.04337	1.01677
1983	1.01928	1.02521	1.00242	1.00750
1984	1.01659	1.03279	1.01029	1.01042
1985	1.02183	1.03968	1.02104	1.02062
1986	1.02578	1.03817	1.02281	1.02453
1987	1.02502	1.05331	1.01970	1.03239
1988	1.03382	1.05759	1.02650	1.04093
1989	1.04433	1.05611	1.03495	1.03932
1990	1.04226	1.03438	1.03122	1.02774
1991	1.02746	1.02115	1.01642	1.01472
1992	1.01367	1.01775	0.99890	1.00000
1993	1.00787	1.01599	1.00141	1.00484
1994	1.00175	1.03147	0.99732	1.00842
1995	1.01166	1.03745	1.00827	1.01074
1996	1.01748	1.03743	1.01774	1.01417
1997	1.01786	1.05670	1.01558	1.03376
1998	1.03668	1.05244	1.02710	1.03266
1999	1.03642	1.05794	1.02739	1.02726
2000	1.03493	1.04600	1.02448	1.03185
2001	1.03615	1.02723	1.02821	1.01646
2002	1.01893	1.01937	1.00987	1.01215
2003	1.01767	1.03300	1.01776	1.02200
2004	1.01431	1.04356	1.00769	1.02446
2005	1.02449	1.04917	1.01906	1.03152
2006	1.03204	1.05217	1.02637	1.03426
2007	1.03388	1.04874	1.02687	1.03402
Average	1.03370	1.05040	1.02710	1.03290

Looking at the entries in Table 1, it can be seen that there is little correspondence between the growth rates for our aggregate capital *services* series $Q_K{}^t/Q_K{}^{t-1}$ and the corresponding KLEMS aggregate capital *services* series $Q_{KO}{}^t/Q_{KO}{}^{t-1}$. There is a bit more correspondence between the growth rates for our aggregate capital *stock* series QK^t/QK^{t-1}

and the corresponding KLEMS aggregate capital *stock* series QK_O^t/QK_O^{t-1} but the series are still not close.[129]

It is possible to explain why the average growth rate of capital *services* should be bigger than the average growth rate of capital *stock* components: basically, the faster growing components of the capital stock (ICT and M&E) have bigger user costs relative to the slower growing components of the capital stock (agricultural land and nonagricultural land) compared to their stock prices. Thus aggregate capital services will tend to grow faster than the corresponding aggregate capital stock.[130] Using our estimates for beginning of the year capital stocks, one plus the average growth rate of ICT stocks is 1.1312 over the years 1962-2007, one plus the average growth rate of machinery and equipment stocks is 1.0379, one plus the average growth rate of nonresidential structure stocks is 1.0299, one plus the average growth rate of inventory stocks is 1.0309. one plus the average growth rate of agricultural land is 0.99930 and one plus the average growth rate of business nonagricultural, nonresidential land is 1.0000. Note that the only average growth rate that exceeds 5% is the average growth rate of the ICT stock at 13% per year. The average growth rate for machinery and equipment is just under 3.8% per year, followed by inventories at 3.1% per year and nonresidential structures at 3.2% per year, and the two land stocks are essentially constant. The average expenditure shares of these 6 capital inputs in total user costs over the years 1961-2007 are 0.05757 (ICT)[131], 0.29108 (M&E), 0.43722 (NR), 0.07858 (BI), 0.05068 (AL) and 0.08486 (BL). An approximation to the overall average year to year (one plus) growth of capital services can be obtained by multiplying each of the 6 component average (one plus) growth rates by the corresponding average services cost shares. When this computation is carried out, we obtain an average growth rate for capital services of 1.03405, which is very close to the average of our top down capital service growth rates, Q_K^t/Q_K^{t-1}, listed in the last row of column 2 of Table 1 above, which was 1.0337. Now carry out the same type of approximate calculation for capital stocks. The average growth rates for the components of the capital stock remain the same but now the shares of each asset in the total asset value of all capital stocks will change. The average shares of these 6 capital assets in the total asset value of capital used by our top down business sector over the years 1961-2007 are 0.02597 ICT), 0.18022 (M&E), 0.42918 (NR), 0.14697 (Business Inventories), 0.08089 (AL) and 0.13675 (BL). An approximation to the overall average year to year (one plus) growth of capital stocks can be obtained by multiplying each of the 6 component average (one plus) growth rates by the corresponding average stock shares. When this computation is carried out, we obtain an average growth rate for the aggregate business sector capital stock of 1.027546, which is very close to the average of our top down capital stock growth rates, QK^t/QK^{t-1}, listed in the last row of column 4 of Table 1 above, which was 1.0271.

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[129] Part of the difference between our lower average rate of growth for the business sector aggregate capital stock as compared to the corresponding KLEMS rate is due to our differing treatments of nonagricultural, nonresidential land input; see the discussion on this point in Appendix 2.
[130] This observation dates back to Jorgenson and Griliches (1967) at least.
[131] Note that the average ICT services cost share is only 5.76% which is not large enough to create a large difference between the rate of growth of capital services and the corresponding capital stocks.

We cannot carry out the same type of exercise for the consistency of the KLEMS capital stock and service flow growth rates because information on the 30 types of asset that the KLEMS program considers has not yet been released. However, it seems unlikely that the capital *services* average growth rate could be close to 5% per year and yet the corresponding aggregate *stocks* could grow at only 3.29% per year.[132]

Recall that the KLEMS business sector includes the services of residential land and structures that are rented whereas our business sector excludes these capital inputs. In what follows, we will make some rough estimates for these excluded capital services and add them to our other capital services in order to determine whether our omission of these residential housing capital services could materially affect our overall rate of growth of capital services.

Recall that in Appendix 2, we made estimates for the total stocks of residential land and residential structures that were used in the Canadian economy in both the Owner Occupied Housing (OOH) sector and the rental sector. Recall that using investment and balance sheet information, we found that an appropriate reconciling depreciation rate for residential structures in year t, δ_{RS}^t, was 0.04 or 4% per year. Using this depreciation rate, a starting value for the stock and the investment information on residential structures (see the series for Q_{IR}^t which is listed in Table 3 of Appendix 2), we formed estimates for beginning of the year t stock of residential structures, QK_{RS}^t, which are listed in Table 14 of Appendix 2. The corresponding year t prices PK_{RS}^t are listed in Table 13 of Appendix 2, which is also equal to the corresponding year t investment price for residential structures, P_{IR}^t, which was listed in Table 2 of Appendix 2. Recall also that in Appendix 2, we used National Balance Sheet information in order to obtain series for the price of residential land, PK_{RL}^t listed in Table 13, and for the corresponding quantity, QK_{RL}^t listed in Table 14 of Appendix 2. There is one more table to recall from Appendix 2 and that is Table 1, which listed the final demand value, quantity and price series for imputed residential rents, V_{IMR}^t, Q_{IMR}^t and P_{IMR}^t respectively, and for paid residential rents, V_{PR}^t, Q_{PR}^t and P_{PR}^t respectively.

The user cost for residential land in year t should be equal to a real rate of return r^t times the stock price of residential land, PK_{RL}^t, and the user cost for residential structures in year t should be equal to the same real rate of return r^t plus the depreciation rate δ_{RS}^t times the stock price of residential structures, PK_{RS}^t.[133] Multiplying these user costs by the corresponding stocks, QK_{RL}^t and QK_{RS}^t respectively, should generate values that are approximately equal to the value of imputed rents in year t, V_{IMR}^t, plus the value of paid residential rents, V_{PR}^t; i.e., the following equation should hold for each year t:

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132 The Statistics Canada KLEMS program uses balancing rates of return by industrial sector whereas our balancing rate of return is a business sector wide rate. It is possible that industrial sectors which are growing very rapidly have very large balancing rates of return and this effect could lead to a larger overall growth rate in the KLEMS capital services aggregate as compared to our aggregated over sectors procedure. This hypothesis could easily be checked if the KLEMS program released more detailed data on its user costs by industrial sector as well as the depreciation rates and balancing rates of return used in each sector.

133 We include property taxes in the rate of return r^t here as opposed to what was done in Appendix 2.

(1) $V_{IMR}^t + V_{PR}^t = r^t\, PK_{RL}^t QK_{RL}^t + (r^t + \delta_{RS}^t)\, PK_{RS}^t QK_{RS}^t$; t = 1961,...,2007.

For each year t, the above equation can be solved for a balancing real rate of return, r^t. The sample average for these real rates was 0.060027 (with a minimum of 0.04045 and a maximum of 0.07137), which is very reasonable considering the fact that r^t also has to include property taxes. Once the balancing real rates of return r^t have been determined, we can postulate that the value of imputed rents V_{IMR}^t is equal to a fraction f_1 of the aggregate value of land rents $r^t\, PK_{RL}^t QK_{RL}^t$ plus another fraction f_2 of the aggregate value of residential structures rents $(r^t + \delta_{RS}^t)\, PK_{RS}^t QK_{RS}^t$. Thus the value of paid rents V_{PR}^t should be equal to the fraction $1 - f_1$ of the aggregate value of land rents $r^t\, PK_{RL}^t QK_{RL}^t$ plus the fraction $1 - f_2$ of the aggregate value of residential structures rents $(r^t + \delta_{RS}^t) PK_{RS}^t QK_{RS}^t$. Thus the following two equations should hold, at least approximately:

(2) $V_{IMR}^t = f_1\, r^t\, PK_{RL}^t QK_{RL}^t + f_2\, (r^t + \delta_{RS}^t)\, PK_{RS}^t QK_{RS}^t$; t = 1961,...,2007;
(3) $V_{PR}^t = (1 - f_1)\, r^t\, PK_{RL}^t QK_{RL}^t + (1 - f_2)(r^t + \delta_{RS}^t)\, PK_{RS}^t QK_{RS}^t$; t = 1961,...,2007.

The parameters f_1 and f_2 in the above two equations were estimated using the Nonlinear option in Shazam; see White (2004). The estimated values for these parameters turned out to be $f_1 = 1.0535$ and $f_2 = 0.64347$ (the standard errors were 0.0217 and 0.00661). This tells us that the OOH sector contains about 64% of the residential structures and 105% of the residential land, which of course, is not sensible.[134] In what follows, we will assume that the OOH sector uses 64% of the stock of residential structures and 90% of the stock of residential land. In Table 2 below, we list the quantity of residential land that is used in the residential housing rental sector, QK_{RRL}^t (which is equal to 0.1 times the total quantity of residential land QK_{RL}^t), the quantity of residential structures used in the rental housing sector (which is equal to 0.36 times the total quantity of residential

[134] This result indicates the difficulties in forming accurate estimates for the amounts of land and structures used in the rental housing market and explains why we excluded rental housing from our business sector value aggregate. Baldwin, Gu and Yan (2007; 43) also reported difficulties in obtaining accurate information on the housing market: "As the output of the lessors of real estate includes the paid rents of rental residential buildings, capital input to the lessors of real estate industry needs to be adjusted to include investment in rental buildings. Data on investment in rental residential buildings are not available. For the annual MFP programs, we divide the total investment in residential building into rental building and owner-occupied dwelling using paid rents for rental buildings and imputed rents for owner occupied dwelling as the split ratios. The investment in residential buildings and paid and imputed rents are available from the Income and Expenditure Accounts. On average, we find that about 30% of total rents are paid rents and the remaining 70% are imputed rents." On the problems associated with obtaining estimates for residential rented land and land by sector in general, Baldwin, Gu and Yan (2007; 43-44) make the following observations: "In the past, the MFP programs assumed that there was little change in the real value of land in the business sector and estimated the real value of land at the industry level, based on the industry distribution of property taxes. We have now adopted the BLS methodology for estimating land stock in the MFP programs of Statistics Canada. The overall effect of adopting the BLS methodology on the business sector MFP growth is small. ... Data on the value of land at the industry level are scarce. We assume that land stock is proportional to the structures stock. The land–structure ratios are derived from the corporate balance sheets by sector which provide data on book values of land and structures by industry for the 1972-to-1987 period (CANSIM Table 180-0002). The real value of land at the industry level is estimated by deflating the nominal value of land using the structure capital's deflators. The final estimates of land stocks in both current and constant dollars at the industry level are benchmarked to the aggregate land stock in the total non-farm business sector."

structures QK_{RL}^t), the stock price of residential land PK_{RL}^t, the stock price of residential structures PK_{RS}^t, the user cost of residential rental housing land U_{RL}^t (which is equal to r^t PK_{RL}^t) and the user cost of residential rental housing structures U_{RS}^t (which is equal to (r^t + δ_{RS}^t) PK_{RS}^t).

Table 2: Rented Residential Housing Quantities, Prices and User Costs of Land and Structures

Year	QK_{RRL}^t	QK_{RRS}^t	PK_{RL}^t	PK_{RS}^t	U_{RL}^t	U_{RS}^t
1961	1068	10336	1.00000	1.00000	0.06470	0.10470
1962	1068	10718	1.06956	1.00504	0.07006	0.10603
1963	1068	11107	1.13138	1.02769	0.07714	0.11118
1964	1068	11510	1.17489	1.07312	0.07962	0.11565
1965	1068	12027	1.28108	1.13368	0.08214	0.11804
1966	1068	12563	1.40204	1.20765	0.08764	0.12380
1967	1068	13032	1.53236	1.28518	0.09715	0.13289
1968	1068	13503	1.65913	1.31431	0.11292	0.14202
1969	1068	14090	1.85677	1.38118	0.12793	0.15041
1970	1068	14805	2.07210	1.42615	0.14789	0.15883
1971	1068	15384	2.29387	1.53179	0.16020	0.16825
1972	1068	16111	2.62704	1.67349	0.16971	0.17505
1973	1068	16930	3.07471	1.97123	0.16714	0.18601
1974	1068	17826	3.74822	2.36134	0.16934	0.20114
1975	1068	18720	4.45312	2.56072	0.20480	0.22020
1981	1068	19551	5.00114	2.76853	0.24696	0.24745
1977	1068	20631	5.54193	2.87768	0.30497	0.27347
1978	1068	21693	6.22122	3.04069	0.35928	0.29723
1979	1068	22729	6.89491	3.28046	0.39398	0.31866
1980	1068	23711	7.82863	3.55455	0.44694	0.34511
1981	1068	24554	9.46319	3.99273	0.52521	0.38131
1982	1068	25472	11.12613	4.08226	0.66445	0.40708
1983	1068	26016	10.97760	4.25350	0.71859	0.44857
1984	1068	26804	11.89508	4.41785	0.78841	0.46953
1985	1068	27579	13.04615	4.55564	0.87292	0.48704
1986	1068	28484	13.36985	4.90827	0.88375	0.52077
1987	1068	29600	15.51626	5.40819	0.94727	0.54650
1988	1068	31005	18.33108	5.78293	1.06724	0.56800
1989	1068	32407	20.91090	6.13195	1.20781	0.59946
1990	1068	33861	24.58825	6.11231	1.45317	0.60573
1991	1068	34967	24.22111	6.32257	1.47289	0.63738
1992	1068	35665	26.88125	6.39710	1.66019	0.65097
1993	1068	36484	28.75011	6.58445	1.74599	0.66325
1994	1068	37193	30.94243	6.76485	1.88192	0.68203
1995	1068	37963	33.02674	6.76717	2.03709	0.68809
1996	1068	38367	32.17431	6.75581	2.07492	0.70591
1997	1068	38939	33.04791	6.87512	2.13258	0.71866

1998	1068	39660	35.07889	6.95993	2.25277	0.72536
1999	1068	40272	36.92678	7.13210	2.34190	0.73760
2000	1068	40937	39.81866	7.29782	2.48867	0.74803
2001	1068	41696	42.39774	7.48766	2.61509	0.76135
2002	1068	42679	47.03668	7.81242	2.79069	0.77601
2003	1068	43997	53.73129	8.21290	2.96812	0.78220
2004	1068	45424	59.30452	8.71618	3.05418	0.79753
2005	1068	47034	68.68515	9.11452	3.26873	0.79834
2006	1068	48697	78.15794	9.78750	3.40378	0.81775
2007	1068	50371	88.93074	10.49192	3.59725	0.84407

Using the above information on the user costs of rental land and structures, U_{RL}^t and U_{RS}^t, and their corresponding quantities, QK_{RRL}^t and QK_{RRS}^t, we formed chained Fisher indexes of these two price and quantity series along with our earlier price and quantity series for aggregate business sector capital services, P_K^t from Table 1 in the main text and Q_K^t from Table 2 in the main text. Denote the new aggregate capital services quantity index by Q_{KN}^t for year t. We then formed (one plus) the growth rates for this augmented capital services aggregate, Q_{KN}^t/Q_{KN}^{t-1}, for the years 1962-2007. We found that the sample average of these growth rates was 1.0337, which is exactly the same average growth rate that we obtained for our capital services aggregate that excluded residential rental housing from the business sector.[135] Thus it appears that the inclusion or exclusion of rental housing in the definition of the Canadian business sector could lead to large differences in the average rate of growth of capital services.

Recall that we defined our capital stock aggregate for year t as QK^t and the corresponding official KLEMS aggregate stock as QK_O^t and the rates of growth for these stock aggregates can be found in Table 1 above. Using the information on the stock prices of rental land and structures, PK_{RL}^t and PK_{RS}^t, and their corresponding quantities, QK_{RRL}^t and QK_{RRS}^t, which are listed in Table 2 above, we formed chained Fisher indexes of these two price and quantity series along with our earlier price and quantity series for aggregate business sector capital services, PK^t and QK^t, whose construction is described above Table 1 in this Appendix. Denote the new aggregate capital services quantity index by QK_N^t for year t. We then formed (one plus) the growth rates for this augmented capital stock aggregate, QK_N^t/QK_N^{t-1}, for the years 1962-2007. We found that the sample average of these growth rates was 1.0280, which is a bit higher than our old average growth rate for the aggregate capital stock used by our business sector, 1.0271.[136] Thus the addition of rented residential property to our old business sector capital stock increased the average growth rate from 2.71% per year to 2.80% per year, which brings us a bit closer to the KLEMS average growth rate for business sector capital stocks of 3.29% per year.

Our conclusion at this point is that the differences in coverage between our definition of the aggregate Canadian business sector cannot explain the differences in the average rate

[135] See the sample average at the bottom of column 2 in Table 1 in this Appendix.
[136] See the last row in column 4 of Table 2 in this Appendix.

of growth of capital services that we obtain for our gross capital model (3.37% per year) and the corresponding KLEMS average rate of growth (5.04% per year).

There is one additional experiment that we can undertake to try and explain our much smaller rate of growth of capital services: since our rental prices for capital do not have asset specific capital gains terms in them, inserting these terms into our user costs should increase the shares of machinery and equipment services in the capital services aggregate and hence lead to a higher average rate of growth of capital services. Thus changing our user cost formula to include ex post asset specific rates of price change should bring us closer to the user cost concept used in the Statistics Canada KLEMS program, which evidently includes some form of asset price appreciation terms in their user costs.[137] We will now explore how much difference adding ex post capital gains terms to the user costs will affect our capital services aggregate growth rate.[138]

We first define the ex post rates of price change for the six assets in our data base; i.e., recalling that PK_{ICT}^t, PK_{ME}^t, PK_{NR}^t, PK_{BI}^t, PK_{AL}^t and PK_{BL}^t are our year t asset prices for ICT capital, machinery and equipment, nonresidential structures, business inventories, agricultural land and nonagricultural, nonresidential business land respectively, the *ex post rates of price change* for these assets are defined as follows:[139]

(4) $\kappa_{ICT}^t \equiv (PK_{ICT}^{t+1}/PK_{ICT}^t) - 1$
(5) $\kappa_{ME}^t \equiv (PK_{ME}^{t+1}/PK_{ME}^t) - 1;$
(6) $\kappa_{NR}^t \equiv (PK_{NR}^{t+1}/PK_{NR}^t) - 1;$

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[137] It is not that easy to determine exactly how the KLEMS program user costs were constructed. Baldwin, Gu and Yan (2007; 24) describe the KLEMS capital service measures as follows. "The asset detail for capital services estimates in the MFP programs consists of 15 types of equipment, and 13 types of structures, and land and inventories for a total of 30 types of assets. The methodology for estimating capital services is documented in Baldwin and Gu (2007a) and Harchaoui and Tarkhani (2002). Here we mention two main features of capital services measures in Canada. First, the capital services measure for Statistics Canada's MFP programs is based on the bottom up approach. This bottom-up approach involves the estimation of capital stock by asset, the aggregation of capital stock of various asset types within each industry to estimate industry capital services, and the aggregation of capital services across industries to derive capital services in the business sector and in the aggregate industry sectors. Second, investment is benchmarked on the estimates of investment included in the input–output tables in order to ensure consistency between capital input measures and output measures. Recent studies by Statistics Canada provide new empirical evidence on the depreciation rate for various types of assets (Statistics Canada 2007). As a result, we have incorporated these new estimates of depreciation rates in the capital service estimates." However, this general introduction to the KLEMS capital services measurement program does not provide us with the details on the exact form of the user cost formula that was used except to refer the reader to Baldwin and Gu (2007). But this latter study contains many user cost variants and none of them appear to match up exactly with what actually appears in the most recent CANSIM tables on Multifactor productivity.

[138] For other studies that explore empirically the differences between various user cost formulae, see Harper, Berndt and Wood (1989), Diewert (2005a) and Baldwin and Gu (2007).

[139] For this model, we changed the definition of the asset prices so that they would better approximate beginning of the period prices; i.e., in this model, the asset prices were set equal to the arithmetic average of the old asset price and the previous period asset price. This averaging leads to slightly smoother user costs. The average rate of growth of the aggregate capital stock using these new stock prices is 2.75% per year which is little changed from the 2.71% average that we obtained using our earlier stock prices; see the last row of column 4 of Table 1 in this Appendix.

(7) $\kappa_{BI}{}^t \equiv (PK_{BI}{}^{t+1}/PK_{BI}{}^t) - 1$;
(8) $\kappa_{AL}{}^t \equiv (PK_{AL}{}^{t+1}/PK_{AL}{}^t) - 1$;
(9) $\kappa_{BL}{}^t \equiv (PK_{BL}{}^{t+1}/PK_{BL}{}^t) - 1$.

The above ex post asset specific rates of price change are listed in Table 4 below for the years 1961-2005. Jorgenson and his coworkers have long maintained that user costs of capital should include the above asset specific rates of price inflation in the formula as a negative contribution term.[140] Thus using this Jorgensonian methodological approach, the old user costs defined by equations (8)-(13) in Appendix 2 should be replaced by the following user costs for our six assets:[141]

(10) $U_{ICT}{}^t \equiv [R^t + \tau_B{}^t + \delta_{ICT}{}^t - (1-\delta_{ICT}{}^t)\kappa_{ICT}{}^t]\, PK_{ICT}{}^t$
(11) $U_{ME}{}^t \equiv [R^t + \tau_B{}^t + \delta_{ME}{}^t - (1-\delta_{ME}{}^t)\kappa_{ME}{}^t]\, PK_{ME}{}^t$;
(12) $U_{NR}{}^t \equiv [R^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t - (1-\delta_{NR}{}^t)\kappa_{NR}{}^t]\, PK_{NR}{}^t$;
(13) $U_{BI}{}^t \equiv [R^t + \tau_B{}^t - \kappa_{BI}{}^t]\, PK_{BI}{}^t$;
(14) $U_{AL}{}^t \equiv [R^t + \tau_B{}^t + \tau_P{}^t - \kappa_{AL}{}^t]\, PK_{AL}{}^t$;
(15) $U_{BL}{}^t \equiv [R^t + \tau_B{}^t + \tau_P{}^t - \kappa_{BL}{}^t]\, PK_{BL}{}^t$

Note that since the asset specific price change terms have been included in the above user costs, the rate of return R^t which appears in those user costs are no longer real rates of return but are *nominal rates* of return; i.e., they have the amount of general inflation which occurred during year t imbedded in them.

A balancing or endogenous nominal rate of return R^t (which includes business income taxes) for the Canadian business sector can be determined by solving the following counterpart to equation (14) in Appendix 2:

$$
\begin{aligned}
(16)\quad & P_C{}^tQ_C{}^t + P_{IG}{}^tQ_{IG}{}^t + P_{IR}{}^tQ_{IR}{}^t + P_{INR}{}^tQ_{INR}{}^t + P_{IME}{}^tQ_{IME}{}^t + P_{II}{}^tQ_{II}{}^t + P_{GN}{}^tQ_{GN}{}^t \\
& \quad + P_{XG}{}^tQ_{XG}{}^t + P_{XS}{}^tQ_{XS}{}^t + P_{MG}{}^tQ_{MG}{}^t + P_{MS}{}^tQ_{MS}{}^t \\
& = P_{L1}{}^tQ_{L1}{}^t + P_{L2}{}^tQ_{L2}{}^t + P_{L3}{}^tQ_{L3}{}^t \\
& \quad + [R^t + \tau_B{}^t + \delta_{ICT}{}^t - (1-\delta_{ICT}{}^t)\kappa_{ICT}{}^t]\, PK_{ICT}{}^t\, QK_{ICT}{}^t \\
& \quad + [R^t + \tau_B{}^t + \delta_{ME}{}^t - (1-\delta_{ME}{}^t)\kappa_{ME}{}^t]\, PK_{ME}{}^t\, QK_{ME}{}^t \\
& \quad + [R^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t - (1-\delta_{NR}{}^t)\kappa_{NR}{}^t]\, PK_{NR}{}^t\, QK_{NR}{}^t + [R^t + \tau_B{}^t - \kappa_{BI}{}^t]\, PK_{BI}{}^t\, QK_{BI}{}^t \\
& \quad + [R^t + \tau_B{}^t + \tau_P{}^t - \kappa_{AL}{}^t]\, PK_{AL}{}^t\, QK_{AL}{}^t + [R^t + \tau_B{}^t + \tau_P{}^t - \kappa_{BL}{}^t]\, PK_{BL}{}^t\, QK_{ICT}{}^t.
\end{aligned}
$$

[140] This choice of user cost formula with ex post asset price changes imbedded in the formula dates back to the pioneering work of Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969).

[141] These are end of the period ex post user costs. To derive this user cost, let P^t be the actual asset price at the beginning of period t, let P^{t+1} be the price of a comparable asset at the beginning of period t+1, let R^t be the one period nominal interest rate or opportunity cost of capital at the beginning of period t (including business income taxes), let δ^t be the applicable period t depreciation rate for the asset and define the ex post period t inflation rate for the asset by $\kappa^t \equiv (P^{t+1}/P^t) - 1$. Note that $P^{t+1} = (1+\kappa^t)P^t$. Following Diewert (1974a) (2005a), define the ex post beginning of the period user cost of the asset by $u^t \equiv P^t - (1+R^t)^{-1}P^{t+1} = P^t - (1+R^t)^{-1}\,(1-\delta t)(1+\kappa^t)P^t$. The end of the period ex post user cost U^t is defined as the beginning of the period user cost u^t multiplied by the discount factor $(1+R^t)$ so $U^t = (1+R^t)u^t = [(1+R^t) - (1-\delta^t)(1+\kappa^t)]P^t = [R^t + \delta^t - (1-\delta^t)\kappa^t]P^t$. We have neglected the property tax term in this derivation.

The gross nominal balancing rates of return R^t are reported in Table 3 below.

Table 3: Before Tax Nominal Balancing Rates of Return and Asset Specific Inflation Rates

Year	R^t	$\kappa_{ICT}{}^t$	$\kappa_{ME}{}^t$	$\kappa_{NR}{}^t$	$\kappa_{BI}{}^t$	$\kappa_{AL}{}^t$	$\kappa_{BL}{}^t$
1961	0.07690	-0.00030	0.00738	0.00296	0.00379	0.02000	0.03478
1962	0.09269	-0.00175	0.03275	0.01621	0.00498	0.04902	0.06855
1963	0.11344	0.00282	0.02545	0.02731	0.00672	0.08411	0.07234
1964	0.12855	0.01370	0.01687	0.04312	0.01453	0.11207	0.08306
1965	0.13798	0.01204	0.02525	0.06027	0.02025	0.11628	0.10875
1966	0.13737	0.02474	0.01166	0.05141	0.01947	0.12500	0.13405
1967	0.10522	0.03782	0.00765	0.02425	0.01393	0.11728	0.12223
1968	0.10625	0.03192	0.01943	0.03331	0.01734	0.06077	0.09477
1969	0.11422	0.03803	0.03917	0.05367	0.02297	0.01563	0.10194
1970	0.11397	0.03539	0.04171	0.05272	0.01816	0.01538	0.11694
1971	0.11316	0.03090	0.02963	0.05611	0.02964	0.05051	0.12515
1972	0.14423	0.03228	0.02841	0.08301	0.07000	0.14904	0.14591
1973	0.23465	0.03392	0.07920	0.14778	0.08901	0.26360	0.18689
1974	0.26054	0.05661	0.13490	0.14779	0.08414	0.28477	0.23559
1975	0.18032	0.03371	0.09702	0.08514	0.07542	0.22165	0.21803
1976	0.14869	-0.00481	0.07233	0.05486	0.06978	0.16667	0.15345
1977	0.1625	-0.01534	0.09570	0.06298	0.08054	0.16998	0.12529
1978	0.18648	-0.02139	0.10874	0.08267	0.09888	0.21329	0.12904
1979	0.22104	-0.06757	0.10842	0.10816	0.10205	0.24968	0.14501
1980	0.20631	-0.09793	0.10595	0.11508	0.09454	0.18451	0.16164
1981	0.16568	-0.02162	0.08820	0.09069	0.08516	0.05766	0.15795
1982	0.09349	-0.04640	0.04516	0.03192	0.06612	-0.01190	0.11311
1983	0.09351	-0.09346	0.03237	0.01524	0.05041	-0.03316	0.05209
1984	0.11212	-0.06575	0.04566	0.03527	0.03619	-0.05266	0.04171
1985	0.10711	-0.07662	0.04302	0.02336	0.01849	-0.06704	0.04950
1986	0.10028	-0.08109	0.01726	0.03059	0.01286	-0.06671	0.05742
1987	0.12869	-0.06815	-0.00239	0.05180	0.01639	-0.03923	0.07644
1988	0.13194	-0.07217	0.01297	0.04912	0.01678	0.02364	0.08850
1989	0.12085	-0.07347	0.02412	0.03731	0.01522	0.07354	0.08887
1990	0.08646	-0.08014	-0.00681	0.00778	0.00739	0.05301	0.07483
1991	0.05106	-0.08932	0.00152	-0.01121	0.02327	0.00340	0.04568
1992	0.07296	-0.04186	0.03981	0.00345	0.02339	-0.00730	0.02214
1993	0.08808	-0.02889	0.04600	0.02293	0.02567	0.02209	0.03300
1994	0.11465	-0.05503	0.04850	0.02302	0.04219	0.05828	0.05110
1995	0.11923	-0.07504	0.03714	0.02151	0.02899	0.07960	0.04417
1996	0.12218	-0.06235	0.02715	0.02743	-0.00624	0.08168	0.04076
1997	0.11813	-0.05253	0.03598	0.02584	-0.01452	0.07072	0.04543
1998	0.11011	-0.07241	0.02620	0.02331	0.01113	0.04905	0.04366
1999	0.11653	-0.06406	0.01163	0.02713	0.01950	0.03446	0.04642
2000	0.13835	-0.03230	0.02275	0.02103	0.02223	0.03209	0.05321
2001	0.12265	-0.02845	0.02309	0.01272	0.01903	0.02994	0.04834
2002	0.11815	-0.06688	-0.02117	0.01841	-0.02075	0.02510	0.04236

2003	0.11957	-0.09715	-0.03930	0.04182	-0.02483	0.02295	0.05336
2004	0.16173	-0.08556	-0.01559	0.06295	0.01106	0.02310	0.07381
2005	0.17294	-0.07239	-0.01473	0.06771	0.01412	0.02453	0.07740
2006	0.16466	-0.04987	-0.02252	0.06253	0.02380	0.02985	0.06792
2007	0.14509	-0.01376	-0.01376	0.02594	0.01539	0.01676	0.03190
Average	0.13151	-0.03174	0.03361	0.04592	0.03138	0.06814	0.08903

The average nominal ex post before tax rate of return earned by the Canadian business sector was 13.15% on average according to the above computations and the after tax average nominal rate of return was 9.50%. These rates of return seem to be too high, particularly in recent years. Again, this could be a reflection of depreciation rates that are too high, investment quantities that are too low or asset valuations for real property that are too low. The average ex post rate of price change for ICT equipment over the period was −3.17%, followed by business inventories at 3.14%, machinery and equipment at 3.36%, nonresidential structures at 4.59%, agricultural land at 6.81% and as might be expected, the rate of price inflation for nonagricultural, nonresidential business land was the highest at 8.90% per year. The high rates of price appreciation for the two land components of the business capital stock will lead to negative ex post user costs for these components for some years. These unadjusted user costs defined by (10)-(15) above are listed in Table 4 below.

Table 4: End of Period Ex Post User Costs for Six Business Sector Capital Stocks

Year	U_{ICT}^t	U_{ME}^t	U_{NR}^t	U_{BI}^t	U_{AL}^t	U_{BL}^t
1961	0.27714	0.22062	0.13435	0.07311	0.07218	0.05740
1962	0.29662	0.21714	0.13877	0.08804	0.06029	0.04096
1963	0.31581	0.25300	0.15220	0.10766	0.04800	0.06262
1964	0.32583	0.28406	0.15732	0.11580	0.03733	0.07255
1965	0.34391	0.29212	0.15758	0.12131	0.04849	0.05794
1966	0.33989	0.31237	0.17703	0.12394	0.04074	0.02740
1967	0.30651	0.28441	0.17912	0.09783	0.00596	-0.00204
1968	0.32724	0.27724	0.17546	0.09661	0.11162	0.05015
1969	0.34437	0.27335	0.16824	0.10087	0.22135	0.05747
1970	0.36276	0.28203	0.17889	0.10835	0.22442	0.02958
1971	0.38213	0.30639	0.18292	0.09616	0.15575	0.00980
1972	0.43477	0.35771	0.20210	0.08752	0.02242	0.03820
1973	0.56721	0.43193	0.26615	0.18481	-0.03420	0.19644
1974	0.60142	0.43718	0.35382	0.24376	-0.03176	0.14445
1975	0.55104	0.41889	0.36124	0.15715	-0.10961	-0.11374
1976	0.57018	0.44146	0.38751	0.12713	-0.02156	0.04877
1977	0.60278	0.46367	0.42678	0.14127	0.03520	0.33114
1978	0.63795	0.53687	0.46962	0.16314	-0.08423	0.51994
1979	0.72463	0.67773	0.53754	0.24351	-0.12481	0.73157
1980	0.68932	0.72057	0.53157	0.25209	0.33971	0.54252
1981	0.50997	0.72574	0.53328	0.19878	1.40356	0.22462
1982	0.44018	0.67997	0.51929	0.07333	1.45044	-0.08877
1983	0.46077	0.74757	0.60123	0.12308	1.69344	0.76565
1984	0.41821	0.79917	0.61211	0.22780	2.08370	1.23517

1985	0.39577	0.82828	0.66254	0.27548	2.07881	1.08941
1986	0.36449	0.91974	0.62358	0.27679	1.86586	0.90484
1987	0.35175	1.10395	0.68337	0.36012	1.75091	1.11888
1988	0.33410	1.06855	0.74864	0.37534	1.12742	1.04136
1989	0.30475	1.00877	0.7895	0.35004	0.57459	0.90578
1990	0.26525	1.00238	0.79019	0.26599	0.47837	0.54736
1991	0.22871	0.83702	0.7123	0.09417	0.65997	0.44817
1992	0.20311	0.80334	0.7515	0.17194	1.02040	1.60173
1993	0.19883	0.87770	0.74285	0.22151	0.86088	1.74678
1994	0.21656	1.02253	0.89864	0.26378	0.77325	2.02201
1995	0.21452	1.13592	0.95293	0.34235	0.62081	2.43227
1996	0.19688	1.23183	0.96393	0.50135	0.68200	2.72319
1997	0.18118	1.21592	0.97992	0.51462	0.83063	2.57365
1998	0.17488	1.26325	0.97837	0.37844	1.09024	2.50227
1999	0.16329	1.39043	1.02216	0.37510	1.46410	2.73197
2000	0.15353	1.47305	1.21290	0.45764	1.88419	3.34455
2001	0.14350	1.42867	1.19237	0.41746	1.71930	3.12413
2002	0.14709	1.63145	1.14942	0.57023	1.76930	3.31492
2003	0.14445	1.68356	1.04776	0.58051	1.87257	3.07552
2004	0.14001	1.73177	1.24282	0.59068	2.65120	4.12939
2005	0.12914	1.75858	1.37033	0.62953	2.88042	4.75128
2006	0.11508	1.72686	1.43864	0.56623	2.69025	5.14932
2007	0.10054	1.56672	1.64156	0.53379	2.63789	6.30792

It can be seen that there were 6 negative ex post user costs for agricultural land (for the years 1973-1976 and 1978-1979) and 3 negative ex post user costs for other business land (for the years 1967, 1975 and 1982). In the computations which follow, we simply set these negative user costs to 0.01.[142]

Using these adjusted Jorgensonian ex post user costs, we calculated a new capital services aggregate as a direct Törnqvist index of the six capital quantities weighted by

[142] Thus when we use these adjusted user costs, it will no longer be the case that the value of inputs equals the value of outputs for the 8 years where adjusted user costs are used Baldwin, Gu and Yan (2007; 25) explained how the KLEMS program dealt with negative user costs as follows: "The second empirical issue involves the way in which we have dealt with negative capital service prices during the estimation procedure. This arises from negative capital income in some periods in a few industries. Capital income is calculated from the input–output system as a residual, and is the difference between nominal value added and labour compensation of paid workers and self-employed workers. Negative capital income and negative capital service prices make aggregation difficult. More importantly, it is not clear that they are in keeping with the spirit of the estimation procedure for capital services. Enterprises are assumed to hire factors to bring the marginal product into equality with these prices. In the case of labour contracts, it is clear what the relevant price is for short-term decisions on hiring. But in the case of capital, the expected long-run capital cost is the relevant concept and short-run fluctuations in return are not likely to heavily influence expectations of long-run rates of returns. Therefore, to construct aggregate capital service input from asset-level capital stock and service prices, we have made adjustments for those assets whose user costs turn negative in the short run. We have set the user costs of the assets with negative user costs equal to the average user costs of the assets across all industries for those assets that are then adjusted for inter-industry differences in the user cost of capital." These adjustments for negative user costs appear to upset the old value of outputs equals value of inputs identity that the KLEMS program used and it is not clear from the above explanation how balance was restored. However, the effects of these adjustments are probably minor.

their new user costs As expected, the new capital services aggregate using ex post user costs grows more rapidly; the average growth rate over the 46 years 1962-2007 turned out to be 3.62% per year, which is a ten percent increase over our old average rate of 3.37% per year (recall the last row of column 2 of Table 1 above). The average productivity growth rate over these years using ex post user costs is 0.93% per year, which is a decrease over our initial average productivity growth rate of 1.01% per year (which used user costs without an asset specific capital gains term). Thus the use of ex post user costs does help to explain a portion of the difference in our estimates of TFP growth as compared to the official KLEMS estimates but there is still a large unexplained component. It is likely that moving from the six asset universe for which data are readily available to the 30 asset framework that is used by the KLEMS program would further narrow the gap between our capital and TFP growth rates and the corresponding KLEMS growth rates but it seems unlikely that aggregation errors in our computations can be the entire explanation for the huge differences in our results compared to the KLEMS results.[143]

It seems appropriate to raise the following issue at this point: is it better to *include* capital gains terms in the user cost formula or to *exclude* them as we have done in this study with the exception of the present Appendix? This is a rather deep question and deserves a lengthy discussion.[144] Suffice it to say here that the present authors favor the inclusion of smoothed or anticipated capital gains terms in the user cost formula[145] but that this question is not completely resolved in the existing user cost literature.

There is one additional factor which could help to explain the differences in our rates of capital services growth as compared to the corresponding Statistics Canada rates of growth: we have only one aggregate business sector in our model (due to data limitations) and hence only one aggregate rate of return for each year whereas the KLEMS program calculates balancing rates of return for each industry in the Statistics Canada list of industries in the input output tables:

"First, aggregate capital services in the business sector are constructed using the so-called 'bottom-up approach'. Baldwin and Gu (2007a) find that there is a large variation in the endogenous rate of return across industries and the endogenous rate of return is positively correlated with capital stock growth across industries. This suggests that the difference in the rate of return across industries is real, and capital tends to move toward those industries that earn relatively high rates of return." J.R. Baldwin, W. Gu and B. Yan (2007; 25).

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[143] Another possible explanation for the discrepancy between our rates of growth for capital services and the corresponding KLEMS estimates is the fact that our treatment of business taxes is an average approach as opposed to the KLEMS marginal approach, which is based on the treatment of tax distortions pioneered by Hall and Jorgenson (1967). Again, it seems unlikely that this factor alone could explain the differences.

[144] For preliminary discussions of this issue, see Diewert (1980; 475-476) (2005a; 492-502) (2006a) and Schreyer (2007).

[145] Given this preference, the reader may well ask: why was this approach not implemented in the present study? The practical problem is to decide *exactly* how to form anticipated asset specific inflation rates in an *objective* and *reproducible* manner. Thus to avoid controversy about the choice of the method for estimating anticipated capital gains terms, we decided to use real interest rates and omit anticipated capital gains terms from our user cost formulae in the main text.

Thus given that high rates of return are associated with rapid rates of capital accumulation, the disaggregated treatment of business sector industries used by the KLEMS program will lead to higher rates of growth of capital services as compared to our estimates.[146] However, in order for this aggregation effect to be substantial, there must be very large differences in the endogenous rates of return across sectors. But the Canadian economy is reasonably competitive and so large differences in sectoral rates of return that persist over time may reflect sectoral measurement errors rather than competitive advantages. It would be very useful for Statistics Canada to publish their sectoral endogenous rates of return so that the credibility of these rates could be established.[147]

One factor which could help to explain the difference between our estimates of Canadian TFP growth and those generated by the KLEMS program was suggested by Wulong Gu of Statistics Canada.[148] He noted that our starting estimates for the stock of nonresidential business structures (which are based on the Statistics Canada balance sheet information) are substantially larger than the estimates used by the KLEMS program and this difference could explain most of the difference in the two sets of estimates. This

[146] We are not able to estimate how significant this factor is due to the lack of published disaggregated data on capital stock and flow components at the industry level. However, Baldwin and Gu (2007; 47) address this issue in Table 11 of their paper. They provide alternative estimates of the average rate of growth of Multifactor productivity growth and for capital services growth for their Canadian business sector for two time periods and for two treatments for the balancing rates of return. The two treatments use either (i) industry specific rates or (ii) a total business sector single rate of return. For the 1961-1981 period, for treatment (i), they estimate average MFP growth at 0.59% per year and capital services growth at 6.19% per year and for treatment (ii), they estimate average MFP growth at 0.91% per year and capital services growth at 5.35% per year. For the 1981-2001 period, for treatment (i), they estimate average MFP growth at 0.12% per year and capital services growth at 3.55% per year and for treatment (ii), they estimate average MFP growth at 0.31% per year and capital services growth at 3.09% per year. Thus simply taking the arithmetic average of the above growth rates as a rough approximation to business sector performance over the period 1961-2001, using the KLEMS disaggregated balancing rate of return approach, MFP growth was about 0.355% per year which is in the neighborhood of the official KLEMS average MFP growth rate from CANSIM II series V41712881 for these years which was 0.49% per year. Taking the average of the treatment (i) capital services growth rates over the two periods gives us an average rate of growth of 4.87% per year, which is somewhat comparable to the official KLEMS average MFP growth rate from CANSIM II series V41713051 for these years which was 5.13% per year. Returning to Table 11 in Baldwin and Gu (2007) and their estimated growth rates using treatment (ii), we see that the average MFP growth rate over the period 1961-2001 was roughly 0.61% per year and the average rate of growth of capital services was 4.22% per year. Thus moving from the disaggregated balancing rates of return to the single business sector balancing rate of return increased their Table 11 KLEMS average rate of MFP growth from 0.355% to 0.61% (an increase of 0.255%) and decreased the Table 11 KLEMS average rate of growth of capital services from 4.87% to 4.22% (a decrease of 0.65% per year). Thus it appears that the KLEMS disaggregation of balancing rates of return could explain perhaps 0.25% of the difference in our much higher TFP growth rates compared to the corresponding KLEMS MFP growth rates.

[147] There are also some very tricky issues associated with aggregating sectoral TFP growth rates into economy wide TFP growth rates; i.e., should capital inputs in a particular sector be regarded as noncompeting inputs or should we look at the allocation of a type of capital across sectors? If we take the second point of view, then our top down approach seems to be appropriate. It would be useful for Statistics Canada to explain exactly how its sectoral productivity estimates were aggregated across sectors.

[148] He discussed a preliminary version of this paper at the 2009 Ottawa Productivity Workshop held at the Bank of Canada on May 6, 2009.

explanation seems very plausible but then the question arises: which set of estimates best reflects reality?

5. Recommendations for the Statistics Canada Productivity Program

There are substantial difficulties in accessing data on the prices and quantities of primary inputs used by the business and nonbusiness sectors from CANSIM. Also it is evident that the coverage of primary input usage by industry by Statistics Canada is not nearly as extensive as the corresponding coverage of gross outputs and intermediate inputs. With the next revision of the System of National Accounts recommending a decomposition of gross operating profits into price and quantity components, it seems time for Statistics Canada to devote more effort into improving measurement with respect to primary inputs used by industries in the Canadian economy. Without accurate information on the flow of labour and capital services by industry, governments and businesses will not be able to plan ahead for Canada's future. It is important to know past trends in TFP growth by industry so that future trends can be anticipated and so that budgetary planning can be carried out on a more rational basis. Hopefully, other national departments interested in Canadian productivity growth (the Bank of Canada, the Department of Finance and Industry Canada to name a few) will support an initiative that will put more resources into the hands of Statistics Canada so that they can provide better information on productivity growth.

Important priorities for improving Statistics Canada's productivity program include the following ones:

- The National Balance Sheet accounts need to be fully integrated with the productivity program; i.e., Statistics Canada collects information on 30 classes of assets with some degree of industry breakdown but publishes only a crude four types of asset by households, corporations and governments breakdown. The household sector needs to be split into a self employed business component and a "consumer of goods and services" component and the corporate sector should be decomposed into industries *with price and quantity information for the 30 classes of asset made available* by quarter and by industry.
- The National Balance Sheet information on the value of land, residential structures and nonresidential structures needs to be greatly expanded so that more information on the *price* and *quantity* of real property by industry is made available.[149] The problems associated with finding adequate constant quality price indexes for residential and nonresidential structures are formidable[150] but given the importance of real property in the Canadian economy, it is necessary to put additional resources into this area of economic measurement.

[149] We have some concerns that the National Balance Sheets are perhaps missing some growth in the value of real assets. Indirect evidence that points in this direction includes declining capital output ratios for the Canadian business sector and substantially increasing nominal and real rates of return earned by the business sector. Part of the problem may be the very high depreciation rates that are being used by the KLEMS program.

[150] For a review of these problems, see Diewert (2007a).

- The Statistics Canada methodology for estimating depreciation rates for reproducible capital stocks seems to be sound but the generated depreciation rates seem high by international standards. These relatively high depreciation rates generate relatively small capital stocks and relatively large endogenous rates of return to capital, which are trending upwards over time. This methodology needs to be carefully checked.
- The KLEMS program has developed very useful price and quantity information on 56 types of labour used by the Canadian business sector but has only made this information generally available in a highly aggregated form with the information on three types of labour service used in this study being made available on CANSIM II. Evidently, the KLEMS program has developed price and quantity information for 56 types of labour by industry for the business sector and it would be extremely useful for this information to be made available to the general community. If it is felt that the disaggregated information is not reliable enough to be released in this form, then it should be aggregated up and released at some level of detail that is more detailed than the present three price and quantity series that are available on CANSIM II. Furthermore, corresponding information on disaggregated labour input by type of worker should also be provided for the nonbusiness sector.[151]
- More information on the incidence of taxes needs to be provided in the input output accounts; i.e., we need to know exactly in which cell of the input output accounts various indirect and direct taxes are applied.[152] Not only is this information required to reconcile final demand indexes with production accounts indexes, it is also required in order to evaluate the efficiency of our tax system.[153]
- This study has shown that over short periods of time, changes in the real price of exports and imports can have substantial effects on living standards. The methodology used here applied only to the aggregate business sector. In Appendix 1, we showed how the methodology can be extended to the industry level but in order to implement this methodology to show the effects of changes in the terms of trade by industry, it will be necessary to expand existing input output tables to include information on exports produced and imports used by industry.[154] Government departments who have an interest in productivity measurement by industry will have to consider whether it would be worthwhile extending the production accounts in this direction. These extended accounts

[151] Statistics Canada has been a pioneer in developing and publishing very detailed information on the prices and quantities of outputs produced and intermediate inputs used by industry back to 1961 in its input output tables. What we are asking here is that these tables be extended to also cover the 56 types of labour input and 30 types of capital input that are being used in the Statistics Canada KLEMS program. Note that extending the input output tables to cover primary input allocations will also involve extensions to the corresponding final demand accounts, which in the case of inputs, will be corresponding household and government supplies of labour and capital.

[152] The reader will recall that in Appendix 2, we were forced to make guesses about the incidence of various consumption, import, property and capital taxes in order to reconcile final demand prices with producer prices. For additional material on how to accomplish this reconciliation, see Diewert (2006b) (2007b).

[153] See Diewert (2001; 97-98) for an elaboration of this point.

[154] Diewert (2007b) (2007c) explains these expanded production accounts in more detail.

would enable researchers to study issues related to outsourcing and globalization in a more scientific manner.

- Baldwin and Gu (2007; 15-22) have a nice discussion about many of the unresolved issues in constructing an appropriate user cost formula in order to price capital services and note that an unambiguous "best practice" measure has not yet emerged. Given this state of affairs, we recommend that Statistics Canada provide not only the actual user costs by asset and year that they used in the KLEMS program but that they provide supplementary information on the various ingredients (interest rates, property taxes, business taxes, asset price appreciation terms and asset prices) that go into the making of the user costs so that researchers can construct their own preferred versions of user costs. Eventually, a view will form on what the "best practice" user cost is but we are not at this point yet and hence it is essential that Statistics Canada provide analysts with information on the various components of user costs.

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