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Wealth Distributions and Power Laws : Evidence from "Rich Lists"

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Wealth distributions and power laws – evidence from “rich lists”

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Preliminary draft

Abstract

We use data from the “rich lists” provided by business magazines like Forbes for several entities (the whole world, the US, China and Russia) to verify if upper tails of wealth distributions follow, as often claimed, power-law behaviour. Using the empirical framework of [Clauset et al. \(2009\)](#) that allows for testing goodness of fit and comparing the power-law model with rival distributions, we found that only in less than one third of cases top wealth distributions are consistent with power-law model. Moreover, even if data are consistent with power-law model, usually they are also consistent with some other rival distribution.

Keywords: power-law model, wealth distribution, goodness of fit, Pareto model

1. Introduction

The search for universal regularities in income and wealth distributions has started over one hundred years ago with the famous work of [Pareto \(1897\)](#). His work suggested that the upper tails of income and wealth distributions follow a power law, which for a quantity x is defined as a probability distribution $p(x)$ proportional to $x^{-\alpha}$, with $\alpha > 0$ being a positive shape parameter known as the Pareto (or power-law) exponent. Pareto’s claim has been extensively tested empirically as well as studied theoretically ([Chakrabarti et al., 2006](#); [Chatterjee et al., 2005](#); [Yakovenko & Rosser Jr, 2009](#); [Yakovenko, 2009](#)). The emerging

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consensus in the empirical econophysics literature is that the bulk of income and wealth distributions seems to follow log-normal or gamma distribution, while the upper tail is best modelled with power-law distribution. Recent empirical studies found power-law behavior in the distribution of income in Australia ([Banerjee et al., 2006](#); [Clementi et al., 2006](#)), Germany ([Clementi & Gallegati, 2005a](#)), India ([Sinha, 2006](#)), Italy ([Clementi & Gallegati, 2005a,b](#); [Clementi et al., 2006](#)), Japan ([Aoyama et al., 2003](#); [Souma, 2001](#)), the UK ([Clementi & Gallegati, 2005a](#); [Drăgulescu & Yakovenko, 2001](#); [Richmond et al., 2006](#)), and the USA ([Clementi & Gallegati, 2005a](#); [Drăgulescu & Yakovenko, 2001](#); [Silva & Yakovenko, 2005](#)). Another group of studies discovered power-law structure of the upper tail of modern wealth distributions in China ([Ning & You-Gui, 2007](#)), France ([Levy, 1998](#)), India ([Jayadev, 2008](#); [Sinha, 2006](#)), Sweden ([Levy, 2003](#)), the UK ([Coelho et al., 2005](#); [Drăgulescu & Yakovenko, 2001](#); [Levy, 2003, 1998](#)), and the USA ([Klass et al., 2007](#); [Levy, 2003](#); [Levy & Solomon, 1997](#); [Levy, 1998](#)). Surprisingly, analogous result were obtained for wealth distribution of aristocratic families in medieval Hungary ([Hegyi et al., 2007](#)) and for the distribution of house areas in ancient Egypt ([Abul-Magd, 2002](#)).

However, as shown recently by [Clauset et al. \(2009\)](#) detecting power-law behaviour in empirical data may be a difficult task. Most of the existing empirical studies exploit the fact that the power-law distribution follows a straight line on a log-log plot with the power-law exponent equal to the absolute slope of the fitted line. The existence of power-law behaviour is often confirmed visually using such a plot, while the exponent is estimated using linear regression. Such approach suffers, however, from several drawbacks ([Clauset et al., 2009](#); [Goldstein et al., 2004](#)). First, the estimates of the slope of the regression line may be very biased. Second, the standard R^2 statistic for the fitted regression line cannot be treated as a reliable goodness of fit test for the power-law behaviour. Third, even if traditional methods succeed in verifying that power-law model is a good fit to a given data set, it is still possible that some alternative model fits the data better. A complete empirical analysis would therefore require conducting a statistical comparison of power-law model with some other candidate distributions.

Using a more refined methodology for measuring power-law behaviour, [Clauset et al.](#)

(2009) have shown recently, among other contributions, that the distribution of wealth among the richest Americans in 2003 as compiled in Forbes’ annual US “rich list” is not fitted well by a power-law model. The present paper tests if their conclusion applies more broadly. We use a large number of data sets on wealth distributions published annually by Forbes and other business magazines concerning wealth of 1) the richest persons in the world, 2) the richest Americans, 3) the richest Chinese, and 4) the richest Russians. The methodology of [Clauset et al. \(2009\)](#) is applied to verify if upper tails of wealth distributions really obey power-law model or if some alternative model fits the data better.

The paper is organized as follows. Section 2 presents statistical framework used for measuring and analyzing power-law behavior in empirical data introduced in [Clauset et al. \(2009\)](#). Section 3 shortly describes our data sets drawn from the lists of the richest persons published by Forbes and other sources, while Section 4 provides the empirical analysis. Section 5 concludes.

2. Statistical methods

In order to detect power-law behaviour in wealth distributions we use a toolbox proposed by [Clauset et al. \(2009\)](#). A density of continuous power-law model is given by

$$p(x) = \frac{\alpha - 1}{x_{min}} \left(\frac{x}{x_{min}} \right)^{-\alpha}. \quad (1)$$

The maximum likelihood estimator (MLE) of the power-law exponent, α , is

$$\hat{\alpha} = 1 + n \left[\sum_{i=1}^n \ln \frac{x_i}{x_{min}} \right], \quad (2)$$

where $x_i, i = 1, \dots, n$ are independent observations such that $x_i \geq x_{min}$. The lower bound on the power-law behaviour, x_{min} , will be estimated using the following procedure. For each $x_i \geq x_{min}$, we estimate the exponent using MLE and then we compute the well-known Kolmogorov-Smirnov (KS) statistic for the data and the fitted model. The estimate \hat{x}_{min} is then chosen as a value of x_i for which the KS statistic is the smallest.¹ The standard

¹The Kolmogorov-Smirnov statistic was also proposed by [Goldstein et al. \(2004\)](#) as a goodness of fit test for discrete power-law model assuming, however, that the lower bound on power-law behaviour is known.

errors for estimated parameters are computed with standard bootstrap methods with 10,000 replications.

The next step in measuring power laws involves testing goodness of fit. A positive result of such a test allows to conclude that the power-law model is consistent with a given data set. Following [Clauset et al. \(2009\)](#) again, we use a test based on semi-parametric bootstrap approach. The procedure starts with fitting a power-law model to data using MLE for α and KS-based estimator for x_{min} and calculating KS statistic for this fit, ks . Next, a large number of bootstrap data sets is generated that follow the originally fitted power-law model above the estimated x_{min} and have the same non-power-law distribution as the original data set below \hat{x}_{min} . Then, power-law models are fitted to each of the generated data sets using the same methods as for the original data set and the KS statistics are calculated. The fraction of data sets for which their own KS statistic is larger than ks is the p -value of the test. The power-law hypothesis is rejected if this p -value is smaller than some chosen threshold. Following [Clauset et al. \(2009\)](#), we rule out the power-law model if the estimated p -value for this test is smaller than 0.1. In our computations, we use 4,999 generated data sets.

If a goodness of fit test rejects power-law behaviour, we may conclude that a power-law has not been found. However, if a data set is well fit by a power law, the question remains if there is other alternative distribution, which is equally good or better fit to this data set. We need, therefore, to fit some rival distributions and compare which distribution gives a better fit. To this end, [Clauset et al. \(2009\)](#) use the likelihood ratio test proposed by [Vuong \(1989\)](#). The test computes the logarithm of the ratio of the likelihoods of the data under two competing distributions, R , which is negative or positive depending on which model fits data better. It also allows to test whether the observed value of R is statistically significant; for details, see [Vuong \(1989\)](#) and [Clauset et al. \(2009, Appendix C\)](#).

Each of the estimators and tests described above has been tested with good effects by [Clauset et al. \(2009\)](#) using Monte Carlo simulations.²

²The Stata software implementing all methods described in this section is available from the author upon

3. Wealth data from the “rich lists”

In several countries business magazines publish annual lists of the richest individuals. The oldest and the most famous one is the Forbes 400 Richest Americans list, which started in 1982. Other “rich lists” published by Forbes include the World’s Billionaires and the 400 Richest Chinese. These lists provide rankings of rich individuals according to their net worth defined as a sum of their assets minus their debts. We use annual data from the Forbes 400 Richest Americans list for the period 1988–2011, from the Forbes World’s Billionaires list for the period 1996–2012 and from the Forbes 400 Richest Chinese list for 2006–2011. In addition, we use 2004–2011 data from the list of top Russian billionaires published by the Russian magazine Finans (www.finansmag.ru). Descriptive statistics for our data sets are presented using beanplots ([Kampstra, 2008](#)) in Appendix A.

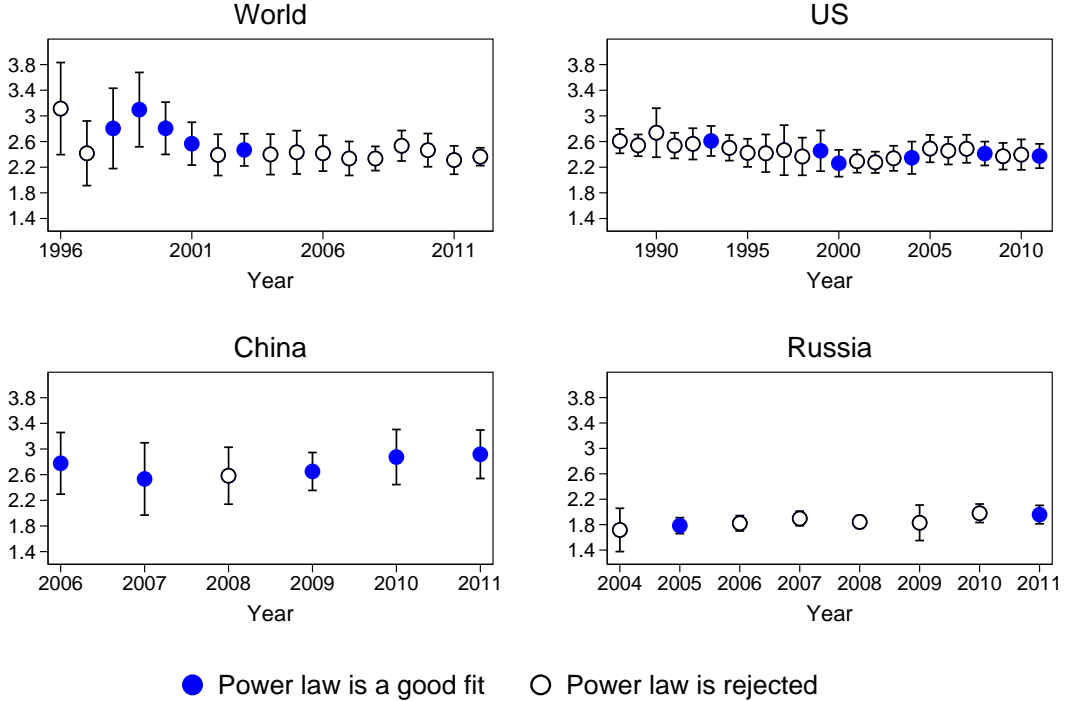
4. Results

Power-law fits to our data sets are shown in Figure 1. The values of the power-law exponent are rather stable over time for all four groups of data sets studied. However, except for Russia, the estimated exponents are substantially higher than usually found in the previous literature on power-law behaviour of wealth distributions.³ This result is a consequence of the fact that previous papers rarely attempted to estimate x_{min} and instead often fitted power-law models to all available observations. However, estimating x_{min} using KS-based approach as described in Section 2 leads to a substantially smaller range of observations that may follow power-law behaviour. For example, for the world richest persons data sets on average only 46% of observations are above \hat{x}_{min} . The most striking conclusion from Figure 1 is that for the three groups of our data sets (the world’s richest, the richest Americans, and the richest Russians) majority of data sets are not fitted

request. The original power-law-testing Matlab and R software written by Aaron Clauset and Cosma R. Shalizi can be obtained from <http://tuvalu.santafe.edu/~aaronc/powerlaws/>.

³[Richmond et al. \(2006\)](#) found that the estimated values of the power-law exponent range from 0.5 to 1.5 for the wealth distribution and from about 1.5 to 3 for income distribution.

Figure 1: Power-law exponents and goodness of fit tests for wealth data

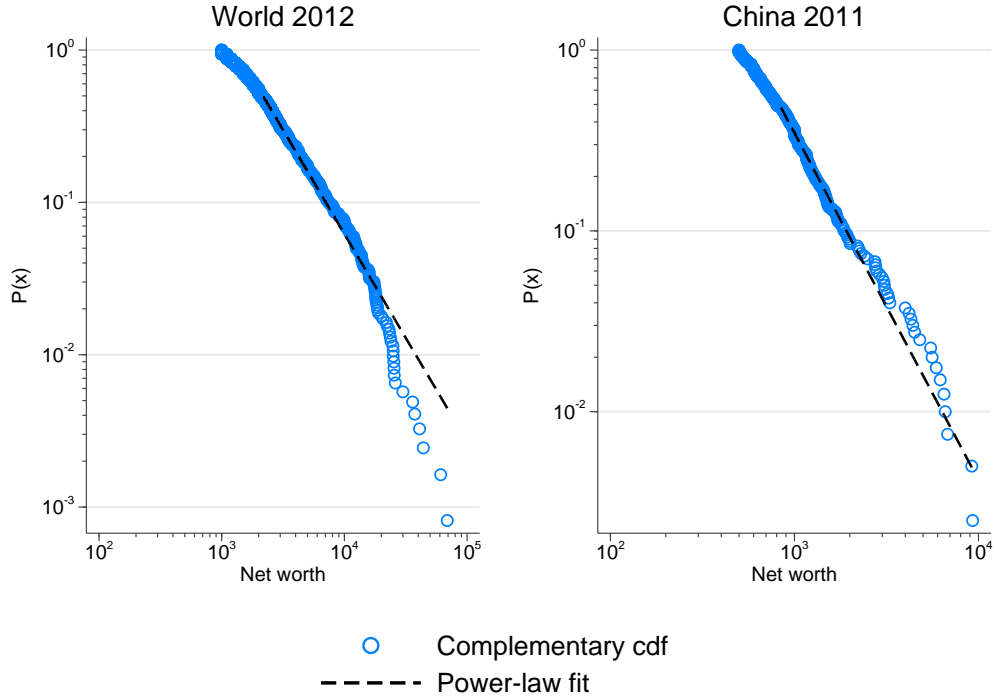


Note: vertical bars show 95% confidence intervals.

well by the power-law model according to the goodness of fit test used. For the US and Russia only one in four data sets seems to have a power-law behaviour, while for the richest persons in the world the number is 29%. Only for China in all but one cases wealth distribution seem to follow power-law model, but the period under study for this country is the shortest. This result suggests that, at least for the data sets drawn from the “rich lists”, wealth distributions often do not follow power-law model and that testing goodness of fit should always precede a declaration that power-law behaviour of wealth distribution was found. Figure 2 shows typical examples of our data sets for the case when power law is not a good fit (left panel, goodness of fit test p -value = 0.02) and for the case when it seems to be a good fit (right panel, p -value = 0.64).

The results of the likelihood ratio tests for all data sets that passed the goodness of fit

Figure 2: The complementary cumulative distribution functions and their power-law fits



test (p -value > 0.1) are given in Table 1. We have followed [Clauset et al. \(2009\)](#) in choosing the following alternative distributions: log-normal, exponential, stretched exponential and power-law with exponential cut-off.⁴ Positive (negative) values of LR mean that the power-law model gives a better (worse) fit to the data compared to a given alternative. If the p -value for the likelihood ratio test is small (e.g., $p < 0.1$), then we may reject the model which gives a worse fit to data. If the p -value is larger than the chosen level, then we are not able to choose between the models.

There are only two data sets (the world richest for 1999 and the richest Chinese for 2011) for which we may conclude that there is good evidence in favour of power-law model. For these data sets, the sign of the likelihood ratio test suggest that power-law model is a better fit over each alternative model.⁵ Three data sets follow rather power-law with exponential

⁴See [Clauset et al. \(2009\)](#) for definitions of these distributions.

⁵The pure power-law model is a subset of power-law with exponential cut-off model and for this reason the

Table 1: Power-law vs. other models for the upper tail of wealth distributions

Data set	Power law		Log-normal		Exponential		Stretched exponential		Power law with cut-off		Support for power law
	p	LR	p	LR	p	LR	p	LR	p		
<i>World</i>											
1998	0.981	0.026	0.979	2.043	0.041	-0.043	0.966	-0.068	0.713	moderate	
1999	0.977	0.483	0.629	1.447	0.148	0.146	0.884	0.000	1.000	good	
2000	0.824	-0.144	0.886	2.313	0.021	-0.158	0.874	-0.141	0.595	moderate	
2001	0.490	-0.544	0.586	2.587	0.010	-0.554	0.580	-0.777	0.212	moderate	
2003	0.154	-0.929	0.353	2.623	0.009	-0.955	0.340	-1.715	0.064	with cut-off	
<i>US</i>											
1993	0.297	-0.680	0.496	3.146	0.002	-0.694	0.488	-0.957	0.167	moderate	
1999	0.506	-0.116	0.908	2.325	0.020	-0.137	0.891	-0.141	0.595	moderate	
2000	0.189	-0.361	0.718	3.630	0.000	-0.361	0.718	-0.660	0.251	moderate	
2004	0.315	-0.431	0.666	2.429	0.015	-0.441	0.659	-0.637	0.259	moderate	
2008	0.268	-0.879	0.379	2.171	0.030	-0.902	0.367	-1.533	0.080	with cut-off	
2011	0.381	-0.307	0.759	4.615	0.000	-0.316	0.752	-0.721	0.230	moderate	
<i>China</i>											
2006	0.377	-0.817	0.414	1.561	0.119	-0.850	0.396	-1.090	0.140	moderate	
2007	0.244	-0.699	0.484	1.394	0.163	-0.705	0.481	-0.800	0.206	moderate	
2009	0.295	-0.940	0.347	2.883	0.004	-0.975	0.330	-1.658	0.069	with cut-off	
2010	0.168	-0.469	0.639	2.739	0.006	-0.479	0.632	-0.656	0.252	moderate	
2011	0.636	0.365	0.715	3.820	0.000	0.084	0.933	-0.119	0.626	good	
<i>Russia</i>											
2005	0.101	-1.596	0.110	3.200	0.001	-1.664	0.096	<i>nc</i>	-	moderate	
2011	0.661	-0.860	0.390	6.270	0.000	-0.886	0.375	<i>nc</i>	-	moderate	

Note: The first column gives p -value for the goodness of fit test for power-law behaviour. For each alternative distribution, a log-likelihood ratio (LR) and a resulting p -value is given. The last column presents the final judgement using the terminology of [Clauset et al. \(2009\)](#): “moderate” means that power law is a good fit but so are some alternatives, “with cut-off” means that the power law with exponential cut-off is favoured over the pure power law, and “good” means that the power law is favoured over all alternatives. “nc” denotes non-convergence of the ML estimator.

cut-off model than the pure power-law model, which means that the very highest wealth observations follow rather exponential than power-law behaviour. However, for each of these data sets log-normal and stretched exponential models are not ruled out as well. The remaining majority of data sets give only moderate support for the power-law behaviour in the sense that some alternatives are also plausible models for these data sets.

In overall, only two out of 55 data sets on wealth distribution analyzed in this paper may be reliably described as following a pure power-law model. In three cases, power-law with exponential cut-off seems to be preferred. In 13 cases, power law is not ruled out, but some other models are also plausible. Among the 37 data sets, which are rejected by the goodness of fit test, six seem to be better fitted by stretched exponential and 18 by power-law with cut-off (detailed results not shown for brevity).

These results suggest that the hypothesis that upper tails of wealth distributions, at least when measured using data from “rich lists”, follow a power-law behavior is statistically doubtful. It seems obvious that this hypothesis should no longer be assumed without empirical analysis of a given data using tools similar to those of [Clauset et al. \(2009\)](#). The existence of popular software implementing such empirical methods should make this task easier. The results of this paper seem also to cast some doubt on the theoretical literature in economics and econophysics that provides a theoretical structure for power-law behaviour of wealth distributions. Theoretical models that make room for some other distributions (especially power-law with exponential cut-off) describing top wealth values may be empirically well-founded.

5. Conclusions

In this paper we have used a large number of data sets on wealth distribution taken from the lists of the richest persons published annually by business magazines like Forbes. Using

former always provides a fit at least as good as the latter. The LR statistic for these models will therefore be negative or zero. P -values for the data sets with “good” support for power-law behaviour show, however, that power-law with exponential cut-off are not favoured over pure power-law.

recently developed empirical methodology for detecting power-law behaviour introduced by [Clauset et al. \(2009\)](#), we have found that top wealth distributions follow pure power-law behaviour only in less than one third of cases. Moreover, even if the data do not rule out the power-law model, usually the evidence in its favour is not conclusive – some rival distributions, most notably power law with exponential cut-off, are also plausible fits to data.

Acknowledgements

I would like to acknowledge gratefully the Matlab and R software written by Aaron Clauset and Cosma R. Shalizi, which implements methods described in Section 2. The software can be obtained from <http://tuvalu.santafe.edu/~aaronc/powerlaws/>. I also thank Moshe Levy for sharing data from Forbes 400 Richest Americans lists. All remaining errors are my own.

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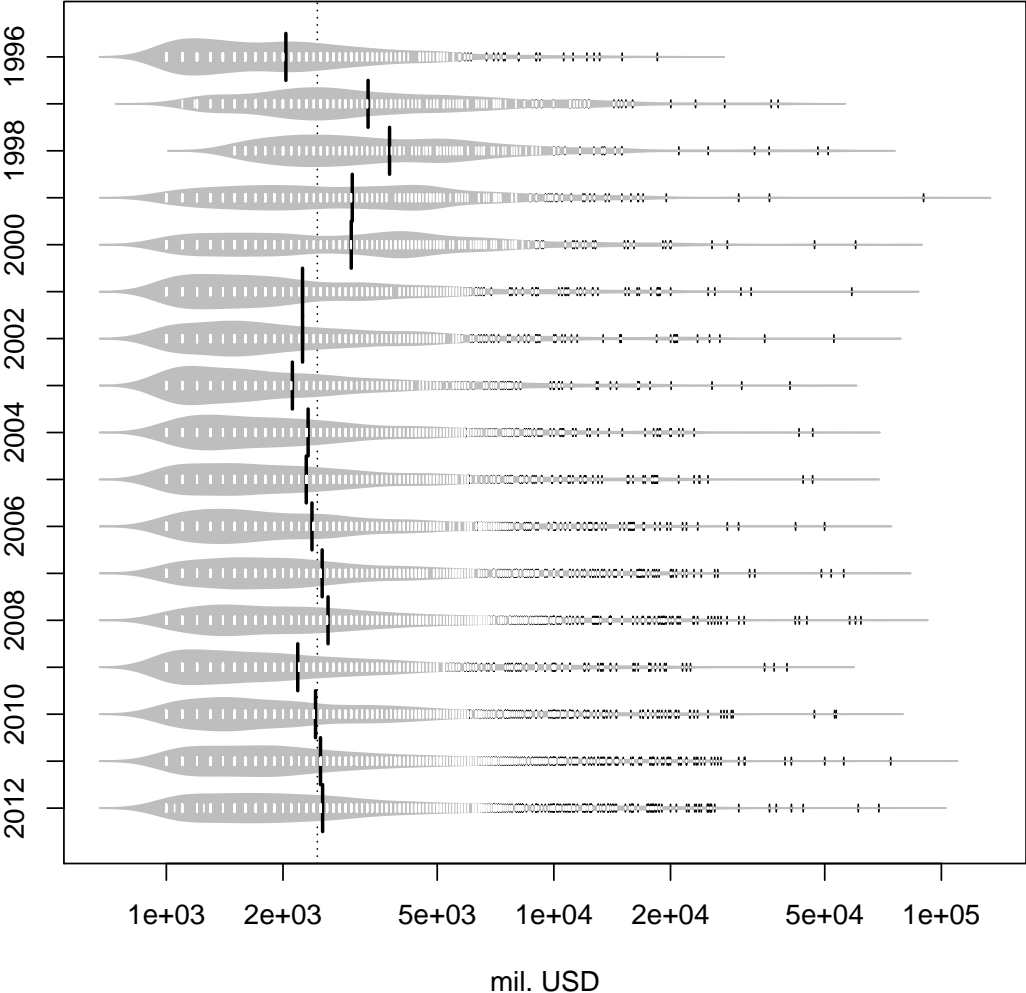
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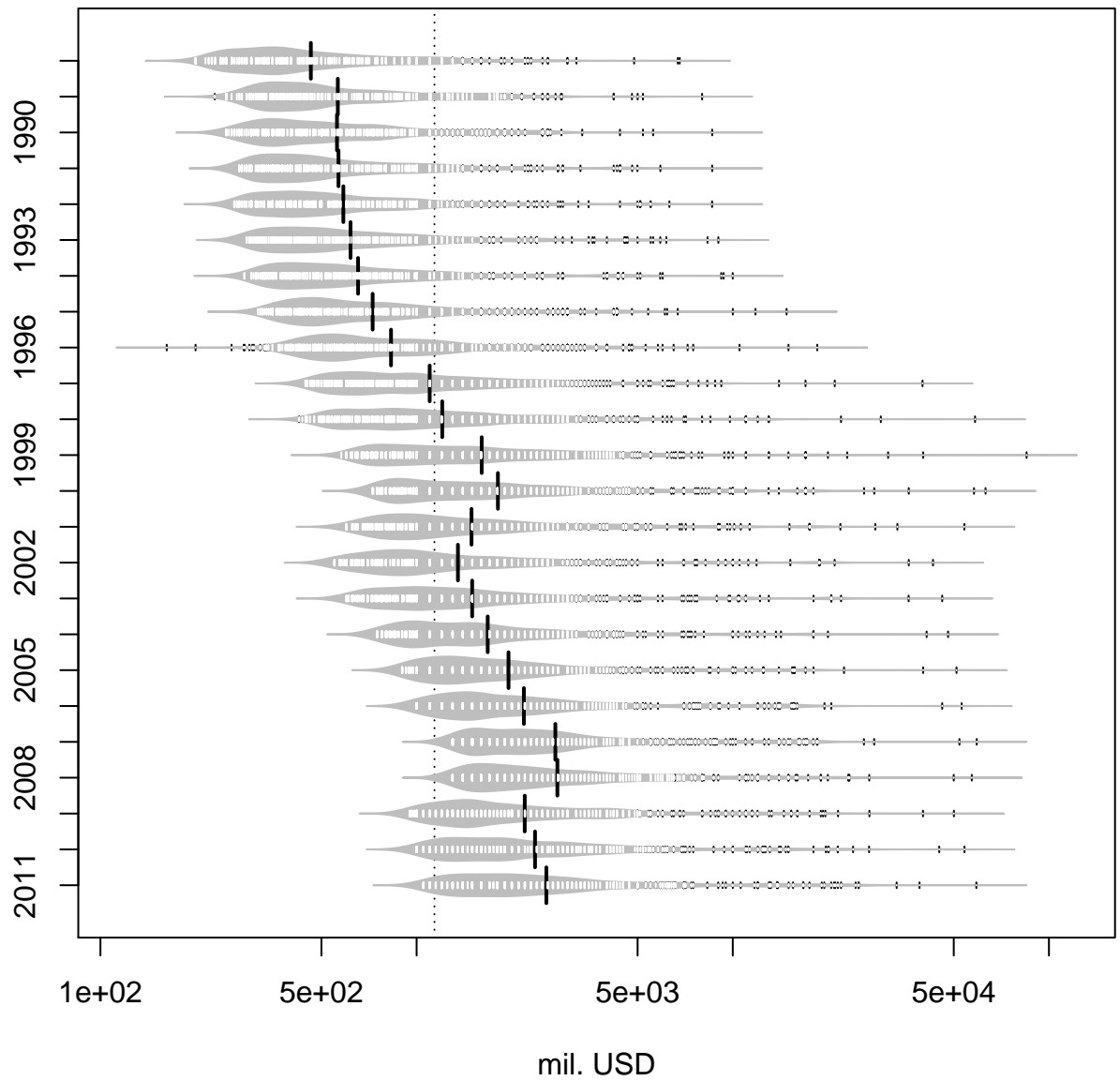
Appendix A. Descriptive statistics for data from “rich lists”

Figure A.3: Beanplots for wealth of world billionaires, Forbes data, 1996–2012



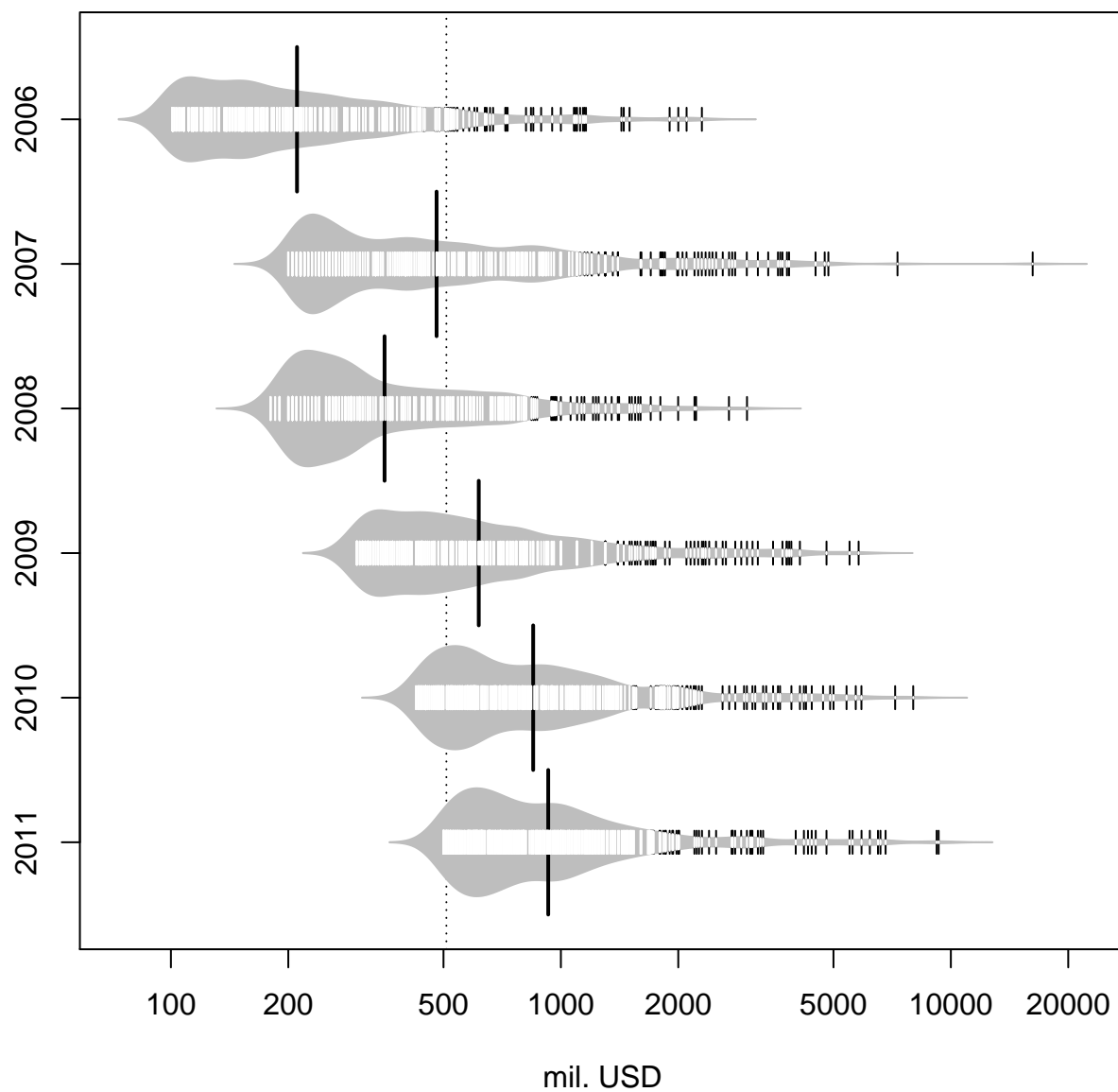
Note: The beanplot for a given year shows on a log scale individual wealth values as short vertical lines with the estimated density shown in gray. The vertical solid black lines show mean net worth for a given year, while the overall vertical dotted line shows the grand mean.

Figure A.4: Beanplots for wealth of the US billionaires, Forbes data, 1988–2011



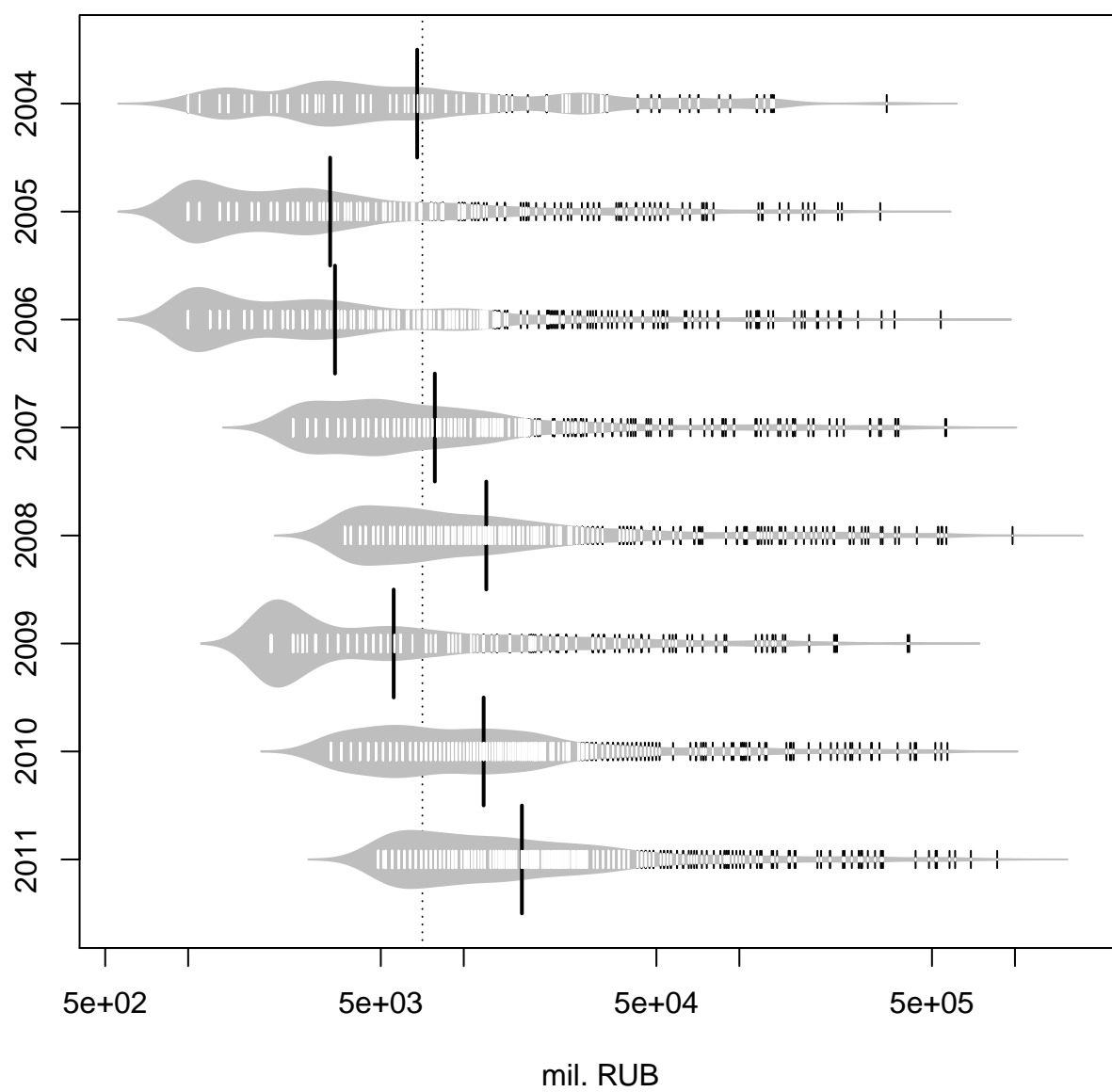
Note: see note to Fig. A.1.

Figure A.5: Beanplots for wealth of the richest Chinese, Forbes data, 2006–2011



Note: see note to Fig. A.1.

Figure A.6: Beanplots for wealth of the richest Russians, Finans magazine data, 2004–2011



Note: see note to Fig. A.1.