Poster Session #2
Time: Thursday, August 9, 2012 PM

Paper Prepared for the 32nd General Conference of
The International Association for Research in Income and Wealth

Boston, USA, August 5-11, 2012

The Empirical Scope of the Inequality Process:
The Statistical Signature of the Inequality Process on Income and Wealth

John Angle

For additional information please contact:

Name: John Angle
Affiliation: The Inequality Process Institute

Email Address: angle@inequalityprocess.org

The Empirical Scope of the Inequality Process: The Statistical Signature of the Inequality Process on Income and Wealth

by

John Angle
angle@inequalityprocess.org
The Inequality Process Institute
http://www.inequalityprocess.org
Post Office Box 215
Lafayette Hill, Pennsylvania 19444
USA

This paper is submitted to the International Association for Research on Income and Wealth (IARIW) in fulfillment of a requirement to make a presentation of findings to the 32nd IARIW General Conference in Boston, August 2012.

The Empirical Scope of the Inequality Process: The Statistical Signature of the Inequality Process on Income and Wealth

ABSTRACT

The Inequality Process (Angle, 1983-2012) is a mathematical model of personal wealth and income dynamics at both the micro level (the person) and the macro level (distributions and other statistics of a population). The Inequality Process (IP) is a stochastic interacting particle system, a class of mathematical model that is canonical in statistical physics, e.g., the particle system model of the Kinetic Theory of Gases (KTG). The IP was not the product of tinkering with the KTG; the relationship to the KTG's particle system was not established until seven years after the first IP was published (Angle, 1990). The IP was abstracted from the Surplus Theory of Social Stratification, as speculatively extended to societies over the continuum of techno-cultural evolution by Gerhard Lenski (1966). The Surplus Theory provides a simple explanation for how egalitarian hunter/gatherer society became the highly inegalitarian chiefdom: competition, already pervaded all human groups, and concentrated wealth in the form of a store of food as soon as the advent of agriculture created a surplus of storeable food. Lenski's speculative extension of the Surplus Theory addresses the question of why did the concentration of wealth of the chiefdom, an extremely inegalitarian society, decrease later as the advance of technology enabled the production of much greater wealth than that of the chiefdom. Lenski offered a number of speculative explanations. The one incorporated into the Inequality Process is that the creation of more wealth requires skilled workers, who control their human capital and can bargain for a larger share of the wealth they create, leaving themselves less exposed in the competition for wealth than their less skilled counterparts. The IP provides a parsimonious model of a wide and growing scope of stable, statistical patterns in well measured income and wealth data from the U.S., quantitatively tested and confirmed, and a number of historical statistical patterns, more loosely and qualitatively confirmed. The IP also passes tests of consistency with its verbal meta-theory. The present paper reviews the most readily explained ten empirical patterns out of fifteen empirical patterns in the IP's published explanandum.

The Empirical Scope of the Inequality Process: The Statistical Signature of the Inequality Process on Income and Wealth

by
John Angle

1. Introduction

The Inequality Process (IP) (Angle, 1983-2012) is a mathematical model of personal income and wealth dynamics at both the micro level (the person) and the macro level (the distribution and other population-level statistics). The statistical signature of the Inequality Process (IP) has been found on its explanandum, the scope of stable empirical patterns in income and wealth data the IP explains. See Table 1 for the IP's published explanandum. The present paper is an expository essay on the part of the IP's published explanandum that follows most immediately from a description of the IP, empirical patterns #1 through #4 in Table 1. Patterns #5 through 16 are not discussed here but there are references for all items in Table 1.

Table 1. The Published Empirical Explanandum of the Inequality Process.

1. The universal pairing (all times, all places, all cultures, all races) of the appearance of extreme social inequality (the chiefdom, society of the god-king) and concentration of wealth after egalitarian hunter/gatherers acquire a storeable food surplus (Angle, 1983, 1986).
2. The pattern of the Gini concentration ratio of personal wealth and income over the course of techno-cultural evolution beyond the chiefdom (Angle, 1983, 1986).
3. The right skew and gently tapering right tail of all distributions of income and wealth (a broad statement of the Pareto Law of income and wealth distribution) (Angle, 1983, 1986).
4. a)The sequence of shapes of the distribution of labor income by level of worker education, b) why this sequence of shapes changes little over decades, and c) why a gamma pdf model works well for fitting the distribution of labor income (Angle, 1990, 2002, 2003, 2006, 2007b);
5. How the unconditional distribution of personal income appears to be gamma distributed at the national level and in successively smaller regions although the gamma distribution is not closed under mixture, i.e., under aggregation by area (Angle, 1996);
6. Why the sequences of Gini concentration ratios of labor income by level of education from low to high recapitulates the sequence of Gini concentration ratios of labor income over the course of techno-cultural evolution (a social science analogue of "ontogeny repeats phylogeny" (Angle, 1983, 1986, 2002, 2003, 2006, 2007b);
7. Why the sequence of shapes of the distribution of labor income by level of education from low to high recapitulates the sequence of shapes of the distribution of labor income over the course techno-cultural evolution (a social science analogue of "ontogeny repeats phylogeny" (Angle, 1983, 1986, 2002, 2003, 2006a, 2006b, 2007b);
8. The old saw, "A rising tide lifts all boats.", expresses the view that most workers regardless of size of their earnings benefit from a business expansion. The IP implies "A rising tide lifts the logarithm of all boats equally.", if 'boat' is taken to mean the size of earnings (Angle, 2003a, 2005, 2007a).
9. The dynamics of the distribution of labor income conditioned on education as a function of the unconditional mean of labor income and the distribution of education in the labor force (Angle, 2002b, 2003a, 2005, 2006, 2007a);
10. The pattern of correlations of the relative frequency of an income smaller than the mean with relative frquencies of other income amounts (Angle, 2005, 2007a);
11. The surge in the relative frequency of large incomes in a business expansion (Angle, 2007a);
12. The "heaviness" of the far right tail of income and wealth distributions being heavy enough to account for total annual wage and salary income in the U.S. National Income and Product Accounts (Angle, 2002c, 2003a).
13. Why and how the distribution of labor income is different from the distribution of income from tangible assets; (Angle, 1997)

14. Why the IP's parameters estimated from certain statistics of the year to year labor incomes of individual workers are ordered as predicted by the IP's meta-theory and approximate estimates of the same parameters from the fit of the IP's stationary distribution to the distribution of wage income conditioned on education; (Angle, 2002a)
15. The Kuznets Curve in the Gini concentration ratio of labor income during the industrialization of an agrarian economy; (Angle, Nielsen, and Scalas, 2009)
16. In an elaboration of the basic IP: if a particle in a coalition of particles has a probability different from 50% of winning a competitive encounter with a particle not in the coalition, this modified IP reproduces features of the joint distribution of personal income to African-Americans and other Americans: a) the smaller median personal income of African-Americans than other Americans; b) the difference in shapes between the African-American distribution of personal income and that of other Americans; this difference corresponds to a larger Gini concentration of the African American distribution; c) the % minority effect on discrimination (the larger the minority, the more severe discrimination on a per capita basis, as reflected in a bigger difference between the median personal incomes of African-Americans and other Americans in areas with a larger % African-American); d) the high ratio of median African-American personal income to the median of other Americans in areas where the Gini concentration ratio of the personal income of other Americans is low; e) the high ratio of median African-American to that of other Americans in areas where the median income of other Americans is high; f) the fact that relationships in d) and e) can be reduced in magnitude by controlling for a measure of economic development of an area or % African-American; g) the greater hostility of poorer other Americans to African-Americans than wealthier other Americans (Angle, 1992). [John Hope Franklin, the late Duke University scholar of African-American history, included this Inequality Process paper on the joint distribution of personal income to African-Americans and other Americans in his collection of his papers now held by the Duke University library.].

The name Inequality Process (IP) in this paper refers to a mathematical model, not to a pervasive empirical process of competition for income and wealth in human populations that the model appears to describe. Reification of the model is avoided. Every application of the IP to explain a stable empirical phenomenon, a pattern in data, is a test of the IP against evidence, a test in which the IP could be disconfirmed. There are three kinds of such tests in the published findings in Table 1. One test is a sharp quantitative test of implications of the IP for empirical pattern(s) in a large public use microdata file published by the U.S. Bureau of the Census, items # 4, 5, 8, 9, 10, 11, 12, 13, 14, 16 in Table 1. A second kind of test of the IP is against broad characterizations of fact from a review of the literature, "stylized facts", items # 1, 2, 3, and 15, tests that are more qualitative. The third kind of test is a mixture of the first two, items #6 and 7 in Table 1. Each of these tests of the IP against data also tests the IP's consistency with its own meta-theory, the verbal social science theory from which it was derived.

2. The Specification of the Inequality Process from a Verbal Cornerstone of Economic Anthropology

The Inequality Process (IP) is a mathematical model specified from the Surplus Theory of Social Stratification, an old theory of economic anthropology that explains why the first appearance of great inequality of wealth in the archeological record of a population appears in the same layer as the first appearance of stores of food, usually due to the invention or adoption of agriculture (Herskovits, 1940; Childe, 1944; Harris, 1959; Dalton, 1960, 1963). This archeological layer corresponds to the transition of the population from being organized as an egalitarian hunter/gatherer group to organization as the extremely inegalitarian chiefdom, the society of the god-king. The IP is also specified from a speculative explanation on the part of Gerhard Lenski (1966) about why the concentration of wealth, extreme in the chiefdom, did not become even more extreme when the advance of technology permitted the creation of greater wealth in technologically more advanced societies. In fact, as Lenski notes the concentration of wealth decreased slowly with technological advance.

The older part of the IP's meta-theory is the Surplus Theory of Social Stratification of economic anthropology. In economic anthropology, the Surplus Theory is viewed as a cornerstone of the field. Marvin Harris (1959: 185), a prominent mid-20th century anthropologist, describes the status of the Surplus Theory in these terms: the "... surplus theory is so widely accepted among anthropologists that many regard it as an innocuous truism". The Surplus Theory runs: 1) people compete for food that is not immediately consumed and is storeable, "surplus wealth", of great utility to people vulnerable to famine, 2) competition generates winners and losers, concentrating the surplus in the hands of winners. The Surplus Theory is a parsimonious verbal theory accounting for the efflorescence of substantial inequality of wealth as soon as a supply of storeable food appears in hunter/gatherer society.

In its recognition of the pervasiveness and power of competition among people for resources of great utility, wealth, the Surplus Theory extends the perspective of population biology to humans. In population biology, members of every non-human species compete for resources (niche). This tenet is reflected in the density dependence term in the Bernoulli, Verhulst, and Lotka-Volterra equations of population increase and decrease (Braun, 1975; Murray, 1993). The concept of intra-species competition in population biology is so fundamental that inter-species competition or symbiosis is modeled via an increase or a decrease, respectively, of the density dependent terms (i.e., for intra-species competition) in the Lotka-Volterra equations of each species (Murray, 1993:78). May (1981) observes that a condition for stable populations in the Lotka-Volterra equations for species in competition is that the terms for intra-species competition in the equation for each species exceed the term for inter-species competition. The Lotka-Volterra equations are fundamental to population biology. The Inequality Process incorporates the hypothesis that there is a pervasive interpersonal competition for wealth in all human populations.

While the Surplus Theory provides a simple, plausible verbal explanation of the universal pairing in archeological strata of the first evidence of substantial institutionalized differences of power and wealth with the first evidence of an abundance of storeable food, the Surplus Theory has no explanation for the course of inequality of wealth in societies more advanced along the continuum of techno-cultural evolution than the chiefdom. A sociologist, Gerhard Lenski writing in **Power and Privilege** (1966) offers several speculative explanations to amend this shortcoming of the Surplus Theory. Lenski asks why, if an abundance of storeable food transforms the most egalitarian societal form into the most inegalitarian one, greater wealth than that produced in a chiefdom in techno-culturally more advanced societies, does not generate even greater inequality than that in the chiefdom. Lenski documents the gradual decrease of inequality in the sense of concentration over the continuum of techno-cultural evolution up through early industrial societies.

One of Lenski's speculative explanations for why the extreme inequality of the chiefdom decrease gradually over later techno-cultural evolution is that the production of more wealth per worker than in the chiefdom requires workers with more specialized skills and training in wealth production and because these workers control their own skills they have more leverage in bargaining for a share of the wealth they produce. The Inequality Process (IP) is a mathematical model of this one of Lenski's several speculative extensions of the Surplus Theory.

The specification of the IP as a mathematical model uses the principle of parsimony as much as possible, that is, the choice of the simplest model is compatible with verbal theory. The steps followed in the specification of the IP are discussed in Angle (1983, 1986, 2002, 2006). For example, except for the elaboration of the Inequality Process in item #15 in Table 1, the principle of parsimony makes all competition among people pairwise, because the Surplus Theory does not discuss the formation of organizations to extract surplus wealth from its producers. Another one of the simplifications of the IP's specification of its meta-theory is representing people as entities so simple, that they are not recognizable as homunculi(ae) economici(ae). While these entities interact competitively they do not make decisions, i.e., are not agents. They are properly called "particles" because they are so simple. The IP's particles have only two traits in the version of the IP presented in this paper. One trait is transient, the particle's wealth, which changes at every interaction with another particle, and the other permanent, the percent of the particle's wealth that it gives up to another particle when it loses a competitive encounter with that other particle, the operationalization of a worker's skill level in Lenski's extension of the Surplus Theory. In this one of several conjectures Lenski made about the trajectory of the concentration of wealth over techno-cultural evolution, the more skilled lose a smaller proportion of wealth in the competition for wealth, making them robust losers. Since winning and losing in the IP is 50/50, in the IP in the long term wealth is transferred to the robust losers. Vince Lombardi's description of American football, "Winning isn't everything; it's the only thing." does not work in the Inequality Process in the long term.

3.0 The Inequality Process Jumps a Disciplinary Boundary into Econophysics

The Inequality Process (IP) is a stochastic interacting particle system, a class of mathematical model that is canonical in statistical mechanics. The first and best known model in this class of model is that of the kinetic theory of gases (KTG), the mechanical basis of gas thermodynamics (Whitney, 1990). The kinetic theory of gases (KTG) may be called the "ideal gas model" in high school physics. The Inequality Process (IP) was not derived from the particle system model of the kinetic theory of gases. Although the first IP paper appeared in 1983, it was not until 1990 that Angle (1990) pointed out the similarities between the IP and the particle system model of the kinetic theory of gases. The IP is isomorphic to that particle system up to two differences.

The IP's potential as scientific law has been enthusiastically recognized in the econophysics literature. A model similar to the IP was published in the econophysics literature in 2000 (Chakraborti and Chakrabarti, 2000). The priority of the IP (the first publication of which is Angle, 1983) was recognized by econophysicists beginning in 2005, when Thomas Lux (2005) informed an international meeting of econophysicists called to celebrate five years of progress on a particle system, produced by tinkering with the KTG and that incorporates one of the two differences of the IP from the KTG (Chakraborti and Chakrabarti, 2000) but is otherwise isomorphic to the Inequality Process, that the five years of progress on that variant of the IP had been anticipated by the IP and its literature. The latest IP paper that Lux (2005) cites was published in 1996. Econophysics is the extension of the models of statistical mechanics in physics to economic and social phenomena. As with the traditional subjects of statistical mechanics the goal of econophysics is the discovery of statistical laws. Table 2 lists papers and books in the econophysics literature that cite the IP.

Econophysicist interest in the Inequality Process (IP) is due in part to its resemblance to the particle system model of the kinetic theory of gases (KTG) and the Chakarborti and Chakrabarti (2000) particle system. So adopting the IP and its empirical confirmations is paradigmatically easy for physicists. But physicists are attracted to the IP by more than paradigmatic familiarity. The IP is parsimonious. Physicists recognize parsimony as a very important characteristic of a model. Parsimony in a mathematical model means that it is a) inflexible, i.e., has a functional form readily falsified by failure to fit data, b) has few parameters to be estimated, c) has precise, testable implications, and d) has wide empirical scope. The IP explains many stable statistical patterns in income and wealth, patterns never before adequately accounted for individually in verbal social science, let alone as joint implications of a simple mathematical model. Physicists also notice that progress on the IP has been made as ordinary progress in science is made:

meta-theory assigning meanings to variables -> abstraction as parsimonious mathematical model -> derivation of hypothesis -> test of hypothesis against data -> empirical confirmation of hypothesis -> derivation of different hypothesis -> test against data -> empirical confirmation -> and so on, widening the scope of empirical phenomena explained by the model -> inductive establishment of model as scientific law

Table 2 Papers in the Econophysics Literature on the Inequality Process (IP) or that Cite the IP

Chen, Shu-Heng and Sai-Ping Li.	2011.	"Econophysics: Bridges over a Turbulent Current"	***International Review of Financial Analysis*** (July, 2011): doi:10.1016/j.irfa.2011.07.001
Kuerten, K.E. and F.V. Kusmartsev.	2011	"Bose-Einstein distribution of money in a free-market economy II".	***Europhysics Letters*** 93, #2, 28003. DOI: 10.1209/0295-5075/93*28003*
Matsuo, M.Y.	2011	"Hierarchical mechanism of development of wealth and structure for a pre-modern local society".	***Physical Review E*** 83, 066110. http://pre.aps.org/abstract/PRE/v83/16/e066110
Sinha, Sitabhra, Arnab Chatterjee, Anirban Chakraborti, and Bikas Chakrabarti	2011	***Econophysics: An Introduction***.	Weinheim, Germany: Wiley-VCH.
Bassetti, F. and G. Toscani.	2010	"Explicit equilibria in a kinetic energy model of gambling".	***Physical Review E*** 83, 066115.
Chakrabarti, Anindya and Bikas K. Chakrabarti	2010	"Statistical theories of income and wealth distribution".	***Economics E-journal*** v. 4. [http://www.economics-ejournal.org/economics/journalarticles/2010-4.
Sokolov, Andrey, Andrew Melatos, and Tien Kieu.	2010	"Laplace transform analysis of a multiplicative asset transfer model".	***Physica A***
Angle, John, François Nielsen, and Enrico Scalas.	2009	"The Kuznets Curve and the Inequality Process". Pp. 125-138. In Banasri Basu, Bikas K. Chakrabarti, Satya R. Chakravarty, Kausik Gangopadhyay, editors, ***Econophysics and Economics of Games, Social Choices and Quantitative Techniques***.	(Proceedings of the Econophys-Kolkata IV Conference, March 2009, Kolkata, India, jointly sponsored by the Indian Statistical Institute and the Saha Institute of Nuclear Physics.) [http://www.isical.ac.in/cmp/econophys4.com/]. Milan: Springer.
Chakrabarti, Anindya and Bikas Chakrabarti	2009	"Microeconomics of the ideal gas like market models"	**Physica A** vol. 388 (#19) pp. 4151-4158. doi:10.1016/j.physa.2009.06.038 . [on-line at http://arxiv.org/abs/0905.3972].
Lopez-Ruiz, Ricardo, Jaime Sanudo, and Xavier Calbet	2009	"Equiprobability, entropy, gamma distributions, and other geometrical questions in multi-agent systems"	**Entropy** 11, 959-971; doi:10.3390/e11040959 [on-line at: http://www.mdpi.com/1099-4300/11/4/959/pdf]
Yakovenko, Victor and J. Barkley Rosser Jr.	2009	"Colloquium: Statistical Mechanics of Money, Wealth, and Income" Reviewing the history of particle system models of income distribution, Yakovenko and Rosser write (p. 3, pagination of on-line version): "Actually, this approach was pioneered by the sociologist John Angle (1986, 1992, 1993, 1996, 2002) already in the 1980s. However, his work was largely unknown until it was brought to the attention of econophysicists by the economist Thomas Lux (2005). Now, Angle's work is widely cited in econophysics literature (Angle, 2006). Meanwhile, the physicists Ispolatov, Krapivsky, and Redner (1998) independently introduced a statistical model of pairwise money transfer between economic agents, which is equivalent to the model of Angle."	***Reviews of Modern Physics*** 81, 1703-1725. [on-line at http://arxiv.org/abs/0905.1518].
Düring, Bertram, Daniel Matthes, Giuseppe Toscani	2008	"A Boltzmann-type approach to the formation of wealth distribution curves"	Institute for Analysis and Scientific Computing, Vienna University Technology.
Bassitti, Frederico, Lucia Ladelli, and Daniel Matthes	2008	"Central Limit Theorem for a Class of One-dimensional Equations"	Institute for Analysis and Scientific Computing, Vienna University of Technology. Report #37 ISBN 978-3-902627-01-8.
Chakraborti, Anirban, and Marco Patriarca.	2008	"Gamma-distribution and wealth inequality"	**Pramana** [Indian Academy of Sciences]. 71(#2): 233-243.
Hayes, Brian	2008	**Group Theory in the Bedroom and Other Mathematical Diversions**. New York: Hill and	New York: Hill and Wang (division of Farrar, Straus, and Giroux).

		Wang (division of Farrar, Straus, and Giroux).	
Landini, Simone, and Mariacristina Uberti	2008	"A statistical mechanic view of macro-dynamics in economics"	**Computational Economics** 32: 121-146
Lux, Thomas	2008	"Applications of Statistical Physics in Economics and Finance" In J. Barkley Rosser Jr., (ed.). **Handbook of Research on Complexity**.	London: Edward Elgar.
Angle, John	2007	"The Macro Model of the Inequality Process and The Surging Relative Frequency of Large Wage Incomes". Pp. 171-196 in A. Chatterjee and B.K. Chakrabarti, (eds.), ***The Econophysics of Markets and Networks***	(Proceedings of the Econophys-Kolkata III Conference, March 2007 [http://www.saha.ac.in/cmp/econophys3.cmp/]). Milan: Springer [ISBN: 978-8847006645]. [on-line at: http://arxiv.org/abs/0705.3430].
Chatterjee, Arnab, Sitabhra Sinha, Bikas Chakrabarti	2007	"Economic Inequality: Is it Natural?".	**Current Science** 92:(#10, May 25, 2007): 1383-1389. [on-line at: http://www.ias.ac.in/currsci/May252007/1383.pdf].
Yakovenko, Victor.	2007	"Econophysics, Statistical Mechanical Approaches to" **Encyclopedia of Complexity and System Science**.	New York: Springer. http://refworks.springer.com/complexity]. [on-line at http://arxiv.org/abs/0709.3662].
Angle, John	2006	"A comment on Gallegati et al.'s "Worrying Trends in Econophysics" ". Pp. 250-253 in A. Chatterjee and B.K. Chakrabarti, (eds.), ***The Econophysics of Stocks and Other Markets.***	(Proceedings of the Econophys Kolkata II Conference, February, 2006 [http://www.saha.ac.in/cmp/econophys2.cmp/]). Milan: Springer. (ISBN: 978-8847005013).
Angle, John	2006	"The Inequality Process as a wealth maximizing algorithm".	***Physica A: Statistical Mechanics and Its Applications*** 367:388-414 (DOI information: 10.1016/j.physa.2005.11.017). A draft version of the paper can be downloaded from the Luxembourg Income Study website: [http://www.lisproject.org/publications].
Chatterjee, Arnab and B.K. Chakrabarti	2006	Abstract of "Heterogeneous agents are responsible for the Pareto tail".	**Applications of Physics to Financial Analysis 5** (June, 2006), 7. [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf].
Foscari, Piero	2006	"Stochastic and deterministic Simulation Techniques for Traffic and Economics".	PhD dissertation, University of Ferrara, Italy. [online at http://eprints.unife.it/27/1/Foscari_2006_PhD_Thesis.pdf]
Gupta, Abhijit Kar	2006	"Econophysics and Sociophysics: Trends and Perspectives". Chakrabarti, B.K., A. Chakraborti, and A. Chatterjee, (eds.), **Econophysics and Sociophysics**.	Weinheim , Germany : Wiley VCH.
Richmond, Peter, Stefan Hutzler, Ricardo Coelho, and Przemek Repetowicz.	2006	"A review of empirical studies and models of income distributions in society". Chakrabarti, B.K., A. Chakraborti, and A. Chatterjee, (eds.), **Econophysics and Sociophysics**.	Weinheim, Germany: Wiley VCH.
Scalas, Enrico, U. Garibaldi and S. Donadio	2006	"Statistical equilibrium in simple exchange games I: methods of solution and application to the Bennati-Dragulescu-Yakovenko (BDY) game".	**The European Physical Journal B** 53(#2, September): 267-272.
Chatterjee, Arnab and B.K. Chakrabarti	2006	Abstract of "Heterogeneous agents are responsible for the Pareto tail".	**Applications of Physics to Financial Analysis 5** (June, 2006), 7. [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf].
Scalas, Enrico	2006	Abstract of "Growth and inequality processes: equilibrium and nonequilibrium models in physics and economics".	**Applications of Physics to Financial Analysis 5** (June, 2006), 43. [http://www2.polito.it/eventi/apfa5/Abstract/abstracts.pdf].
Lux, Thomas	2005	"Emergent statistical wealth distributions in simple monetary exchange models: a critical review". Pp. 51-60 in A. Chatterjee, S. Yarlagadda, and B.K. Chakrabarti, (eds.), **Econophysics of Wealth Distributions**.	Milan, Italy: Springer. (the proceedings volume of the International Workshop on the Econophysics of Wealth Distributions, March, 2005, Kolkata, India [http://www.saha.ac.in/cmp/econophysics/]). [on-line version of paper at: [http://arxiv.org/abs/cs/0506092].
Kleiber, Christian and Samuel Kotz.	2003	**Statistical Size Distributions in Economics and Actuarial Science.** Pages 162, 163, 167, and 168.	New York: Wiley.

4.0 The Inequality Process (IP) as Mathematical Model

4.1 The IP's Defining Equations

The Inequality Process (IP) is defined by the equations for the transfer of wealth between two randomly paired particles, its transition equations:

$$x_{it} = x_{i(t-1)} + d_t\, \omega_{\theta j}\, x_{j(t-1)} - (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)}$$
$$x_{jt} = x_{j(t-1)} - d_t\, \omega_{\theta j}\, x_{j(t-1)} + (1-d_t)\, \omega_{\psi i}\, x_{i(t-1)}$$

(1a,b)

where,

$x_{it} \equiv$ particle i's wealth at time t

$x_{j(t-1)} \equiv$ particle j's wealth at time $t-1$

$0 < \omega_{\theta j} < 1.0$ fraction lost in loss by particle j

$0 < \omega_{\psi i} < 1.0$ fraction lost in loss by particle i

$$d_t = \begin{cases} 1 & \text{with probability of } .5 \\ 0 & \text{otherwise} \end{cases}$$

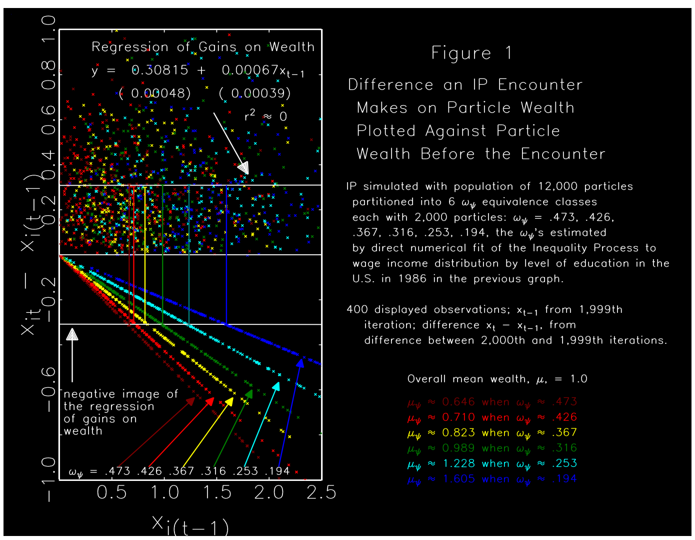

Figure 1

4.2 Figure 1: The Way to Understand the IP's Transition Equations

The way to understand the Inequality Process (IP) and its properties is the graph of change in the wealth of particles as a result of a competitive encounter against their wealth before the encounter. See Figure 1. There are six subsets of the population of particles in the simulation of the IP in Figure 1. Each subset has the same number of particles. Each particle in each subset has the same value of ω as that of other particles in that subset, i.e., ω_ψ is shared by all particles in the ψ^{th} subset. Call the subset in which all particles have parameter ω_ψ the ω_ψ equivalence class of particle. In Figure 1, each ω_ψ equivalence class of particle has a distinctive color. These colors range from deep red for the subset with the largest ω_ψ to purple for the subset with the smallest ω_ψ, i.e., from intense (hot) competition to less intense (cool) competition.

Figure 1 graphs the change in particle wealth between time point t-1 and time point t against wealth at time t-1. Change is on the y-axis. Wealth at the previous time point, t-1, is on the x-axis. Gains are above the x-axis, losses under it. There is a dot of color at each (x,y) coordinate for a small sample of particles in each ω_ψ equivalence class. The color indicates the ω_ψ equivalence class of the particle experiencing the gain or loss.

Note that while winning or losing is a 50/50 coin toss (a 0,1 uniform discrete probability function, or Bernoulli variable). The wealth gained from a win is itself random from the winning particle's point of view, because of the random pairing of particles. The amount of wealth a particle loses in a loss, on the other hand, once the fact of a loss is determined, is known to the particle before the encounter. This amount is the product of its parameter, ω_ψ, and its wealth at time point t-1.

Consequently, gains are a multi-colored cloud in Figure 1 while losses all line up on separate lines by color code radiating downward from the origin. Gains and losses in the Inequality Process are asymmetric. There is information in a loss (the revelation of the particle's parameter) but not in a win. Since winning is 50/50, in the IP wealth is transferred on average from particles that lose a larger share of their wealth when they lose to particles that lose a smaller share.

4.3 Consistency with Lenski's Extension of the Surplus Theory

In Figure 1 mean wealth in each ω_ψ equivalence class scales inversely with ω_ψ or, conversely, mean wealth in each ω_ψ equivalence class scales directly with $(1-\omega_\psi)$. Vertical, color coded bars mark the mean wealth of particles in each ω_ψ equivalence class. The Inequality Process is consistent with Lenski's extension of the Surplus Theory and the common experience that more skilled workers are wealthier: particles that hang on to a larger share of their wealth when they lose in competition, the operationalization of worker skill in the Inequality Process, have higher mean wealth. In the Inequality Process, on average, wealth is transferred to particles that lose less when they lose, the robust losers.

4.3.1 Symmetries Evident in Figure 1

While gains and losses are asymmetric, there are symmetries evident in Figure 1. Since gains are random, their expectation is the same for all particles regardless of their ω_ψ or their wealth at time point t-1. Thus in Figure 1 the regression line of gains at time point t regressed on wealth at time point t-1 has near zero slope. The negative image of this regression line intersects the $y = -\omega_\psi x_{\psi(t-1)}$ line of uniformly colored dots formed by losses in the ω_ψ equivalence class at mean wealth, $\mu_{\psi(t-1)}$, in the ω_ψ equivalence class, showing that $\omega_\psi \mu_{\psi(t-1)} \approx$ the expectation of gain regardless of ω_ψ equivalence class.

4.4 The Stationary Distribution of Wealth in Each ω_ψ Equivalence Class

Figure 2 displays the stationary distribution of wealth in each ω_ψ equivalence class of the Inequality Process (IP) of Figure 1. The larger ω_ψ the more the mass of the stationary distribution (corresponding to particle wealth in that equivalence class) is bunched up over small wealth amounts. However as ω_ψ becomes small, the mode, the median, and the mean of the distribution move over larger wealth amounts and the distribution becomes less right skewed, more symmetric, closer to a normal distribution.

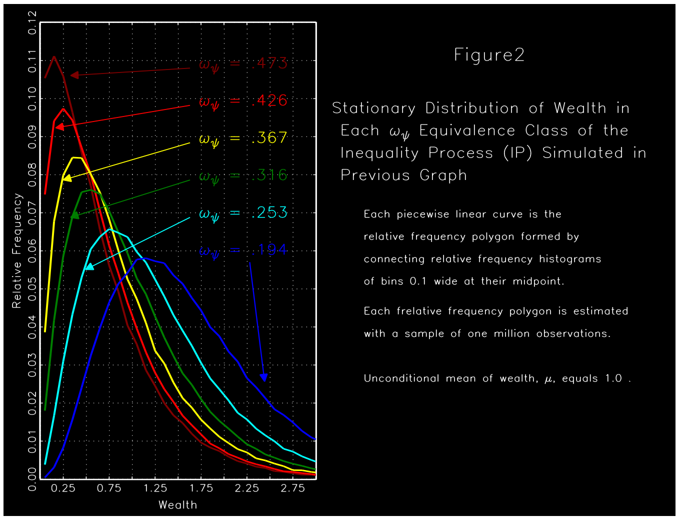

Figure 2

The simulation of the IP in Figures 1 and 2 has six ω_ψ equivalence classes. All six equivalence classes have the same number of particles. The Gini concentration ratio of the Inequality Process' stationary distribution has a simple relationship to ω_ψ if there is only one ω_ψ equivalence class. See Figure 3. The x-axis of Figure 3 is ω that characterizes all particles in a simulation of the IP. The curve in Figure 3 is the Gini concentration ratio of this simulation of the IP a single equivalence class all of whose particles have their parameter equal to ω. The straight line in Figure 3 is the y = x line. The curve of the IP's Gini concentration ratio hugs the y = x line, departing from it in a symmetric way.

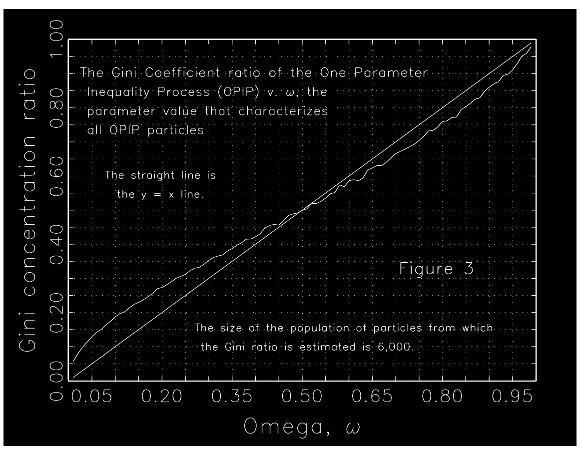

Figure 3

5.0 The Solution of the IP's Transition Equations for the Wealth of the General Particle in the ψ^{th} equivalence class

The Inequality Process' (IP's) solution for the wealth of the general particle, particle i in the ω_ψ equivalence class at time point t, $x_{i\psi t}$, is found by solving the transition equations of the Inequality Process in terms of particle i's parameter, ω_ψ, particle i's history of past wins and losses, Bernoulli variables, i.e., the d_i's, d_{it}, $d_{i(t-1)}$, $d_{i(t-2)}$, ..., and the wealth and parameters of the particles it has competed with in the past, $x_{j(t-1)}$, $x_{k(t-2)}$, $x_{l(t-3)}$, ... The solution technique is backward substitution. The solution of x_{it} is:

$$x_{it} = \begin{pmatrix} \omega_{\theta j} x_{j(t-1)} d_{it} \\ + \omega_{\phi k} x_{k(t-2)} d_{i(t-1)} [1 - \omega_{\psi} (1 - d_{it})] \\ + \omega_{\xi l} x_{l(t-3)} d_{i(t-2)} [1 - \omega_{\psi} (1 - d_{it})][1 - \omega_{\psi} (1 - d_{i(t-1)})] \\ + \ldots\ldots\ldots \end{pmatrix} \quad (2)$$

(2) is the solution of x_{it} in (1a,b). (2) is run-like, that is, x_{it} is largely a function of the length of its run of wins backward in time: d_{it}, $d_{i(t-1)}$, $d_{i(t-2)}$, ... if they equal 1.0 (i.e., are wins). x_{it} is what particle i won in the past from competitors and did not lose in later competitive encountes. When ω_ψ is almost 1.0, then any loss at time-point t almost wipes out x_{it}, particle i's wealth, because in (2) all previous gains would be multiplied by $(1-\omega_\psi)$. So when ω_ψ is near 1.0, nearly all particle wealth in the Inequality Process (IP) is the result of a straight run of wins in (2) extending from the present into the past. A long straight run of wins is rare since there is a .5 chance of losing at each time-point. Consequently, only a few lucky particles hold most of the wealth at any one time while the great majority of particles have little wealth, i.e., the IP with ω_ψ near 1.0 concentrates surplus wealth. Thus the IP accounts for the efflorescence of great inequality when hunter/gatherers acquire a store of food (surplus wealth), the phenomenon that the Surplus Theory of Social Stratification explains verbally, provided that large ω_ψ is associated with workers who are less skilled than workers in technologically more advanced economies who produce more wealth per capita (the Lenski extension of the Surplus Theory).

When ω_ψ is small, a particle's wealth can sustain a loss without being obliterated. So when ω_ψ is small, a run of wins backward in time can tolerate a loss without terminating the run, and wealth is less concentrated in a few lucky particles, those with a long run of wins. Since the probability of a long run of wins is quite small, wealth is transferred from particles in the equivalence class of particles with larger ω_ψ to particles in the equivalence class of particles with smaller ω_ψ, the robust losers, those by hypothesis of the IP's meta-theory, are more productive of wealth.

6.0 The Macro Model of the Inequality Process (IP), an Approximation to the IP's Stationary Distribution

6.1 The Negative Binomial Probability Function (pf)

The run-like character of the solution to the Inequality Process (IP), (2), suggests approximating its stationary distribution by a negative binomial probability function (pf), the distribution of the length of a run tolerating N losses, that is, the probability of k wins before the N^{th} loss:

$$p(k) \equiv \binom{N+k-1}{N-1} p^N q^k$$
$$p \equiv \textit{probability of a loss}$$
$$q \equiv (1-p), \textit{ probability of a win}$$
$$N \equiv \textit{number of losses}; N = 1, 2, \ldots$$
$$k \equiv \textit{number of wins before } N \textit{ losses}; \quad k = 0, 1, 2, \ldots$$

(3)

Inspection of the right hand side (RHS) of (2) shows that its approximation by a sum of k wins (Bernoulli variables equal to 1) has to be stopped by N+1 losses. This approximation has a condition: the ω_ψ's of all equivalence classes need to be clustered sufficiently near the weighted mean of the ω_ψ's (weighted by the number of particles in each class) so that the amount of of wealth that particle i gains from each particle can be approximated by the mean of wealth gained from all particles when they lose and factored out of the RHS of (2). That leaves the RHS of (2) as a sum of Bernoulli variables weighted by $(1-\omega_\psi)$ raised to the power of the number of losses a particle suffers later in time. The discrete approximation to the RHS of (2) after mean gain from wins is factored out is a sum of k Bernoulli variables each equal to 1 and a sum of N+1 Bernoulli variables each equal to zero. The number of terms in this sum is k + N + 1. N+1 has to approximate the sum of the geometric series of $(1-\omega_\psi)$ raised to successively higher integer powers.

$$(N+1) \approx \sum_{\tau}^{\infty} (1-\omega_\psi)^\tau = \frac{1}{\omega_\psi}$$
$$N \approx \frac{1-\omega_\psi}{\omega_\psi}$$

(4a,b)

6.2 The Two Parameter Gamma PDF Approximation to the Negative Binomial PF

The two parameter gamma probability density function (pdf) is a limit of the negative binomial pf as a single win's addition to the length of a run of wins tolerating a given number of losses before ending becomes small relative to the mean of wins (Bartko, 1961). A negative binomial pf can be approximated by a gamma pdf. The gamma pdf has been a model for income distribution since the late 19th century (March, 1898; Salem and Mount, 1974; Cowell, 1977; and McDonald and Jensen, 1979). In the gamma pdf limit of the negative binomial pf, the negative binomial's N parameter, becomes the gamma pdf's shape parameter, alpha, α. It can be readily demonstrated numerically that N changes the shape of the negative binomial pf in a way similar to that of the α shape parameter of the gamma pdf. The two parameter gamma pdf is:

$$
\begin{array}{lcl}
f(x) & \equiv & \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)}\, x^{\alpha-1}\, e^{-\lambda x} \\
x & \equiv & wealth\ (in\ this\ application) \\
x & > & 0 \\
\alpha & \equiv & shape\ parameter;\ \alpha > 0 \\
\lambda & \equiv & scale\ parameter;\ \lambda >
\end{array}
\qquad (5)
$$

6.3 The Macro Model of the Inequality Process

The Macro Model of the Inequality Process (IP) is the two parameter gamma pdf with shape parameter, alpha, α, equal to the expression in (4b) and its scale parameter, lambda, λ, derived from (4b) and the gamma pdf expression for the mean of x in terms of its parameters, α/λ (an easy integration or Salem and Mount, 1974). The Macro Model of the Inequality Process is:

$$
\begin{array}{lcl}
f(x) & \equiv & \dfrac{\lambda_{\psi t}^{\ \alpha_{\psi}}}{\Gamma(\alpha_{\psi})}\, x^{\alpha_{\psi}-1}\, e^{-\lambda_{\psi t} x} \\
x & \equiv & wealth\ in\ the\ \omega_{\psi}\ equivalence \\
 & & class \\
x & > & 0 \\
\alpha_{\psi} & \equiv & shape\ parameter \ \approx\ \dfrac{1-\omega_{\psi}}{\omega_{\psi}} \\
\lambda_{\psi t} & \equiv & scale\ parameter \ \approx\ \dfrac{1-\omega_{\psi}}{\tilde{\omega}_t\ \mu_t} \\
\tilde{\omega}_t & \equiv & harmonic\ mean\ of\ \ \omega_{\psi}'s
\end{array}
\qquad (6)
$$

Unconditional wealth, μ_t, is, without loss of generality, assigned the value 1.0, a value that facilitates computations.

The Macro Model of the Inequality Process approximates the stationary distribution of wealth in the ω_{ψ} equivalence class well but with noticeable imperfection. The IP's stationary distribution's mode in the ω_{ψ} equivalence class is below that of the corresponding Macro Model. See Figure 4, Figure 2 with Macro Model approximations in each ω_{ψ} equivalence class.

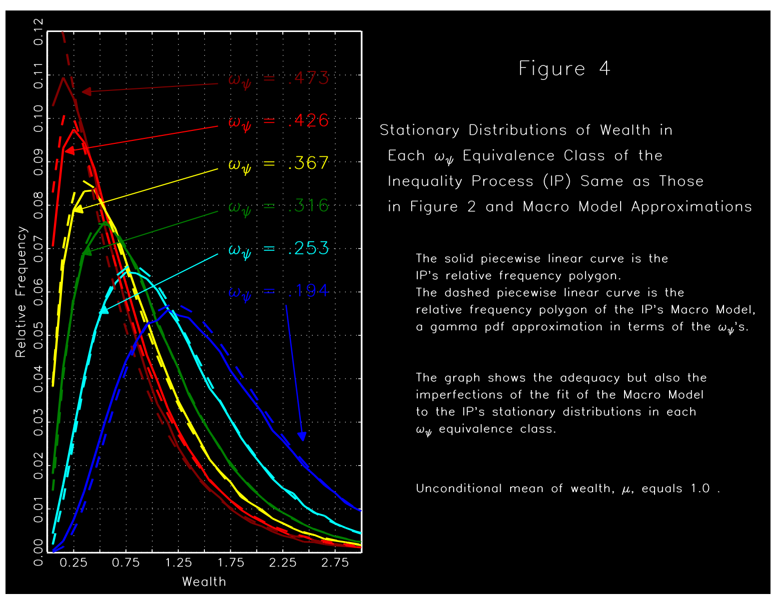

Figure 4

Note that the dynamics of the Macro Model are driven by its scale parameter, $\lambda_{\psi t}$. Its shape parameter, α_ψ, is treated as constant over time because of the IP's meta-theory. The IP's operationalization of worker skill level is $(1-\omega_\psi)$. There is no element of change in this proposition. So, the Macro Model's shape parameter, α_ψ, a sole function of ω_ψ, is constant. This aspect of the IP's meta-theory is testable when there is a fit of the IP to data.

The dynamics of the Macro Model come from two terms, the unconditional mean of wealth, μ_t, and the harmonic mean of the particles' ω_ψ parameter, the operationalization of the average skill level of workers. These two variables enter the expression for $\lambda_{\psi t}$ as a product, $(\tilde{\omega}_t\ \mu_t)$. In an industrial economy experiencing a business expansion and a demand for more skilled workers, one might expect these two exogenous drivers of the Macro Model's dynamics to move in opposite directions. A rising education level in the labor force would decrease the harmonic mean of the ω_ψ's, while a business expansion would be expected to increase μ_t. The question of whether $\lambda_{\psi t}$ increases or decreases from time period to time period is thus a question of whether the proportional increase in μ_t exceeds the proportional decrease in the harmonic mean of the ω_ψ's. Their product is the exogenous driver of the dynamics of the Macro Model, which has no endogenous dynamics since it does not model a feedback loops from μ_t to $\tilde{\omega}_t$ and vice versa. So given the expression for the mean of a random variable in the two parameter gamma pdf, the Macro Model's estimator of the mean of particle wealth in the ω_ψ equivalence class is, $\mu_{\psi t}$, is:

$$\mu_{\psi t} = \frac{\alpha_\psi}{\lambda_{\psi t}} \approx \frac{(\tilde{\omega}_t\ \mu_t)}{\omega_\psi} \tag{7}$$

When $\lambda_{\psi t}$ increases the mean of particle wealth in the ω_ψ equivalence class, $\mu_{\psi t}$, decreases, and vice versa. More than the mean moves when $\lambda_{\psi t}$ changes. A decrease in $\lambda_{\psi t}$ stretches the whole Macro Model distribution to the right over larger wealth amounts, i.e., all percentiles of a distribution of income or wealth gain. Such a change given (6) can only occur when $(\tilde{\omega}_t\ \mu_t)$ becomes larger. Conversely, when $(\tilde{\omega}_t\ \mu_t)$ gets smaller, $\lambda_{\psi t}$ gets bigger, and the whole Macro Model distribution is compressed to the left over smaller wealth amounts, i.e., all percentiles of a distribution of income or wealth decrease.

6.4 The Dynamics of the IP's Macro Model

The IP's Macro Model is a gamma pdf approximation to the stationary distribution of wealth in the ω_ψ equivalence class. The previous section of the paper pointed out that the dynamics of the Macro Model are exogenous and entirely driven by the product of the harmonic mean of particles' ω_ψ's and the unconditional mean of wealth, μ_t, set at 1.0 in the IP simulation of Figures 1, 2, and 4. The partial derivative of the Macro Model in the ω_ψ equivalence class with respect to the product of its exogenous drivers of change, its sole source of dynamics, is:

$$\frac{\partial\ f_{\psi t}(x_0)}{\partial(\tilde{\omega}_t \mu_t)} = f_{\psi t}(x_0)\ \lambda_{\psi t} \left(\frac{x_0 - \mu_{\psi t}}{\tilde{\omega}_t \mu_t} \right)$$

$$= f_{\psi t}(x_0)\ \frac{(1-\omega_\psi)}{(\tilde{\omega}_t\ \mu_t)^2}\ (x_0 - \mu_{\psi t}) \tag{8}$$

(8) shows how the Macro Model density at x_0 changes as a function of an increase in the product $(\tilde{\omega}_t \ \mu_t)$. If that product decreases, the partial derivative reverses sign. The dynamics of the Macro Model's density at x_0 is the result of multiplying the density of the Macro Model at x_0 by three factors:

a) $\frac{1}{(\tilde{\omega}_t \, \mu_t)^2}$

This term, a), contains the exogenous drivers of the dynamics of the Macro Model. If the product $(\tilde{\omega}_t \ \mu_t)$ increases, the density of the Macro Model at x_0 decreases, regardless of x_0 as the whole distribution is stretched to the right. This effect is pronounced; it is a quadratic function of $(\tilde{\omega}_t \ \mu_t)$. When $(\tilde{\omega}_t \ \mu_t)$ gets smaller, the converse occurs.

b) $(x_0 - \mu_{\psi t})$

This term, b), determines the sign and degree of change depending on which side of the mean of wealth in the stationary distribution of the ω_ψ equivalence class x_0 is located. If x_0 is smaller than the mean, the density at x_0 decreases when $(\tilde{\omega}_t \ \mu_t)$ increases, i.e., there will be fewer wealth or income amount smaller than the mean (and the mean will increase). Similarly, when $(\tilde{\omega}_t \ \mu_t)$ increases there will be more wealth or income amounts greater than the former mean. Consequently, b) implies the pattern of correlation of changes in the relative frequencies of labor income of all amounts, for example, over time that has been tested for and confirmed (Angle, 2007a).

b) implies another interesting effect in the dynamics of income and wealth distributions that has been tested for and confirmed (Angle, 2007a). Since the distribution of income or wealth is right skewed there is a non-trivial relative frequency at income or wealth amount farther above mean income or wealth than the smallest income or wealth is below mean income or wealth. Thus the Macro Model implies a surge in the relative frequency of large amounts of income and wealth when $(\tilde{\omega}_t \ \mu_t)$ increases.

c) $(1-\omega_\psi)$

c) is intensified the degree of effect of change in the exogenous driver of change $(\tilde{\omega}_t \ \mu_t)$ in the stationary distribution of wealth in the ω_ψ equivalence class with smaller ω_ψ, by hypothesis of the IP's meta-theory that of workers with more skills.

7.0 The Statistical Signature of the Inequality Process (IP) on Income and Wealth

The explanandum of the Inequality Process (IP) is the set of stable empirical patterns that the IP explains in income and wealth statistics. Collectively, the explanandum shows that the statistical signature of the IP is all over stable patterns in the statistics of income and wealth and points to a pervasive process of interpersonal competition for income and wealth in human populations, an inference of some economic importance hiding in plain sight.

This section of the paper, Section #7.0, points out how items #1 to #4 in Table 1, the published explanandum of the Inequality Process (IP), are accounted for by the IP.

Item #1. The universal pairing (all times, all places, all cultures) of the appearance of extreme social inequality (the chiefdom, society of the god-king) and concentration of wealth sone after egalitarian hunter/gatherers acquire a surplus of storeable food (Angle, 1983, 1986).

In Lenski's extension of the Surplus Theory of Social Stratification, the producers of the first surplus of storeable food are not highly skilled in the production of wealth, i.e., there is little need of training since there is little technology to learn. This proposition is operationalized in the IP by a small (1-ω), i.e., large ω. In the IP a large ω yields a large Gini concentration ratio of wealth. See Figure 3. The IP implies that soon after a hunter/gatherer population finds itself producing a storeable food surplus, competition for wealth in a population with a large ω (particles sustain a big loss in proportional terms when they sustain a loss; a low skill population) yields a distribution of wealth with a large Gini concentration ratio. Hence the universality of the pairing of the first evidence of a storeable food surplus and an extremely inegalitarian distribution of wealth. The IP is consistent with the Surplus Theory as interpreted and extended to all societies by Gerhard Lenski.

Item #2. The pattern of the Gini concentration ratio of personal wealth and income over the course of techno-cultural evolution beyond the chiefdom (Angle, 1983, 1986).

While the Surplus Theory provides a simple verbal explanation for the appearance of great inequality with the introduction of a storeable food surplus into a hunting and gathering population, the original Surplus Theory has no explanation for why the concentration of income and wealth decreases over the course of techno-cultural evolution. Lenski posits that societies higher on the techno-cultural scale than the chiefdom produce more wealth per capita than a chiefdom via a more advanced technology of wealth production and the vesting of wealth production skills in workers and that is why the concentration of wealth decreases in societies higher up on the scale of techno-cultural evolution than the chiefdom. A more skilled labor force is operationalized in the Inequality Process (IP) by smaller ω. Lenski speculates that more skilled workers have greater bargaining leverage to obtain a greater share of the wealth they produce and are more sheltered in the competition for wealth. Figure 3 shows that the Gini concentration ratio of the stationary distribution of wealth in the IP hugs the y = x line, i.e., is approximately, although not exactly, equal to ω. The IP is consistent with the Lenski extension of the Surplus Theory: Figure 3 shows that the IP's operationalization of more skilled labor, smaller ω, is associated with a smaller Gini concentration ratio.

Item #3. The right skew and gently tapering right tail of all distributions of income and wealth (a broad statement of the Pareto Law of income and wealth distribution) (Angle, 1983, 1986).

As Figure 2 shows, the IP's stationary distribution of wealth in the ω_ψ equivalence class is right skewed. The stationary distributions of wealth in all the ω_ψ equivalence classes in Figure 2 are right skewed with gently tapering right tails. Figure 4 shows that each IP stationary distribution in Figure 2 can be approximated by a gamma pdf. The two parameter gamma pdf is right skewed for finite values of its shape parameter, α.

Item #4. a) The sequence of shapes of the distribution of labor income in an industrial economy by level of worker education, b) why this sequence of shapes changes little over decades, and c) why a gamma pdf model works well for fitting the distribution of labor income (Angle, 1990, 2002, 2003, 2006, 2007a, 2007b);

4a) Most of the wealth of an industrial economy is tied up in human capital, the investment in worker skills and health. This fact is ascertained by capitalizing the income stream from labor. Labor income is an index to human capital. So it is appropriate to model labor income by the stock form of wealth in the Inequality Process (IP). Lenski's speculative extension of the Surplus Theory of Social Stratification to societies beyond the chiefdom on the scale of techno-cultural evolution runs that 1) the larger amount of wealth produced in these societies than in the chiefdom requires increasingly skilled labor as one moves up the techno-cultural scale, and 2) more skilled labor has the leverage to bargain for a larger share of the wealth it produces, that is, more skilled labor is more sheltered in the free-for-all competition for surplus wealth. This latter proposition is modeled in the IP as a smaller ω_ψ. Consequently, a crucial test of the Inequality Process (IP) is whether the sequence of its stationary distributions with smaller ω_ψ corresponds to the sequence of shapes of labor income distributions of workers at successively higher levels of education. If the IP fails this test, it is clearly disconfirmed. However, the IP passes the test. See Figure 5.

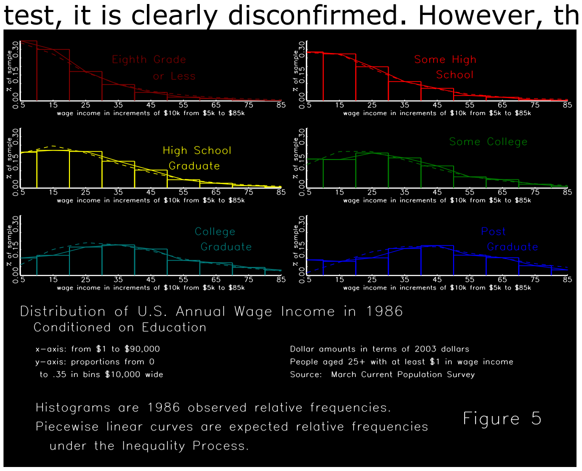

Figure 5

Figure 5 is a direct fit of the Inequality Process (IP) to annual wage and salary income in 1986 in the U.S., as reported on the March 1987 Current Population Survey (CPS), conducted by the U.S. Bureau of the Census. By 'direct fit' is meant a search over the IP's parameter vector of six cells to find the vector that best fits the six partial distributions of wage and salary income by level of education. The measure of fit is squared error, weighted by the fraction of the labor force at each level of education. The method of search is modified simulated annealing (Kirkpatrick, Gelatt, and Vecchi, 1983). The fitted IP stationary distributions of Figure 5 are the mean fit of 36 independent fits of the IP to the six relative frequency distributions of wage and salary income conditioned on education in 1986. See Table 3.

Table 3. Estimated Particle Parameters $\hat{\omega}_{\psi t}$ and the Estimated Mean of Wealth, $\hat{\mu}_{\psi t}$, in Each IP ω_ψ Equivalence Class Based on 36 Fits of the IPDO to the Distribution of Wage Income Conditioned on Education in U.S. in 1986				
education	estimated $\hat{\omega}_{\psi t}$	mean standard error of estimate of $\hat{\omega}_{\psi t}$ in 36 replications of fit to 1986 data	estimated $\hat{\mu}_{\psi t}$ where μ = 1.0 (mean of 36 independent replications of fit to 1986 data)	mean standard error of estimate of $\hat{\mu}_{\psi t}$ in 36 replications of fit to 1986 data
eight years or less	.4733	.0200	0.6571	.0280
some high school	.4261	.0173	0.7826	.0273
high school graduate	.3674	.0096	0.8602	.0130
some college	.3162	.0104	1.0046	.0234
college graduate	.2528	.0090	1.2568	.0353
some post-graduate education or more	.1940	.0078	1.6152	.0402

Figure 6 puts the squared errors of the 36 independent fits of the IP to 1986 income data into the context of 36 similar fits in each of 49 other years of data (1961 through 2010). Figure 6 graphs the 36 sums of squared error in each year from 1961 through 2010, data from the March 1962 through March 2011 CPS'. The lines in Figure 6 connecting observations connect the first, second, third,... etc. fits in one year to the first, second, ...etc. fits in the next year. It's a graphical convenience; there is no significant correlation between the sequence of fits and the goodness of fit. There is however a close relationship between goodness of fit and the number of workers whose data are reported in a CPS. 1986 was a year of good fits but not the best and not substantially different from other years in the 1980's or the last half of the 1990's.

Note that the six estimated ω_ψ's in Table 3 are used in the IP simulation that produced Figures 1, 2, and 4.

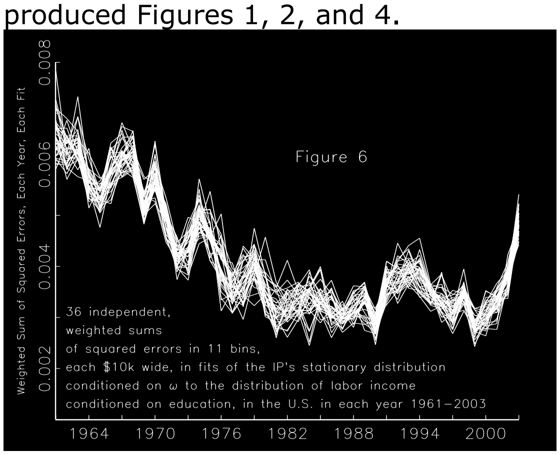

Figure 6

4b) The meta-theory of the Inequality Process (IP) asserts that worker skill level is what allows a worker to bargain for a fraction of wealth produced. The greater the skill level, the bigger the fraction, the more the worker is sheltered from competition for surplus wealth. This is one of Lenski's (1966) speculative extensions of the Surplus Theory of Social Stratification to societies beyond the chiefdom on the scale of techno-cultural evolution. There is no endogenous dynamic in Lenski's speculation about the effect of vesting more human capital in a worker and the worker's ability to keep more of the wealth that worker produces, so the principle of parsimony used in the specification of the IP requires the constancy over time of the effect on a worker's ability to retain wealth of a given level of worker skill. In the case of the IP fits in Figure 5, the ω_ψ's estimated by partitioning the IP's particles into six ω_ψ equivalence classes, one per level of worker education distinguished with the fraction of particles in each ω_ψ equivalence class matched to workers with a given level of education equal to the fraction those workers make in the sample, are not only all ordered in each year at expected under the IP's meta-theory (Lenski's extension of the Surplus Theory), the estimated ω_ψ's are nearly constant, despite the rising level of education in the U.S. labor force from 1961 through 2010. See Figure 7.

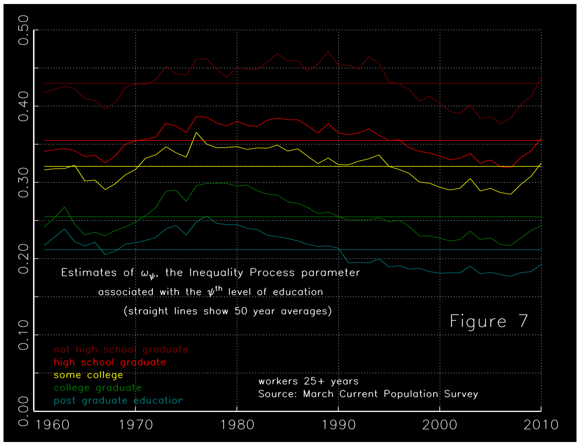

Figure 7

4c) A gamma pdf model works well for fitting the distribution of labor income, a fact demonstrated in a 1898 presentation to the Paris Statistical Society by Lucien March (March, 1898). A two parameter gamma pdf with shape and scale parameters equal to the expressions for those quantities in the Macro Model of the Inequality Process (IP) fits the stationary distribution of the Inequality Process in the IP's ω_ψ equivalence class. See Figure 4. The ω_ψ's of the IP stationary distributions of Figures 1, 2, and 4 are estimated from the direct fit of the IP to the distribution of annual wage and salary income in the U.S. in 1986 shown in Figure 5. The fits of the IP's Macro Model and the direct fits of the IP's stationary distributions to the empirical distributions of Figure 5 are essentially indistinguishable.

8.0 Discussion:

There is a tenuous thread in the history of ideas that links the concepts of the Surplus Theory of Social Stratification, its speculative extension by Gerhard Lenski (1966), and perhaps even their operationalization in the Inequality Process backward in time through 20th century social science to the social science of Karl Marx in the 19th (cf. Engels, 1972[1884]). There is enough of a connection to that 19th century thinker that some economists are leery of the Inequality Process (IP) because of its provenance. No matter that no work on the IP addresses issues of government policy or political action or that the IP is based on the work of apolitical social scientists in industrial democracies, so great is the adversion among some economists to any concept traceable however remotely to Karl Marx. Other economists see decision making by individual participants in markets as the essence of their field and see the absence of both in the IP as presently specified as grounds for dismissing the IP as irrelevant to economics. Still other economists, e.g., Gallegati, Keen, Lux, and Ormerod (2006), think that the class of particle system models that includes the Inequality Process is incapable of explaining economic growth and thus is irrelevant to contemporary industrial economies. This particular criticism has been refuted (Angle, 2006e). Many economists, in the author's experience, view model parsimony as a problem rather than a virtue and the growing explanandum of the IP as an unimportant curiosity rather than a sign post toward an exciting scientific frontier.

It is true that the IP advances by the explanation of one empirical pattern after the other and thus does not necessarily address major concepts of the current mainstream paradigm of economics. However, it is also true that the IP implies a number of familiar verbally maintained propositions about personal income and wealth of mainstream economics. These are propositions that have never been put on a firm mathematical and empirical foot singly let alone jointly. See Table 4 for six such propositions. Anyone who thinks that the IP is a resurrection of Karl Marx' doctrines of radical egalitarianism should pay attention to items #1 and 2 in Table 4. Pareto's and Gibrat's contributions to the study of income distribution, findings that implied the unlikelihood that a radically egalitarian income distribution is feasible, were inspired by antagonism toward egalitarian advocacy of such an income distribution. Pareto was a well known political conservative. Their findings were likely ensconced in the mainstream economic canon for ideological reasons: something to refute redistributionist egalitarian arguments with. The IP implies the same conclusion that Pareto and Gibrat reached but without ideological motivation.

Table 4. The Inequality Process (IP) Jointly Puts Well Accepted Verbally Justified Views in Economics on a Firm Scientific Footing for the First Time

Widely Accepted Proposition in Economics	Inequality Process' Explanation
1) All distributions of labor income are right skewed with tapering right tails; hence the impossibility of radical egalitarianism, the inference motivating Pareto's study of income and wealth distribution.	The IP generates right skewed distributions shaped like empirical distributions of labor income or personal assets (depending on the value of the particle parameter).
2) Differences of wealth and income arise easily, naturally, and inevitably via a ubiquitous stochastic process; cf. the most general statement of Gibrat's Law; hence the impossibility of radical egalitarianism.	In the IP, differences of wealth arise easily, naturally, and inevitably, via a ubiquitous stochastic process.
3) A worker's earnings are tied to that worker's productivity [i.e., a central tenet of economics since Aesop's fable of the ant and the grasshopper was all there was to economics] but there is a wide distribution of returns to similarly productive workers.	In the IP's Macro Model, an approximation to its stationary distribution, a particle's expected wealth is determined by the ratio of mean productivity in the population to that of an individual. There is a distribution of wealth around this expectation.
4) Labor incomes small and large benefit from a business expansion strong enough to increase mean labor income, i.e., there is a community of interest between all workers regardless of their earnings in a business expansion. A conclusion encapsulated in the saying, "A rising tide lifts all boats."	In the IP's Macro Model, an increase in the unconditional mean of wealth increases all percentiles of the stationary distribution of wealth by an equal factor. In pithy statement form: "A rising tide lifts the logarithm of all boats equally.".
5) Competition transfers wealth to the more productive of wealth via transactions without central direction, i.e., via parallel processing.	In the IP, competition between particles causes wealth to flow via transactions from particles that are by hypothesis and empirical analogue less productive of wealth to those that are more productive of wealth, enabling the more productive to create more wealth, explaining economic growth without a) requiring knowledge of how wealth is produced or b) central direction, i.e., with a minimum of information, two reasons why the IP may have been naturally selected. These features enable the IP to operate homogeneously over the entire course of techno-cultural evolution independently of wealth level.
6) Competition and transactions maximize societal gross product and over the long run drive techno-cultural evolution.	The Inequality Process operates as an evolutionary wealth maximizer in the whole population of particles, given a relaxation of the zero-sum constraint on wealth transfers within the model, by transferring wealth to the more productive.

9.0 References

See also Table 2.

Angle, John. 1983. "The surplus theory of social stratification and the size distribution of Personal Wealth." ***1983 Proceedings of the American Statistical Association***, Social Statistics Section**.** Pp. 395-400. Alexandria, VA: American Statistical Association.

_____. 1986. "The surplus theory of social stratification and the size distribution of Personal Wealth." ***Social Forces*** 65:293-326.

_____. 1986. "Coalitions in a stochastic process of wealth distribution." ***1986 Proceedings of the American Statistical Association***, Social Statistics Section. Pp. 259-263. Alexandria, VA: American Statistical Association.

_____. 1990. "A stochastic interacting particle system model of the size distribution of wealth and income." ***1990 Proceedings of the American Statistical Association*,** Social Statistics Section. Pp. 279-284. Alexandria, VA: American Statistical Association.

_____. 1992. "The Inequality Process and the distribution of income to blacks and whites". ***Journal of Mathematical Sociology*** 17:77-98. [Included in the Inventory of the John Hope Franklin Papers, 1889-1998 and undated, bulk 1970-1998, Rare Book, Manuscripts, and Special Collections Library, Duke University Libraries, http://library.duke.edu/digitalcollections/rbmscl/franklinjohnhope/inv/].

_____. 1993a. "Deriving the size distribution of personal wealth from 'the rich get richer, the poor get poorer'". ***Journal of Mathematical Sociology*** 18:27-46.

_____. 1993b. "An apparent invariance of the size distribution of personal income conditioned on education." ***1993 Proceedings of the American Statistical Association*,** Social Statistics Section . Pp. 197-202. Alexandria, VA: American Statistical Association.

_____. 1996. "How the gamma law of income distribution appears invariant under aggregation". ***Journal of Mathematical Sociology***. 21:325-358.

_____. 1997. "A theory of income distribution". ***1997 Proceedings of the American Statistical Association*,** Social Statistics Section. Pp. 388-393. Alexandria, VA: American Statistical Association.

____. 1998. "Contingent forecasting of the size of the small income population in a recession". ***1998 Proceedings of the American Statistical Association*,** Social Statistics Section**.** Pp. 138-143. Alexandria, VA: American Statistical Association.

____. 1999. "Evidence of pervasive competition: the dynamics of income distributions and individual incomes". ***1999 Proceedings of the American Statistical Association*,** Social Statistics Section. Pp. 331-336. Alexandria, VA: American Statistical Association.

____. 1999. "Contingent forecasting of the size of a vulnerable nonmetro population". ***Proceedings of the 1999 Federal Forecasters' Conference***. Pp. 161-169. (Awarded "Best Paper" by Organizing Committee of the 2000 Federal Forecasters' Conference, 9/14/00). Washington, DC: U.S. Government Printing Office.

____. 2000. "The binary interacting particle system (bips) underlying the maxentropic derivation of the gamma law of income distribution". ***2000 Proceedings of the American Statistical Association*,** Social Statistics Section. Pp. 270-275. Alexandria, VA: American Statistical Association.

____. 2001. "Modeling the right tail of the nonmetro distribution of wage and salary income". ***2001 Proceedings of the American Statistical Association*,** Social Statistics Section. [CD-ROM], Alexandria, VA: American Statistical Association.

____. 2002a. "The statistical signature of pervasive competition on wages and salaries". ***Journal of Mathematical Sociology***. 26:217-270. [abstracted in Cambridge Science Abstracts and Sociological Abstracts].

____. 2002b. "Modeling the dynamics of the nonmetro distribution of wage and salary income as a function of its mean". ***Proceedings of the 2002 Joint Statistical Meetings*,** (American Statistical Association, Business and Economic Statistics Section). [CD-ROM], Alexandria, VA: American Statistical Association.

____. 2002c. "Contingent forecasting of bulges in the left and right tails of the nonmetro wage and salary income distribution". ***Proceedings of the 2002 Federal Forecasters' Conference***. (Awarded "Best Paper" by Organizing Committee of the 2003 Federal Forecasters' Conference, 9/18/03). Washington, DC: U.S. Government Printing Office.

____. 2003a. "The dynamics of the distribution of wage and salary income in the nonmetropolitan U.S.". ***Estadistica***.55: 59-93.

____. 2003b. "Inequality Process, The". An entry in T. Liao, et al., (eds.), ***The Encyclopedia of Socil Science Research Methods****.* Volume 2: 488-490. Thousand Oaks, CA: Sage.

____. 2003c. "Imitating the salamander: a model of the right tail of the wage distribution truncated by topcoding". November, 2003 ***Conference of the Federal Committee on Statistical Methodology***, [http://www.fcsm.gov/events/papers2003.html].

____. 2005. "The U.S. Distribution of Annual Wage and Salary Income since 1961: the Perceived Inequality Trend". Population Association of America Annual Meeting. Philadelphia, April, 2005. Downloadable from: [http://paa2005.princeton.edu/download.aspx?SubmissionID=50379].

____. 2006a. "Not a Hollowing Out, A Stretching: Trends in U.S. Nonmetro Wage Income Distribution, 1961-2003". Self published as a downloadable *.pdf file on RePEc/MPRA. [on-line at: http://mpra.ub.uni-muenchen.de/10111/].

____. 2006b (received 8/05; electronic publication: 12/05; hardcopy publication 7/06). "The Inequality Process as a wealth maximizing algorithm"'. ***Physica A: Statistical Mechanics and Its Applications*** 367:388-414 (DOI information: 10.1016/j.physa.2005.11.017). A draft version of the paper can be downloaded from the Luxembourg Income Study website: [http://www.lisproject.org/publications].

____. 2006c. "A measure of intergroup discrimination: color and wage income in the nonmetropolitan U.S.". ***Proceedings of the 2006 Joint Statistical Meetings*, (**American Statistical Association, Social Statistics Section), August, 2006. Pp, 1889-1894. [CD-ROM], Alexandria, VA: American Statistical Association.

____. 2006d. “The Inequality Process as an evolutionary process”. ***Proceedings (late breaking papers) of the Genetic and Evolutionary Computation Conference (GECCO), July, 2006***. Late breaking paper (lbp) #137.

____. 2006e. “A comment on Gallegati et al.’s “Worrying Trends in Econophysics” ”. Pp. 247-253 in A. Chatterjee and B.K. Chakrabarti, (eds.), ***The Econophysics of Stocks and Other Markets*** (Proceedings of the Econophys Kolkata II Conference, February, 2006 [http://www.saha.ac.in/cmp/econophys2.cmp/]). Milan: Springer. (ISBN: 978-8847005013).

____. 2007a. “The Macro Model of the Inequality Process and The Surging Relative Frequency of Large Wage Incomes”. Pp. 171-196 in A. Chatterjee and B.K. Chakrabarti, (eds.), ***The Econophysics of Markets and Networks*** (Proceedings of the Econophys-Kolkata III Conference, March 2007 [http://www.saha.ac.in/cmp/econophys3.cmp/]). Milan: Springer [ISBN: 978-8847006645]. [on-line at: http://arxiv.org/abs/0705.3430].

____. 2007b. “The Inequality Process is an evolutionary process”. ***The Constructal Theory of Social Dynamics***. Adrian Bejan and Gilbert Merkx, eds. (Proceedings of the Conference on the Constructal Theory of Social Dynamics, Duke University, April 2006). New York: Springer [ISBN: 978-0387476803].

____. 2007c. “A mathematical sociologist’s tribute to Comte: sociology as science”. **Footnotes** [monthly newsletter of the American Sociological Association] 35(No. 2, February): 10,11. [on-line at: http://www2.asanet.org/footnotes/feb07/fn9.html . Its web-log is on-line at http://members.asanet.org/Forums/view_forum.php?id=11].

____, François Nielsen, and Enrico Scalas. 2009. “The Kuznets Curve and the Inequality Process”. Pp. 125-138. In Banasri Basu, Bikas K. Chakrabarti, Satya R. Chakravarty, Kausik Gangopadhyay, editors, ***Econophysics and Economics of Games, Social Choices and Quantitative Techniques***. (Proceedings of the Econophys-Kolkata IV Conference, March 2009, Kolkata, India, jointly sponsored by the Indian Statistical Institute and the Saha Institute of Nuclear Physics.) [http://www.isical.ac.in/cmp/econophys4.com/]. Milan: Springer.

____. 2009. “A Test of Two Similar Particle System Models of Wage Income Distribution Conditioned on Education”. ***Proceedings of the 2009 Joint Statistical Meetings,*** (American Statistical Association, Business and Economic Statistics Section), Pp. 1003-1017. CD-Rom. Alexandria, VA: American Statistical Association.

____. 2010. "The Inequality Process as an Evolutionary Dynamic Attractor". ***Proceedings of the 2010 Joint Statistical Meetings,*** (American Statistical Association, Section and Statistical Learning and Data Mining), Joint Statistical Meetings, August 2010.

____. 2012. “The Inequality Process v. The Saved Wealth Model: Which is the More Likely to Imply an Analogue of Thermodynamics in Social Science”. ***Journal of Mathematical Sociology***. 36: 156-182. An earlier version of this paper **"** The Inequality Process vs. The Saved Wealth Model. Two Particle Systems of Income Distribution; Which Does Better Empirically?” is on RePEc/MPRA. [http://mpra.ub.uni-muenchen.de/20835/], a revision of “A Test of Two Similar Particle System Models of Wage Income Distribution Conditioned on Education”, 2009 Joint Statistical Meetings, in its proceedings volume.

Bartko, J. 1961. “The negative binomial distribution: a review of properties and applications”. ***The Virginia Journal of Science***. 12: 18-37.

Braun, M. 1978. ***Differential Equations and Their Applications: An Introduction to Applied Mathematics.*** New York: Springer.

Chakraborti, Anirban and Bikas Chakrabarti. 2000. “Statistical mechanics of money: how saving affects its distribution”, ***The European Physical Journal B*** 17: 167-170.

Childe, V. Gordon. 1944. “Archeological ages as technological stages”. ***Journal of the Royal Anthropological Institute of Great Britain and Ireland***. 74: 7-24.

Cowell, Frank. 1977. ***Measuring Inequality: Techniques for the Social Sciences***. New York: Wiley.

Dalton, G. 1960. “A note of clarification on economic surplus”. ***American Anthropologist***. 62: 483-490.

_____. 1963. “Economic surplus, once again”. ***American Anthropologist***. 65: 389-394.

Engels, Frederick. 1972 [1884]. ***The Origin of the Family, Private Property, and the State in the Light of the Researches of Lewis H. Morgan***. Eleanor B. Leacock (ed.). New York: International Publishers.

Gallegati, Mauro, Steven Keen, Thomas Lux, and Paul Ormerod. 2006. “Worrying trends in econophysics”. ***Physica A***. 370: (1) October, 1-6.

Gibrat, Robert. 1931. ***Les Inégalités Economiques.*** Paris: Sirey.

Harris, Marvin. 1959. “The economy has no surplus?”. ***American Anthropologist***. 61: 185-199.

Herskovits, Melville. 1940. ***The Economic Life of Primitive Peoples***. New York: Knopf.

Kirkpatrick, S., C.D. Gelatt, M.P. Vecchi. 1983. “Optimization via simulated annealing”. ***Science*** 220: 671-680.

Lenski, Gerhard. 1966. ***Power and Privilege***. New York: McGraw-Hill.

McDonald, J. and B. Jensen. 1979. “An analysis of some properties of alternative measures of income inequality based on the gamma distribution function”. ***Journal of the American Statistical Association***. 74: 856-860.

March, Lucien. 1898. “Quelques exemples de distribution de salaires”. ***Journal de la Societe Statistique de Paris.*** June and July issue: 193-206, 241-248.

May, Robert. 1981. “Models for two interacting populations”. In R. May (ed.), ***Theoretical Ecology***. Pp. 78-104. 2nd edition. Oxford: Blackwell Scientific.

Morgan, Lewis H. 1963 [1877]. ***Ancient Society***. Eleanor B. Leacock (ed.). New York: World Publishers.

Murray, J. 1993. ***Mathematical Biology*** (2nd ed.). Berlin: Springer.

Pareto, Vilfredo. 1897. ***Cours d’Economie Politique***. Vol. 2. Lausanne: Rouge.

Pearson, Harry. 1957. “The economy has no surplus: critique of a theory of development”. Pp. 320-341 in ***Trade and Market in the Early Empires***. Karl Polanyi, Conrad Arensberg, and Harry Pearson. Glencoe, IL: Free Press.

Salem, A. and T. Mount. 1974. “A convenient descriptive model of income distribution: the gamma density”. ***Econometrica***: 42: 1115-1127.

Volterra, Vito. 1939. “The general equations of biological strife in the case of historical actions”. In F. Scudo and J. Ziegler, 1978. ***The Golden Age of Theoretical Ecology, 1923 – 1940: A Collection of the Works of V. Volterra, V.A. Kostitzin, A.J. Lotka, and A.N. Kolmogoroff***. Lecture Notes in Biomathematics, volume 22. Berlin: Springer.

Whitney, Charles. 1990. ***Random Processes in Physical Systems***. New York: Wiley.