Decomposing the Foster-Greer-Thorbecke Index of Vulnerability to Poverty

Martina Celidoni Department of Economics - University of Padua

2011 IARIW-OECD Conference on Economic Insecurity: Measurement, Causes, and Policy Implications

November 22-23, 2011

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 ◇◇◇

Decomposing the Foster-Greer-Thorbecke Index of Vulnerability to Poverty

- Focus Risk is quite different in size, likelihood and frequency over time. Different features correspond to different implications for the ability to cope with them as well as for policy (Dercon, 2001; similar idea in Morduch (2000)) → Need of more information.
- **This paper** Highlights vulnerability as a function of 3 contributing factors (characteristics): expected incidence, expected intensity and expected downward variability.

- Offers an empirical illustration using BHPS (intertemporal) and SHIW (across regions).

The three Is of Poverty: Incidence, Intensity and Inequality

(Sen, 1976; Foster et al., 1984; Jenkins and Lambert, 1997)

$$P_{\alpha} (\mathbf{y}; \mathbf{z}) = \frac{1}{N} \sum_{h=1}^{Q} \left[\frac{z - y_h}{z} \right]^{\alpha}, \quad (1)$$

$$P_{\alpha=2} (\mathbf{y}; \mathbf{z}) = H \left[l^2 + (1 - l)^2 C V_{\rho}^2 \right], \quad (1)$$

$$P_{\alpha=2} (\mathbf{y}; \mathbf{z}) = H \left[l^2 + (1 - l)^2 C V_{\rho}^2 \right], \quad (2)$$

$$H = Q/N, \quad (3)$$

$$l = \frac{1}{Q} \sum_{h=1}^{Q} \left[\frac{z - y_h}{z} \right], \quad (4)$$

$$C V_{\rho}^2 = \frac{1}{Q} \sum_{h=1}^{Q} \frac{(\mu_{\rho} - y_h)^2}{\mu_{\rho}^2}. \quad (5)$$

$$Cumulative sum of poverty gaps per capita$$

$$INTENSITY (beight)$$

$$INTENSITY (beight)$$

$$INTENSITY (beight)$$

$$INTENSITY (beight)$$

$$INEQUALITY (curvature)$$

$$INCIDENCE (length)$$

$$INCIDENCE (l$$

イロト イヨト イヨト イヨト 「日」 シック

The three vulnerability contributing factors

Expected Incidence, Expected Intensity and Expected Downward variability

- \mathbf{y}_s^h vector of possible income values at t+1 for the household h
- ► The vector ỹ^h_s represents a permutation of y^h_s, so that the elements are non-decreasingly ranked, i.e. for all ỹ^h_s, ỹ^h₁ ≤ ỹ^h₂ ≤ ... ≤ ỹ^h_{S_h}... ≤ ỹ^h_N.

$$V_{\alpha=2,h}\left(\tilde{\mathbf{y}};\mathbf{z}\right) = \sum_{s=1}^{S_h} p_s \left[\frac{z - \tilde{y}_s^h}{z}\right]^2 = EH_h \left[EI_h^2 + (1 - EI_h)^2 ECV_h^2\right], \quad (6)$$

$$EH_h = \frac{S_h}{N} \tag{7}$$

$$EI_{h} = \sum_{s=1}^{S_{h}} p'_{s} \frac{(z - \tilde{y}_{s}^{h})}{z}, \quad p'_{s} = \frac{1}{S_{h}}$$
 (8)

$$ECV_h^2 = \sum_{s=1}^{S_h} \rho_s' \frac{(\mu - \tilde{y}_s^h)^2}{\mu^2}, \quad \rho_s' = \frac{1}{S_h}$$
(9)

An example



Examples of Poverty Gaps Pattern

Table: Vulnerability to poverty and its contributing factors

	$V_{\alpha=2,h}$	EH _h	Elh	ECV_h^2
Ind1	0.3611	0.5	0.83	0.0408
Ind2	1.3681	1.0	2.25	0.5352
Ind3	1.8125	1.0	2.58	1.1488

Data and empirical strategy

Data British Household Panel Survey (BHPS) 1991-2004 Survey on Household Income and Wealth (SHIW) 1989-2004

Method - Intertemporal comparison for England (income) I period: 1991-1997, II period: 1998-2004

> - Interregional comparison for Italy (consumption) North, Central and South (ISTAT classification)

Background England: Welfare reform implemented in the late 1990s

Italy: Characterised by a strong territorial difference in poverty rates

England: Welfare reform

Reform objectives:

- ► to increase economic activity, limit welfare dependency and, at the same time, reduce poverty (Gregg, 2008)
- reduction of poverty amongst pensioners and households with children, especially during 2000/01-2004/05 (Brewer et al., 2006)

Strategy:

- Introduction of and, later increases in, the National Minimum Wage (NMW)
- Child Tax Credit and Working Tax Credit
- Personalized Welfare-to-work Support (lone parents, sick and disabled)
- Minimum Income Guarantee for pensioners (then Pension Credit in 2003)

Remarks:

Good outcomes in terms of a lower pensioner poverty and higher replacement rates at the bottom of the income distribution (Gregg, 2008).

Table: Vulnerability to poverty and its contributing factors - England

BHPS (1997-2004)					
$V_{\alpha=2,h}$ (s.d.) EH_h (s.d.) EI_h (s.d.) ECV_h^2 (s.d.)					
t = I	0.0216 (0.065)	0.1657 (0.293)	0.0869 (0.158)	0.0171 (0.094)	
t = II	0.0169 (0.054)	0.1355 (0.255)	0.0773 (0.161)	0.0228 (0.158)	

Notes: $V_{\alpha=2,h}$ is the average vulnerability. Period I: 1991-1997. Period II: 1998-2004

Table: England - Paired t-tests

Vulnerability to poverty			
	Obs	Mean	SD
fgtl	1973	0.0216096	0.0649725
fgtll	1973	0.0169134	0.0543854
diff	1973	0.0046962	0.063896
$Ho: \ mean(diff) = 0 Ha: \ mean(diff) \neq 0 Pr(\mid \mathcal{T} \mid > \mid t \mid) = 0.0011$			



England - The 3 vulnerability contributing factors

Expected Incidence				
	Obs	Mean	SD	
Elncidencel	1973	0.165653	0.2926451	
EIncidencell	1973	0.1355345	0.2552977	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr($ T > t $) = 0.0000				
Expected Intensity				
Obs Mean SD				
EIntensityl	1973	0.086887	0.1584452	
EIntensityII	1973	0.0772852	0.1614714	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr(T > t) = 0.0164				

Table: England - Vulnerability among targeted group - Paired t-tests

	Obs	Mean	SD	
Vulnerability to poverty - HHs with children				
fgtl	480	0.0256242	0.0660988	
fgtll	480	0.0120929	0.0363083	
Ho: mean(diff)	= 0 Ha: mean(dif	$ff) \neq 0$ $Pr(T > $	t) = 0.0000	
Vulnerability to poverty - HHs whose head is retired				
fgtl	715	0.0259855	0.0576641	
fgtll	715	0.0232811	0.0526733	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr($\mid T \mid > \mid t \mid$) = 0.2250				
Vulnerability to poverty - low-income HHs				
fgtl	278	0.0752275	0.1012279	
fgtll	278	0.0585404	0.0829827	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr(T > t) = 0.0022				



England - Vulnerability among HHs with children

Expected Intensity				
	Obs	Mean	SD	
EIntensityl	480	0.0837655	0.1522128	
EIntensityII	480	0.0672556	0.1303707	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr($ T > t $) = 0.0182				



England - Vulnerability among HHs whose head is retired

Expected Incidence				
	Obs	Mean	SD	
Elncidencel	715	0.195651	0.3129282	
Elncidencell	715	0.1449684	0.2550703	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr(T > t) = 0.0000				
Expected Intensity				
Obs Mean SD				
EIntensityl	715	0.0949148	0.1583907	
EIntensityII	715	0.0785798	0.1573525	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr($\mid T \mid > \mid t \mid$) = 0.0165				



England - Vulnerability low-income HHs

Expected Intensity				
	Obs	Mean	SD	
EIntensityl	251	0.2546807	0.1715719	
EIntensityII	251	0.2028458	0.1570548	
Ho: mean(diff) = 0 Ha: mean(diff) \neq 0 Pr(T > t) = 0.0000				

The Shapley decomposition

Which aspect contributed the most in explaining the change in vulnerability

(Shapley, 1953; Shorrocks, 1999; Chakravarty et al., 2008)

$$\Delta V_{\alpha=2,h} = f(\Delta EH_h, \Delta EI_h, \Delta ECV_h^2)$$
(10)

$$C(\Delta EH) = \frac{2}{6} \left[\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^{2} \neq 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^{2} \neq 0) \right] + \\ + \frac{1}{6} \left[\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^{2} \neq 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^{2} \neq 0) \right] + \\ + \frac{1}{6} \left[\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^{2} = 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^{2} = 0) \right] + \\ + \frac{2}{6} \left[\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^{2} = 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^{2} = 0) \right],$$

$$(11)$$

Decomposing the Foster-Greer-Thorbecke Index of Vulnerability to Poverty Results - England II

The Shapley decomposition II

How does it work?

► Let's denote $\Delta V = I$, $\Delta EH = a$, $\Delta EI = b$, and $\Delta ECV^2 = c$, so that I = C(a) + C(b) + C(c).

How to obtain C(a), C(b) and C(c)?

- Let's focus on C(a)
- ► C(a) can be obtained:
 - 1. $I(a,b,c) I(b,c) \times 2$ (a,b,c and a,c,b) 2. $I(a,b) - I(b) \times 1$ (c,a,b)
 - 3. $I(a,c) I(c) \times 1 (b,a,c)$
 - 4. $I(a) I \times 2$ (b,c,a and c,b,a)
- the order in which the factors are $eliminated^1$ matters,
- the final contribution will be a sum of all the contributions, weighted by the number of possible permutations (3!),
- C(b) and C(c) will be computed in the same way.

¹kept constant

The Shapley decomposition: Results

Table: Shapley decomposition.

England				
$\Delta V_{\alpha=2,h}$ (s.d.)	$C(\Delta EH_h)$ (s.d.)	$C(\Delta El_h)$ (s.d.)	$C(\Delta ECV_h^2)$ (s.d.)	
-0.0047 (0.064)	-0.0016 (0.037)	-0.0035 (0.032)	0.00033 (0.023)	
100%	43.9%	36.7%	19.4%	



<ロト <回ト < 巨ト < 巨ト < 巨ト 三 三 つへで



<ロト <回ト < 巨ト < 巨ト < 巨ト 三 三 つへで

Conclusions

- ► Risk is different in size, likelihood and frequency → Different features correspond to different implications for policy (Dercon, 2001).
- Decomposition of the Foster-Greer-Thorbecke Index of Vulnerability to Poverty as description of poverty risk:
 - Expected Incidence,
 - Expected Intensity,
 - Expected Downward Variability.
- Possibility of comparing vulnerability among periods and across regions/countries.

<ロ> <用> <用> < => < => < => < => <000</p>

Thanks for your attention!