Decomposing the Foster-Greer-Thorbecke Index of Vulnerability to Poverty

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Abstract

This paper applies the decomposition of the Foster-Greer-Thorbecke poverty index to the measurement of the individual vulnerability to poverty. I highlight that poverty risk can be expressed as a function of expected incidence, expected intensity and expected downward variability. An empirical illustration is provided using the British Household Panel Survey (BHPS) and the Survey on Household Income and Wealth (SHIW).

Keywords: Vulnerability, Poverty risk, Decomposition, Foster-Greer-Thorbecke index
JEL classification: D63, I30, I32

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1 Introduction

Poverty analysis usually focuses on indexes that are sensitive to the number of people below the poverty line, the poverty gap and the distribution of income among the poor; these three poverty aspects are usually defined in literature as the three *Is* of poverty (Jenkins and Lambert, 1997). The description of the phenomenon based on these three components has been widely used because it helps in disentangling different sources of changes in poverty, allowing richer inter-temporal, inter-regional, cross-national or inter-group comparisons.

I propose to adopt the same approach to vulnerability to poverty, that is the probability, today, of being in poverty or to fall into deeper poverty in the future. Differently from the standard analysis of poverty, vulnerability is related to poverty risk with a more forward-looking perspective rather than an ex post lowness of income assessment. This concept is important because it can be considered an ex ante information source that allows the design of better protection policies to prevent households and individuals from experiencing severe welfare losses, rather than cure them when they are already poor (Chaudhuri *et al.* 2002, Zhang and Guanghua 2008, Jamal 2009).

Similarly to decomposing poverty as a function of incidence, intensity and inequality of income among the poor people, individual vulnerability to poverty in its Foster-Greer-Thorbecke (FGT) version, can be rewritten in terms of three potential sources of risk: the possible states of the world in which poverty is experienced (expected incidence), the expected poverty gap and a measure of the downward income volatility.

Each of these three components describes a particular aspect of poverty risk that can potentially lead to different risk-management policies. Being prone to poverty can increase because there are more possibilities that the income falls below a chosen poverty threshold, independently from the magnitude of the negative income shock. This source of risk recalls in some sense the incidence in the poverty decomposition framework, where the number of poor is substituted by the possible *contingencies* that an individual faces. Very close to intensity there is instead the expected poverty gap. If the latter increases also vulnerability is higher. The third contributing factor is downward variability of income: the higher this volatility the more unpredictable is the risk faced by the individual. The focus especially on negative shocks aims at separating out threats from the overall expectations, i.e. downward risks from uncertainty in general.

This view in terms of contributing factors that I propose meets the need, highlighted by Dercon (2001), of describing the different types of risk faced by individuals. He argues that risk is quite different in size, likelihood and frequency over time and different features correspond to different implications for the ability to cope with them as well as for policy purposes. Also Morduch (2000) says that it is important considering some of the patterns related to risk, since they have quite different impacts on the ability to cope with them for individuals, households, communities and other institutions. For instance it is possible to distinguish between *catastrophic versus non-catastrophic risks* according to the size of the shock. The former could be very unlikely with nonetheless a large impact so that it takes a long time before recovering from them. Different patterns of risk could also have different effects on the decision-making of individuals about investments in education or health.

This approach to vulnerability to poverty provides information that could be useful for policy makers who follow especially the World Development Report 2000/01's directions, where it is argued how optimal design should aim to strengthen, complement and replace existing coping strategies. It is stressed also the importance of overcoming the traditional *safety net policies*, which allow households to survive the consequences of poor outcomes in favor of welfare drops prevention. From this point of view therefore it is worthwhile examining poverty risk measures also in terms of their contributing components, to provide more accurate information about the ex ante risk faced by households.[1] If for instance poverty risk is due mostly to volatility and the inability of smoothing consumption (i.e. large expected downward volatility), risk-insurance programs or incentives for self-protecting savings are the candidates for helping households avoiding poverty. If instead rare catastrophic events are poverty trigger (i.e. large expected intensity), adequate financial support is needed to recover faster from them. When, on the contrary, there are several poverty episodes (i.e. large expected incidence) and the phenomenon becomes *structural*, the solution cannot be only financial but also based on non-monetary strategies. In this paper I will also present two empirical applications using British and Italian data.

[1] In the process proposed by Dercon (2001) for optimal policy design, this analysis is related especially to the first step about understanding the poverty risk.

2 The three vulnerability contributing factors

In poverty analysis the FGT family of poverty indexes (1) includes the *headcount ratio*, H, if $\alpha = 0$, the *poverty gap ratio*, I, if $\alpha = 1$. When $\alpha = 2$, (1) can be expressed as a function of headcount ratio, the poverty gap ratio and the squared *coefficient of variation* of income among the poor, CV^2, as inequality index[2]

$$P_\alpha(\mathbf{y}; \mathbf{z}) = \frac{1}{N} \sum_{h=1}^{Q} \left[\frac{z - y_h}{z} \right]^\alpha, \tag{1}$$

$$P_{\alpha=2}(\mathbf{y}; \mathbf{z}) = H \left[I^2 + (1 - I)^2 CV_p^2 \right], \tag{2}$$

$$H = Q/N, \tag{3}$$

$$I = \frac{1}{Q} \sum_{h=1}^{Q} \left[\frac{z - y_h}{z} \right], \tag{4}$$

$$CV_p^2 = \frac{1}{Q} \sum_{h=1}^{Q} \frac{(\mu_p - y_h)^2}{\mu_p^2}. \tag{5}$$

In the expressions (1)-(5), Q represents the number of households whose income y_h is below the chosen poverty line, z, N is the dimension of the society and μ_p is the average income of poor households. The parameter α can be considered the weight attached to extreme poverty, the higher this value the greater the aversion for deep poverty.

$$V_{\alpha=2,h}(\tilde{\mathbf{y}}; \mathbf{z}) = \sum_{s=1}^{S_h} p_s \left[\frac{z - \tilde{y}_s^h}{z} \right]^2. \tag{6}$$

The analogous in the vulnerability framework when $\alpha = 2$ is contained in (6). Differently from the poverty context, it focuses on the individual level rather than on the society. Instead of considering a vector of actual household incomes, $\mathbf{y} = (y_1, y_2, .., y_N)$, as the poverty index does, in the vulnerability analysis there is a vector of possible income values at $t + 1$ for the household h, $\mathbf{y}_s^h = (y_1^h, y_2^h, .., y_N^h)$, where N are the *possible states of the world* that the household could face.[3] Let us consider a new vector $\tilde{\mathbf{y}}_s^h$, which represents a permutation of $\mathbf{y}_s^h$, so that the elements are non-decreasingly ranked, i.e. for all $\tilde{y}_s^h$,

[2] An alternative decomposition is described in Aristondo *et al.* (2010).

[3] For expositional convenience, I assume that the number of possible states of the world for each household is the same, but nothing changes if N is substituted by N_h.

$\tilde{y}_1^h \leq \tilde{y}_2^h \leq \ldots \leq \tilde{y}_{S_h}^h \ldots \leq \tilde{y}_N^h$. I denote S_h the number of states in which the welfare measure is expected to fall below the poverty threshold, z, and p_s the probability that the s^{th} state occurs. The FGT index of vulnerability for the household h will be a sum of possible poverty gaps in $t+1$, weighted by the their probability.

The decomposition proposed by Foster *et al.* (1984), applied to vulnerability to poverty, can be performed as follows: EH is the *expected incidence*, i.e. the number of states in which the household is expected to be poor; the aggregate poverty gap is substituted by EI, the *expected intensity* or expected poverty gap, and finally ECV^2 replaces the inequality among the poor and describes in this context the *expected downward variability* for the household income, where μ_h is the expected average income for the household h during poverty,

$$V_{\alpha=2,h}\left(\tilde{\mathbf{y}};\mathbf{z}\right) = EH_h\left[EI_h^2 + (1-EI_h)^2 ECV_h^2\right] \tag{7}$$

$$EH_h = \frac{S_h}{N} \tag{8}$$

$$EI_h = \sum_{s=1}^{S_h} p_s' \frac{(z-\tilde{y}_s^h)}{z}, \quad p_s' = \frac{1}{S_h} \tag{9}$$

$$ECV_h^2 = \sum_{s=1}^{S_h} p_s' \frac{(\mu_h-\tilde{y}_s^h)^2}{\mu_h^2}, \quad p_s' = \frac{1}{S_h}. \tag{10}$$

It is possible to derive also an expression for the change of the FGT vulnerability index, which will depend on the variations of its three contributing factors. To show this more explicitly, the subscripts 1 and 0 are used referring to the period in which vulnerability is measured. The change of $V_{\alpha=2,h,t}$ between the values at times 0 and 1 can then be expressed as

$$\begin{aligned}\Delta V_{\alpha=2,h} = EH_{h,1}\left[EI_{h,1}^2 + (1-EI_{h,1})^2 ECV_{h,1}^2\right] - \\ - EH_{h,0}\left[EI_{h,0}^2 + (1-EI_{h,0})^2 ECV_{h,0}^2\right],\end{aligned} \tag{11}$$

$$\Delta V_{\alpha=2,h} = f(\Delta EH_h, \Delta EI_h, \Delta ECV_h^2) \tag{12}$$

where the operator Δ denotes the variation between times 0 and 1 of $V_{\alpha=2,h}$ and the three factors that appear in (12). In Appendix A I describe the Shapley decomposition of (11) to derive the contributions of ΔEH_h, ΔEI_h and ΔECV_h^2 to the overall change in the FGT vulnerability index, $V_{\alpha=2,h}$, as suggested by Chakravarty *et al.* (2008).

3 Data

I will estimate vulnerability to poverty and its three components using data of the British Household Panel Survey (BHPS) to show an inter-temporal comparison and the Italian Survey on Household Income and Wealth (SHIW) for an inter-regional empirical illustration.

The BHPS follows a representative sample of British households yearly; I consider especially the period 1991-2004. Additional sub-samples were added in 1997 and 1999, respectively Scotland-Wales and Northern Ireland, to increase the relative small Scottish and Welsh samples size and to cover Norther Ireland properly, for a UK analysis rather than England only.[4] In the empirical application I do not include those sub-samples in order to allow a more straightforward inter-temporal comparison, therefore the focus will be on England only. The disposable annual equivalized household income is used as welfare measure; this information is provided in the survey for those households in which all eligible adults gave a full interview. The equivalence scale used is the square root of the household size and all values have been expressed in real terms (deflated to January 1998 prices). The final sample is composed by 1973 households,[5] whose characteristics are summarized in Table 1.

- Table 1 here -

For an inter-regional illustration, the SHIW is used; it collects information for a representative sample of the Italian population about the households disposable income and consumption.[6] In this case in which both income and consumption are available, I use the latter as welfare measure since it incorporates the risk-management strategies of the household.[7] The Italian survey is slightly different from the BHPS because it is conducted every

[4]For a more detailed description of the data see http://www.iser.essex.ac.uk/bhps.

[5]I selected those households that were present in the panel for at least three times in the periods 1991-1997 and 1998-2004, to have sufficient observations for the vulnerability computation and the inter-temporal comparison. Moreover, I do not use sample weights provided in the BHPS because related to a rather special sample in the dataset.

[6]See http://www.bancaditalia.it/statistiche/indcamp/bilfait for a more detailed description of the data.

[7]Consumption is deflated to 1991 prices.

two years;[8] the time period that I will consider for the analysis is 1989-2004.[9] For the SHIW, the final sample size is 2519 households[10] and it is described in Table 1.

For England the FGT vulnerability index will be computed in two periods of time, splitting the dataset in two parts with equal number of waves, 1991-1997 and 1998-2004, then vulnerability will be computed using data up to 1997 and compared with that of the second period, for each household. By doing this, I assume implicitly that, within the period, I observe for each household income values drawn from the same distribution. The poverty lines used are the 60% of the median values respectively in 1997 and 2004. For England I propose also the Shapley decomposition, in order to understand which factor, among the three listed (8, 9 and 10), contributed the most in explaining the changes in poverty risk.

The FGT version of vulnerability to poverty is computed using as possible income values those already experienced by the household in the past, assuming that the data are informative about all the possible idiosyncratic shocks; the probabilities, p_{s_h}, are given by $1/d$, where d is the number of observations for each household. Very similar is the computation of vulnerability in the Italian case, with the only difference that I consider only one period, because I am interested in comparing the poverty risk across regions. The poverty line is computed as the 60% of the median equivalised household consumption in 2004.

4 Empirical Illustrations

The decomposition described is now applied to England and Italy as illustrative examples respectively for an inter-temporal and inter-regional comparison of the poverty risk and its contributing factors.

This type of analysis is interesting in the British case because of the welfare reform implemented in the late 1990s. According to Gregg (2008), the objective of the government in 1996/1997 was to increase economic activity, limit welfare dependency and, at the same time, reduce poverty. To meet these goals, the government proposed a strategy based on

[8] The data are collected every two years from 1987, with an exception for the year 1998 when information was gathered three years after 1995.

[9] Even if the Bank of Italy provides data from 1977, the longitudinal component starts only from 1987, but I restrict the time period analyzed to 1989-2004 because, as already pointed out in literature (Biagi *et al.*, 2009), two few households remain in the panel from 1987 to 1989.

[10] The sample selection in this case is different from the previous case, since I am interested only in comparing vulnerability across regions, I therefore selected those households that were present in the year 2004 and observed for at least three times.

the following measures: incentives to work, welfare payments conditional on behavioral requirements, minimum income secure for vulnerable groups and incentives for self-protecting savings among low income groups. Also Brewer *et al.* (2006) report that the reduction of poverty amongst pensioners and households with children has formed an important part of the Labour government's agenda, especially during its second term in office (2000/01-2004/05). Poverty, measured as the number of families whose income is below the 60% of the median equivalized income, fell by 2.1 %, considering incomes after housing costs, during the Labour's first term (1996/97-2000/01), and slightly faster during the second term (2.5%).

In more details, a particularly relevant measure was the introduction of and, later increases in, the National Minimum Wage (NMW). The previous industry specific minimum wage system, set by the Wages Councils, was introduced in 1917 and abolished in 1993. In 1998 a new NMW was proposed by the Low Pay Commission for the whole country. The minimum level was not raised much above prices until 2001, after which a sharp increase occurred until 2006. The effects of this measure can be noticed, according to Gregg (2008), looking at the growth by decile of the earnings distribution. Prior to the introduction of the NMW, the growth in earnings was slower in lowest decile and faster at the top of the distribution. By contrast, after the introduction, the most rapid growth in earnings was registered at the lowest paid part of the distribution, while the upper part has continued in a very similar fashion as before.

While the NMW focused especially on the pay of all low paid workers, independently from the family structure, the innovations in the Tax and Benefit System tried to account for families with dependent children. The government proposed an expansion of the Tax Credit system (then called Family Credit) in two directions: the Working Tax Credit and the Child Tax Credit. Before 1998 support for children came from four sources whose generosity was increased starting with the March 1998 budget. According to Gregg (2008), this reform partly reflects the Government thought that poverty was concentrated among families with younger children. The overall impact of the new Childrens Tax Credit was that families with children, independently from their marital status, received around twice as much as before while married childless couples lost an extra tax allowance.

At the same time The Working Families Tax Credit (WFTC) was announced, and became available to claimants from October 1999. Compared to its predecessor, it increased support for those in full-time or better paid part-time work (i.e. earning more than £92.90) and extended eligibility to in-work support to a large number of families. For a detailed description, see Gregg (2008), who reports that for lower earnings individuals there was also a significant reduction in income tax and National Insurance (NI) contributions.

Specifically targeted for vulnerable groups, the government introduced also the so-called *Personalized Welfare-to-work Support* that is the delivering of a support services package tailored to the individual's needs of lone parents, sick and disabled. For pensioners instead, the Labour government chose to support the poorest individuals by increasing the value of means-tested benefits. The Minimum Income Guarantee was introduced in 1999, then changed to Pension Credit in 2003. These reforms have had relatively good outcomes in terms of a lower pensioner poverty and higher replacement rates at the bottom of the income distribution (Gregg, 2008).

Given all these innovations in the British welfare system in favour of low-pay workers, families with children, vulnerable groups and pensioners, England offers an interesting illustrative example for the inter-temporal analysis of poverty risk and its factors.

The aim of this empirical application is not to test causal effects or to evaluate the effectiveness of these policies, but to describe how the poverty risk has evolved in a period of relevant changes.[11]

- Table 2 here -

Looking at Table 2 where the averages of the whole index and its contributing factors are reported, it is possible to observe that vulnerability to poverty has decreased between the two periods, from 0.0246 to 0.0189 on average. This difference is statistically different from zero according to the paired t-test[12] in Table 3 where it is shown the rejection of the null hypothesis, i.e. equality in poverty risk between the two periods analyzed.

[11] Piachaud *et al.* (2000) attempt to evaluate the potential impacts of the government initiatives on child poverty. Using micro-simulation modeling, they estimated an increase in incomes of the poorest more than those better-off and of households with children more than others. They also simulated a decrease in the proportion of children in poverty (living in households with equivalized disposable income below 50% of mean value) from 26% to 20% and a reduction in the size of the poverty gap. Moreover Gregg (2008) argues that there has been a decline in poverty among families with children which came about partly through increased employment and partly through the increased generosity of benefits.

[12] The test takes into account that the two samples are not independent.

- Table 3 here -

After having decomposed the vulnerability index, it is possible to notice that the reduction in poverty risk is driven by the expected incidence that decreases from 0.1728 to 0.1280. Downward variability and expected intensity stay quite constant between the two periods, in fact in Table 3 we accept the null hypothesis of equality in the paired t-tests. This result is confirmed also looking at Table 4, where the contributions of each factor variation has been estimated using the Shapley decomposition. It can be noticed that the expected incidence, i.e. the number of periods in which the household could experience poverty, explains on average about the 86% of the inter-temporal variation measured with the FGT vulnerability index. The whole index has decreased because of a reduction in the possible states in which the household experiences poverty but understanding which policy has especially driven this result remains to be explored. Even if the causal effect must be documented, the attempt to favor work participation or to condition financial support to active job search seems to be a possible successful strategy for reducing expected incidence through earnings.

- Table 4 here -

Since some welfare reforms were particularly targeted for specific groups, it is interesting looking more in details at those. I consider therefore families with children, pensioners and low-income households.

- Table 5 -

Table 5 reports the vulnerability index and its contributing factors in the two periods for households with at least one child. If the paired t-test are performed, it is possible to notice how the reduction is always statistically significant on average, with a lower level of confidence for the expected downward variability.

If the focus is on households whose head is retired, there is not a statistically significant change in the overall index, but in only one of its contributing factors, the expected incidence that decreases between the two periods (Table 6).

- Table 6 here -

- Table 7 here -

Table 7 reports the poverty risk indexes for those households that were in the lowest[13] part of the income distribution in both periods analyzed. The t-tests suggest a statistically significant decrease in the overall vulnerability index, driven by the expected incidence.

I propose also a second example: the inter-regional comparison of vulnerability to poverty using Italian data. According to the Italian National Institute of Statistics (ISTAT), Italy is characterized by a strong territorial difference in poverty rates; from 1997 to 2006 in the South the incidence of poverty is about five times higher than the North. Italy therefore represents an interesting example for an inter-regional comparison to highlight how risk changes according to regions or groups of regions. In this case I consider three groups of regions: those in the North-, Centre- and South-Italy.[14]

- Table 8 -

As expected, Table 8 shows how the poverty risk in the sample is mainly concentrated in the South-regions, the index is in fact more than six times higher than North- and Centre-Italy. In Table 9, the t-tests suggest that the poverty risk between North- and Centre-Italy is not statistically different, while it does increase if we compare the South with them.

- Table 9 -

For a more detailed description of poverty risk, it is possible to look at the three contributing factors: expected incidence is on average five times higher in the South than the other Italian regions, the expected poverty gap is about 0.1351 compared to 0.0240 and 0.0268 respectively in the North and in the Centre and finally also the downward variability is much larger in the South. See Appendix C for a more detailed regional breakdown. By performing the equality tests, the null hypothesis is accepted always when comparing North- and Centre-Italy while the South always registers higher statistically significant values (Table 10, 11 and 12).

- Table 10 -

- Table 11 -

- Table 12 -

[13] I define as the lowest part of the income distribution up to the 25^{th} percentile.

[14] I include the islands in the South-Italy category.

This picture of vulnerability in Italy confirms the strong territorial component of the poverty phenomenon, characterized by a persistent large gap between poverty risk in the North-/Centre-Italy and the South. In this illustration I adopted a national relative poverty line for simplicity; this choice is appropriate as long as there are not substantial differences in the cost of living across regions. On the contrary, if the cost of living is not homogeneous in the country, by using a national relative poverty line, the consequences are an underestimation of the poverty risk where the cost is higher and an overestimation where that cost is lower. The example that I propose is just a simple illustration about the vulnerability index decomposition, that can be easily adjusted to regional differences in the poverty line if the focus is on accurate poverty risk measurement.

5 Conclusions

For a more complete description of the phenomenon, poverty is usually described in terms of the number of people below the poverty line, the poverty gap and the distribution of income among the poor, as Sen (1976) proposed.

Using the decomposition of one of the FGT poverty index ($\alpha = 2$) (Foster *et al.*, 1984), I suggest to express also individual vulnerability to poverty as function of three contributing factors, expected incidence, expected intensity and downward variability. This approach to poverty risk can be useful as information source for policies design, since different patterns of risk faced by individuals could lead to different risk management policies (Dercon, 2001).

References

Aristondo, O., De la Vega, C. L. and Urrutia, A. (2010) A new multiplicative decomposition for the Foster-Greer-Thorbecke Poverty Indices, *Bulletin of Economic Research*, **62**, 259–267.

Biagi, F., Giraldo, A. and Rettore, E. (2009) Gli effetti dell'attrito sulla stima della disuguaglianza in italia, in *Dimensioni della disuguaglianza in Italia: povertà, salute, abitazione* (Eds.) B. A. S. C. and A. Schizzerotto, Il Mulino, Bologna.

Brewer, M., Goodman, A., Shaw, J. and Sibieta, L. (2006) Poverty and inequality in britain: 2007, institute for Fiscal Studies: IFS Commentary, No. 73.

Chakravarty, S. R., Deutsch, J. and Silber, J. (2008) On the Watts multidimensional poverty index and its decomposition, *World Development*, **36**, 1067–1077.

Chaudhuri, S., Jalan, J. and Suryahadi, A. (2002) Assessing household vulnerability to poverty from cross-sectional data: A methodology and estimates from Indonesia, Discussion Papers 0102-52, Columbia University, Department of Economics.

Dercon, S. (2001) Assessing vulnerability to poverty, Jesus College, Oxford and Centre for the Study of African Economies (CSAE), Department of Economics, Oxford University.

Foster, J., Greer, J. and Thorbecke, E. (1984) A class of decomposable poverty measures, *Econometrica*, **52**, 761–766.

Gregg, P. (2008) Uk welfare reform 1996 to 2008 and beyond: A personalised and responsive welfare system?, The Centre for Market and Public Organisation 08/196, Department of Economics, University of Bristol, UK.

Jamal, H. (2009) Assessing vulnerability to poverty: Evidence from pakistan, social Policy and Development Centre (SPDC): Research Report N.80.

Jenkins, S. P. and Lambert, P. J. (1997) Three 'I's of poverty curves, with an analysis of UK poverty trends, *Oxford Economic Papers*, **49**, 317–327.

Morduch, J. (2000) Between the state and the market: Can informal insurance patch the safety net?, *World Bank Research Observer*, **14**, 187–207.

Piachaud, D., Sutherland, H. and Centre, U. I. R. (2000) How effective is the british government's attempt to reduce child poverty?, Tech. rep.

Sen, A. K. (1976) Poverty: an ordinal approach to measurement, *Econometrica*, **44**, 219–231.

Shapley, L. S. (1953) A value for n-person games, in *Contributions to the theory of games II, Annals of mathematics studies* (Eds.) H. W. Kuhn and A. W. Tucker, Princeton University Press, Princeton.

Shorrocks, A. F. (1999) Decomposition procedures for distributional analysis. a unified framework based on the Shapley value, University of Essex: mimeo.

Zhang, Y. and Guanghua, W. (2008) Can we predict vulnerability to poverty?, WIDER Research Paper N. 2008/82.

A The Shapley decomposition

The Shapley decomposition technique (Shapley, 1953) was for the first time applied in game theory, then Shorrocks (1999) used this method in distributional analysis to decompose also income inequality indexes. In this paper I propose, as in Chakravarty *et al.* (2008), the Shapley decomposition to understand the factors contributions to the change over time in the value of the indicator $V_{\alpha=2,h,t}$. I denote $\Delta V = I$ the change of $V_{\alpha=2,h,t}$ and ΔEH, ΔEI, and ΔECV^2 represent respectively the variations over time of the three determinants EH, EI, and ECV^2. Since the change in the vulnerability index, I, can be expressed as a function of three variables $\Delta EH = a$, $\Delta EI = b$, and $\Delta ECV^2 = c$, the contribution $C(a)$ of a in explaining I, can be expressed by the following

$$C(a) = \frac{2}{6}\left[I(a,b,c) - I(b,c)\right] + \frac{1}{6}\left[I(a,c) - I(c)\right] + \frac{1}{6}\left[I(a,b) - I(b)\right] + \frac{2}{6}\left[I(a)\right], \qquad (13)$$

where the order in which a,b and c are eliminated is taken into account. Similarly it is possible to determine the marginal contribution $C(b)$ of b and $C(c)$ of c and then find out that

$$I(a,b,c) = C(a) + C(b) + C(c). \qquad (14)$$

In order to clarify that in case analysed a, b and c represent changes in the contributing factors, I rewrite the marginal contribution of a as follows

$$\begin{aligned} C(\Delta EH) = & \frac{2}{6}\left[\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^2 \neq 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^2 \neq 0)\right] + \\ & + \frac{1}{6}\left[\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^2 \neq 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^2 \neq 0)\right] + \\ & + \frac{1}{6}\left[\Delta V(\Delta EH \neq 0, \Delta EI \neq 0, \Delta ECV^2 = 0) - \Delta V(\Delta EH = 0, \Delta EI \neq 0, \Delta ECV^2 = 0)\right] + \\ & + \frac{2}{6}\left[\Delta V(\Delta EH \neq 0, \Delta EI = 0, \Delta ECV^2 = 0) - \Delta V(\Delta EH = 0, \Delta EI = 0, \Delta ECV^2 = 0)\right], \end{aligned} \qquad (15)$$

where $\Delta EH = 0$ means that, when the change in $V_{\alpha=2,h,t}$ is computed, I assume that the expected incidence did not change between time 0 and 1, whereas $\Delta EH \neq 0$ will mean that the expected incidence changed. Similar interpretations hold for ΔEI and ΔECV^2.

B The decomposition of Vulnerability to Poverty

$$
\begin{aligned}
V_{\alpha=2,h}\left(\tilde{\mathbf{y}};\mathbf{z}\right) &= \frac{1}{N}\sum_{s=1}^{S_h}\left[\frac{z-\tilde{y}_s^h}{z}\right]^2 \\
&= \frac{S_h}{N}\sum_{s}^{S_h}\frac{1}{S_h}\left[\frac{z-\mu+\mu-\tilde{y}_s^h}{z}\right]^2 \\
&= \frac{S_h}{N}\sum_{s=1}^{S_h}\frac{1}{S_h}\left[\frac{(z-\mu)^2}{z^2}+\frac{2(z-\mu)(\mu-\tilde{y}_s^h)}{z^2}+\frac{(\mu-\tilde{y}_s^h)^2}{z^2}\right] \\
&= \frac{S_h}{N}\left[\sum_{s=1}^{S_h}\frac{1}{S_h}\left(\frac{(z-\mu)^2}{z^2}\right)+\sum_{s=1}^{S_h}\frac{(\tilde{y}_s^h)^2}{S_h z^2}-\frac{\mu^2}{z^2}\right] \\
&= \frac{S_h}{N}\left[\left(\frac{(z-\mu)^2}{z^2}\right)+\sum_{s=1}^{S_h}\frac{1}{S_h}\frac{(\tilde{y}_s^h-\mu)^2}{z^2}\right] \\
&= \frac{S_h}{N}\left[\left(\frac{(z-\mu)^2}{z^2}\right)+\sum_{s=1}^{S_h}\frac{1}{S_h}\frac{(\tilde{y}_s^h-\mu)^2\mu^2}{z^2\mu^2}\right] \\
&= \frac{S_h}{N}\left[\left(\frac{(z-\mu)^2}{z^2}\right)+\sum_{s=1}^{S_h}\frac{1}{S_h}\left(\frac{-2z^2+2z\mu+2z^2-2z\mu+\mu^2}{z^2}\right)\frac{(\tilde{y}_s^h-\mu)^2}{\mu^2}\right] \\
&= \frac{S_h}{N}\left[\left(\frac{(z-\mu)^2}{z^2}\right)+\left(-2\frac{z-\mu}{z}+1+\frac{z^2-2z\mu+\mu^2}{z^2}\right)\sum_{s=1}^{S_h}\frac{1}{S_h}\frac{(\tilde{y}_s^h-\mu)^2}{\mu^2}\right]
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
V_{\alpha=2,h}\left(\tilde{\mathbf{y}};\mathbf{z}\right) &= \frac{S_h}{N}\left[\left(\frac{(z-\mu)^2}{z^2}\right)+\left(-2\frac{z-\mu}{z}+1+\frac{z^2-2z\mu+\mu^2}{z^2}\right)\sum_{s=1}^{S_h}\frac{1}{S_h}\frac{(\tilde{y}_s^h-\mu)^2}{\mu^2}\right] \\
&= EH_h\left[EI_h^2+(1-EI_h)^2 ECV_h^2\right]
\end{aligned} \tag{17}
$$

$$EH_h = \frac{S_h}{N} \tag{18}$$

$$EI_h = \sum_{s=1}^{S_h} p'_s\frac{(z-\tilde{y}_s^h)}{z}, \quad p'_s = \frac{1}{S_h} \tag{19}$$

$$ECV_h^2 = \sum_{s=1}^{S_h} p'_s\frac{(\mu-\tilde{y}_s^h)^2}{\mu^2}, \quad p'_s = \frac{1}{S_h} \tag{20}$$

C Italy: inter-regional comparison

Italy - FGT vulnerability index $\alpha = 2$

Italy - Expected Incidence, EH

Italy - Expected Intensity, EI

Italy - Expected Downward variability, ECV^2

D Tables

Table 1: *Sample Characteristics*

	UK - BHPS (1991-2004)		Italy - SHIW (1989-2004)	
Household Head's age:	Obs	%	Obs	%
$\leq$ 34	89	4.51	66	2.62
35-44	392	19.87	373	14.81
$\geq$ 45	1492	75.62	2080	82.57
Education:	Mean	S. D.	Mean	S. D.
% O-level or lower in HH	0.36	0.42	0.70	0.36
% A-level or equivalent in HH	0.20	0.31	0.22	0.30
% Degree or higher in HH	0.12	0.27	0.08	0.21
% Earners in HH	0.45	0.40	0.32	0.32
% Children in HH	0.10	0.20	0.09	0.17

Table 2: *Vulnerability to poverty and its contributing factors - England*

BHPS (1997-2004)				
	$V_{\alpha=2,h}$ **(s.d.)**	EH_h **(s.d.)**	EI_h **(s.d.)**	ECV_h^2 **(s.d.)**
t = I	0.0246 (0.069)	0.1728 (0.303)	0.0989 (0.168)	0.0171 (0.093)
t = II	0.0189 (0.055)	0.1280 (0.247)	0.0970 (0.181)	0.0210 (0.150)

Notes: $V_{\alpha=2,h}$ is the average vulnerability. Period I: 1991-1997. Period II: 1998-2004

Table 3: *England - Paired t-tests*

Vulnerability to poverty			
	Obs	**Mean**	**SD**
fgtI	1973	0.0246372	0.0687573
fgtII	1973	0.0188850	0.0546004
diff	1973	0.0057522	0.0653570
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr($\mid T \mid > \mid t \mid$) = **0.0001**			
Expected Incidence			
	Obs	**Mean**	**SD**
EIncidenceI	1973	0.172791	0.3034075
EIncidenceII	1973	0.1279608	0.2471147
diff	1973	0.0448302	0.2434716
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr($\mid T \mid > \mid t \mid$) = **0.0000**			
Expected Intensity			
	Obs	**Mean**	**SD**
EIntensityI	1973	0.0989191	0.167721
EIntensityII	1973	0.0970404	0.1810023
diff	1973	0.0018787	0.1877558
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr($\mid T \mid > \mid t \mid$) = 0.6568			
Expected downward variability			
	Obs	**Mean**	**SD**
EDownVariabilityI	1973	0.017066	0.0928077
EDownVariabilityII	1973	0.0210122	0.1503004
diff	1973	-0.0039462	0.1406362
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr($\mid T \mid > \mid t \mid$) = 0.2128			

Table 4: *Vulnerability to poverty decomposition - England*

BHPS - Contributing factors			
$\Delta V_{\alpha=2,h}$ **(s.d.)**	**C**(ΔEH_h) **(s.d.)**	**C**(ΔEI_h) **(s.d.)**	**C**(ΔECV_h^2) **(s.d.)**
-0.0057 (0.065)	-0.0049 (0.042)	-0.0007 (0.0322)	-0.0001 (0.018)

Table 5: *Vulnerability among households with children - England - Paired t-tests*

	Obs	**Mean**	**SD**
Vulnerability to poverty			
fgtI	480	0.0256242	0.0660988
fgtII	480	0.0120929	0.0363083
diff	480	0.0135313	0.0641141
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0000**			
Expected Incidence			
EIncidenceI	480	0.1662897	0.2976191
EIncidenceII	480	0.0881548	0.1936527
diff	480	0.0781349	0.2352904
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0000**			
Expected Intensity			
	Obs	**Mean**	**SD**
EIntensityI	480	0.0968764	0.1673252
EIntensityII	480	0.073816	0.149735
diff	480	0.0230604	0.1699051
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0031**			
Expected downward variability			
	Obs	**Mean**	**SD**
EDownVariabilityI	480	0.0132477	0.0482428
EDownVariabilityII	480	0.0084687	0.0435847
diff	480	0.004779	0.0621858
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0929**			

Table 6: *Vulnerability among households whose head is retired - England - Paired t-tests*

	Obs	**Mean**	**SD**
Vulnerability to poverty			
fgtI	715	0.0259855	0.0576641
fgtII	715	0.0232811	0.0526733
diff	715	0.0027044	0.0595475
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = 0.2250			
Expected Incidence			
EIncidenceI	715	0.2348685	0.3377639
EIncidenceII	715	0.2053413	0.3039077
diff	715	0.0295271	0.2627566
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0028**			
Expected Intensity			
	Obs	**Mean**	**SD**
EIntensityI	715	0.118607	0.1673879
EIntensityII	715	0.1250038	0.1778733
diff	715	-0.0063968	0.185038
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = 0.3556			
Expected downward variability			
	Obs	**Mean**	**SD**
EDownVariabilityI	715	0.0165163	0.0813092
EDownVariabilityII	715	0.0179087	0.1117814
diff	715	-0.0013924	0.1214059
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = 0.7592			

Table 7: *Vulnerability among low-income households - England - Paired t-tests*

	Obs	**Mean**	**SD**
Vulnerability to poverty			
fgtI	278	0.0752275	0.1012279
fgtII	278	0.0585404	0.0829827
diff	278	0.0166871	0.0899281
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0022**			
Expected Incidence			
EIncidenceI	278	0.58568	0.3416604
EIncidenceII	278	0.4925916	0.339211
diff	278	0.0930884	0.3598192
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = **0.0000**			
Expected Intensity			
	Obs	**Mean**	**SD**
EIntensityI	278	0.248794	0.1595086
EIntensityII	278	0.24636	0.1707445
diff	278	0.002434	0.1720944
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = 0.8137			
Expected downward variability			
	Obs	**Mean**	**SD**
EDownVariabilityI	278	0.0473399	0.1368262
EDownVariabilityII	278	0.0465264	0.1734583
diff	278	0.0008135	0.1770532
Ho: mean(diff) = 0 Ha: mean(diff) $\neq 0$ Pr($\mid T \mid > \mid t \mid$) = 0.9390			

Table 8: *Vulnerability to poverty and its contribution factors - Italy*

ITALY (1989-2004)					
	Obs.	$V_{\alpha=2,h}$ **(s.d.)**	EH_h **(s.d.)**	EI_h **(s.d.)**	ECV_h^2 **(s.d.)**
North	1155	0.0033 (0.020)	0.0511 (0.144)	0.0240 (0.073)	0.0011 (0.012)
Centre	564	0.0030 (0.015)	0.0582 (0.150)	0.0268 (0.073)	0.0009 (0.007)
South	803	0.0219 (0.047)	0.2749 (0.327)	0.1351 (0.172)	0.0080 (0.023)

Table 9: *Vulnerability to poverty - Italy - T-tests*

North- and Centre-			
	Obs	**Mean**	**SD**
North	1155	0.0032655	0.0198168
Centre	564	0.0029624	0.014822
		Mean	**SE**
diff		0.0003031	0.0008541
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = 0.7227			
North- and South-			
	Obs	**Mean**	**SD**
North	1155	0.0033005	0.018727
South	803	0..0219552	0.0465589
		Mean	**SE**
diff		-0.0186896	0.0017434
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			
Centre- and South-			
	Obs	**Mean**	**SD**
Centre	564	0.0029624	0.014822
South	803	0.0219552	0.0465589
		Mean	**SE**
diff		-0.0189928	0.0017576
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			

Table 10: *Expected Incidence - Italy - T-tests*

North- and Centre-			
	Obs	**Mean**	**SD**
North	1155	0.0510596	0.1445176
Centre	564	0.058251	0.1490538
		Mean	**SE**
diff	-0.0071914	0.0075812	
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = 0.3430			
North- and South-			
	Obs	**Mean**	**SD**
North	1155	0.0510596	0.1445176
South	803	0.2749392	0.3271318
		Mean	**SE**
diff		-0.2238796	0.0123025
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			
Centre- and South-			
	Obs	**Mean**	**SD**
Centre	564	0.058251	0.1490538
South	803	0.2749392	0.3271318
		Mean	**SE**
diff		-0.2166882	0.0131401
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			

Table 11: *Expected Intensity - Italy - T-tests*

North- and Centre-			
	Obs	**Mean**	**SD**
North	1155	0.0240035	0.0725959
Centre	564	0.0268039	0.0728896
		Mean	**SE**
diff		-0.0028004	0.0037394
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = 0.4541			
North- and South-			
	Obs	**Mean**	**SD**
North	1155	0.0240035	0.0725959
South	803	0.1351524	0.1722431
		Mean	**SE**
diff		-0.1111489	0.0064427
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			
Centre- and South-			
	Obs	**Mean**	**SD**
Centre	564	0.0268039	0.0728896
South	803	0.1351524	0.1722431
		Mean	**SE**
diff		-0.1083485	0.0068093
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			

Table 12: *Expected Downward Variability - Italy - T-tests*

North- and Centre-			
	Obs	**Mean**	**SD**
North	1155	0.0010561	0.0125243
Centre	564	0.0009499	0.006831
		Mean	**SE**
diff		0.0001062	0.0004675
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = 0.8203			
North- and South-			
	Obs	**Mean**	**SD**
North	1155	0.0010561	0.0125243
South	803	0.0079493	0.0227018
		Mean	**SE**
diff		-0.0068932	0.0008818
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			
Centre- and South-			
	Obs	**Mean**	**SD**
Centre	564	0.0009499	0.0068319
South	803	0.0079493	0.0227018
		Mean	**SE**
diff		-0.0069994	0.0008512
Ho: mean(diff) = 0 Ha: mean(diff) $\neq$ 0 Pr(| T |>| t |) = **0.0000**			