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PPP's for SDR's? Towards A Coherent Measure of Global Inflation

Utz-Peter Reich

For additional information please contact:

Name: Utz-Peter Reich
Affiliation: Mainz University of Applied Sciences

Email Address: utz.reich@gmail.com

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Paper submitted for Session 8A
by
Utz-Peter Reich
Mainz University of Applied Sciences
utz.reich@gmail.com

Abstract
If it is a lesson learnt from the current financial-economic crisis that global markets and global money require global regulation then it is also true that global regulation requires global economic data. Purchasing power parities (PPP's) are well suited to meet this new need, because they are being established at regular time intervals, and their scope is world-wide, in principle. Their use has been restricted to observing the real economy so far, providing international volume comparisons for products at the elementary level of aggregation, for national industries at a higher level, and finally for domestic product and national income for countries as a whole.

At the national level of aggregation the bridge to the financial economy is reached, because the general price level, which serves as the measure of national inflation is equal (grosso modo) to the inverse of the purchasing power of the national currency. The paper develops this track.

The recently aired political proposal to replace the US-dollar in its role as a universal means of payment by some more diversified system such as the Special Drawing Rights employed by the IMF warrants an equally universe measure of monetary variation. Such a measure – so the claim of the paper – may be based on existing PPP's with relatively few adaptations in concept and some extension in data coverage. The paper is essentially of a theoretical nature with some experimental calculations for illustration.

World Financial System

The recent financial crisis and its spillover into economic recession have confronted the world with a long-standing, but still unanswered question: what kind of international reserve currency do we need to secure global financial stability and facilitate world economic growth? There were in history various institutional arrangements, including the Silver Standard, the Gold Standard, the Gold Exchange Standard and the Bretton Woods system. The underlying problem, however, as the ongoing financial crisis demonstrates, is far from being solved, and proves an inherent weakness of the current international monetary system. The crisis calls for creative reform of the existing international monetary system towards an international reserve currency with a stable value, rule-based issuance and manageable supply, so as to achieve the objective of safeguarding global economic and financial stability. What we need is, in the words of the governor the People's Bank of China, "an international reserve currency that is disconnected from individual nations and is able to remain stable in the long run". (Zhou Xiaochuan 2009)

Countries issuing currencies which are used as reserve by others are constantly confronted with the dilemma between achieving their domestic monetary policy goals and meeting other countries' demand for reserve currencies. On the one hand, the monetary authorities cannot simply focus on domestic goals without carrying out their international responsibilities; on the other hand, they cannot pursue different domestic and international objectives at the same time. They may either fail to adequately meet the demand of a growing global economy for liquidity as they try to ease inflation pressures at home, or create excess liquidity in the global markets by overly stimulating domestic demand.

The dollar reserve system may not be the only source of global financial instability, but it contributes to it. The question is, will the global economy lurch from the current system to another – such as the two currency reserve system towards which the world now seems to be moving – equally beset with problems? There is a remarkably simple solution, one which was recognized long ago by Keynes: the international community can provide a new form of fiat money to act as reserves. The countries of the world would agree to exchange the fiat money for their own currency, in times of crisis. (Stiglitz 2007, p.260) The desirable goal of reforming the international monetary system is to create an international reserve currency that is disconnected from individual nations and is able to remain stable in the long run, thus removing the inherent deficiencies caused by using credit-based national currencies.

Special consideration may be given to the SDR in playing a greater role. The SDR has the features and potential to act as a super-sovereign reserve currency. The scope of using the SDR should be broadened, so as to enable it to fully satisfy the member countries' demand for a reserve currency. The SDR, which is now only used between governments and international institutions, could become a widely accepted means of payment in international trade and financial transactions. (Zhou Xiaochuan 2009)

Any currency, whether national or international needs a control for inflation. At present, these are submitted implicitly by the currencies employed in the Special Drawing Rights (US$, €, Pound Sterling, Yen). But using the national rates of inflation of these countries as control implies that inflation of all other countries goes unnoticed, as long as exchange rates respond and shield it from outside. Purchasing power parities remain unaffected. A world money requires a measure of world inflation. It can be based on the already existing system of international purchasing power parities, which must be completed into incorporating a measure of change over time. The paper provide a theoretical structure for such a concept.

Purchasing power parities

The compilation of purchasing power parities began as a research project at the University of Pennsylvania and has since grown to become a regular activity of official statistical bodies, such as OECD, EUROSTAT and UN. Its purpose is to provide a sound empirical basis for international comparison of different national gross domestic products.(OECD 2005, p.1f) Each GDP being compiled and expressed in its own national currency, such comparison requires a rule of transforming different currencies into each other. The straight-forward way is to resort to official exchange rates, as they are determined either on the markets of foreign exchange or by the respective governments. This has been general practice before the advent of purchasing power parities. Exchange rates, however, are highly volatile, making a comparison of yearly GDPs dependent on a daily changing index, and in addition they only allow comparison of nominal GDP, and not of its complement real GDP, which is the universal measure of domestic production. Between the two there is the national price level, which differs between countries. In order to eliminate differences in national price levels purchasing power parities are required.

In their simple form purchasing power parities (PPPs) are price relatives that show the ratio of the prices in national currencies of the same commodity-product in different countries.[1] For example, if the price of a hamburger in France is 2.84 euros and in the United States it is 2.20 dollars, the PPP for hamburgers between France and the United States is 2.84 euros per 2.20 dollars or 1.29 euros to the dollar. In other words, for every dollar spent on hamburgers in the United States, 1.29 euros would have to be spent in France in order to obtain the same quantity or quality – or volume – in hamburgers. (OECD 2005 p.2)

Determination of such elementary PPPs forms the first stage of the compilation process. The second stage is situated at the level of product groups where the price relatives calculated for products in the group are combined in an unweighted average for the group as a whole. The third stage is attained at the level of aggregate GDP where the expenditures of each GDP component are compiled. Comparing GDP components (e.g. capital formation) at purchasing power parity means comparing the same volume of aggregate product.[2]

This paper addresses the third stage of aggregation only. It is at this stage the real economy and its monetary complement meet in the statistical system so that an extension of PPPs going beyond GDP to measuring a world price level, and its movement, the world rate of inflation, must start from here. The rationale for the extension is simple. The axiom that nominal GDP equals real GDP multiplied by the general price level is accepted world wide and obvious in countries which take the implied GDP deflator as the measure of their general price level. In other countries, using rather the Consumer Price Index, the relationship applies with a small transformation. The present concept of a world GDP established by means of purchasing power parities may generate its own complement of a world price level, and - more interesting,- its change over time, a world rate of monetary inflation.

As said above, the institutional backup for implementing such an extension is hardly visible, at present. But to the visionaries, such as quoted above, it appears at the horizon of the future, taking Special Drawing Rights from the IMF as point of departure. Whatever the currencies, on which such monetary proxy is based will be, it is hardly acceptable that only their price level will enter into the concern of global monetary policy. All national price levels must be included in such a measure in a fair share.

Index number theory

Searching for a measure of the general price level, or worse, a rate of inflation, one invariably runs into an index number problem. The problem was invented by Irving Fisher in the early 20th century when the science of physics stood at its height of glamour and served as the

[1] By commodity-product we mean a good or service which is being produced in an institutional unit and acquired for use through a market, ignoring the SNA distinction between market and non-market production at this stage.

[2] OECD 2005 cannot abstain from asserting that both volumes "will, in principle, provide equivalent satisfaction or utility" (p.2). The advantage of applying such microeconomic variables, which are unobservable by definition, to macroeconomic aggregates such as exports or capital formation has never really been proven. We prefer, therefore, to stay with the concept of product "volume" as defined in statistical systems. By the way, avoiding the term "equal" and using the term "equivalent" instead, the authors themselves admit to a certain uneasiness in this direct micro-macro identification.

model of every other science. Economics, in particular, learned to pursue a mathematical track which, for good or for bad, it has never left since. Fisher imitated physics, - not only by building a physical model of circulating fluids of an economy, still to be admired at Harvard University,- but by introducing the axiomatic method of building a theory. He postulated five axioms an economic index number would have to obey. In a second step he tried several index number formulae against these axioms, and finally decided for one index, which has been called "ideal" ever since. He did so in spite of the fact that the index violates one of the axioms, erected previously, a procedure that would be unthinkable in physics. Nevertheless, it established the index number problem and with it a new discipline of index number theory, which has flourished as an independent research field, although one has learned not to "search for the holy grail of index number theory" (Balk 2008) anymore.

In this paper we avoid the index number problem, and stay at a relatively low level of technical sophistication. The issue of a world price level and a corresponding rate of inflation is still at a stage of infancy where clarification of concepts, and design of statistical procedures are discussed rather than the choice of a certain index number formula. Were we to take our path through index number theory we would have to consider and choose between seven methods for making multilateral comparisons: the star method, the democratic and plutocratic weight methods, the GEKS method, the own share method, the average basket method and the GK method (Diewert 2008, p. 208). Fortunately, only two of these are of relevance in practice, and we will discuss these only. A similar situation prevails in respect to the dimension of time. The list of possible index number formulae suggested for comparison over time has grown almost to infinity, for all practical purposes, (as well as the number of tests supporting them) where again only a few have found their way into statistical organisations. It is unlikely that other formulae than the one's in use already will be applied when a world rate of inflation is to be determined
.

The aggregation system

The calculation of aggregate purchasing power parities is based on the assumption that physically idenical products represent the same economic value allmover the world ("a potato is a potato"). Hence it makes sense to invent a way of calculationg a world price as an analytical means to compare national price levels and to eliminate the effect of varying exchange rates on the product comparison. The procedure requires each country to provide a set of national annual prices for a selection of representative and comparable products chosen from a common basket of goods and services that covers the whole range of final expenditure on GDP and a detailed breakdown of final expenditure on GDP according to a common classification. (EUROSTAT, OECD 2005, p. 223) Let $v_i^j(t)$ be the value of a product flow in classification group *i* of country *j* at time *t*. It is denominated in the currency of country *j*, which is the currency in which its national accounts are compiled. Let $p_i^j(t)$ be a corresponding price index number furnished by the department of price statistics. It is then possible to derive, as a complement, a volume index number

(1) $$q_i^j(t) = \frac{v_i^j(t)}{p_i^j(t)}.$$

Both, the price index number and the volume index number are dimensionless variables. They acquire their economic meaning when attached to some base year *t=0*, which is selected arbitrarily. It is worth noting that in respect to their meaning price and volume index numbers are not symmetric. Price is an intensive variable; when two national markets of the same price are added price of the combined market remains the same. The two volumes, however, double. They are what in physics is called an extensive variable, a distinction which pure mathematical index number theory does not yet recognize.

The last stage of the PPP compilation process is reached by inserting these data into the following system of equations and solving for the unknowns $\pi_i\,(t)$ and $\varepsilon^j(t)$,

$$\pi_i \sum_j q_i^j - \sum_j \varepsilon^j v_i^j = 0\ ,\ i = 1,...$$
$$\sum_i \pi_i q_i^j - \varepsilon^j \sum_i v_i^j = 0\ ,\ j = 1,...$$

(2)

In equations 2, $\pi_i\,(t)$ defines a world price index number of product *i* [SDR/unit of product] at time t, and $\varepsilon^j(t)$ the exchange prevailing at purchasing power parity of the currency of country *j* [SDR/unit of national currency]. This is the stage with which we are concerned here. Equations 2 form a system of lienar homogeneous equations. Their number is given by the sum of the number of commodity classes figuring in the comparison and the number of countries participating. It is a Geary-Khamis (GK) system such as which is being used by the United Nations[3]. We have opted against the Eltetö-Köves-Szulc (EKS) system, the common alernative, for the following reason.

It is well known that the conflict between the two systems is one of the properties of transitivity versus characteristicity in multilateral and bilateral comparisons. (Daban, Doménech, Molinas 1997, p. 33) The GK system assures transitivity of comparison which means that a direct comparison of two countries gives the same figures as constructing the comparison through a third country. The EKS system, on the other hand, refers to the optimal character of the basket of products taken as representative of the patterns of expenditure in two countries, and also requires that the weights used in the comparison be solely based on spending patterns of these two countries. The weakness of the GK system is therefore that its representative basket changes with every country joining the group.

The UN is an organisation comprehending all nations of the world. Even if not all nations participate in the PPP project, yet, this is the perspective, and the number of countries is then naturally fixed. The situation is different for exclusive “clubs” such as EU and OECD where the selection of countries is open to variation, and rivalry between two similar countries is more at the order of the day. In solving for a unique world price index number for every single world product-commodity and an exchange rate that establishes a unique general price level through all countries of the world the GK system recommends itself as the appropriate tool for transforming a set of national economies into one world economy, at the accounting level.

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[3] Symbols and definition of variables are slightly at variance wirh offical PPP notation. For establishing the connection, see the appendix.

Normalising the system

Equations 2 form a linear homogeneous system. It can be solved if its rank is lower than the number of equations, which is assured by its construction. But if it can be solved its solution is determined up to a scaling factor only. The system delivers a set of proportions, not of absolute values. The absolute value of the unknowns is fixed by adding an equation of normalisation. Such normalisation is not an arbitrary decision; for it implies more than simply finding a numeraire. It defines the standard of measurement.

A simple rule such as setting the exchange rate of one country equal to one does not suffice ere. For assume country *k* is that country. Taking its currency as numeraire means

(3) $$\varepsilon^k = 1 \ .$$

It follows from the second set of equations 2 and equation 1 that for this particular country the system reduces to

(4) $$\sum_i \pi_i q_i^k = \sum_i v_i^k = \sum_i p_i^k q_i^k \quad .$$

GDP of country k valued at world prices equals GDP valued at its national prices. World prices would be tied to the price level of country k, which is non-sense. We must look for a more sophisticated normalisation rule.

In actual PPP work one transforms different national values into values of one nation (usually the US) by means of the nominal exchange rates established on corresponding markets. If e^j is the market exchange rate (units of SDR/units of national currency) nominal world GDP in this currency is then given by

(5) $$Y^k = \sum_j \frac{e^j}{e^k} \sum_i v_i^j \ .$$

On the other hand, world GDP is given within the GK system by

(6) $$Y = \sum_i \pi_i \sum_j q_i^j = \sum_j \varepsilon^j \sum_i v_i^j \ .$$

Normalising equation 6 to equation 5 means

(7) $$\sum_j \varepsilon^j \sum_i v_i^j = \sum_j \frac{e^j}{e^k} \sum_i v_i^j \ .$$

It follows that at given prices and volumes at the national level world GDP may differ depending on the market exchange rates between the numeraire currency and all others. This is unimportant as long as one remains within one point of time, which is the present application of the GK system. But in a coherent combination of space and time, as is envisaged in a complete world accounting system normalisation to the currency of one country creates an unwarranted statistical bias.

The way out towards a sensible normalisation can be found by following up a theorem exposed by Diewert. After showing that the level approach to index number theory is impossible he concludes that “instead of trying to decompose the value of the aggregate into price and quantity components for a single period, we instead attempt to decompose a value ratio pertaining to two periods…into a price change component P and a quantity change component Q.” (Diewert 2008, p.191) All variables figuring in the GK system are variables depending on time. Their time series are regularly observed and recorded in statistical offices. While actual measurement makes them discrete it is admissable, for theoretical purposes, to treat them as continuous variablesin time which simplifies the argument. Let a dot above the variable denote the first derivative with respect to time. Applying the operation to equation 6 yields

(8) $$\dot{Y} = \sum_j \varepsilon^j \sum_i \dot{v}_i^j + \sum_i v_i^j \sum_j \dot{\varepsilon}^j \ .$$

The new normalisation proposed here with the purpose of unifying inflation accounting across space and time is described by equation 9:

(9) $$\sum_j \dot{\varepsilon}^j \sum_i v_i^j = 0 \ .$$

This is a symmetric normalisation assigning no special role to any country. The rationale behind is the following. A change in nominal GDP is measured in Special Drawing Rights, - all exchange rates being defined as SDRs per unit national currency. A movement in nominal world GDP may occur for two reasons. Either one of the volumes q_i^j moves or one of the prices p_i^j . A mere variation in exchange rates between national currencies cannot cause an increase in nominal GDP, as exchange rates compare currencies and not production.

If equation 9 holds there is a certain base year value of world GDP determined by valuing all national GDPs in SDRs of that year using existing market exchange rates, and from the base year on equation 9 takes over. The yearly changes in parity exchange rates are gauged in such a way that their weighted sum is zero. The procedure can be illustrated by resorting back to the discrete case. A discrete approximation to equation 9 is given by

(10) $$\sum_{i,j} v_i^j(t) d\varepsilon^j \cong \sum_{i,j} v_i^j(t) \Delta\varepsilon(t)^j = 0 \ .$$

where the finite difference $\Delta\varepsilon$ is defined as

(11) $$\Delta\varepsilon^j(t) = \varepsilon^j(t) - \varepsilon^j(t-1) \ .$$

Inserting equation 11 into equation 10 yields

(12) $$\sum_{i,j} \varepsilon^j(t) v_i^j(t) = \sum_{i,j} \varepsilon^j(t-1) v_i^j(t) \ .$$

This is a Laspeyres type index for describing the movement of parity exchange rates; its Paasche complement would be just as legitimate to use, of course. The new nominal world GDP is the sum of the new national GDPs valued at last year’s parity exchange rates, which value sets the scale for this year’s parity exchange rates. To repeat the rationale, a shift in exchange rates can indicate neither a movement in world product nor in world inflation, by itself, because an exchange rate expresses a relationship between currencies only.

With normalisation 12, GK system 2 is determined and we may theoretically define world growth and world inflation in the following simple way,

(13) $$\frac{dQ}{Q} = \sum_{i,j} \frac{\pi_i dq_i^j}{\sum_{k,l} \pi_k q_k^l}$$

(14) $$\frac{dP}{P} = \sum_{i,j} \frac{q_i^j d\pi_i}{\sum_{k,l} \pi_k q_k^l}$$

The definition combines valuation at purchasing power parities with accounting for inflation in a systematic and coherent way. It means that the measure of growth determine within a worlde system differs fromits measure at the national level (loss of characticity). So does themeasure of inflation. Measured with the world system it looks different than when measured at the national level. The reason is clear. Varibales are defined within a theoretical system based on certain assumption. The crucial assumption here is the value of a commodity product. In worlde system this value is deemd to be the same in all countries, which is definitely not true when you compile growth and inflation on the bases of purely national systems. Whether the assumption is reasonable is a matter of judgement. When a world currency is being thought about as a realistic possibility the assumption that the same product-commodities incorporate equal economic value, independent of space and time, is a necessary theoretical implication.

Example of compilation

Table 1 provides a simple application of the model to an economy of two countries and two products. Volumes are fixed at $q_i^j = 100$ for all products and countries in order to demonstrate the pure effect of price changes. The value of world product Y is assumed to be 1000 S.D.R.s in the base year $t = 0$. SDSs of the year are thus defined as the measurung rod of value throughout the example. Price indexes begin at $p_i^j = 1.0$ which is the usual convention for a base year. The flow values $v_i^j = p_i^j \; q_i^j$ are therefore equal to 100. Equation system 2 is then specified as in equation 15, for t = 0, namely

(15) $$\begin{aligned} &\pi_1(100+100) - (\varepsilon^A \times 100 + \varepsilon^B \times 100) = 0 \\ &\pi_2(100+100) - (\varepsilon^A \times 100 + \varepsilon^B \times 100) = 0 \\ &(\pi_1 \times 100 + \pi_2 \times 100) - \varepsilon^A(100+100) = 0 \\ &(\pi_1 \times 100 + \pi_2 \times 100) - \varepsilon^B(100+100) = 0 \end{aligned}$$

The system is normalised by equation 16 on the basis of equation 6

(16) $$Y = 1000\,.$$

Equations 15 and 16 are solved by all unknowns acquiring a value of 2.5, an expression of the triviality of the example. The following year (t = 1),the price index of product 1 in country A rises be 10 percent. The nominal value of the follows v_1^A followsan ris from100 to 110, as the volume q_1^A has not altered. The corresponding PPP system looks as follows for t = 1:

(17)
$$\begin{aligned}
&\pi_1(100+100)-(\varepsilon^A\times 110+\varepsilon^B\times 100)=0\\
&\pi_2(100+100)-(\varepsilon^A\times 100+\varepsilon^B\times 100)=0\\
&(\pi_1\times 100+\pi_2\times 100)-\varepsilon^A(110+100)=0\\
&(\pi_1\times 100+\pi_2\times 100)-\varepsilon^B(100+100)=0
\end{aligned}$$

The system is normalised in accordance with equation 12 is

(18)
$$\begin{aligned}
&Y(1)=\sum_j \varepsilon^j(1)\sum_i v_i^j(1)=\sum_j \varepsilon^j(0)\sum_i v_i^j(1)\\
&=2.5\times(110+100)+2.5\times(100+100)=1025
\end{aligned}$$

World price of product 1 increases, as a result of the national price increase ($\pi_1(1)=2.624$), while world price of product 2 remains almost constant ($\pi_2(1)=2.501$). Parity exchange rates, however, both move. The currency of country A is devalued, the currency of country B is revalued. World nominal GDP increases to 1025; as real GDP has not changed, by definition, this is a pure change of the world price level. World inflation is measured at 2.5 percent per year. If world GDP measured at exchange rates had increased more, say to 1040 SDR of year 1, the balance of 15 to the compiled PPP GDP of 1025 would signify a devalution of the world currency against all national currencies. Our normalisation rule

In year t = 2 product 2 raises its price in country A, and in years t = 3 and t = 4 country B follows (table 1). The final outcome after four years of price movement is a price level of 1.102 which is slightly more than if all movements had been performed together. A joint national price rise by 10 percent would result in an equal price rise globally, because the GK system is linear. The compiled difference expresses the fact that history matters. The measurement of an aggregated price level is path dependent, which is a well known puzzle at the national level and needs not be treated here.

Table 1

Price level of a world of two countries and two goods at constant volumes ($q_i^j = 100$)				
	National price indexes (p_i^j)		World price index (π_i)	World GDP at PPP [$\pi_i (q_i^A + q_i^B)$]
	Country A	Country B		
t = 0				
Product 1	1.0	1.0	2.500	500
Product 2	1.0	1.0	2.500	500
Parity exchange rate (ε^j)	2.500	2.500	X	**1000**
t = 1				
Product 1	**1.1**	1.0	2.624	525
Product 2	1.0	1.0	2.501	500
Parity exchange rate (ε^j)	2.440	2.563	X	**1025**
t = 2				
Product 1	1.1	1.0	2.624	525
Product 2	**1.1**	1.0	2.624	525
Parity exchange rate (ε^j)	2.385	2.624	X	**1050**
t = 3				
Product 1	1.1	**1.1**	2.753	551
Product 2	1.1	1.0	2.625	525
Parity exchange rate (ε^j)	2.445	2.561	X	**1076**
t = 4				
Product 1	1.1	1.1	2.753	556
Product 2	1.1	**1.1**	2.753	556
Parity exchange rate (ε^j)	2.503	2.503	X	**1102**

Conclusion

The system of equations introduced above extends the ordinary measurement of purchasing power parities between nations into a model of the global economy based on the law of one single price for every product, wherever it is produced or consumed. The extension is simple in mathematical terms, because it concerns only the normalisation of the Geary-Khmis system. The rule suggested is to set the aggregate change in parity exchange rates equal to zero, for the reason that a mere movement of exchange rates must not affect measurement neither of product growth nor of money inflation.

The example used to illustrate the model shows that a world price level is just as path dependent as a national price level, which is due to the fact that the weights with which national price changes enter into the global figure are variable over time. Also national price levels and rates of inflation willchange when they are emboried in a global system of accounts.

When applied to statistical practice, the suggested mathematical model allows definition of a world price level and a world rate of inflation that is independent of any particular choice of a numeraire country and includes all countries at a fair economic share.

Appendix

References

Balk, Bert M. (2008), Searching for the holy grail of index number theory, in: Journal of Economic and Social Measurement 33, p. 19-25.

Daban, Doménech, Molinas (1997), p. 33

Diewert, W. Erwin (2008), Index numbers, in: The New Palgrave Dictionary of Economics, Second Edition Volume 4, Palgrave, Macmillan: Basingstoke, New York, p.190-214.

EUROSTAT, OECD (2005), PPP methodological manual, Luxembourg, Paris.

Stiglitz, Joseph (2007), Making globalization work, Penguin: London.

Zhou Xiaochuan (2009), Statement on reforming the international monetary system, The People's Bankof China, Beijing.