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The Physiological and Physical Purchasing Power (PhPP) of Money

A Green and Walrasian Paradigm for Real Wealth Measurement

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Abstract

Current Purchasing Power Parity (PPP) comparisons have lost the link to walrasian analysis and do not sufficiently take account of today's lack of sustainability. Both concerns are remedied by PhPP, which stands likewise for "physiological purchasing power" or "physical purchasing power" of a currency. PhPP is the estimated exchange rate of a currency, at a given moment in time, with respect to a certain quantity of human metabolic life. Human metabolic life is defined in energetic terms and expressed in megajoules per year (MJ/a). The basic quantity measured in economics and finance is wealth, defined by Walras in terms of utility and limited availability. Applying the concepts and procedures used in scientific metrology to measurement of utility and limited availability results in a new green and walrasian paradigm of real wealth measurement. If real wealth is a cardinal quantity, it needs a scalar measurement unit. As utility is usually thought to be ordinal, cardinality of real wealth must come from limited availability. Limited availability refers to physical flows and is at the heart of sustainability analysis. Among all physical flows entering the economy, energy is the only one of extra-terrestrial origin, whose user efficiency determines the long term level of overall sustainable activity on the globe.

Empirical PhPP estimation is made by hedonic regression on food and energy price data collected for CPI and PPI. A pilot study for Switzerland in 2003 shows per capita real GDP to be 577 times the minimum cost of life. Choosing materially compatible units for real wealth, energy and utility allows formulating new viability conditions. Thus, maximum viable energy intensity (energy-to-GDP ratio) is bound by PhPP, and any viable homo economicus agent must have a minimum utility-to-wealth ratio. PhPP requires much less data than PPP and is therefore easier to implement than PPP. This could facilitate estimation namely for developing countries.

1. What is the physiological purchasing power PhPP of a currency?

Physiological purchasing power PhPP of a currency is its theoretically estimated exchange rate, at a given moment in time and at a given place, with respect to a certain quantity of human physiological life. PhPP answers the question:

"What quantity of human physiological life does one unit of a given currency (e.g. USD) buy in average at a given moment in time (e.g. in January 2010) and at a given place (e.g. Denver, Colorado)?" Physiology is the specialization of biology analyzing functions of living systems.

Human life is defined here in its biological or more precisely, physiological sense, by asking the question: what is the difference between a living and a dead human body? The living human body has a so-called metabolic activity which the dead body does not have and which is usually expressed in terms of joules per second (= Watt) or kilocalories (kcal) per day or megajoules (MJ) per year.

We can then define a "unit human physiological life" by taking the basal metabolic rate (BMR) or resting energy expenditure (REE) of a reference person with a reference activity, sex, age, weight and height, during a given period of time in a neutrally temperate environment. For reasons that are explained further down, the "unit human physiological life" is chosen to correspond to the basal metabolism of a sleeping person of female sex, aged 20 years, weighing 53 kg, of height 162 cm, during one year. For that reference person and its reference activity, which could be more familiarly called "sleeping beauty", the resting energy expenditure calculated with the Mifflin equation (1990) often used in calorie calculators lies between 60 and 65 joules per seconds (= 60 to 65 W) or between 1250 and 1300 kcal a day or between 1900 and 2000 MJ a year. The Mifflin equation has been estimated for males and females separately and has the following formulae:

REE (kcal/day, males) = 10 x weight (kg) + 6.25 x height (cm) – 5 x age (y) + 5

REE (kcal/day, females) = 10 x weight (kg) + 6.25 x height (cm) – 5 x age (y) – 161

With non-metric units (pounds, feet, inches) the coefficients have to be adjusted accordingly. For persons not at sleep the effective caloric daily need depends on effective activity. REE has to be multiplied by a factor ranging from 1.2 for sedentary activity to 1.9 for extra high activity. As the year is the predominant time period used in accounting, it is convenient to define also annual REE, meaning that the daily REE shown above has to be adjusted accordingly.

In this interpretation the PhPP approach considers that economic activity serves a superior purpose of conserving human physiological life.

2. What do we measure in economics and finance?

It is important to realize that human physiological life is not exactly the fundamental quantity measured in economics and finance. Fundamental quantities used in economics and finance are cost, value and wealth.

Cost and value are almost the same, their only difference being that they depend on the economic role of the agent they refer to. In any transaction-based economic system, the cost of products occurs with a producer (production cost) or a buyer (acquisition cost) whereas the value of products occurs with a consumer (user value) or a seller (market value). Products in this sense are goods or services; for a list of products see e.g. the Central Product Classification.

Cost and value are also similar to wealth, except that cost and value, both refer to products (e.g. “the cost of a house”, “the value of an hour of my labor”), whereas wealth refers to agents (e.g. “the wealth of Mr. Smith”, “the wealth of the ABC corporation”, “the wealth of nations”). Agents in this sense are households, firms or governments. Wealth is not specific to the economic role of producer, buyer, consumer or seller and is in this sense more neutral than cost or value.

A succinct description of the quantity “social wealth” in terms of scarcity has been given by Leon Walras in the following terms:

“By social wealth I mean all things, material or immaterial … that are scarce, that is to say on the one hand useful to us and, on the other hand, only available in a limited quantity” [2].

Note that useful alone does not mean wealth. The oxygen in the air is certainly very useful, but it becomes wealth only if its availability is sufficiently limited so that it gets a price. The same can be said for limited availability. A dangerous virus may not be available in large quantity, but it is not wealth as long as it is not useful.

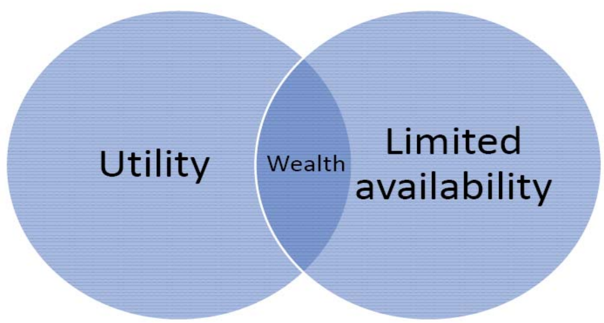

Fig. 1, Walrasian wealth

Wealth may be attributed to individuals or collectivities, but it is social in the sense that wealth can be exchanged or sold against other wealth. A special kind of wealth is money, the sum of quantitative rights that the society recognizes to be the property of an agent. In the absence of property laws, there is no wealth; there are only various levels of control.

Walras noted that in his general equilibrium theory, there is exactly one (scalar) numeraire, to be freely chosen among all products. The numeraire is the accounting or measurement unit. If the choice of the numeraire is money, his equations describe nominal wealth; all the other choices of numeraire describe real wealth. If all relative prices were always the same, the economy would be in perpetual equilibrium, in which case the choice of the real numeraire would not matter. As this is not the case, the choice of the real numeraire matters. PhPP chooses “unit physiological life of one year” as real numeraire.

[2] L. Walras, Elements of Pure Economics or the Theory of Social Wealth, translated by William Jaffé, Orion Editions, Philadelphia, PA, 1984, p. 65

Wealth is not reducible to any other quantity. This is independent from the choice of numeraire. Imagine for instance choosing coconuts as numeraire. The total wealth of, say, Mr. Smith would then amount to x coconuts. Yet he does not necessarily possess one single coconut, but merely the equivalent of x coconuts, i.e. an amount of wealth that could be exchanged for x coconuts. Wealth is always the equivalent of the numeraire used to account for it.

3. Metrological concepts and principles

The science of measurement and its application is called metrology. The internationally standardized definitions of basic and general metrological concepts and principles can be found in the International vocabulary of metrology (VIM) available on the website of the Bureau international des poids et mesures BIPM. PhPP applies these for measurement of real wealth, real cost and real value in general and purchasing power of currencies in particular.

According to the VIM, a quantity is a property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference (1.1.)[3]. The reference can be a measurement unit, i.e. a real scalar quantity defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number (1.9.), often combined with a measurement procedure (2.6.) describing the measurement in detail.

As seen above, (real) wealth is a property recognized with economic agents. (Real) Wealth is however not directly measurable; its measurement is the result of a complicated process requiring at least the following elements:

A measuring instrument (3.1.) in general, i.e. a device used for making measurements. In the special case of wealth, cost or value, the measuring instrument is money or, more precisely, the currency serving as legal tender at the place and moment of measurement. There exists therefore a clear difference between using different weight units (e.g. kilogram, pound) or different currencies: The first are fix whereas the latter are variable, varying in time (inflation) and in space (PPP). The nominal prices of goods or services are therefore indications (4.1.) of measurements made by using a given currency. Indications are only quantity values provided by a measuring instrument; they are not the desired measurement results (2.9). The desired measurement results are the real prices. In order to get these, we must know the calibration (2.39) of the measurement instrument, i.e. the PhPP of the currency at the place and moment of the transaction. PhPP is the proportionality factor transforming nominal wealth, cost or value to real wealth, cost or value.

The set of all currencies used in international trade make up the measuring system (3.2.) of economics and finance. Measures of monetary policy aiming at stabilizing the PhPP of a currency correspond in metrological terms to the adjustment (3.11) of the measuring instrument or the measuring system. If a chosen real numeraire is legally pegged to the nominal numeraire as was the case in the gold standard, this corresponds in metrological terms to a measurement standard (5.1.). Legal pegs of currencies usually break down in the long term unless supported

[3] Numbers in brackets refer to numbers of the respective definition in the International vocabulary of metrology – Basic and general concepts and associated terms (VIM), 3rd edition, Joint Committee for Guides in Metrology, 2008, available for free on the BIPM website.

by wide-ranging adjustment measures (3.11), as the breaking up of the gold standard has proven. Monetary policy is also used to stabilize the business cycle by anti-cyclical measures.

Most prices are measured on markets. Price surveys are the usual measurement method (2.5.). The market (i.e. the set of all exchange possibilities of a good or service of a given agent at a given place and moment in time) is the most frequently used measurement principle (2.4.) for measuring cost, value or wealth. Not all markets are equal; some allow quite fair competition, while others are seller-dominated (oligopolies) and again others are buyer-dominated (oligopsonies). The type of market usually influences the price level of goods and services.

Real wealth of an agent is the aggregation or sum of storable real-priced inventories of that agent. This aggregation is usually made within a framework of generally accepted accounting principles GAAP. The existing International Financial Reporting Standard IFRS accounts nominal wealth and are therefore characterized by money illusion. Constant purchasing power accounting CPPA is only required in times of hyperinflation. A necessary prerequisite for the generalized use of real wealth accounting is the definition of a real wealth unit conforming to the rules of the International System of Units (SI). This will be done further down.

4. Metrological assessment of past and present approaches to real wealth

The major insufficiency of existing approaches to real wealth is that all of them have failed to generalize the practice of real wealth accounting. This is not astonishing, as in a 30’000 items rich consumer basket there are 30’000 potential real numeraires and hence as many ways to calculate real wealth. If these items may be assembled to baskets that may also be chosen as numeraires, the number of possible ways to calculate real wealth becomes astronomical. Real wealth accounting is not possible without further specification of what is exactly of interest.

One of the simplest known choices of a real wealth unit has been the Big Mac, known in the form of Big Mac Index. It is a quick check of purchasing power parity. Its simplicity has the merit to satisfy the metrological principle that measurement units are scalar (1.9.). Its problem is to be based upon a single good. One Big Mac contains 485 kcal. PhPP can therefore be seen as a generalization of the Big Mac Index to all energy-bearing goods.

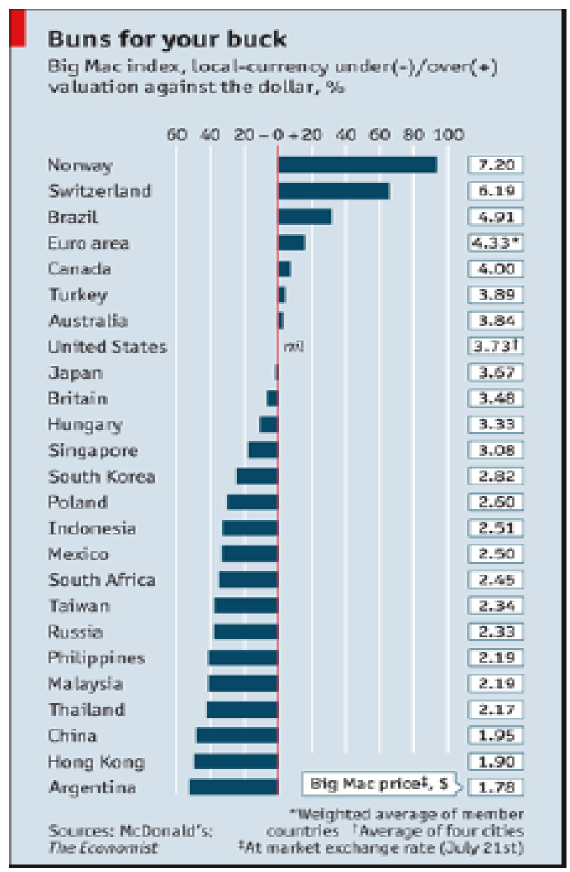

Table 1, Big Mac Index Purchasing Power

The early writers of the 19th century, especially Ricardo and Marx, focused attention on the theory of labor value. This merits mentioning in the metrological context as it was an attempt to consider the labor hour as a scalar value unit. The authors thought of labor value as a natural law. Socialist or centrally planned economies of the 20th century de facto disproved the natural law aspect as they implemented it in form of enacted social laws. Had they restricted themselves to implementing a pure measurement standard for labor value, they would have fixed a single hourly wage rate for everyone and have let all other prices fluctuate freely. Such systems are implemented in various forms of time banking. In the walrasian approach the labor value standard is merely a special choice of a real numeraire. This numeraire choice is problematic as it suppresses competition: The hourly income is the essential result or reward of a competitive economic process. Standardizing it by decree is like organizing Olympic Games and obliging all athletes to achieve exactly a standard performance set by the organizers.

The state of the art of purchasing power parity (PPP) calculation is the International Comparison Program ICP. It is an index-based method expressing the purchasing power parity PPP of any local currency unit (LCU) per international dollar in a reference year (e.g. 2005). The international dollar is a US dollar that takes account of the price level difference between a country and the base country chosen to be the US. In the US the international dollar is identical to the national dollar. There is a similarity with inflation measurement. A consumer price index CPI also expresses the current purchasing power of a currency in terms of the purchasing power of that currency unit in a chosen base year (e.g. 2000). In both, PPP and inflation measurement, the numeraire is a consumer basket. As development levels and consumer baskets vary greatly between countries, there are considerable differences in methodology between CPI calculations of different countries. Harmonization is greater within the EU, where the relative development levels of member countries is similar so that a Harmonized Index of Consumer Prices HICP has been elaborated.

In metrological terms, PPP converts indications from one non-calibrated measurement instrument into theoretical indications of another such instrument. Consumer baskets are vectors with variable coefficients (weights) that do not satisfy the metrological property of scalar measurement units. Comparability between different baskets is difficult as the methodological problems of price indices show. In particular, chain indices used in inflation, linking bilaterally each measured point to its most immediate neighbor, give in general different results from star indices used in PPP, linking multilaterally all points to one single one. For this reason it is usually impossible to achieve numerical consistency between inter-temporal inflation and international PPP comparisons. Inconsistency arises when the results of several years of cross-cutting PPP calculations are compared with the consumer price index of each country. In that case numerical consistency only exists for the base country, i.e. the USA (see table 4). Diewert has shown[4] that no multilateral index method satisfies all criteria it is expected to satisfy.

This problem can be treated by better specifying what is exactly measured. Such analysis clearly differentiates between the measuring (the known, reference or fix quantity, for convenience defined in a measurement unit) and the measured (the unknown, measurand or variable quantity found in scientific phenomena). Confusion between the measuring and the measured constitutes a type of a circular definition, which is present in definitions like "one dollar is one dollar" or "a consumer basket is a consumer basket" and should normally be avoided if clarity is the purpose.

[4] Diewert, National Bureau of Economic Research, Working Paper Nb. 5559,1996

The theoretical concept underlying PPP is utility. If utility is taken as cardinal quantity, it is a scalar and its application to wealth and value measurement could in principle overcome the difficulties that vector-type consumer baskets with variable coefficients (weights) have in failing to satisfy the scalar nature for measurement units. Utility has however itself to overcome a metrological problem. Most contemporary authors refuse the concept of cardinal utility and argue that utility is only ordinal. Indeed, Debreu has proved that order or preference relations of economic agents suffice to create general equilibrium[5]. If all utility were ordinal, wealth would be ordinal, too, as it would inherit this property from utility. Wealth may however inherit cardinality from limited availability, its other conceptual component. That would mean utility is ordinal except where it is wealth, i.e. where it has limited availability and is therefore cardinal.

5. Measurement unit for real wealth and its interpretation

It is convenient to define units for quantities frequently used. In the format of the international system of units, units are sometimes named after eminent scholars having favored the scientific progress of their discipline. To distinguish units from the respective scholar, units are written in lower case letters.

Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
plane angle	radian [b]	rad	1 [b]	m/m
solid angle	steradian [b]	sr [c]	1 [b]	m^2/m^2
frequency	hertz [d]	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, amount of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiant flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge, amount of electricity	coulomb	C		s A
electric potential difference, electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
electric resistance	ohm	Ω	V/A	$m^2\ kg\ s^{-3}\ A^{-2}$
electric conductance	siemens	S	A/V	$m^{-2}\ kg^{-1}\ s^3\ A^2$
magnetic flux	weber	Wb	V s	$m^2\ kg\ s^{-2}\ A^{-1}$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$
Celsius temperature	degree Celsius [e]	°C		K
luminous flux	lumen	lm	cd sr [c]	cd
illuminance	lux	lx	lm/m^2	m^{-2} cd
activity referred to a radionuclide [f]	becquerel [d]	Bq		s^{-1}
absorbed dose, specific energy (imparted), kerma	gray	Gy	J/kg	$m^2\ s^{-2}$
dose equivalent, ambient dose equivalent, directional dose equivalent, personal dose equivalent	sievert [g]	Sv	J/kg	$m^2\ s^{-2}$
catalytic activity	katal	kat		s^{-1} mol

Table 2, Examples of some SI units bearing names of eminent scholars

Source: http://www.bipm.org/utils/common/pdf/si_brochure_8_en.pdf

[5] G. Debreu, Theory of Value, an Axiomatic Analysis of Economic Equilibrium, Cowles Foundation Monograph 17, Yale University Press, 1959.

In honor of Leon Walras, whose definition of wealth we have taken, we name the measurement unit for real wealth walras and describe it in familiar language as "the theoretical minimum quantity of real wealth consumed in one year by the metabolism of the sleeping beauty" [6]. Further down we will give a more precise definition of the walras. In order to eliminate money illusion the definition explicitly refers to real wealth, not to wealth, as "wealth" is more often understood as "nominal" rather than "real" wealth.

Due to the similarity between wealth, cost and value it is possible to show interpretations of the unit of real wealth in terms of real cost and real value.

In terms of cost, the walras expresses the theoretical minimum real cost of conserving the physiological human life of the sleeping beauty during one year.

In terms of value, the walras expresses the theoretical minimum real value of the energetic characteristic consumed by the sleeping beauty in goods or services during one year.

The interpretation in terms of value uses the fact that energy is not a single good or service, but a characteristic[7] that may be found in many goods and services. The new approach to consumer theory proposed by Lancaster considers that a consumer demands a good or service mainly because of characteristics it contains. Often a consumer might e.g. not demand sugar as such, but the characteristic that sugar is sweet. In this case sweetness and calories are two distinct characteristics of sugar. In developed countries with large consumer baskets there are more goods than characteristics, whereas in developing countries it might be the inverse. Empirically characteristics are estimated by using so-called hedonic regression techniques.

It is worth while looking at the possible functions the walras could have in relation to the existing functions of money. Usually money is said to have the four following functions:

Money is the unit of account for (nominal) value. The walras could certainly become a unit of account for real value; this would be its primary function.

Money is a standard for denominating deferred (nominal) debts. The walras could become a standard for denominating real debts. In this role it could replace the Special Drawing Right SDR.

Money is a store of (nominal) value. The walras could only become a store of real value to the extent that the underlying energetic goods can be stored.

Money is a medium of exchange of (nominal) value. The walras cannot perform the role of medium of exchange, as it is only a measurement unit, but not a measurement instrument. Of course one could introduce a currency called walras, but that would have the same deficiencies as all other currencies. Exchange rates among currencies are set by markets, whereas the exchange rates between currencies and the walras, i.e. the calibration (2.39) or PhPP of currencies, are estimated.

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[6] See also: S. Defilla, Physica A (2007), 42 - 51

[7] See Kevin Lancaster, New Approach to Consumer Theory, Journal of Political Economy, 1966, for the introduction of characteristics in consumer theory.

6. Metrology of utility and productivity

As wealth is the intersection of utility and limited availability, it is worth while describing the concept of utility. Anything that applies to utility also applies to wealth (but not vice-versa). Utility is also closely related to productivity.

Utility of any product may be extremely variable and even its sign (positive or negative) may depend on such things like the kind of interaction of the economic agent with a product. Utility of a gun is different (has opposite sign) for the owner than what it is for the victim. The victim experiences a so-called external effect or externality different from the utility of the owner. As a general rule utility should be defined with respect to the owner.

What has been stated for utility also applies to marginal utility, which is mathematically derived from utility by the first order derivative with respect to the good owned. By taking a first order derivative the nature and the dimension of the quantity in question changes. If, say, the quantity in question is a speed measured in meters per second, its derivative with respect to time gives an acceleration measured in meters per square seconds. The two are completely different quantities. If - to take another example - nominal wealth is measured in USD and derivable with respect to a good - say a Big Mac - , then the nominal price of the marginal good can be seen as first order derivative of nominal wealth with respect to the good. It will be expressed in USD per unit Big Mac.

Marginal utility has become the leading concept in value theory at the end of the 19th century. Irving Fisher[8] proposed the util as measurement unit for marginal utility but did not give an explicit definition, noting only that it was proportional to (nominal) price and that marginal utility of a product decreases with consumption. Thus, e.g. marginal utility of the 100th loaf of bread bought during a period is larger than the marginal utility of the 150th loaf of bread bought during the same period. If what has just been mentioned about the change of dimension by taking the first order derivative applies, there can be no single unit for marginal utility with dimension util. Marginal utility of product x must have a different unit from marginal utility of product y.

Utility is mostly used in maximization problems. Take a consumer having the choice between two goods, x and y, with (nominal) prices P_x and P_y respectively, and a utility function U(x,y) being smooth, increasing and strictly quasiconcave. Utility maximization under a (nominal) linear income constraint $B = x P_x + y P_y$ results in the ratio of the marginal utility of x, noted U_x, to P_x equaling the ratio of the marginal utility of y, noted U_y, to P_y.[9] This widely known result from consumer theory is mostly written as $U_x / P_x = U_y / P_y = \lambda_i$ where λ_i, the optimum value of the Lagrange multiplier, is a specific real number >0 representing the consumer's marginal utility of (budget) money when his utility is maximized, i.e. his inputs (*x*,*y*) optimized. The second order sufficient condition (bordered Hessian > 0) is always satisfied if the utility function has the above-stated properties. The optimum then is a utility maximum and is unique for one given consumer.

The analogical result can be stated for a cost-minimizing producer under a production constraint. A cost-minimizing firm under production constraint[10] may have the choice between two inputs, x and y, with (nominal) prices P_x and P_y, respectively, and a production or, more precisely, a revenue function R(x, y) relating the (nominal) receipts earned (if all production is sold) to the

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[8] I. Fisher, Mathematical Investigations in the Theory of Value and Prices, 1892, p. 18

[9] This demonstration is well explained in student textbooks such as e.g. Alpha C. Chiang, Fundamental Methods of Mathematical Economics, Third Edition, Mc Graw Hill, 1984, pp. 400 ss

[10] Alpha C. Chiang, Fundamental Methods of Mathematical Economics, Third Edition, Mc Graw Hill, 1984, p. 418

corresponding inputs of the production process. R(x,y) is assumed to be smooth, increasing and strictly quasi-concave. Minimization of the (nominal) linear cost function $C = x P_x + y P_y$, under the (nominal) production function constraint R(x, y) results in the ratio of marginal productivity of x (noted R_x) to P_x equaling the ratio of marginal productivity of y (noted R_y) to P_y. This equally well known result is mostly written as $P_x / R_x = P_y / R_y = \mu_i$, where μ_i, the optimal value of the Lagrange multiplier, is a specific real number >0 characterizing producer i and representing for each of his optimal inputs (x, y) the marginal cost per unit of receipts, i.e. the cost to revenue ratio for each input. The second order sufficient condition (bordered Hessian <0) is always satisfied for production functions having the stated properties, the optimization therefore yields a unique cost minimum for the given firm. For interpretation purposes we prefer using the reciprocal formulation of this result and write it as $R_x / P_x = R_y / P_y = 1/\mu_i = \nu_i$, where R_x and R_y, respectively, are the marginal (nominal) revenue or turnover contribution of the input x and y at the optimal point ν_i.

A high λ_i means a happier consumer in the sense that his budget constraint gives him more utility in a given set of prices than a low λ_i. A high ν_i means a more efficient producer that earns more revenue in a given set of prices than a lower ν_i. Attaining optimized values of λ_i and ν_i means that an agent attains highest utility or least cost combinations of inputs x and y among those combinations available to him, but does neither mean that the agent is competitive (has sufficiently high λ_i or ν_i to remain in the market) nor even that he is economically viable (that he produces more output than input (both in nominal terms). Admit as supplementary hypothesis that the production function is homogenous of degree one in goods and services or in characteristics (e.g. Cobb-Douglas production function). In this case, by virtue of the Euler theorem, the sum of marginal productivities of goods and services gives total production R. An economically viable agent must have an optimum ν_I greater or equal to 1. The marginal revenue of each input exceeds the price he has to pay for acquiring it if and only if his ν_I is greater or equal to 1. Viability depends on the adaptation of outputs to inputs, i.e. on availability of resources. If both resources (x and y) are scarcer, their prices P_x and P_y will increase, requiring higher R_x and R_y for a same ν_I.

From the metrological point of view both, utility maximization under budget constraint as well as cost minimization under production constraint can be considered as a kind of examples of measurement models (VIM 2.48). Such models relate measurement inputs (P_x, P_y) to measurement outputs called measurands, i.e. quantities to be measured, in our case λ_i and ν_i, but also marginal utilities U_x and U_y as well as marginal revenues R_x and R_y and the associated optimal proportions of x and y used by agent i.

In both cases above, prices of goods x and y (P_x and P_y) as well as the functions B (budget constraint), C (cost function) and R (production function), are expressed in nominal, not real terms. This way to proceed corresponds to the metrological reality, as only nominal data can be measured with the available measurement instruments (currencies). Prices therefore bear the dimension [currency / unit good] and the functions B, C and R the dimension [currency]. Marginal revenues R_x and R_y are the first order derivative of R with respect to x or y respectively and therefore have dimension [currency / unit good]. This means that ν_i is a dimensionless number. Now it becomes again clear why a single unit for marginal utility of any good or service cannot exist. If marginal utility U_x and U_y each had the dimension [util], integration with respect to x would give dimension [util * unit good x] and integration with respect to y would give dimension [util * unit good y], both applicable to the same concept: utility.

It is essential to look whether these conclusions also hold if (nominal) prices and (nominal) revenue are substituted for real prices or real revenue. Mathematically this substitution means dividing them by the price of the real numeraire. The change to real values is operated on the optimization results only, as the agents most probably do optimization on the basis of nominal values. In the simplest case, if e.g. the good x is chosen as real numeraire, then the optimum for agent i simplifies to $U_x / (P_x / P_x) = U_x = U_y / (P_y / P_x) = \Lambda_i$ and $R_x / (P_x / P_x) = R_x = R_y / P_y = N_i$ respectively. The optimum Λ_i equals U_x, the marginal utility of the real numeraire, and the optimum N_i equals R_x, the marginal (nominal) revenue of the real numeraire, whose inverse is M_i. As R_x has dimension [currency / unit real numeraire], that means that N_i has necessarily the same dimension. The same applies for the dimension of marginal utility of the numeraire, U_x, which has the same dimension as Λ_i. Real prices have dimension [(currency / unit good) / (currency / unit real numeraire)] which simplifies to [unit real numeraire / unit good y] for good y and is dimensionless for the real numeraire.

These observations are valid independently of the particular choice of the real numeraire. Take now the special choice of the walras as real numeraire. N_i and has the dimension [currency / walras] and receives a concrete interpretation. It shows how much agent i earns (in terms of currency) during one year by using one supplementary (i.e. marginal) unit of annual physiological life. This is the optimum money-to-walras ratio of agent i. Furthermore, as the walras does not refer to a single good, but to a characteristic of goods, no real price of a single good or service is dimensionless; all real prices of single goods or services take the dimension [walras / unit good]. For an agent to be economically viable, his N_i must be greater or equal than the price of one walras, which is equivalent to stating that his M_i, or optimum walras-to-money ratio must be smaller or equal than PhPP.

Define now the util as the utility of the energetic characteristic owned and consumed in goods or services during one year by a specific consumer of female gender, 53 kg weight and 162 cm height, at sleep.

Where utility is ordinal, the util describes a reference utility. For that purpose the energetic characteristic consumed during one year is taken with unit "one" so that the marginal and mean utility of the first unit coincide. Where utility is cardinal (i.e. where it is also wealth), a stronger hypothesis has to be made, namely that marginal utility of the energetic characteristic is admitted to be constant. This means three things:

Firstly, aggregation over time: the consumer that is adding one more year of his life (at sleep) gets the same marginal satisfaction as each year lived (more precisely: slept) before.

Secondly, aggregation over energy consumption rate: if a consumer doubles energy spending (e.g. by being awake and more active), this gives twice as much satisfaction as sleeping. Increased activity can take place by using any energy form, including technical energy consumed by energy equipment (e.g. washing machines or cars) owned by the consumer.

Thirdly, aggregation over consumers: if a second identical consumer is added to a first one, independently of whether they form a collectivity (e.g. a firm or other grouping) or not, total satisfaction received by both consumers exactly doubles.

These three consequences of constant marginal utility may seem restrictive. For an individual good such as the Big Mac these consequences are most probably not satisfied, as one can reasonably assume that consuming a second Big Mac during the same time period gives less incremental satisfaction than consuming a first one. For the energy characteristic as a whole there exist however enough intra-energy substitution possibilities making constant constant marginal utility of the energy characteristic appear a reasonable assumption. If a reader questions this, he should also question the constant marginal utility of a consumer basket: He should then argue that owning a second fully equipped house gives a consumer less incremental satisfaction than the first such house. If marginal utility of the consumer basket was not constant, neither PPP nor inflation calculation would give accurate results.

The marginal utility of good x, U_x, is the first order derivative of U with respect to x and has the dimension [util / (marginal) unit of good x], and analogous for marginal utility of good y. The optimum λ_i has the dimension [(util / unit good x) / (currency / unit good x)] which simplifies to [util / currency]. Utility maximization in nominal terms shows consumer satisfaction of money of agent i, i.e. his optimum utility-to-currency ratio. In real terms, if x is chosen to be the walras, then Λ_i has the dimension [util / walras] and receives a concrete interpretation, too. It shows how much consumer satisfaction agent i receives during one year by consuming one unit of annual physiological life. This is his <u>optimum util-to-walras ratio</u>.

Having introduced the above definition of the util that is materially compatible with the definition of the walras allows spelling out a special interpretation of utility. Admit that the utility function is homogenous of degree one in goods and services. In this case, by virtue of the Euler theorem, the sum of marginal utilities of goods and services gives total utility. As with productivity (R) of a producer, total utility U of a consumer must have a minimum threshold for a consumer to survive. With our choice of units, the optimum util-to-walras ratio must then be numerically greater or equal to 1 if the consumer is to survive. If a consumer has less utility than that threshold, consumption in general gives him too little satisfaction for motivating him to acquire essential goods, services or characteristics, let alone superior ones. This describes a psychologically unhealthy person. As economics does not deal with psychological illness, the minimum utility-to-wealth ratio could be added to the usual definition of the homo economicus. Recall that this viability condition is in principle valid for any choice of measurement units, but it is numerically visible only by choosing compatible measurement units for wealth and utility.

Empirically utility of consumption by agents could possibly be measured by questioning agents about their preferences as compared with their utility of one year physiological life. If such empirical research was undertaken regularly, systematically and complementary to consumer price surveys made for consumer price indices (CPI), the util, in conjunction with the walras, could possibly play a role in measurement of welfare.

7. Exact physical definition of the walras

Before looking at the measurement of limited availability, it is appropriate to give an exact definition of the walras. In metrological terms, metabolic activity is a quantity of the same kind as energy use. The international metrology vocabulary (VIM) defines “kind of quantity” as “the aspect common to mutually comparable quantities” (1.2). Quantities of the same kind can be added to each other. The rigorous and quantitatively precise way to define the walras is in terms of the physical concept of energy use.

The above-mentioned interval of 1900 to 2000 MJ per year for the resting annual energy expenditure (REE) is too large for a precise definition of a measurement unit. Precision of the measurement unit determines precision of the measurement results. Within the mentioned interval there is a quantity of 1956.1 MJ also known as Planck energy, i.e. the Planck unit for energy[11]. Planck units are the so-called natural units of the Universe that are based upon fundamental physical constants. As Planck, who discovered them in 1899, wrote, they are "constant for all times and all civilizations, even for non-human ones"[12]. Since then their quantitative determination has greatly improved. Planck units simplify theoretical physics and sometimes designate extremes such as the smallest or the biggest quantity of their kind to be found in the Universe. Planck energy is an exception as it is of human scale: Besides being within the interval of the annual resting energy expenditure, 1956.1 MJ corresponds also approximately to a 60 W lamp burning during one year or to a car tank filling of 52 liter diesel oil, i.e. to quantities used in everyday transactions. From its magnitude, Planck energy is ideally suited to be taken to define a measurement unit for wealth. The energy quantity referenced in the definition of the walras is therefore made to coincide with the Planck unit.

A wealth measurement unit defined according to the conventions of the SI system would be defined in language form, not as a mathematical formula. This is so because wealth is a base quantity, as wealth is not reducible to any other quantity found elsewhere. In other words, wealth is a quantity sui generis that cannot be defined by a definite mathematical formula as a function of any other physical quantity, including energy. Wealth is not energy, just as it is not identical to any other potential numeraire. In the SI system, all base units are defined in language form, e.g. the kilogram, defined as the "mass of the international prototype of the kilogram". The derived units are expressed as mathematical formula using base units, e.g. the energy unit joule (J = kg m^2 s^{-2}). In SI terminology, the walras would have the following exact definition:

1 walras (Wal) is the real wealth in, or the real value of, 1956.1 MJ of the energy characteristic available in an environment at physical and chemical equilibrium[13].

The proposed abbreviation for the walras is Wal. It has already been mentioned earlier that energy is not a single good, but a characteristic found in many goods. The addition "in an environment at physical and chemical equilibrium" is necessary for the precise definition of the available energy which is only defined with respect to an environment in equilibrium[14]. For the specialist, there is in fact a small difference between "energy" and "available energy". Energy is a quantity that can enter or leave a system, but that remains constant in time at the level of the Universe, whereas available or Gibbs free energy, more recently called exergy, can enter or leave a system just like energy, but that can only decrease in time at the level of the Universe. Available energy takes account of the thermodynamic quality of energy and is closely related to negentropy, a quantity that can only decrease in time at the level of the Universe. With some minor differences, mainly concerning the thermodynamic quality of heat, available energy corresponds to the popular understanding of "energy".

The study of the interaction between economics and physics is also the subject matter of econophysics and thermoeconomics.

[11] Planck mass energy equivalent is defined in terms of three fundamental physical constants: c (speed of light in vacuum), h-bar (Planck constant over 2 pi) and G (Newtonian constant of gravitation) by the formula $\sqrt{\hbar c^5/G}$

[12] M. Planck, Sitzungsberichte der Preussischen Akademie der Wissenschaften 5, 479 (1899).

[13] See also: S. Defilla, Physica A (2007), 42 - 51

[14] See Diederichsen Ch.: Referenzumgebungen zur Berechnung der chemischen Exergie. Fortschritt-Ber. VDI-Reihe 19, Nb. 50. Düsseldorf, VDI, 1991

8. Measuring Limited Availability with PhPP

As wealth has two components, utility and limited availability, it is also appropriate to briefly look at the way limited availability can be measured. Whatever applies to limited availability (or scarcity) also applies to wealth.

Limited availability refers to limited physical flows within an economy. This is different from availability used in the above definition of the walras, where it relates to energy, hence to a state function or a stock of limited availability, taking into account thermodynamic quality of that stock. Stocks are however less relevant for economic availability given that all economic inputs (except capital) are flows. Stocks are often maintained at minimum level, sometimes called security level, whereas flows are maximized. All quantities on markets such as supply and demand are flows. Also the gross domestic product GDP and all its components (consumption, investments and exports minus imports) are flows expressed on annual basis.

A major cause of limited availability is often set by the environment. It is therefore important, also for wealth measurement, to take the environment into account if limited availability is not to be ignored. The environment is taken into account in sustainability analysis. Sustainability was defined first in 1987 by the Brundtland Commission as meeting the needs of the present without compromising the need of future generations. Unsustainable consumption is therefore consuming today the revenue of tomorrow. Later three pillars of sustainability have been distinguished, namely the economy, the society and the environment. The influence of the environment upon the economy is also being studied in ecological economics.

Walrasian general equilibrium may be seen as a very first attempt to characterize economic sustainability. It is difficult to imagine a sustainable economy that is far off economic (walrasian) equilibrium. For Walras the economy was possibly never actually in equilibrium but continuously trial-touching around a near-equilibrium state ("tâtonnement").

Natural resource flows can originate from different parts of the environment. Many natural resources stem from the the biosphere in form of animals, plants or parts thereof. Other natural resources come from the atmosphere, for instance the oxygen used in respiration and combustion. A third kind of natural resources come from the hydrosphere, especially sweet water used by all living organisms on land. A fourth type of natural resources stems from the lithosphere and concerns land and all minerals, metals and stones, which include fossil and fissile energy.

There is one extraterrestrial resource that is usually not classified among natural resources because of its special nature. It is solar radiation that provides most energy used to day, as fossil energy is solar energy that has been accumulated over the past. Among all physical flows entering the economy, energy is the only one that has extraterrestrial origin. In a system of flows such as an economy or a triple system composed of economy, society and environment, a flow from outside these three is the only external constraint to availability and its user efficiency therefore plays the role of a limiting factor for overall sustainable activity on the globe. If limited availability of flows is a criterion for defining wealth and choosing the real numeraire, energy naturally imposes itself.

The intensity of solar energy is determined by the so-called solar constant. At the distance of the earth from the sun, this amounts to 174 PW (Petawatt or 10^{15} W) equaling 1367 W per square meter cross section. Given that the surface of the earth is four times its cross section as the surface of any sphere is exactly four times its maximal cross section, this makes 342 W of solar energy per square meter of earth surface, averaged over day and night, all seasons and all locations. These values apply at the outer atmosphere. At the earth surface, there remains about 122 PW or 239 W per square meters at sunny conditions and 89 PW or 175 W per square meter at average weather conditions. The resulting solar energy flow per square meter is of the same order of magnitude as the REE of our reference person, which is 62 W.

By far not all the incoming solar energy is used. Most of it is transformed to heat and rejected back in the form of infrared radiation into space. The efficiency factor between input and output is called energy efficiency. It can be 100% or less. Most conventional electricity our of coal or fissile fuels is being produced with only 30% efficiency and most commercially used photovoltaic solar panels produce electricity at 10% efficiency. Green plants produce energy in the 0.1 to 1% efficiency range (cf. table 3).

Plant species or biotope	**Production (g / m^2 / year)**	**Production (W / m$^{2)}$**	**Energetic Efficiency (%)**
C4-plants peak performance	19700	9.9	4
C4-plants all year	*8000*	*4*	*1.7*
C3-plants peak performance	7300	3.7	1.5
C3-plants all year	2000	1	0.4
Tropical rain forest	990	0.5	0.2
Deciduous wood	580	0.3	0.12
Savanna	410	0.2	0.09
Maize	396	0.2	0.08
Coniferous wood	360	0.18	0.07
Continents (average)	347	0.17	0.07
Agricultural land, cereals	290	0.15	0.06
Grass (moderate climate)	200 - 270	0.1 - 0.14	0.04 - 0.05
Ocean (continental shelf)	160	0.08	0.03
Earth (average)	157	0.079	0.03
Ocean (average)	78	0.04	0.016
Ocean (open)	56	0.028	0.011

Table 3, Energy efficiency of plants and ecosystems

Source: S. Defilla, Energiepolitik, technische und wirtschaftliche Grundlagen, Anhang.

As herbivore animals feed themselves from green plants with 10% efficiency, the overall efficiency of a herbivore economy lies in the 0.01% to 0.1% range and decreases by a further factor 10 for carnivores. As human beings are partly herbivores and partly carnivores, and fossil energy is mainly solar energy stored geologically in earlier ages, the average energy efficiency of the human economy lies in the 0.001% to 0.01% range. Availability decreases with each supplementary chain link, if its efficiency is less than 100%.

It is now possible to show how PhPP is relevant for signaling limited availability. For that purpose we investigate the relationship between the limited availability of the extraterrestrial resource and the way it affects consumption of (real) wealth.

Overall wealth consumption is co-determined by overall output of goods and services, which in turn is indicated by the GDP. Global GDP equals global consumption plus global investment. The investment-to-GDP ratio at global level is around 20%, meaning that the consumption-to-GDP ratio is around 80%. If we were to determine a maximum GDP attainable with a constant external energy inflow we would have to know a maximum GDP-to-energy ratio, i.e. a ratio that indicates the maximum GDP that can be attained with a given energy availability. This ratio is a kind of maximum overall economic efficiency. It can be nominal (nominal GDP-to-energy) or real (real GDP-to-energy). The relationship between the two is as follows:

Real GDP-to-energy ratio = nominal GDP-to-energy ratio times PhPP

This exemplifies the general way PhPP is applied: it is the proportionality factor transforming nominal to real data. We could have used the inverse of PhPP, i.e. the walras price (a currency-to-walras ratio) instead. In that case the formula would have to be written as:

Real GDP-to-energy ratio = nominal GDP-to-energy ratio divided by the walras price

The important fact to bear in mind is that PhPP and walras price are both variable with availability of the real numeraire; PhPP increases with increasing availability of the real numeraire (i.e. energy) and decreases in the opposite case; for the walras price the variability takes place in the opposite direction. Thus, in the PhPP approach, the real GDP and therefore the real GDP-to-energy ratio automatically decrease in case of limited availability of the real numeraire, which indicates a decrease of real overall economic efficiency.

The difference with traditional inflation can now be made clear. In traditional inflation, if prices of the entire consumer basket or a large part of it rise, this is above all the effect of increased money supply, i.e. of a higher sum of income and credits received by the economy, and not the effect of limited availability of natural or extraterrestrial resources. Consumer baskets with variable coefficients allowing for substitution have almost unlimited availability. If we deflate nominal GDP in a context of traditional inflation, this means that we eliminate the effect of monetary policy, not of limited resource availability. The limited availability aspect is only present in form of too little availability of the consumer basket with respect to money supply.

From the point of view of traditional inflation, PhPP concentrates on non-core inflation, which is defined as inflation without energy and food items. Only non-core inflation may signal the limited availability of the energetic numeraire. PhPP requires much less data than PPP and it is also more efficient than PPP from the points of view of metrology and sustainability because it is based upon a measurement unit and therefore directly signals limited availability of the energetic numaraire referenced in the definition of the measurement unit. It remains to be seen how sustainable monetary policy should be defined in the PhPP context.

If the GDP-to-energy ratio is measured in units that are materially compatible with the units of the money-to-walras viability threshold, it can directly be compared with it. We have already seen that cost minimization of agents yields an optimum N_i which, if expressed in real terms, is the optimum money to walras ratio of agent i. For an agent to be economically viable, his N_i must be greater or equal than the price of one walras, and equivalently his M_i (i.e. the inverse of N_i) must be smaller or equal than PhPP. Unit compatibility is given if the GDP is measured in nominal money units and if there exists a fix relationship between energy and the walras. This relationship is established by using the energy unit referenced in the definition of the walras. Conversion of energy units used in this article can be made with the help of the conversion table in the annex.

The economic efficiency referred to in the optimum N_i indicates a firm's (nominal) turnover per real wealth unit consumed. In the case of firms, efficiency is usually not measured as turnover, but as value added that is only a part of turnover. The ratio of value added to turnover depends on the specific branch of activity of a firm. On national level the sum of value added gives the GDP. If the ratio of value added to GDP is, say, 0.5, then GDP amounts to half the size of total turnover, the other half being spent on acquiring intermediary inputs. As most firms exist for the purpose of producing value added, it is justified to think of cost minimization under value added (instead of turnover) constraint. The optimum can be called H_i and is an optimum ratio of (nominal) value added per real wealth unit consumed. Its inverse can be named Γ_i. If value added is about half of turnover, a viability condition with respect to H_i or Γ_i is about twice as strong as the earlier viability condition indicated by N_i or M_i.

For empirical investigation of these relations we can derive (nominal) GDP-to-energy ratios from their inverse energy-to-(nominal)-GDP ratios also called energy intensities. These are known for all countries and are published annually on the internet site of the International Energy Agency IEA in the key world energy statistics. Remark that households have no value added, as labor is not considered to be a service in the usual sense. When taking the energy-to-GDP ratio, the question arises whether, for consistency, energy consumption of households should also be excluded, or on the contrary, whether all national energy consumption should be included. Energy intensities at national level are usually taken to include all national energy consumption.

Viability can be expressed as upper threshold of energy intensity. The upper threshold for energy intensity is PhPP. If energy intensity is taken to include all energy consumption (including by households), the viability condition is stronger than if it were done by excluding household energy consumption.

Columns two and three of the table below indicate energy intensities for 2003 in original units published by the International Energy Agency. Columns three and four give the inverse GDP-to-energy ratio where energy has been converted to Planck units (PE), whereas GDP has been left in 2000 USD and 2000 USD using PPP respectively. Further down, the figure of 424.65 USD / PE referring to Switzerland in table 4 will be compared with the PhPP estimated for this country in 2003.

2003 data	Energy / GDP TOE / 1000 USD	Energy / GDP TOE / 1000 USD PPP	GDP / energy USD / PE	GDP / energy USD PPP / PE
World	0.32	0.21	145.97	222.44
OECD	0.2	0.19	233.56	245.85
USA	0.22	0.22	212.33	212.33
Australia	0.26	0.2	179.66	233.56
Japan	0.11	0.15	424.65	311.41
Switzerland	0.11	0.12	424.65	389.26
South Korea	0.35	0.23	133.46	203.09
Russia	2.09	0.51	22.35	91.59
South Africa	0.86	0.26	54.32	179.66
Brazil	0.31	0.15	150.68	311.41
China	0.92	0.23	50.77	203.09
Bangladesh	0.4	0.09	116.78	519.02

Table 4, Energy-to-GDP ratios, source: IEA

9. Pilot estimation of PhPP and the hedonic walras price

PhPP and its inverse, the walras price, can be estimated by multiple hedonic regression involving the energetic characteristic alongside with other characteristics and with price. As the estimation is hedonic, the resulting walras price should be called hedonic walras price (HWP). It is a conditional mean price. Price data are taken from individual energy and food prices from price surveys of consumer price indices (CPI) and producer price indices (PPI).

On the basis of over 24'000 individual 2003 CPI and PPI energy price data for Switzerland and the corresponding meta data received from the Swiss Federal Office of Statistics, an identical number of transactions has been reconstructed, each involving several physical characteristics as well as money paid in exchange. Several hundreds of different regression specifications have been tested. The best one was found to be a log-log specification involving besides the energy characteristic (or numeraire N) and price P also the characteristic of physical mass M and one dummy variable D for each physically identical good:

$$\ln N = \gamma_0 + \gamma_P \ln P + \gamma_M \ln M + \gamma_1 D_1 + \ldots + \gamma_6 D_6 + \eta$$

After estimation it has been found that physically similar goods can easily be grouped in coarser categories without loss of estimation efficiency. Best estimates give adjusted R squared of 99.5% and very significant t-ratios for all coefficients (see results in the annex).

On the basis of the best equation retained, PhPP has been calculated as first order partial derivative of N with respect to P at given other covariates. The natural choice of the standard values of the other covariates is the one corresponding to the energy of highest thermodynamic quality that can be shown to be electricity. The PhPP was found to be 0.010 Wal / CHF. HWP was calculated as inverse of PhPP (1/PhPP) and found to be 102.36 CHF / Wal, indicating the theoretical minimal cost of "pure sleeping" during one year. Conversion of the Swiss per capita GDP of 2003 to Wal gives 577 Wal per person per year, meaning that the total per capita output of Switzerland in 2003 is 577 times the minimum biological cost of physiological life.

The log-log specification allows in principle to detect non-linearity of money in wealth. The pilot study has shown that non-linearity is not statistically significant, as the respective coefficient is not significantly different from 1. Still, the mentioned 102.36 CHF / Wal are only exact for a real transaction size of 1 Wal. For smaller and larger transactions the HWP is slightly different. This shows that the choice of the wealth unit especially matters in a context of non-linearity.

For comparing the hedonic walras price of 102.36 CHF / Wal with the nominal GDP to energy ratio of 424.65 USD 2000 / Planck energy stated in table 4 above we use the definition of one walras, being the real wealth of one Planck unit of energy. The following corrections have to be made: The 424.65 USD figure has to be converted to USD 2003 by multiplying with the corresponding inflation index (1.06858, source: IMF WEO), and then converted to CHF at nominal 2003 exchange rates (1.34 CHF per USD, source: IMF WEO), giving 611.07 CHF / Planck energy. This is roughly six times as high as the HWP of 102.36 CHF/ Wal. As six is greater than one, the Swiss economy in 2003 was highly viable.

The equivalent calculation could be made with the respective inverse quantities. In that case, (nominal) energy intensity, expressed in Planck units per (nominal) GDP, would have to be less or equal than PhPP. The factor 6 represents a kind of overall harvest factor of the Swiss economy. It is a significant factor determining the degree of sustainability of an economy. These results exist with any measurement unit, but they can only be made numerically explicit if the units of energy and real wealth are materially compatible with each other. The PhPP approach clearly indicates which energy price (namely the hedonic price of one Planck energy, which has a fixed proportionality with the HWP) is to be used for making the comparison with this ratio.

10. Conclusion

PhPP has been shown to be a natural way for estimating the purchasing power of currencies. It follows metrological principles and considers money as main measurement instrument and the market as main measurement principle for measuring real wealth and value.

Walrasian wealth, at the intersection of utility and limited availability, receives its cardinality from limited availability, meaning that utility is necessarily both, cardinal (where it is wealth) and ordinal (elsewhere). On the basis of energy, a resource that is indispensable for human physiological life and is also the major physical limiting factor for sustainable consumption, the walras has been defined as unit for real wealth or real value, and the util as reference utility materially compatible with the walras. The double interpretation of PhPP in terms of physiological and physical purchasing power has thus been illustrated.

Material compatibility of measurement units allows for a direct description of economic viability. Viable economic agents must have a minimum utility-to-wealth ratio, and their maximum energy intensity will be limited by PhPP. If PhPP decreases with decreasing availability of the real numeraire, energy intensity has to follow the decreasing path.

Estimation of PhPP is less data intensive and more relevant for limited availability than PPP. These are advantages to bear in mind each time when improvements of PPP are sought.

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12. Annex

Statistical results of the PhPP pilot study

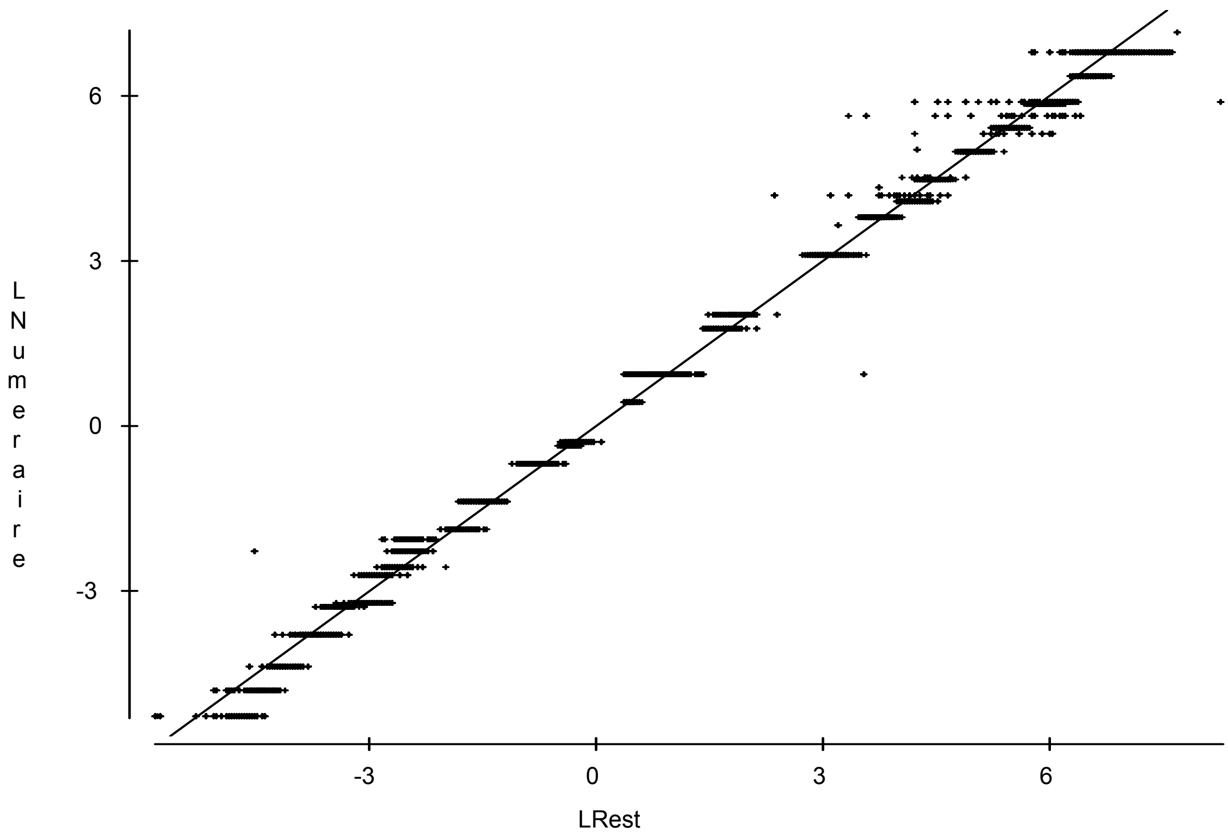
Dependent variable is: **LNumeraire**
No Selector

R squared = 99.5% R squared (adjusted) = 99.5%
s = 0.2607 with 24249 - 9 = 24240 degrees of freedom

Source	**Sum of Squares**	**df**	**Mean Square**	**F-ratio**
Regression	321798	8	40224.8	591878
Residual	1647.38	24240	0.067961	

Variable	**Coefficient**	**s.e. of Coeff**	**t-ratio**	**prob**
Constant	2.76023	0.0993	27.8	≤ 0.0001
Lprice	0.997242	0.0016	636	≤ 0.0001
Lmass	0.143091	0.0019	76.5	≤ 0.0001
DistrHeating	-12.2567	0.1447	-84.7	≤ 0.0001
CarFuel	-10.1382	0.1345	-75.4	≤ 0.0001
ELHOil	-9.62431	0.1395	-69.0	≤ 0.0001
RawWood	-9.52903	0.1430	-66.6	≤ 0.0001
Gas	-9.25687	0.1348	-68.7	≤ 0.0001
DryWood	-8.38461	0.1435	-58.4	≤ 0.0001
Electricity	0	0	•	•

Graphical fit



Conversion table of energy units; example: one unit Planck energy = 1956.1 MJ

	kcal	MJ	kWh	Wyear	Planck Energy	TOE
kcal	1	0.0041876	0.001163317	0.000132708	2.14079E-06	1E-07
MJ	239	1	0.2778	0.031690623	0.000511221	2.39E-05
kWh	860	3.6	1	0.114077116	0.001840249	8.6E-05
Wyear	7535	31.6	8.766	1	0.016131627	0.000754
Planck Energy	467117	1956.1	543.4	61.99	1	0.046712
TOE	10000000	41876	11633	1327	21.4	1