Session Number: Parallel Session 8D
Time: Friday, August 27, PM

Paper Prepared for the 31st General Conference of
The International Association for Research in Income and Wealth

St. Gallen, Switzerland, August 22-28, 2010

Measuring Economic Insecurity

Walter Bossert and Conchita D'Ambrosio

For additional information please contact:

Name: Conchita D'Ambrosio
Affiliation: Universit`a di Milano-Bicocca, DIW Berlin and Econpubblica, Universit`a Bocconi

Email Address: dambrosio@unibocconi.it

Measuring Economic Insecurity*

Walter Bossert
Department of Economics and CIREQ, University of Montreal
walter.bossert@umontreal.ca

Conchita D'Ambrosio
Università di Milano-Bicocca, DIW Berlin and Econpubblica, Università Bocconi
conchita.dambrosio@unibocconi.it

This version: July 4, 2010

Corresponding Author: Walter Bossert
Department of Economics
University of Montreal
P.O. Box 6128, Station Downtown
Montreal QC H3C 3J7
Canada
Phone: (514) 343 6539
FAX: (514) 343 7221
e-mail: walter.bossert@umontreal.ca

* We thank an associate editor, two referees, Miguel A. Ballester, Lars Osberg, Erik Thorbecke, John A. Weymark and participants at several conferences for helpful suggestions and comments. Financial support from MIUR (Prin 2007), the Fonds de Recherche sur la Société et la Culture of Québec and the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

Abstract. We provide an axiomatic treatment of the measurement of economic insecurity, assuming that individual insecurity depends on the current wealth level and its variations experienced in the past. Current wealth could also be interpreted as incorporating the individual's evaluation of future prospects. Variations in wealth experienced in the recent past are given higher weight than experiences that occurred in the more distant past. Two classes of measures are characterized with sets of plausible and intuitive axioms. *Journal of Economic Literature* Classification No.: D63.

Keywords: Insecurity, Wealth Streams, Social Index Numbers.

1 Introduction

Economic insecurity is a phenomenon that, while appearing frequently in public-policy debates, has not yet been analyzed from a thorough theoretical perspective. To the best of our knowledge, there are few contributions in the economics literature that are devoted to defining and measuring economic insecurity. Osberg [9] notes that a considerable amount of research has been done on economic risk but, in contrast, insecurity has not received much attention. The need of a theoretically sound measure of the phenomenon has also been stated recently by the Commission on the Measurement of Economic Performance and Social Progress; see Stiglitz, Sen and Fitoussi [15, p.58].

Clearly, economic insecurity is a multi-faceted issue and a comprehensive formal definition that subsumes all possible aspects of it is likely to remain elusive for some time to come. However, specific aspects of the notion of insecurity appear to be identifiable. In particular, it seems to us that there is a clear link with past, current and expected future wealth levels. An individual may be insecure regarding the future evolution of her or his wealth profile. The resources available to an individual in the present are important because a higher current wealth level corresponds to a larger buffer stock to be relied upon. Experiences play a crucial role in shaping individual perceptions regarding the ability to cope with possible adverse events. Clearly, economic agents remember gains and losses in resources experienced over time, and the more recent these variations are, the more vivid are these memories. People frequently decide from experience as opposed to deciding from description, that is, individual choices are often based on previous personal experience rather than on the description of all possible outcomes and their probabilities. For a comparison among these two decision methods, see, among others, Hertwig, Barron, Weber and Erev [6]. As this discussion regarding the incorporation of an individual's experiences suggests, our interpretation of an insecurity index is descriptive.

While there are, of course, many aspects of life that may play an important role in assessing the economic insecurity faced by an agent, it seems to us that an adequate (and, from an applied perspective, realistic) option is to use a comprehensive notion of wealth as the relevant variable. By doing so, we abstract from determinants of insecurity that cannot be captured by a monetary variable (see Stiglitz, Sen and Fitoussi [15, pp.53–54] for a discussion and examples). However, this simplification seems to be less of a problem if the notion of wealth employed is comprehensive. Wealth is assumed to encompass everything that may help an individual in coping with adverse occurrences. It includes, for instance, claims on governments, family, friends etc. Sen [13] refers to these claims as entitlements—consumption bundles available to an agent given her or his rights and opportunities; see also Sen [14, p.497].

It is important to note that the current wealth level could also be interpreted as the present value of all expected changes in future wealth. In this case, this value will take into account an individual's evaluation of future prospects. Because we do not want to commit to any specific theory of choice under uncertainty, we leave the way that expectations about the future are integrated into current wealth open and focus on the aggregation problem that remains: the way we might want to solve trade-offs involving wealth variations in the past and current (including expectations) wealth levels. Thus, the measures of individual insecurity we propose in this paper have as their domain wealth streams of varying lengths. The length of these streams is not assumed to be fixed because individuals are of different ages in a given time period and, moreover, the availability of data may impose restrictions on how far back in the past we can go when assessing economic insecurity.

Although the design of social rather than individual measures of economic insecurity also is an issue of considerable interest, we focus on the individual index number problem in this paper. We justify this choice by appealing to the observation that economic insecurity is very much a sentiment experienced by each individual. To draw a parallel to other questions involving economic index numbers, consider the measurement of deprivation as a prominent example. As illustrated in Yitzhaki [17] and much of the subsequent literature, it is natural to first obtain an individual value of deprivation for any income distribution and then, in a second stage, aggregate these individual deprivation values into a social deprivation index. This second stage is frequently performed by calculating the arithmetic mean of the individual deprivation values, and the more substantive problem is that of designing the individual index. The measurement of economic insecurity is similar in this respect: once an individual index of insecurity is established, a social index can easily be obtained by applying a (possibly but not necessarily arithmetic) mean to the individual insecurity values. Note that this contrasts with economic measures of inequality: there is no 'individual' inequality because the phenomenon in itself is defined in terms of the disparity present in a distribution.

We propose a set of properties that we think a measure of economic insecurity should possess and use them to characterize specific linear measures of insecurity. According to these indices, insecurity is given by the current wealth level multiplied by minus one plus weighted sums of the wealth gains (losses) experienced in the past. Two sequences of coefficients are employed—one applies to gains, the other to losses. The coefficients are such that recent experiences are given higher weight than experiences that have occurred in the more distant past. A subclass of these measures is obtained by giving higher weights to the absolute values of past losses than to those of past gains, thereby reflecting an attitude that we may label loss aversion in analogy to risk aversion in models of individual decision making under uncertainty.

The first axiom we propose is a natural monotonicity property with respect to the addition of another past period to a given wealth stream. If this new period is associated with a level of wealth that induces an additional gain (a loss, no change, respectively), insecurity is assumed to decrease (increase, remain unchanged, respectively) as a consequence. Our second property expresses the fundamental assumption that gains and losses in the past matter more the closer to the present period they occur. Furthermore, we employ standard notions of homogeneity and translatability that are familiar from economic index number theory. A temporal aggregation property serves two purposes: first, it ensures that the influence of the past on individual insecurity is limited to the gains and losses experienced and, furthermore, it imposes a separable (recursive) structure on the index. Finally, an axiom that can be thought of as the analogue of an attitude of risk aversion is added in order to obtain our second characterization result.

The next section presents the formal framework and the properties we consider important for a wealth based measure of economic insecurity. In section 3 we characterize a class of linear measures and an important subclass. Section 4 concludes with a discussion of some open questions as well as brief remarks on the links between our approach and criteria employed in inequality measurement and the analysis of decision making under risk.

2 Wealth streams and individual insecurity

For any $T \in \mathbb{N}_0$, let $\mathbb{R}^{(T)}$ be the $(T+1)$-dimensional Euclidean space with components labeled $(-T, \ldots, 0)$. Zero is interpreted as the current period and T is the number of past periods taken into consideration. We allow T to vary because people alive in the current period may have been born (or have become economic agents) in different periods. A measure of individual insecurity is a sequence of functions $V = \langle V^T \rangle_{T \in \mathbb{N}_0}$ where, for each $T \in \mathbb{N}_0$, $V^T \colon \mathbb{R}^{(T)} \to \mathbb{R}$. This index assigns a degree of insecurity to each individual (net) wealth stream $w = (w_{-T}, \ldots, w_0) \in \bigcup_{T \in \mathbb{N}_0} \mathbb{R}^{(T)}$. We employ a comprehensive notion of wealth and we allow net wealth to be negative.

It is possible to think of w_0 as encompassing not only the current wealth level of the individual but also her or his assessment of (uncertain) future levels of net wealth. We do not want to commit to a specific method of forming expectations about the future which is why we choose this general formulation. For convenience, we will continue to refer to w_0 as current wealth, keeping in mind that this figure may include expectations regarding future wealth levels.

We now introduce axioms that are intended to capture essential properties of wealth based insecurity measures. The approach we follow is descriptive in nature and, thus, the axioms are to be interpreted accordingly.

Our first property is the following difference monotonicity axiom.

Difference monotonicity. For all $T \in \mathbb{N}$, for all $w \in \mathbb{R}^{(T-1)}$ and for all $\gamma \in \mathbb{R}$,

$$V^T(w_{-(T-1)} + \gamma, w) \geq V^{T-1}(w) \;\Leftrightarrow\; \gamma \geq 0.$$

Difference monotonicity requires a decrease in insecurity as a consequence of the *ceteris paribus* addition of another period $-T$ which introduces a gain between periods $-T$ and $-(T-1)$, thus allowing past gains to work against insecurity. Analogously, the measure of insecurity is assumed to increase if a period $-T$ is added in a way such that wealth decreases, *ceteris paribus,* when moving from $-T$ to $-(T-1)$. Finally, if the addition of period $-T$ involves a wealth level identical to that of period $-(T-1)$, insecurity is unchanged. This is a monotonicity requirement that appears to be essential in capturing the notion of increased (decreased, unchanged, respectively) insecurity as a response to additional losses (additional gains, no changes, respectively) in past wealth levels. Note that the axiom does not imply that gains and losses have to be treated symmetrically; it is possible, for instance, that adding a gain of a certain magnitude, *ceteris paribus,* decreases insecurity by less than a loss of the same magnitude increases insecurity. We will return to this issue in more detail at the end of this section.

Next, we state a property that captures the observation that recent experiences carry a higher weight than experiences that occurred in the more distant past.

Proximity property. For all $T \in \mathbb{N} \setminus \{1\}$, for all $w \in \mathbb{R}^{(T)}$ and for all $\tau \in \{1, \ldots, T-1\}$,

$$\begin{aligned} & V^T(w_{-T}, \ldots, w_{-(\tau+1)}, w_{-(\tau+1)}, w_{-(\tau-1)}, \ldots, w_0) \;\geq \\ & V^T(w_{-T}, \ldots, w_{-(\tau+1)}, w_{-(\tau-1)}, w_{-(\tau-1)}, \ldots, w_0) \\ \Leftrightarrow \quad & w_{-(\tau+1)} \geq w_{-(\tau-1)}. \end{aligned}$$

The proximity property ensures that a gain (loss) of a given magnitude reduces (increases) insecurity, *ceteris paribus,* to a higher extent the closer to the present this gain (loss) occurs. That is, changes in wealth from one period to the next have a more severe impact the closer they are to the present period.

A common property in the design of economic index numbers is homogeneity, an axiom that ensures that proportional changes in wealth are mirrored in the corresponding insecurity values. Thus, homogeneity requires insecurity to be measured by means of a ratio scale.

Homogeneity. For all $T \in \mathbb{N}_0$, for all $w \in \mathbb{R}^{(T)}$ and for all $\lambda \in \mathbb{R}_{++}$,

$$V^T(\lambda w) = \lambda V^T(w).$$

An analogous property applies to absolute instead of proportional changes. Formulated for insecurity measures, it is defined as follows. We use $\mathbf{1}_r$ to denote the vector consisting of $r \in \mathbb{N}$ ones.

Translatability. For all $T \in \mathbb{N}_0$, for all $w \in \mathbb{R}^{(T)}$ and for all $\delta \in \mathbb{R}$,

$$V^T(w + \delta \mathbf{1}_{T+1}) = V^T(w) - \delta.$$

Translatability differs from the usual translation scale property in that the value of δ is subtracted from the level of insecurity when δ is added to the wealth level in each period. This is a consequence of the inverse relationship between wealth and insecurity.

The conjunction of homogeneity and translatability implies that $V^0(w_0) = -w_0$ for all $w_0 \in \mathbb{R}$. Thus, V^0 is a decreasing linear function of w_0. We state this observation, to be used in the proof of our main theorem, in the following lemma.

Lemma 1. *If a measure of individual insecurity V satisfies homogeneity and translatability, then*

$$V^0(w_0) = -w_0 \quad \textit{for all } w_0 \in \mathbb{R}. \tag{1}$$

Proof. Setting $T = 0$ and $w_0 = 0$, homogeneity implies

$$V^0(0) = V^0(\lambda \cdot 0) = \lambda V^0(0) \quad \text{for all } \lambda \in \mathbb{R}_{++}$$

and, substituting any $\lambda \neq 1$, it follows that

$$V^0(0) = 0. \tag{2}$$

Setting $T = 0$ and $\delta = -w_0$ in the definition of translatability and using (2), we obtain

$$V^0(0) = V^0(w_0 + (-w_0)) = V^0(w_0) + w_0 = 0 \quad \text{for all } w_0 \in \mathbb{R}. \tag{3}$$

Clearly, the last equality in (3) is equivalent to (1). ■

Note that the full force of homogeneity and translatability is not needed for the above lemma; as is evident from the proof, it is sufficient to use the respective properties that are obtained by restricting the scopes of the axioms to the cases in which $T = 0$.

The next axiom combines a recursivity condition with the assumption that the role of past wealth enters through wealth differences only. Thus, in addition to the separability property encompassed by the axiom, this aggregation property expresses the assumption that past gains and losses are what matters to an agent; see also the discussion in the introduction.

Temporal aggregation property. For all $T \in \mathbb{N}\setminus\{1\}$, there exists a function $\Phi^T : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that, for all $w \in \mathbb{R}^{(T)}$,

$$V^T(w) = \Phi^T\left(w_{-T} - w_{-(T-1)}, V^{T-1}(w_{-(T-1)}, \ldots, w_0)\right).$$

The temporal aggregation property is a separability condition that allows a measure of insecurity to be calculated by recursively moving back from the current period to the earliest relevant period where, in the step involving period $-t$, the part of insecurity that takes into consideration all periods from $-t$ to the current period is obtained as an aggregate of the insecurity resulting from considering periods $-(t-1)$ to period zero only and the change experienced in the wealth level between periods $-t$ and $-(t-1)$; see Blackorby, Primont and Russell [2] for a detailed discussion of various recursivity properties.

Finally, we introduce the requirement that *ceteris paribus* losses of a certain magnitude in a given period have at least as strong an impact on insecurity as *ceteris paribus* gains of the same magnitude in the same period. This assumption is captured in the following axiom.

Weak loss priority. For all $T \in \mathbb{N}$, for all $w \in \mathbb{R}^{(T-1)}$ and for all $\gamma \in \mathbb{R}_{++}$,

$$V^T(w_{-(T-1)} + \gamma, w) - V^T(w_{-(T-1)}, w) \;\geq\; V^T(w_{-(T-1)}, w) - V^T(w_{-(T-1)} - \gamma, w).$$

Weak loss priority can be interpreted as an insecurity analogue of weak risk aversion in the context of individual choice under uncertainty. However, since we do not want to exclude alternative attitudes towards the comparison of gains and losses, we provide a characterization of all measures that satisfy all of the above axioms except for the last and, in a second axiomatization, derive the additional restrictions that are obtained if weak loss priority is added to the list of properties.

3 Linear insecurity measures

The classes of insecurity measures that we consider in this paper involve two sequences of parameters—one sequence that applies to past period-to-period losses in wealth and one sequence that is used for period-to-period gains. The sequences need not be the same but, within each sequence, some natural restrictions apply. Let $\alpha = \langle \alpha_{-t} \rangle_{t \in \mathbb{N}}$ and $\beta = \langle \beta_{-t} \rangle_{t \in \mathbb{N}}$ be two sequences of parameters such that

$$\left[\alpha_{-t} > \alpha_{-(t+1)} > 0 \quad \text{and} \quad \beta_{-t} > \beta_{-(t+1)} > 0\right] \quad \text{for all } t \in \mathbb{N}. \tag{4}$$

The set of all sequences α such that $\alpha_{-t} > \alpha_{-(t+1)} > 0$ for all $t \in \mathbb{N}$ is denoted by $\mathcal{C}$. $\mathcal{C}^2$ is the Cartesian product of $\mathcal{C}$ with itself, that is, $\mathcal{C}^2$ is the set of all pairs of sequences (α, β) satisfying (4). A measure of individual insecurity corresponding to a pair of sequences $(\alpha, \beta) \in \mathcal{C}^2$, $V_{(\alpha,\beta)} = \left\langle V^T_{(\alpha,\beta)} \right\rangle_{T \in \mathbb{N}_0}$, is defined by letting, for all $T \in \mathbb{N}_0$ and for all $w = (w_{-T}, \ldots, w_0) \in \mathbb{R}^{(T)}$,

$$V^T_{(\alpha,\beta)}(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0.$$

The measures defined above bear some formal resemblance to specific social evaluation functions that have appeared in the context of the ethical approach to income inequality measures (see Kolm [7], Atkinson [1] and Sen [12]) and models of choice under risk (see, for instance, Rothschild and Stiglitz [10,11]). The Gini [5] index is one of the most established and well-known measures of income inequality. Mehran [8] and Weymark [16] independently introduced the generalized Ginis. These measures retain the linear structure of the Gini in rank ordered subspaces of the space of income distributions but allow for alternative degrees of inequality aversion by generalizing the coefficients to any rank ordered sequence of parameters. A subclass of the generalized Ginis is given by the single series Ginis, characterized in Bossert [3]. They are generalized Ginis such that the sequence of coefficients is the same for all population sizes. Because we work with a different domain and axioms that are suited to the environment discussed here, a single sequence of coefficients no longer is sufficient to adequately express the notion of insecurity that we intend to capture here. Moreover, because we examine wealth levels assigned to time periods rather than to individuals, many properties and methods that are appropriate in the context of inequality or risk measurement are not suitable in our setting.

Our first result characterizes the class of measures based on two sequences of parameters as defined above.

Theorem 1. *A measure of individual insecurity V satisfies difference monotonicity, the proximity property, homogeneity, translatability and the temporal aggregation property if and only if there exists $(\alpha, \beta) \in \mathcal{C}^2$ such that $V = V_{(\alpha,\beta)}$.*

Proof. 'If.' Let $(\alpha, \beta) \in \mathcal{C}^2$. That $V_{(\alpha,\beta)}$ satisfies homogeneity and translatability is immediate. Difference monotonicity follows from the positivity of the coefficients α_{-t} and β_{-t}; see the definition of $\mathcal{C}$. The proximity property is satisfied because of the inequalities that apply to the sequences of parameters; see, again, the definition of $\mathcal{C}$. To see that the temporal aggregation property is satisfied, define, for all $T \in \mathbb{N} \setminus \{1\}$, the function $\Phi^T : \mathbb{R}^2 \to \mathbb{R}$ by letting, for all $(x, y) \in \mathbb{R}^2$,

$$\Phi^T(x, y) = \begin{cases} \alpha_{-T}\, x + y & \text{if } x > 0 \\ y & \text{if } x = 0 \\ \beta_{-T}\, x + y & \text{if } x < 0. \end{cases}$$

'Only if.' Suppose V satisfies the required axioms. We prove the relevant implication by inductively constructing a pair of sequences $(\alpha, \beta) \in \mathcal{C}^2$ such that

$$V^T(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0 \qquad (5)$$

for all $T \in \mathbb{N}_0$ and for all $w \in \mathbb{R}^{(T)}$.

If $T = 0$, (5) is satisfied for all $w = (w_0) \in \mathbb{R}^{(0)}$ (trivially, for *any* pair $(\alpha, \beta) \in \mathcal{C}^2$ and, in particular, for the pair of sequences to be constructed below) because of (1).

Now let $T = 1$.

If $w \in \mathbb{R}^{(1)}$ is such that $w_{-1} = w_0$, difference monotonicity and (1) together imply

$$V^1(w) = V^0(w) = -w_0. \qquad (6)$$

If w is such that $w_{-1} > w_0$, translatability with $\delta = -w_0$ implies

$$V^1(w_{-1} - w_0, 0) = V^1(w_{-1} - w_0, w_0 - w_0) = V^1(w_{-1}, w_0) + w_0 = V^1(w) + w_0$$

and, therefore,

$$V^1(w) = V^1(w_{-1} - w_0, 0) - w_0. \tag{7}$$

Applying homogeneity with $\lambda = w_{-1} - w_0 > 0$, it follows that

$$V^1(w_{-1} - w_0, 0) = V^1((w_{-1} - w_0) \cdot 1, (w_{-1} - w_0) \cdot 0) = (w_{-1} - w_0)V^1(1, 0)$$

and, together with (7),

$$V^1(w) = \alpha_{-1}(w_{-1} - w_0) - w_0 \tag{8}$$

where $\alpha_{-1} = V^1(1, 0)$. By difference monotonicity, $\alpha_{-1} > 0$.

If w is such that $w_{-1} < w_0$, a parallel argument yields

$$V^1(w) = \beta_{-1}(w_{-1} - w_0) - w_0 \tag{9}$$

where $\beta_{-1} = -V^1(-1, 0) > 0$.

Combining (6), (8) and (9), we obtain

$$V^1(w) = \sum_{\substack{t \in \{1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0$$

for all $w \in \mathbb{R}^{(1)}$.

Now suppose that $T \in \mathbb{N} \setminus \{1\}$ and

$$V^{T-1}(w) = \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T-1\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0 \tag{10}$$

for all $w \in \mathbb{R}^{(T-1)}$ where $(\alpha_{-(T-1)}, \ldots, \alpha_{-1})$ and $(\beta_{-(T-1)}, \ldots, \beta_{-1})$ are such that $\alpha_{-1} > \ldots > 0$ and $\beta_{-1} > \ldots > 0$. We have to show that there exists $(\alpha_{-T}, \beta_{-T})$ such that

$$\alpha_{-1} > \ldots > \alpha_{-T} > 0 \quad \text{and} \quad \beta_{-1} > \ldots > \beta_{-T} > 0 \tag{11}$$

and

$$V^{T}(w) = \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} > w_{-(t-1)}}} \alpha_{-t} \left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t \in \{1,\ldots,T\}: \\ w_{-t} < w_{-(t-1)}}} \beta_{-t} \left(w_{-t} - w_{-(t-1)}\right) - w_0 \tag{12}$$

for all $w \in \mathbb{R}^{(T)}$.

Together with (10), the temporal aggregation property implies the existence of a function $\Phi^T \colon \mathbb{R}^2 \to \mathbb{R}$ such that

$$\begin{aligned} V^T(w) &= \Phi^T\left(w_{-T} - w_{-(T-1)}, V^{T-1}(w_{-(T-1)}, \ldots, w_0)\right) \\ &= \Phi^T\Bigg(w_{-T} - w_{-(T-1)}, \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ w_{-t}>w_{-(t-1)}}} \alpha_{-t}\left(w_{-t} - w_{-(t-1)}\right) \\ &\qquad + \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ w_{-t}<w_{-(t-1)}}} \beta_{-t}\left(w_{-t} - w_{-(t-1)}\right) - w_0\Bigg) \end{aligned} \tag{13}$$

for all $w \in \mathbb{R}^{(T)}$.

First, consider $w \in \mathbb{R}^{(T)}$ such that $w_{-T} = w_{-(T-1)}$. Difference monotonicity and (10) together imply

$$\begin{aligned} V^T(w) &= V^{T-1}(w_{-(T-1)}, \ldots, w_0) \\ &= \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ w_{-t}>w_{-(t-1)}}} \alpha_{-t}\left(w_{-t} - w_{-(t-1)}\right) + \sum_{\substack{t\in\{1,\ldots,T-1\}:\\ w_{-t}<w_{-(t-1)}}} \beta_{-t}\left(w_{-t} - w_{-(t-1)}\right) - w_0 \end{aligned}$$

and it follows that

$$\Phi^T(0, y) = y \quad \text{for all } y \in \mathbb{R}^2. \tag{14}$$

Now consider the case in which w is such that $w_{-T} > w_{-(T-1)}$. Homogeneity implies that Φ^T satisfies

$$\Phi^T(\lambda x, \lambda y) = \lambda \Phi^T(x, y) \quad \text{for all } \lambda, x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R} \tag{15}$$

and translatability implies

$$\Phi^T(x, y - \delta) = \Phi^T(x, y) - \delta \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } \delta, y \in \mathbb{R}. \tag{16}$$

Letting $\delta = y$, (16) implies $\Phi^T(x, 0) = \Phi^T(x, y) - y$ and, thus,

$$\Phi^T(x, y) = \Phi^T(x, 0) + y \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R}. \tag{17}$$

Letting $\lambda = x > 0$, (15) implies

$$\Phi^T(x, 0) = \Phi^T(x \cdot 1, x \cdot 0) = x\Phi^T(1, 0) \quad \text{for all } x \in \mathbb{R}_{++}$$

and, together with (17), we obtain

$$\Phi^T(x, y) = \alpha_{-T}\, x + y \quad \text{for all } x \in \mathbb{R}_{++} \text{ and for all } y \in \mathbb{R} \tag{18}$$

with $\alpha_{-T} = \Phi^T(1,0)$. By difference monotonicity, $\alpha_{-T} > 0$ and by the proximity property, $\alpha_{-T} < \alpha_{-(T-1)}$ and, thus,

$$\alpha_{-1} > \ldots > \alpha_{-T} > 0. \tag{19}$$

If w is such that $w_{-T} < w_{-(T-1)}$, an argument parallel to that used above to derive (18) can be employed to obtain

$$\Phi^T(x,y) = \beta_{-T}\,x + y \quad \text{for all } x \in \mathbb{R}_{--} \text{ and for all } y \in \mathbb{R} \tag{20}$$

with $\beta_{-T} = -\Phi^T(-1,0)$. By difference monotonicity, $\beta_{-T} > 0$ and by the proximity property, $\beta_{-T} < \beta_{-(T-1)}$ and, thus,

$$\beta_{-1} > \ldots > \beta_{-T} > 0. \tag{21}$$

Combining (14), (18) and (20), it follows that

$$\Phi^T(x,y) = \begin{cases} \alpha_{-T}\,x + y & \text{if } x > 0 \\ y & \text{if } x = 0 \\ \beta_{-T}\,x + y & \text{if } x < 0 \end{cases}$$

for all $(x,y) \in \mathbb{R}^2$. Substituting back into (13), we obtain (12) for all $w \in \mathbb{R}^{(T)}$. Because $\alpha_{-1} > 0$ and $\beta_{-1} > 0$ and, moreover, (19) and (21) are satisfied for all $T \in \mathbb{N} \setminus \{1\}$, the pair of sequences (α, β) thus constructed satisfies (11) and therefore is in $\mathcal{C}^2$ as required.
■

The definition of the measures characterized in theorem 1 is silent about the relationship between the current wealth level and the coefficients α_{-1} and β_{-1} applied to the most recent gains and losses. This lack of specification is intentional: it is not obvious to us how current levels are to be traded off against past differences. Among other things, this may depend on the interpretation of w_0 and, particularly, on whether this variable represents current wealth only or an aggregate of current wealth and future expectations on the part of an agent.

The result of the previous theorem does not impose any restrictions on the relationship between the sequences α and β. A plausible assumption appears to be the requirement that *ceteris paribus* losses of a certain magnitude in a given period have at least as strong an impact on insecurity as *ceteris paribus* gains of the same magnitude in the same period. This assumption is captured in the weak loss priority axiom. If this property is added to those of theorem 1, the parameter values must be such that losses carry a weight that is at least as high as that for gains in each period, which leads to a subclass of the measures identified in the previous characterization. To define this loss averse class formally, let $\mathcal{D}$ be the set of all pairs of sequences $(\alpha, \beta) \in \mathcal{C}^2$ such that

$$\alpha_{-t} \geq \beta_{-t} \quad \text{for all } t \in \mathbb{N}.$$

Adding weak loss priority to the axioms of theorem 1 leads to a characterization of these loss averse measures. This is stated in the following theorem, the proof of which is straightforward and left to the reader.

Theorem 2. *A measure of individual insecurity V satisfies difference monotonicity, the proximity property, homogeneity, translatability, the temporal aggregation property and weak loss priority if and only if there exists $(\alpha, \beta) \in \mathcal{D}$ such that $V = V_{(\alpha,\beta)}$.*

Theorem 2 identifies the class of insecurity measures that we advocate in this paper. As an example, consider the measure obtained by choosing the sequences α and β so that

$$\alpha_{-t} = \frac{1}{2t-1} \quad \text{and} \quad \beta_{-t} = \frac{\alpha_{-t}}{2}$$

for all $t \in \mathbb{N}$. Clearly, $(\alpha, \beta) \in \mathcal{D}$. The coefficients according to the sequence α are the inverses of the coefficients corresponding to the Gini social evaluation function; see, for instance, Donaldson and Weymark [4] and Weymark [16].

4 Concluding remarks

The measures of individual insecurity characterized in this paper share a linear structure with the generalized Gini social evaluation functions used in ethical approaches to inequality measurement. Furthermore, they resemble rank ordered decision criteria employed in theories of choice under uncertainty. This linearity clearly depends crucially on the combination of homogeneity and translatability that we use in our characterizations. There is some scope for the examination of alternatives that lead to different structures and different analogues in inequality measurement such as quadratic functional forms that may relate to variance based inequality measures.

Our insecurity indices are sufficiently flexible to admit the use of any model describing the formation of individual expectations. We intentionally do not pick a specific theory of decision making under uncertainty in order to allow for a wide variety of possible applications. The exploration of such applications constitutes a natural next step in this area of research.

We provide a thorough treatment of individual insecurity based on wealth considerations in this paper. This leaves open the problem of aggregating individual insecurity values into a social index. Thus, an explicit study of economic insecurity for society as a whole is another task to be undertaken in future work. In addition, one may want to explore the possibility of including (non-monetary) variables other than wealth in order to arrive at a more comprehensive notion of insecurity.

References

[1] A.B. Atkinson, On the measurement of inequality, J. Econ. Theory 2 (1970) 244–263.

[2] C. Blackorby, D. Primont, R.R. Russell, Duality, Separability, and Functional Structure: Theory and Economic Applications, North-Holland, Amsterdam, 1978.

[3] W. Bossert, An axiomatization of the single-series Ginis, J. Econ. Theory 50 (1990) 82–92.

[4] D. Donaldson, J.A. Weymark, A single-parameter generalization of the Gini indices of inequality, J. Econ. Theory 22 (1980) 67–86.

[5] C. Gini, Variabilità e mutabilità, Studi Economico-Giuridici della Facoltà di Giurisprudenza dell' Università di Cagliari a.III (1912) parte II.

[6] R. Hertwig, G. Barron, E.U. Weber, I. Erev, Decision from experience and the effects of rare events in risky choice, Psychological Science 15 (2004) 534–539.

[7] S.-C. Kolm, The optimal production of social justice, in: J. Margolis and S. Guitton (Eds.), Public Economics, Macmillan, London, 1969, pp. 145–200.

[8] F. Mehran, Linear measures of income inequality, Econometrica 44 (1976) 805–809.

[9] L. Osberg, Economic insecurity, Social Policy Research Centre Discussion Paper 88, 1998.

[10] M. Rothschild, J.E. Stiglitz, Increasing risk I: a definition, J. Econ. Theory 2 (1970) 225–243.

[11] M. Rothschild, J.E. Stiglitz, Increasing risk II: its economic consequences, J. Econ. Theory 3 (1971) 66–84.

[12] A. Sen, On Economic Inequality, Oxford University Press, Oxford, 1973.

[13] A. Sen, Famines as failures of exchange entitlements, Economic and Political Weekly 11 (1976) 1273–1280.

[14] A. Sen, Resources, Values and Development, Basil Blackwell, Oxford, 1984.

[15] J.E. Stiglitz, A. Sen, J.-P. Fitoussi, Report by the Commission on the Measurement of Economic Performance and Social Progress, available from http://www.stiglitz-sen-fitoussi.fr, 2009.

[16] J.A. Weymark, Generalized Gini inequality indices, Math. Soc. Sci. 1 (1981) 409–430.

[17] S. Yitzhaki, Relative deprivation and the Gini coefficient, Quart. J. Econ. 93 (1979) 321–324.