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Measuring poverty accounting for time

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Abstract

In this paper we make a methodological proposal to measure poverty accounting for time by proposing a new index that aims at reconciling the way poverty is measured in a static and a dynamic framework. Our index is able to consider the duration of the poverty spell and the social preference for equality in well-being given that, in contrast with others that have been previously proposed, it is sensitive to the level of inequality between individual complete poverty experiences over time. Moreover, other indices in the literature can be interpreted as special cases of our more general measure.

Keywords: intertemporal poverty, duration, equality, poverty measurement
JEL classification: D31, D63, I32

Introduction

During the twentieth century the literature on the measurement of poverty has constructed a sound analytical framework in which there is a large consensus on the set of properties that poverty indices should verify. The seminal work by Sen (1976) focused the discussion in the three main dimensions of poverty: incidence, intensity and inequality, and drew the path through which subsequent research has followed. In these first stages poverty measurement was generally linked to a static view of the poverty phenomenon because most of the data was of a cross-sectional nature. The increasing availability of longitudinal data in a variety of countries has stimulated the advances in undertaking a more dynamic view of the issue.

The first approaches to considering the time dimension in poverty measurement were centered in analyzing poverty transitions and the duration of poverty spells. All these contributions put forward the importance of flows into and out of poverty and the significant heterogeneity in the poverty dynamics pattern of different populations. More recently, there is an increasing research interest in developing new methodologies that take into account individual income profiles along time in constructing a consistent aggregate indicator. Two main approaches have been proposed in the literature, the *components approach* and the *spells approach*. The first of these approaches contributed to underlying the relevance of permanent income in poverty analysis related to the possibility of compensating low and high income periods. The use of a permanent income concept allows this approach to easily adapt the standard poverty measurement tools to a dynamic setting. In contrast with this methodology, the spells approach is able to incorporate duration as a poverty dimension while considering incidence in the aggregate poverty indicator. Also, recent contributions have permitted the consideration of poverty intensity within this aggregate poverty measure, not only considering the poverty gap but also taking into account that the concatenation of poverty periods may aggravate poverty. In this context and despite the many advances in the literature on poverty dynamics there is still no consensus on a measure of poverty that adequately summarizes the information provided by a panel of individuals.

From our point of view, these recent contributions within the spells approach allow to adequately include both the incidence and the intensity of poverty dimensions in a

dynamic framework. In particular, the incidence of poverty in a dynamic setting is based in a double cut-off approach where, apart from the poverty line, a certain number of periods below the poverty threshold are necessary to identify those who are longitudinally poor. In the case of intensity it is not only a larger poverty gap that increases poverty but also the concatenation of periods below the poverty threshold could make poverty more severe. This considers the relevance of poverty spell duration.

Regarding the third relevant dimension in the measurement of poverty, the inequality of poverty experiences has been also partially incorporated in the aggregate intertemporal poverty indicators. A way to include inequality is through indices that are sensible to the equalization of individual per-period poverty gaps; however we believe that the way in which this has been implemented up to now can be significantly improved by considering an index that takes into account the inequality between complete individual poverty patterns in time. This, together with spell duration, leads to rule out what in the literature is known as *path independence*. Path independence implies that aggregating first across individuals and then across time periods should be equivalent to aggregating in the reverse order (i.e. first across time periods for each individual and then across the population). In our view, given that inequality should be evaluated across individuals it is necessary that we first summarize the complete individual information in time and then construct an aggregate index that takes into account a social preference for equality among individuals. In this way we move a step forward in the direction of constructing an integrated framework in the measurement of poverty which is consistent for both a static and a dynamic setting. Therefore, we can incorporate time into all Sen (1976)'s three dimensions while cross-sectional poverty could be understood as a particular case of a more general dynamic framework.

The aim of the paper is first to discuss the relevant properties we believe that an aggregate intertemporal poverty index should verify to be consistent with our view of longitudinal poverty and then propose a new index strongly rooted in the previous literature that verifies these properties. In fact, some of the other intertemporal poverty indices recently proposed can be viewed as special cases of our index.

The structure of the paper is the following. The first section presents the different approaches in the literature of poverty measurement accounting for time. The second section details the relevant properties an aggregate intertemporal poverty index should

verify and discusses the way in which they are fulfilled by previous indices. The third section introduces a new aggregate intertemporal poverty index that is consistent with the relevant properties. The last section summarizes the main contribution of the paper.

1. Poverty and time: the different approaches in the literature

Poverty dynamics has experienced a large development in income distribution analysis in recent years. Two main approaches to modeling intertemporal poverty dynamics can be distinguished as noted by Yaqub (2000). The first has been labeled as the *components approach* and focuses on estimating the permanent and transitory components of poverty. The second has been labeled as the *spells approach* and focuses on poverty spell duration and transitions into and out of poverty.[1] The crucial distinction between these approaches in the study of intertemporal poverty is that the components approach assumes compensation between low and high income periods and then the identification of who is poor each period of time becomes unnecessary while in the spells approach no compensation is allowed and one needs to identify who is poor each period.

More specifically, an important part of this literature has aimed to find a way to summarize the total population income pattern along a time interval into an aggregate intertemporal poverty index. The way this index has been constructed can be generally interpreted as following a two step procedure. A first step aims to obtain an inter-temporal wellbeing (or lack of) indicator for each individual based on her income profile while a second step aggregates all these in a single inter-temporal poverty index for the society as a whole. In implementing this two-stage procedure each of the aforementioned approaches follows a different strategy.

In the components approach the individual intertemporal wellbeing indicator is constructed in order to capture the permanent component of income. This indicator can be used to identify who is chronically poor (those individuals for whom its value is below the poverty line). To approximate this permanent component Jalan and Ravallion (1998), for example, average individual income-to-needs ratio (income relative to the

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[1] Throughout this paper, following Bane and Ellwood (1986), we will consider a poverty spell as the set of consecutive periods during which income falls below the poverty line.

per period poverty line) over the complete period of analysis, while Duncan and Rodgers (1991) calculate an income-to-needs ratio for the whole period by summing up incomes and poverty lines in time. Alternatively Duncan and Rodgers (1991) and Gaiha and Deolalikar (1993) approximate the permanent component by the predicted income (or income-to-needs) estimated using a fitted econometric model, and Rodgers and Rodgers (1993) use the maximum sustainable annual consumption level that the agent could achieve with her actual income stream over the years if she could save and borrow at a constant interest rate. The components approach generally implies assuming a perfect substitution of income over time and is insensitive to the number of periods that individuals spend below the poverty line. Thus a single period of high income either at the beginning or at the end of the observed time span can offset several periods below the poverty line, which could be reasonable to assume only in absence of liquidity constraints.[2] The assumption of a perfect substitution of income in time was recently relaxed by Foster and Santos (2009) and by Porter and Quinn (2008). The former by using a parameterized generalized mean that allows for a whole range of alternative and explicit degrees of income substitutability between time periods. The latter by assuming an increasing elasticity of substitution using a linear combination of CES functions.

In the second step, the components approach typically calculates an aggregate intertemporal poverty index over the distribution of individual indicators: the headcount ratio (Duncan and Rodgers, 1991, Gaiha and Deolalikar, 1993), the squared poverty gap index of Foster *et al.* (1984) (Rodgers and Rodgers, 1993 and Jalan and Ravallion, 1998), or the Clark *et al.* (1981) index (Foster and Santos, 2009). These last two indices, as in the static approach to measuring poverty, take all three dimensions of poverty into account: incidence, intensity and inequality among the poor. Therefore, in essence, this approach constructs an intertemporal income measure over which a poverty index satisfying the standard axioms is then computed.

In the spells approach, instead, the first step consists in computing an individual intertemporal poverty index with desirable properties based on the per-period individual

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[2] Some authors (Rodgers and Rodgers, 1993; Slesnick, 2001) maintained that poor individuals can smooth consumption by borrowing and saving over time. However Jappelli (1990) showed the existence of liquidity constraints affecting mostly low income households, while others (Kempson, 1996; Azpitarte, 2008) obtained that there is a significant correlation between income and assets.

poverty indicators. Then, in the second step, an aggregated index is obtained averaging these over the whole population. In general, per-period poverty indicators are based on each period individual poverty gap. In the earliest and most simple examples of this methodology such as Duncan *et al.* (1984), Duncan and Rodgers (1991), Duncan *et al.* (1993) or Gaiha and Deolalikar (1993) the intertemporal (chronic) poverty index takes the value 1 if the number of periods below the poverty line exceeds a certain number out of the total periods of observation. These papers do not take into account per-period poverty intensity or intertemporal variability (inequality of per-period individual poverty gaps along time). More recently, Foster (2007, 2009) generalizes this approach presenting a new family of chronic poverty measures based on the classical *FGT* family of poverty indices constructed over the per-period normalized gaps of the chronically poor. This group is identified using a dual cutoff approach: an individual is considered to be chronically poor if the percentage of time she spends below the poverty line exceeds a certain threshold. His approach, in our view, is equivalent to constructing an individual intertemporal poverty measure using a *FGT* index on each individual income profile and then computing the arithmetic mean over the population. Note also that this last methodology verifies a path independence property. This means that once individuals are identified as chronically poor, all their per-period observations are assumed to be independent from each other. This derives from the fact that Foster's index is the mean of the elements of a matrix with column vectors listing individuals and row vectors listing years, where the typical element is the normalized gap to the power of the poverty aversion parameter when the individual is chronically poor and zero otherwise.

Obviously the relative degree of overall poverty when comparing two individuals over time depends also on the role and evaluation of persistence. Since Bane and Ellwood (1986) it appears clear in the literature that the longer a person has been poor the less likely it is that she will escape poverty.[3] Foster (2007) underlines that his approach rules out the possibility that those continuous periods below the poverty line create greater harm than the same periods in poverty interspersed with non poverty periods. He actually says that it is not entirely clear whether and how the time-ordering of income

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[3] Even if a part of this result may come about due to individual heterogeneity it is also likely that poverty itself makes it more difficult to leave. This last effect is generally referred to in the literature as *true state dependence*.

should impact in aggregation or identification of chronic poverty and he decides to take the extreme position by ignoring the time-orderings entirely. Most recently, a variety of papers such as Calvo and Dercon (2007), Hoy and Zheng (2008), Bossert *et al.* (2008) or Mendola *et al.* (2009) have incorporated the sensitivity to poverty persistence within the individual intertemporal poverty index in different ways. These papers propose a list of desirable properties for measuring poverty across time focusing on trajectories of poverty rather than on the set of periods of poverty at different points in time.[4]

It is important to note that papers within the spells approach, unlike those within the components approach, generally construct the aggregate poverty index in a way that is inconsistent with the classical approach to measuring poverty. This is because computing a mean of individual intertemporal poverty indicators does not make the aggregate index sensitive to the distribution across individuals. The only exception to this inconsistency that we know of is Hoy and Zheng (2008) who discuss the possibility that individual lifetime poverty measures are aggregated using a function reflecting society's preferences for equality of individual poverty deprivations, in a similar way to other social welfare measures. This is in line with what has been done not only in the measurement of poverty and inequality but also in another form of deprivation like unemployment (Paul, 1992, Riese and Brunner, 1998, Borooah, 2002, Sengupta, 2009, Shorrocks, 2009a,b). Reconciling the properties of the aggregated intertemporal poverty experiences within the spells approach with those widely accepted for static poverty measures is precisely our main aim in this paper. Using the spells approach to construct a time-sensitive poverty measure is advantageous in order to incorporate the sensitivity to poverty trajectories and the duration of spells which is not easily captured by the components approach.

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[4] Calvo and Dercon (2007) apply discount factors to incorporate sensitivity to the income trend in their measures and they also introduce poverty duration sensitivity taking into account the impact of the poverty gap the previous year. Hoy and Zheng (2008) measure of lifetime poverty increases with poverty spells experienced at the early stages of life and with the accumulation of poverty periods in time (not necessarily consecutive). Bossert *et al.* (2008) propose a measure of poverty that increases with the duration of spells in time.

2. Relevant properties in the measurement of poverty accounting for time

2.1 Preliminary notation and definitions

Consider a society consisting of N individuals observed T periods of time represented by a NxT matrix Y which elements are per-period individual equivalized income (or consumption). For each individual i we can denote the raw vector $y_i = (y_{i1}, y_{i2}, ..., y_{iT})$ representing her non-negative income profile in time. Matrix Y may be written as:

$$Y = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1T} \\ y_{21} & y_{22} & \cdots & y_{2T} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ y_{N1} & y_{N2} & \cdots & y_{NT} \end{pmatrix}$$

An individual i is poor in period t if and only if her income falls below the corresponding poverty line z_t.[5] Let $g_i^\gamma = (g_{i1}^\gamma, g_{i2}^\gamma, ..., g_{iT}^\gamma)$ be the raw vector of normalized poverty gaps[6] to the power of γ (where $\gamma \geq 0$) for individual i, which elements are given by:

$$g_{it}^\gamma = \begin{cases} \left(\dfrac{z_t - y_{it}}{z_t} \right)^\gamma & \text{if} & y_{it} < z_t \\ \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Let s_{it} be the number of periods of the particular (poverty or non poverty) spell the ith individual is in at period t, and $q_i(y_i; z)$ the total number of time periods that individual i is poor ($g_{it}^1 > 0$).

An individual intertemporal poverty index is a function $p_i(y_i; z)$ that maps each income profile y_i into $\Re_+$ (where $\Re_+$ is the nonnegative real number set) for a given poverty

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[5] This poverty line may be constant over time, as it is common in the measurement of absolute poverty.

[6] The normalization of poverty gaps is not essential in our framework. Non normalized poverty gaps could also be used.

line vector $z = (z_1, z_2, ..., z_T)$. Thus, p_i is a function whose value indicates the degree of intertemporal poverty level of individual i, associated with her income profile in time, y_i, and z. Let $p = (p_1, p_2 ..., p_N)$ denote the vector of individual intertemporal poverty indicators for the society, and $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_N)$ the vector of ordered intertemporal individual poverty experiences, where $\tilde{p}_1 \geq \tilde{p}_2 \geq ... \geq \tilde{p}_N$, being $q(p)$ the number of intertemporal poor individuals ($p_i > 0$).

An aggregate poverty index is a function $P(Y; z)$ which given the poverty line vector z maps each income matrix $Y = (y_1, y_2, ..., y_N)$ into $\Re_+$. The value of $P(Y; z)$ represents the aggregate poverty level of a particular society accounting for time.

2.2 Desirable properties of an aggregate intertemporal poverty index

Given that within the spells approach the index is constructed in two steps, in order to discuss the desirable properties of the aggregate intertemporal poverty index P, we need, in a first stage, to discuss the desirable properties of the individual intertemporal poverty indicator p_i. It is reasonable to assume that p_i should verify the equivalent continuity, focus, monotonicity and scale invariance axioms that are usual in the standard poverty measurement case. Now, however, these properties refer to time observations of the same individual instead of referring to different individuals at the same moment in time.

The **continuity axiom** requires that $p_i(y_i; z)$ is continuous function of y_i for any given z. The **focus axiom** requires that any increase in income at a period in which the individual is non-poor should not affect the measured level of her individual intertemporal poverty. Accepting this property implies assuming that it is not possible to compensate poverty gaps with incomes over the poverty threshold at periods when the individual is out of poverty, something common within the spells approach. The **monotonicity axiom** requires that a decrease in the income at any period in which the individual is below the poverty line should lead to an increase in the measured level of intertemporal poverty. The **scale invariance axiom** requires that if both the poverty line

vector and every income are scaled up or down by the same factor the intertemporal poverty level should remain unchanged.[7]

However, in our view p_i should not verify the time version of the anonymity or symmetry axiom, and the transfer axiom should not be directly incorporated in this setting given that both are, in some cases, incompatible with a relevant property pointed out by the literature within the spells approach. Indeed given that longer poverty spells reduce the probability of a poverty exit, we want the measure of individual intertemporal poverty to be sensitive to poverty spells duration. We propose a **poverty spell duration sensitivity axiom** that requires that given any two poverty spells (a certain number of concatenated periods of poverty) the index should be higher when both of the spells are consecutive. Thus, the concentration of periods of poverty in a fewer number of poverty spells will increase the individual intertemporal poverty index.[8]

Assuming an equivalent transfer axiom in this intertemporal setting would imply that an income transfer between two periods in which the individual is poor would always increase poverty if the income loss takes place in the period with a lower income level. This would happen whatever the length of the poverty spell in which each period is inserted. Clearly, this becomes a problem if we want to assume the spell duration sensitivity axiom in the case that the income loss takes place in a shorter spell compared to that of the income gain. This is because the regressive transfer is reducing the poverty gap of a period in which the individual is closer to the poverty line which should increase poverty but, at the same time, this period is inserted in a longer poverty spell which should decrease poverty. Thus it is not clear if, in this particular case, intertemporal poverty should always increase. Therefore, we propose an **intertemporal regressive transfer axiom** that implies that if the previous regressive transfer reduces the income in a period inserted in an equal or larger spell compared to that of the income gain, then poverty should increase.

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[7] Note that this last property only makes sense in the measurement of relative poverty.

[8] A more general property was proposed by Hoy and Zheng (2008), called *the chronic-poverty axiom*, which implies that the closer two spells are from one another, the greater lifetime poverty is. In the same line Mendola *et al.* 2009 defended that poverty should be decreasing in the time distance between spells. Hojman and Kast (2009) considered that it would also be desirable that poverty should reflect a preference for improving sequences of outcomes.

We also believe that in our setting it is not interesting that an individual intertemporal index verifies the equivalent to the anonymity axiom. Accepting this axiom would imply that permutations of per-period incomes across time would not affect intertemporal poverty. As far as one believes that the intertemporal poverty index should be sensitive to the individual poverty profile or trajectory across time (for example, to the length of the poverty spells) anonymity should not be imposed because permutations may affect trajectories in a way that they may aggravate intertemporal poverty.

In the second stage an aggregate intertemporal poverty measure $P(Y;z)$ is defined to be a continuous function of individual intertemporal poverty indices reflecting society's preference about the distribution of intertemporal poverty deprivations and can be written as:

$$P(Y;z) = f\left(p_1(y_1;z),...,p_N(y_N;z)\right)$$

In general we would like P to be consistent with the desirable properties of static poverty measures. Thus the function f must verify the following properties. The **continuity** property requires that the P index is a continuous function of p_i. The **Anonymity** or **symmetry** property implies that the P index remains unchanged after whichever permutation of p_i (which corresponds to permutations of row vectors in matrix Y). The **replication invariance** property requires that the P index remains unchanged under r replications of the original population where the dimension of the new matrix will be $(rN)xT$, thus allowing for the comparison of populations with different sizes. **Monotonicity** imposes that P decreases whenever there is a decrease in p_i. **Preference for intertemporal poverty equality** requires that P decreases whenever there is an equalization of p_is. This is the analogue of the Pigou-Dalton principle of transfers used in the inequality and welfare analysis of income distributions, and commonly required in the measurement of cross-sectional poverty in line with Sen (1976)'s seminal discussion of poverty dimensions. This property in our case can be formalized as follows. Let p and p' be two individual intertemporal poverty vectors respectively associated to the income matrices Y and Y' of equal size NxT. If p' is obtained from p such that there are two individuals i and j so that:

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$$p_i > p'_i > p_j; p'_i + p'_j = p_i + p_j; p'_s = p_s \text{ for all } s \neq i, j$$

where, at least, individual i is intertemporally poor, then $P(Y';z) < P(Y;z)$.

We consider this set of axioms as the minimum requirement for a measure of intertemporal poverty. Obviously, verifying other properties could be of interest. For instance, we could require the index to be normalized, to range between zero and one, or to be additively decomposable by population subgroups.

2.3 Properties of previous aggregate intertemporal indicators

After detailing the desirable properties that a measure of intertemporal poverty should comply, it is interesting to analyze to what extent the indices that have been proposed in the literature verify them so far. The literature following the components approach does not properly capture the relevance of poverty duration. In particular this approach leads to the violation of the focus, spell duration and intertemporal transfer axioms because periods with income above the poverty line can compensate poverty gaps. Within the spells approach the earliest and most simple contributions do incorporate the relevance of duration in the measurement of longitudinal poverty but fail to capture other important dimensions such as intensity of poverty experiences and their distribution across the population. This generally implies that often measures that have been proposed within this approach do not fulfill neither the monotonicity in the income of the poor axiom nor the intertemporal transfer axiom.

The most recent contributions within the spells approach have proposed indices that take better into account the different dimensions of intertemporal poverty. One seminal contribution that centered the main issues on the measurement of poverty in a longitudinal setting was Foster (2007, 2009)'s intertemporal poverty index that can be rewritten in our notation as:[9]

$$K_\gamma(Y;z) = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} g_{it}^\gamma \quad \gamma \geq 0$$

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[9] This is a measure of chronic poverty, for this reason a person will be considered (chronically) poor if spending a minimum percentage τ of the total time of observation below the poverty line. In the subsequent discussion of the properties of this index we will leave this out for simplicity, thus assuming that $\tau = \frac{1}{T}$.

This index for $\gamma > 1$ verifies most of the properties we have previously discussed. However, it fails to satisfy some axioms such as the preference for intertemporal poverty equality and spell duration.

The first axiom is not satisfied because Foster's index does not take into consideration the whole income profile of individuals (once they are identified as chronic poor). Let us consider the following example with two income matrices and their corresponding normalized poverty gap matrices (given a constant poverty line $z_t = 5$ for all t):

Example 1

$$Y = \begin{pmatrix} 0 & 4 & 6 & 6 \\ 4 & 0 & 6 & 6 \end{pmatrix} \quad \Rightarrow \quad Y' = \begin{pmatrix} 0 & 0 & 6 & 6 \\ 4 & 4 & 6 & 6 \end{pmatrix}$$

$$g^1 = \begin{pmatrix} 5/5 & 1/5 & 0 & 0 \\ 1/5 & 5/5 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad g^{1\,\prime} = \begin{pmatrix} 5/5 & 5/5 & 0 & 0 \\ 1/5 & 1/5 & 0 & 0 \end{pmatrix}$$

where in the first row of the Y and Y' matrices we have the incomes of individual i and in the second row we have the income of individual j both during four periods of time (being both considered chronic poor). The Y' matrix can be obtained from Y by a simple permutation of incomes of both individuals in the second period. Foster's index will remain constant after this permutation. However, it seems reasonable to consider that aggregate intertemporal poverty has increased as long as one cares about equality between the different individual poverty experiences. This is because after the permutation intertemporal poverty is more concentrated in individual i (first row in matrix Y and Y').

Foster (2007) proposed a transfer axiom in the measurement of intertemporal poverty to take into account the sensitivity to inequality among the poor pointed out by Sen (1976) in a cross-sectional setting. This axiom roughly says that a per-period equalization of all poverty gaps of (chronic) poor individuals should reduce intertemporal poverty. This seems reasonable but does not embrace all possible equalizations between two given individuals (as we will see in example 2). On the other hand Foster's index verifies a more general property, according to which any equalization of poverty gaps g_i^1s (of any individual and period) reduces intertemporal poverty. The reason why we do not

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consider this general property as desirable is because it implies a decrease in poverty, regardless of the profile of poverty gaps of the individuals involved in the income transfer. In particular it ignores any information about the poverty gaps they experienced in any of the other periods. Let us consider the following example:

Example 2

$$Y = \begin{pmatrix} 1 & 4 & 6 & 4 \\ 4 & 1 & 6 & 1 \end{pmatrix} \quad \Rightarrow \quad Y' = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 3 & 1 & 6 & 1 \end{pmatrix}$$

$$g^1 = \begin{pmatrix} 4/5 & 1/5 & 0 & 1/5 \\ 1/5 & 4/5 & 0 & 4/5 \end{pmatrix} \quad \Rightarrow \quad g^{1\,'} = \begin{pmatrix} 3/5 & 1/5 & 0 & 1/5 \\ 2/5 & 4/5 & 0 & 4/5 \end{pmatrix}$$

Here the Y' matrix can be obtained from Y by a transfer from individual j (second row in matrix Y and Y') to individual i (first row in matrix Y and Y') in the first time period so that incomes in that period are equalized. This transfer implies a decrease in Foster's intertemporal poverty index K_γ for $\gamma > 1$. In our example $K_2(Y;z) = 0.255$ while $K_2(Y';z) = 0.235$. However this index does not take into account the information contained in the matrices regarding other time periods different from the first one. In this particular example it would be reasonable to consider the donor, j, as intertemporally poorer than the receiver, i, despite being less poor in the first period. This is because they are both poor the same number of periods, while j has a lower total income in poverty. In this setting we would expect the intertemporal poverty index to increase.

The violation of the second axiom, regarding spell duration sensitivity, comes from the fact that Foster's index does not take into account the length of the spell to which each period belongs. Let us consider the following example:

Example 3

$$Y = \begin{pmatrix} 0 & 0 & 6 & 0 \\ 6 & 6 & 6 & 6 \end{pmatrix} \quad \Rightarrow \quad Y' = \begin{pmatrix} 0 & 0 & 0 & 6 \\ 6 & 6 & 6 & 6 \end{pmatrix}$$

$$g^1 = \begin{pmatrix} 5/5 & 5/5 & 0 & 5/5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad g^{1\,'} = \begin{pmatrix} 5/5 & 5/5 & 5/5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In this case, the Y' matrix is obtained from Y by joining the two poverty spells of individual i with durations of two and one periods into a three-period single poverty spell. Again, Forster's index does not change after this concatenation of spells while if one agrees that longer spells aggravate poverty, as a substantial part of the literature on poverty dynamics does, then the aggregate intertemporal poverty index should increase.[10] This problem will be shared by all other indices that do not incorporate sensitivity to spell duration.

Bossert *et al.* (2008) improved the existing proposals by introducing an index that is sensitive to the duration of spells. Their proposal weights each individual per-period poverty by the length of the spell to which that period belongs to.[11] Their aggregate intertemporal poverty measure P^* is an average of individual intertemporal poverty indices P_i^*, and in the case of using the *FGT* index as the per-period poverty indicator it can be rewritten (in our notation) as:

$$P^* = \frac{1}{N}\sum_{i=1}^{N} P_i^* = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} g_{it}^{\gamma} s_{it} \quad \gamma \geq 0$$

In the case of example 3, this index increases after joining the two spells, as our spells duration sensitivity axiom requires, because it weights each period in the new and longer spell more than it did in the original spells. However this index fails to solve the counterintuitive result of Foster's index in examples 1 and 2. Given that in these examples the length of the poverty spells of both individuals is the same in both matrices, the permutation in the first example does not modify the aggregate index while poverty is reduced by the partial equalization in the second one from a value of P^* for $\gamma = 2$ of 0.425 for matrix Y to 0.385 for matrix Y'. This results from the fact that this index does not consider individuals' complete intertemporal profiles thus it ignores the concentration of poverty in one individual (example 1) or her lower income in the remaining periods below the poverty line (example 2). This will be a common

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[10] Note that, as mentioned before, Foster (2007, page 17) took an extreme position adopting his time anonymity axiom as it was not clear to him whether and how the time-ordering of incomes should impact the aggregation of poverty.

[11] This is not the only case of a spell duration sensitive measure. Calvo and Dercon (2007) have previously proposed an index of intertemporal poverty where the contribution of a period of poverty is larger if that period is followed after another period of poverty. Mendola *et al.* (2009) have alternatively proposed an index that accounts for the relative distances between poverty periods.

feature of those indices that construct an aggregate intertemporal poverty measure averaging over either individual or per-period indicators.

Therefore, in order to obtain an aggregate intertemporal poverty index that fulfills all the properties discussed above, we need an indicator that is constructed taking into consideration the individuals' complete time profile.

3. A new distribution-sensitive aggregate intertemporal poverty measure

Our main aim in this paper is to reconcile the way of measuring poverty in a cross-section and a panel, thus taking the individual as the reference. In order to do this it appears most reasonable to follow the two-step procedure mentioned above. First we construct an individual intertemporal poverty indicator aggregating per-period poverty.[12] Secondly our aggregate intertemporal poverty measure is based on the distribution of these individual indicators.

In the first stage we define the individual intertemporal poverty indicator p_i as being a modified *FGT* index defined over time:

$$p_i(y_i;z) = \frac{1}{T}\sum_{t=1}^{T} g_{it}^{\gamma} w_{it}$$
(2)

where γ is the usual *FGT* parameter. As far as $\gamma > 1$ this parameter introduces here the sensitivity of the individual intertemporal poverty index to higher inequality of poverty experiences over time. Thus, for any given time-averaged poverty gap, a higher variability of the normalized gaps over time increases individual poverty. Each poverty gap (to the power of γ) is furtherly weighted according to the duration of the corresponding poverty spell in which that period is involved. In particular, weights w_{it} can be expressed as:

$$w_{it} = \left(\frac{s_{it}}{T}\right)^{\beta}$$

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[12] Note that this framework can be easily accommodated to measure chronic poverty in line with Foster (2007, 2009) by just introducing the temporal cut-off τ in the definition of the normalized poverty gaps (expression 1) and with a minimum adaptation of the axioms.

where as far as $\beta > 0$, this parameter increases the relative weight of larger spells consistently with the idea that the continuous accumulation of poverty periods aggravates the individual poverty experience. This is a generalization of weights proposed by Bossert *et al.* (2008) allowing for different degrees of sensitivity to duration given that we believe that this is a value judgement, not different to, for example, the sensitivity to inequality in poverty indices.

In a second stage, unlike most of the previous literature, we summarize individual intertemporal poverty indices over the whole population by constructing an aggregate intertemporal poverty measure P, which is consistent with the way that poverty is usually measured in a cross-section of individuals:

$$P(Y;z) = \frac{1}{N} \sum_{i=1}^{N} p_i^{\alpha} \tag{3}$$

where

$$p_i^{\alpha} = \begin{cases} (p_i)^{\alpha} & \text{if } p_i > 0 \\ 0 & \text{if } p_i = 0 \end{cases}$$

If $\alpha > 1$, this parameter allows for sensitivity of the aggregate intertemporal poverty index to the distribution of intertemporal individual poverty experiences. Thus, a higher inequality of poverty experiences over the population will increase aggregate poverty. This would be consistent with a version of the "equality-preferring axiom" by which the society prefers the individual poverty deprivations to be equally distributed, as pointed out by Hoy and Zheng (2008). This is in line with Shorrocks (2009a, b) who proposed a normative framework to measure a deprivation dimension such as unemployment in an intertemporal context.[13] We believe that being poverty another form of deprivation, intertemporal poverty should be measured within this framework.

The complete expression for the aggregate intertemporal poverty index integrating expression (2) in (3) is the following:

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[13] Other authors who made proposals for measuring unemployment in a similar way were Paul (1992), Riese and Brunner (1998), Borooah (2002) and Sengupta (2009).

$$P(Y;z) = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\left(\dfrac{1}{T}\displaystyle\sum_{t=1}^{T}g_{it}^{\gamma}\left(\dfrac{s_{it}}{T}\right)^{\beta}\right)^{\alpha} & \text{if} \quad \alpha > 0 \\[4mm] \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}p_i^0 = \dfrac{q}{N} & \text{if} \quad \alpha = 0 \end{cases} \qquad\qquad \textbf{(4)}$$

One of the advantages of our framework is that it encompasses some of the previous literature within the spells approach. For example, Foster (2007, 2009)'s K_{γ} measure can be obtained as an extreme case of our measure when, $\beta = 0$ and $\alpha = 1$, that is, when normalized poverty gaps are not weighted by the poverty spell duration and the aggregate intertemporal poverty index is simply the average of individual intertemporal indicators over the population, therefore insensitive to the indicators' distribution.[14] Bossert *et al.* (2008)'s index can be expressed as a scalar transformation (by *T*) of our index by fixing $\beta = \alpha = 1$, that is, when normalized poverty gaps are weighted proportionally to spell duration and the aggregation over the population is also an average. Moreover, $P(Y;z)$ can be viewed as a generalization of the popular *FGT* measure to the dynamic framework, given that the latter can be obtained from the former when $T = 1$ and $\gamma = 1$.

It is easy to check that our individual intertemporal poverty measure p_i defined in (2) satisfies all desirable properties discussed in Section 2.2 as far as $\gamma > 1$ and $\beta > 0$. In particular, the fulfillment of the focus and the monotonicity axioms come straightforward from the definition of the poverty gaps. The scale invariance axiom is verified as a consequence of the normalization of the poverty gaps. As far as $\beta > 0$, concatenated spells increase individual poverty, thus verifying the spell duration sensitivity axiom. The higher β the larger the penalty to longer spells, ceteris paribus. Therefore, β can be understood as a parameter reflecting the degree of aversion to spell duration, which was implicit in other proposals. Similarly, $\gamma > 1$ guarantees the fulfillment of the Pigou-Dalton principle of transfers adapted to the intertemporal framework (the intertemporal transfer axiom). In the case where $\gamma = 1$, p_i depends on the individual time-averaged normalized gap (weighted by spell duration) and not on its

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[14] This is true provided that $\tau = \frac{1}{T}$. Note that higher values of τ could be easily accommodated in our framework by using a double cut-off in defining normalized poverty gaps in (1).

distribution within spells, while when $\gamma = 0$, p_i depends on the proportion of periods the individual spends below the poverty threshold (weighted by spell duration), but neither on the average magnitude of poverty gaps nor on their distribution over time. For values of γ higher than one, the parameter reflects the aversion to inequality of poverty gaps over time when measuring individual intertemporal poverty. Additionally, p_i is normalized to lie between 0 and 1, taking the lowest value when an individual is never poor, and the largest when she is always poor with zero income. Similarly it is immediate to verify that p_i does not satisfy the time anonymity axiom proposed by Foster (2007), according to which the individual index should be invariant to permutations of incomes across time. Note also that p_i does not verify the direct translation to the time dimension of the replication invariance property usually required in cross-sectional poverty measurement, which is not a problem given that it does not appear reasonable to compare distributions with different *T*.

Our aggregate measure *P* is also normalized to lie between 0 and 1, taking the lowest value when nobody in the population is ever poor, and taking the largest value when everybody is always poor and their income is always zero. Further, it fulfills all the axioms that were also discussed in Section 2.2 as far as $\gamma > 1, \beta > 0$ and $\alpha > 1$, including the preference for intertemporal poverty equality which is generally not verified by other indices that have been proposed in the literature so far. This property is the consequence of the parameter α being greater than one. Then α can be understood as a parameter reflecting the extent of aversion to inequality of intertemporal poverty across individuals, which was implicit in other aggregate poverty measures. Note that a consequence of this property and the weighting scheme w_{it} is that *P* does not verify the path independence axiom. This is consistent with our view that time periods and individuals are not symmetric dimensions as far as we give relevance to the role played by poverty persistence and to social preference for equality between individual intertemporal poverty experiences.[15] While persistence was already taken into account by other approaches (for example Calvo and Dercon, 2007, Bossert *et al.,* 2008, and

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[15] Note that both parameters γ and α do not need to take the same value because they are capturing different features of intertemporal poverty. For example, one could be interested in analyzing the distribution (then $\alpha > 1$) of the percentage of time spent in poverty by a given population (thus fixing $\gamma = 0$).

Mendola *et al.*, 2009 among others) equality of individual complete profiles has not been considered so far. It is important to underline that the three parameters in expression (4) make the inevitable value judgements about measuring longitudinal poverty explicit, i.e. by choosing each parameter value they allow one to choose to what extent penalize: the variability of individual well-being over time, the duration of poverty spells, and the inequality of intertemporal poverty across the population.

Moreover, the *P* index also satisfies the additive decomposability by subpopulations property, which is of particular interest for empirical analysis. Let $Y = \left(Y^1, Y^2, ..., Y^K\right)'$ an exhaustive partition of the population into *K* mutually exclusive demographic groups, with $\pi = \left(\pi^1, \pi^2, ..., \pi^K\right)'$ their respective population shares, then:

$$P(Y;z) = \sum_{k=1}^{K} P\left(Y^k;z\right)\pi^k$$

Finally, our aggregate indicator *P* is consistent with a partial ordering that comes from dominance criteria based on modified TIP (*Three I's of Poverty*) curves defined over the vector of ordered intertemporal individual poverty experiences $\tilde{p} = \left(\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_N\right)$ where $\tilde{p}_1 \geq \tilde{p}_2 \geq ... \geq \tilde{p}_N$, instead of over that of ordered individual poverty gaps as in Jenkins and Lambert (1997). Then our intertemporal TIP curve (ITIP) for each value of $s = m/N$ can be expressed as:

$$ITIP_s\left(\tilde{p}\right) = \sum_{j=1}^{m} \frac{\tilde{p}_j}{N}$$

where *m* is any integer number such that $m \leq N$. $ITIP(\tilde{p})$ accumulates intertemporal individual poverty levels, from higher to lower intertemporal poverty, divided by *N*. Similar to conventional TIP curves, $ITIP(\tilde{p})$ shows i) the *incidence* of intertemporal poverty (the proportion of ever poor), ii) the *intensity* of intertemporal poverty experiences (that for each individual depends on the level and distribution over time of normalized poverty gaps and on spells duration), and iii) the *inequality* of intertemporal poverty across the population. The dominance in these curves (i.e., when the curve of a distribution is always equal or below that of another one) allow to identify partial

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orderings of aggregate intertemporal poverty which are robust to the choice of a particular aggregate poverty indicator verifying our set of axioms.

4. Conclusions

This paper proposes a new aggregate intertemporal poverty index that verifies a set of relevant properties. These properties are based on those of the standard static poverty analysis adapted to a dynamic framework. Thus, our index measures poverty in a dynamic setting in a way that is consistent with the core dimensions of poverty measurement proposed by Sen (1976). Our index is strongly rooted in the previous literature embracing other intertemporal poverty indices that have been recently proposed as special cases.

The main contribution of our index is that it incorporates social preference for equality between individual complete poverty experiences over time, while maintaining other desirable features of previous indices such as spell duration sensitivity. Obviously, we consider the implications of the complete poverty pattern in time on our index in a particular and practical way. One could think of alternative ways of including other relevant aspects of individual time trajectories discussed by other authors due to their different implications in terms of welfare. These could be, for instance, the distance between poverty spells, the preference for upward trajectories versus downward ones, the implications of an excess of variability in well-being, or the point within the life-cycle when the spells take place, among others.

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