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An Inter-temporal Relative Deprivation Index

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An Inter-temporal Relative Deprivation Index

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Abstract

The paper provides the axiomatic characterization of a new relative deprivation index. The concept of relative deprivation is here extended towards the inter-temporal framework.

In fact, if we agree that deprivation is a relative concept, we should also believe that individuals not only take care of their relative position with respect to others, but also of their relative position with respect to their own past. The index main contribution is to bridge two streams of the well-being literature: it is a generalization of existing measures of relative deprivation on one hand and of mobility on the other.

The new index is illustrated with an application to EU countries.

Keywords: Relative Deprivation, Intertemporal Measurement, Distribution, Axioms

JEL Classification: I32, D31, D63, D71, D81

Introduction

Deprivation has been always considered as an inter-personal concept: it is the feeling an individual experiences when she realizes to be worse-off than someone else in the society (Runciman [1966], Bossert et al. [2007]). But, going back to the overcited definition of deprivation by Runciman [1966], we find out a neglected point: a person is relatively deprived when she realizes not to have something that other persons, “*which may include himself at some previous or expected time*”, have (Runciman [1966], p.9). Inspired by Runciman seminal work, we believe that deprivation is also inter-temporal, meaning that some individual’s reference group is made not only by the other individuals, but also by her own history.

Note that, even when time has been taken into account in the deprivation literature (Bossert et al. [2008], Bossert et al. [2007]), the reference group has been always other-related. In particular, when determining the deprivation of some individual, Bossert et al. [2008] stress the importance of taking into account, in a two periods framework, the number of individuals overtaking her. On the other hand, Bossert et al. [2007] bring time into the analysis to consider other-regarding deprivation in different times.

While Bossert et al. [2007] shift attention from deprivation to social exclusion when adding time to the analysis (“*Social Exclusion*” is “*being in a state of deprivation over time*” p.777), we believe that time is in fact another dimension of deprivation, which can be explained as restricting some individual reference group to her own past.

In this work, therefore, we embed also a concept of time-regarding deprivation: individuals are not deprived just because their incomes are lower then someone else today, but also because their incomes are lower than what it was used to be in the past.

In particular, we do not bring in the analysis the persistence in the state of deprivation (as in Bossert et al. [2007] or Bossert et al. [2008]), but we rather look at the direction of the individual income path (as it is done in Ceriani [2009]), defining as history-regarding

deprived the individual whose income is falling over time. In this sense, our index is inter-temporal and not multi-temporal.

In some sense, this work is motivated by the vast prospect theory literature (Kahneman and Tversky [1979] and Kahneman and Tversky [2000]): outcomes *per se* do not have any meaning, but they become relevant only when interpreted as gain or loss. In fact, each individual current outcome works as reference point which serves as zero point of the value scale. While in traditional relative deprivation framework, this zero point is used to evaluate the relative position only with respect to other individuals, in our work we stretch this idea and we also measure the deviations from that reference point to the individual previous period outcomes.

Finally, in the aggregate deprivation index, we consider the interrelations between each individual other-regarding deprivation and her history-regarding deprivation, and we take advantage of existing literature about multivariate and multidimensional welfare analysis (Gigliarano and Mosler [2009], Bourguignon and Chakravarty [2003]).

Note that the term *deprivation* has been used in the literature in a somehow confusing manner. On one side, deprivation is considered as the number of functionings from which a person is excluded (see, for example, Chakravarty and D'Ambrosio [2006]) or as lack of resources, in particular, employment, access to education, childcare, healthcare facilities, and social participation, (Eurostat [2010]). On the other side, deprivation is used to denote the sense of difference, or depression, an individual feels by comparing her situation with the desired one (Runciman [1966], Bossert et al. [2007], Chakravarty [2007], Mukerjee [2001], Yitzhaki [1979]). We follow the latter approach, since the former concept is strictly related to and hardly separable from the idea of multidimensional poverty.

In empirical paragraph we apply the new index to a selection of EU countries, stressing the information we can gain from our approach.

The paper is organized as follows: Section 1 provides the characterization of our individual inter-temporal relative deprivation index, and contains the main result; Section 2 sum up

the individual deprivation in an aggregate inter-temporal relative deprivation index; Section 3 presents the results of the empirical test of the index, based on EU-Silc longitudinal dataset; Section 4 concludes.

1 Individual inter-temporal relative deprivation

1.1 Framework

Consider a population N of individuals $i = 1, 2, \dots, n$, $n \in \mathbb{N}$, over a period of time T of length $(p + 1)$, made of $\tau = t - p, \dots, t - 1, t$ moments, where t is today, $p \in \{0, 1, 2, \dots, \bar{p}\}$ is fixed, it represents the lag between today and the past, while $\bar{p} \leq t$ depends on the data availability or research purposes.

We are interested in analyzing the distribution of income of the population N along the time period T . Therefore, the population can be represented by a matrix X of dimension $n \times (p + 1)$ where all entries are non negative reals and a generic entry $x_{i\tau}$ represents the income level of the i -th individual in the τ -th period of time.

$$X = \left[\begin{array}{ccc|c} x_{1t-p} & \dots & x_{1t-1} & x_{1t} \\ x_{2t-p} & \dots & x_{2t-1} & x_{2t} \\ \vdots & \vdots & \vdots & \vdots \\ \hline x_{it-p} & \dots & x_{it-1} & x_{it} \\ \hline \vdots & \vdots & \vdots & \vdots \\ x_{nt-p} & \dots & x_{nt-1} & x_{nt} \end{array} \right]$$

The matrix X belongs to the class \mathcal{X} of all the $(n \times (p + 1))$ matrixes with non-negative entries. We are interested in defining individual i -th deprivation level at a generic time t . For this purpose, we need three sets of information:

- x_{it} , individual i -th income level at time t ;
- $X_{(\sim i)t}$, the vector of income levels of all other individuals in the population at time t ;
- $X_{i(\sim t)}$, the vector of income history of individual i -th up to t .

For simplicity of exposition, we group all the information we need to know in a single vector defined as:

$$X_{it} = [x_{it}, X_{i(\sim t)}, X_{(\sim i)t}] = [x_{it}, X_{\sim(it)}],$$

where $X_{\sim(it)}$ is the vector of all incomes in X_{it} but x_{it} .

The class \mathcal{X}_{it} is the set of all the vectors of length $(p + n)$ of the type X_{it} .

Let \succeq be a complete, transitive and continuous individual deprivation order defined over \mathcal{X}_{it} . An individual deprivation index D_i^{p+n} is a function $D_i^{p+n} : \mathbb{R}_+^{p+n} \rightarrow \mathbb{R}_+$ such that, given two generic vectors X_{it} and X'_{it} belonging to \mathcal{X}_{it} , $D_i^{p+n}(X_{it}) \geq D_i^{p+n}(X'_{it})$ if and only if $X_{it} \succeq X'_{it}$. For sake of simplicity we will avoid the use of superscript $p + n$ in labelling the individual relative deprivation index D_i .

1.2 Separability by means of Independence Axiom

Aim of this work is to provide the axiomatic characterization of the new relative deprivation index D_i based upon an extension of the concept of deprivation towards an inter-temporal framework.

The first axiom explicitly refers to the central idea embedded in the deprivation concept: deprivation is a self-referent notion. This axiom allows to obtain additive separability and to express deprivation as a function of differences between the reference income and other incomes (see Neilson [2006]).

Axiom 1.1 (Self-referent Independence).

Consider any nonempty $X_{\sim(it)}$ and $X'_{\sim(it)}$ obtained by the combination of two sub-vectors, as follows: $X_{\sim(it)} = [Z, W]_{\sim(it)}$, $X'_{\sim(it)} = [Z', W']_{\sim(it)} \in \mathbb{R}_+^{p+n-1}$, where Z, Z' are vectors of at least two elements. For any $x_{it} \in \mathbb{R}_+$:

- (i) If $[x_{it}, x_{it} + Z, W] \succeq [x_{it}, x_{it} + Z', W]$, then $[x'_{it}, x'_{it} + Z, W'] \succeq [x'_{it}, x'_{it} + Z', W']$ for any $x'_{it} \in \mathbb{R}_+$ and for any W' .
- (ii) If $[x_{it}, Z, x_{it} + W] \succeq [x'_{it}, Z', x'_{it} + W]$, then $[x_{it}, Z, x_{it} + W'] \succeq [x'_{it}, Z', x'_{it} + W']$, for any W' .

To grasp the meaning of Self-referent Independence, consider the following example. Let us assume we want to define the deprivation order related to the first realization of a vector of length three $X_{it} = [10, 30, 5]$. Condition (i) of Self-referent Independence says that if $[10, 30, 5] \succeq [10, 25, 10]$ (note that it is the same as writing $[10, 10 + 20, 10 - 5] \succeq [10, 10 + 15, 10 + 0]$), then $[y, y + 20, y - 5] \succeq [y, y + 15, y + 0]$, for any $y \in \mathbb{R}_+$, where the first element of the vector corresponds to individual i -th income. Therefore, condition (i) stresses the importance of considering the distance between the referent income and the other incomes in the vector, regardless the level of the referent income. Condition (ii) of Self-referent Independence states that if $[10, 30, 5] \succeq [11, 25, 6]$ (note that is the same as writing $[10, 30, 10 - 5] \succeq [11, 25, 11 - 5]$) then $[10, 30, 10 - k] \succeq [11, 25, 11 - k]$. In other words, keeping constant the distance between the referent income and the third income, the deprivation order only depends on the levels of the first two incomes. While the first part of the axiom stresses the importance of the relative concept of deprivation (individual i -th level of income is in fact irrelevant), the second part allows for the separability between *relevant* and *irrelevant* reference groups and for deprivation comparison of individuals with different incomes.

Self-Referent Independence alone gives us the first important result:

Theorem 1.1. *Suppose $n - 1 + p \geq 2$. The deprivation ordering \succeq satisfies Self-Reference Independence if and only if there exists a function D_i of the form:*

$$D_i(X_{it}) = \sum_{\tau=t-p}^{t-1} d_\tau(x_{i\tau} - x_{it}) + \sum_{j=1}^n d_j(x_{jt} - x_{it}) \quad (1)$$

The functions d_j and d_τ are unique up to a joint increasing affine transformation, that is if δ_j and δ_τ also represent the deprivation ordering \succeq , then $\delta_j = ad_j + b$ and $\delta_\tau = ad_\tau + b$ for some scalar $a > 0$ and some scalar b .

Proof. The proof builds upon Neilson [2006].

\implies : it is straightforward.

\impliedby : consider the function $f : \mathbb{R}_+^{p+n+1} \rightarrow \mathbb{R}_+^{p+n+1}$ as $f(X_{it}) = [x_{it} - x_{it}, x_{it-p} - x_{it}, \dots, x_{it-1} - x_{it}, x_{1t} - x_{it}, x_{2t} - x_{it}, \dots, x_{nt} - x_{it}]$.

Let \succeq^* be a derived deprivation ordering defined as $f(X_{it}) \succeq^* f(X'_{it})$ if and only if $X_{it} \succeq X'_{it}$. Recall that \succeq^* keeps transitivity and completeness belonging to \succeq . By Debreu [1959], since \succeq^* is complete and transitive there exists a continuous function d_f representing \succeq^* , that is, if $X_{it} \succeq^* X'_{it}$, then $f(X_{it}) \geq f(X'_{it})$.

Since $f([x_{it}, x_{it} + Z, W]) = [0, Z, W]$, then by condition (i):

$[0, Z, W] \succeq^* [0, Z', W]$ implies $[0, Z, W'] \succeq^* [0, Z', W']$ for any W' .

Since $f([x_{it}, Z, x_{it} + W]) = [0, Z - x_{it}, W]$ then, by condition (ii):

$[0, Z - x_{it}, W] \succeq^* [0, Z' - x'_{it}, W]$ implies $[0, Z - x_{it}, W'] \succeq^* [0, Z' - x'_{it}, W']$ for any W' .

Combining the two cases (namely, rename X_{it} by $[0, S, W]$ where $S = Z$ in (i) and $S = Z - x_{it}$ in (ii)) we get that $[0, S, W] \succeq^* [0, S', W]$ implies $[0, S, W'] \succeq^* [0, S', W']$ for any W' . By Debreu [1959], theorem 3, the function d^* representing \succeq^* can be

written as:

$$d^*(\Xi) = \sum_{k=1}^{p+n} d_k(\xi_k) \quad (2)$$

where d_k are unique up to a joint increasing affine transformation.

Let $\Xi = f(X_{it})$, and remember that $X_{it} = [x_{it}, X_{i(\sim t)}, X_{(\sim i)t}]$

$$\begin{aligned} D_i(X_{it}) &= d^*(f(X_{it})) \\ &= d^*\left(\overbrace{0}^{\xi_1}, \overbrace{x_{it-p} - x_{it}}^{\xi_2}, \overbrace{x_{it-p+1} - x_{it}}^{\xi_3}, \dots, \overbrace{x_{it-1} - x_{it}}^{\xi_{p+1}}, \overbrace{x_{1t} - x_{it}}^{\xi_{p+2}}, \dots, \overbrace{x_{it} - x_{it}}^{\xi_{p+n}}\right) \\ &= d(0) + \sum_{\tau=t-p}^{t-1} d_\tau(x_{i\tau} - x_{it}) + \sum_{j=1}^{n-1} d_j(x_{jt} - x_{it}) \\ &= \sum_{\tau=t-p}^{t-1} d_\tau(x_{i\tau} - x_{it}) + \sum_{j=1}^n d_j(x_{jt} - x_{it}) \end{aligned} \quad (3)$$

□

Theorem 1.1, therefore, contains a substantial result: individual i -th deprivation index is going to be an additive function of the differences between her income and all the other incomes in vector $X_{\sim(it)}$. Moreover, since $X_{\sim(it)}$ is made up by the two subvectors $X_{(\sim i)t}$ and $X_{i(\sim t)}$, by construction the deprivation index is defined by two terms. The first one is an additive function of the differences between individual i -th income and incomes in her past, while the second one is an additive function of the differences between individual i -th income and all other individuals' incomes in the population at time t . The novelty of this results relies on two facts. First, in the literature income differences are usually obtained by imposing some Invariance Axioms (see, for instance, Ebert and Moyes [2000] and Chakravarty [2007]). Second, additivity is normally exogenously imposed by means of some Additivity Axiom (see Ebert and Moyes [2000], Chakravarty [2007] and Bossert et al. [2007]). In our opinion, this common practice of assuming *a priori* the structure of the

index weakens the axiomatic approach which should be based, instead, on more primitive hypotheses.

In the following we will further characterize the individual deprivation index $D_i(X_{it})$ in (1), knowing that it is an additive function of two components: an *History-regarding* component defined as:

$$d_p(x_{it}, X_{i(\sim t)}) = \sum_{\tau=t-p}^{t-1} d_\tau(x_{i\tau} - x_{it}) \quad (4)$$

and an *Other-regarding* component defined as:

$$d_n(x_{it}, X_{(\sim i)t}) = \sum_{j=1}^n d_j(x_{jt} - x_{it}). \quad (5)$$

1.3 Transformation Axioms

Since individual i -th feeling of deprivation arises from the comparison with incomes higher than hers, let us now introduce the definitions of the relevant reference groups. In our approach, as already mentioned, we will have two referent groups: not only individuals who are better-off today, but also past spells where individual i -th's income was higher than today.

Definition 1.1 (Other-regarding reference group). *Individual i -th other-regarding reference group at time t is defined as the set of incomes at time t higher than individual i -th income at time t :*

$$X_{it}^{BO} = \{x_{jt} \in X_{(\sim i)t} | x_{jt} > x_{it}\} \quad (6)$$

Let B^O be a subset of N such that:

$$B^O = \{j \neq i, j \in N | x_{jt} \in X_{it}^{BO}\}.$$

Definition 1.2 (History-regarding reference group). *Individual i -th history-regarding reference group at time t is defined as the set of individual i -th incomes at periods $\tau < t$ that are higher than individual i -th income at time t :*

$$X_{it}^{BH} = \{x_{i\tau} \in X_{i(\sim t)} | x_{i\tau} > x_{it}\} \quad (7)$$

Let B^H be a subset of T such that:

$$B^H = \{\tau < t, \tau \in T | x_{i\tau} \in X_{it}^{BH}\}.$$

Definition 1.3 (Reference incomes). *The union of the incomes belonging to the other-regarding reference group and the incomes belonging to the history-regarding reference group gives the vector of individual i -th reference incomes:*

$$X_{it}^B = \{x_{jt} \in X_{it}^{BO}, x_{i\tau} \in X_{it}^{BH}\} \quad (8)$$

And let $B = B^O \cup B^H$ be the reference group of individual i -th at time t .

Definition 1.4 (Not-reference incomes). *The complement of the set of reference incomes not including x_{it} is the set of not-reference incomes for individual i -th at time t :*

$$X_{it}^{\sim B} = \{x_{j\tau} \in X_{it}, x_{j\tau} \notin X_{it}^B, x_{j\tau} \neq x_{it}\} \quad (9)$$

Using a sloppy notation, let us consider the set X_{it}^{BO} , X_{it}^{BH} , X_{it}^B and $X_{it}^{\sim B}$ as vectors.

Note that $[X_{it}^B, X_{it}^{\sim B}, x_{it}] = X_{it}$.

Now we will introduce some axioms which are standard in income distribution analysis.

The first one asserts that only the reference group is relevant for the analysis.

Axiom 1.2 (Better-off Focus). *For any x_{it} , X_{it}^{BO} , X_{it}^{BH} , and $X_{it}^{\sim B}$*

$$D_i([x_{it}, X_{it}^{BO}, X_{it}^{BH}, X_{it}^{\sim B}]) = D_i([x_{it}, X_{it}^{BO}, X_{it}^{BH}, Y_{it}^{\sim B}]) \quad (10)$$

for any $Y_{it}^{\sim B}$.

The next axiom imposes a lower bound to the possible values of the index.

Axiom 1.3 (Normalization). *(i) $D_i(X_{it}) \geq 0$ and (ii) $D_i(X_{it}) = 0$ if and only if $B = \emptyset$.*

Normalization states that deprivation is always non-negative, and it is zero if individual i -th reference group is empty (i.e. nobody in the population is better-off than individual i -th at time t and individual i -th has not experienced higher income in the past) .

Axiom 1.4 (Population Proportionality).

$$D_i(X_{it}) = d_p(x_{it}, X_{i(\sim t)}) + d_{\alpha n}(\underbrace{(x_{it}, X_{(\sim i)t}), (x_{it}, X_{(\sim i)t}), \dots, (x_{it}, X_{(\sim i)t})}_{\alpha \text{ times}}) \quad (11)$$

Population Proportionality states that, if we replicate α -times the population at time t , keeping fixed the history-length p , the individual deprivation remains unchanged. This allows for comparisons of population of different size: otherwise, an individual living in a highly populated country would always feel more deprived than another individual facing the same income distribution but in a smaller country.

Note that we are not imposing any sort of “Time-Proportionality” (as, instead, we have done with Population Proportionality). In fact, we believe that frameworks that have a different number of past spells are not directly comparable, since they bring along a different set of information.

Axiom 1.5 (Scale Invariance). *For each $\lambda_p, \lambda_n \in \mathbb{R}_{++}$:*

$$D_i(X_{it}) = d_p(x_{it}, X_{i(\sim t)}) + d_n(\lambda_n x_{it}, \lambda_n X_{(\sim i)t}) \quad (12)$$

$$D_i(X_{it}) = d_p(\lambda_p x_{it}, \lambda_p X_{i(\sim t)}) + d_n(x_{it}, X_{(\sim i)t}) \quad (13)$$

The individual deprivation index is invariant with respect to the unit of measurement. Scale Invariance may appear unusual in the deprivation literature, where usually linear homogeneity is assumed (Ebert and Moyes [2000] and Bossert et al. [2007]). However in the empirical application, while comparing deprivation in different countries, or at different times, one needs to homogenize incomes by means of Purchasing Power Parities or exchange

rates. In our opinion, an explicit statement of Scale Invariance is preferred to the common practice. Note, moreover that Scale invariance allows us to start the empirical analysis with a transformed matrix, where all incomes are made comparable in real terms. In any case, our contribution is not isolated, for other scale-invariant deprivation indexes, see Chakravarty [1997], Mukerjee [2001] and D'Ambrosio and Frick [2007].

Axiom 1.6 (Individual Anonymity). *Let Π be a $(n \times n)$ permutation matrix and $X^\Pi = \Pi X$, where $X, X^\Pi \in \mathcal{X}$. Therefore*

$$D_i(X_{it}) = D_{\pi(i)}(X_{\pi(i)t}) \quad (14)$$

where $X_{\pi(i)t}$ is defined as X_{it} starting from X^Π and $\pi(i)$ is the label denoting individual i -th in the new permuted matrix.

The deprivation index is indifferent to the labeling of individuals: names do not matter, but just current levels of income and histories. Note that we are permuting the rows of the original matrix X : therefore, each individuals keep her own history. This property rules out the importance of the rank between individuals.

Theorem 1.2. *An individual deprivation index $D_i(X_{it})$ defined as in (1) satisfies Better-off Focus, Normalization, Population Proportionality, Scale Invariance, Individual Anonymity if and only if it can be written as:*

$$D_i(X_{it}) = \sum_{\tau \in B^H} d_\tau \left(\frac{x_{i\tau} - x_{it}}{\mu_p} \right) + \sum_{j \in B^O} \frac{1}{n} d \left(\frac{x_{jt} - x_{it}}{\mu_n} \right) \quad (15)$$

Proof.

\implies : it is straightforward.

\impliedby : By Better-off focus we can replace any income in $X_{it}^{\sim B}$ with x_{it} . Therefore, the index

$D_i(X_{it})$ reduces to:

$$D_i(X_{it}) = \sum_{\tau \in B^H} d_\tau(x_{i\tau} - x_{it}) + \sum_{j \in B^O} d_j(x_{jt} - x_{it}) \quad (16)$$

By Normalization, if $B = \emptyset$ then $D_i(X_{it}) = 0$. This allows us to well define the individual deprivation index in case of an empty reference group. On the other hand, if $B \neq \emptyset$ then, $D_i(X_{it}) > 0$.

By Population proportionality:

$$D_i(X_{it}) = d_p(X_{\tau < t}, x_{it}) + d_{\alpha n}(\underbrace{(x_{it}, X_{(\sim i)t}), (x_{it}, X_{(\sim i)t}), \dots, (x_{it}, X_{(\sim i)t})}_{\alpha \text{ times}}). \quad (17)$$

By theorem 4 in Shorrocks [1980], this forces $D_i(X_{it})$ to be of the form:

$$D_i(X_{it}) = \sum_{\tau \in B^H} d_\tau(x_{i\tau} - x_{it}) + \frac{1}{\alpha} \sum_{j \in B^O} d_j(x_{jt} - x_{it}). \quad (18)$$

Choosing $\alpha = n$:

$$D_i(X_{it}) = \sum_{\tau \in B^H} d_\tau(x_{i\tau} - x_{it}) + \frac{1}{n} \sum_{j \in B^O} d_j(x_{jt} - x_{it}). \quad (19)$$

By Scale Invariance, for each $\lambda_p, \lambda_n \in \mathbb{R}_{++}$:

$$D_i(X_{it}) = d_p(\lambda_p x_{it}, \lambda_p X_{\tau < t}) + d_n(\lambda_n x_{it}, \lambda_n X_{(\sim i)t}) \quad (20)$$

Without loss of generality, let us choose $\lambda_p = \frac{1}{\mu_p}$, where $\mu_p = \frac{\sum_{\tau=t-p}^t x_{i\tau}}{p+1}$, and $\lambda_n = \frac{1}{\mu_n}$, where $\mu_n = \frac{1}{n} \sum_{i=1}^n x_{it}$. Moreover, by Individual Anonymity, $d_j = d$ for each $j \neq i$.

Therefore,

$$D_i(X_{it}) = \sum_{\tau \in B^H} d_\tau \left(\frac{x_{i\tau} - x_{it}}{\mu_p} \right) + \sum_{j \in B^O} \frac{1}{n} d \left(\frac{x_{jt} - x_{it}}{\mu_n} \right) \quad (21)$$

□

Notice that one interesting feature of index (15) is that if we concentrate on the uni-temporal case, it boils down to Chakravarty index (Chakravarty [1997]).

1.4 Sensitivity Axioms

Following Paul [1991] suggestions, we believe that individual deprivation should be sensitive to income transfers taking place among persons being richer than individual i -th. Paul [1991] believes that an individual feels less envious with respect to an increase in the income of a rich person.

Definition 1.5 (Other-regarding Regressive Transfer).

Let $X_{it} = [x_{it}, X_{i(\sim t)}, x_{1t}, \dots, x_{jt}, \dots, x_{kt}, \dots, x_{nt}]$. We say that Y_{it} is obtained from X_{it} by means of an other-regarding regressive transfer if $Y_{it} = [x_{it}, X_{i(\sim t)}, x_{1t}, \dots, x_{jt} - \epsilon, \dots, x_{kt} + \epsilon, \dots, x_{nt}]$ for all $x_{it} \leq x_{jt} - \epsilon$ and $x_{jt} \leq x_{kt}$

Definition 1.6 (History-regarding Regressive Transfer).

Let $X_{it} = [x_{it}, \dots, x_{it-p_j}, \dots, x_{it-p_k}, \dots, X_{(\sim i)t}]$. We say that Y_{it} is obtained from X_{it} by means of an history-regarding regressive transfer if $Y_{it} = [x_{it}, \dots, x_{it-p_j} - \epsilon, \dots, x_{it-p_k} + \epsilon, \dots, X_{(\sim i)t}]$ for all $x_{it} \leq x_{it-p_j} - \epsilon$ and $x_{it-p_j} \leq x_{it-p_k}$

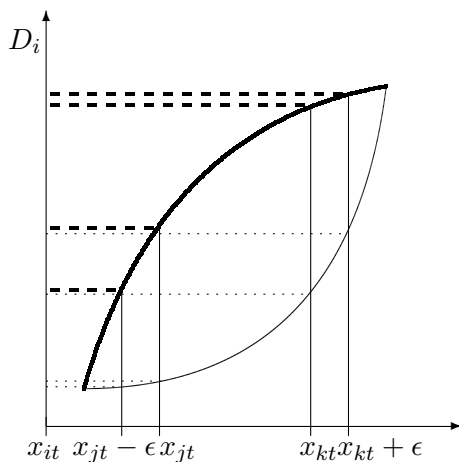
Axiom 1.7 (Regressive (Progressive) Transfer Principle). Let $X_{it} = [x_{it}, X_{i(\sim t)}, X_{(\sim i)t}]$ and Y_{it} obtained from X_{it} by means of an other-regarding regressive transfer and an history-regarding regressive transfer, then:

- Distant Principle: $D_i(X_{it}) \leq D_i(Y_{it})$
- Neighbor Principle: $D_i(X_{it}) \geq D_i(Y_{it})$

Note that, Paul [1991] allows only for indexes that satisfy Neighbor Principle, while we take a more general approach in a twofold way. First of all, we consider also transfers occurring in different past periods. Second and more important, we do not constrain *a priori* the researcher belief in evaluating marginal changes. If we assume Distant Principle, individual i -th feels more deprived if she faces an increase in incomes well greater than hers. And these incomes may be in her own past or of other individuals today. In this case, the researcher believes that the wealthiest individuals in the society act as benchmarks toward which all the other individuals aspire. Moving further this threshold increases individual

i -th feeling of deprivation. On the other hand (Neighbor Principle), researchers may think that individual i -th is more affected by changes in incomes closer to hers. On the basis of this reasoning, it lays the idea each individual compares herself with her alike.

Figure 1: Other-regarding Distant (Neighbor) Principle



Theorem 1.3. *An individual deprivation index $D_i(X_{it})$ defined as in (15) satisfies Distant (Neighbor) Principle if and only if d and d_τ are strictly convex (concave) on \mathbb{R}_+ .*

Proof. Distant (Neighbor) Principle is equivalent to assume that $\sum_{j \in B^O} d(\cdot)$ and $\sum_{\tau \in B^H} d_\tau(\cdot)$ are Schur-convex (concave), see Kolm [1976], p. 82.

By Marshall and Olkin (Marshall and Olkin [1979], theorem C.1.a., p.64) this condition is equivalent to $d(\cdot)$ and $d_\tau(\cdot)$ being strictly convex (concave), for each $\tau \in B^H$. \square

1.5 Time-Discounting Axioms

The following set of axioms is inspired by Ok and Masatlioglu [2007] and al Nowaihi and Dhimi [2006] and it concerns the characterization of the history-regarding deprivation.

Axiom 1.8 (Memory Sensitivity). *For any ξ , z_{it}, y_{it} , and for any period of time t and for any $p_1 \in \mathbb{N}$, there exists a $p_2 \geq p_1$ such that:*

$$D \left(\left[x_{it}, \underbrace{\xi, \xi, \xi, \dots, z_{it-p_1}, \dots, \xi}_{Z_{i\tau < t}}, X_{\sim(i)t} \right] \right) \geq D \left(\left[x_{it}, \underbrace{\xi, \dots, y_{it-p_2}, \dots, \xi, \xi, \xi}_{Y_{i\tau < t}}, X_{\sim(i)t} \right] \right) \quad (22)$$

where z_{it-p_1} and y_{it-p_2} are the income level z_{it} and y_{it} owned respectively at time $t - p_1$ and $t - p_2$.

This axiom states that there always exists a period of time ($t - p_2$) which is so far in the past that individual i -th income in such past period becomes irrelevant for the feeling of deprivation, regardless of the income amount. In other words, we assume that the flow of time weakens the memories of the past experiences. Note that this axiom is similar to Impatience in al Nowaihi and Dhami [2006] and to Time sensitivity in Ok and Masatlioglu [2007].

Example 1.1. *Take two scenarios A and B , where $A = [x_{it}, Z_{i\tau < t}, X_{\sim(i)t}] = [10, 10, 10, 20, X_{\sim(i)t}]$ and $B = [x_{it}, Y_{i\tau < t}, X_{\sim(i)t}] = [10, 25, 10, 10, X_{\sim(i)t}]$. Memory Sensitivity states that deprivation of the latter vector is not necessary higher than deprivation of the former one. In fact, even if in B individual i -th has experienced an higher past income than in A (25 versus 20), it is possible that her memory about 25 is weaker because is more remote than her memory about 20.*

In other words, Memory Sensitivity states that, regardless of the size of the difference in current and past incomes, it is always possible to find a period which is so far in the past that such difference becomes irrelevant. The next axiom, instead, states that, regardless of how far in the past we look at, it is always possible to find an income level which is so high that it is still important in determining the feeling of deprivation.

Axiom 1.9 (Income Sensitivity). *For any ξ , y_{it} , for any period of time t and for any p_1 ,*

$p_2 \in \mathbb{N}$, such that $p_1 \leq p_2$, there exists $z_{it}, w_{it} \neq y_{it}$, and $z_{it}, w_{it} > x_{it}$, such that:

$$D \left(\left[\begin{array}{c} x_{it}, \underbrace{\xi, \dots, y_{it-p_2}, \dots, \xi, \xi}_{Y_{i\tau < t}}, X_{\sim(i)t} \end{array} \right] \right) \geq \quad (23)$$

$$D \left(\left[\begin{array}{c} x_{it}, \underbrace{\xi, \xi, \dots, z_{it-p_1}, \dots, \xi}_{Z_{i\tau < t}}, X_{\sim(i)t} \end{array} \right] \right) \geq D \left(\left[\begin{array}{c} x_{it}, \underbrace{\xi, \dots, w_{it-p_2}, \dots, \xi, \xi}_{W_{i\tau < t}}, X_{\sim(i)t} \end{array} \right] \right)$$

where $z_{it-p_1}, y_{it-p_2}, w_{it-p_2}$ are the income level z_{it}, y_{it} and w_{it} owned respectively at time $t - p_1$ and $t - p_2$.

Even if the past period $t - p_2$ is far enough so that the income level w_{it} is negligible with respect to having z_{it} in a more recent past (as stated in Memory Sensitivity), we can find an income level so high (y_{it}) that memories back to $t - p_2$ is still strong enough to overcome z_{it} .

Example 1.2. Take three scenarios A , B and C , where $A = [x_{it}, Y_{i\tau < t}, X_{\sim(i)t}] = [10, 50, 10, 10, X_{\sim(i)t}]$, $B = [x_{it}, Z_{i\tau < t}, X_{\sim(i)t}] = [10, 10, 10, 20, X_{\sim(i)t}]$ and $C = [x_{it}, W_{i\tau < t}, X_{\sim(i)t}] = [10, 25, 10, 10, X_{\sim(i)t}]$. Income Sensitivity states that deprivation of the latter vector is not necessary higher than deprivation of the second one. In fact, even if in C individual i -th has experienced an higher past income than in B (25 versus 20), it is possible that her memory about 25 is weaker because is more remote than her memory about 20. On the other hand, that past period which is far enough to forget an income equal 25 is not far enough to forget about an income of 50. Therefore, deprivation in framework C is not higher than in B .

Ceteris paribus, a marginal increase in some past income cannot decrease deprivation, as stated in the following axiom.

Axiom 1.10 (Past Incomes Monotonicity). For any $\xi, x_{it}, X_{\sim(i)t}$ and for any $\epsilon > 0$:

$$D \left([\xi, \xi, \xi + \epsilon, \dots, \xi, x_{it}, X_{\sim(i)t}] \right) \geq D \left([\xi, \xi, \xi, \dots, \xi, x_{it}, X_{\sim(i)t}] \right) \quad (24)$$

Axiom 1.11 (Time-Income Monotonicity). *For any ξ , y_{it}, w_{it}, z_{it} , for any period of time t and for any $p_1, p_2, p_3 \in \mathbb{N}$, if $p_2 \leq p_3$, and $w_{it} \geq y_{it}$, then:*

$$\begin{aligned}
 D([x_{it}, \xi, \dots, y_{it-p_3}, \dots, \xi, \xi, \xi, X_{\sim(i)t}]) &\geq D([x_{it}, \xi, \xi, \xi, \dots, z_{it-p_1}, \dots, \xi, X_{\sim(i)t}]) \\
 &\text{implies} \\
 D([x_{it}, \xi, \xi, \dots, w_{it-p_2}, \dots, \xi, \xi, X_{\sim(i)t}]) &\geq D([x_{it}, \xi, \xi, \xi, \dots, z_{it-p_1}, \dots, \xi, X_{\sim(i)t}])
 \end{aligned} \tag{25}$$

For sure we get a higher deprivation if individual i -th experience a higher income in a more recent past.

Axiom 1.12 (Time-Income Separability). *For any y_i, w_i, z_i, v_i , for any period t and for any $p_1 \leq p_2 \leq p_3 \leq p_4$*

if

$$\begin{aligned}
 D([\xi, \xi, \dots, y_{it-p_1}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, w_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) \\
 D([\xi, \xi, \dots, z_{it-p_1}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, v_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) \\
 D([\xi, \xi, \dots, y_{it-p_3}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, w_{it-p_4}, \dots, \xi, x_{it}, X_{\sim(i)t}])
 \end{aligned} \tag{26}$$

Then

$$D([\xi, \xi, \dots, z_{it-p_3}, \dots, \xi, x_{it}, X_{\sim(i)t}]) = D([\xi, \xi, \dots, v_{it-p_4}, \dots, \xi, x_{it}, X_{\sim(i)t}])$$

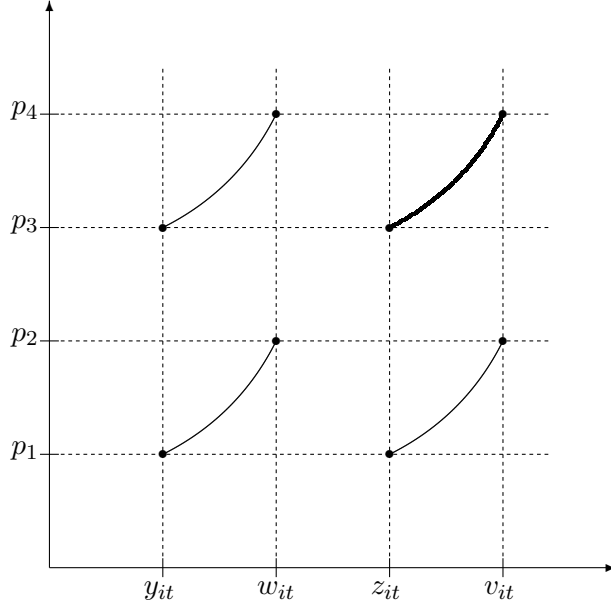
This axiom will become clearer, looking at the following figure.

Time-Income Separability gives independence between levels of income and periods of time.

In other words, keeping fixed incomes, it possible to order the deprivation when this incomes are experienced at different periods of time. Analogously, keeping fixed the time, we can order the deprivation of different levels of income experienced at the same time.

The following axiom, instead, states that if the deprivation felt by having y at $t - p_1$ does

Figure 2: Time-Income Separability (*thin lines stated, thick line implied*)



not change if we experience at $t - p_3$ an outcome of v , then deprivation must be the same either if we have z at $t - p_1$ and $w - z + z - x$ at time $t - p_3$. This means that moving from $t - p_1$ to $t - p_3$, we should change the outcome by $v - y$ to have the same level of deprivation. Without the following axiom, Time-Income Separability alone would impose an additive relation between the relevance of incomes and the relevance of time. Path-Independence, instead, allows for a multiplicative structure. For a more detailed discussion, we refer to Ok and Masatlioglu [2007].

Axiom 1.13 (Path-Independence). *For any y_i, w_i, z_i, v_i , for any period t and for any $p_1 \leq p_2 \leq p_3$*

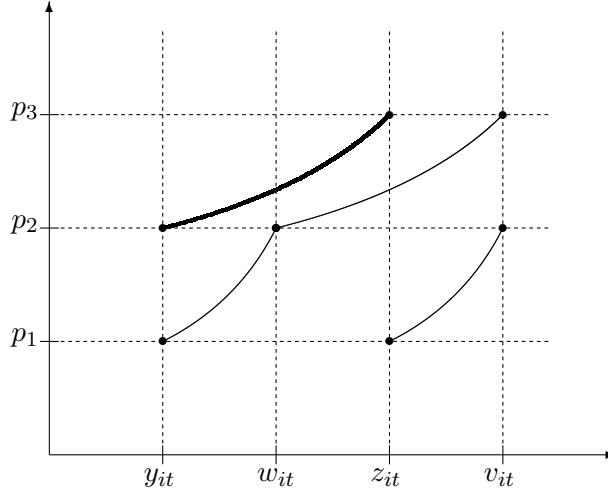
if

$$\begin{aligned}
D([\xi, \xi, \dots, y_{it-p_1}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, w_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) \quad (27) \\
D([\xi, \xi, \dots, z_{it-p_1}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, v_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) \\
D([\xi, \xi, \dots, w_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) &= D([\xi, \xi, \dots, v_{it-p_3}, \dots, \xi, x_{it}, X_{\sim(i)t}])
\end{aligned}$$

Then

$$D([\xi, \xi, \dots, y_{it-p_2}, \dots, \xi, x_{it}, X_{\sim(i)t}]) = D([\xi, \xi, \dots, z_{it-p_3}, \dots, \xi, x_{it}, X_{\sim(i)t}])$$

Figure 3: Path-Independence (*thin lines stated, thick line implied*)



Theorem 1.4. *An individual deprivation index $D_i(X_{it})$ defined as in (Theorem 3) satisfies Memory-Sensitivity, Income-Sensitivity, Past Incomes Monotonicity, Time-Income Monotonicity, Time-Income Separability and Path-Independence if and only if there exists a continuous function $\omega : \mathbb{N}^2 \rightarrow \mathbb{R}_{++}$ such that for each $X_{it} \in \mathcal{X}_{it}$:*

$$D_i(X_{it}) = \sum_{\tau \in B^H} \omega(\tau, t) \delta_\tau \left(\frac{x_{i\tau} - x_{it}}{\mu_p} \right) + \sum_{j \in B^O} \frac{1}{n} d \left(\frac{x_{jt} - x_{it}}{\mu_n} \right) \quad (28)$$

where $\omega(\cdot, t)$ is increasing with $\omega(-\infty, t) = 0$ and $\omega(\tau, t) = 1/\omega(t, \tau)$

Proof. See Ok and Masatlioglu [2007], theorem 1. □

Theorem 1.4 allows for separating time and outcomes. Note that the History-regarding deprivation has been decomposed into the product of $\omega(\tau, t)$ which acts as discount factor and tells how the researchers disvalue the past incomes, and δ_τ which is a function of the income gaps. The discount factor is bounded between 0 and 1 and it is equal to 1 when $\tau = t$. Note that the role of this discount factor is not related to the need of deflating past incomes to let them comparable at time t . Since the last set of axioms (Memory Sensitivity, Income Sensitivity, Past Incomes Monotonicity, Time-Income Monotonicity, Time-Income Separability and Path-Independence) require that incomes can be compared regardless of the time when they occur, implicitly we are stating that the original data are already in real terms.

In order to have an explicit parametrization of the deprivation-aversion towards others, and of the relative importance between the history-related deprivation and the other-regarding deprivation, we choose, as a special case of the measure as defined in theorem (1.4), $\omega(\tau, t) = \left(\frac{1}{1+\beta p}\right)^{\frac{\gamma}{\beta}}$, $\delta_\tau(s) = \epsilon s^\alpha$, $d(s) = (1-\epsilon)s^\alpha$, where $\alpha \in \mathbb{R}_+$, $\epsilon \in [0, 1]$. Therefore, (15) becomes:

$$D_i(X_{it}) = \epsilon \sum_{\tau \in B^H} \left(\frac{1}{1+\beta p}\right)^{\frac{\gamma}{\beta}} \left(\frac{x_{i\tau} - x_{it}}{\mu_p}\right)^\alpha + (1-\epsilon) \sum_{j \in B^O} \frac{1}{n} \left(\frac{x_{jt} - x_{it}}{\mu_n}\right)^\alpha \quad (29)$$

Note that if $\alpha \in [0, 1]$ $D_i(X_{it})$ satisfies Progressive-transfer principle, that is, it values more the increase in more similar incomes than x_{it} , while, if $\alpha > 1$, $D_i(X_{it})$ satisfies Regressive-transfer principle, that is we give more weight to marginal increases in the highest incomes.

We have chosen to have a general characterization of the History-regarding deprivation, which includes several common discounting models, such as the exponential and the hyperbolic.

2 Aggregate Inter-temporal relative deprivation

Here, starting from the individual inter-temporal deprivation found in the previous section, we sum up over the entire population to obtain the aggregate inter-temporal deprivation (inspired by Bossert et al. [2008]).

The characterization of index is based on the following axioms: Independence axiom, to justify the additive form of the index, Anonymity to treat all the individual deprivation symmetrically, Scale Invariance that allows cross countries comparisons, Monotonicity to be sure that an increase in some individual deprivation does not decrease the aggregate deprivation, and Transfer, to evaluate more the more deprived individual in the society.

Let us define an Aggregate Inter-temporal relative deprivation measure as a function $AD : \mathcal{D}^n \rightarrow \mathbb{R}_+$, where \mathcal{D}^n is the class of all vectors D^n made up by the individual relative deprivations of all the individuals in the population N .

Axiom 2.1 (Independence). *Let $D_1^n = [D_{11}, D_{12}, \dots, D_{1j}, \dots, D_{1n}]$ and $D_2^n = [D_{21}, D_{22}, \dots, D_{2j}, \dots, D_{2n}]$ two deprivation profiles. Let \tilde{D}_1^n and \tilde{D}_2^n be derived by D_1^n and D_2^n in the following way: $\tilde{D}_k^n = [D_{k1}, D_{k2}, \dots, \beta, \dots, D_{kn}]$, for $k = 1, 2$.*

If $AD(D_1^n) = AD(D_2^n)$ then $AD(\tilde{D}_1^n) = AD(\tilde{D}_2^n)$, for all $\beta \in \mathbb{R}_+$

Axiom 2.2 (Anonymity). *For all $D^n \in \mathcal{D}^n$: $AD(D^n) = AD(D^{\pi(n)})$*

where $D^{\pi(n)} \in \mathcal{D}^n$ and $D^{\pi(n)} = [D_{\pi(1)}, D_{\pi(2)}, \dots, D_{\pi(n)}]$, and π is a permutation function defined on $N = \{1, 2, \dots, n\}$.

Axiom 2.3 (Scale Invariance). *For any $D^n \in \mathcal{D}^n$ and $\lambda > 0$:*

$$AD(D^n) = AD(\lambda D^n)$$

where $\lambda D^n \in \mathcal{D}^n$ and $\lambda D^n = [\lambda D_1, \lambda D_2, \dots, \lambda D_n]$.

Axiom 2.4 (Monotonicity). *Let $D_1^n = [D_{11}, D_{12}, \dots, D_{1i}, \dots, D_{1n}]$ and let D_2^n be derived from D_1^n in the following way: $D_2^n = [D_{11}, D_{12}, \dots, \beta, \dots, D_{1n}]$, where $\beta > D_{1i}^n$. Then, for all $D_1^n, D_2^n \in \mathcal{D}^n$: $AD(D_2^n) \geq AD(D_1^n)$.*

Axiom 2.5 (Transfer). *Let $D_1^n = [D_{11}, D_{12}, \dots, D_{1i}, \dots, D_{1j}, \dots, D_{1n}]$ and let D_2^n be de-*

rived from D_1^n in the following way: $D_2^n = [D_{11}, D_{12}, \dots, D_{1i} + \epsilon, \dots, D_{1j} - \epsilon, \dots, D_{1n}]$, where $D_{1i} \geq D_{1j}$.

Then $AD(D_2^n) \geq AD(D_1^n)$.

Theorem 2.1. *An aggregate Inter-temporal relative deprivation measure AD satisfies Independence, Scale Invariance, Anonymity, Monotonicity and Transfer if and only if it can be written as:*

$$AD(D^n) = \frac{1}{n} \sum_{i=1}^n D_i^\theta(X_{it}) \quad (30)$$

Proof. By Independence, AD has an additive representation of the form:

$$AD = \sum_{i=1}^n f_i(D_i) \quad (31)$$

where the $f_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ are unique up to a joint increasing affine transformation.

By Anonymity $f_i = f$ for each $i = 1, 2, \dots, n$. By Monotonicity f is increasing in its argument and by Transfer f is convex. By Scale Invariance, $AD(\lambda D_n) = AD(D_n)$. Without loss of generality, we choose $\lambda = \frac{1}{n}$ and $f(D_i) = D_i^\theta$, $\theta \geq 1$. \square

It is worth to underline the role of parameter θ : the higher θ , the more we weigh the most deprived individuals in the society. Moreover, notice that when $\theta = 1$ we are assuming some degree (regulated by ϵ) of substitutability between History-regarding and Other-regarding deprivation.

3 An empirical test

The empirical test is based on EU-Silc Longitudinal 2007 Dataset. We provide the comparison between our new measure, the relative form of the Yitzhaki [1979] index and the Gini index in a cross-countries analysis.

We first deflate incomes using the harmonized consumer price index provided by Eurostat (2010), in order to have real incomes, comparable at time 2005, and we clean the dataset dropping all negative incomes. Using the Gini-index we find out (see table1) the well known ranking among European countries: in 2007, the most unequal countries are the mediterranean countries (Portugal Italy and Spain), followed by the central-Europe (France, Belgium and Austria) and then the Scandinavian countries (Norway and Sweden). Moving from inequality to deprivation, the Yitzhaki relative index slightly changes the figure, since Austria becomes the least deprived and Norway rises up to the mid-range.

Table 1: Traditional Inequality and Deprivation Measures

	Gini	$p90/p10$	Yitzhaki
Austria	0.240	2.825	0.238
Belgium	0.251	3.112	0.250
Spain	0.297	4.030	0.289
France	0.273	3.340	0.276
Italy	0.314	3.982	0.323
Norway	0.228	2.736	0.281
Portugal	0.374	5.166	0.324
Sweden	0.228	2.723	0.246

We now move to explore how our new deprivation measure can enrich the picture. Recall that the aggregate deprivation measure is:

$$AD(X) = \frac{1}{n} \sum_{i=1}^n \left[\underbrace{\epsilon \sum_{\tau \in B^H} \left(\frac{1}{1 + \beta p} \right)^{\frac{\gamma}{\beta}} \left(\frac{x_{i\tau} - x_{it}}{\bar{\mu}_i} \right)^\alpha + (1 - \epsilon) \sum_{j \in B^O} \frac{1}{n} \left(\frac{x_{jt} - x_{it}}{\mu_n} \right)^\alpha}_{D_i(X_{it})} \right]^\theta \quad (32)$$

Parameter ϵ defines the relative importance of the history-regarding deprivation towards

the other-regarding deprivation: therefore $\epsilon = 0.5$ weighs equally the two components, while by setting $\epsilon = 0$ we boil down to the traditional definition of deprivation.

Parameter α , instead, controls for the individual index $D(X_{it})$ to satisfy Progressive-transfer principle (if $\alpha \in [0, 1]$) or Regressive-transfer principle (if $\alpha > 1$). Remember that in the first case we evaluate more the increase in more similar incomes than x_{it} , while, in the second case, we give more weight to marginal increases in the highest incomes.

Parameter θ is tasked to regulate the importance of the more deprived individuals: the higher θ the more they are weighted. Note the difference between the role of α and θ : the former weighs each individual gaps, while the latter each individual deprivation (which is a function of all the gaps).

We keep fixed $\gamma = 1$ and $\beta = 0.0006$, as in Yi et al. [2006].

Table 2 show the results when $\theta = 1$, for different values of α and ϵ . When $\alpha = 1$ and $\epsilon = 0$, our index boils down to the relative-Yitzhaki index. The higher is α , the higher the values of the index. Note that the marginal contribution of the history-regarding deprivation ($\epsilon = 1$) is lower than the marginal contribution of the other-regarding deprivation ($\epsilon = 0$): this appears reasonable given both the short time period taken into account and the fact that the distribution of individuals' incomes along time is less unequal than the distribution of incomes of the population at a given time.

Italy and Spain are always among the most deprived countries regardless of the specification of the parameters. Belgium and Sweden appears to change their ranking, worsening their position, if we give more importance to the smaller gaps ($\alpha = 0.5$) both in history-regarding ($\epsilon = 1$) and in other-regarding deprivation ($\epsilon = 0$). It means that there are not severely deprived individuals, but the large majority of deprived individuals face short distances from their reference group. France and Portugal shows the opposite situation, since we see a consistent decrease in their deprivation when we evaluate more the smaller gaps ($\alpha = 0.5$): therefore it seems that large majority of deprived individuals face wide distances from their reference group. Moreover, in Portugal, the history regarding deprivation ($\epsilon = 1$)

is very small in comparison to the other countries: we should therefore conclude that the portuguese are not worsening their economic situation over time (either they are growing or they are staying constant). Norway, on the other hand, ends to the top of the deprivation scale if we concentrate on the history-regarding deprivation, giving more importance to the largest gaps ($\epsilon = 1$, $\alpha = 2$): some individuals must have experienced a harsh drop in their income.

Table 2: Inter-temporal Aggregate Relative Deprivation Index - $\theta = 1$

	$\theta = 1$								
	$\alpha = 1$			$\alpha = 0.5$			$\alpha = 2$		
	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$
	Austria	0,238	0,143	0,049	0,0012	0,0008	0,0005	0,551	0,290
Belgium	0,249	0,144	0,040	0,0017	0,0014	0,0010	0,483	0,252	0,020
Spain	0,289	0,174	0,059	0,0026	0,0022	0,0018	0,700	0,371	0,042
France	0,276	0,161	0,046	0,0001	0,0001	0,0001	0,670	0,350	0,030
Italy	0,323	0,186	0,049	0,0036	0,0025	0,0014	0,885	0,461	0,036
Norway	0,281	0,159	0,038	0,0005	0,0002	0,0000	0,521	0,276	0,032
Portugal	0,324	0,180	0,036	0,0005	0,0003	0,0002	1,490	0,753	0,016
Sweden	0,246	0,134	0,021	0,0009	0,0008	0,0006	0,402	0,208	0,015

Table 3 replicates the former exercise, by setting $\theta = 2$. We can appreciate few changes in the results as comparing to the Table 2.

4 Conclusion

In this paper we provide the axiomatic characterization of a new deprivation index: the Inter-Temporal Relative Deprivation Index. This index is based on a more general and complete idea of deprivation: people compare themselves not only with other individuals but also with their own past history. Differently from previous contributions, the index additivity form is not imposed *a priori*, but it is obtained by means of an independence axiom

Table 3: Inter-temporal Aggregate Relative Deprivation Index - $\theta = 2$

	$\alpha = 1$			$\alpha = 0.5$			$\alpha = 2$		
	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$	$\epsilon = 0$	$\epsilon = 0.5$	$\epsilon = 1$
Austria	0,084	0,033	0,011	0,001	0,0006	0,0002	8,0316	2,019	0,013
Belgium	0,093	0,032	0,008	0,002	0,0010	0,0006	1,0028	0,257	0,005
Spain	0,124	0,050	0,018	0,002	0,0017	0,0011	2,6059	0,675	0,028
France	0,108	0,039	0,011	0,000	0,0001	0,0001	3,3751	0,859	0,013
Italy	0,149	0,052	0,013	0,003	0,0017	0,0007	6,5883	1,670	0,019
Norway	0,119	0,043	0,014	0,000	0,0001	0,0000	3,2700	0,838	0,026
Portugal	0,138	0,044	0,007	0,000	0,0002	0,0001	44,5738	11,152	0,003
Sweden	0,093	0,030	0,006	0,001	0,0007	0,0004	0,9076	0,236	0,016

which brings along also the relative concept embedded in the idea of deprivation. Moreover, we introduce sensitivity to transfer allowing for different effects (progressive or regressive) on deprivation status of these transfers. The axiomatic approach takes also advantage of the literature about time-discounting (Ok and Masatlioglu [2007] and Loewenstein and Prelec [1991]) in order to obtain a flexible weighting system. The index main contribution is to bridge two streams of the well-being literature: relative deprivation on one hand and mobility on the other. In fact, if we restrain the index only to Other-regarding deprivation, it becomes a generalization of existing deprivation indexes, such as Yitzhaki [1979] and Chakravarty [1997]; while if we focus only on History-regarding deprivation, we obtain a measure of individual mobility (the *directional movement* in Fields [2007]). The empirical exercise about shows how the new index contributes to disentangle the different faces of relative deprivation.

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