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On the "Pro-Poorness" of Growth in a Multidimensional Context

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On the "Pro-Poorness" of Growth in a Multidimensional Context

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Abstract

This paper represents a first attempt to gather the issues of growth "pro-poorness" and multidimensional poverty assessments. More precisely, we suggest the use of sequential dominance procedures (Bourguignon, 1989, Atkinson, 1992, Jenkins and Lambert, 1993) to test the "pro-poorness" of observed growth spells when poverty is measured on the basis of income and some other discrete well-being attribute. Sequential procedures are also used to get graphical tools that are consistent with the spirit of Chen and Ravallion's (2003) growth incidence curve and Son's (2004) poverty growth curve. Contrary to traditional unidimensional tests, our methodology allows to take into account the importance of deprivation correlations at the individual level and thus may reverse results observed with the traditional tools of growth "pro-poorness" check. An illustration of our approach is finally given using Turkish data for the period 2003-2005.

JEL classification: I32, C00.

Key words: "Pro-poor" growth, growth incidence curve, sequential stochastic dominance.

1 INTRODUCTION

The definition of the Millennium Development Goals in 2000 by the international community was a major breakdown with the previous paradigm of the Washington consensus and its implicit reference to "trickle down" theories. One remarkable feature was the rehabilitation of Chenery, Ahluwalia, Bell, Duloy, and Jolly's (1974) advocacy in favor of introducing redistributive concerns into growth policies in the developing

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world.[1] Indeed, since late 1990s, many social researchers have forcefully argued in favor of assigning to growth only an instrumental role with respect to poverty issues. In other words, poverty alleviation should not be regarded as a desirable side effect of growth but the ultimate goal to be reached in the spirit of former World Bank's president Robert McNamara desire to shift the focus towards targeted poverty reduction. However, how best this goal can be met was an open and complex question, and researchers have spent many efforts in yielding lessons from the empirics of growth and poverty. In particular has been developed a literature related to the identification of "pro-poor" growth spells, *i.e.* growth spells that corresponds to a marked improvement regarding the state of poverty. The nature of this bias in favor of the poor has entailed many debates, notably concerning the desirabilty of observing a poverty alleviation effect of inequality reduction to tag a growth pattern as "pro-poor" (Kakwani and Pernia, 2000, Ravallion, 2004, Zepeda, 2004, Osmani, 2005), but the theoretical framework to be used for empirical assessment is now well defined (Duclos, 2009).

The commitment of the international community to the achievement of the height Millennium Development Goals was also an official recognition of the multidimensional nature of poverty. It is well known (see for instance Sen, 1987, 1992, Streeten, 1994) that the linkages between income (or expenditure) and well-being are not straightforward and hinge on many determinants like idiosyncratic characteristics or market factors. As a result, the efficiency of poverty reducing policies should also be assessed on the basis of the satisfaction of non-income needs like health, education or participation to social life. If poverty has to be thought and measured taking a multidimensional approach, it is then necessary to have a look at the "pro-poor" nature of growth beyond the sole monetary aspects of poverty. The recent abundant literature on these two concepts has evolved in a parallel way. Surprisingly, very few attemps have been carried out in order to include the additional information aasociated with other dimensions of well-being alongside the monetary one within the assessment of the "pro-poor" nature of growth. At our knowledge, the only studies that deal with that issue are Klasen (2008) and Grosse, Harttgen, and Klasen (2008) that suggest making use of the tools developed for "pro-poor" growth tests to investigate the distribution of changes with respect to non-income attributes.

The non-income growth incidence curve proposed in these studies allows for widening the scope of "pro-poor" growth analyses and may highlight potential discrepencies between progresses in the monetary and non-monetary dimensions. However, these graphical tools only focus on the marginal distributions of well-being attributes and thus, do not take into account the additional information provided by the joint distribution of that attributes. Many authors (Atkinson and Bourguignon, 1982, Tsui, 2002, Bourguignon and Chakravarty, 2002) have stressed the importance of correlations between the distributions of the different attributes in multidimensional poverty

[1]Today, such policies are generally called inclusive growth policies.

measurement. Indeed, if poverty indices are based on individualistic welfare functions that are not separable with respect to the different attributes (Kolm, 1977), poverty may raise or decrease without any changes occuring in the marginal distribution of the attributes if some attributes are substitutes or complements with respect to well-being.

However, the literature on sequential stochastic dominance offers a promising way to address this issue without requiring strong assumptions about how dimensions of poverty should exactly be related, an assumption about which there may not be a wide agreement. Originally, sequential stochastic dominance has been introduced in order to address comparisons of income distributions with households differing by their composition and size. Despite that the use of equivalence scales makes it possible to obtain homogeneous distributions of equivalent incomes, it entails several problems as it relies on strong normative assumptions. Consequently, the distributional income comparisons may be highly sensitive to the choice of equivalence scale. Sequential stochastic dominance techniques (Bourguignon, 1989, Atkinson, 1992, Jenkins and Lambert, 1993, Chambaz and Maurin, 1998, Duclos and Makdissi, 2005) addresses that kind of issues, since they highlight the conditions to be met so that the results of poverty comparisions are robust to the choice of this equivalence scale. As noted in recent studies like Duclos and Échevin (2009), the method can naturally be extended to cases where household size can be replaced by non-income poverty dimensions.

In the present paper, we go a step further and, using this sequential stochastic dominance framework, suggest a new way of testing the "pro-poor" nature of growth for poverty measures based on both income and other characteristics that can be summed up using some ordinal index. It is worth noting that the proposed dominance criteria are related to classes of poverty measures complying with axioms upon which it is reasonable to think that agreement should unambiguously be met. Our different propositions rely in particular on two crucial assumptions. The first one is that the income poverty line may vary with the level reached by the ordinal variable and can be set to zero above some values of that index. Thus, the approach is compatible with different rival approaches of poverty identification. The second assumption is that the marginal contribution of income to well-being decreases with the level of the non-income attribute. Using that minimal set of assumptions, we define criteria that are robust to choices in both the value of the poverty lines and the functional form of the bidimensional poverty measure.

The paper is organized as follows. In section 2, we describe the "classical" tools used for "pro-poor" growth check and extend them in section 3 to multidimensional approaches of poverty using sequential stochastic dominance criteria. In section 4, the methodology is illustrated to the case of Turkey, taking into account alongside the income dimension the educational level achievement of each individual. Finally, section 5 concludes.

2 "Pro-poorness" with unidimensional poverty

Let $y_i \in \mathbb{R}$ be the level of some monetary variable, like income or expenditure, for the ith person of a given population of size $n \in \mathbb{N}^*$.[2] The distribution of income among the population can then be described by the n-vector $\boldsymbol{y} := \{y_1, \ldots, y_n\}$. In order to ease the comparisons between distributions of different sizes, it is often preferable to use the univariate cumulative distribution function (*cdf*) $F(z; \boldsymbol{y})$. That *cdf* returns the probability $p \in [0,1]$ of picking out of $\boldsymbol{y}$ an income which value is less than the threshold z. It is worth noting that, in the context of poverty analysis, the *cdf* corresponds to the widely used poverty measure known as the headcount index Θ_0.

In the present section, monetary poverty is first assessed using the following class Π_1 of additive poverty measures:

$$\Theta(\boldsymbol{y}, z) := \int_0^z \theta(y, z) \; dF(y; \boldsymbol{y}) \tag{1}$$

with $\theta(z,z) = 0$, $\partial\theta/\partial z \geqslant 0$, $\partial\theta/\partial y \leqslant 0$ if $y < z$, $\partial\theta/\partial y = 0$ if $y \geqslant z$ so that the measure complies with the traditional axioms of focus, weak monotonicity, continuity, anonymity, population, non-decreasingness with respect to the poverty line and subgroup additivity.[3] That class of subgroup additive poverty measures (Foster and Shorrocks, 1991) is very general and includes the most widely used poverty measures like the one suggested by Watts (1968) and Foster, Greer, and Thorbecke (1984).[4] Here we would like to stress the particular importance of the anonymity axiom that states that income is the sole relevant variable to be used to discriminate people for poverty analysis. In equation (1), the respect of the anonymity axiom then entails that the individual poverty function θ is the same for each individual. This crucial assumption will be partially slackened in section 3 when individuals with different needs will be considered.

As stressed in the literature (Kakwani and Pernia, 2000, Chen and Ravallion, 2003, Kraay, 2006), whether an observed growth pattern is "pro-poor" or not crucially depends on the social evaluator's definition of what may be a "pro-poor" growth. In particular, it relies on the way any additional income should be shared between the different members of the population so as to get a growth pattern that is neither "pro-poor" nor "anti-poor," but ethically "neutral." As our work is orthogonal with respect to this specific point, we refer here to the general definition of "pro-poor" growth proposed by Duclos (2009). For Duclos, the assessment of the "pro-poorness" of growth between years t and $t+1$ always implies the comparison of the poverty level in $t+1$ with the level that would have been observed for some conterfactual distribution defined

[2] For the sake of simplicity, we will consider that y denotes the income level, but this choice does not preclude using any other concepts that would be relevant to assess monetary poverty.

[3] See Zheng (1997) for a comprehensive review of the axiomatic framework used for unidimensional poverty analysis.

[4] In the first case, the individual function is defined by $\theta(x,z) := \log x - \log z$. With the Foster, Greer, and Thorbecke's (1984) class of poverty measures, that function becomes $\theta(x,z) := (1 - x/z)^\alpha$, $\alpha \geqslant 0$.

by $\boldsymbol{y}_t$ and some real-valued function γ that relates to the social evaluator's definition of "pro-poorness." In other words, growth is deemed "pro-poor" with respect to some given poverty measure Θ and some poverty line z if and only if:

$$\Theta(\boldsymbol{y}_{t+1}, z) - \Theta\big(\gamma(\boldsymbol{y}_t), z\big) \leqslant 0. \tag{2}$$

Duclos (2009) argues that the definition of γ may be ruled by either ethical, statistical or administrative arguments, and that this diversity explains the heterogeneity of feelings with respect to what could be a "pro-poor" growth. As noted in Kakwani and Son (2008), empirical investigations generally focuses on three rival definitions of γ, thereafter called the "poverty reducing," the "relative," and the "absolute" approaches of "pro-poor" growth. In the first case, growth is deemed "pro-poor" if poverty has decreased over the period of interest, so that $\gamma^p(\boldsymbol{y}_t) := \boldsymbol{y}_t$. On the contrary, with the two remaining approaches it is supposed that growth should be associated with a decrease in inequality for the benefit of the poor, in order to observe "pro-poor" growth. With the "relative" approach (Baulch and McCulloch, 1998, Kakwani and Pernia, 2000), income inequalities are thought in relative terms and the counterfactual distribution is simply $\gamma^r(\boldsymbol{y}_t) := \boldsymbol{y}_t \frac{\mu_{t+1}}{\mu_t}$ where μ_t is the mean value of $\boldsymbol{y}_t$. On the other hand, using the "absolute" view means that income inequalities are thought on the basis of absolute income differences among the population, hence $\gamma^a(\boldsymbol{y}_t) := \boldsymbol{y}_t + (\mu_{t+1} - \mu_t)$. While the first approach does not impose any restriction on Θ, it is important to stress that the use of the "relative" and "absolute" approaches confines the analysis to poverty measures that are respectively scale-invariant and translation-invariant.[5]

A traditional problem with the criterion defined in equation (2) is that using any other poverty measure Θ or changing the poverty line z may reverse the statement made about the "pro-poorness" of growth over a given period. It is then necessary to assess the robustness of the results using stochastic dominance criterions (Atkinson, 1987, Foster and Shorrocks, 1988).[6] As shown in Duclos (2009), the use of first-order dominance properties for the issues lead to the following result:

Proposition 1. *For a given counterfactual scenario γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of the poverty measure among the family Π_1 and the value of the poverty line z if and only if:*

$$F(z; \boldsymbol{y}_{t+1}) - F\big(z; \gamma(\boldsymbol{y}_t)\big) \leqslant 0 \quad \forall z \leqslant z^+, \tag{3}$$

[5] Regarding, the family of poverty measures defined in equation (1), Θ is scale-invariant if and only if $\theta(\lambda y, \lambda z) = \theta(y, z)$, $\forall \lambda \in \mathbb{R}_{++}$, and translation-invariant if and only if $\theta(y+\varepsilon, \lambda+\varepsilon) = \theta(y, z)$, $\forall \varepsilon \in \mathbb{R}$. On this issue, see notably Bresson and Labar (2007).

[6] We should be cautious with the use of the term "robustness" since this type of robustness check does not take into account the issues of sampling errors.

with a least one value $z^* \in [0, z^+]$ such that:

$$F(z^*; \boldsymbol{y}_{t+1}) - F\big(z^*; \gamma(\boldsymbol{y}_t)\big) < 0. \tag{4}$$

Chen and Ravallion (2003) have also demonstrated that first-order stochastic dominance can also be easily assessed using a single graph of the observed growth rates for each percentile of the population over the period of interest. Instead of the *cdf*, these authors then prefer working with the quantile function, or Pen's parade, F^{-1} that, using Gastwirth's (1971) definition, is simply:

$$F^{-1}(p; \boldsymbol{y}_t) := \min\{y_{it} \in \boldsymbol{y}_t | F(y_{it}; \boldsymbol{y}_t) \geqslant p\}. \tag{5}$$

Using equation (5), the growth incidence curve (GIC) proposed by Chen and Ravallion (2003) is obtained by plotting for each $p \in [0, 1]$ the value of the the function:[7]

$$g_1(p; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) := \frac{F^{-1}(p; \boldsymbol{y}_{t+1})}{F^{-1}(p; \boldsymbol{y}_t)} - 1. \tag{6}$$

Then, the evaluation of "pro-poor" growth is performed when comparing that function with the function $g_1\big(p; \gamma(\boldsymbol{y}_t), \boldsymbol{y}_{t+1}\big)$ up to the percentile that corresponds to the highest admissible value z^+ for the poverty line.

Corollary 1. *For some given criterion γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choices of the poverty measure among the family Π_1 and the value of the poverty line z if and only if:*

$$g_1(p; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) - g_1\big(p; \gamma(\boldsymbol{y}_t), \boldsymbol{y}_{t+1}\big) \geqslant 0 \quad \forall p \in \big[0, F(z^+, \boldsymbol{y}_t)\big], \tag{7}$$

with a least one value $p^* \in [0, F(z^+, \boldsymbol{y}_t)]$ such that:

$$g_1(p^*; \boldsymbol{y}_t, \boldsymbol{y}_{t+1}) - g_1\big(p^*; \gamma(\boldsymbol{y}_t), \boldsymbol{y}_{t+1}\big) > 0. \tag{8}$$

Proposition 1 and corollary 1 are appealing since they mean that the condition (2) is fulfilled for all poverty measures from the class defined by equation (1) and all poverty lines in the range $[0, z^+]$ (Duclos, 2009). As a consequence, it is very robust from an ethical point of view since it requires minimal agreement for the assessment of "pro-poor" growth for a given benchmark scenario γ.

However, it is well known that first-order dominance tests are likely to be not conclusive in a significant number of cases. It is then necessary to add further restrictions on the type of poverty measures used for "pro-poor" growth assessments and to turn to higher-order stochastic dominance conditions. For instance, if the class of poverty

[7] It is worth noting that the idea of performing welfare comparisons on the basis of the quantile functions is not new and can be traced at least to Mahalanobis (1960).

measures defined in equation (1) is restricted to indices that respect $\partial^2\theta/\partial y^2 \geqslant 0$, we get the class of poverty measures Π_2 that belongs to Π_1 and complies with the weak transfer axiom (Sen, 1976). According to the weak transfer axiom, an income loss for a poor individual does not raise poverty if it is at least compensated by an increase of the same amount for a poorer person. Robustness tests based on this axiom are then more powerful than first order stochastic conditions as they do not require income improvement at each quantile of the population during the period of interest. More precisely, second order dominance tests require the use of the poverty gap function G such that:

$$G(z;\boldsymbol{y}) := \int_0^z (z-y)\,dF(y;\boldsymbol{y}). \tag{9}$$

That function simply returns the average shortfall with respect to the poverty line z given the income distribution $\boldsymbol{y}$. The relationship between growth "pro-poorness" and the class of poverty measures Π_2 is then summarized by the following proposition:

Proposition 2. *For a given counterfactual scenario γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of the poverty measure among the family Π_2 and the value of the poverty line z if and only if:*

$$G(z;\boldsymbol{y}_{t+1}) - G\big(z;\gamma(\boldsymbol{y}_t)\big) \leqslant 0 \quad \forall z \leqslant z^+, \tag{10}$$

with a least one value $z^* \in [0, z^+]$ such that:

$$G(z^*;\boldsymbol{y}_{t+1}) - G\big(z^*;\gamma(\boldsymbol{y}_t)\big) < 0. \tag{11}$$

A "pro-poorness" test proposed by Son (2004) and related to the class of poverty measures Π_2 is based on the poverty growth curve (PGC) that plots the growth rate of the mean income of the bottom p percent of the population when individuals are ranked by increasing order of income. More formally, the PGC is defined by:

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1}) := \int_0^p \frac{F^{-1}(u;\boldsymbol{y}_{t+1})}{F^{-1}(u;\boldsymbol{y}_t)} - 1\,du. \tag{12}$$

It can easily be checked that the comparison of the observed PGC with the one corresponding to the counterfactual distribution yields a criterion that is equivalent to the one presented in proposition 2.[8]

Corollary 2. *For some given criterion γ and a given maximum value z^+ for the poverty line, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choices of the poverty measure among the*

[8]On the power of the PGC for "pro-poorness" tests, see Davis (2007).

family Π_2 *and the value of the poverty line* z *if and only if:*

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1}) - g_2\big(p;\gamma(\boldsymbol{y}_t),\boldsymbol{y}_{t+1}\big) \geqslant 0 \quad \forall p \in \big[0, F(z^+;\boldsymbol{y}_t)\big]\,, \tag{13}$$

with a least one value $p^* \in [0, F(z^+;\boldsymbol{y}_t)]$ *such that:*

$$g_2(p;\boldsymbol{y}_t,\boldsymbol{y}_{t+1}) - g_2\big(p;\gamma(\boldsymbol{y}_t),\boldsymbol{y}_{t+1}\big) > 0. \tag{14}$$

3 "PRO-POORNESS" WITH MULTIDIMENSIONAL POVERTY

The previous section has reviewed the conditions to be met in order to get a judgment that is not likely to be contingent to choices for the functional form of the poverty measure or for the value of the poverty line. However, the results depend on the crucial assumption made in the previous section that poverty should be thought only in monetary terms. Yet, most researchers agree that other dimensions of poverty like education, health, access to public services or real freedoms should be taken into account for the analysis of poverty. The inclusion of such elements logically changes the definition of poverty indices and induces a slackening of the anonimity axiom.[9] Moreover, in most cases, it may entails that the marginal contribution of income to poverty is determinated by the the level of the other dimensions chosen to assess poverty.

In order to take into account this aspect, we propose to make use of the tools of sequential stochastic dominance. Originally, this methodology has been developed in order to assess robust comparisons of income distributions when households differ in needs. Although differences in househods' needs are most of the time based on differences in their size or composition in the studies using the framework of sequential stochastic dominance, other characteristics that are of interest for social welfare analysis or multidimensional approaches to poverty can also be taken into account as proxies of needs.

For this purpose, we will consider that the additional information to be included in the poverty measure can be summed up by the variable x, though our framework can easily be extended so as to use more additional variables. In many situations, the satisfaction of non-monetary needs cannot be assessed by continuous variables. Here, we will assume that the variable x is discrete and takes $K \in \mathbb{N}^*\backslash 1$ values that can be ordered in the following manner $x^1 \leqslant x^2 \leqslant \ldots x^K$. That index may have a cardinal content, but, for our present purpose, we are only interested by its ordinal properties.[10] We also assume that the well-being of the i-th person increases with the

[9]However, it is worth stressing, as noted by Kolm (1977), that this slackening may ease the agreement on that property of equal treatment for the equal.

[10]In empirical applications of the propositions suggested in the present section, we may face some difficulties in ranking the some categories of individuals. A first solution is to gather together categories which ranking is not straightforward. A more robust solution proposed by Atkinson (1992) is to use the sequential criteria for all relevant orderings of the categories used for x. For instance, let the members of

value of that index. The distribution of that variable in the population is described by the n-vector $\boldsymbol{x} := \{x_1, \dots, x_n\}$ which elements x_i are ordered in the same manner as in $\boldsymbol{y}$. Let $\boldsymbol{X}$ be the $n \times 2$-matrix obtained by placing side by side the vectors $\boldsymbol{y}$ and $\boldsymbol{x}$ and the ith line of that matrix summarizes all the relevant characteristics of person i.

Considering many attributes for the analysis of poverty also implies a change in the definition of the poverty domain. In the multidimensionnal poverty measurement literature, many rival definitions have been suggested (Bourguignon and Chakravarty, 2002, Duclos, Sahn, and Younger, 2006, Alkire and Foster, 2007). In this section, we will consider a very general definition of the poverty domain that is close to the one used in Duclos, Sahn, and Younger (2006). Regarding the income dimension, we assume that that the income poverty line is a non-increasing function of the value of the index x_i.

For instance, let us consider income y and health x as the relevant dimensions for poverty analysis, and two poor individuals A and B with the same income $y_A = y_B < \tilde{z}$, but different health levels. More precisely, suppose that individual B does not suffer from any deprivation with respect to health while individual A is handicapped, *i.e.* $x_A < x_B$. Indeed, this health shortage implies that A is poorer than B since he suffers from deprivation in that dimension, but we may go a step further and consider that person A's handicap has also consequences in the income dimension. The handicap generates specific expenditures (long-term medical treatments, protheses...) and increases the cost of other expenditures like transport. As a consequence, we may conclude that the income poverty line $\tilde{z}$ is inappropriate for person A as its income level cannot yield the same consumption level as for individual B. Thus, while A and B share the same income level, we may also consider that A suffers from a larger degree of deprivation than B in the income dimension. Thus, we should observe $z(x_A = x^k) > z(x_B = x^{k+s})$, $k \in \{1, \dots, K-1\}$, $s \in \{1, \dots, K-k\}$. In order to save space space, let z_k denote the value of income poverty line for those individuals which value of x_i is equal to x^k. Income deprivations are then assessed using the K-vector $\boldsymbol{z} := \{z_1, \dots, z_K\}$ such that $z_1 \geqslant z_2 \cdots \geqslant z_K \geqslant 0$.

It is important to stress that we do not need for our analysis to specify explicitly a poverty line for the non-income dimension since all the relevant information concerning the shape of the poverty domain is included in the vector $\boldsymbol{z}$. For instance, the traditional "intersection" approach of poverty identification is obtained for $z_k = c \in \mathbb{R}_{++}$, $k \in \{1, \dots, j\}$, and $z_k = 0$, $k \in \{j+1, \dots, K\}$, $1 \leqslant j \leqslant K$. On the other hand, a "union" approach can be used by imposing $z_k = +\infty$, $k \in \{1, \dots, j\}$, and $z_k = c$, $k \in \{j+1, \dots, K\}$, $1 \leqslant j \leqslant K$. These two cases are illustrated on figure 1 where the poverty domain is depicted each time by the set of horizontal thick lines.

We then show how growth "pro-poorness" can be robustly assessed using sequen-

the population be of types x^a, x^b and x^c. Assuming that the individuals of types x^a are the neediest but that x^b and x^c cannot easily be compared, it would then be necessary to perform our tests for $x^a < x^b < x^c$ and $x^a < x^c < x^b$.

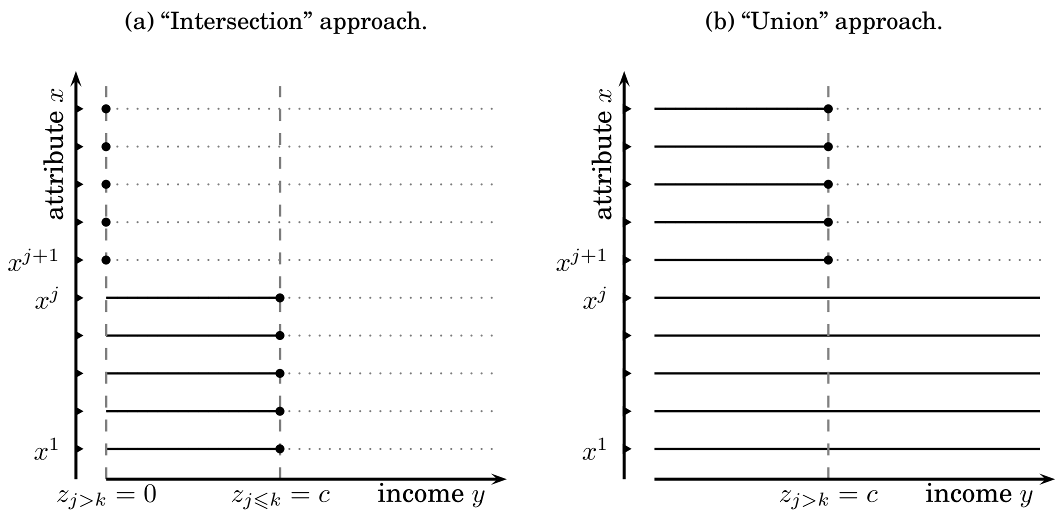

Figure 1: The definition of the poverty domain under different rival approaches.

tial dominance procedures that mirror the first and second-order stochastic dominance conditions expressed in the previous section.[11]

3.1 First-order stochastic sequential dominance and k-GIC

The theoretical developments in this section are very close to the one realized in Gravel and Moyes (2008), Duclos and Échevin (2009). Define $F_k(y; \boldsymbol{y})$ as the income *cdf* of those individuals for which level of x_i is equal to x^k. Poverty is then assessed using the following class of additive poverty measures:

$$\Theta(\boldsymbol{X}_t, \boldsymbol{z}) := \sum_{k=1}^{K} q_k(\boldsymbol{x}_t) \int_0^{z_k} \theta(y, x, z_k)\, dF_k(y; \boldsymbol{y}_t), \tag{15}$$

with $q_k(\boldsymbol{x}_t)$, $\sum_{k=1}^{K} = 1$, being the share of the population belonging to group k. Here, we consider the class of poverty measures $\bar{\Pi}_1$ such that θ in equation (15) satisfies the following properties: $\theta(z_k, x^k, z_k) = 0$, $\partial\theta/\partial z_k \geqslant 0$, $\partial\theta/\partial y \leqslant 0$ if $y < z_k$, $\partial\theta/\partial y = 0$ if $y \geqslant z_k$. Thus, as in the case of the monetary poverty, the indices are supposed to comply with the multidimensional counterparts of the focus, weak monotonicity, continuity, anonymity, population, non-decreasingness with respect to the poverty line

[11] In the present paper, we do not explore higher-order dominance sequential dominance conditions. However, our results can easily be extended to third-order stochastic dominance tests using Lambert and Ramos's (2002) results.

and subgroup additivity axioms.[12] Moreover, it is also assumed that:

$$\frac{\partial}{\partial y_i}\theta(y_i, x^k, z_k) \leqslant \frac{\partial}{\partial y_i}\theta(y_i, x^{k+1}, z_{k+1}), \quad \forall k \in \{1, \dots, K-1\}. \tag{16}$$

It can easily be seen that $\Pi_1 \subset \bar{\Pi}_1$. That class of unidimensional poverty measures is obtained for $z_k = z \ \forall k \in \{1, \dots, K\}$ and when the value of $\partial\theta/\partial y$ does not vary with the level of x_i. Finally, the condition expressed in equation (16) is standard in the literature on multidimensional inequalities and poverty (Atkinson and Bourguignon, 1982, Tsui, 2002) and is related to the axiom known as the non-decreasingness under correlation switches. That axiom stipulates that, given two individuals with endowments (y_A, x_A) and (y_B, x_B), a permutation of the values of these two vectors so that A can be said unambiguously poorer than B, should not lower the poverty level, other things being equal.

We now turn to the issue of "pro-poorness" evaluation. In the previous section, the assessment of the "pro-poorness" of growth was performed on the basis of a counterfactual income distribution $\gamma(\boldsymbol{y}_t)$. That definition of "pro-poorness" is consistent with the income-based approach of poverty but may not be appropriate when other attributes are taken into account. Indeed, in the context of our setting, we are concerned with the evolution of the whole matrix $\boldsymbol{X}$, so that growth between the years t and $t+1$ will be deemed "pro-poor" for a given counterfactual benchmark Γ, a given poverty measure Θ and a given set of poverty lines $\boldsymbol{z}$ if and only if:

$$\Theta(\boldsymbol{X}_{t+1}, \boldsymbol{z}) - \Theta\big(\Gamma(\boldsymbol{X}_t), \boldsymbol{z}\big) \leqslant 0. \tag{17}$$

The main difference with the definition corresponding to equation (2) consists in the definition of the counterfactual scenario that gives more latitude for the social evaluator. Indeed, as we may observe simultaneous variations of the vectors $\boldsymbol{y}$ and $\boldsymbol{x}$, it is then necessary to ask whether the evaluation should be performed on the basis of a counterfactual distribution for the distribution of the index x. We then have to distinguish the situations in which the counterfactual matrix $\Gamma(\boldsymbol{X}_t)$ is obtained from $\boldsymbol{X}_t$ by simply changing its income vector $\boldsymbol{y}_t$, and cases in which the non-income vector $\boldsymbol{x}_t$ is not necessarily leaved unchanged. Let the first situation be called the "income pro-poorness" of growth and the second one "well-being pro-poorness" of growth. To avoid confusion, let γ_y and γ_x respectively denote the functions used to define the counterfactual distributions of the income and non-income variables.

The case of "well-being pro-poorness" deserves some interest because the counterfactual distribution $\gamma_x(\boldsymbol{x}_t)$ of the non-income item may be slightly more complex that the one corresponding to individual incomes. The most important question is whether $\gamma_x(\boldsymbol{x}_t)$ should be exogeneously or endogeneously defined with respect to the observed

[12] For a comprehensive review of the axioms used for multidimensional poverty measurement, see Bresson (2009).

growth pattern.

In the former case, we may choose to define $\gamma_x(\boldsymbol{x}_t)$ using the initial and final distributions $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+1}$, and dissociate it from observed changes in the income dimension. It is worth emphasizing that, due to the particular nature of the variable x, the choice of γ_x is obviously knottier than with income. Hence the relative and absolute counterfactual functions γ^r and γ^a cannot be used with our general setting as the variable x is ordinal — it would not make sense for instance to apply a given growth rate on qualitative data. In fact, this criticism prevails for all conceptions of the counterfactual scenario that rely on a distributional neutral approach of growth and uses mean-based definitions of inequality. With respect to that issue, a promising solution is the use of median-based approaches with inequality being thought in terms of "distance" to the median value (Allison and Foster, 2004).[13] Finally, with the counterfactual function γ^p, the non-income vector is simply the initial distribution $\boldsymbol{x}_t$ so that "income pro-poorness" is a particular case of "well-being pro-poorness."

On the contrary, we may feel that $\gamma_x(\boldsymbol{x}_t)$ should be computed on the basis of some statistical or theoretical relationship between x and y, and the counterfactual distribution of income $\gamma_y(\boldsymbol{y}_t)$. More complex designs can also be chosen, using CGE models with micro-simulations exercices so as to fully take the effects of economic growth into account. Finally, whatever the chosen procedure, the nature of the variable used for the index x has also to be taken into account as it may be bounded (Klasen, 2008).

In order to save space, we now introduce the following notation:

$$\Delta^{\gamma}_{t,t+1} F_k(z;\boldsymbol{y}) := q_k(\boldsymbol{x}_{t+1}) F_k(z;\boldsymbol{y}_{t+1}) - q_k\big(\gamma_x(\boldsymbol{x}_t)\big) F_k\big(z;\gamma_y(\boldsymbol{y}_t)\big), \tag{18}$$

with $q_k(\boldsymbol{x})$ being the share of observations from $\boldsymbol{x}$ which values are equal to x^k. The properties of the class of poverty measures $\bar{\Pi}_1$ then lead to the following result:

Proposition 3. *For a given counterfactual scenario Γ and a given vector $\boldsymbol{z}^+$ of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of a poverty measure from the class $\bar{\Pi}_1$ and the value of the poverty lines $\boldsymbol{z}$ if and only if:*

$$\sum_{k=1}^{j} \Delta^{\gamma}_{t,t+1} F_k(z;\boldsymbol{y}) \leqslant 0 \quad \forall z \leqslant z_j^+,\ j \in \{1,\dots,K\}, \tag{19}$$

with a least one integer $j^* \in \{1,\dots,K\}$ and one value $z^* \in [0, z_{j^*}^+]$ such that:

$$\sum_{k=1}^{j^*} \Delta^{\gamma}_{t,t+1} F_k(z^*;\boldsymbol{y}) < 0. \tag{20}$$

The criterion suggested in proposition 3 refers to the one first suggested in Bour-

[13] Here, the word "distance" does not refer to the traditional euclidean distance but to the number of categories separating two values of the index x.

guignon (1989) and developed by Atkinson (1992), Jenkins and Lambert (1993), and Chambaz and Maurin (1998), but applied to the question of the assessment of "pro-poor" growth. The second difference with respect to these studies is that the heterogeneity of the population is not grasped by the household size, but by any set of individual characteristics that can be considered as relevant dimensions of poverty.

In the previous section, we have seen that the conditions to be met to conclude in a robust manner whether growth has been "pro-poor" could also be expressed with the help of the GIC (*cf* corollary 1). In most cases, that equivalence cannot be observed, except when the counterfactual distribution $\gamma_x(\boldsymbol{x}_t)$ is the same as the distribution of x observed in year $t+1$. For that particular case, it is necessary to define the k partial quantile function as:

$$F_k^{-1}(p;\boldsymbol{X}) := \min\left\{y_{it} \in \boldsymbol{y}_t^k | F\left(y_{it};\boldsymbol{y}_t^k\right) \geqslant p\right\}, \tag{21}$$

with $\boldsymbol{y}_t^k$ being the subset of values from $\boldsymbol{y}_t$ corresponding to individuals which value of the index x is not greater than x^k. The function $F_k^{-1}(p;\boldsymbol{X})$ returns the value of income y corresponding to the p-th centile of the subpopulation of type 1 to k ranked by increasing value of income. For $k = K$, that function simply becomes the traditional quantile function presented in equation (5). Using that instrument, we can then propose the use of the following k-GIC:

$$g_{1,k}(p;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) := \frac{F_k^{-1}(p;\boldsymbol{X}_{t+1})}{F_k^{-1}(p;\boldsymbol{X}_t)} - 1, \tag{22}$$

that corresponds to the income growth rate of the p-th percentile of the subpopulation of type 1 to k considering the non-income attribute. Dominance can then assessed by comparing the values of that function with the corresponding k-GIC for the counterfactual distribution $\Gamma(\boldsymbol{X}_t)$ for the bottom part of the population. Our results are then summarized by the following corollary:

Corollary 3. *For a given counterfactual scenario γ and a given vector $\boldsymbol{z}^+$ of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of poverty measure among the family $\bar{\Pi}_1$ if and only if:*

$$g_{1,k}(p;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) - g_{1,k}\big(p;\Gamma(\boldsymbol{X}_t),\boldsymbol{X}_{t+1}\big) \geqslant 0 \quad \forall p \leqslant F(z_k^+,\boldsymbol{y}_t^k), k \in \{1,\dots,K\}, \tag{23}$$

with a least one integer $j^* \in \{1,\dots,K\}$ and one value $p^* \in \left[0, F(z_{j^*}^+,\boldsymbol{y}_t^{j^*})\right]$ such that:

$$g_{1,j^*}(p^*;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) - g_{1,j^*}\big(p^*;\Gamma(\boldsymbol{X}_t),\boldsymbol{X}_{t+1}\big) > 0. \tag{24}$$

The condition suggested with corollary 3 suits best situations such that the distribution of the non-monetary variable x is time-invariant. However, that result can easily be extended to the case of variable distributions of that index x. Indeed it can be shown that a sufficient, but not necessary, condition for growth to be deemed "pro-poor" between t and $t+1$ given the courterfactual scenario Γ and the set of poverty lines $\boldsymbol{z}^+$ is to comply simultaneously with the conditions expressed in corollary 3 and:

$$F(x^j; \boldsymbol{x}_{t+1}) \leqslant F\big(x^j; \gamma_x(\boldsymbol{x}_t)\big) \quad \forall j \in \{1, \dots, K\} \text{ such that } z_j > 0. \tag{25}$$

3.2 SECOND-ORDER STOCHASTIC SEQUENTIAL DOMINANCE AND k-PGC

As in the case of homogenous populations, the test suggested in the previous section may be unconclusive. In order the increase the power of the test, it is then necessary to turn to a reduced set of poverty measures $\bar{\Pi}_2$. Starting with the conditions used to define the class $\bar{\Pi}_1$, we impose the following additional restriction:

$$\frac{\partial^2}{\partial y^2}\theta(y_i, x^k, z_k) \geqslant \frac{\partial}{\partial y^2}\theta(y_i, x^{k+1}, z_{k+1}) \geqslant 0, \quad \forall k \in \{1, \dots, K-1\}. \tag{26}$$

The condition (26) can be decomposed in two parts. The first one relates to the non-negativity of the second-order derivative of the function θ with respect to income. This non-concavity assumption is well-known in the poverty and inequality literature, and significates that progressive transfers of income — a transfer is said progressive if it reduces inequalities — within the set of individuals with the same value of the index x do not raise the poverty level. The second part of condition (26) is the non-increasingness of $\partial^2\theta/\partial y^2$ with respect to the value of x. That assumption indicates that there are diminising returns of progressive transfers as we move to less needy individuals for given levels of income.[14,15]

Let $G_k(z; \boldsymbol{y})$ denote the value of $G(z; \boldsymbol{y})$ when $F(z; \boldsymbol{y})$ is replaced by $F_k(z; \boldsymbol{y})$ in equation (9). That function indicates the value of the average income gap among individuals of the k-th type for a given income poverty line z. Using the following notation:

$$\Delta^{\gamma}_{t,t+1} G_k(z; \boldsymbol{y}) := q_k(\boldsymbol{x}_{t+1}) G_k(z; \boldsymbol{y}_{t+1}) - q_k\big(\gamma_x(\boldsymbol{x}_t)\big) G_k\big(z; \gamma_y(\boldsymbol{y}_t)\big), \tag{27}$$

we get the "pro-poorness" condition expressed in proposition 4 when poverty measures of the class $\bar{\Pi}_2$ are considered.

Proposition 4. *For a given counterfactual scenario Γ and a given vector $\boldsymbol{z}^+$ of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice*

[14]As emphasized in Lambert and Ramos (2002), it is worth mentionning that a class of poverty measures that belongs to $\bar{\Pi}_1$ and includes $\bar{\Pi}_2$ can also be used if only the non-concavity of θ is assumed. It is then necessary to turn to the sequential dominance criterion proposed by Bourguignon (1989) to get a robust evaluation of growth "pro-poorness" using that intermediate class of poverty measures.

[15]For a discussion on that generalization of the Pigou-Dalton transfer principle, see in particular Ebert (2000).

of poverty measure among the family $\bar{\Pi}_2$ if and only if:

$$\sum_{k=1}^{j} \Delta^{\gamma}_{t,t+1} G_k(z;\boldsymbol{y}) \leqslant 0 \quad \forall z \leqslant z^{+}_{j},\, j \in \{1,\dots,K\}, \tag{28}$$

with a least one integer $j^* \in \{1,\dots,K\}$ and one value $z^* \in [0, z^{+}_{j^*}]$ such that:

$$\sum_{k=1}^{j^*} \Delta^{\gamma}_{t,t+1} G_k(z^*;\boldsymbol{y}) < 0. \tag{29}$$

As in the previous section, it may be interesting to look for an alternative way of expressing proposition 4 when the marginal distributions $\boldsymbol{x}_{t+1}$ and $\gamma_x(\boldsymbol{x_t})$ do not differ. Let $g_{2,k}$ be the k-PGC, that is the function that returns the mean growth rate of the bottom p percents of the subpopulation of type 1 to k considering the non-income attribute, *i.e.*:

$$g_{2,k}(p;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) := \int_0^p \frac{F_k^{-1}(u;\boldsymbol{X}_{t+1})}{F_k^{-1}(u;\boldsymbol{X}_t)} - 1\, du. \tag{30}$$

Corollary 4. *For a given counterfactual scenario γ and a given vector $\boldsymbol{z}^+$ of maximum values for the specific poverty lines, the statement that the growth pattern observed between years t and $t+1$ is "pro-poor" is weakly robust with respect to the choice of poverty measure among the family $\bar{\Pi}_2$ if and only if:*

$$g_{2,k}(p;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) - g_{2,k}\big(p;\Gamma(\boldsymbol{X}_t),\boldsymbol{X}_{t+1}\big) \geqslant 0 \quad \forall p \leqslant F(z^{+}_{k},\boldsymbol{y}^{k}_{t}), k \in \{1,\dots,K\}, \tag{31}$$

with a least one integer $j^* \in \{1,\dots,K\}$ and one value $p^* \in \left[0, F(z^{+}_{j^*},\boldsymbol{y}^{j^*}_{t})\right]$ such that:

$$g_{2,j^*}(p^*;\boldsymbol{X}_t,\boldsymbol{X}_{t+1}) - g_{2,j^*}\big(p^*;\Gamma(\boldsymbol{X}_t),\boldsymbol{X}_{t+1}\big) > 0. \tag{32}$$

4 ILLUSTRATION : TESTING THE PRO-POORNESS OF GROWTH IN TURKEY 2003-2005

The proposed methodology is now applied using data from the 2003, 2004 and 2005 Turkish household consumption and expenditure surveys (HICES) provided by the Turkish Statistics Institute (Turkstat). Turkey is an interesting case over which to test the pro-poorness of growth. After the 2001 crisis, Turkey entered a period of high growth and structural transformations. Following a rebound in 2001, annual growth rates averaged nearly 7% over the years 2003-2007. According to the international standards, poverty is low in comparison with other MENA countries, but inequalities remain high and are to a large extent driven by high differentials across regions. Moreover, despite improvements in social indicators, education records pretty and weak levels in comparisons with countries with equivalent levels of GDP per capita (Akkoyunlu-Wigley, 2008). The country also faces wide income and education gaps between urban and rural areas (World Bank, 2005 and 2008) and education seems to hold an important role in understanding discrepancies of development within the country (Duman, 2008).

In order to illustrate the usefulness of our methodology, poverty is here defined using education alongside the more traditional income component. The income component corresponds to the disposable equivalent individual income adjusted by the OECD equivalence scale. In order to take inflation into account, all incomes are expressed in reference to the 2003 consumer price index provided by Turkstat. Education deprivations are measured on the basis of education level attainments. Our datasets allow the distinction between the following six categories: illiterate, literate but without completing school, primary school, primary education, secondary education and occupational education equal to secondary school, high school and higher studies. Since children have not achieved their final educational level, the analysis focuses on the adult population (older than 20 years). In the spirit of our framework, we put the reasonable assumptions that well-being is an increasing function of education attainments and that income improves well-being the more at low educational levels. Consequently, each sample has been split into six groups of educational levels ranked by decreasing needs with respect to income: illiterate persons are thus associated with highest needs and high school and higher studies with the lowest ones. Regarding the pro-poorness of growth, the illustrations rely on a very traditional counterfactual scenario, that is a relative approach of pro-poor growth for the income dimension ($\gamma_y(\boldsymbol{y}_t) = \gamma^r(\boldsymbol{y}_t) = \boldsymbol{y}_t\frac{\mu_{t+1}}{\mu_t}$) while using just the observed changes for education ($\gamma_x(\boldsymbol{x}_t) = \boldsymbol{x}_t$).

In section 3, we mentionned that the classes of bidimensionnal poverty measures used for "pro-poorness" checks implies the definition of different monetary poverty lines for each value of the non-monetary attributes. For the sake of simplicity, we define a general income poverty line expressed as some percentage of the median income as usually done for the analysis of poverty in OECD countries, and consider that this poverty line is appropriate for the least deprived group regarding education. More precisely, as stochastic dominance tests are designed to assess the robustness of poverty comparisons to the level of the poverty line, we have opted for a very conservative maximum value of that income poverty line, that is 90% of the median income for the whole population. With the choice of a strictly positive value of the poverty line for persons with high education endowments, we have assumed that improving the education level of any individual should raise its level of well-being in a significant manner but never results in a move out of poverty.

For the remaining educational groups, instead of choosing some particular values, we prefered leaving that issue unanswered *a priori* since one can hardly conclude how important should be the income level of a poorly educated individual so as to escape poverty regarding the standard for a well-educated person. Nethertheless, we still required these specific income poverty lines to be never inferior to the ones corresponding to better educated groups ($z_k \geqslant z_{k+1}\ \forall k = 1, \ldots, K-1$ refering to the eduction level). Agregating sequentially the population by the educational level attainment, we then have estimated each time the income level $\hat{z}_k^+$ such that the sign of the dominance curve changed and considered this value as a maximum for the definition of the poverty frontier such that the “pro-poor" judgement still holded. As a consequence, we let the data show what was the bound for the poverty domain (the “critical set" in Duclos, Sahn, and Younger, 2006). If the poverty line chosen for the better educated subpopulation was included in that set ($z_6 \leqslant \hat{z}_k\ \forall k = 1, \ldots, 6$), we did not reject the possibility of accepting a “pro-poor" judgement, but added that the result was valid only for sets of poverty lines below the critical poverty frontier ($z_k \leqslant \hat{z}_k\ \forall k = 1, \ldots, 6$).

From the several comparisons carried out using the data from the three household surveys mentioned above, we extract three cases that allow highlighting the relevancy of our methodology.

Our first illustration is related to the contrast between the urban and rural areas in Turkey considering the 2003-2004 growth spell. For both populations, we used the first order sequential dominance procedure so that our results refer to the very inclusive class of multidimensional poverty measures $\bar{\Pi}_1$. As shown in section 3.1, the sequentiel first order stochastic dominance procedure consists in comparing the value of the multidimensional headcount index, *i.e.* the share of the population which income and education level attainment are less than the chosen values, for each couple of income and education levels included in the poverty domain. As the non-income is described by a discrete variable, the most appropriate approch consists in beginning with the susbset of illiterate individuals and to estimate the difference in the share of person with income less than a given value between the final distribution and the counterfactual distribution up to the value where the sign of that difference changes. Then, if that value if consistent with the income poverty line chosen for that group, that is any value above the income poverty line for the highly educated persons in our setting, we then can add the set of individuals belonging to the second education group and then perform our comparison of multidimensional headcount indices along the range of income values and so on.

In practice it may be desirable first to perform the more traditional first-order stochatic dominance procedure for the whole population and then to contrats the results with our sequential procedure. In figure 2, the difference in the headcount index between the final and the counterfactual distributions is depicted by the thick continuous curve for the whole population. We can observe that the traditional first-order stochastic dominance was satisfied for both the urban (figure 2a) and the rural (figure 2b) areas between 2003 and 2004 since the corresponding curves are in both cases below the horizontal axis up to the income poverty line (represented by the dashed vertical line). Consequently, with a traditional monetary approach of poverty, we can conclude that growth has been “pro-poor," in the relative sense, in the urban and rual areas.

(a) Urban population. (b) Rural population.

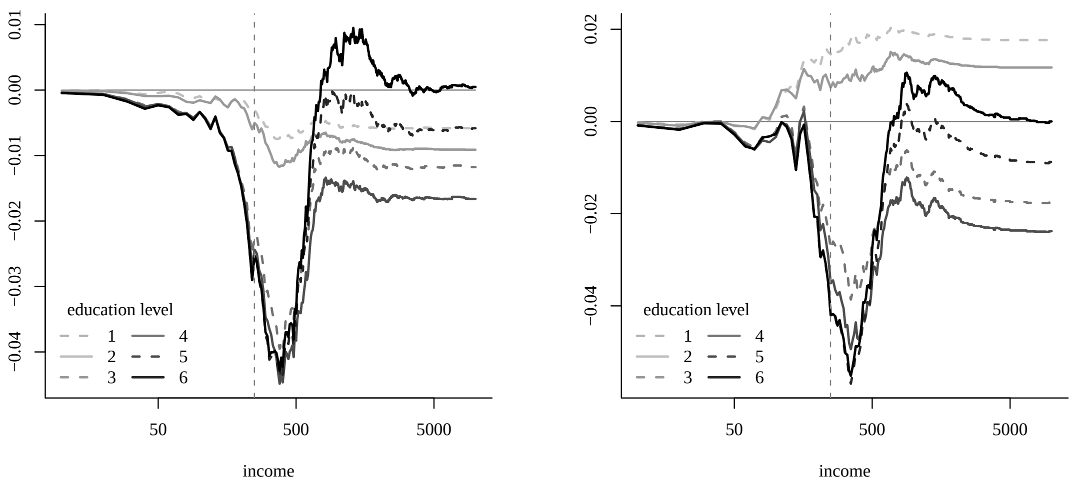

Figure 2: Pro-poor growth check : first order sequential dominance checks for Turkey, 2003-2004, urban and rural populations.

However, the picture becomes slightly different once we turn to multidimensional poverty with the inclusion of the education dimension. Considering the urban area, each curve is below zero for the bottom part of the income range and the estimated values $\hat{z}_k^+$ are always greater than the value chosen for the highly educated individuals, so that the classical "pro-poor" result is confirmed by the multidimensional analysis. For the rural area, relying solely on the standard monetary analysis without putting some emphasis on the poorly educated households would lead to the wrong conclusion that the growth pattern was biased in favor of the neediest between 2003 and 2004. Indeed, focusing on the first two groups of education attainment (the continuous and dashed light gray curves in figure 2b) show that the share of low income and low education individuals has not deacreased as much as it would have been the case with a "neutral" growth pattern during the period.

The second illustration is related to the usefulness of a second-order sequential stochastic dominance check. Looking at figure 3a, it can be seen that we cannot conclude wheter growth was "pro-poor" in a robust manner considering the whole Turkish population for the 2004-2005 growth spell since dominance curves are sometime above and sometime below zero for income levels below the income poverty line corresponding to the highly educated households. However, since the curves are above that level for the very bottom part of the income range, it may be interesting to focus on the more limited set $\bar{\Pi}_2$ of distribution-sensitive multidimensional poverty measures and consequently to turn to the second-order sequential stochastic dominance procedure. Contrary to the first-order procedure, the second-order one relies on the use of the

(a) First order dominance.

(b) Second order dominance.

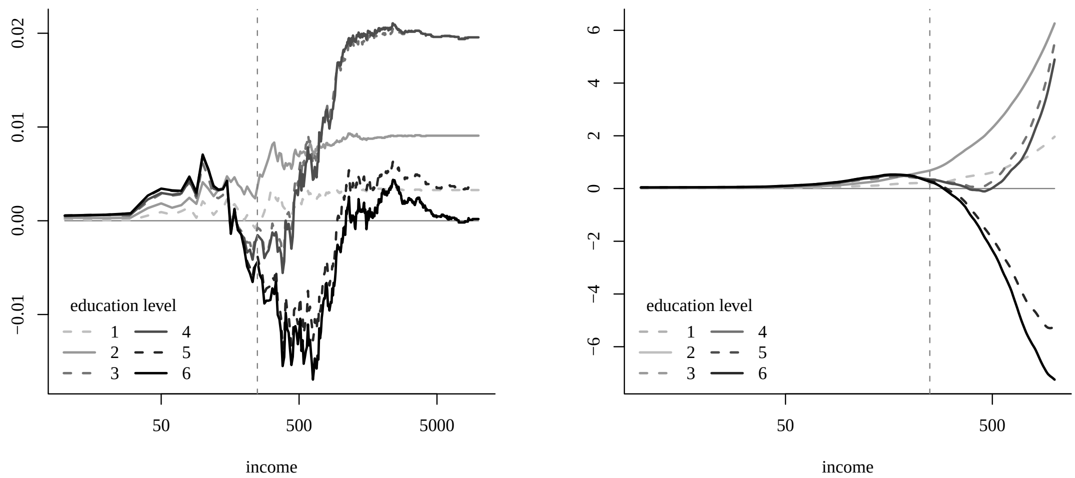

Figure 3: Pro-poor growth check : first and second order sequential dominance for Turkey, 2004-2005, whole population.

income gaps, that is the extent of income shortfalls with respect to the poverty line times the value of the corresponding multidimensionnal headcount index.

The results, plotted in figure 3b show that the joint distribution of education and income in 2005 is dominated by the corresponding counterfactual based on the 2004 distribution up to some admissible poverty frontier. In other words, we can conclude in a robust manner that growth can be deemed “anti-poor", in the relative sense, in Turkey during the period 2004-2005 considering indices within the set of distribution-sensitive multidimensional poverty measures. It is worth noting that the main results are similar to the one obtained with the traditionnal first- and second-order dominance checks in that case (*c.f.* the thick black curves in figure 3a and 3b), but our approach yields more informations on the distribution of the economic growth “cake" as it shows unambiguous welfare improvement at the firt order for the groups of illiterate and literate but without completing school individuals.

5 CONCLUDING REMARKS

In this paper, we have proposed to extend the use of sequential stochastic dominance techniques in order to assess robust judgments of the “pro-poorness” of growth within the framework of multidimensional approach to poverty measurement. Indeed the traditional tools used to check for “pro-poor” growth focus on the sole monetary aspect of poverty. As is well-known, the inclusion of other dimensions of poverty induces a change in the poverty definition. In particular, the anonymity axiom that monetary index of poverty should satisfy is not always true and is not ethically any more acceptable. Here, we propose to use the sequential dominance procedures suggested by Bourguignon (1989) and developed by many authors like Atkinson (1992) and Jenkins and Lambert (1993) in order to define first-order and second-order dominance criteria that make it possible to assess weak robustness of "pro-poor" growth in income as well as in other well-being attributes for a class of poverty measures and a wide range of poverty lines.

Unlike to the traditional studies using sequential stochastic dominance, the heterogeneity of the population is not defined on the basis of the households sizes and compositions. On the contrary, individuals' needs differ according to non-income attributes as in the study of Duclos and Échevin (2009) for poverty measurement. Unlike the attempt made by Grosse, Harttgen, and Klasen (2008) who extend Chen and Ravallion's (2003) growth incidence curve (GIC) to non-monetary dimensions of poverty, our methodology takes into account the changes in the joint distribution of the well-being attributes. For this purpose, it only adds two weak conditions to the traditional mathematical conditions used for unidimensional poverty measurement. The first one is that the income poverty line does not increase with the level of the non-monetary indicator. The second one imposes the marginal contribution of income to well-being to decrease with the level of non-income attributes. As a special case of our approach, it is possible to define the equivalence of GIC and PGC curves, named k-GIC and k-PGC curves, that are based on partial quantile functions and may be used to get robust conclusions when the marginal distribution of the non-monetary attribute is left unchanged. It is worth noting that the use of these curves can be extended so as to take changes in the distribution of the non-income attributes into account. Finally, though the social evaluator has more latitude to define the counterfactual situation in order to make judgments of the "pro-poorness" of growth, the definition of that counterfactual is challenging from an empirical point of view as it entails considering the relationships between income and the non-income attributes. Our feeling is that this issue should be a matter of scrutiny for further empirical studies.

A Proof of propositions and corollaries 3 to 4

A.1 First-order stochastic sequential dominance

The proof proposition 3 is similar to Jenkins and Lambert (1993, proposition 1). We first introduce the following notation:

$$\Delta^{\gamma}_{t,t+1} f_k(z, \boldsymbol{y}) := q_k(\boldsymbol{x}_{t+1}) f_k(z, \boldsymbol{y}_{t+1}) - q_k\big(\gamma_x(\boldsymbol{x}_t)\big) f_k\big(z, \gamma_y(\boldsymbol{y}_t)\big), \tag{33}$$

with f being the density function, that is the first-order derivative of F. Using equation (15) and integrating by parts yields:

$$\Theta(\boldsymbol{X}_{t+1}, \boldsymbol{z}) - \Theta\big(\Gamma(\boldsymbol{X}_t), \boldsymbol{z}\big) = \sum_{k=1}^{K} \int_0^{z_k} \theta(y, x, z_k) \Delta_{t,t+1}^{\gamma} f_k(y; \boldsymbol{y})\, dy, \tag{34}$$

$$= \sum_{k=1}^{K} \Big[\theta(y, x, z_k) \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y})\Big]_0^{z_k}$$

$$- \sum_{k=1}^{K} \int_0^{z_k} \frac{\partial \theta(y, x, z_k)}{\partial y} \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y})\, dy. \tag{35}$$

By definition $\Delta_{t,t+1}^{\gamma} F_k(0; \boldsymbol{y}) = 0$. Since, by assumption, the class of poverty measures satisfies $\theta\big(z_k, z_k\big) = 0$, the first term in the last expression can be dropped. Using the additive properties of integral calculus, the second term can be expressed as:

$$\sum_{k=1}^{K} \int_0^{z_k} \frac{\partial \theta(y, x, z_k)}{\partial y} \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y})\, dy = \int_0^{z_1} \sum_{k=1}^{K} \frac{\partial \theta(y, x, z_k)}{\partial y} \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y})\, dy. \tag{36}$$

Moreover, it can easily be shown that $\sum_{k=1}^{K} a_k b_k = a_K \sum_{k=1}^{K} b_k + (a_{K-1} - a_K) \sum_{k=1}^{K-1} b_k + \cdots + (a_1 - a_2) b_1 = a_K \sum_{k=1}^{K} b_k + \sum_{j=1}^{K-1} \Big((a_j + a_{j+1}) \sum_{k=1}^{j} b_k\Big)$. Noting $h_j(y, x, z_j) = \frac{\partial \theta(y,x,z_j)}{\partial y} - \frac{\partial \theta(y,x,z_{j+1})}{\partial y}$, the right-hand term of equation (36) can then be rewritten as:

$$\int_0^{z_1} \left(\frac{\partial \theta(y, x^K, z_K)}{\partial y} \sum_{k=1}^{K} \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y}) + \sum_{j=1}^{K-1} \left(h_j(y, x, z_j) \sum_{k=1}^{j} \Delta_{t,t+1}^{\gamma} F_k(y; \boldsymbol{y}) \right) \right) dy. \tag{37}$$

By assumption, both $\frac{\partial \theta(y,x,z_K)}{\partial y} \leqslant 0$ and $h_j(y, x, z_j) \leqslant 0$, $\forall j \in \{1, \ldots, K-1\}$. A sufficient condition for $\Theta(\boldsymbol{X}_{t+1}, \boldsymbol{z}) - \Theta\big(\Gamma(\boldsymbol{X}_t), \boldsymbol{z}\big)$ to be negative is then:

$$\sum_{k=1}^{j} \Delta_{t,t+1}^{\gamma} F_k(y, \boldsymbol{y}) \leqslant 0 \quad \forall y \leqslant z_j,\ j \in \{1, \ldots, K\}. \tag{38}$$

For the necessary part of the proof, see Chambaz and Maurin (1998).

Concerning corollary 3, it can easily be seen that the left-hand term in equation (38) can be rewritten as :

$$\sum_{k=1}^{j} \Delta_{t,t+1}^{\gamma} F_k(y, \boldsymbol{y}) = \sum_{k=1}^{j} q_k(\boldsymbol{x}_{t+1}) \Big(F_k(z; \boldsymbol{y}_{t+1}) - F_k\big(z; \gamma_y(\boldsymbol{y}_t)\big) \Big), \tag{39}$$

if we observe $q_k(\boldsymbol{x}_{t+1}) = q_k\big(\gamma(\boldsymbol{x}_t)\big)$. Multiplying each term in (39) by $\sum_{k=1}^{j} q_k(\boldsymbol{x}_{t+1})$, the second term simply becomes $F(z; \boldsymbol{y}_{t+1}^j) - F\big(z; \gamma_y(\boldsymbol{y}_t^j)\big)$. Condition (38), can then be rewritten as:

$$F(z; \boldsymbol{y}_{t+1}^j) \leqslant F\big(z; \gamma_y(\boldsymbol{y}_t^j)\big) \quad \forall y \leqslant z_j,\ j \in \{1, \ldots, K\}, \tag{40}$$

that, using definition (21), can also be expressed in the following manner:

$$F_j^{-1}(p;\boldsymbol{y}_{t+1}^j) \geqslant F_j^{-1}\big(p;\gamma_y(\boldsymbol{y}_t^j)\big) \quad \forall p \leqslant F(z_j;\boldsymbol{y}_t^j),\ j \in \{1,\dots,K\}. \tag{41}$$

Dividing each term of equation (41) by $F_j^{-1}(p;\boldsymbol{y}_t^j)$ and substracting one yields the comparison of the k-GIC used for corollary 3.

A.2 Second-order stochastic sequential dominance

Regarding proposition 4, the proof now refers to Chambaz and Maurin (1998, proposition 4). Let Ξ denote the last term in equation (35). Integrating this expression by parts then yields:

$$\begin{aligned}
\Xi &= -\sum_{k=1}^{K}\int_0^{z_k}\frac{\partial\theta(y,x,z_k)}{\partial y}\Delta_{t,t+1}^{\gamma}F_k(y;\boldsymbol{y})\,dy, &(42)\\
&= \sum_{k=1}^{K}\left(\int_0^{z_k}\frac{\partial^2\theta(y,x,z_k)}{\partial y^2}\Delta_{t,t+1}^{\gamma}G_k(y;\boldsymbol{y})\,dy - \left[\frac{\partial\theta(y,x,z_k)}{\partial y}\Delta_{t,t+1}^{\gamma}G_k(y;\boldsymbol{y}_t)\right]_0^{z_k}\right), &(43)\\
&= \sum_{k=1}^{K}\left(\int_0^{z_k}\frac{\partial^2\theta(y,x,z_k)}{\partial y^2}\Delta_{t,t+1}^{\gamma}G_k(y;\boldsymbol{y})\,dy - \frac{\partial}{\partial y}\theta(z_k,x,z_k)\Delta_{t,t+1}^{\gamma}G_k(z_k;\boldsymbol{y}_t)\right). &(44)
\end{aligned}$$

Using the same manipulations as for equations (36) and (37), we then obtain:

$$\begin{aligned}
\Xi = &\int_0^{z_1}\left(\frac{\partial^2\theta(y,x^K,z_K)}{\partial y^2}\sum_{k=1}^{K}\Delta_{t,t+1}^{\gamma}G_k(y;\boldsymbol{y}) + \sum_{j=1}^{K-1}\left(\frac{\partial}{\partial y}h_j(y,x,z_j)\sum_{k=1}^{j}\Delta_{t,t+1}^{\gamma}G_k(y;\boldsymbol{y})\right)\right)dy\\
&-\left(\frac{\partial}{\partial y}\theta(z_K,x^K,z_K)\sum_{k=1}^{K}\Delta_{t,t+1}^{\gamma}G_k(z_k;\boldsymbol{y}) + \sum_{j=1}^{K-1}\left(h_j(z_j,x,z_j)\sum_{k=1}^{j}\Delta_{t,t+1}^{\gamma}G_k(z_k;\boldsymbol{y})\right)\right)
\end{aligned} \tag{45}$$

By assumption, the class $\frac{\partial\theta}{\partial y} \leqslant 0$, $\frac{\partial^2\theta}{\partial y^2} \geqslant 0$, $h_j(y,x,z_j) \leqslant 0$, and $\frac{\partial}{\partial y}h_j(y,x,z_j) \geqslant 0$ forall poverty measure $\Theta \in \bar{\Pi}_2$. It can then be easily seen that a sufficient condition for $\Theta(\boldsymbol{X}_{t+1},\boldsymbol{z}) - \Theta\big(\Gamma(\boldsymbol{X}_t),\boldsymbol{z}\big)$ to be negative is then:

$$\sum_{k=1}^{j}\Delta_{t,t+1}^{\gamma}G_k(y,\boldsymbol{y}) \leqslant 0 \quad \forall y \leqslant z_j,\ j \in \{1,\dots,K\}. \tag{46}$$

For the necessary part of the proof, see Chambaz and Maurin (1998).

Regarding corollary 4, the left-hand term in equation (46) can be rewritten as :

$$\sum_{k=1}^{j}\Delta_{t,t+1}^{\gamma}G_k(y,\boldsymbol{y}) = \sum_{k=1}^{j}q_k(\boldsymbol{x}_{t+1})\Big(G_k(z;\boldsymbol{y}_{t+1}) - G_k\big(z;\gamma_y(\boldsymbol{y}_t)\big)\Big), \tag{47}$$

if $q_k(\boldsymbol{x}_{t+1}) = q_k\big(\gamma(\boldsymbol{x}_t)\big)$ holds $\forall k \in \{1, \ldots, j*\}$. Multiplying each term in (39) by $\sum_{k=1}^{j} q_k(\boldsymbol{x}_{t+1})$, the second term simply becomes $G(z; \boldsymbol{y}_{t+1}^j) - G\big(z; \gamma_y(\boldsymbol{y}_t^j)\big)$. Condition (46), can then be rewritten as:

$$G(z; \boldsymbol{y}_{t+1}^j) \leqslant G\big(z; \gamma_y(\boldsymbol{y}_t^j)\big) \quad \forall y \leqslant z_j,\, j \in \{1, \ldots, K\}. \tag{48}$$

Moreover, integrating $G(z; \boldsymbol{y}^j)$ by parts yields:

$$G(z; \boldsymbol{y}^j) = \int_0^z (z - y)\, dF(y; \boldsymbol{y}^j), \tag{49}$$

$$= \int_0^z F(y; \boldsymbol{y}^j)\, dy. \tag{50}$$

Pluging (50) in relation (48) and inverting F, we get:

$$\int_0^p F^{-1}(u; \boldsymbol{y}_{t+1}^j)\, du \geqslant \int_0^p F^{-1}\big(u; \gamma_y(\boldsymbol{y}_t^j)\big)\, du \quad \forall p \leqslant F(z_j; \boldsymbol{y}_t),\, j \in \{1, \ldots, K\}. \tag{51}$$

Dividing each term of equation (51) by $\int_0^p F_j^{-1}(u; \boldsymbol{y}_t^j)\, du$ and substracting one yields the comparison of the k-PGC used for corollary 4.

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