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Measuring group disadvantage with indices based on relative distributions

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Measuring group disadvantage with indices based on relative distributions

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Abstract

A long literature on between-group inequality in Social Science and Statistics has developed statistical tools for measuring the extent of inequality of opportunity or, more narrowly, gender inequality. In this paper I propose a family of new indices that are sensitive to inequality between pairs of groups whenever that inequality implies relative disadvantage for a group of concern. The indices are based on cumulative relative distributions and the disadvantage is captured through indirectly measuring differences between the quantiles of the compared distributions. The indices are advocated to study topics like gender inequality. They are suitable for continuous variables; but, with a random transformation, the indices can also be applied to discrete variables. The indices are applied to study gender inequality in Chile over several dimensions of well-being. Gender differences turn out most detrimental to Chilean women in dimensions of earnings, dignity and life satisfaction.

Introduction

The concern for differences in the distribution of wellbeing characteristics among groups within societies has earned a long-standing interest in the Social Sciences and Political Philosophy. This concern has often emphasized the potential presence of socio-economic discrimination of different natures. (e.g. Becker, 1971; Phelps, 1972; Arrow, 1973) In general, it has been associated with concepts of inequality of opportunities.[1] The normative view for between-groups differences related to ethnicity or gender states that they are intrinsically unfair (particularly when the groups are defined over characteristics beyond the individuals' control), and instrumentally detrimental to individuals and societies (e.g. Arneson, 1989; Cohen, 1989; Nussbaum and Glover, 1995; Roemer, 1998; Fleurbaey, 2001; Sen, 2001).

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[1]For a good review of the main conceptual issues and the literature on inequality of opportunity see Fleurbaey (2008). Also Roemer (1998).

From a quantitative perspective, one way of measuring the extent of differences in well-being between groups is to use statistical indicators that capture between-group inequalities and that declare the total absence of between-group inequality if and only if the conditional distributions of wellbeing are identical across groups.[2] There is also an interest in quantifying between-group inequalities with a focus on capturing inequality if and when it is detrimental to one specific group and not to other(s). Gastwirth (1975), Butler and McDonald (1987) and Dagum (1987) provide some examples of the latter. The literature on multidimensional gender inequality indices features some recent examples of indices that are insensitive to inequalities whenever they are detrimental to women (e.g. Hausmann, Tyson, and Zahidi, 2007; Permanyer, 2009).

I propose new indices of between-group inequality that have a "focus axiom", i.e. that are sensitive only to inequality whenever it renders a specific group at disadvantage with respect to other(s). These indices are based on the theory of relative distributions (e.g. see Handcock and Morris, 1999) and are defined, in principle to compare two groups over one dimension of well-being. Extensions to compare one group against several others and two groups over several dimensions are also proposed. Unlike the existing indices that focus on one group's disadvantage, the indices proposed in this paper do not rely on one standard of the distribution (usually the mean). They rather draw information from the whole conditional distributions of well-being. The indices are most suitable for continuous variables but they can also be applied to multinomial, discrete variables using a standard random transformation. The indices are based on cumulative relative distributions and the disadvantage is captured indirectly through differences between the quantiles of the distributions. Another interesting trait is that the extreme values of the indices are related to well-established concepts of distributional comparisons such as first-order stochastic dominance and relative overlap of distributions.

In the next section I introduce the basic indices after a brief introduction to cumulative relative distributions. A subsection describes the application of these indices to discrete variables. Then a second subsection proposes relative versions of the indices. The third subsection suggests ways of combining the indices in order to perform comparisons involving several groups or several dimensions of wellbeing. The following section sketches out the asymptotic distribution of some of the indices, which is useful to perform inference with analytical standard errors and relatively large sample sizes, as an alternative to bootstrapping methods. The fourth section is devoted to a comparative discussion of the indices vis-a-vis other indices in the literature of inequality indices based on specific group disadvantage. The fifth section provides an empirical application to gender inequality in Chile. The application bears special interest since, in 2009, OPHI carried out an addendum to the CASEN Chilean household survey featuring special modules for quality of employment, agency and empow-

[2] This condition is consistent with a literalist definition of inequality of opportunity by Roemer (1998, p. 15-6) combining his assumption of charity with Fleurbaey's notion of equal well-being for equal responsibility (Fleurbaey, 2008, p. 25). It is also consistent with Fleurbaey's concept of circumstance neutralization (*ibid.*, p. 26). There are alternative ways of measuring between-group inequality. For instance, it could be measured as the residual inequality after within-group inequality has been suppressed (e.g. by replacing individual's wellbeing values with those of their group mean). Such approach has been followed, among others, by Roemer (2006); Elbers, Lanjouw, Mistiaen, and Ozler (2008); Ferreira and Gignoux (2008); Lanjouw and Rao (2008).

erment, physical security, dignity and life satisfaction and subjective wellbeing. The indices are helpful in showing that the most prominent inequalities detrimental to Chilean women appear in the areas of income and earnings, dignity and life satisfaction (and subjective wellbeing). By contrast areas like security or employment either do not exhibit inequality detrimental to women or do not show any between-group inequality at all. The paper finishes with concluding remarks.

Indices of group disadvantage based on cumulative relative distributions

The notion of relative distributions, and its derived statistical tools, is several decades old.[3] For the indices of this paper the relevant concept is that of cumulative relative distributions (CRD). CRDs map the proportion of a so-called *reference distribution* into the proportion of a so-called *compared distribution*. Let F_B be the cumulative density function (CDF) of reference group B and F_A be the CDF of the compared group A. Then the cumulative relative density of A, compared to B is:

$$G_{A/B}(r) \equiv F_A\left(F_B^{-1}(r)\right), \qquad 0 \leq r \leq 1,$$

where r is the proportion of individuals of group B who have a value of the wellbeing attribute not higher than $F_B^{-1}(r)$. Notice that:

$$\frac{\partial G_{A/B}(r)}{\partial r} = \frac{\frac{\partial G_{A/B}(r)}{\partial y}}{\frac{\partial r}{\partial y}} = \frac{f_A\left(F_B^{-1}(r)\right)}{f_B\left(F_B^{-1}(r)\right)}.$$

The derivative of G is equal to the ratio of marginal densities of A over B, hence it is positive. However, unlike the case of the Lorenz Curve, the CRD curve is neither convex nor concave a priori. Several indices and statistical tools based on these distributions have been proposed. Le Breton, Michelangeli, and Peluso (2008) propose several of these measures. An index derived from one of their families, which is relevant for this paper is the following average absolute distance (AAD) indicator:

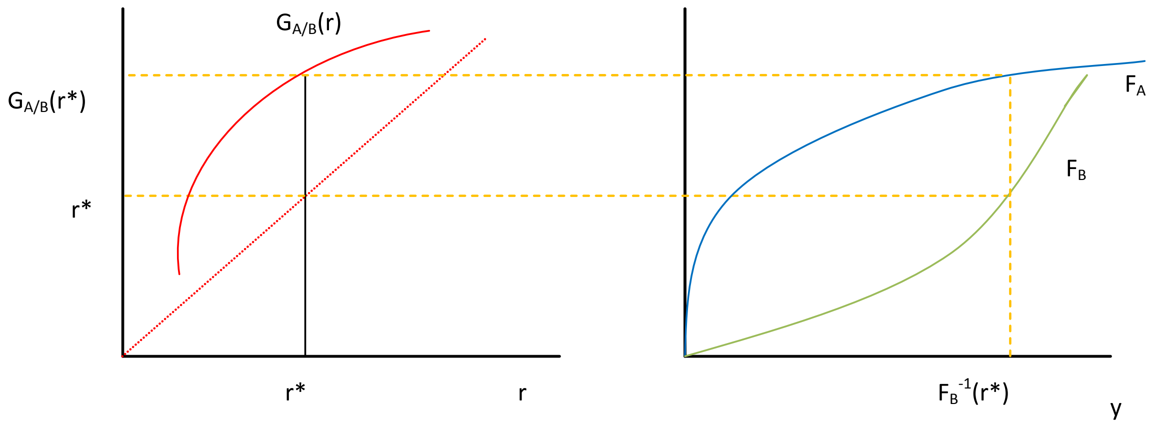
$$AAD = 2\int_0^1 \left|G_{A/B}(r) - r\right| dr.^4 \qquad (1)$$

Indices like the AAD capture indirectly the dissimilarity between F_A and F_B, as is apparent from Figure (1).

[3] For a brief history see Handcock and Morris (1999, Chapter 2).

[4] In practice estimating an index like the AAD requires choosing a number of percentile proportions, r, for comparisons. For instance, with N_p equally spaced proportions, the empirical estimation of AAD is: $\frac{2}{N_p}\sum_{r=1/N_p}^{1}\left|G_{A/B}(r) - r\right|$. The choice of number of proportion may affect the value of the indicator. This requires robustness analysis, involving the the computation of the indices with several choices of numbers of proportions. In the empirical application below I show that the indices do not vary significantly with different such choices.

Figure 1: Correspondence between differences in cumulative distributions and differences between the CRD and the egalitarian relative distribution



That is: $AAD \propto \int_{F_B^{-1}(0)}^{F_B^{-1}(1)} |F_A(y) - F_B(y)|\, dy$. An index like AAD is useful to measure the degree of dissimilarity between the two distributions. However it is not informative as to whether this dissimilarity favours any group in particular. The assessment of detrimental dissimilarity, using CRDs, can be performed instead with the new indices proposed in this paper.The new indices are the following:

$$I^{\alpha}_{A/B} \equiv (\alpha+1)\int_0^1 \left(G_{A/B}(r) - r\right)^{\alpha} I\left(G_{A/B}(r) \geq r\right) dr = (\alpha+1)\int_0^1 \left(G_{A/B}(r) - r\right)^{\alpha}_{+} dr, \;\; \alpha \in \mathbb{N}, \quad (2)$$

where I is an indicator function equal to one if the statement in parenthesis is true; otherwise, it is equal to zero. The indices $I^{\alpha}_{A/B}$ are only sensitive to gaps in percentile proportions when $G_{A/B}(r) \geq r$. They compare indirectly the quantiles of the two distributions, i.e. $F_A^{-1}(r)$ against $F_B^{-1}(r)$. In this paper the focus is on $I^0_{A/B}$ and $I^1_{A/B}$. Different values of α lead to different interpretations for these comparisons. For instance, $I^0_{A/B}$ measures the proportion of quantiles in A that are lower tha B's. If A and B have the same population it gives the percentage of ranked people poorer in A than in B.[5] Whenever $I^0_{A/B} = 1$ the distribution of B first-order stochastically dominates A's. This extreme represents maximum disadvantage (against group A) for $\alpha = 0$. On the other extreme, whenever $I^0_{A/B} = 0$ either both distributions are identical or A first-order stocastically dominates B. Such situation reflects null disadvantage against group A. Discriminating between these two options, whenever $I^0_{A/B} = 0$, is easy: Estimate $G_{B/A}(r)$ and then compute $I^0_{B/A}$. If $I^0_{B/A} = I^0_{A/B} = 0$ then both distributions are identical (and viceversa). If $I^0_{B/A} > I^0_{A/B} = 0$ then A first-order stocastically dominantes B (and viceversa).

I^1 measures indirectly the gaps between the quantiles of the two distributions. To see

[5] That is, the poorest person in A is compared against the poorest person in B and so on until the richest in each group.

this notice that for a given range of proportions of B, $[\underline{r}, \overline{r}]$, such that $G_{A/B}(r) \geq r$:

$$\int_{\underline{r}}^{\overline{r}} \left(G_{A/B}(r) - r\right)_+ dr \propto \int_{F_B^{-1}(\underline{r})}^{F_B^{-1}(\overline{r})} \left[F_A(y) - F_B(y)\right]_+ dy,$$

And:

$$\int_{F_B^{-1}(\underline{r})}^{F_B^{-1}(\overline{r})} \left[F_A(y) - F_B(y)\right]_+ dy = \int_{\underline{r}}^{\overline{r}} \left(F_B^{-1}(r) - F_A^{-1}(r)\right)_+ dr.$$

When $I_{A/B}^1 = 0$, A is at a minimum (null) disadvantage vis-a-vis B. On the other extreme $I_{A/B}^1 = 1$ implies an absolute lack of overlap between the distributions of A and B such that the richest person in A is poorer than the poorest person in B (and viceversa). Interpretations become less straightforward for $\alpha \geq 2$. For instance, $I_{A/B}^2$ emphasizes the bigger gaps between $G_{A/B}(r)$ and r, by squaring them.[6] For different values of α the indices are related to each other via the following implications: $I_{A/B}^{\alpha} = 0 \leftrightarrow I^{\alpha+1} = 0$, $\forall \alpha \in \mathbb{N}_+$, which implies $0 < I_{A/B}^{\alpha} < 1 \leftrightarrow 0 < I^{\alpha+1} < 1$, $\forall \alpha \in \mathbb{N}_+$; and $I^{\alpha+1} = 1 \rightarrow I^{\alpha} = 1$, $\forall \alpha \in \mathbb{N}_+$.

The indices are normalized between 0 and 1.[7] The indices also satisfy properties of population replication invariance and ratio scale invariance. The former means that if the population of A is multiplied by λ_A ($\lambda_A \in \mathbb{R}_{++}$) and that of B is multiplied by λ_B ($\lambda_B \in \mathbb{R}_{++}$) then $G_{A/B}(r) = G_{A/B}(r; \lambda_A, \lambda_B)$. This is easily confirmed by looking at the empirical versions of $G_{A/B}(r; \lambda_A, \lambda_B)$ and $F_B^{-1}(r; \lambda_B)$, i.e. $\widehat{G}_{A/B}(r; \lambda_A, \lambda_B)$ and $\widehat{F}_B^{-1}(r; \lambda_B)$. $\widehat{F}_B^{-1}(r; \lambda_B) = \left[\min y \mid \widehat{F}_B(y; \lambda_B) \geq r\right]$. And $\widehat{F}_B(y; \lambda_B) \equiv \frac{1}{\lambda_B N_B} \sum_{i=1}^{N_B} \lambda_B I(y_i \leq y)_i = \frac{1}{N_B} \sum_{i=1}^{N_B} I(y_i \leq y)_i \equiv \widehat{F}_B(y)$. Therefore $\widehat{F}_B^{-1}(r; \lambda_B) = \widehat{F}_B^{-1}(r)$. Then $\widehat{G}_{A/B}(r; \lambda_A, \lambda_B) \equiv \frac{1}{\lambda_A N_A} \sum_{i=1}^{N_B} \lambda_A I\left(y_i \leq \widehat{F}_B^{-1}(r; \lambda_B)\right)_i = \widehat{G}_{A/B}(r)$.[8] Ratio scale invariance means that if all the outcome values, y_i, are multiplied by λ ($\lambda \in \mathbb{R}_{++}$) then $G_{A/B}(r) = G_{A/B}(r; \lambda)$. This result is also easily checked from the empirical formulas: $\widehat{F}_B\left(\widehat{F}_B^{-1}(r); \lambda\right) \equiv \frac{1}{N_B} \sum_{i=1}^{N_B} I\left(\lambda y_i \leq \lambda \widehat{F}_B^{-1}(r)\right)_i = \frac{1}{N_B} \sum_{i=1}^{N_B} I\left(y_i \leq \widehat{F}_B^{-1}(r)\right)_i \equiv \widehat{F}_B\left(\widehat{F}_B^{-1}(r)\right)$. Therefore $\widehat{F}_B^{-1}(r; \lambda) = \lambda \widehat{F}_B^{-1}(r)$ and then $\widehat{G}_{A/B}(r; \lambda) \equiv \frac{1}{N_A} \sum_{i=1}^{N_B} I\left(\lambda y_i \leq \lambda \widehat{F}_B^{-1}(r; \lambda_B)\right)_i = \widehat{G}_{A/B}(r)$.

For discrete variables

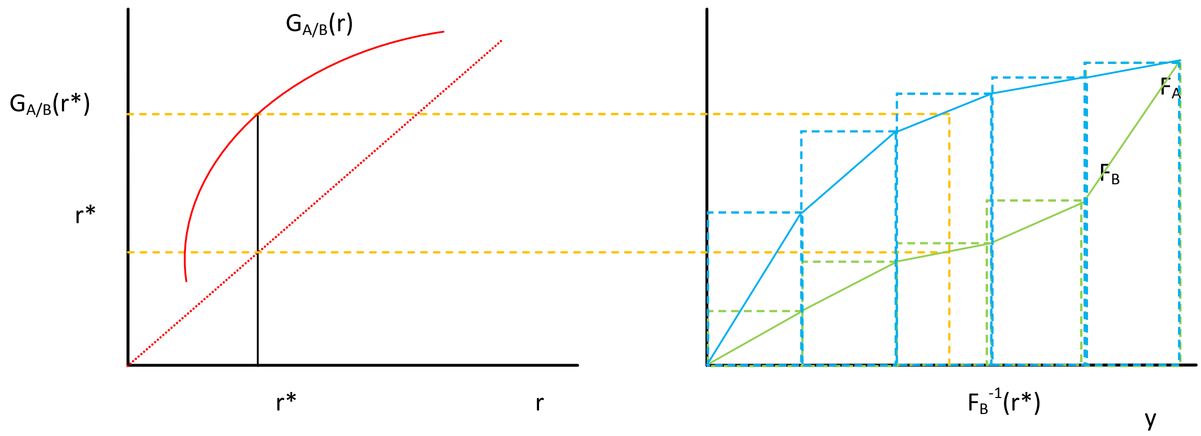
Handcock and Morris (1999) explain the estimation of relative distributions for discrete variables. They propose using a random transformation that attributes cumulative probability

[6] Like the P^2 measure of Foster, Greer, and Thorbecke (1984).

[7] It can not be negative since it only adds the gaps $G_{A/B}(r) - r$ only when $G_{A/B}(r) \geq r$; and it can not be higher than one because: $\int_0^1 (1-r)^{\alpha} dr = \frac{1}{\alpha+1}$, and $\alpha \in \mathbb{N}$.

[8] N_A and N_B stand for the population sizes of A and B. The indicator functions I are sub-indexed by i in order to highlight that they correspond to every individual, who in turn is being multiplied λ_A (or λ_B) times.

Figure 2: Mapping of differences between CDFs into a cumulative relative distribution for discrete variables using the uniform random transformation



mass to values in between those of the multinomially distributed, discrete variables according to the following rule:

$$F^d(x) = U\left[F(x_{i-1}), F(x_i)\right], \quad x_{i-1} \leq x \leq x_i, \quad i = 1, ..., Q,$$

where the number of multinomial categories is $Q+1$ and U denotes the uniform distribution. Therefore $F^d(x)$ takes a value from a uniform distribution bounded between $F(x_{i-1})$ and $F(x_i)$.[9] . With this transformation $G_{A/B}(r)$ is derived the following way:

$$G_{A/B}(r) = \left[r - F_B(x_{i-1})\right]\frac{p_A(x_i)}{p_B(x_i)} + F_A(x_{i-1}), \qquad F_B(x_{i-1}) \leq r \leq F_B(x_i), \quad i = 1, ..., Q, \tag{3}$$

where $p_A(x_i)$ and $p_B(x_i)$ are the probabilities of being in state x_i for the respective density functions of A and B. The derivation of $G_{A/B}(r)$ with discrete variables, under the uniform random transformation has the following graphical representation:

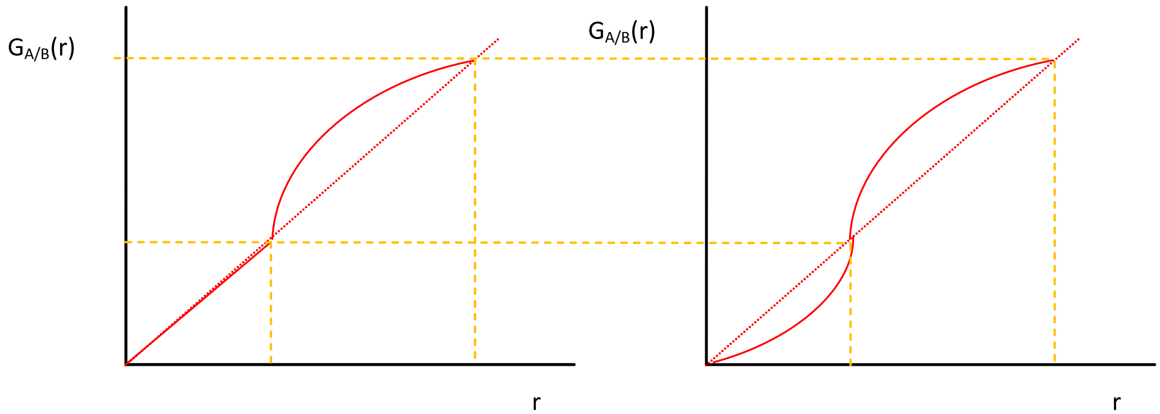
Using $G_{A/B}(r)$ from (3) the indices from (2) can be calculated for discrete variables.

Relative indices

Combinations of the indices in (2) with indices that capture dissimilarity without a focus on specific group disadvantage (e.g. the AAD) yield additional interesting information. For instance, defining $AAD^{\alpha} = (\alpha+1)\int_0^1 \left|G_{A/B}(r) - r\right|^{\alpha} dr$, the following family of relative indices:

[9] That is: $F^d(x) = F(x_{i-1}) + \frac{F(x_i)-F(x_{i-1})}{x_i - x_{i-1}}(x - x_{i-1}) = F(x_{i-1}) + \frac{p(x_i)}{x_i - x_{i-1}}(x - x_{i-1})$

Figure 3: Left panel: the two CDFs overlap in part of their common support. Right panel: the two CDFs cross once.



$$R^{\alpha}_{A/B} \equiv \frac{I^{\alpha}_{A/B}}{AAD^{\alpha}} = \frac{\int_0^1 \left(G_{A/B}\left(r\right) - r\right)^{\alpha}_{+} dr}{\int_0^1 \left|G_{A/B}\left(r\right) - r\right|^{\alpha} dr}, \ \alpha \in \mathbb{N}, \tag{4}$$

provide a measure of the proportion of the dissimilarity between the two distributions that is detrimental to group A. [10] For instance, a measure like R^{α} is helpful to compare the following two cases in Figure (3):

$I^{\alpha}_{A/B} \forall \alpha \in \mathbb{N}_+$ ranks the two cases equally. However the case on the left side of Figure (3) exhibits less distributional dissimilarity. All its dissimilarity is detrimental to group A, whereas in the case on the right side of Figure (3) part of the dissimilarity is detrimental also to group B. Hence $R^{\alpha} = 1 \ \forall \alpha \in \mathbb{N}_+$ for the case on the left side, whereas for the case on the right side: $0 < R^{\alpha} < 1 \ \forall \alpha \in \mathbb{N}_+$. Depending on the question of concern, e.g. detrimental dissimilarity for one group versus proportion of total dissimilarity detrimental to one group, it is most appropriate to rely on the rankings of $I^{\alpha}_{A/B}$ or $R^{\alpha}_{A/B}$ respectively.

Composite indices for comparisons involving several groups or several dimensions

The indices presented above compare two groups of a population over one dimension of well-being. A handful of extensions to two groups and several dimensions and to one dimension and several groups are possible. Each extension is related to a different line of inquriy into dissimilarity between groups and/or across dimensions. The following are some proposals:

[10] This focus on the proportion of distributional differences that is detrimental to one group is present in the proposals of new gender inequality indices by Permanyer (2009).

Two groups and several dimensions

Let $I^{\alpha}_{A/B,d}$ denote the disadvantage index of A over B for dimension d. One way to aggregate these indices over D dimensions is by applying weighted generalized means:[11]

$$\begin{aligned} S^{\beta,\alpha}_{A/B} &= \left[\sum_{d=1}^{D} w_d \left(I^{\alpha}_{A/B,d}\right)^{\beta}\right]^{\frac{1}{\beta}} \ \forall\beta \in \mathbb{R}/\left\{0\right\}, \\ &= \prod_{d=1}^{D} \left(I^{\alpha}_{A/B,d}\right)^{w_d}, \ \beta = 0 \end{aligned} \tag{5}$$

An interesting feature of $S^{\beta,\alpha}_{A/B}$ when $\beta \neq 0$ and $-\infty < \beta < \infty$ is that $S^{\beta,\alpha}_{A/B} = 1$ if and only if B's distribution first-order dominates A's in every dimension.[12] Under the same conditions for β, $S^{\beta,1}_{A/B} = 1$ if and only if the richest person in A is poorer than the poorest person in B for every dimension. On the other extreme $S^{\beta,\alpha}_{A/B} = 0$ if and only if either there is no dissimilarity or it is not detrimental to group A for every dimension. [13] The importance of every dimension is controlled by w_d ($w_d \geq 0 \wedge \sum_{d=1}^{D} w_d = 1$). As with weighted generalized means, more negative values of β increasingly attach more weight in the determination of $S^{\beta,\alpha}_{A/B}$ to the dimensions with the lowest value of $I^{\alpha}_{A/B,d}$. More positive values of β do the same but for dimensions with the highest $I^{\alpha}_{A/B,d}$.

Several groups

A similar composite indicartor to (5) can be proposed to summarize the comparisons of one group of a society against all the other groups over one dimension of well-being. Such composite indicator, defined in reference to the comparison of one group against the others, provides a ranking of relative disadvantage of groups within a society:

$$\begin{aligned} S^{\beta,\alpha}_{A} &= \left[\sum_{g=1}^{G} w_g \left(I^{\alpha}_{A/g}\right)^{\beta}\right]^{\frac{1}{\beta}} \ \forall\beta \in \mathbb{R}/\left\{0\right\}, \\ &= \prod_{g=1}^{G} \left(I^{\alpha}_{A/g}\right)^{w_g}, \ \beta = 0 \quad , \end{aligned} \tag{6}$$

where $I^{\alpha}_{A/g}$ is the group disadvantage index comparing group A versus group g; G is the total number of groups, and $S^{\beta,\alpha}_{A}$ is the index summarizing the degree of relative disadvantage

[11] For recent examples of composite indicators based on generalized means see, e.g. Foster, Lopez-Calva, and Szekely (2005); Seth (2009).

[12] That is, every marginal distribution of B dominates A's. This result does not imply stochastic dominance over the joint distribution.

[13] Some alternative conditions for β are less interesting or informative. For instance, with $\beta = -\infty \vee \beta = \infty$, $S^{\beta,\alpha}$ is determined exclusively by just one of the I^{α}_{d} (the minimum or the maximum, respectively). Also $\beta = 0$ implies that $S^{\beta,\alpha} = 0$ if $\exists k | I^{\alpha}_{k} = 0$, even if $I^{\alpha}_{k} > 0 \ \forall d \neq k$.

of group A against the others in its society. The importance of every reference group to which A is compared is controlled by w_d $(w_d \geq 0 \wedge \sum_{d=1}^{D} w_d = 1)$ and the discussion of the sensitivity of the index (5) to β also applies to (6). When $\beta \neq 0$ and $\infty < \beta < \infty$, $S_{A/B}^{\beta,\alpha} = 1$ if and only if all other groups in society first-order stocastically dominate A in the considered dimension. Under the same conditions for β, $S_{A/B}^{\beta,1} = 1$ if and only if the richest person in A is poorer than the poorest person in every other group. By contrast, $S_{A/B}^{\beta,\alpha} = 0$ if and only no existing dissimilarity with respect to any group is detrimental to group A. This situation includes the possibility that in some or all of the comparisons there is no dissimilarity between group A's distribution and some (or all) of the other groups'. Total similarity over the dimension of wellbeing exists if and only if $S_g^{\beta,\alpha} = 0 \forall g \in \{1, ..., G\}$. For instance, if the groups are defined over combinations of circumstances beyond the individuals control then the latter condition can signal perfect equality of opportunity in a society.[14]

Analogues to the indices in (5) and (6) can also be constructed replacing, respectively, $I_{A/B,d}^{\alpha}$ with $R_{A/B,d}^{\alpha}$, and $I_{A/g}^{\alpha}$ with $R_{A/g}^{\alpha}$.

Inference

Several bootstrapping techniques are available for performing inference and deriving confidence intervals for this paper's indices. [15] In this section I sketch out analytical approximations to the asymptotic distribution of some of these indices, combining the results of Handcock and Morris (1999, Chapter 9) with the Delta Method.

Firstly, consider the joint distribution of several differences, $\left(\widehat{G}_{A/B}(r) - r\right)$, defined over different values of r. Let $\widehat{G_p}$ be the column vector of dimension p, containing $\left(\widehat{G}_{A/B}(r) - r\right)$ estimated for p proportions of sample B. The vector of p statistics $\sqrt{N_A}\left(\widehat{G_p} - G_p\right)$ is asymptotically normally distributed with a zero mean and an asymptotic covariance matrix, $V(G)_{pxp}$. This result is based on Handcock and Morris (1999, p. 143). N_A is the population size of A and G_p is a column vector of dimension p, containing $\left(G_{A/B}(r) - r\right)$ for p proportions. The diagonal elements of $V(G)_{pxp}$ are:

$$V(G)_{ii} = G_{A/B}(i)\left[1 - G_{A/B}(i)\right] + \frac{N_A}{N_B} i(1-i)\left[\frac{f_A\left(\widehat{F}_B^{-1}(r)\right)}{f_B\left(\widehat{F}_B^{-1}(r)\right)}\right]^2, \qquad 0 < i < 1,$$

where f_A is the marginal density function of population A. The off-diagonal elements are the following:

[14] At least according to one definition in Roemer (1998) and also in terms of Fleurbaey's *circumstance neutralization*, i.e. the inability of circumstances to explain distributions of well-being outcomes Fleurbaey (2008).

[15] For examples of these bootstrapping techniques Mooney and Duval (see e.g. 1993).

$$V(G)_{jk} = G_{A/B}(j)\left[1 - G_{A/B}(k)\right] + \frac{N_A}{N_B} j(1-k) \frac{f_A\left(\widehat{F}_B^{-1}(j)\right)}{f_B\left(\widehat{F}_B^{-1}(j)\right)} \frac{f_A\left(\widehat{F}_B^{-1}(k)\right)}{f_B\left(\widehat{F}_B^{-1}(k)\right)}, \quad \forall j \leq k.$$

Secondly, consider the empirical verison of the index $I^{\alpha}_{A/B} : \widehat{I}^{\alpha}_{A/B,p} \equiv \frac{\alpha+1}{p} \sum_{r=i/p}^{1} \left[\widehat{G}_{A/B}(r) - r\right]_{+}^{\alpha}$.[16] Its variance is:

$$Var\left(\widehat{I}^{\alpha}_{A/B,p}\right) = \left(\frac{\alpha+1}{p}\right)^2 \left[\begin{array}{c} \sum_{r=i/p}^{1} var\left(\left[\widehat{G}_{A/B}(r) - r\right]_{+}^{\alpha}\right) \\ + \sum_{r=i/p}^{1} \sum_{s\neq r}^{1} covar\left(\left[\widehat{G}_{A/B}(r) - r\right]_{+}^{\alpha} \left[\widehat{G}_{A/B}(s) - s\right]_{+}^{\alpha}\right) \end{array} \right] \tag{7}$$

And let $\widehat{G^{\alpha}_p}$ be the column vector of dimension p, containing $\left(\widehat{G}_{A/B}(r) - r\right)^{\alpha}$ estimated for p proportions of sample B. Using the Delta Method the asymptotic distribution of the vector of p statistics $\sqrt{N_A}\left(\widehat{G^{\alpha}_p} - G^{\alpha}_p\right)$ can be approximated as being normal with zero mean and an asymptotic covariance maxtrix, $A_{pxp} V(G)_{pxp} A'_{pxp}$. A_{pxp} is a diagonal matrix whose elements are: $A_{ii} \equiv \alpha\left(\widehat{G}_{A/B}(i) - i\right)^{\alpha-1}$. For $\alpha \geq 1$ the empirical counterparts to the elements in $A_{pxp} V(G)_{pxp} A'_{pxp}$ serve as approximations to the variances and covariances in (7) and hence to the standard errors of $\widehat{I}^{\alpha}_{A/B,p}$. Moreover because $\widehat{I}^{\alpha}_{A/B,p}$ is the sum of statistics that are themselves asymptotically normally distributed then $\widehat{I}^{\alpha}_{A/B,p}$ is also asymptotically normally distributed. Therefore z-tests can be performed with it.

Comparison with other approaches

The literature offers other indices of between-group inequality which are sensitive to inequalities that render specific groups of society at a relative disadvantage with respect to others. In this section I compare the indices proposed in this paper with Gartswirth's PROB measure (Gastwirth, 1975), Butler and McDonald's Pietra functions (Butler and McDonald, 1987), Dagum's relative economic affluence measure, D (Dagum, 1987), and the Gender Gap indices of Hausmann, Tyson, and Zahidi (2007) and Permanyer (2009).[17]

[16] Generally, the index can be constructed using several choices of numbers of proportions for comparison, p. In the empirical application I estimate the indices with different choices to show the robustness of their values to different sensible choices (in the sense of involving relatively high p). Also different spacings between the proportions can be considered. I have written the sum in $\widehat{I}^{\alpha}_{A/B}$ restricting it to choices in which the proportions are equally spaced between each other.

[17] I want to acknowledge motivation and knowledge of other approaches to write this section to Le Breton, Michelangeli, and Peluso (2008).

Comparison with the PROB measure of Gartswirth

The PROB measure of Gastwirth (1975) is defined as: $PROB \equiv \int_0^\infty \left[1 - F^A(x)\right] f^B(x)\, dx$. It measures the probability of finding an individual in A having at least as much of x as a random individual in B. Whenever $f^A = f^B$, $PROB = 0.5$. When $PROB < 0.5$ the distribution of B has some advantage over A's such that the probability of finding someone in A having at least as much of x as a randomly chosen person from B is lower than the probability that would ensue from identical distributions. A similar interpretation, favouring A's distribution over B's, ensues when $PROB > 0.5$. On the extremes: $PROB = 0 \leftrightarrow F^A\left(x^B_{\min}\right) = 1$. When the measure is equal to 0 it means that the richest person in A is not better off than the poorest person in B (whose value of x is denoted by $x^B_{\min}$). $PROB = 1 \leftrightarrow F^A\left(x^B_{\max}\right) = 0$: When the measure is equal to 1 the poorest person in A is richer than the richest person in B. In both extreme cases the distributions do not overlap.

The following correspondences hold between $PROB$ and I^α. Firstly, $f^A = f^B \rightarrow \left(PROB = 0.5 \wedge I^\alpha_{A/B} = 0\right) \forall\alpha \in \mathbb{N}$. Secondly, $I^\alpha_{A/B} = 0 \forall\alpha \in \mathbb{N} \rightarrow PROB \geq 0.5$, thirdly, $PROB = 0 \leftrightarrow I^\alpha_{A/B} = 1 \forall\alpha \in \mathbb{N}_+$. Finally, $PROB = 1 \leftrightarrow \left(I^\alpha_{A/B} = 0 \wedge I^\alpha_{B/A} = 1\right) \forall\alpha \in \mathbb{N}_+$. However, when $0 < I^\alpha_{A/B} < 1$ nothing can be concluded about $PROB$ because the latter is a sum of both positive and negative gaps between $G_{A/B}(r)$ and r, whereas $I^\alpha_{A/B}$ only considers the positive gaps.[18] That is, $PROB$ does not have an exclusive focus on inequality detrimental to one specific group. It rather allows for compensation between parts of the distribution in which one group is favoured and parts in which the other group has the advantage. This has an additional implication in the way $PROB$ and I^α identify identical distributions. $PROB = 0.5 \nrightarrow f^A = f^B$, therefore it can not distinguish between a situation of two identical distributions and one in which the advantage of one distribution at lower values of the variable is perfectly compensated by the advantage of the other distribution at higher values of the variable. By contrast I^α can identify a situation of a identical distributions since: $f^A = f^B \leftrightarrow \left(I^\alpha_{A/B} = 0 \wedge I^\alpha_{B/A} = 0\right)$.

Comparison with the indices by Butler and McDonald

Butler and McDonald (1987) propose the following Pietra indices:

[18] The fact that $PROB$ considers both positive and negative gaps can be ascertained by noticing that:

$\int_0^1 \left(G_{A/B}(r) - r\right) dr = \frac{1}{2} - PROB$, since $f^B(x)\, dx = dF^B(x) \equiv r$ and $PROB = 1 - \int_0^1 G_{A/B}(r)\, dr$. Le Breton, Michelangeli, and Peluso (2008) also show this result.

$$P(0,0) = F_B\left(\int_0^1 F_A^{-1}(r)\,dr\right) - F_A\left(\int_0^1 F_B^{-1}(r)\,dr\right),$$

$$P(1,1) = \frac{\int_0^{F_B\left(\int_0^1 F_A^{-1}(r)dr\right)} F_B^{-1}(r)\,dr}{\int_0^1 F_B^{-1}(r)\,dr} - \frac{\int_0^{F_A\left(\int_0^1 F_B^{-1}(r)dr\right)} F_A^{-1}(r)\,dr}{\int_0^1 F_A^{-1}(r)\,dr}.$$

The first Pietra index measures the the difference between the fraction of population B who have a value of the variable equal or less than the mean of population A and the fraction of A with a value equal or less than the mean of B. The second Pietra index measures the difference between the fraction of total holdings of the variable in B held by those with a value equal or less than the mean of A and the fraction of total holdings of the variable in A held by those with a value equal or less than the mean of B.[19]

A first comparison is that $I^{\alpha}_{A/B} = 0 \rightarrow P(0,0) \geq 0$ and$\left(I^{\alpha}_{A/B} = 0 \wedge I^{\alpha}_{B/A} = 0\right) \rightarrow P(0,0) = 0$.The first relationship is true because $I^{\alpha}_{A/B} = 0 \leftrightarrow F_B \geq F_A \forall x$ (hence also $\int_0^1 F_A^{-1}(r)\,dr \geq \int_0^1 F_B^{-1}(r)\,dr$). The second one is true because $\left(I^{\alpha}_{A/B} = 0 \wedge I_{\alpha B/A} = 0\right) \rightarrow F_B = F_A \forall x \rightarrow P(0,0) = 0$. The reverse relationships are not true. For instance, $P(0,0) = 0$ does not differentiate between two identical distributions and, for instance, another pair which are not identical but happen to be symmetrical, centered around the same mean value and with different kurtosis. Because of the different kurtosis of this counter-example, $P(0,0) = 0 \nrightarrow \left(I^{\alpha}_{A/B} = 0 \vee I^{\alpha}_{B/A} = 0\right)$.It follows also that $P(0,0) < 0 \rightarrow I^{\alpha}_{A/B} > 0$. Finally, whenever $I^{\alpha}_{A/B} = 1 \rightarrow P(0,0) = -1$.The reverse is not true because $I^{\alpha}_{A/B} = 1$ if and only if the distributions do not overlap (and the poorest individual in B is better off than the richest in A), however $P(0,0) = -1$ does not imply lack of overlap (although it is implied by the latter).

[19] Butler and McDonald also propose:

$$P(0,1) = F_B\left(\int_0^1 F_A^{-1}(r)\,dr\right) - \frac{\int_0^{\int_0^1 F_B^{-1}(r)dr} F_A^{-1}(r)\,dr}{\int_0^1 F_A^{-1}(r)\,dr}$$

$$P(1,0) = \frac{\int_0^{\int_0^1 F_A^{-1}(r)dr} F_B^{-1}(r)\,dr}{\int_0^1 F_B^{-1}(r)\,dr} - F_A\left(\int_0^1 F_B^{-1}(r)\,dr\right)$$

These measures are less intuitive but have a particular interpretation in their discussion of an interdistributional welfare function (p. 15-6).

Similarly, $I^{\alpha}_{A/B}=0\rightarrow P(1,1)\geq 0$ (hence $P(1,1)<0\rightarrow I^{\alpha}_{A/B}>0$), becuase $F_B\geq F_A\forall x$ implies both that $\int_0^1 F_B^{-1}(r)\,dr\leq\int_0^1 F_A^{-1}(r)\,dr$ and $\int_0^{F_B\left(\int_0^1 F_A^{-1}(r)dr\right)} F_B^{-1}(r)\,dr\geq \int_0^{F_A\left(\int_0^1 F_B^{-1}(r)dr\right)} F_A^{-1}(r)\,dr$. Moreover, $\left(I^{\alpha}_{A/B}=0\wedge I^{\alpha}_{B/A}=0\right)\rightarrow P(1,1)=0$.The opposite however is not true. For instance consider two distributions for which $\mu=\int_0^1 F_B^{-1}(r)\,dr=\int_0^1 F_A^{-1}(r)\,dr$ and $F_B<F_A\forall x\leq\mu$, and $F_B>F_A\forall x\geq\mu$. For some of these two distributions it is possible that $P(1,1)>0$ and $I^{\alpha}_{A/B}>0$. Finally, whenever $I^{\alpha}_{A/B}=1\rightarrow P(1,1)=-1$, yet the reverse is not true for the reasons mentioned with respec to $P(0,0)$.

Comparison with the index by Dagum

Dagum (1987) proposed a measure of relative economic affluence (REA) which, in this paper's notation, is defined as:$D_{A/B}=1-\frac{d_A}{d_B}$, where:

$$\begin{aligned} d_B &= \int_0^{\infty} dF_B(y)\int_0^{y}(y-x)\,dF_A(x), \\ d_A &= \int_0^{\infty} dF_A(x)\int_0^{x}(x-y)\,dF_B(y). \end{aligned}$$

$D_{A/B}=0$ whenever $F_B=F_A\forall x$ (because in that case $d_B=d_A$) but the opposite is not true. $D=1$ if and only if the two distributions do not overlap and the poorest person in B is better-off than the richest person in A. Hence $D_{A/B}$ has a focus on the relative economic advantage of group B over A; like $I^{\alpha}_{A/B}$. As Dagum shows, $D_{A/B}>0\leftrightarrow\int_0^{\infty} ydF_B(y)>\int_0^{\infty} ydF_A(y)$ (1987; p. 6).When $\int_0^{\infty} ydF_B(y)<\int_0^{\infty} ydF_A(y)$, $D_{B/A}$ can be used instead of $D_{A/B}$. This two variants of D are analogous to $I^{\alpha}_{A/B}$ and $I^{\alpha}_{B/A}$, although in the case of $I^{\alpha}_{A/B}: I^{\alpha}_{A/B}>0\leftrightarrow\exists x|F_A(x)>F_B(x)$.

Whenever $I^{\alpha}_{A/B}=0$, $D_{A/B}\leq 0$, because when $I^{\alpha}_{A/B}=0$ either the two distributions are identical (in which case $D_{A/B}=0$) or A first-order stochastically dominates B (in which case $\int_0^{\infty} ydF_B(y)<\int_0^{\infty} ydF_A(y)$ and $D_{A/B}<0$, by implication). Therefore $I^{\alpha}_{A/B}=0\leftrightarrow D_{A/B}\leq 0$. Also $I^{\alpha}_{A/B}=1\leftrightarrow D_{A/B}=1$: The two indices hit their maxima when the poorest person in B is better-off than the richest person in A (in the case of $D_{A/B}$, $d_A=0$). By contrast the indices perform differently in identifying pairs of identical distributions from other pairs of distributions. In the case of $I^{\alpha}: F_B=F_A\forall x\leftrightarrow\left(I^{\alpha}_{A/B}=0\wedge I^{\alpha}_{B/A}=0\right)$. However in the case of $D: F_B=F_A\forall x\rightarrow\left(D^{\alpha}_{A/B}=0\wedge D^{\alpha}_{B/A}=0\right)$. Therefore $\left(I^{\alpha}_{A/B}=0\wedge I^{\alpha}_{B/A}=0\right)\rightarrow\left(D^{\alpha}_{A/B}=0\wedge D^{\alpha}_{B/A}=0\right)$.That is, I^{α} is helpful in identifying identical pairs of distributions while D can take the same value when the two distributions are identical and, for instance, when they have identical means but they are otherwise different (e.g. symmetric distributions centered around the same mean but with different kurtosis); because $d_A = d_B \leftrightarrow \int_0^\infty y dF_B(y) = \int_0^\infty y dF_A(y)$ (Dagum, 1987, p. 6 equations 4 and 5). Thereby $D_{A/B}$ implicitly compensates the distributional differences favouring A with distributional differences favouring B, whereas $I^\alpha_{A/B}$ is only sensitive to distributional differences (e.g. mesaured by differences between quantiles) favouring group B.[20]

Comparison with the gender gap indices

A recent concern for quantifying the degree of gender inequality has led to several proposals of gender gap indices.[21] Most of these are exclusively sensitive to inequalities that are detrimental to women. One such index is the Global Gender Gap Index (Hausmann, Tyson, and Zahidi, 2007). The index is the weighted sum of subindices for different dimensions. Within each dimension and for each considered variable, the average achievement of women is divided by that of men. If the ratio is higher than 1 then it is capped so that variables benefitting women do not compensate for those detrimental for women.[22] This index therefore works with one single standard of the distributions: the average attainment. Unlike I^α, it does not consider information on the distribution of the variable for men and women. The ratio gaps in average attainment are certainly informative and interesting in themselves.

However the average attainment gap itself is uninformative as to other aspects, e.g. whether the richest woman is poorer than the poorest man, which is known to happen when $I^1 = 1$. On the other hand I^α does not say anyhting about the magnitude of the gaps in average attainment, although it does say something about the relationship between the two average attainmnets. For instance, $I^0 = 1$ implies that the average attainment of A is lower than B's; the opposite however is not necessarily true. $I^0 = 0$ implies that that the average attainment of A is at least as high as B's; but the reverese is not necessarily true. Therefore I^α and the Global Gender Gap Index provide complementary information. While the latter focuses on average attainment comparisons, the former provides different indirect measures of the gaps between several quantiles of the distribution that are detrimental to women.

A similar complementarity ensues from comparing this paper's indices with the index by Permanyer (2009), which itself is based on other indices he proposed (his equations 8 to 10). His main index (11) is the product of all the average attainment ratios (where the lowest attainment is on the numerator, and the highest on the denominator), powered by a function that depends on an inequality aversion ratio and an index that measures the balance between the gaps in opposite directions (i.e. some favouring women; while others, men). This balance function is similar to $R^1_{A/B}$: The former works with ratio gaps in average attainment while the latter works indirectly with gaps between the cumulative distribution functions.

[20] Dagum (1987) is aware of this potential compensation (see p. 7).

[21] For a good review see Permanyer (2009).

[22] They use the world-wide standard deviations of the variables to estimate their weights in the index, in order to give more weights to gender gaps in variables with relatively lower world viariability, e.g. primary enrolment rate (Hausmann, Tyson, and Zahidi, 2007, p. 5).

Empirical application: studying the extent of gender inequality in Chile

The Oxford Poverty and Human Development Initiative (OPHI) collected an addendum to the 2006 Chile's National Household Survey (CASEN); prompted by OPHI's advocacy for collecting information on dimensions of poverty and wellbeing, which are not surveyed on a systematic basis. These dimensions are deemed fundamental for a comprehensive assessment of wellbeing (e.g. see Alkire, 2007), and even valued as important by the poor themselves (e.g. see Narayan and Walton, 2000; Narayan, Walton, and Chambers, 2000). The 2009 OPHI questionnaire[23] contains information on the following so-called missing dimensions of poverty:[24] Employment, particularly its quality and informality (Lugo, 2007); agency and empowerment, that is the ability to advance goals or values one has reason to value (Ibrahim and Alkire, 2007); physical safety, i.e. security from violence to property or self including its perception (Diprose, 2007); the ability to go about without shame, including dignity, respect and freedom for humilliation (Zavaleta, 207); and psychological and subjective wellbeing, including meaning in life and satisfaction (Samman, 2007).[25]

Even though these dimensions have been brought to attention to broaden the analysis of multidimensional poverty, the availability of data on them represents an opportunity to broaden also the analysis of multidimensional inequality, including between-group inequality. The purpose of this empirical applcation is to broaden the analysis of gender inequality in Chile considering inequality over these missing dimensions, using the OPHI dataset. The Chilean government itself is interested in monitoring gender inequality. Indeed their recent report, "Indice the inequidad territorial de Genero" (MIDEPLAN) using CASEN 2006 estimates the Global Gender Gap Index of Hausmann, Tyson, and Zahidi (2007). Unlike the MIDEPLAN report this section does not calculate a composite indicator of gender inequality over several dimensions. Rather it seeks to identify dimensions where there is more between-group inequality and within these, those where inequality are the most detrimental to women. The latter is accomplished using the I^{α} indices.

Data and selection of questions

The OPHI dataset covers 2,058 households from the 2006 CASEN. The questions on the missing dimensions were asked to adults at least 18 years old, which renders a maximum sample of 5,627 people. Due to the nature of some of the questions and the interview, response rates vary significantly across modules. For every dimension there is a module, with exception of agency questions which are located in every module. That is, there are agency questions about different aspects of life, e.g. agency in health care, physical security, religious observance, employment and so forth. I estimate inequality indices for these modules and

[23] OPHI and de Chile (2009).

[24] The authors in parenthesis developed internationally comparable questionnaires for the corresponding dimension.

[25] The latter is not strictly deemed a dimension of poverty but several scholars advocate its systematic measurement.

also for more traditional questions of educational attainment, earnings per hour and monthly income (including non-labour income sources).

Since the questionnaire offers several questions for every module I restrict the number of questions, over which I estimate the I^{α} indices, by estimating indices of between-group inequality that are sensitive to any dissimilarity in the male and female distributions, i.e. not just to those which are only detrimental to women, for instance. For this purpose I estimate the following Pearson-Cramer index:

$$H = \frac{\sum_{t=1}^{T} \sum_{a=1}^{A} w^t \frac{\left(p_a^t - p_a^*\right)^2}{p_a^*}}{\min\left(T-1, A-1\right)}, \tag{8}$$

where p_a^t is the probability of attaining the value a of an outcome conditional on belonging to group t, and p_a^* is the probability of attaining that same value, but for the whole population. T and A are, respectively, the number of groups (two in the case of gender) and the number of possible values of the outcome. The percentage of the sample belonging to group t is: w^t. $H = 0$ if and only if the conditonal distributions are all identical and it equals 1 if and only if there is complete association between groups and sets of outcome values. I also test the null hypothesis that the conditional distributions are homogeneous using the statistic: $X = \sum_{t=1}^{T} \sum_{a=1}^{A} N^t \frac{\left(p_a^t - p_a^*\right)^2}{p_a^*}$, where N^t is the sample size of group t. The statistic has an asymptotic chi-square distribution with $(T-1)(A-1)$ degrees of freedom (e.g. Hogg and Tanis, 1997). I also report the overlap measure: $O = \sum_{a=1}^{A} \min\left(p_a^1, ..., p_a^T\right)$, which is equal to 1 if and only if the conditional distributions perfectly overlap; and 0, if and only if they do not overlap at all. For continuous variables I report the ratio of between-group inequality to total inequality of the mean log deviation index[26] and the AAD from (1). I use these indices, in particular the homogeneity test results, to select among all the questions those whose between-group inequality are most salient, in order to apply the I^{α} indices to them.[27]

The results for this first stage of selection of questions with salient between-group inequality are in Appendix 1.[28] Most dimensions exhibit several questions in which there is statistically significant dissimilarity between the gender-conditioned distributions. Questions on security and violence are the least heterogeneous among men and women whereas, on the other extreme, almost all questions on dignity exhibit statistically significant differences between men's and women's distributios.

[26] The mean log deviation for two groups is: $MLD = \sum_{i=1}^{\sum^T N^t} \log\left(\frac{\mu}{x_i}\right)$, where μ is the total-population mean of variable x. The between-group component is: $BGI = \sum_{t=1}^{T} \log\left(\frac{\mu}{\mu_t}\right)$, where μ_t is the mean of group t. The ratio therefore is: $S = \frac{BGI}{MLD}$.

[27] Since I am using only two continuous variables I consider them for the estimation of I^{α} without prior selection.

[28] To read the actual wording and categories of the questions please use the question codes in the tables and refer to http://www.ophi.org.uk/subindex.php?id=chile.

Results

Using the first-stage results as selection criteria, I calculate $I^0_{Women/Men}$, $I^1_{Women/Men}$, and $I^2_{Women/Men}$ for the questions with most salient dissimilarities. The results are in Appendix 2, grouped and ranked by the value of $I^0_{Women/Men}$. Then, within each of these groups, questions are ranked according to their values of $I^1_{Women/Men}$, and $I^2_{Women/Men}$. For monthly income and hourly earnings estimations were made with 20, 50 and 100 equally-distanced proportions. For the rest, discrete variables, estimations were performed with 10,20 and 50 equally-distributed proportions. According to the results, income and earnings, as well as the dimensions of dignity and most dimensions of life satisfaction and meaning in life exhibit high, and relatively the highest, levels of $I^0_{Women/Men}$, often equal to 1. In other words, for all these dimensions the distribution of men often first-order stochastically dominates women's. That is, the wellbeing of women over these dimensions is worse than men's for any additive, univariate wellbeing function that is increasingly monotonic to the level of the variable (e.g. income level).

Among these dimensions with high levels of $I^0_{Women/Men}$, those that exhibit relatively the widest gaps (among quantiles) are income (also with the highest $I^2_{Women/Men}$) and earnings, and four questions on dignity (frequencies of feeling blushing, of feeling disabled, of feeling repressed and of feeling humiliated). Self-reported health, full-time employment, life satisfaction with family and dwelling and happiness also appear high in terms of $I^0_{Women/Men}$, and $I^1_{Women/Men}$. A second group characterized by $0.5 < I^0_{Women/Men} < 1$ has several dimensions of agency in religion and health and life satisfaction at work. Among these, the one with highest $I^1_{Women/Men}$, and $I^2_{Women/Men}$ is the question on not practicing religion because expected not to. Finally, a group in which $I^0_{Women/Men} \leq 0.5$ includes several dimensions of quality of employment (maternity leave, toilet and potable water facilities, uncomfortable positions at work), some dimensions on security, control over day-to-day decisions and agency in employment, household chores. The difference between these dimensions and those not selected to the second stage is that the former exhibit some significant between-group inequality, but this inequality is not necessarily detrimental to women; whereas the latter do not exhibit statistically significant between-group inequality.

Concluding remarks

This paper builds on the theory of relative distributions to propose indices of group disadvantage. That is, these indices belong to a strand of the between-group inequality literature characterized by placing a focus in the relative affluence or disadvantage of one group with respect to other(s). The indices by Garstwirth, Butler and McDonald, Dagum belong to this familly along with the gender gap indices. Like the latter, the I^α indices are only sensitive to distributional differences only whenever these are detrimental to a group of interest (e.g. women). That is, they have an embedded focus axiom. The other indices of relative advantage are also computed from the perspective of a group. However the I^α indices bear a conceptual distinction over these other indices in that the latter compensate distributional differences which are detrimental to one group with differences which are detrimental to another group.

Besides the I^{α} indices provide interesting and rich information about the nature and relative extent of detrimental inequality. For instance, the I^0 are informative as to whether there is first-order stochastic dominance of one group's distribution over the other. The combination of $I^0_{A/B}$ and $I^0_{B/A}$ allows to identify situations in which the two distributions are identical; a useful trait lacking in other indices. I^1 is likewise helpful to determine whether the disadvantage is such that, on the extreme, the richest person in the disadvantaged group is worse-off than the poorest person in the other group. $I^1_{A/B}$ is also proportional to the area between the two cumulative distribution functions whenever $F_A > F_B$, therefore it is sensitive to the gaps between the quantiles of the two distribution whenever $F_A^{-1}(r) < F_B^{-1}(r)$. In fact I^{α} takes positive values if and only if at least for one proportion r the respective quantile in B is greater than in A. Hence I^{α} for $\alpha \geq 1$ measures the gaps between these quantiles as a proportion of the maximum possible gap (which happen whenever there is no overlap between the distributions).

in the empirical application to gender inequality over the missing dimensions of poverty in Chile, the indices I^{α} prove useful in showing the existence of significant between-group inequality detrimental to women in certain dimensions. The most salient ones are the traditional dimensions of income and earnings; and the less traditional ones of dignity, life satisfaction and some measures of agency in religious activity and health care. Some questions from quality of employment also appear with some disadvantage against women although it is only quantitatively significant in the case of full-time employment opportunities. Several other questions exhibit significant between-group inequality yet not necessarily to the detriment of women. Such questions are mostly from the module on security and violence and about some working conditions like sanitary facilities, maternity leave, and agency over work and household chores decisions.

References

Alkire, S. (2007): "The missing dimensions of poverty data: Introduction to the special issue," *Oxford Development Studies*, 35(4), 347–59.

Arneson, R. (1989): "Equality and equal opportunity for welfare," *Philosophical studies*, 56, 77–93.

Arrow, K. (1973): *Discrimination in the labor market*chap. The theory of discrimination. Princeton University Press.

Becker, G. (1971): *The economics of discrimination*. The University of Chicago Press.

Butler, R., and J. McDonald (1987): "Intedistributinal income inequality," *Journal of Business and Economic Statistics*, 5(1), 13–7.

Cohen, G. (1989): "On the currency of egalitarian justice," *Ethics*, 99(4), 906–44.

Dagum, C. (1987): "Measuring the economic affluence between populations of income receivers," *Journal of Business and Economic Statistics*, 5(1), 5–8.

Diprose, R. (2007): "Physical safety and security: a proposal for internationally comparable indicators of violence," *Oxford Development Studies*, 35(4), 431–58.

Elbers, C., P. Lanjouw, J. Mistiaen, and B. Ozler (2008): "Reinterpreting between-group inequality," *Journal of Economic Inequality*, 6, 231–45.

Ferreira, F., and J. Gignoux (2008): "The measurement of inequality of opportunity: theory and application to Latin America," Policy Research Working Paper Series.

Fleurbaey, M. (2001): "Egalitarian opportunities," *Law and Philosophy*, 20(5), 499–530.

——— (2008): *Fairness, Responsibility and Welfare*. Oxford University Press.

Foster, J., J. Greer, and E. Thorbecke (1984): "A class of decomposable poverty measures," *Econometrica*, 52(3), 761–6.

Foster, J., L. F. Lopez-Calva, and M. Szekely (2005): "Measuring the distribution of human development: methodology and application to Mexico," *Journal of Human Development*, 6, 5–29.

Gastwirth, J. (1975): "Statistical measures of earnings differentials," *The American Statistician*, 29(1), 32–5.

Handcock, M., and M. Morris (1999): *Relative distribution methods in the social sciences*, Statistics for Social Science and Public Policy. Springer.

Hausmann, R., L. Tyson, and S. Zahidi (2007): *Global Gender Gap Report*chap. The Global Gender Gap Index 2007. UNDP.

Hogg, R., and E. Tanis (1997): *Probability and statistical inference*. Prentice Hall.

Ibrahim, S., and S. Alkire (2007): "Agency and empowerment: a proposal for internationally comparable indicators," *Oxford Development Studies*, 35(4), 379–403.

Lanjouw, P., and V. Rao (2008): "Revisiting between-group inequality measurement: an application to the dynamics of caste inequality in two Indian villages," manuscript.

Le Breton, M., A. Michelangeli, and E. Peluso (2008): "Wage discrimination measurement: in defense of a simple but informative statistical tool," Econpubblica Working Paper 128.

Lugo, M. A. (2007): "Employment: a proposal for internationally comparable indicators," *Oxford Development Studies*, 35(4), 361–78.

Mooney, C., and R. Duval (1993): *Bootstrapping. A nonparametric approach to statistical inference*, Quantitative applications in the social sciences. Sage.

Narayan, D., and M. Walton (2000): *Can anyone hear us? (voices of the poor)*. Oxford University Press.

Narayan, D., M. Walton, and R. Chambers (2000): *Crying out for change (voices of the poor)*. Oxford University Press.

Nussbaum, M. C., and J. Glover (1995): *Women, culture and development: a study of human capabilities*, WIDER studies in Development Economics. Clarendon Press.

OPHI, and U. de Chile (2009): "Encuesta - Otras dimensiones de la calidad de vida en los hogares," Discussion paper, OPHI and Universidad de Chile.

Permanyer, I. (2009): "The measurement of multidimensional gender inequality: continuing the debate," *Social Indicators Research*.

Phelps, E. (1972): "The statistical theory of racism and sexism," *The American Economic Review*, 62(4), 659–61.

Roemer, J. (1998): *Equality of Opportunity*. Harvard University Press.

——— (2006): "Economic development as opportunity equalization," Cowles Foundation Discussion Paper 1583.

Samman, E. (2007): "Psychological and subjective well-being: a proposal for internationally comparable indicators," *Oxford Development Studies*, 35(4), 459–86.

Sen, A. (2001): *Development as freedom*. Oxford University Press.

Seth, S. (2009): "A class of association sensitive multidimensional welfare indices," OPHI Working Paper 27.

Zavaleta, D. (207): "The ability to go about without shame: a proposal for internationally comparable indicators of shame and humilliation," *Oxford Development Studies*, 35(4), 405–30.

1 Appendix: OPHI survey questions with salient gender inequality

Some traditional dimensions

[29] Bootstrapped 95% confidence intervals.

[30] F/M means that the female distribution is the compared one and the male distribution is the reference one. 20 is the number of equally spaced proportions used for the estimation.

[31] Earnings per hour, includes all labour sources.

[32] Monthly income, includes non-labour soruces.

[33] Pearson-Cramer index.

[34] Overlap index.

Table 1: Between-group inequality of earnings and income

Question[29]	Mean log deviation		Average Absolute deviation[30]					
	Uweighted	Weighted	F/M (20)	M/F (20)	F/M (50)	M/F (50)	F/M (100)	M/F (100)
Earnings[31]	0.014	0.020	0.160	0.144	0.158	0.148	0.158	0.147
	[0.007,0.024]	[0.007-0.039,]	[0.113,0.208]	[0.099,0.196]	[0.111,0.207]	[0.101,0.195]	[0.109,0.207]	[0.099,0.196]
Income[32]	0.031	0.047	0.257	0.227	0.256	0.231	0.255	0.228
	[0.020,0.043]	[0.026,0.074]	[0.207,0.300]	[0.181,0.276]	[0.206,0.299]	[0.180,0.273]	[0.204,0.297]	[0.178,0.269]

Table 2: Between-group inequality of educational levels

Question	H[33]	O[34]
Education levels	0.031	0.981

Table 3: Employment and household chores

Question[35]	H[36]		O	
	value	rank	value	rank
Contractual relation (E17)	0.125***	7	0.935	13
Work for wage (E19)	0.032	23	0.984	3
Full time/ part time (E20)	0.137***	4	0.912	19
Layoff insurance (E21)	0.112***	9	0.883	24
Medical leave (E22)	0.040	21	0.964	7
Paid vacation (E23)	0.044*	19	0.957	10
Maternity leave (E24)	0.101***	11	0.900	21
Health problem at work (E30)	0.026	24	0.988	2
Health problem affected work (E31)	0.064	17	0.958	9
Permanent effect most serious incident (E32)	0.137***	4	0.900	21
Most serious problem (E33)	0.421***	1	0.690	26
Potable water at work (E34a)	0.123***	8	0.922	17
Proper toilet facilities at work (E34b)	0.143***	2	0.903	20
Uncomfortable work positions (E34c)	0.071***	15	0.928	16
Satisfactory purpose at work (E37a)	0.020	25	0.981	4
Motivation at work (E37b)	0.011	26	0.990	1
Autonomy, self-organization at work (E37c)	0.052	18	0.956	11
Concern about being harmed by work (E35)	0.137***	4	0.891	23
Work because need the income (E451a)	0.075*	13	0.930	15
Work because forced to (E451b)	0.085**	12	0.932	14
Work because expected to (E451c)	0.075*	13	0.940	12
Work because important for self (E451d)	0.042	20	0.963	8
Chores because necessary (E452a)	0.112**	9	0.872	25
Chores because forced to (E452b)	0.070	15	0.970	6
Chores because expected to (E452c)	0.039	22	0.974	5
Chores because important to self (E452d)	0.138***	3	0.915	18

[35] The number of the questions in the questionnaire are in parentheses.
[36] ***: Statistically significant at 99%. **: Statistically significant at 95%. *: Statistically significant at 90%.

Table 4: Health and empowerment

Question	H		O	
	value	rank	value	rank
Self-perceived health status (S1)	0.106***	1	0.933	5
Inability to tackle health differently (S11a)	0.066**	3	0.937	4
Do what forced to do to tackle health (S11b)	0.082**	2	0.959	1
Do what expected to when tackling health (S11c)	0.047	5	0.955	3
Do what I deem important for health (S11d)	0.051	4	0.959	1

Table 5: Perceptions about religion

Question	H		O	
	value	rank	value	rank
Practice of religion (EMP6)	0.126***	3	0.876	9
Importance of religion in life (EMP7)	0.142***	2	0.882	8
Have to practice religion already practiced (EMP101a)	0.025	9	0.979	2
Practice religion becaused forced to (EMP101b)	0.027	8	0.995	1
Practice religion because expected to (EMP101c)	0.024	10	0.978	3
Practice religion because importance (EMP101d)	0.074*	7	0.967	4
Cannot practice religion (EMP102a)	0.097	6	0.926	5
Do not practice religion because forced to (EMP102b)	0.169***	1	0.850	10
Do not practice religion because expected to (EMP102c)	0.118***	4	0.898	7
Do not practice religion because important (EMP102d)	0.118***	4	0.902	6

Table 6: Perceptions about decision making

Question	H		O	
	value	rank	value	rank
Control over day-to-day activities (EMP1)	0.295***	1	0.767	2
Change things in the community (EMP14)	0.053	2	0.959	1

Table 7: Perceptions about subjective wellbeing and life satisfaction

Question	H		O	
	value	rank	value	rank
Happiness (MV1)	0.089***	3	0.934	13
General life satisfaction (MV2a)	0.057	7	0.947	11
Life satisfaction: nourishment (MV2b)	0.043	11	0.978	1
Life satisfaction: dwelling (MV2c)	0.074 ***	5	0.935	12
Life satisfaction: income (MV2d)	0.054	8	0.951	10
Life satisfaction: health (MV2e)	0.047	9	0.964	6
Life satisfaction: work (MV2f)	0.081 ***	4	0.934	13
Life satisfaction: local security (MV2g)	0.040	13	0.963	7
Life satisfaction: friends (MV2h)	0.038	14	0.973	4
Life satisfaction: family (MV2i)	0.099 ***	1	0.905	16
Life satisfaction: education (MV2i)	0.029	16	0.975	3
Life satisfaction: freedom to choose (MV2k)	0.041	12	0.977	2
Life satisfaction: dignity (MV2l)	0.045	10	0.956	8
Life satisfaction: neighbourhood (MV2m)	0.093 ***	2	0.915	15
Life satisfaction: ability to help others (MV2n)	0.038	14	0.973	4
Life satisfaction: spiritual beliefs (MV2o)	0.068 **	6	0.954	9

Table 8: Perceptions about meaning in life

Question	H		O	
	value	rank	value	rank
Life has meaning (EMP15)	0.044	3	0.976	2
Life has clear purpose/sense (MV3a)	0.072 **	1	0.943	4
Found satisfying sense for life (MV3b)	0.063 *	2	0.944	3
Clear idea of what gives meaning to life (MV3c)	0.028	4	0.985	1

Table 9: General autonomy

Question	H		O	
	value	rank	value	rank
Freedom to decide how to live own life (MV4a)	0.090 ***	1	0.954	3
Freedom to express ideas and opinions (MV4b)	0.027	2	0.991	1
To be honest with one self (MV4c)	0.017	3	0.985	2

2 Appendix: Female disadvantage in Chile among the dimensions with salient gender inequality

[37]The ranges of values for the I^a indices correspond to the minimum and maximum value from the three estimations with different numbers of proportions chosen (see Results section).

[38]Bootstrapped 95% confidence intervals in brackets, estimated for 100 equally-spaced proportions in the

Table 10: Competence

Question	H		O	
	value	rank	value	rank
People say I am capable (MV5a)	0.034	3	0.981	1
Most times I feel I deliver in what I do (MV5b)	0.039	2	0.971	2
In general I feel very capable (MV5c)	0.060 *	1	0.960	3

Table 11: Relationships

Question	H		O	
	value	rank	value	rank
Get along well with people in contact (MV6a)	0.058	2	0.977	2
I regard people I relate to as close (MV6b)	0.060 *	1	0.950	3
Peope around me care about me (MV6c)	0.017	3	0.984	1

Table 12: Dignity

Question	H		O	
	value	rank	value	rank
Feeling embarrased (SH3a)	0.060	10	0.956	4
Feeling ridiculous (SH3b)	0.114 ***	5	0.903	8
Feeling repressed/intimidated (SH3c)	0.135 ***	2	0.900	9
Feeling humiliated (Sh3d)	0.118***	4	0.896	10
Feeling foolish (SH3e)	0.084 ***	7	0.947	5
Feeling childish (SH3f)	0.047	12	0.960	3
Feeling invalid/paralysed (SH3g)	0.135 ***	2	0.889	11
Feeling blushing (SH3h)	0.150 ***	1	0.866	12
Feeling being laughed at (SH3i)	0.077 ***	8	0.941	6
Feeling repellent to others (SH3j)	0.072 **	9	0.975	1
Feeling being treated with respect (SH4)	0.054	11	0.973	2
Feeling being treated unjustly (SH5)	0.086 ***	6	0.918	7

case of continuous variables and 50 equally-spaced proportions for discrete variables.

[39] The ranges of values for the I^a indices correspond to the minimum and maximum value from the three estimations with different numbers of proportions chosen (see Results section).

[40] Bootstrapped 95% confidence intervals in brackets, estimated for 100 equally-spaced proportions in the case of continuous variables and 50 equally-spaced proportions for discrete variables.

[41] The ranges of values for the I^a indices correspond to the minimum and maximum value from the three estimations with different numbers of proportions chosen (see Results section).

[42] Bootstrapped 95% confidence intervals in brackets, estimated for 100 equally-spaced proportions in the case of continuous variables and 50 equally-spaced proportions for discrete variables.

Table 13: Security

Question	H		O	
	value	rank	value	rank
Entering house without permission (V1Aa)	0.004	13	0.998	1
Took something by force (V1Ab)	0.020	9	0.992	4
Stealing something from your property (V1Ac)	0.078 ***	1	0.967	10
Stealing animals or crops (V1Ad)	0.052 **	5	0.992	4
Deliberate damaging the house (V1Ae)	0.015	12	0.996	3
Being assaulted without weapon (V2Aa)	0.019	10	0.989	6
Being assaulted with weapon (V2Ab)	0.066 ***	2	0.984	7
Being shot with firearm (V2Ac)	0.019	10	0.998	1
Perception of future victimhood (next year) (V3)	0.051	6	0.952	13
Inability to prevent or reduce crime in different way (V10a)	0.056	3	0.955	12
To prevent crime I do what forced to by others (V10b)	0.034	8	0.967	10
To prevent crime I do what I am expected to do (V10c)	0.039	7	0.978	8
To prevent crime I do what I deem important (V10d)	0.056	3	0.970	9

Table 14: Ranking of questions with $I^0 = 1$ by I^1 and I^2

Question[37][38]	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Income	1 [0.99-1]	0.255-0.257 [0.204-0.297]	0.058-0.059 [0.037-0.078]
Earnings	1 [0.96-1]	0.158-0.160 [0.109-0.207]	0.023-0.024 [0.011-0.039]
Frequency of feeling blushing (SH3h)	1 [0.98-1]	0.134-0.138 [0.093-0.181]	0.017 [0.008-0.030]
Frequency of feeling disabled (SH3g)	1 [0.98-1]	0.132-0.134 [0.097-0.171]	0.018 [0.009-0.029]
Frequency of feeling repressed (SH3c)	1 [0.98-1]	0.124-0.128 [0.086-0.167]	0.015 [0.007-0.027]
Frequency of feeling humilliated (SH3d)	1 [0.98-1]	0.123-0.125 [0.083-0.165]	0.015 [0.007-0.026]
Self-reported health (S1)	1 [0.98-1]	0.110-0.112 [0.062-0.161]	0.011 [0.003-0.022]
Life satisfaction: family (MV2i)	1 [0.98-1]	0.102-0.103 [0.061-0.149]	0.010 [0.004-0.021]
Full-time employment (E20)	1 [0.98-1]	0.095-0.098 [0.072-0.126]	0.010 [0.005-0.016]
Life satisfaction: dwelling (MV2c)	1 [0.98-1]	0.095-0.096 [0.049-0.138]	0.008 [0.002-0.017]
Happiness (MV1)	1 [0.98-1]	0.092-0.094 [0.052-0.135]	0.008 [0.002-0.016]
Frequency of feeling ridiculous (SH3b)	1 [0.98-1]	0.090-0.092 [0.055-0.132]	0.008 [0.003-0.017]
Free to decide how to live (MV4a)	1 [0.98-1]	0.077-0.080 [0.033-0.121]	0.005-0.006 [0.001-0.013]
Life satisfaction: dignity (MV2l)	1 [0.97-1]	0.067-0.068 [0.025-0.111]	0.004 [7x10-4-0.012]
Frequency of feeling treated unjustly (SH5)	1 [0.94-1]	0.067-0.068 [0.02-0.111]	0.004 [5x10-4-0.012]
Frequency of feeling laughed at (SHi)	1 [0.97-1]	0.053 [0.015-0.088]	0.003 [2x10-4-0.008]
Life has clear purpose (MV3a)	1 [1-1]	0.043-0.044 [0.002-0.087]	0.002 [1x10-5-0.007]

Table 15: Ranking of questions with $0.5 < I^0 < 1$ by I^1 and I^2

Question[39,40]	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
No religion because expected to (EMP102c)	0.92-1 [0.670-1]	0.083-0.084 [0.011-0.166]	0.007-0.008 [2x10e−4-0.029]
Life satisfaction: work (MV2f)	0.9-0.98 [0.54-1]	0.057-0.058 [0.016-0.104]	0.004 [4x10e−4-0.011]
No religion because forced to (EMP102b)	0.9 [0.730-1]	0.095-0.096 [0.018-0.168]	0.010 [3x10e−4-0.030]
No religion because important (EMP102d)	0.76-0.8 [0-0.98]	0.019 [0-0.096]	0.001 [0-0.012]
Do what forced to do for health (S11b)	0.60-0.64 [0.339-1]	0.027 [2x10e−4-0.070]	0.001 [2x10e−70.006]
Tackle health differently (S11a)	0.54-0.60 [0.26-1]	0.020 [0.001-0.067]	8x10e-4 [6x10e−6-0.004]

Table 16: Ranking of questions with $0.5 \leq I^0 \leq 1$ by I^1 and I^2

Question[41,42]	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
Maternity (at work) (E24)	0.01-0.02 [0-0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Toilet (at work) (E34b)	0.01-0.02 [0-0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Uncomfortable (E34c)	0.01-0.02 [0-0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Concerned damage (E35)	0.01-0.02 [0-0.02]	0 [0]	0 [0]
Work forced to (E451b)	0.01-0.02 [0-0.01]	0 [0-0.049]	0 [0-0.002]
Chores because important (E452d)	0.01-0.02 [0-0.01]	0 [0-0.066]	0 [0-0.005]
Control over day-to-day decisions (EMP1)	0.01-0.02 [0-0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Stealing something from property (V1ac)	0.01-0.02 [0-0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Being assaulted with weapon (V2Ab)	0 [0−.0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]
Potable water (at work) (E34a)	0 [0−.0.02]	0 [0-8x10e-8]	0 [0-3x10e-33]