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**Measuring Globalization Using Weighted Network Indexes**

Kam Ki Tang and Amy Wagner

For additional information please contact:

Name: Kam Ki Tang

Affiliation: University of Queensland

Email Address: [kk.tang@uq.edu.au](mailto:kk.tang@uq.edu.au)

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# Measuring globalization using weighted network indexes

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Kam Ki Tang\*<sup>#</sup> and Amy Wagner

School of Economics, University of Queensland

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## *Abstract*

A defining feature of globalization is that individuals, organizations, countries, or regions are increasingly integrated into worldwide networks of exchange. International trade is considered particularly important in determining the pace of globalization. However, most commonly used de facto measurements of international trade globalization, such as trade intensity index (exports plus imports divided by GDP) focus exclusively on the volume or intensity of trade flows, completely ignoring the network aspect of the exchange.

Recently, there is an emerging literature using network analysis to quantify the system of international trade and financial capital flows. Specifically, weighted network measurements have been applied to study globalization, economic integration and contagion. However, these measurements are largely a replica from those used in other disciplines, usually sociology, which has a long history of social network analysis. Due to the fundamental differences between economic and social networks, the adaption may not be appropriate. In this paper, we address this deficiency using the axiomatic approach. A total of seven axioms are established for the construction of a globalization index for economic networks. No existing network indexes are found to satisfy all these axioms. As a consequence, a new index to that effect has been developed. The new index is applied to international trade, financial capital, and human capital flow data in order to quantify these three dimensions of globalization. Specifically, we measure the extent to which individual countries are integrated into various economic networks (i.e. the degree of globalization) and how their degrees of integration change over time (i.e. the pace of globalization).

\* Corresponding author. School of Economics, University of Queensland, Brisbane, QLD 4072, Australia. [kk.tang@uq.edu.au](mailto:kk.tang@uq.edu.au).

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## 1 Introduction

Trade intensity or dependency index, constructed as exports plus imports divided by GDP, is one of the most commonly used de facto measures for trade openness. With the surge in empirical studies of globalization, the index is also featured prominently in the measurement of globalization (e.g. see Jaumotte, Lall & Papageorgiou 2008, Dreher 2006). However, such application of the trade intensity index (TII) is of deficiency in that it suffers from a size bias<sup>1</sup> in measuring openness and further omits connectivity in measuring globalization. Against this background, this paper aims first to improve the conventional index as a measure of trade openness, and then to further develop the subsequent openness index into a measure of trade globalization. The proposed globalization index can be readily used to measure other aspects of globalization. In this paper, it is also used to measure the globalization of financial capital and human capital flows respectively.

The size bias of the TII refers to the fact that the index tends to understate the openness of large economies relative to small economies. In response, some researchers either switch to de jure measures such as tariff rates (e.g. Agenor 2004) or seek modification to the TII formula.<sup>2</sup> This paper considers the latter approach only (the reason of not looking at the former becomes clear later). A number of correction methods have been proposed in the literature. However, in focusing on the symptom rather than the source of the problem, these corrections appear to be ad hoc. Our approach, in contrast, is to tease out the internal inconsistency of the TII that underlies the bias, which once revealed, will automatically point to the appropriate correction. This leads to the development of a more generalized measure of trade openness that embodies the TII as a special case. The correction is important not only for measuring trade openness but also for measuring trade globalization – the main purpose of the paper – because the improved trade openness measure forms the building block of the subsequent trade globalization measure.

A defining feature of globalization is that individuals, organizations, countries, or regions are increasingly integrated into worldwide *networks of exchange*. However, de facto openness measurements such as the TII, including its various modifications, focus exclusively on the volume or intensity of flows of goods and services while completely ignore the network aspect of the exchange. On the other hand, de jure openness measures such as tariff rates only

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<sup>1</sup> Another widely recognized limitation is that it is sensitive to short run macroeconomic fluctuations. However, one can use the HP filter to separate short run and long run components.

<sup>2</sup> However, it is fair to say that most researchers do not seek remedies to this problem at all, continue to use the TII as a measure of trade openness.

inform a country's import regime but are silent on the extent to which it has integrated into the international trade network.<sup>3</sup>

Recently there is an emerging literature using network analysis to gain better understanding of the system of international trade flows known as the World Trade Web (WTW) and, as a natural extension, to quantify trade globalization (e.g. Fagiolo, Reyes & Schiavo, 2007a&b; Kali and Reyes, 2007; Kastle, Liesch & Steen, 2006).<sup>4</sup> However, these measurements are largely a replica from those used in other disciplines, usually sociology, which has a long history of social network analysis. Due to the fundamental differences between economic and social networks, a direction adaption of 'off-the-shelf' network indexes may not be appropriate (Borgatti 2005). In this paper, we address this issue using the axiomatic approach. This approach has been used extensively in the inequality and poverty literature.<sup>5</sup> However, to the best of our knowledge, this is the time it is used in the network analysis of trade or globalization.

A number of desirable properties for a trade globalization index (TGI) – denoted as the axioms – are first developed. The axioms are then used to pin down the appropriate network indexes from the pool of existing indexes. However, no existing network measurements are found to satisfy all the axioms. As a consequence, a new index to that effect is developed on the basis of the improved trade openness measure proposed in this paper. The new TGI can be applied to international trade data to quantify the extent to which individual countries are integrated into the WTW (i.e. the degree of globalization) and how their degrees of integration change over time (i.e. the pace of globalization).

Trade is not the only aspect of globalization. According to the glossary of the International Monetary Fund (IMF), globalization is “[t]he process through which an increasingly free flow of ideas, people, goods, services, and capital leads to the integration of economies and societies.” Given that the axioms are generic to many economic networks, the TGI can be easily modified to measure other aspects of globalization provided that appropriate data are available. As a demonstration, we show how similar indexes can be constructed to measure the globalization of financial assets and human capital flows. This allows us to examine how the three aspects of globalization are related to or different from each other.

The rest of the paper is organized as follows. Section 2 explains the problem of the size bias for the TII and methods to correct it. Section 3 introduces the network analysis framework and develops the axioms for trade globalization measures. Section 4 applies the axioms to develop a new index of trade globalization. It then explains how the index can be extended to

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<sup>3</sup> De jure openness measures have other limitations as well. See Anderson & van Wincoop (2004) for a survey of various methods to measure trade barriers and their limitations.

<sup>4</sup> For a recent survey of network analysis in economics more generally, see Schweitzer et al. (2009).

<sup>5</sup> See, e.g. Cowell & Kuga (1981a, 1981b), Gajdos & Maurin (2003), Sen (1976, 1983), and Foster, Greer, and Thorbecke (1984).

measure globalization in financial asset and human capital flows respectively. Section 5 describes the data sets used for the empirical work. Section 6 compares and contrasts the results of using the new and conventional measures of trade globalization, as well as the three different dimensions of globalization. The last section concludes the paper.

## 2 Trade Openness Indexes

### 2.1 The trade intensity index (TII)

The TII is computed as the total-trade-to-GDP ratio:

$$TII_i = \frac{X_i + M_i}{Y_i} \quad (1)$$

where  $X_i$  and  $M_i$  are respectively the total export value and total import value of country  $i$ ;  $Y_i$  is Purchasing Power Parity (PPP)-based Gross Domestic Product (GDP). All variables are measured in the same unit, e.g. current US\$.

A key rationale of dividing the total trade value by GDP is to control for country size as large countries are likely to trade more in absolute terms. Also, since both trade and GDP are measured in the same unit, there is no need to adjust for inflation in temporal comparison or for exchange rates in cross country comparison.

The reason for using PPP-based GDP measures is that, due to the Balassa-Samuelson effect, the prices of non-tradable goods and services in higher income countries will be bigger than those in lower income countries, leading to underestimation of the economic sizes of lower income countries. The use of PPP-based GDP in constructing openness measures is straightforward due to the Penn World Table and is seen in a number of studies, including Alcalá & Ciccone (2004), Dollar and Kraay (2003), Spilimbergo, Londoño & Székely (1997). Still, this is not a universal practice; some cross-country studies, such as Yanikkaya (2003)<sup>6</sup> and Cavallo & Frankel (2008), continue to use non-PPP-based GDP in constructing their trade openness measures. For our dataset, the average time series correlation of the two TIIs using PPP-based and non-PPP-based GDP respectively is less than 0.6, and the average cross sectional correlation is slightly more than 0.8,<sup>7</sup> indicating that using PPP-based GDP can make substantial differences to the TII.

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<sup>6</sup> However, his study uses a large number of trade openness measures, many of them are not related to trade flows.

<sup>7</sup> For each of the 182 countries in our dataset, we compute the time series correlation of the two TIIs, the average of the 182 time series correlation coefficients is equal to 0.59. For each of the 47 years, we compute the cross sectional correlation of the two TIIs, the average of the 47 cross-sectional correlation coefficients is equal to 0.82.

## 2.2 A generalized trade openness index (TOI)

If the purpose is to measure trade intensity or trade dependency, then the TII will be an appropriate measure. However, if the purpose is to measure trade openness, it has a limitation of being biased against large economies. To fix the idea, consider a world of two countries,  $p$  and  $q$ , where  $Y_p = 2Y_q = 2Y$ , and  $X_p = X_q = M_p = M_q = X$ . That is, country  $p$  is twice the size of country  $q$ , and both countries maintain a zero trade balance.<sup>8</sup> The TII will *always* indicate that country  $p$  is twice as open as country  $q$  regardless of the volume of trade flows between them.

Many previous studies are aware of this size bias and have attempted to correct it. Table 1 lists the standard TII, a number of modified measures found in the literature, and the outcome they bring in the above two-country example.

Table 1. Alternative measures of trade openness for a two-economy case

Measure	Formula	Openness measure for the larger economy, $p$	Openness for the smaller economy, $q$
Trade intensity	$\frac{X_i + M_i}{Y_i}$	$X / Y$	$2X / Y$
Anderson (1994)	$\frac{X_i + M_i}{2(Y_i + M_i)}$	$X / (2Y + X)$	$X / (Y + X)$
Li, Morck, Yang & Yeung (2004) <sup>9</sup>	$\frac{M_i}{Y_i} - \left( 1 - \frac{Y_i}{\sum_{j=1}^N Y_j} \right)$	$(X / 2Y) - (1/3)$	$(X / Y) - (2/3)$
Squalli & Wilson (2006)	$\left( \frac{X_i + M_i}{Y_i} \right) \left[ \frac{(X_i + M_i)N}{\sum_{j=1}^N (X_j + M_j)} \right]$	$X / Y$	$2X / Y$
Ferrieri (2006)	$\left( \frac{V_i}{V_{\max}} \right)^{(1-\theta_i)(1-\pi_i)}$ $V_i = \frac{X_i + M_i}{Y_i}$ , $V_{\max} = \max \{V_i\}, i = 1, 2, \dots, N$ , $\theta_i = \frac{X_i + M_i}{\sum_{j=1}^N (X_j + M_j)}$ , $\pi_i = \frac{Y_i}{\sum_{j=1}^N Y_j}$	$(0.5)^{1/6} = 0.89$	1

<sup>8</sup> The assumption of zero trade balance is merely for simplicity and is not essential for the discussion.

<sup>9</sup> According to Li et al (2004), the measure was originally suggested by Frankel (2000). However, we are not able to identify such an formula from Frankel (2000), perhaps this is because different versions have been consulted.

Anderson's (1994) measure was originally developed to accommodate entrepot economies such as Hong Kong and Singapore, which have large imports for re-exports. According to this measure, the smaller country,  $q$ , is still more open than the larger one,  $p$ . On the other hand, according to the measure of Li et al. (2004), the larger country will be more open if  $X/Y < 2/3$ . The threshold of  $2/3$  indicates arbitrariness in this correction; also, for countries with very small imports, the index value could become negative. Squalli & Wilson (2006) suggest a composite trade intensity index that consists of the product of two components, one is the conventional trade intensity measure, and the other one is the ratio of the country's trade flow to the world's average trade flow. The second component aims to account for the fact that small countries have small trade flows in absolute terms. However, in the above example, their measure yields exactly the same results as the conventional one. Ferrieri (2006) proposes yet another way to modify the conventional measure. There is no clear explanation in the paper why a power function is used. In the above example, this measure indicates country  $q$  is still more open than country  $p$ , though the gap is reduced to about 10%.

Graff (1999) takes the very different approach of using a parametric method. Specifically, he regresses the conventional trade openness measure against GDP and a constant, and then takes the residual as the trade openness measure. His method can be considered as a limited case of using gravity models to measure trade barriers – an opposite measure of trade openness. In the literature of measuring trade barriers, it is common to estimate a gravity model that incorporates, amongst others; the economic sizes of two trading countries, their distance, dummies for common borders, common languages, past or current colonial relationships etc., and then take the residual as a measure of trade barriers.<sup>10</sup> While this is in the right direction, like the other aforementioned methods, the correction appears to be ad hoc and to lack a clear theoretical foundation.

We would like to argue that, the fundamental problem with the TII as an openness measure lies with two implicit assumptions embodied in the measure that are logically inconsistent and thus, once that inconsistency is resolved, the size bias will naturally disappear.

The TII can be written as a linear combination of an export openness measure and an import openness measure:

$$TII_i = \frac{X_i + M_i}{Y_i} = \frac{X_i}{Y_i} + \frac{M_i}{Y_i} \quad (2)$$

Recall that the rationale of normalizing exports by GDP is that a large country, due to its bigger size, can export more than a small country, and therefore it is necessary to adjust its

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<sup>10</sup> For a survey of various ways to measure trade barriers, including gravity models, see Anderson & van Wincoop (2004).

export volume with GDP to give a more meaningful measure of its export openness. Likewise, the import volume should be adjusted with GDP as a large country can also import more than a small country. In other words, the GDP works as a proxy of the country's supply capacity in the export openness measure and as a proxy for its demand capacity in its import openness measure. Yet, this implicitly implies that the country of concern is not constrained by the demand capacity of the potential destination countries for its exports or by the supply capacity of the potential sourcing countries for its imports. In the aforementioned two-country world, the internal inconsistency of this assumption becomes very clear: if country  $p$ 's size constrains its exports, it must also constrain country  $q$ 's imports simultaneously, and vice versa.<sup>11</sup> In a multiple-country world, the implicit assumption is likely to hold if the country of concern is a small economy. However, when it comes to cross country comparison involving both small and large economies, this assumption will become invalid and cause downward bias in the openness measure for the large economies.

Once we realize that the problem of the conventional openness measures lies with the asymmetric treatment of exports and imports, the correction reveals itself immediately: both exports and imports have to be adjusted for supply and demand capacities in a symmetric way. Hence we propose the following formula:

$$TOI_i = \frac{X_i + M_i}{[(Y_i)^r + (RWY_i)^r]^{1/r}}, \quad -\infty > r \geq 1 \quad (3)$$

where  $TOI_i$  is our trade openness index for country  $i$ , and  $RWY_i = \sum_{j=1, j \neq i}^N Y_j$  is the total PPP-based GDP measure of the rest of the world for country  $i$ .<sup>12</sup> Here  $RWY_i$  serves as a measure of the external supply and demand capacity constraints for country  $i$ .

This formula is not only theoretically sounder than the conventional one and the alternatives listed in Table 1, but also consistent with the strong empirical evidence associated with gravity models, which suggests that bilateral trade flow is determined by, amongst other factors, the sizes of both the exporting and the importing countries. In the above two-country example, both countries  $p$  and  $q$  will have the same measure of openness regardless the value of  $r$ . This result is intuitive because give the two countries trade the same amount of goods under exactly the same constraints, they should be regarded as equally open.

The denominator in formula (3) is a constant-elasticity-of-substitution (CES) function with homogeneity of degree one, so that the TOI remains scale free. The elasticity of substitution,

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<sup>11</sup> Amongst the studies enlisted in Table 1, Li et al. (2004) and Ferrieri (2006) do indirectly account for the external capacity as their modifications include a measure of relative economic size. However, the specifications of their formulas appear to be rather ad hoc as discussed previously.

<sup>12</sup>  $RWY_i$  includes the GDP of countries that have zero trade with country  $i$  as it is a measure of the capacity of the country's potential trading partners.



$s$ , is equal to  $1/(1-r)$ . The formula is in fact a generalized measure of trade openness as it compresses many other measures as special cases. Table 2 shows various specific forms of  $TOI_i$  for different values of  $r$ , including the special cases of linear ( $r = 1$ ), Cobb-Douglas ( $r \rightarrow 0$ ), and Leontief functions ( $r \rightarrow -\infty$ ).

In the case of the linear function, the denominator simply becomes the world GDP. Since the world GDP is common to all countries, using  $TOI_i(r = 1)$  to compare trade openness across countries is the same as comparing their gross trade volumes. On the other hand, in the case of the Leontief function, as  $RWY_i$  is bigger than  $Y_i$  for any single country in practice,  $TOI_i(r = -\infty) = (X_i + M_i)/Y_i = TII_i$ . That is, the conventional openness measure, TII, is a special and, in fact, an extreme case of the more generalized TOI proposed in this paper.

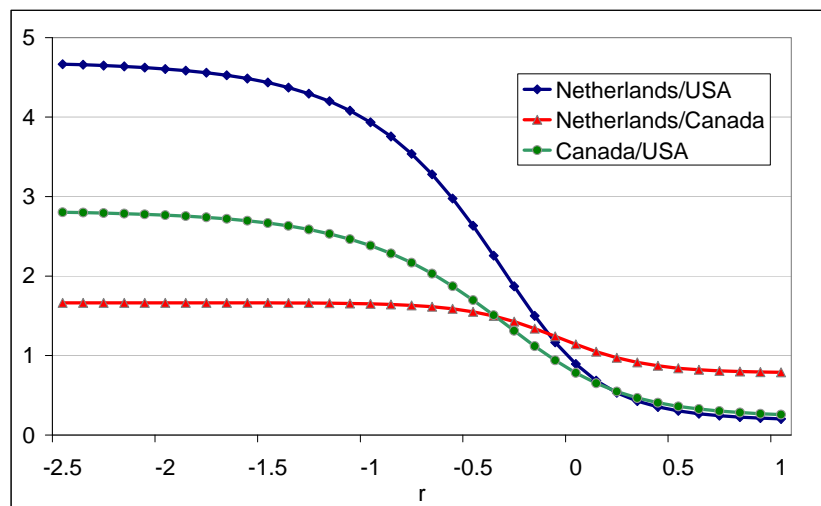
Table 2. Specific forms of TOI for different elasticities of substitution

$r$	$s$	$TOI_i$	Openness measure for $p$ and $q$
1 (Linear)	$\infty$	$\frac{X_i + M_i}{(Y_i + RWY_i)}$ $= \frac{X_i + M_i}{WY}, WY = \text{world GDP}$	$2X / 3Y$
0 (Cobb-Douglas)	1	$\frac{X_i + M_i}{(Y_i \times RWY_i)^{1/2}}$	$2^{1/2} X / Y$
-1	0.5	$\frac{X_i + M_i}{[(Y_i)^{-1} + (RWY_i)^{-1}]^{-1}}$ $= \frac{(X_i + M_i)WY}{(Y_i \times RWY_i)} \text{ or } (X_i + M_i)[(Y_i)^{-1} + (RWY_i)^{-1}]$	$3X / Y$
$-\infty$ (Leontief)	0	$\frac{X_i + M_i}{\min(Y_i, RWY_i)}$ $= \frac{X_i + M_i}{Y_i} \text{ if } Y_i \leq RWY_i$	$2X / Y$

Figure 1 shows the relative trade openness of a small (the Netherlands), a medium (Canada), and a large (United States) country in 2006 against the value of  $r$ . In terms of PPP-based GDP, the size of the US economy is over 10 times of Canada's and over 23 times of the Netherlands's. As expected, the wider the GDP gap between two countries, the larger the variation in the relative openness measure. The relative trade openness measure of any given pair of countries is bounded; the upper bound corresponds to the ratio of two countries' TIIs, while the lower bound corresponds to the ratio of their trade volumes. Thereby, the ratio of maximum to minimum values of the relative openness measures of two countries is exactly the inverse of the ratio of their GDPs.

Two important messages come out of Figure 1. Firstly, there is no definite correction of the size bias. Except for the extreme cases of  $r = 1$  and  $r = -\infty$ , it is not easy to rule out (or in) any other particular value for  $r$  without further theoretical or empirical considerations. Secondly and accordingly, in cross country analyses involving economies of heterogeneous sizes, the choice of the value for  $r$  could potentially make large quantitative differences to their (relative) trade openness measures and subsequently other results that follow.

Figure 1. Relative trade openness values of Singapore, Canada, and the US in 2006



It is interesting to see that when  $r = -1$ , the resulting expression of TOI is associating more closely with many other trade openness measures than when  $r$  is equal to other values. Firstly, for small countries  $RWY_i \approx WY$  and thus, the value of  $TOI_i(r = -1)$  will be very close to that of the  $TII_i$ ; only in the case of large countries would the value of  $TOI_i(r = -1)$  differs noticeably from that of the  $TII_i$ . This sits well with the previous argument that the TII is logically consistent in the small country scenario. Secondly, the measure can be expressed as  $\ln TOI_i(r = -1) = \ln[(X_i + M_i)/Y_i] - \ln(1 - Y_i/WY)$ , which closely resembles the formula of Li et al. (2004) except that all terms are in logarithmic. Thirdly, the measure can be considered as a non-parametric version of Graff's measure. This is because  $\ln TII_i = \ln[(X_i + M_i)/Y_i] = -\ln WY + \ln RWY_i + \ln TOI_i(r = -1)$ . Since  $\ln RWY_i$  is negatively related to  $\ln Y_i$ , if we regress  $\ln TII_i$  against  $\ln Y_i$  and a constant, the residual will closely resemble  $\ln TOI_i(r = -1)$ .<sup>13</sup> In the empirical part, we set  $r = -1$ .

Formula (3) is useful when bilateral flow data are either not readily available (e.g. migration) or available at a high financial cost (e.g. foreign direct investment). This is less an issue for trade flows as bilateral trade data can be obtained from the IMF's Direction of Trade dataset

<sup>13</sup> However, we are not suggesting that because of these,  $-1$  is a preferred value for  $r$ .

at a reasonable cost. When bilateral data are available, a more elaborate measure of trade openness that bases on the same principle will be a weighted average of bilateral openness measures:

$$BTOI_i(\theta_{ij}) = \sum_{j=1; j \neq i}^N \frac{\theta_{ij}(X_{ij} + M_{ji})}{[(Y_i)^r + (Y_j)^r]^{1/r}} \quad (4)$$

where  $BTOI_i$  is the bilateral trade openness index for country  $i$ ;  $X_{ij}$  is the exports from country  $i$  to country  $j$ ;  $M_{ji}$  is the imports of country  $i$  from country  $j$ ; and  $\theta_{ij}$  is the weight for country  $j$ .

At first instance one may nominate GDP share as the weight, i.e.  $\theta_{ij} = Y_j / RWY_i$ . However, there is no reason why a larger trading partner should be weighted more heavily than a smaller one. One may argue that the trade flows with the larger partner are likely to be bigger in absolute terms and therefore it should be weighted more. If trade volume is the reason, then the weight should be trade share instead, i.e.  $\theta_{ij} = (X_{ij} + M_{ji}) / (X_i + M_i)$ . Yet, since (GDP-adjusted) trade flows are the one being weighted, using trade share as a weight implies that the stronger the trade link, the larger the weight on the link, resulting in an upward bias of the index. In conclusion, neither GDP share nor trade share is appropriate weight. In fact, given that the bilateral openness measure already accounts for trade volume and economic size, there is no need to account for them twice. As a result, we opt for a constant weight:  $\theta_{ij} = 1/(N-1)$ . In this case, the BTOI becomes

$$BTOI_i = \frac{1}{N-1} \sum_{j=1; j \neq i}^N \frac{(X_{ij} + M_{ji})}{[(Y_i)^r + (Y_j)^r]^{1/r}} \quad (5)$$

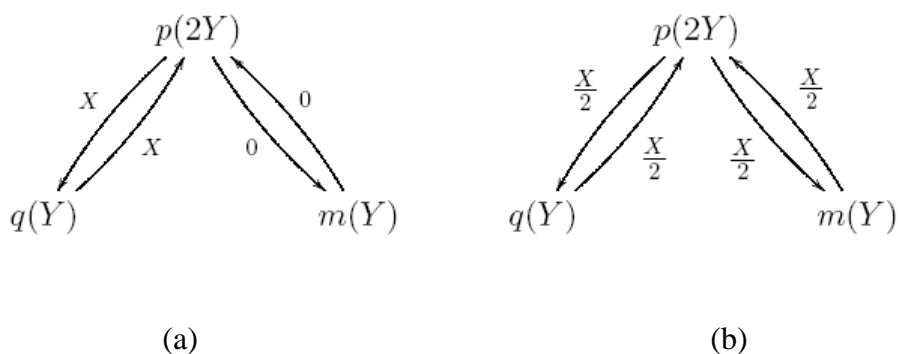
As it will be seen later, the use of bilateral instead of aggregate data makes a big difference to the outcome of trade openness measures. This is because although the TOI is constructed to correcting for the size bias, the actual extent to which the bias is mitigated in equation (3) depends on the value of  $r$ . When  $r$  is very small (i.e. toward negative infinity), the TOI becomes the TII and the size bias will remain the same. On the contrary, in (5), regardless the value of  $r$ , the same GDP-adjusted trade flow item will enter the respective openness measures of the importing and exporting countries, completely removing any bias due to size differences between the two countries. This was already demonstrated previously in the two-country example.

The BTOI formula (5) is the building block of our trade globalization measure.

### 3 Network Analysis

If the objective is to measure trade openness as in contrast to trade globalization, then the TOI developed in Section 2 will be a valid measure. However, as a trade globalization measure, it is still deficient in that it focuses entirely on the GDP-adjusted trade volume of a country but not its connectivity with other countries.<sup>14</sup> To clarify the argument, let's extend the previous example into a three-country world. The additional country,  $m$ , is of the same size as country  $q$ , i.e.  $Y_m = Y$ . Now consider the following two scenarios in Figure 2.

Figure 2. A three-country example



In scenario (a) of Figure 2, country  $p$  trades only with country  $q$  and their trade flows remain the same as before. In scenario (b), country  $p$  shifts half of its trade to country  $m$ . The total trade volumes of  $p$  are the same in both scenarios (i.e.  $2X$ ), but it trades with twice as many countries in the second scenario as in the first one. Intuitively one would consider country  $p$  to be more globalized in the second scenario than in the first one. Yet, in both scenarios, the BTOI for  $p$  is the same. To address this limitation of the TOI, we deploy network analysis.

#### 3.1 A brief introduction to relevant graph theory

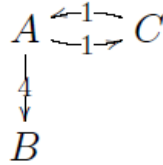
Here it is useful to first briefly review the relevant graph theory, which is the mathematical foundation of network analysis.

The graphical representation of the WTW is a weighted directed graph. A graph consists of a set of nodes or vertices that are connected by a set of edges or links. Directed graphs are those in which the edges are associated with a particular direction. In weighted graphs the edges are associated with a numerical value known as a weight. Figure 3 is an example of a weighted graph, in which  $A$  and  $C$  have links in both direction with unity strength, while  $A$  and  $B$  have a link in only one direction with a strength of four units. A weighted directed

<sup>14</sup> Since the trade flow is GDP-adjusted, the distribution of trade flows does affect the value of  $BTOI_i$  if country  $i$ 's trading partners are of different sizes.

graph can be represented by  $G = (V, E, x)$  where  $V = \{v_1, v_2, \dots, v_N\}$  is the set of vertices or nodes in the graph,  $E = \{(v_a, v_b) | a, b \in \{1, 2, \dots, N\}\}$  is the set of edges or links connecting them and  $x$  is a function that relates each edge to a weight:  $x: E \rightarrow \mathbb{R}^+$ .

Figure 3. Example of a weighted graph



There are a variety of measures of node centrality (node importance) in graphs. Two of the simplest and most commonly used are node degree and node strength. The degree of node  $i$  is the number of links connected to node  $i$ . The strength of node  $i$  is the sum of the weights of all links connected to node  $i$ . In this paper the degree and strength of node  $i$  in a directed network are denoted  $k_i^D$  and  $s_i^D$  respectively.

In this paper, the distribution of link weights is considered to be an important factor in determining the connectivity of a node. Therefore, it is necessary to clarify what is meant by nodes having the same distribution of link weights. Let  $P$  and  $Q$  be sets of link weights where  $Q$  has at least as many elements as  $P$ .  $Q$  is said to have the same distribution as  $P$  if and only if one of the following holds:

1.  $Q$  is a scaling of  $P$ , e.g.  $P = \{1, 2, 2, 4\}$  and  $Q = \{3, 6, 6, 12\}$ ;
2.  $Q$  is a population replication of  $P$ , e.g.  $P = \{1, 2, 2, 4\}$  and  $Q = \{1, 1, 2, 2, 2, 2, 4, 4\}$ ;
3.  $Q$  is both a scaling and a population replication of  $P$ , e.g.  $P = \{1, 2, 2, 4\}$  and  $Q = \{3, 3, 6, 6, 6, 6, 12, 12\}$ .

### 3.2 The World Trade Web

The global goods trading system is known as the World Trade Web (WTW) in the network analysis literature. The WTW is an example of a complex network. Complex networks are graphs that contain too many nodes and links to be effectively illustrated or analysed using standard graph theory. Such systems thereby have to be analysed in terms of their statistical properties. For surveys of these methods, see Albert and Barabasi (2002) and Newman (2003).

In recent years there have been a number of studies that have applied network analyses to the WTW (e.g. see Reyes, Schiavo & Fagiolo, 2008; Li, Jin & Chen, 2003; and the citations in the introduction). A motivation of all these studies is that, while standard statistics such as

import/export volumes can only describe the first-order trade relationships between countries, network analysis can permit the description of the second-order relationships. Examples of the second-order relationships include whether a group of countries form a trade cluster, or the importance of a country as a ‘hub’ in the WTW. These studies almost exclusive focus on the topology of the WTW, i.e. the statistical properties of the whole or parts of the WTW.<sup>15</sup> In relation to the current paper, one of the common findings of the literature is that many structural properties of the WTW display a remarkable stationarity over a long period of time (e.g. two decades), leading to some researchers to cast doubt on the notion that trade globalization has intensified over the recent decades (Fagiolo, Reyes and Schiavo 2007a).

A limitation of the current literature, as summarized well by Fagiolo, Reyes and Schiavo (2007a), is that “the majority of existing works have only aimed at providing another (albeit more powerful) way of *describing* trade patterns, but they have not succeeded in showing how these descriptions can help in *explaining* macroeconomic dynamics” (emphases original). But recently, Kali and Reyes (2007, 2009) have made progress in that direction, applying trade networking measures in macroeconomic economic analysis. Their first study shows that a country’s position in the WTW, after controlling for its trade openness, still has a substantial implication for economic growth, even though the study is based on cross sectional data without controlling for reverse causality. Their second study shows that the network characteristics of the WTW can help explain why financial shocks originating from highly connected countries are more contagious.

In the WTW, countries are referred to nodes and the flows between them as links. Following formula (5), the trade globalization index (TGI) developed in this paper uses the GDP-adjusted bilateral trade flows as a measure of link weight:

$$x_{ij} = \frac{X_{ij}}{[(Y_i)^r + (Y_j)^r]^{1/r}} \quad (6)$$

$x_{ij}$  represents the GDP-adjusted export volume from country  $i$  to country  $j$ . Since here we focus on *international* trade flow,  $x_{ii} = 0 \forall i$ . Also, because the WTW is modelled as a directed network, import flows,  $x_{ji}$ , are considered to be positive flows in the other direction. Therefore,  $x_{ij} \geq 0 \forall i, j$ .

We consider countries that are more integrated into the WTW to be more globalized. We define the degree of integration into the WTW and, hence, the TGI as a function of (1) the strength of the country; (2) the degree of the country; and (3) the distribution of its flows (i.e.

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<sup>15</sup> Because of this, many studies make use of the global (instead of local) network measures, such as centralization index, clustering coefficient, network density, assortative mixing, degree correlation; see Kali and Reyes (2007) for further explanation.

link weights) with other countries. This definition is rather important because by restricting the arguments of the TGI function, we effectively exclude some network indexes (this will be made clearer later).

Numerous network indexes have been developed in the literature, some pertain to the characteristics of the entire network (e.g. node density) and some pertain to the characteristics of individual nodes (e.g. node strength). Given that the purpose of the current paper is to measure how globalized individual countries are, it is the indexes that characterize the connectivity of individual nodes in a network that are appropriate here. Still, there are a fair number of indexes that fall into this category. One possible approach is to compute the value of all these indexes, and then examine if they all point to similar conclusions. An obvious hurdle of this approach is how to set the threshold of similarity and what to do if the conclusions are not similar. Another possible approach is to combine all relevant indexes into a composite index using, e.g., principal component methods. A limitation of this approach is that the weighting assigned in principal component methods may not be an appropriate one. An even bigger but hidden danger of taking this approach is that, some of the existing network indexes may not be applicable to the specific network system of interest. As Borgatti (2005) pointed out, different index formulas make different implicit assumptions about the way in which an item flows in a network, and since index formulas are network specific, applying a formula to incompatible networks could get misleading results.

Considering these problems, here we propose to use an axiomatic approach to the development of a TGI. An axiomatic approach requires first to develop a number of properties, i.e. axioms, that a TGI should exhibit, and then to use the axioms to select appropriate formulas for the TGI.

### 3.3 Axioms of a trade globalization index

In what follows, we list each of the axioms and then discuss its implications and/or rationale.

**Axiom 1.** *Scale independent.*

This axiom is automatically satisfied because the link strength is measured in GDP-adjusted trade flows and, thus, it is unit free.

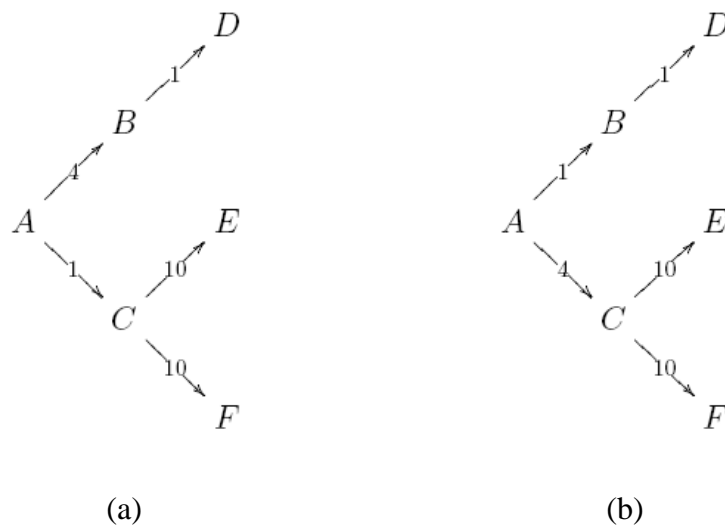
**Axiom 2.** *Nonnegativity.*

Even though  $x_{ij} \geq 0 \forall i, j$ , it does not necessarily guarantee an index constructed using  $x_{ij}$  terms as arguments must be nonnegative. There is nothing to gain from allowing the index to be negative. At the same time, this axiom is not very restrictive as any index values can easily be rescaled to non-negative values.

**Axiom 3.** *Interchanging the weights of any two links of a country will not affect the index value of the country.*

In panel (a) of Figure 4, the link weight between A-B is 4 and that between A-C is 1, while C is better connected than B. Exchanging the weights between A-B and A-C, resulting in panel (b), should have no effect on the index for A according to this axiom. This implies that all incoming and outgoing links with any nodes are weighted equally. This axiom has quite a strong implication. It implies that a country's own globalization measure is independent of the connectivity of its trading partners (and the connectivity of the trading partners' partners and so forth).

Figure 4. Interchanging the weights of links

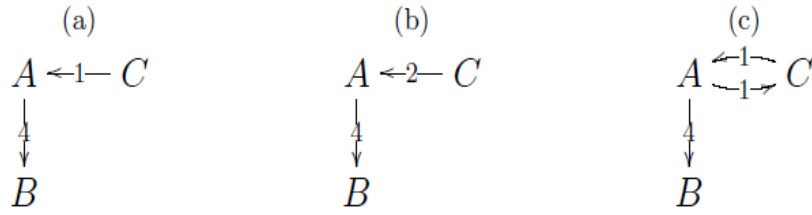


**Axiom 4.** *Strongly monotonically increasing in  $x_i = (x_{i1}, \dots, x_{iN}, x_{N1}, \dots, x_{Ni})$ , the vector of weights for all possible links with country  $i$ .*

If a single  $x_{ij} > 0$  or  $x_{ji} > 0$  is increased, this corresponds to the weight of one link with country  $i$  increasing, holding the total number of links with  $i$  and the weights of the other links with  $i$  constant. This implies country  $i$  is integrated more strongly into the WTW. For instance, in Figure 5 the index for A in panel (b) should be strictly greater than the index for A in panel (a). Additionally, if a single  $x_{ij} = 0$  or  $x_{ji} = 0$  is increased to be greater than zero, this corresponds to an extra link being formed, holding the weights and number of the other links with  $i$  constant. This corresponds to an increase in the connectivity of  $i$  and hence it should also be considered more integrated into the WTW. This is demonstrated in Figure 5. The index for A in panel (c) should be strictly greater than the index for A in panel (a).



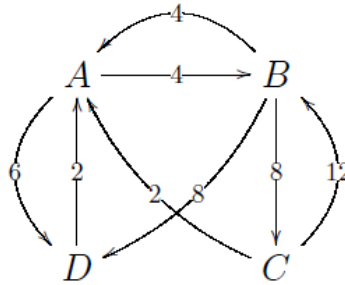
Figure 5. Strongly monotonically increasing in  $x_i$



**Axiom 5.** *Linear homogeneity in country strength, holding the degree and the distribution of link weights constant.*

If the strength of a country is doubled, keeping the degree and the distribution of link weights constant, it should be considered twice as integrated into the WTW. This is displayed in Figure 6. The strength of A,  $s_A^D$  is 18 and the strength of B,  $s_B^D$ , is 36. All else about countries A and B is the same. Therefore for Axiom 3 to be satisfied, the index for B should be twice the index for A.

Figure 6. Linear homogeneity in country strength



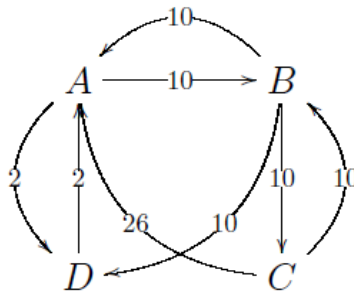
The requirement to hold the degree and distribution of link weights constant means that this axiom can be restated as linear homogeneity in  $x_i$ . Note that Axioms 3 and 5 do not imply Axiom 4. For example, let  $x$  and  $y$  be the weights of links with country  $i$ . Therefore,  $x, y > 0$ .  $f(x, y) = x^2 y^{-1}$  is nonnegative and linearly homogeneous in  $(x, y)$  but is monotonically *decreasing* in  $y$ .

**Axiom 6.** *Strictly increasing in the evenness of the distribution of link weights, holding the strength and the degree constant.*

The index should be of a larger value if trade flows with a country are more evenly distributed across its links, holding its strength and the degree constant. A country that has moderate volumes of trade flows with a number of other countries is more globalized than a country that has nearly all its trade flows with one particular country and hence should have a

higher index value. For an example of this axiom refer to Figure 7,  $s_A^D = s_B^D = 50$ ,  $k_A^D = k_B^D = 5$ , the set of link weights with A is  $\{2,2,10,10,26\}$  and the set of link weights with B is  $\{10,10,10,10,10\}$ . The link weights with B are more evenly distributed than the link weights with A. Therefore the index for B should be strictly greater than the index for A.

Figure 7. Strictly increasing in the evenness of the distribution of link weights

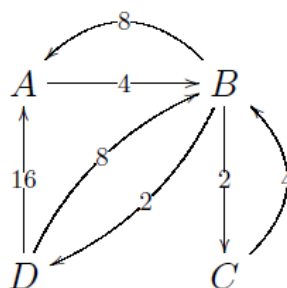


It should be noted since the trade flows of a country are normalized by its own GDP as well as the GDP of its trading partners, evenly distributed link weights do not necessary imply evenly distributed gross trade flows. For instance, if a country trades with a large country as well as with a small country, then an even distribution of its trade flows means that it will trade more with the larger country and trade less with the smaller one.

**Axiom 7.** *Linearly homogenously increasing in country degree, holding the strength and the distribution of link weights constant.*

All else held constant, a country that is connected to many other countries is more globalized than one that is connected to just a few other countries. In the example in Figure 8,  $s_A^D = s_B^D = 28$ ,  $k_B^D = 6 = 2k_A^D$ . The set of link weights with B is  $\{2,2,4,4,8,8\}$ , which has the same distribution as the set of link weights with A,  $\{4,8,16\}$ . Therefore the index for B should be twice the index for A.

Figure 8. Linear homogenously increasing in country degree



## 4 A Trade Globalization Index

In this section, we apply Axioms 1 to 7 to select appropriate network indexes as our TGI.

### 4.1 Classes of centrality indexes from network analysis

Since the purpose of constructing a TGI is to compare and contrast, both in cross-section and over time, the extent to which countries are integrated in the WTW, it is indexes of node centrality that are relevant. Centrality captures the idea that some nodes within a network are more important or influential than other nodes (Koschützki, Lehmann, Peeters, Richter, Tenfelde-Podehl & Zlotowski 2005). There are three main classes of centrality indexes. Degree centrality indexes measure the interaction or popularity of the node in the network (Freeman 1979, Newman 2005). Closeness centrality indexes measure the ability of the node to interact with other nodes in the network without relying on intermediaries. This can also be interpreted as the efficiency or speed with which a node is able to interact with other nodes (Freeman 1979). Betweenness centrality indexes measure how much power the node has in controlling the interaction between other nodes (Freeman 1979). Closeness and betweenness centrality index values for a given node depend on not only the connectivity of the node itself, but also the connectivity of its partners, and of the partners of its partners and so forth (see Borgatti 2005; Reyes, Schiavo & Fagiolo 2008). As a result, they will not satisfy Axiom 3.<sup>16</sup> Thereby, in what follows we only compare the TGI with existing degree class indexes of centrality.

Degree centrality indexes can be sub-divided into binary network analysis indexes and weighted network analysis indexes.<sup>17</sup> Only those that are applicable to directed networks are considered here.

### 4.2. Degree class centrality indexes.

Binary network analysis is the original form of network analysis. All binary network analysis indexes are based on the adjacency matrix. Each element of the adjacency matrix,  $a_{ij}$ , takes a value of one if there is a link from node  $i$  to node  $j$  and a value of zero if there is no link from  $i$  to  $j$ . Existing degree centrality indexes for binary networks include node degree centrality

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<sup>16</sup> For instance, suppose node  $A$  is well connected with other nodes but node  $B$  is not, and node  $C$  is initially connected to node  $A$  but not to node  $B$  (i.e. the strength of the  $C$ 's link with  $A$  is positive and that with  $B$  is zero). Exchanging the strength between the links with  $A$  and  $B$  will therefore reducing the index value for  $C$ .

<sup>17</sup> Binary network analysis is considered the first level analysis. Weighted network analysis, by taking the strength of links into account, is considered as the second level analysis. Most recently there has emerged another (i.e. the third) level of analysis in which the nodes themselves are assigned a degree of freedom known as "fitness" (e.g. see Glattfelder & Battiston 2009). The fitness of a node measures its ability to connect with other nodes. For instance, Garlaschelli & Loffredo (2004) suggest that in the WTW the fitness of a country is determined by its economic size – despite the obvious endogeneity problem.

(Nieminen 1974), relative node degree centrality (Freeman 1979), average nearest neighbour degree (e.g. Fagiolo, Reyes & Schiavo 2008), and eigenvector centrality using the adjacency matrix (e.g. Jackson 2008, Newman 2008). Node degree centrality index simply measures the total number of links that a node has. Relative node degree centrality index is equal to the node degree centrality index divided by maximum potential degree, i.e. the number of nodes in the network minus one.

All binary network measures of centrality focus only on the number of links, ignoring volume and distributional considerations. Due to this, they all fail to satisfy Axioms 4, 5 and 6 (refer to Table 3). Moreover, average nearest neighbour degree and eigenvector centrality measures, like closeness and betweenness measures, depend on the connectivity of a node's partners and thus fail to satisfy Axiom 3. Additionally, in the WTW some links may be hundreds of times smaller than others and yet binary network analysis treats them all as equally important. A natural progression from this is to consider weighted network analysis centrality indexes.

Table 3. Axioms satisfied by existing degree class centrality indexes for binary networks

Axiom	Formula	1	2	3	4	5	6	7
Node degree centrality	$k(i) = \sum_{j=1}^N (a_{ij} + a_{ji})$	✓	✓	✓	✗	✗	✗	✓
Relative node degree centrality	$\frac{k(i)}{N-1}$	✓	✓	✓	✗	✗	✗	✓
Average nearest neighbour degree	$\frac{1}{k(i)} \sum_{j=1; j \neq i}^N k(j)$	✓	✓	✗	✗	✗	✗	✗
Eigenvector centrality using the adjacency matrix	$\lambda \mathbf{x} = \mathbf{A} \mathbf{x}$ $\lambda =$ the largest eigenvalue of adjacency matrix $\mathbf{A}$ $\mathbf{x} =$ vector of centrality	✓	✓	✗	✗	✗	✗	✗

The formulae for weighted network analysis statistics are based on the weight matrix. The elements of the weight matrix are denoted by  $w_{ij}$ .<sup>18</sup> Existing degree class centrality indexes for weighted networks include strength, relative strength, weighted average nearest neighbour degree (Barrat, Barthélemy, Pastor-Satorras & Vespignani 2004), average nearest neighbour strength, and eigenvector centrality using the weight matrix (e.g. Jackson 2008, Newman 2004).

Existing weighted network indexes all fail to account for the distribution of link weights, and therefore fail to satisfy Axiom 6 (refer to Table 4). Moreover, all of them also fail to satisfy

<sup>18</sup> In the case of the TGI,  $w_{ij} = x_{ij}$ .

Axiom 7. Average nearest neighbour strength and eigenvector centrality measures, once again, fail to satisfy Axiom 3.

Table 4. Axioms satisfied by existing degree class centrality indexes for weighted networks

Axiom <sup>19</sup>	Formula	1	2	3	4	5	6	7
Node strength	$s(i) = \sum_{j=1}^N (w_{ij} + w_{ji})$	✓	✓	✓	✓	✓	✗	✗
Relative node strength	$\frac{2s(i)}{\sum_{j=1}^N s(j)}$	✓	✓	✓	✓	✓	✗	✗
Weighted average nearest neighbour degree	$\sum_{j=1}^N \frac{(w_{ij} + w_{ji})}{s(i)} k(j)$	✓	✓	✓	✓	✗	✗	✗
Average nearest neighbour strength	$\frac{1}{k(i)} \sum_{j=1; j \neq i}^N s(j)$	✓	✓	✗	✗	✗	✗	✗
Eigenvector centrality using the weight matrix	$\lambda \mathbf{x} = \mathbf{W} \mathbf{x}$ $\lambda =$ the largest eigenvalue of weight matrix $\mathbf{W}$ $\mathbf{x} =$ vector of centrality	✓	✓	✗	✓	✗	✗	✗

In addition, Fagiolo, Reyes & Schiavo (2007a) use the Herfindahl-Hirschman concentration index to measure how uneven the distribution of link weights with a country is. The inverse of this index satisfies Axiom 6. It also satisfies Axioms 1-3. However, it does not satisfy the remaining axioms.

### 4.3 A New Trade Globalization Index

Given that no existing network indexes satisfy all of the specified axioms, we develop a new TGI. The new TGI is a function of the *weight matrix* containing GDP-adjusted bilateral trade flow,  $\mathbf{X} = [x_{ij}]_{N \times N}$  where  $x_{ij} = X_{ij} [(Y_i)^r + (Y_j)^r]^{-1/r}$ , and satisfies all of the specified axioms.

Initially the most general set of functions was taken. This set was subjected to each of the axioms in turn and functions were eliminated from the set with each repetition. The first set of results is stated as Proposition 1.

**Proposition 1.** *The Trade Globalization Index satisfies all of Axioms 1 to 7 if*

$$TGI(x_i) = \frac{1}{N-1} \left\{ \sum_{j=1}^N [(x_{ij})^{1/2} + (x_{ji})^{1/2}] \right\}^2 \quad (7)$$

<sup>19</sup> Node strength and average nearest neighbour strength measures satisfy Axiom 1 (i.e. being scale free) only if the input itself is scale free, which is the case here.

where  $x_i = (x_{i1}, x_{i2}, \dots, x_{iN}, x_{1i}, x_{2i}, \dots, x_{Ni})$ , and  $x_{ij} = \frac{X_{ij}}{[(Y_i)^r + (Y_j)^r]^{1/r}}$ .

Proof. Refer to Appendix A.

It should be emphasized that  $TGI(x_i)$  as shown in formula (7) is not an arbitrary CES function. It is the *only* CES function that satisfies all of Axioms 1 to 7. In fact, the authors are not able to identify any other functional forms that satisfy Axioms 1 to 7. However, it is incorrect to conclude that (7) is the unique formula for the TGI, as it is explained next.

#### 4.4 Alternative specifications

If we define the link weight as the absolute trade flow without adjusting for GDP:<sup>20</sup>

$$x_{ij}^* = X_{ij} = M_{ji} \quad (8)$$

then the following index also satisfies all the seven axioms:

$$TGI(x_i^*, Y_i, RWY_i) = \frac{1}{N-1} \frac{\left\{ \sum_{j=1}^N [(x_{ij}^*)^{1/2} + (x_{ji}^*)^{1/2}] \right\}^2}{[(Y_i)^r + (RWY_i)^r]^{1/r}} \quad (9)$$

The functional form of the numerator guarantees that Axioms 2-7 are satisfied, while the GDP component in the denominator acts as a normalization factor, guaranteeing that Axiom 1 is satisfied.

This alternative TGI formula can be expressed as the product of two indexes:

$$\begin{aligned} TGI(x_i^*, Y_i, RWY_i) &= \frac{1}{N-1} \frac{\left\{ \sum_{j=1}^N [(x_{ij}^*)^{1/2} + (x_{ji}^*)^{1/2}] \right\}^2}{(X_i + M_i)} \frac{(X_i + M_i)}{[(Y_i)^r + (RWY_i)^r]^{1/r}} \\ &= TCI_i \times TOI_i \end{aligned} \quad (10)$$

where  $TCI_i = \left\{ \sum_{j=1}^N [(x_{ij}^*)^{1/2} + (x_{ji}^*)^{1/2}] \right\}^2 [(X_i + M_i)(N-1)]^{-1}$  is a trade connectivity index, and  $TOI_i$  is the trade openness measure using aggregate trade data. Here the TCI measures the evenness of the distribution of a country's trade flows while the TOI measures the trade openness of the country.

<sup>20</sup> Fagiolo, Reyes & Schiavo (2007b) uses the same definition of link weight.

The main difference between (7) and (9) is in how the evenness of the distribution of trade flows is assessed. In (7), it is the evenness of GDP-adjusted trade flows that matters, while in (9), it is the evenness of the absolute trade flows that matters regardless the distribution of economic sizes of a country's trading partners.

Furthermore, before specifying the axioms, it has been assumed the TGI is a function of the strength, the degree, and the distribution of the link weights of the country. It thereby excludes other possible arguments such as the strength and degree of the country's trading partners, which are important in defining the closeness and betweenness centrality indexes. If we assume the TGI function to have those factors as arguments at the outset, the resulting axioms and index could be very different. For instance, the current set of axioms is appropriate if the purpose is to examine the effect of trade globalization on growth or inequality<sup>21</sup>; however, if the purpose is to examine how financial globalization may increase countries' vulnerability to contagion, then the connectivity of a country's partners will become of paramount importance and thus Axiom 3 should be replaced with an axiom that reflects this.<sup>22</sup>

#### 4.5 Financial globalization

The axioms developed above are quite general and therefore can be applied to many other economic (or non-economic) networks once the link weight is appropriately defined. An obvious extension is to apply the index to measure financial globalization. Assuming that we have some measures of bilateral financial capital flows (see the data section for a discussion of the data issue), a financial globalization index (FGI) can be constructed in a very similar way as the TGI:

$$FGI_i = \frac{1}{N-1} \left\{ \sum_{j=1}^N \left[ (v_{ij})^{1/2} + (v_{ji})^{1/2} \right] \right\}^2 \quad (11)$$

where  $v_{ij} = \frac{\text{financial capital flows from country } i \text{ to country } j}{[(Y_i)^r + (Y_j)^r]^{1/r}}$ ; and  $v_{ii} = 0, \forall i$ .

The only difference between the FGI and the TGI is that the link weight for the former is GDP-adjusted financial capital flows instead of trade flows.

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<sup>21</sup> Yet, if one hypothesizes that export globalization and import globalization may have different effects on growth and inequality (e.g. as in Jaumotte, Lall & Papageorgiou 2008), it is straightforward to modify the TGI to measure only exports or imports.

<sup>22</sup> Also, it is not clear if it is possible to construct an index that satisfies other axioms in this circumstance.

## 4.6 Human capital globalization

Besides trade and capital flows, the international movement of human capital is also a fixture of globalization. In the era of knowledge economy, human capital flow is of particular importance. Brain drain has been a big issue for developing countries. In this sense one may argue that the measurement of human capital globalization is misleading as some countries experience mostly outflows of human capital but little inflows. However, recent literature suggests that labour sourcing countries will also benefit from brain gain or brain exchange in the long run, as some (even though not the majority) outgoing people eventually return home after gaining overseas education or working experience. Also, the development of information and telecommunication technology also allows diasporas to contribute to their mother countries' development without physically relocating back home. For example, it has been reported that there is a new trend that African academics and other professionals abroad use digital technology to help their universities, schools or individuals back home to catch up with the developed countries (BBC, 29 May 2010).

Again, assuming that we have some measures of bilateral human capital flows, a human capital capitalization index (HGI) can be constructed accordingly:

$$HGI_i = \frac{1}{N-1} \left\{ \sum_{j=1}^N [(s_{ij})^{1/2} + (s_{ji})^{1/2}] \right\}^2 \quad (12)$$

where  $s_{ij} = \frac{\text{human capital flows from country } i \text{ to country } j}{[(POP_i)^r + (POP_j)^r]^{1/r}}$ ;  $s_{ii} = 0, \forall i$ ; and  $POP_i$  is the population of country  $i$ .

## 5 Data

### 5.1 Bilateral merchandise trade flows

Trade globalization index is constructed using bilateral trade data from the International Monetary Fund Direction of Trade Statistics, 2009 (DoT). There are multiple measures of trade in DoT database, including (1) exports of a country to the world by partners; (2) imports (f.o.b. – free on board) of a country from the world by partners; (3) imports (c.i.f. – cost, insurance, and freight) of a country from the world by partners.

In principle, items (1) and (2) should be identical as both do not include c.i.f. adjustment, i.e.  $X_{ij} = M(f.o.b.)_{ji}$ . However, typically there are inconsistency in the records between the importing and the exporting countries due to various reasons, such as differences in classification concepts and detail, time of recording, valuation, and coverage, as well as processing errors. For instance, the discrepancy between the US and China regarding their



bilateral trade balance is well-knowingly large due to their differences in the treatment of re-exports via Hong Kong. In the literature of capital flight, researchers will use trade records of developed countries as the benchmark measure for trade flow between developed and developing countries. Since the main focus of this paper is on the methodology of measuring globalization instead of the precise trade balance between individual pair of countries, we do not intend to reconcile every single difference, instead we resort to taking simple average of items (1) and (2) if both are available. If the data for (1) is missing, then we use (2) only, and vice versa. If both (1) and (2) are missing, then we assume that country  $i$ 's export to country  $j$  is zero. This assumption makes sense because if the export volume is discernible, then either the exporting or the importing country should have a record.

The dataset covers 189 countries for 47 years from 1960 to 2006. Thus, it covers almost all countries in the world except some island states. The availability of trade figures is wider than that of GDP figures. If the GDP figures for a country is missing but its trade figures are available, the latter will still be used in the computation of the indexes for its trading partners, even though its own indexes cannot be computed. Indexes can be computed for 182 countries in total.

## 5.2 Purchasing Power Parity based GDP

The PPP-based GDP (current US\$) are sourced from the Penn World Table 6.3 (Heston et al., 2009). For China, the Penn World Table has two series of data, one is based on the official growth rates for the whole period while the other one uses a modification of the official rate. We use the second series (code CH2) as the authors of the Penn World Table suggest that it “provides a more consistent recent economic history of China relative to other countries.”

## 5.3 Bilateral financial asset trade

For financial globalization, the bilateral financial asset trade data are sourced from the Coordinated Portfolio Investment Survey (CPIS) conducted by the IMF. This measure is a stock instead of flow measure.<sup>23</sup> However, there are no other options for bilateral investment data. Another, perhaps even greater, limitation of this dataset is that there are lot more missing observations than trade flows, probably due to confidentiality. But it is important to emphasise that the main purpose here is to demonstrate the method of measuring financial globalization using network analysis. The same dataset was used previously by Fagiolo, Reyes & Schiavo (2007b) to examine international financial integration. The dataset covers a much shorter period of 2001 to 2006.

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<sup>23</sup> A very small number of observations in the original dataset are of negative values. In those cases, we assume the figure refers to capital flows in the opposite direction. More specially, we then set  $V_{ij} = 0$  and add  $-V_{ij}$  to  $V_{ji}$ , where  $V_{ij}$  is the stock of portfolio investment from country  $i$  in country  $j$ .

Besides foreign portfolio investment, foreign direct investment (FDI) and bank loans<sup>24</sup> are two other major types of international investment. They are not considered here mainly due to data limitation. While bilateral FDI data are available at UNCTAD, the cost of obtaining them, even for a single year, is prohibitive for the authors.<sup>25</sup>

#### 5.4 Bilateral tertiary student flows

Due to the lack of bilateral migration data<sup>26</sup>, we use bilateral tertiary student flow data as a proxy for human capital flows. The data were obtained from the UNESCO. The data covers the period of 1999 to 2007. The dataset contained bilateral tertiary student flow data for about 200 countries. 125 countries received student inflows during the sample period while 195 countries had outflows for the same period. Overall, approximately 10% of international (or internationally mobile) student inflows from unidentified source countries. However, for some host countries, such as Norway and Belgium, the proportion of inflowing students with no specific country of origin can be as large as 30% in some years. We allocate those students to countries according to the distribution of the students with specified country of origin.

A host country that has no detail of the country of origin of international students at all is China. However, although China is a main source of international students, it hosts only 42,138 international students, which is a very small number compared to its huge population (about 1.2 billion people). As a result, in practical sense, it is equivalent to assume that China does not host any international students.

## 6 Results

The following results are obtained based on  $r = -1$ .

### 6.1 Trade globalization

Since our dataset is a panel, we can examine the impact of the new measure over time for any given country or cross-country for any given time. Table 5 summarize the times series and cross-sectional correlation coefficients between the TII, TOI, BTOI and TGI. The time series correlations are computed in the following way: for each of the 182 countries, we compute the correlation coefficients between the four indexes, and then take the average of these coefficients across all the countries. The figures in the brackets are the standard deviations.

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<sup>24</sup> Bank loans are classified as “other investments” in the International Financial Statistics glossary.

<sup>25</sup> Information about the cost is available at: <http://www.unctad.org/Templates/Page.asp?intItemID=3206>.

<sup>26</sup> According to the World Bank, bilateral migration data are available only for a few selected OECD countries, but are not available for the vast majority of countries. For details see the Data Note of Migration and Remittances Factbook at [www.worldbank.org/prospects/migrationandremittances](http://www.worldbank.org/prospects/migrationandremittances). Mayda (2007) obtains bilateral data on migrates inflowing into 14 OECD countries over 1980-1995 from the International Migration Statistics for OECD countries.

The cross-sectional correlations are computed in the opposite way: for each year of 1960 to 2006, we compute the cross sectional correlation coefficients between the four indexes, and then take the average of these coefficients across all the years.

Table 5. Correlation Coefficients of TII, TOI, BTOI and TGI

47 year; 182 countries	TII-TOI	TII-BTOI	TOI-BTOI	TGI-TII	TGI-TOI	TGI-BTOI
Time series	> 0.999 ( $< 0.001$ )	0.840 (0.205)	0.840 (0.204)	0.715 (0.236)	0.716 (0.236)	0.702 (0.230)
Cross section	> 0.999 ( $< 0.001$ )	0.194 (0.149)	0.202 (0.147)	0.014 (0.011)	0.016 (0.012)	0.849 (0.129)

In general, the times series correlation is much larger than the cross-sectional one, with the exceptions of TII-TOI and TGI-BTOI. The very high correlations for TII-TOI is related to the use of  $r = -1$ . As shown previously in Figure 1, the difference between the two indexes depends very much on the choice of the value for  $r$ . However, as far as the objective of this paper is concerned, the correlations between the TGI and TII are most important, and the figures show that while their time series correlation is moderately large, their cross-sectional correlation is close to zero.

To further explore the cross sectional difference between different indexes, Table 6 shows the top 10 countries based on the average values of the four indexes respectively over 2001-2006.<sup>27</sup> The values of the top most countries are normalized to 100. The list based on the TII is dominated by small countries as expected. When the TOI is used, there are some changes in the composition of the top 10 but the list remains dominated by small countries. However, then the BTOI is used, there are significant changes in that the top 10 list now includes some of the largest countries. This is because the use of bilateral data in the BTOI completely removes the size bias embodied in openness measures that are based on aggregate data. The use of the TGI sees the large countries moving further upward and expelling small countries like Liberia out of the list. This result is due to the fact that, large countries in general have much more extensive trading networks than small countries and the TGI captures this character successfully. Only two top 10 TII countries can maintain a presence in the top 10 TGI list, namely Belgium and Singapore. As the TGI in general reverses the orders of large and small countries based on the TII, that is why the cross sectional correlation between the two indexes is very low. On the other hand, as relative sizes and connectivity typically change only gradually, that is why the temporal correlation between the two indexes is high.

<sup>27</sup> 2001-2006 is the longest period of time for which we can compare the TGI, FGI and HGI later.

Table 6. Top 10 countries according to the TII, TOI, BTOI and TGI, 2001-2006

TGI		BTOI		TOI		TII	
United States	100	United States	100	Liberia	100	Liberia	100
Germany	62.5	Germany	72.2	Singapore	36.6	Singapore	24.1
France	47.5	Liberia	60.0	Bermuda	36.4	Hong Kong	19.6
United Kingdom	39.3	France	56.1	Belgium	29.7	Malaysia	15.8
China	39.1	Japan	47.1	Hong Kong	29.6	Djibouti	15.1
Italy	35.7	China	46.4	Netherlands	21.7	Guyana	14.2
Japan	33.9	Singapore	42.8	Ireland	21.6	Maldives	13.7
Netherlands	29.6	South Korea	41.5	St. Vincent & the Grenadines	19.7	Belgium	13.4
Belgium	26.0	United Kingdom	41.0	Bahamas	19.5	Antigua & Barbuda	12.6
Singapore	20.1	Italy	37.3	Luxembourg	19.4	Bahamas	12.0

To get a better ‘feel’ of the differences that the TGI makes as compared to the conventional index TII, we look more closely at the results of, once again, the Netherlands, Canada, and the US. We plot the times series of the two indexes for the three countries in Figure 9. The values of both series at 1960 are normalized to one.

The pace of globalization of each of the countries varies quite substantially depending on whether the TGI or the TII is used. For all three countries, the TGI indicates a more volatile process of trade globalization than the TII does, but it also indicates a more rapid pace of globalization for Canada and the US over the whole sample period.

Given that each country’s initial index values are normalized to one in Figure 9, the series cannot be compared across countries. To that end, we plot the relative index values for the three countries in Figure 10. The top panel shows that the TII greatly exaggerates the Netherlands’s degree of trade globalization relative to the US’s. According to the TII, the Netherlands was over 5 times more globalized than the US on average, while the TGI indicates that the US was over 3.5 times more globalized than the Netherlands. The big difference between the two results is due to fact that the TII has a size bias against the US as well as it does not account for connectivity. On the contrary, the TGI raises the Netherlands’s degree of trade globalization relative to Canada by about 50% on average. Accordingly, the TII also greatly overstates Canada’s extent of trade globalization as compared to the US. Based on the TII, Canada is about 3.5 times more globalized than the US, however, based on the TGI, the US is 10 times more globalized than Canada.

A preliminary conclusion that can be drawn from this section is that, accounting for size bias and connectivity in measuring trade globalization is paramount importance in cross country comparison, but is also important in temporal comparison for some individual countries.

Figure 9. Trade globalization measures for the Netherlands, Canada, and the US

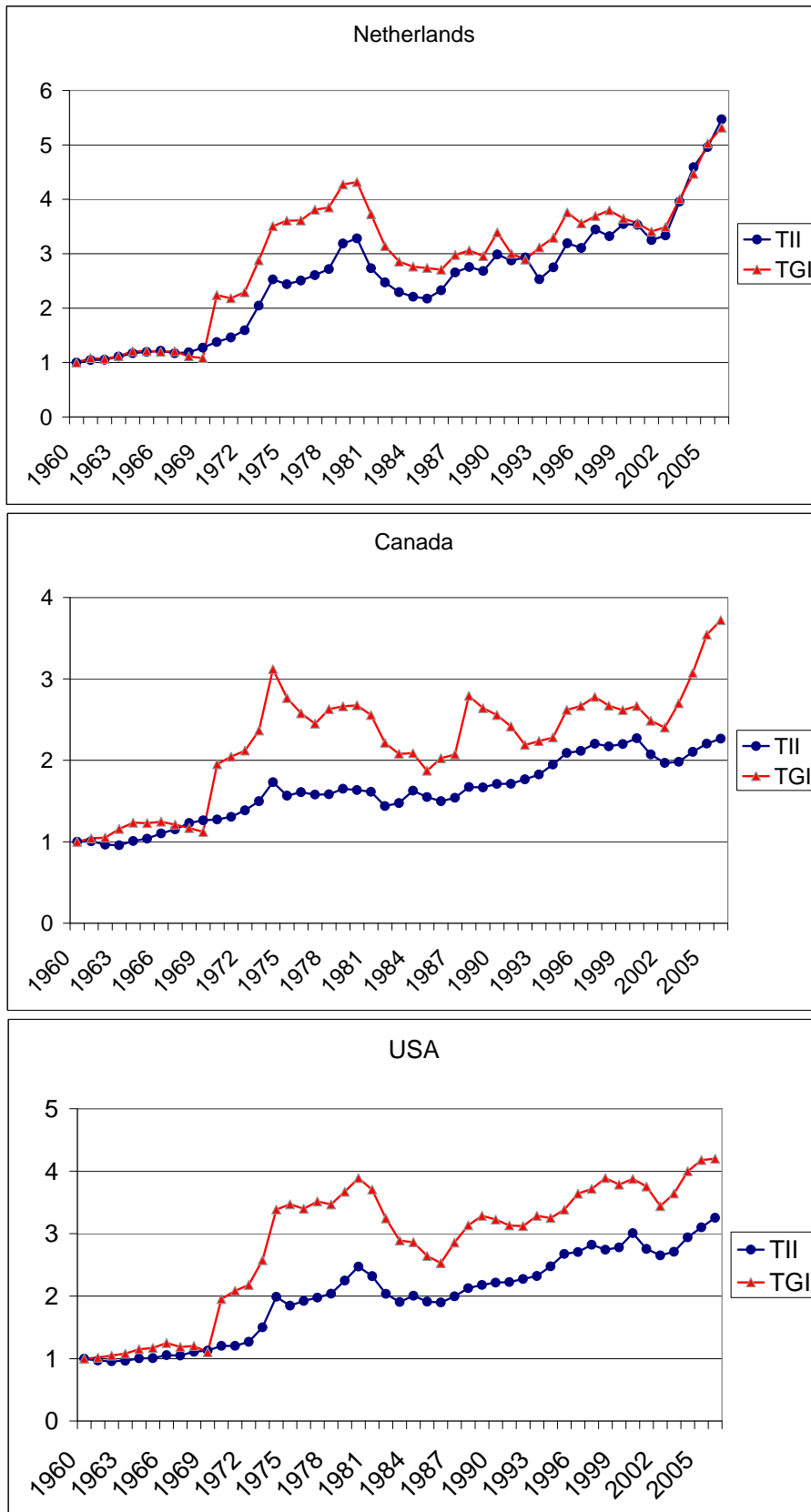
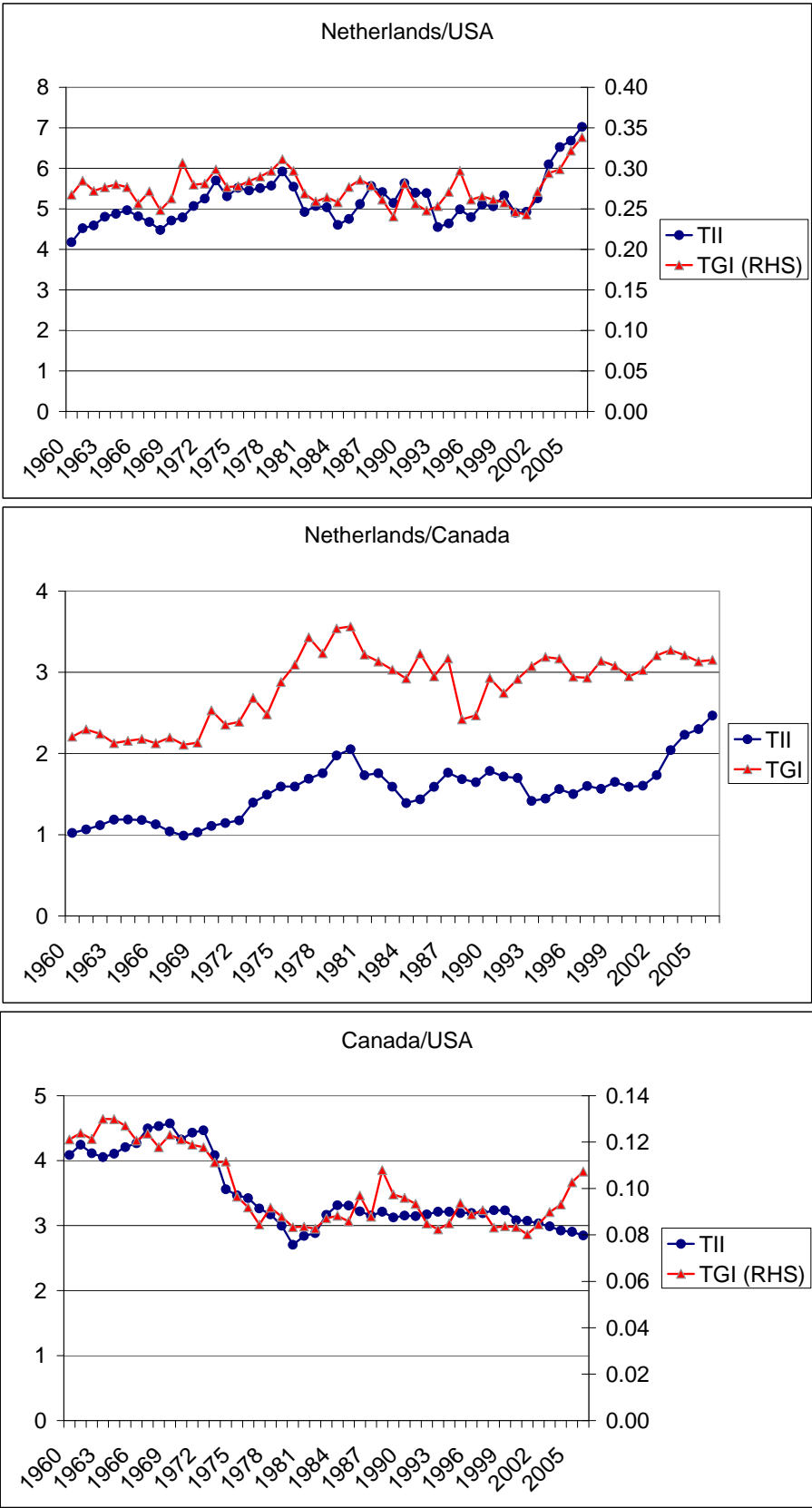


Figure 10. Relative degrees of trade globalization between the Netherlands, Canada, and the US



## 6.2 Comparing trade, financial and human capital globalization

Although we have computed the globalization indexes using portfolio investment stock data (FGI) and tertiary student flow data (HGI) for each of the years in which respective data are available, we do not look at their results individually due to space limit. Instead, we focus on comparing TGI with FGI and HGI.

Table 7 reports the time series and cross-sectional correlation coefficients for the TGI, FGI and HGI. Again, the figures in the brackets are standard deviations. Due to data limitation, the correlations are computed over 2001-2006 and 164 countries only.

Table 7. Correlation Coefficients of HGI, FGI and TGI, 2001-2006

7 years; 164 countries	FGI-HGI	FGI-TGI	HGI-TGI
Time series	0.349 (0.320)	0.556(0.321)	0.376 (0.299)
Cross section	0.062 (0.031)	0.164 (0.028)	0.182 (0.034)

Like the results in Table 5, the time series correlations between the three measures of globalization are larger than the cross sectional one. However, the time series correlation coefficients are not as high as those in Table 5, and the cross sectional correlation coefficients are not as low as their counterparts too. Yet, these results still imply that there are substantial differences in the pace of globalization along these three dimensions of globalization over time and even more so across countries.

Table 8 shows the top 10 most globalized countries according to the 2001-2006 average values of TGI, FGI, and HGI respectively. The index values are normalized such that the largest figure for each index is equal to 100.

Table 8. Top 10 countries according to the TGI, FGI and HGI, 2001-2006

TGI		FGI		HGI	
United States	100	Luxembourg	100	Cyprus	100
Germany	62.5	Bermuda	86.5	United Kingdom	81.9
France	47.5	United States	63.1	Australia	78.1
United Kingdom	39.3	United Kingdom	20.4	Switzerland	73.5
China	39.1	Germany	16.5	Norway	70.1
Italy	35.7	Ireland	14.6	France	69.4
Japan	33.9	France	12.7	Germany	58.8
Netherlands	29.6	Netherlands	11.2	Iceland	55.3
Belgium	26.0	Italy	8.3	Sweden	48.6
Singapore	20.1	Switzerland	7.9	Ireland	47.2

A number of findings are worth highlighting. Firstly, the majority of the top 10 countries are west European countries. Secondly, there are quite a number of countries appearing in at least two lists, including Germany, France, the UK, the US, Italy, the Netherland, Ireland,

Switzerland, and the first three countries appear on all three lists. This suggests that countries that are highly globalized in one aspect also tend to be highly globalized in other aspects. The top positions of Luxembourg and Bermuda on the FGI list are consistent with the fact that the two countries are well-known offshore financial centres.<sup>28</sup> Likewise, the high positions of the UK and Australia on the HGI list are probably related to the fact that both countries are major players in international education markets. Lastly, the index values fall quite quickly within the top 10 lists. This suggests that there is a hierarchical order of globalization in that a small number of countries being much more globalized than the others. However, due to the short time period of data and the large number of missing observations, especially for the FGI, the results have to be viewed with caution.

A preliminary conclusion can still be drawn from this section in that, the pace of globalization along different dimensions could be very different both across countries and over time. As a result, using globalization measures in one or even a few areas as a proxy for the multifaceted phenomenon of globalization may be misleading. This thereby suggests that multidimensional measures of globalization are highly warranted.

## 6 Conclusion

How do measures of trade globalization measure up? In this paper we argue that one of most commonly used de facto trade globalization measures, the trade intensity index (TII), is unfit for the purpose. The TII is deficient for two reasons: first it is bias again large country; and second, it does not account for the connectivity of countries. To address the size bias problem, we propose a trade openness index (TOI) if aggregate trade data are used and a bilateral trade openness index (BTOI) if bilateral trade data are available. While the TOI is a generalized index in that it incorporates the TII as a special case, the extent to which it mitigates the size bias depends on the choice of the value for parameter  $r$ . On the contrary, the BTOI can eliminate the bias regardless the value of  $r$ . Based on the BTOI, we then develop a new trade globalization index (TGI) using network analysis and an axiomatic approach. A total of seven axioms have been developed in order to identify appropriate formulas for the TGI. To the best of our knowledge, this is the first time the axiomatic approach is used in the network analysis of trade or globalization. The seven axioms are generic to many other economic networks, including international finance and human capital flows. An application of similar indexes to these networks leads to the development of a financial globalization index (FGI) and a human capital globalization index (HGI).

It was found that using the TGI instead of the TII as a measure of trade globalization can make a big difference in temporal comparison for individual countries and a very big

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<sup>28</sup> Despite their names, offshore financial centres (OFCs) do not have to be physically offshore. The Financial Stability Forum (2000) characterizes OFCs as jurisdictions that attract a high level of non-resident financial activity (relative to domestic financial activity) due to favorable economic regimes.



difference in cross country comparison. Furthermore, the relative pace of globalization in goods, financial assets, and human capital respectively is found to be quite different both across countries and over time as well.

These findings have a number of implications. Firstly, since globalization analysis typically involves cross sectional or panel data, the choice of globalization measures could significantly affect the results, at least in quantitative sense. Secondly, it is not quite plausible to use globalization measures in just one or two dimensions, typically trade, as a proxy for the multifaceted globalization phenomenon. Thirdly, researchers can be presented with a dilemma that, when bilateral data of certain aspects of globalization (e.g. for information flows) are not available, whether they should use aggregate data in those aspects or focus on the other aspects with bilateral data. Given that a large share of the differences between the TII and TGI is due to size bias, the second best solution is perhaps to include all the essential aspects of globalization but correct for the size bias when aggregate measures are used.

Lastly, the current version of the paper is still work in progress. There are two natural directions of extension. Firstly, the empirical results reported in this paper are based on the choice of  $r = -1$  and therefore some sensitivity tests on the value of  $r$  are needed. Secondly, the current paper only develops some arguably better measures of globalization without demonstrating how they can aid our understanding of the impacts of globalization on, e.g. inequality or growth; thereby, some empirical applications of the new measures are in order.

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## Appendix A. Proof

**Axioms 1-3.** Immediate.

**Axiom 4.** Let  $x_{i,t}$  be the vector of all potential links with country  $i$  in period  $t$ . Let  $x_{i,2} \geq x_{i,1}$  such that  $x_{i,2} \neq x_{i,1}$ .  $f(x) = x^{1/2}$  is strictly increasing in  $x$ . Therefore

$\sum_{j=1}^N [(x_{ij,2})^{1/2} + (x_{ji,2})^{1/2}] > \sum_{j=1}^N [(x_{ij,1})^{1/2} + (x_{ji,1})^{1/2}]$ . Since  $f(x) = x^2$  is increasing in  $x$  where  $x \in \mathbb{R}^+$ ,  $TGI(x_{i,2}) > TGI(x_{i,1})$ .

**Axiom 5.** Let  $\alpha > 0$ . The only way to multiply country strength holding country degree and the distribution of link weights constant is to multiple  $x_i$  by  $\alpha$ .

$$TGI(\alpha x_i) = [1 / (N - 1)] \left\{ \sum_{j=1}^N [(\alpha x_{ij})^{1/2} + (\alpha x_{ji})^{1/2}] \right\}^2 = \alpha TGI(x_i) \quad (13)$$

**Axiom 6.** Let  $x_{i,1} = (x_{i1}, x_{i1}, \dots, x_{ab}, \dots, x_{cd}, \dots, x_{iN}, x_{Ni})$  (where one of  $a$  and  $b$  is  $i$  and one of  $c$  and  $d$  is  $i$ ) such that  $0 < x_{ab} < x_{cd}$ . Let  $\bar{x}$  be the average of  $x_{ab}$  and  $x_{cd}$ . Let  $0 < \delta < (x_{cd} - \bar{x}) = (\bar{x} - x_{ab})$ . Therefore  $x_{ab} < (x_{ab} + \delta) \leq \bar{x} \leq (x_{cd} - \delta) < x_{cd}$ . Let  $x_{i,2} = (x_{i1}, x_{i1}, \dots, (x_{ab} + \delta), \dots, (x_{cd} - \delta), \dots, x_{iN}, x_{Ni})$ .<sup>29</sup>

$$TGI(x_{i,1}) = [1 / (N - 1)] \left[ (x_{i1})^{1/2} + (x_{i1})^{1/2} + \dots + (x_{ab})^{1/2} + \dots + (x_{cd})^{1/2} + (x_{iN})^{1/2} + (x_{Ni})^{1/2} \right]^2 \quad (14)$$

$$\begin{aligned} & TGI(x_{i,2}) \\ &= [1 / (N - 1)] \left[ (x_{i1})^{1/2} + (x_{i1})^{1/2} + \dots + (x_{ab} + \delta)^{1/2} + \dots + (x_{cd} - \delta)^{1/2} + (x_{iN})^{1/2} + (x_{Ni})^{1/2} \right]^2 \end{aligned} \quad (15)$$

$f(x) = x^{1/2}$  is strictly concave in  $x$  so

$$(x_{ab} + \delta)^{1/2} - (x_{ab})^{1/2} > (x_{cd})^{1/2} - (x_{cd} - \delta)^{1/2} \Rightarrow (x_{ab} + \delta)^{1/2} + (x_{cd} - \delta)^{1/2} > (x_{ab})^{1/2} + (x_{cd})^{1/2}$$

Therefore  $TGI(x_{i,2}) > TGI(x_{i,1})$ .

**Axiom 7.** Let  $\alpha > 0$ . The only way to multiply the degree of a country by  $\alpha$ , holding the country strength and the distribution of link weights constant, is to divide the weight of all links by  $\alpha$  and replicate the population of the resulting link weights  $\alpha$  times.

$$TGI(x_i) = TGI(s_i, k_i).$$

$$TGI(s_i, \alpha k_i) = [1 / (N - 1)] \left\{ \sum_{j=1}^N [\alpha (x_{ij} / \alpha)^{1/2} + \alpha (x_{ji} / \alpha)^{1/2}] \right\}^2 = \alpha TGI(s_i, k_i) \quad (16)$$

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<sup>29</sup> Adding  $\delta$  to  $x_{ab}$  and subtracting it from  $x_{cd}$  ensures that the total strength of country  $i$  is held constant and that the distribution of link weights is more even.