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Intertemporal Poverty Measurement: Tradeoffs and Policy Options

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# Intertemporal Poverty Measurement: Tradeoffs and Policy Options\*

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## Abstract

This paper makes three contributions to the literature on intertemporal poverty measurement, in particular the aggregation of a measure of wellbeing over time and across people. Firstly we conduct an exhaustive analysis of properties of intertemporal poverty measures, identifying relationships between the functional form and properties of ITPMs and identifying the tradeoffs and compatibilities that exist between the properties. We link this to the normative choices a poverty analyst must make when measuring intertemporal poverty. We also determine a ‘recipe’ which may be used to construct poverty measures with properties desired by the poverty analyst. Second, we apply the recipe to construct a new family of intertemporal poverty measures with the desired property of increasing compensation, a property that has thus far not been discussed in the literature. Third, we calculate measures from the new family and compare them to other measures recently proposed in the literature, to evaluate poverty in rural Ethiopia in the period 1994 - 2004.

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# 1 Introduction

Quantitative measurement of poverty began with Rowntree (1901)'s innovative study of poor householders in York. Following the seminal contribution of Sen (1976), the quantitative unidimensional poverty measures developed in this literature aggregate cross-section or 'snapshot' data on a measure of wellbeing<sup>1</sup> across those members of a sample of individuals or households identified as poor. Economists have now reached a reasonable level of consensus on what the desirable properties of such a measure should be, including how to identify the poor and aggregate the data, incorporating depth of poverty and distribution amongst the poor appropriately into the measure (see Atkinson (1987), Ravallion (1996) and Dercon (2006) for progressive reviews). Measures satisfying these properties include those suggested by Chakravarty (1983) and Foster, Greer, and Thorbecke (1984) (subsequently FGT), although the headcount measure – which does not satisfy Sen's *monotonicity* or *transfer* properties – still finds frequent application in policy work.

Both economists and non-economists studying the lives of the poor have criticised this approach, observing that poverty is a multidimensional state, characterised by lack of assets, feelings of insecurity or vulnerability, and deprivation in many dimensions and over extended durations<sup>2</sup>. Whilst much research has agreed in principle with this critique, in practice many challenges arise when attempting to be true to the broader definition of poverty using economic tools and the available empirical data.

Data on many dimensions have been included in policy-relevant analysis, for example, the Human Development Index (UNDP<sup>3</sup>). The theoretical challenges that arise when aggregating data across dimensions of deprivation are analysed in the growing literature on multidimensional poverty measurement, see the review by Atkinson (2003) as an introduction as well as and Bourguignon and Chakravarty (2003) and more recent work by Foster and Alkire (2007).

In this paper, though, we focus on the challenges that arise when attempting to take into account the time dimension. Time-relevant facets of poverty include duration or chronicity, systematic changes, variation and risk or vulnerability. Chronicity can be seen as a key hardship for a number of reasons, firstly, intrinsically and intuitively: we might postulate that longer is simply worse when you are in a difficult situation. Secondly, it may be that being longer in poverty reduces the chances of being able to climb out of poverty; possible mechanisms are that assets are depleted (including body mass) and morale is lower (Narayan-Parker, Patel, Schaff, Rademacher, and Koch-Schulte, 2000). Alternatively, it may be argued that periods in poverty can be compensated for if future welfare improves.

The absence of panel data meant that until recently such propositions could not be tested empirically, but when more than one time period of data is available the possibilities for analysis increase, and it is possible to research questions around poverty dynamics. If someone is observed as poor in a particular survey how representative is that of their

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<sup>1</sup>Typically income or consumption.

<sup>2</sup>On policy, see Bank (2001) and Narayan-Parker, Patel, Schaff, Rademacher, and Koch-Schulte (2000). For philosophical perspectives see Nussbaum and Sen (1999) and the various writings of Amartya Sen (1982), (1997), (2001).

<sup>3</sup>Various years, see <http://hdr.undp.org/en/>

lifetime welfare? If we have poverty measures from two cross sections at the beginning and end of a time period, what percentage of the poor are included in both of these sets?

One strand of empirical literature, mainly in developed countries has analysed poverty ‘spells’, for example estimating the probability of entering or exiting poverty over time (Bane and Ellwood (1986), Stevens (1999)), confirming empirically that longer time spent in poverty increases the probability of being poor in the future<sup>4</sup>. As data became available on two time periods for developing countries, economists calculated transition matrices<sup>5</sup> to conclude that much of observed poverty is transient (see Baulch and Hoddinott (2000) for an overview). The literature further broadened in the 1990’s into analyses of vulnerability, “churning” and transient poverty, and the conceptual analysis of chronic poverty (see Hulme and Shepherd (2003), McCulloch and Calandrino (2003), McKay and Lawson (2003), Chaudhuri and Ravallion (1994) and CPRC (2004) for analysis and empirical applications).

Another strand of the literature on the intertemporal facets of poverty is that of normative poverty measurement, a direct extension of the quantitative poverty measurement discussed above, to the intertemporal case. Several authors have proposed methods to characterise poverty over time, usually into a single number or composite poverty index, generally termed a ‘chronic poverty’ measure. The approach proposed by Jalan and Ravallion (2000) involves decomposing an FGT-style intertemporal poverty measure into ‘chronic’ and ‘transient’ components, where the ‘chronic’ component – which has been most widely adopted and applied in the empirical literature – focuses on those who are poor *on average* during the period under scrutiny. This approach implicitly assumes perfect substitutability of wellbeing across time, both above and below the poverty line. In contrast, the measure proposed by Foster (2007) has its basis in the ‘spells’ or counting approach described above, weighting the depth of poverty experienced by those thereby identified as chronically poor according to an FGT-style transformation before aggregating over time and individuals. Calvo and Dercon (2007) generalise a particular case of this approach and show that it can also be discounted over time. Foster (2007)’s and Calvo and Dercon (2007)’s measures allow some degree of substitution between wellbeings in different periods provided they are all below the poverty line, but none across it. The measure proposed by Foster and Santos (2006) takes an approach intermediate between this and Jalan and Ravallion (2000)’s approach, allowing imperfect substitution both below and across the poverty line.

The current paper builds on this normative poverty measurement literature, contributing a systematic analysis of the properties of intertemporal poverty measures, including a number of key observations on the complementarities and potential tradeoffs between certain desirable properties of the measures. In any application, the properties of the chosen measure must reflect the particular facet(s) of intertemporal poverty the poverty analyst is attempting to capture, and the existence of tradeoffs between properties mean that a measure appropriate for one facet will not necessarily possess properties making it appropriate for another; this perhaps accounts for the current lack of consensus in the literature. We build on the formal analysis to suggest a recipe for construction of

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<sup>4</sup>This has parallels in the unemployment literature, which has shown that longer duration can often send negative signals to employers (thus duration can increase duration even further).

<sup>5</sup>The transition matrix cross-tabulates categories, usually “poor” and “non poor” for both survey periods to analyse what percent of the observations fall into each category

intertemporal poverty measures with desired properties, and apply the recipe to construct a new class of measures which possess an increasing-compensation property. Finally, we apply our new measure, together with others suggested in the recent literature, to the analysis of poverty in rural Ethiopia between 1994 and 2004, using the Ethiopian Rural Household Survey.

## Overview

Section 2 contains a systematic analysis of potential properties of intertemporal poverty measures, and the restrictions they impose on the functional form of the measure. We show that mild and intuitively acceptable properties impose a great deal of structure, within which there are flexible components. The choice of these allows the poverty analyst to determine the ordinal and cardinal properties of the measure, which we assert she should do according to the context of application and the facets of intertemporal poverty she wishes to capture.

Importantly we distinguish groups of properties among which there is no conflict or trade-off (for example, INCREASING COST OF HARDSHIP and INTERTEMPORAL TRANSFER, and groups among which there must be a tradeoff (for example, DURATION SENSITIVITY and INTERTEMPORAL TRANSFER). These results will inform the choice of measures which are appropriate to apply to measurement of a particular facet of intertemporal poverty, for example measurement of the degree of chronic poverty. In fact, we show that several measures previously suggested as appropriate for the measurement of ‘chronic poverty’ do not in fact capture chronicity, but rather, other facets of intertemporal poverty.

The analysis in section 2 provides a method for construction of intertemporal poverty measures with desired properties, and in section 3 we apply this method to construct a new class of measures satisfying the INCREASING COMPENSATION property.

In section 4 we apply our results to the analysis of poverty in some villages in rural Ethiopia between 1994 and 2004, using the Ethiopian Rural Household Survey panel data set and some descriptive logit regressions. We find that the assumptions (both explicit and implicit) in the duration-adjusted poverty measures make a considerable difference to the identification of the poorest households in the sample. In terms of individual characteristics, it appears that agricultural shocks (such as crop failure through pests and trampling) as well as illness are correlated with higher poverty. Similar findings to the static poverty literature are larger household size, less educated household heads and low initial assets. There are, however, a number of econometric challenges that are presented if one wishes to attempt any kind of modelling of this kind of duration-adjusted poverty measure; studies of consumption using panel data usually control for household fixed-effects or unobservables correlated with the average level of consumption over time. When we study duration poverty we are trying to understand exactly those characteristics, and we have only one observation for each household in the ten year period. This is one of our challenges for further analytic work.

Section 5 concludes and notes possibilities for further work.

## 2 Analysis of Intertemporal Poverty Measures and their Properties

In this section we discuss properties of an intertemporal poverty measure, some of which have been suggested in the recent literature and others which we introduce. Taking certain of these properties as fundamental and axiomatic, we are able to restrict significantly the space of possible intertemporal poverty measures, as well as providing a framework in which a poverty analyst may construct a measure with properties that she regards as necessary for applied context in which she is working. In particular we show how the properties of the trajectory ordering, trajectory-distribution ordering and cardinal properties may be determined recursively by the composition of suitable functions.

To put this in the context of the extant literature on intertemporal poverty measures, Jalan and Ravallion (1998) do not discuss the properties of their chronic or transient poverty measures. Foster and Santos (2006) and Foster (2007) do determine and discuss many of the properties satisfied by their measures. Calvo and Dercon (2007) make a broader discussion of possible properties and their connection with the functional form of the measure; however, without formal justification they restrict attention to certain functional structures (thus excluding the possibility of achieving some desirable properties) and their analysis is informal. This paper presents the first general and rigorous discussion, in the intertemporal context, of the connections between properties and the form of the measure. We follow a similar approach to that taken by, for example Foster and Shorrocks (1991) in the context of static poverty measures.

We shall illustrate the analysis by reference to the intertemporal poverty measures proposed and favoured by the above authors.

### 2.1 Notation and Definitions

For the purpose of our theoretical analysis, we assume the availability of data  $x \in \mathbb{R}_+$  on a cardinal, interpersonally and intertemporally comparable indicator of wellbeing<sup>6</sup> for each of a set of  $N \in \mathbb{N}$  homogeneous individuals<sup>7</sup> labelled  $i = 1, 2, \dots, N$  in each of a set of  $T \in \mathbb{N}$  time-periods labelled  $t = 1, 2, \dots, T$ . We thus have an  $(N \times T)$  matrix of data

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<sup>6</sup>We will never have a perfect measure of wellbeing, but in practice must use a proxy. It may be argued that value of consumption is typically the best indicator of achieved wellbeing available, certainly better than income which may be subject to intertemporal smoothing. This is discussed further in section 4 in the context of poverty in rural Ethiopia. Of course, the arguments made in the literature on multidimensional poverty measurement, for example ? and Bourguignon and Chakravarty (2003) will apply here, and so  $x$  may itself be a multidimensional index. However, the introduction of several dimensions raises issues that are not dealt with in this paper, for example the order of intertemporal, multidimensional and social aggregation.

<sup>7</sup>Though data may be available just at the household level; in practice, in order to maintain comparability (for example between households of different composition, or between individuals with different characteristics) the data may need to be transformed. These issues are discussed further in section 4.

points,

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1T} \\ x_{21} & x_{22} & \cdots & x_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NT} \end{pmatrix}. \quad (2.1)$$

(We are operating in a world of certainty, with a fixed number of individuals, whose wellbeing is observed in every time period of interest.) The data matrix  $X$  is an element of the set

$$\mathcal{X}_{NT} = \mathbb{R}_+^{N \times T}. \quad (2.2)$$

Our aim will be to compute from the wellbeing data  $X$  a real-valued measure or index  $P$  which represents information about the poverty experienced by the population of individuals whose wellbeing is measured. We may be interested in comparing populations of different sizes, or for which data is available for a different number of time periods, so the domain of the index function must be

$$\mathcal{X} = \bigcup_{N=1}^{\infty} \bigcup_{T=1}^{\infty} \mathcal{X}_{NT}.$$

**Definition 1.** An *Intertemporal Poverty Measure (ITPM)* is a function

$$P : \mathcal{X} \rightarrow \mathbb{R} \quad \text{such that} \quad P : X \mapsto P(X) \quad (2.3)$$

whose properties, as defined and discussed below, are congruent with the intertemporal measurement of poverty.

We shall abuse notation, letting  $N$  represent the set  $\{1, 2, \dots, N\} \subset \mathbb{N}$  and  $T$  represent the set  $\{1, 2, \dots, T\} \subset \mathbb{N}$ .

We may wish to refer to the **trajectory** of wellbeings experienced by a particular individual  $i \in N$ , that is, a row of the data matrix, for which we shall use the notation  $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iT})$ .

We may then think of a data matrix  $X \in \mathcal{X}$  as representing a **distribution of trajectories**. The set  $\mathcal{X}$  represents all possible distributions of trajectories.

We may wish to refer to the **distribution of wellbeings** experienced by the population in a particular time period  $t \in T$ , that is, a column of the data matrix, for which we shall use the notation  $X_t = (x_{1t}, x_{2t}, \dots, x_{Nt})'$ .

We shall also be interested in families of intertemporal poverty measures, indexed by a particular level<sup>8</sup> of wellbeing  $z \in \mathbb{R}_+$ .

**Definition 2.** A *wellbeing-indexed ITPM* is a function

$$P : \mathcal{X} \times \mathbb{R}_+ \rightarrow \mathbb{R} \quad \text{such that} \quad P : (X, z) \mapsto P(X; z) \quad (2.4)$$

whose properties, as defined and discussed below, are congruent with the intertemporal measurement of poverty.

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<sup>8</sup> $z$  typically has the interpretation of a ‘poverty line’ though that need not be the case.



## Intertemporal Poverty Measures in the Literature

We shall refer to a number of intertemporal poverty measures suggested in the recent literature; for clarity we write these measures following the notational convention defined above.

Jalan and Ravallion (2000)'s 'total poverty' measure  $P_{JRT}$  is essentially FGT-2 (squared poverty gap) aggregated over time periods as well as individuals,

$$P_{JRT}(X; z) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \left(1 - \frac{x_{it}}{z}\right)^2 \mathbb{I}(x_{it} \leq z) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \left(\max \left[0, 1 - \frac{x_{it}}{z}\right]\right)^2 \quad (2.5)$$

while their 'chronic poverty' measure  $P_{JRC}$  is FGT-2 applied to each individual's mean wellbeing,

$$P_{JRC}(X; z) = \frac{1}{N} \sum_{i=1}^N \left(1 - \frac{\bar{x}_i}{z}\right)^2 \mathbb{I}(\bar{x}_i \leq z) = \frac{1}{N} \sum_{i=1}^N \left(\max \left[0, \frac{1}{T} \sum_{t=1}^T \left(1 - \frac{x_{it}}{z}\right)\right]\right)^2. \quad (2.6)$$

Foster (2007)'s measure is similar to  $P_{JRT}$ , but he incorporates a 'poverty line'  $\tau$  in the time dimension so that a household's wellbeings only enter if that household is below the wellbeing poverty line  $z$  in a proportion of periods greater than  $\tau$ . (He also permits a flexible power parameter  $\alpha$ ; for comparability, and because it yields attractive properties, we shall take  $\alpha = 2$ ; in our empirical section we take  $\tau = 0.5$ .)

$$P_F(X; z) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \left(1 - \frac{x_{it}}{z}\right)^\alpha \mathbb{I}(x_{it} \leq z) \mathbb{I}\left(\sum_{t=1}^T \mathbb{I}(x_{it} \leq z) \geq \tau T\right) \quad (2.7)$$

Calvo and Dercon (2007) analyse a great variety of individual poverty measures, obtained by permuting three operations applied to the wellbeing data: focus, convex decreasing transformation, and linear aggregation over time. They discuss the properties of the measures thus generated, and prefer that obtained by the sequence focus, transformation, aggregation. As a representative measure they apply this with an FGT transformation to obtain (incorporating linear social aggregation),

$$P_{CD}(X; z) = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \beta^{T-t} \left(1 - \frac{x_{it}}{z}\right)^\alpha \mathbb{I}(x_{it} \leq z) \quad (2.8)$$

$$= \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \beta^{T-t} \left(\max \left[0, 1 - \frac{x_{it}}{z}\right]\right)^\alpha, \quad (2.9)$$

equivalent to Jalan and Ravallion (2000)'s total poverty measure, generalised from  $\alpha = 2$  and by the inclusion of the discount factor. In our empirical analysis we restrict attention

to the case with  $\alpha = 2$  and  $\beta = 1$ , identical to  $P_{JRT}$ . Calvo and Dercon (2007) also consider measures which are not additively separable over time.

The approach taken by Foster and Santos (2006) is somewhat different from the above; a generalised mean or constant elasticity of substitution aggregation over time yields their family of measures,

$$P_{FS}(X; z) = \begin{cases} \frac{1}{\beta} \frac{1}{n} \frac{1}{T} \sum_{i=1}^n \max \left[ 0, \sum_{t=1}^T \left( 1 - \left( \frac{\hat{x}_{it}}{z} \right)^\beta \right) \right] & \text{for } \beta \leq 1; \beta \neq 0, \\ \frac{1}{\beta} \frac{1}{T} \sum_{i=1}^n \max \left[ 0, \sum_{t=1}^T (\ln z - \ln \hat{x}) \right] & \text{for } \beta = 0. \end{cases} \quad (2.10)$$

All of the above measures (except Jalan and Ravallion (2000)'s 'total poverty' measure) are referred to by their authors as 'chronic poverty' measures; as we shall discuss below, their properties enable them to capture particular intertemporal aspects of poverty, though not necessarily chronicity *per se*.

## 2.2 Induced Orderings and Functional Structure

An intertemporal poverty measure  $P$  (2.3) is a real-valued function and thus induces a natural total preorder<sup>9</sup>  $\preceq$  on its domain set  $\mathcal{X}$ , such that

**Definition 3** (Induced Trajectory-Distribution Ordering). *For any  $X, Y \in \mathcal{X}$ ,*

$$X \preceq Y \quad \text{if and only if} \quad P(X) \leq P(Y). \quad (2.11)$$

As elements of  $\mathcal{X}$  are distributions of trajectories, and  $P$  is an intertemporal poverty measure,  $X \preceq Y$  should be read 'distribution  $Y$  contains at least as much intertemporal poverty as distribution  $X$ '. What 'intertemporal poverty' means in this context will of course depend on the properties of  $P$  as discussed below.

Given  $N$  and  $T$ , an individual  $i \in N$  and trajectories  $\mathbf{w}_j$  for each  $j \neq i$ , an ITPM  $P$  induces a total preorder of the trajectory space  $\mathbb{R}_+^T$ :

**Definition 4** (Induced Trajectory Ordering  $iNTX$ ). *Given  $N, T, i \in N$  and  $X_{-i} = [\mathbf{w}_j]_{j \neq i}$ , for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$ ,*

$$\mathbf{x} \preceq_{iNTX} \mathbf{y} \quad \text{if and only if} \quad P(X) \leq P(Y). \quad (2.12)$$

where

- $[X]_{it} = x_t$  and  $[Y]_{it} = y_t$  for each  $t \in T$ , and
- $[X]_{jt} = [Y]_{jt} = [\mathbf{w}_j]_t$  for each  $j \neq i$  and each  $t \in T$ .

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<sup>9</sup>A total preorder is a total (complete) transitive binary relation.

**Definition 5.** Given  $N, T$  and  $i \in N$ , say that  $P$  is **separable**<sup>10</sup> in  $i$  given  $N$  and  $T$  if the induced trajectory ordering  $iNTX$  is independent of the  $\mathbf{w}_j$ . In that case the induced trajectory ordering may be represented by  $\lesssim_{iNT}$ .

**Definition 6.** [SEPARABILITY] An ITPM  $P$  is SEPARABLE if, given  $N$  and  $T$ , it is separable in each  $i \in N$ .

**Proposition 1.** Given  $N$  and  $T$ , any SEPARABLE ITPM may be written in the form

$$P(X) = A(p_1(\mathbf{x}_1), p_2(\mathbf{x}_2), \dots, p_N(\mathbf{x}_N)) \quad (2.13)$$

where  $A : \bigcup_{N \in \mathbb{N}} \mathbb{R}^N \rightarrow \mathbb{R}$  is strictly increasing in each of its arguments, and each  $p_i : \mathbb{R}_+^T \rightarrow \mathbb{R}$ . Conversely, any function of the form (2.13) is SEPARABLE.

*Proof.* Converse: the function  $p_i$  establishes the ordering  $\lesssim_{iNT}$ , which is preserved in  $P$  by monotonicity of  $A$ .  $\square$

SEPARABILITY is a natural regularity or independence property; given that an ITPM reflects an ordering over trajectories for each individual in the distribution, we require that ordering to be invariant to changes in the other individuals' trajectories. That is, if trajectory  $\mathbf{x}$  experienced by individual  $i$  is 'no poorer than' trajectory  $\mathbf{y}$ , this ordering should not be reversed if there are changes in the trajectories experienced by individuals other than  $i$ .

We now strengthen this property to introduce symmetry across individuals in the trajectory ordering; if trajectory  $\mathbf{x}$  is 'no poorer than' trajectory  $\mathbf{y}$  for some individual  $i$ ,  $\mathbf{x}$  should be 'no poorer than' trajectory  $\mathbf{y}$  for any other individual  $j$ .

**Definition 7.** [TRAJECTORY-ORDERING] An ITPM  $P$  is TRAJECTORY-ORDERING if it is SEPARABLE and, given  $T$ , the induced trajectory orderings  $\lesssim_{iNT}$  are identical for each  $N \in \mathbb{N}$  and each  $i \in N$ .

Notation: we represent the trajectory orderings induced by a TRAJECTORY-ORDERING ITPM by  $\lesssim_T$ .

For  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$ , we interpret  $\mathbf{x} \lesssim_T \mathbf{y}$  as ' $\mathbf{y}$  is at least as poor as  $\mathbf{x}$ '. For each  $T \in \mathbb{N}$  we may define two derived relations on  $\mathbb{R}_+^T$ :

- $\mathbf{x} \sim_T \mathbf{y}$  if  $\mathbf{x} \lesssim_T \mathbf{y}$  and  $\mathbf{y} \lesssim_T \mathbf{x}$ , ' $\mathbf{y}$  is equally as poor as  $\mathbf{x}$ '.
- $\mathbf{x} \prec_T \mathbf{y}$  if  $\mathbf{x} \lesssim_T \mathbf{y}$  and  $\mathbf{y} \not\lesssim_T \mathbf{x}$ , ' $\mathbf{y}$  is strictly poorer than  $\mathbf{x}$ '.

All intertemporal poverty measures suggested in the literature are TRAJECTORY-ORDERING, although the property has not previously been stated explicitly.<sup>11</sup> Our motivation for introducing the TRAJECTORY-ORDERING property is to elucidate the relationship between the functional form of the measure and its properties.

<sup>10</sup>In the sense of Gorman (1968).

<sup>11</sup>Note that it is not equivalent to the ubiquitous property of anonymity or symmetry across individuals; that does not restrict interactions between individuals whereby orderings may change according to trajectories experienced by others. Also it is purely an ordinal property while full symmetry is cardinal. However, we shall later impose symmetry as well.

**Proposition 2.** 1. Given  $T$ , any TRAJECTORY-ORDERING ITPM may be written in the form

$$P^{(T)}(X) = A^{(T)}(p^{(T)}(\mathbf{x}_1), p^{(T)}(\mathbf{x}_2), \dots, p^{(T)}(\mathbf{x}_N)) \quad (2.14)$$

where  $A^{(T)} : \bigcup_{N \in \mathbb{N}} \mathbb{R}^N \rightarrow \mathbb{R}$  is strictly increasing in each of its arguments, and  $p^{(T)} : \mathbb{R}_+^T \rightarrow \mathbb{R}$ .

2. The ordering induced by  $p^{(T)}$  on  $\mathbb{R}_+^T$  is the trajectory ordering  $\lesssim_T$ .

*Proof.* 1. Separability yields form as above, TRAJECTORY-ORDERING yields  $p_1 = p_2 = \dots = p_N = p^{(T)}$ .

2. Follows directly from  $A$  strictly increasing in all arguments. □

**Corollary 3.** Without loss of generality, any TRAJECTORY-ORDERING ITPM may be written in the form

$$P(X) = A(p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_N)) \quad (2.15)$$

where  $A : \bigcup_{N \in \mathbb{N}} \mathbb{R}^N \rightarrow \mathbb{R}$  is strictly increasing in each of its arguments, and  $p : \bigcup_{T \in \mathbb{N}} \mathbb{R}_+^T \rightarrow \mathbb{R}$ .

*Proof.*  $P(X)$  is trajectory ordering and therefore for each  $T$  has a representation (2.14). The  $A^{(T)}$  are increasing functions of each of their arguments, and trajectory orderings are preserved under increasing transformations, therefore any differences in  $A^{(T)}$  across  $T$  may be captured by making appropriate transformations of the  $p^{(T)}$ , which, collected over  $T$ , form  $p$ . □

We have thus extended the trajectory ordering to  $\bigcup_{T \in \mathbb{N}} \mathbb{R}_+^T$ .

Note that the function  $A$  need not be symmetric in each of its arguments, consistent with our observation above that TRAJECTORY-ORDERING does not entail full symmetry. Note also that each of the ITPMs suggested in the literature are TRAJECTORY-ORDERING and thus may be represented in this form. In fact, their representations above (2.5) – (2.10) are all of this form.

**Corollary 4.** Any TRAJECTORY-ORDERING ITPM may be written in the form

$$P(X) = G(S(p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_N))) \quad (2.16)$$

where  $G : \mathbb{R} \rightarrow \mathbb{R}$  is strictly increasing and  $S : \bigcup_{N \in \mathbb{N}} \mathbb{R}^N \rightarrow \mathbb{R}$  induces the same trajectory-distribution ordering as  $A$ .

*Proof.* Without loss of generality, write  $A = G(S)$  for appropriate functions  $G : \mathbb{R} \rightarrow \mathbb{R}$  and  $S : \bigcup_{N \in \mathbb{N}} \mathbb{R}^N \rightarrow \mathbb{R}$ . Existence is trivial; we may take  $G(x) = x$  and  $S = A$ . □

## 2.3 A Recipe for Construction of Intertemporal Poverty Measures

The above results enable us to suggest a simple recipe for construction of intertemporal poverty measures with desirable properties:

- **Step 1:** Choose a function  $p$  which induces a trajectory ordering with the desired properties.
- **Step 2:** Choose a social aggregation function  $S$  to aggregate over individuals in a way that yields an ordering of distributions of trajectories with desired properties.
- **Step 3:** If necessary, choose a transformation function  $G$  to yield a poverty measure with the desired cardinal properties.

Of course, in practice things may not be so simple, as application of this recipe requires the existence (and discovery) of suitable functions  $p$ ,  $S$  and  $G$ . In section 3 below we apply this recipe to construct a new class of intertemporal poverty measures with an increasing compensation property of the trajectory ordering.

Decomposing  $P$  in this way enables us to distinguish clearly between the different types of properties: properties of the trajectory ordering, properties of the distribution ordering, and cardinal properties of the measure. This will facilitate our analysis greatly, enabling us to clarify tradeoffs and compatibilities between different properties, in a more complete and coherent way than has yet appeared in the literature.

This section continues with an analysis of these three classes of properties. Some properties, which we believe are justified in all circumstances and thus take as axiomatic, enable us to make restrictions on the classes of functions suitable for  $p$ ,  $S$  and  $G$ . Other properties will depend on the particular information which the poverty measure is intended to represent; in some cases we are able to show how the functions may be chosen to incorporate the chosen properties.

## 2.4 Properties of the Trajectory Ordering

Through most of this section we take  $T$  as fixed, and discuss possible properties of the trajectory orderings  $\succsim_T$ . As the properties of  $\succsim_T$  are entirely determined by the function  $p$  we also analyse the relationships between the form of  $p$  and these properties. This will facilitate the choice of suitable functions  $p$  to yield intertemporal poverty measures with desired properties. The function  $p$  of course determines an ordering over the entire trajectory space  $\bigcup_{T=1}^{\infty} \mathbb{R}_+^T$  and so we conclude the section by briefly discussing the properties of the ordering in this dimension.

The first property we consider is continuity of the trajectory ordering.

**Definition 8.** [CONTINUITY] *Given  $T$ ,  $\succsim_T$  is continuous on  $\mathbb{R}_+^T$  if, for any trajectory  $\mathbf{x} \in \mathbb{R}_+^T$  and sequence of trajectories  $\mathbf{y}_s \in \mathbb{R}_+^T$  with limit  $\mathbf{y} \in \mathbb{R}_+^T$  such that  $\mathbf{x} \succsim_T \mathbf{y}_s$  for each  $s \in \mathbb{N}$ ,  $\mathbf{x} \succsim_T \mathbf{y}$ .*

**Proposition 5.** *Given  $T$ ,  $\succsim_T$  is continuous on  $\mathbb{R}_+^T$  if and only if  $p$  is a continuous function on  $\mathbb{R}_+^T$ .*

*Proof.* Topological argument (see Debreu (1960)). □

CONTINUITY is essentially a regularity property, required for a ‘well-behaved’ trajectory ordering. An important argument in its favour is that a non-continuous poverty measure would be excessively sensitive to measurement error at any point of discontinuity. We therefore restrict attention to measures satisfying CONTINUITY. Of the ITPMs suggested in the literature, all satisfy CONTINUITY except  $P_F$ .

It will prove useful to note here a standard result about continuous total preorders on a topological space:

**Lemma 6.** *If  $\succsim_T$  is continuous on  $\mathbb{R}_+^T$  then the ‘indifference’ sets  $I(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}_+^T : \mathbf{y} \sim_T \mathbf{x}\}$  are connected.*

### Sensitivity to Wellbeing: Monotonicity, Focus and Identification

We now consider properties of the trajectory ordering in relation to changes in wellbeing, primarily in a single time period.

We start with a property which is fundamental to the concept of *poverty* measurement and is ubiquitous in the poverty measurement literature. All other things equal, an increase in wellbeing of one individual in one time period should not increase the intertemporal measure of poverty  $P$ . In the context of the trajectory ordering, a trajectory  $\mathbf{x}$  which differs from a trajectory  $\mathbf{y}$  only by having greater wellbeing in a single time period should be ordered as weakly less poor than  $\mathbf{y}$ .

**Definition 9.** [WEAK MONOTONICITY] *Given  $T$ ,  $\succsim_T$  satisfies WEAK MONOTONICITY if, for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $x_t > y_t$  for some  $t \in T$  and  $x_\tau = y_\tau$  for all  $\tau \neq t$ ,  $\mathbf{x} \succsim_T \mathbf{y}$ .*

**Proposition 7.**  *$\succsim_T$  satisfies WEAK MONOTONICITY if and only if  $p$  is a weakly decreasing function of each of its arguments at each point in  $\mathbb{R}_+^T$ .*

*Proof.* ‘If’: weakly decreasing  $p$  yields weakly monotone  $\succsim_T$ . ‘Only if’: any  $p$  consistent with weakly monotone  $\succsim_T$  must be weakly decreasing throughout  $\mathbb{R}_+^T$ . □

It will prove useful to note here that TRAJECTORY-ORDERING, CONTINUITY and WEAK MONOTONICITY together enable us to identify, for every trajectory, a **constant-wellbeing equivalent**<sup>12</sup> trajectory.

**Definition 10** (Constant-Wellbeing Trajectories). *Given  $T \in \mathbb{N}$ , define the set of constant-wellbeing trajectories  $C_T \subset \mathbb{R}_+^T$  as  $C_T = \{\mathbf{c} \mid \mathbf{c} \in \mathbb{R}_+^T, c_1 = c_2 = \dots = c_T\}$ .*

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<sup>12</sup>A similar approach is taken in, for example, ?, though in the context of populations rather than trajectories.

Note that  $C_T$  is the  $T$ -dimensional generalisation of the familiar ‘45-degree line’. It has a natural total order  $\leq$  where for  $\mathbf{c}, \mathbf{d} \in C_T$ ,  $\text{vecc} \leq \mathbf{d}$  if and only if  $c_1 \leq_{\mathbb{R}} d_1$  where  $\leq_{\mathbb{R}}$  is the usual total order of the reals.

**Definition 11** (Constant-Wellbeing Equivalent of a Trajectory). *Given an ITPM  $P$  satisfying TRAJECTORY-ORDERING, CONTINUITY and WEAK MONOTONICITY with induced trajectory orderings  $\{\preceq_T\}$ , define the **constant-wellbeing equivalent** of a trajectory  $\mathbf{x} \in \mathbb{R}_+^T$  as  $\mathbf{c}(\mathbf{x})$  where  $\mathbf{c} : \mathbb{R}_+^T \rightarrow C_T$  such that  $\mathbf{c}(\mathbf{x}) = \min\{\mathbf{c} \in C_T \mid \mathbf{c} \sim_T \mathbf{x}\}$ . (Minimum under the natural total order of  $C_T$  defined above.)*

**Proposition 8.** *For all  $\mathbf{x} \in \mathbb{R}_+^T$ ,  $\mathbf{c}(\mathbf{x})$  exists.*

*Proof.*  $C_T$  is totally ordered and therefore if  $\{\mathbf{c} \in C_T \mid \mathbf{c} \sim_T \mathbf{x}\}$  has a minimal element that minimal element is unique and is thus the minimum. WEAK MONOTONICITY and CONTINUITY are sufficient for  $\{\mathbf{c} \in C_T \mid \mathbf{c} \sim_T \mathbf{x}\}$  to be non-empty; CONTINUITY may be invoked again to show that if non-empty,  $\{\mathbf{c} \in C_T \mid \mathbf{c} \sim_T \mathbf{x}\}$  has a minimal element.  $\square$

WEAK MONOTONICITY is satisfied by all poverty measures suggested in the literature, and is essentially equivalent to, for example, Bourguignon and Chakravarty (2003)’s ‘axiom’ MN. However, it is a weak property and does not ensure any *sensitivity* of the measure to lack of wellbeing. One remedy would be simply to strengthen the property, requiring  $\mathbf{x} \prec_T \mathbf{y}$  rather than  $\mathbf{x} \preceq_T \mathbf{y}$ . However, this would yield a ‘poverty measure’ that was sensitive to the wellbeings of all individuals, even those who are consistently very well-off.

Following Sen (1976) it is conventional in the poverty measurement literature to distinguish between *identification* of the poor among the population being studied, and *aggregation* of information available about those identified as poor to construct the index of poverty. It follows that the index should not be sensitive to the level of wellbeing of those not identified as poor (the ‘focus principle’). This makes intuitive sense; we are unlikely to want our measure of poverty to decrease if, all other things equal, an already well-off person becomes better off. This principle may be formalised as a FOCUS property.

For static, unidimensional measures it is conventional to choose a poverty line  $z$ , and demand that the measure is not sensitive to wellbeings  $x$  which lie above this line.<sup>13</sup> It is not entirely straightforward to extend this concept to several dimensions or multiple time periods. Bourguignon and Chakravarty (2003) define two possibilities for the multidimensional context, whose analogues in the intertemporal context, expressed as properties of the trajectory ordering,<sup>14</sup> we give here:

<sup>13</sup>If  $z$  is chosen independently of the observed distribution of wellbeings, this corresponds to the concept of absolute rather than relative poverty, an approach that we shall maintain throughout the paper. Note that identification could, in principle, be based on information different from that incorporated in the poverty measure. We shall ignore this possibility; if such data were available and informative about poverty, we maintain that it should be incorporated in the wellbeing measure.

<sup>14</sup>Bourguignon and Chakravarty (2003) state the properties as properties of the poverty measure itself, the direct analogues being STRONG FOCUS: The trajectory-indexed ITPM  $P$  satisfies STRONG FOCUS relative to the set of trajectories  $Z$  if, for any  $N$  and  $T$ , for any  $i \in N$  and  $t \in T$ , and for any  $X, Y \in \mathcal{X}_{NT}$  such that

- $x_{it} > y_{it}$  and  $y_{it} > z_t^T$ ,
- $x_{i\tau} = y_{i\tau}$  for all  $\tau \neq t$ , and

**Definition 12.** [STRONG FOCUS] *Given  $T$ ,  $\succsim_T$  satisfies STRONG FOCUS relative to  $z \in \mathbb{R}_+$  if, for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $x_t > y_t > z$  for some  $t \in T$  and  $x_\tau = y_\tau$  for all  $\tau \neq t$ ,  $\mathbf{x} \sim_T \mathbf{y}$ .*

**Definition 13.** [WEAK FOCUS] *Given  $T$ ,  $\succsim_T$  satisfies WEAK FOCUS relative to  $z \in \mathbb{R}_+$  if, for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $x_t > y_t > z$  for some  $t \in T$  and  $x_\tau = y_\tau > z$  for all  $\tau \neq t$ ,  $\mathbf{x} \sim_T \mathbf{y}$ .*

Note that STRONG FOCUS entails WEAK FOCUS.

It follows directly from the definitions that if  $\succsim_T$  satisfies STRONG FOCUS relative to  $z \in \mathbb{R}_+$  then for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $x_t > y_t > z$  for some  $t \in T$  and  $x_\tau = y_\tau$  for all  $\tau \neq t$ ,  $p(\mathbf{x}) = p(\mathbf{y})$ . If  $\succsim_T$  satisfies WEAK FOCUS relative to  $z \in \mathbb{R}_+$  then, for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $x_t > y_t > z$  for some  $t \in T$  and  $x_\tau = y_\tau > z$  for all  $\tau \neq t$ ,  $p(\mathbf{x}) = p(\mathbf{y})$ .

Intuitively, under WEAK FOCUS, the function  $p$  (and thus the intertemporal poverty measure  $P$ ) are not sensitive to changes in the wellbeing in any period, for an individual whose level of wellbeing lies above a ‘poverty line’  $z$  in *every* period. Under STRONG FOCUS, this property is strengthened, so that  $p$  (and thus  $P$ ) are not sensitive to changes in wellbeing in any period above the poverty line, for any individual (even if that individual’s wellbeing lies below the poverty line in other periods).

We note here that all of the intertemporal poverty measures suggested in the literature satisfy WEAK FOCUS, while  $P_F$  and  $P_{CD}$  additionally satisfy STRONG FOCUS.

These alternative focus properties do not ensure sensitivity, but simply restrict it. In order to define a property that ensures sensitivity of the measure to poverty or lack of wellbeing, we must return to the idea of identification. In the intertemporal context, we take this to mean identification of trajectories of wellbeings as ‘intertemporally poor’.

Given the concept of the trajectory ordering, we have a natural way to identify *poor* and *non-poor* trajectories.

**Definition 14.**  $\succsim_T$  identifies the trajectory  $\mathbf{x} \in \mathbb{R}_+^T$  as **non-poor** if  $\mathbf{x} \sim_T \mathbf{y}$  where  $y_t$  is arbitrarily large for each  $t \in T$ .

**Definition 15.**  $\succsim_T$  identifies the trajectory  $\mathbf{x} \in \mathbb{R}_+^T$  as **poor** if  $\mathbf{y} \prec_T \mathbf{x}$  where  $\mathbf{y}$  is any non-poor trajectory.

- 
- $x_{j\tau} = y_{j\tau}$  for all  $j \neq i$  and all  $\tau \in T$ ,

$P(X) = P(Y)$ , and WEAK FOCUS: The trajectory-indexed ITPM  $P$  satisfies WEAK FOCUS relative to the set of trajectories  $Z$  if, for any  $N$  and  $T$ , for any  $i \in N$  and  $t \in T$ , and for any  $X, Y \in \mathcal{X}_{NT}$  such that

- $x_{it} > y_{it}$  and  $y_{it} > z_t^T$ ,
- $x_{i\tau} = y_{i\tau}$  and  $y_{i\tau} > z_\tau^T$  for all  $\tau \neq t$ , and
- $x_{j\tau} = y_{j\tau}$  for all  $j \neq i$  and all  $\tau \in T$ ,

$P(X) = P(Y)$ .



Weak monotonicity yields considerable structure: if  $\mathbf{x}$  is non-poor under the WEAKLY MONOTONE trajectory ordering  $\lesssim_T$  then any trajectory  $\mathbf{y}$  with wellbeings  $y_t \geq x_t$  in each period  $t \in T$  must also be non-poor. Similarly, if  $\mathbf{x}$  is poor under the WEAKLY MONOTONE trajectory ordering  $\lesssim_T$  then any trajectory  $\mathbf{y}$  with wellbeings  $y_t \leq x_t$  in each period  $t \in T$  must also be poor.

It will be convenient to label the space of trajectories identified as poor; let  $\Phi^T = \{\mathbf{x} \in \mathbb{R}_+^T : \mathbf{x} \text{ is poor under } \lesssim_T\} \subset \mathbb{R}_+^T$ .

We are now able to strengthen the monotonicity property in a fairly general way that does not conflict with the focus principle:

**Definition 16.** [STRICT MONOTONICITY] *Given  $T$ ,  $\lesssim_T$  satisfies STRICT MONOTONICITY if, for any  $\mathbf{x}, \mathbf{y} \in \Phi^T$  such that  $x_t > y_t$  for all  $t \in T$ ,  $\mathbf{x} \prec_T \mathbf{y}$ .*

A stronger monotonicity property satisfied by Jalan and Ravallion (2000)'s and Foster and Santos (2006)'s measures though not by Calvo and Dercon (2007)'s nor Foster (2007)'s is:

**Definition 17.** [STRONG MONOTONICITY] *Given  $T$ ,  $\lesssim_T$  satisfies STRONG MONOTONICITY if, for any  $\mathbf{x}, \mathbf{y} \in \Phi^T$  such that  $x_t > y_t$  for some  $t \in T$ ,  $\mathbf{x} \prec_T \mathbf{y}$ .*

We can also define a general focus property:

**Definition 18.** [FOCUS] *Given  $T$ ,  $\lesssim_T$  satisfies FOCUS if  $\Phi^T \neq \emptyset$  and there exists a non-poor trajectory  $\mathbf{x}^T \in \mathbb{R}_+^T$  comprising finite wellbeings in every period.*

Each of the intertemporal poverty measures suggested in the literature satisfy STRICT MONOTONICITY and FOCUS(though the 'headcount' version of Foster's measure does not satisfy STRICT MONOTONICITY).

We now seek a more practical approach to identification.

**Proposition 9.** *Given  $T$  and a trajectory ordering  $\lesssim_T$  which satisfies CONTINUITY, STRICT MONOTONICITY and FOCUS, there exists a hypersurface (connected space of dimension  $T - 1$ )  $Z$  of trajectories  $\mathbf{x} \in \mathbb{R}_+^T$  bounding  $\Phi^T$ ; this is intersected exactly once by the space of constant-wellbeing trajectories, and thus may be labelled by that wellbeing  $z \in \mathbb{R}_+^T$  such that  $\mathbf{z} = (z, z, \dots, z) \in Z$ . Without loss of generality we may represent  $\lesssim_T$  with a function  $p$  where  $p(\mathbf{z}) = 0$ ; then  $p(\mathbf{x}) > 0$  for all poor trajectories  $\mathbf{x}$  and  $p(\mathbf{x}) = 0$  for all non-poor trajectories.*

*Proof.* Topological argument. □

The wellbeing level  $z$  represents that constant level of wellbeing which is on the margin of being regarded as ‘intertemporally poor’. It is important to be clear that it is not directly analogous to the poverty line in static, unidimensional poverty measurement; a better analogue for the poverty line is the entire space of marginally poor trajectories  $Z$ .

It is helpful to consider the shape of  $Z$  for the measures suggested in the literature. For  $P_{JRC}$  it is the simplex passing through  $\mathbf{z}$ , for  $P_{CD}$  it is an extended L-shape, while for  $P_{FS}$  it is an isoquant of the CES function on which the measure is based. As  $P_F$  is not in general continuous,  $Z$  may not be connected; its shape is very complex and does not intuitively correspond to an idea of ‘marginally poor’ trajectories.

## Intertemporal Intrapersonal Transfer

We now consider the sensitivity of the poverty measure to changes in wellbeing in more than one time period. We shall see that the choices made here are central to the concept of intertemporal poverty measurement, and lead to strong restrictions on the form of the function  $p$ . We identify desirable but incompatible properties, between which the poverty analyst must make a normative or empirical choice, according to the context of application.

To what extent can a relatively high level of welfare in one period compensate for low welfare in another? This is a crucial consideration in the choice of intertemporal poverty measure, and one in which there is no consensus as yet in the literature. As we noted in the introduction, Jalan and Ravallion (2000)’s measure averages wellbeings across time and thus allows perfect compensation between periods while both Foster (2007) and Calvo and Dercon (2007) only allow (imperfect) compensation between periods both below the poverty line; Foster and Santos (2006)’s measure does allow imperfect compensation both below and across the poverty line.

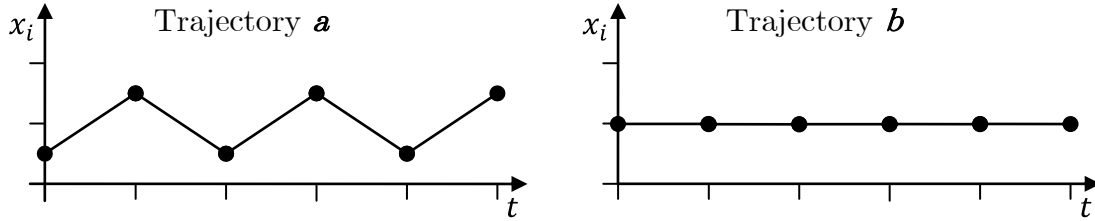
The intertemporal analogue of Sen (1976)’s *transfer* axiom, applied to the trajectory ordering, is as follows.

**Definition 19.** [INTERTEMPORAL TRANSFER]<sup>15</sup> Given  $T \geq 2$ ,  $\succsim_T$  satisfies INTERTEMPORAL TRANSFER if, for any  $\delta > 0$ ,  $\mathbf{x}, \mathbf{y} \in \Phi^T$  such that  $x_t > x_s$ ,  $y_t = x_t + \delta$ ,  $y_s = x_s - \delta$  for some  $t, s \in T$  and  $x_\tau = y_\tau$  for all  $\tau \neq t, s$ ,  $\mathbf{x} \prec_T \mathbf{y}$ .

Intuitively, INTERTEMPORAL TRANSFER reflects the idea that a period of elevated wellbeing cannot *fully* compensate for a period of depressed wellbeing. It would seem an appropriate normative choice when the poverty analyst aims to capture the total burden of poverty over time. It is also closely related to the concept of fluctuation or variance aversion, which is natural if the measure is to reflect preferences for smoothing of wellbeing.

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<sup>15</sup>This is similar to Foster (2007)’s *transfer* property which requires chronic poverty to decrease given a smoothing of incomes among those identified as chronically poor. (Foster does not distinguish between smoothing over time and over people.) Note that the measures suggested by Foster (2007) do not in general satisfy the property. Calvo and Dercon (2007)’s first, dismissed, suggestion for ‘*increasing cost of hardship*’ is essentially the same.



We may extend this concept, recognising that the resistance to compensation will be stronger, the lower the level of wellbeing experienced.<sup>16</sup> That is, we should allow a greater (or at least, not lesser) marginal rate of compensation between a pair of periods when wellbeing is greater in both.

**Definition 20.** [NON-DECREASING COMPENSATION] *The marginal rate of intertemporal compensation between an individual's welfare in two periods should not decrease, as the period wellbeings increase in proportion. Equivalently, the elasticity of compensation should not decrease as wellbeing increases.*

We may require a stronger version of this property:

**Definition 21.** [INCREASING COMPENSATION] *The marginal rate of intertemporal compensation between an individual's welfare in two periods should increase, as the period wellbeings increase in proportion, given that their trajectory is identified as 'poor'. Equivalently, the elasticity of compensation should not decrease as wellbeings increase in proportion.*

**Proposition 10.** 1. STRONG FOCUS and INCREASING COMPENSATION are incompatible properties.

2. The only trajectory ordering satisfying STRONG FOCUS and NON-DECREASING COMPENSATION is the 'Rawlsian' ordering.

*Proof.* 1. Assume that  $p(\mathbf{x})$  satisfies STRONG FOCUS. Consider poor trajectories  $\mathbf{x}$  and  $\alpha\mathbf{x}$ ,  $\alpha > 1$ , such that all elements of  $\mathbf{x}$  are strictly less than  $z$  while at least one element of  $\alpha\mathbf{x}$  is strictly greater than  $z$ . Elasticity of compensation at  $\alpha\mathbf{x}$  is zero and at  $\mathbf{x}$  is greater than or equal to zero, therefore  $P(X)$  does not satisfy INCREASING COMPENSATION.

2. Consider  $p(\mathbf{x})$  that induces the 'Rawlsian' ordering and thus satisfies STRONG FOCUS. Consider poor trajectories  $\mathbf{x}$  and  $\alpha\mathbf{x}$ ,  $\alpha > 1$ , such that all elements of  $\mathbf{x}$  are strictly less than  $z$  while at least one element of  $\alpha\mathbf{x}$  is strictly greater than  $z$ . Elasticity of compensation at  $\alpha\mathbf{x}$  is zero and at  $\mathbf{x}$  is zero, therefore  $P(X)$  does not satisfy INCREASING COMPENSATION. The 'Rawlsian' ordering is the unique ordering satisfying this for all  $\mathbf{x}$ .

□

<sup>16</sup>This is intuitively desirable but of course is an empirically testable proposition; we are not aware of a study which has established this.

**Proposition 11.** *If (and only if)  $p$  satisfies INCREASING COMPENSATION its lines of constant MRC diverge faster than homothetic.*

This provides us with a simple condition to test whether a function  $p(x)$  satisfies the property.

## Permutations of Wellbeings over Time

We consider here the impact on the trajectory ordering of permutations of wellbeings between different time periods.

The simplest approach, which has been taken (implicitly or explicitly) by most authors in this literature, is to impose perfect time symmetry.

**Definition 22.** [TIME SYMMETRY] *Given  $T$ ,  $\succsim_T$  satisfies TIME SYMMETRY if, for any  $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^T$  such that  $\mathbf{x} = M\mathbf{y}$  for some permutation matrix  $M$ ,  $\mathbf{x} \sim_T \mathbf{y}$ .*

**Proposition 12.**  *$\succsim_T$  satisfies TIME SYMMETRY if and only if  $p$  is a symmetric function of the component wellbeings.*

Of course, imposing such a property does not allow for various aspects of intertemporal poverty which the poverty analyst might want to capture, including systematic changes (for example a trajectory with a systematic downward trend might be considered ‘worse’ than an equivalent trajectory with an upward trend) or asymmetric transfer properties (for example the elasticity of compensation may be greater between successive periods than between those separated by a considerable time). This is an especially important consideration when the intervals between data periods are not regular.

Calvo and Dercon (2007) apply discount factors to incorporate sensitivity to trend in their measures. This is an important contribution which may be used as an alternative to TIME SYMMETRY if demanded by the context of application.

## Duration or Chronicity of Poverty

We now return to one of the motivating concepts for the measurement of intertemporal poverty; the attempt to capture information about *chronicity* in a quantitative measure. Our main result here is that it is difficult to do this whilst maintaining the transfer properties discussed above. In fact the poverty analyst must make a normative choice among these properties, according to the context in which she applies the measure, and whether she aims to measure the total burden of poverty experienced, or chronicity of poverty.

Most of the discussion in the policy literature (CPRC, 2004), and in much of the economic literature (Calvo and Dercon, 2007) has focused around the concept of chronicity of poverty: prolonged periods below the poverty line must be thought of as worse than

shorter, other things equal. The fundamental idea is that prolonged periods of low well-being may have an adverse effect over and above that due to the depth of poverty alone. We attempt to reflect this in a duration-sensitivity property.<sup>17</sup>

**Definition 23.** [DURATION SENSITIVITY] *Given  $T$  and trajectories  $\mathbf{x}, \mathbf{y}$  with identical average wellbeing, but with strictly more periods in  $\mathbf{x}$  spent below the poverty line,  $p(\mathbf{x}) > p(\mathbf{y})$ .*

**Proposition 13.** *No  $p$  satisfies both INTERTEMPORAL TRANSFER and DURATION SENSITIVITY.*

*Proof.* Consider  $\mathbf{x}, \mathbf{y}$  with identical wellbeings in all periods except  $t = 1$  and  $t = 2$ , where  $x_1 = z + \delta$ ,  $y_1 = z - \delta$ ,  $x_2 = z/2 - \delta$  and  $y_2 = z/2 + \delta$  where  $\delta \in (0, z/4)$ . If  $p$  satisfies DURATION SENSITIVITY (1) then  $p(x) < p(y)$ . If  $p$  satisfies INTERTEMPORAL TRANSFER then  $p(x) \geq p(y)$ , a contradiction. Therefore no function  $p$  satisfies both INTERTEMPORAL TRANSFER and DURATION SENSITIVITY.  $\square$

The tradeoff between INTERTEMPORAL TRANSFER and DURATION SENSITIVITY means that the poverty analyst must choose between them when choosing an intertemporal poverty measure. In fact the only measures proposed thus far in the literature that do satisfy duration sensitivity is the duration extended headcount measures (special cases of the poverty measure proposed by Foster (2007) with  $\alpha = 0$  and  $\tau < 1$  in the intertemporal FGT framework). Thus, whilst the concept of chronic poverty in the sense of long duration may be intuitive, in fact imposing this property precludes some other desirable properties.<sup>18</sup>

## Temporal Homogeneity

Given a TRAJECTORY-ORDERING ITPM  $P(X) = A(p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_N))$  the trajectory function  $p : \bigcup_{T \in \mathbb{N}} \mathbb{R}_+^T \rightarrow \mathbb{R}$  induces a total preorder on  $\bigcup_{T=1}^{\infty} \mathbb{R}_+^T$ , the space of trajectories of different durations. The properties discussed in detail above all relate to the orderings over the spaces of same-duration trajectories  $\mathbb{R}_+^T$ . We now consider how these are connected; a natural approach is to require the constant-wellbeing trajectories to be equivalent across trajectory lengths.<sup>19</sup>

**Definition 24.** [TIMESPAN COMPARABILITY]  $\preceq$  *satisfies TIMESPAN COMPARABILITY if, for all wellbeings  $x \in \mathbb{R}_+$  and all  $T_1, T_2 \in \mathbb{N}$ ,  $(x, x, \dots, x_{T_1}) \sim (x, x, \dots, x_{T_2})$ .*

<sup>17</sup>For clarity, although Foster (2007)'s *time monotonicity* is motivated in a similar way, it does not capture this idea but is a consequence of strong monotonicity.

<sup>18</sup>There is an analogy here to the static literature in which the intuitive proposition 'A population with a greater proportion of poor people is worse off than one with a lesser proportion of poor people' conflicts with sensitivity to inequality among the poor, or the transfer principle.

<sup>19</sup>Whilst it is possible to think of contexts in which this property would not be desired, perhaps if the greater information in a longer series of observations is in itself informative, such exceptional situations seem quite contrived.

**Proposition 14.**  $\succsim$  satisfies TIMESPAN COMPARABILITY if and only if for all wellbeings  $x \in \mathbb{R}_+$  and all  $T_1, T_2 \in \mathbb{N}$ ,  $p(x_{(1)}, x_{(2)}, \dots, x_{(T_1)}) = p(x_{(1)}, x_{(2)}, \dots, x_{(T_2)})$ .

*Proof.* Trivial. □

With the assumption of TIMESPAN COMPARABILITY we are now able to prove a result which will be useful in the subsequent analysis.

**Lemma 15.** Given a function  $p$  which satisfies CONTINUITY, WEAK MONOTONICITY and TIMESPAN COMPARABILITY, there exists for every  $\mathbf{x} \in \bigcup_{T=1}^{\infty} \mathbb{R}_+^T$  an **equivalent constant wellbeing**  $c(\mathbf{x})$  such that  $p(c(\mathbf{x})) = p(\mathbf{x})$ .

*Proof.* Topological argument. □

Note that the equivalent constant wellbeing may not be unique. However, we may define a function  $c : \bigcup_{T=1}^{\infty} \mathbb{R}_+^T \rightarrow \mathbb{R}_+$  by taking  $c$  to be the *minimum* equivalent constant wellbeing for each  $\mathbf{x}$ .

## Conclusion

We have conducted a fairly exhaustive analysis of fundamental and desirable properties of the trajectory ordering, and provided results which enable these properties to be represented by a function  $p$ . Construction of a suitable  $p$  is the first stage of our recipe for construction of an intertemporal poverty measure; we now turn to the subsequent stages.

We note here that although some of the properties are fundamental and will subsequently be taken as axiomatic, others are the choice of the poverty analyst and must be chosen to reflect those facets of intertemporal poverty that she wishes her measure to capture. How she makes this choice will depend on the application, and may be driven by normative considerations. Alternatively a more welfarist approach may be taken, and she may attempt to determine empirically the preferences over trajectories of wellbeings held by the subjects of her study. This approach would raise many practical challenges which we shall not explore further in the present paper.

We will note briefly, however, that the precise specification of the properties of the measure should in either case reflect the observed variable which is being used to proxy for wellbeing. For example, if income were used, the poverty analyst should impose a higher degree of intertemporal substitution than if consumption were used, to allow for intertemporal consumption smoothing.

## 2.5 Properties of the Trajectory-Distribution Ordering

Having discussed and analysed properties of the trajectory ordering induced by a TRAJECTORY ORDERING intertemporal poverty measure  $P$  we now turn to properties of the ordering it induces on  $\mathcal{X}$ , that is, the ordering of *distributions* of trajectories.

Throughout this section we assume that the poverty analyst has chosen the properties of the trajectory ordering, subject to the restrictions discussed above, and has found a suitable function  $p$  which embodies her chosen properties. We assume also that the properties chosen include WEAK MONOTONICITY, CONTINUITY and TIMESPAN INDEPENDENCE so that by Lemma 15 for every trajectory  $\mathbf{x} \in \bigcup_{T \in \mathbb{N}} \mathbb{R}_+^T$  there exists an equivalent constant wellbeing  $c(\mathbf{x})$ .

This greatly simplifies the analysis, as we are thus able to focus entirely on the properties of the ordering of constant-wellbeing trajectories induced by the ITPM. Intuitively, the ITPM 'treats' any trajectory  $\mathbf{x}$  in exactly the same way as its equivalent constant-wellbeing trajectory  $\mathbf{c}(\mathbf{x})$ .<sup>20</sup> Drawing the obvious analogy between constant-wellbeing trajectories and individual wellbeings in the context of static poverty measurement enables us to draw directly on that mature literature for properties and results. In particular we make use of Foster and Shorrocks (1991)'s results which invoke Gorman (1968) to characterise the class of subgroup-consistent poverty indices.

As the literature on static, unidimensional poverty measurement has reached a broad consensus on the desirable properties of such a measure, we shall not discuss alternative properties in any detail here, but simply establish the appropriate analogy with the intertemporal case.

### Analogy with Static, Unidimensional Poverty Measurement

We shall let  $c_i = c(\mathbf{x}_i)$  and regard this, the individual's equivalent constant wellbeing, as analogous to the individual wellbeing<sup>21</sup> in the static, unidimensional case, notated  $x_i$  in Foster and Shorrocks (1991). A distribution of equivalent constant wellbeings  $c = (c_1, c_2, \dots, c_n) \in \bigcup_{N=1}^{\infty} \mathbb{R}_+^N$  is then analogous to Foster and Shorrocks (1991)'s distribution  $x$ . We take Foster and Shorrocks (1991)'s distribution space  $\mathcal{D}$  to be the image of  $\mathcal{X}$  under  $c$ , that is, the space of all vectors of equivalent constant wellbeings under  $P$ . (The structure of this space is consistent with the definition of  $\mathcal{D}$  given by Foster and Shorrocks (1991); if  $P$  satisfies FOCUS then  $D = [0, z]$  while if  $P$  does not satisfy FOCUS  $D = [0, \infty)$ .)

We start with the 'basic properties' listed by Foster and Shorrocks (1991), stating them in the context of equivalent constant wellbeings and clarifying their relationship to more general properties of intertemporal poverty measures. Some are equivalent to properties that we shall impose on the trajectory-distribution ordering, while others follow directly from properties already established of the trajectory ordering. We note that for Foster and Shorrocks (1991) an index  $P$  is in sections 2 and 3 a family of poverty measures indexed by poverty lines  $z \in D$  while in section 4 it is a particular poverty measure. For clarity, consistency with the rest of our paper and because their main results are not dependent on their assumption of the focus property we follow the latter, dropping the argument  $z$  in our statement of their properties.

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<sup>20</sup>In fact, we may go further than this; the existence of equivalent constant wellbeings  $c(\mathbf{x})$  enables us to compare trajectories of different durations and thus distributions of trajectories of a variety of durations; in practice this will be useful and straightforward to implement but the formal analysis would require an extension of the foundations so we set the task aside for now.

<sup>21</sup>The literature typically refers to 'income' or 'consumption'; see our discussion above and in section 4.

**Definition 25.** [FS1-SYMMETRY]  $P$  satisfies FS-SYMMETRY if  $\tilde{P}(c) = \tilde{P}(d)$  whenever  $c \in \mathcal{D}$  is obtained from  $d \in \mathcal{D}$  by a permutation.

The natural symmetry property for intertemporal poverty measures, a property of the trajectory-distribution ordering, is as follows:

**Definition 26.** [POPULATION SYMMETRY]  $P(X) = P(Y)$  whenever  $X \in \mathcal{X}$  is obtained from  $Y \in \mathcal{X}$  by a permutation of trajectories across individuals, that is,  $X = \Pi Y$  where  $\Pi$  is an  $N \times N$  matrix of ones and zeros, each row and each column summing to one.

This kind of anonymity or symmetry property is standard in the social welfare literature; in the context of intertemporal poverty measurement the only information about each individual that impacts on the intertemporal poverty measure  $P$  should be the trajectory of measured wellbeings. It is straightforward to show that if  $P$  satisfies POPULATION SYMMETRY it also satisfies FS1-SYMMETRY.

**Definition 27.** [FS2-REPLICATION INVARIANCE]  $\tilde{P}(c) = \tilde{P}(d)$  whenever  $c \in \mathcal{D}$  is obtained from  $d \in \mathcal{D}$  by a replication.

This is essentially homogeneity of degree zero in population size. If  $P$  is to measure the per-capita impact<sup>22</sup> of intertemporal poverty, then it must satisfy this kind of homogeneity property. That is, applying the measure to a different population with the same *distribution* of wellbeing trajectories, should yield the same result. This property may be formalised as follows for the intertemporal case:

**Definition 28.** [POPULATION-SIZE INVARIANCE]  $P$  satisfies POPULATION-SIZE INVARIANCE if for any  $N_1, N_2$  and  $T$  and for any  $X \in \mathcal{X}_{N_1 T}, Y \in \mathcal{X}_{N_2 T}$  such that  $X$  and  $Y$  represent an identical distribution of trajectories,  $P(X) = P(Y)$ .

Again, it is straightforward to show that if  $P$  satisfies POPULATION-SIZE INVARIANCE it also satisfies FS2-REPLICATION INVARIANCE.

**Definition 29.** [FS3-MONOTONICITY]  $\tilde{P}(c) \leq \tilde{P}(d)$  whenever  $c \in \mathcal{D}$  is obtained from  $d \in \mathcal{D}$  by an increment to a poor person.

This property follows directly from WEAK MONOTONICITY which we have assumed  $P$  to satisfy.

**Definition 30.** [FS4-FOCUS]  $\tilde{P}(c) = \tilde{P}(d)$  whenever  $c \in \mathcal{D}$  is obtained from  $d \in \mathcal{D}$  by an increment to a nonpoor person.

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<sup>22</sup>The poverty analyst may, of course, in some applications wish to measure the total rather than per-capita burden of poverty in a population, necessitating a measure which is homogeneous of degree 1 in population size. Such measures will be directly related to the class of decomposable measures.



This property would follow directly from FOCUS; we have not imposed FOCUS on the trajectory ordering, though of course  $P$  may satisfy it; we note however that Foster and Shorrocks (1991)'s main results do not depend upon the assumption of this property.

**Definition 31.** [FS5-RESTRICTED CONTINUITY]  $\tilde{P}(c)$  is continuous as a function of  $c_i$  on  $[0, z]$  where  $z$  is the poverty line.

This follows directly from CONTINUITY which we have assumed  $P$  to satisfy; in fact CONTINUITY is stronger and will allow us to apply those results of Foster and Shorrocks (1991) which depend on their stronger continuity property.<sup>23</sup>

The main result in Foster and Shorrocks (1991) establishes a general functional form for poverty measures satisfying a subgroup consistency property, which we shall wish to extend to the intertemporal case. This property requires that if the level of poverty rises in any subset of a population whilst remaining fixed in the complementary subset, the overall level of poverty shall rise. In the context of equivalent constant wellbeings the property is:

**Definition 32.** [FS-SUBGROUP CONSISTENCY] A poverty index  $P$  is FS-SUBGROUP CONSISTENT if, for any  $N_1, N_2 \in \mathbb{N}$ ,  $c, c' \in D^{N_1}$  and  $d, d' \in D^{N_2}$ ,  $\tilde{P}(c, d) > \tilde{P}(c', d')$  whenever  $\tilde{P}(c) > \tilde{P}(c')$  and  $\tilde{P}(d) = \tilde{P}(d')$ .

The natural equivalent in the intertemporal case is:

**Definition 33.** [POPULATION SUBGROUP CONSISTENCY] A poverty index  $P$  is POPULATION SUBGROUP CONSISTENT if, for any  $T \in \mathbb{N}$ ,  $N_1, N_2 \in \mathbb{N}$ ,  $X, X' \in \mathcal{X}_{N_1 T}$  and  $Y, Y' \in \mathcal{X}_{N_2 T}$ ,  $P(X, Y) > P(X', Y')$  whenever  $P(X) > P(X')$  and  $P(Y) = P(Y')$ .

$P$  is POPULATION SUBGROUP CONSISTENT if and only if it satisfies FS-SUBGROUP CONSISTENCY and so we are able to apply the results of Foster and Shorrocks (1991) to determine a general form for POPULATION SUBGROUP CONSISTENT intertemporal poverty indices.

**Proposition 16.** Let  $P : \mathcal{X} \rightarrow \mathbb{R}$  satisfy TRAJECTORY-ORDERING, WEAK MONOTONICITY, CONTINUITY and TIMESPAN INVARIANCE. Then  $P$  satisfies POPULATION SYMMETRY, POPULATION-SIZE INVARIANCE and POPULATION SUBGROUP CONSISTENCY if and only if there exist functions  $\phi : D \rightarrow \mathbb{R}$  and  $F : \phi(D) \rightarrow \mathbb{R}$  such that

$$P(X) = F \left[ \frac{1}{N(X)} \sum_{i=1}^{N(X)} \phi(c_i) \right] \quad (2.17)$$

where  $F$  is continuous and increasing,  $\phi$  is continuous and non-increasing, and  $c_i = c(\mathbf{x}_i)$  where  $c$  is the equivalent constant wellbeing from Lemma ?? above.

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<sup>23</sup>Our CONTINUITY property is weaker than Foster and Shorrocks (1991)'s stronger continuity property, however it is strong enough for their results still to hold.

*Proof.* Follows directly from Lemma ?? above and Foster and Shorrocks (1991) Proposition 1, relaxing the assumption of FOCUS.  $\square$

We note that the ordinal properties of  $P$  are independent of the function  $F$ , which establishes the cardinal properties of the poverty measure; we discuss these further below. Of course there may be other properties of the trajectory-distribution ordering that the poverty analyst wishes  $P$  to satisfy; these must be established by choice of the function  $\phi$  if we are to retain the properties listed above.

In particular, a property that we have not yet discussed is INTERPERSONAL TRANSFER or sensitivity to inequality among the poor; following Sen (1976) this is conventionally adopted in the static, unidimensional poverty measurement literature. Informally, its analogue in the intertemporal context can be ensured by convexity of the function  $\phi$ .<sup>24</sup>

## 2.6 Cardinal Properties of the Intertemporal Poverty Measure

If the poverty analyst wishes to assign an interpretation to the numerical value taken by her poverty measure (rather than just using it for comparison) she must choose its cardinal properties to reflect the interpretation she wishes to assign. We assume that she has found suitable functions to construct a poverty measure  $P'$  with the ordinal properties she desires (as discussed above) and note that any strictly increasing transformation  $f : \mathbb{R} \rightarrow \mathbb{R}$  yields a poverty measure  $P = f(P')$  with the same ordinal properties. The final task is therefore to find a transformation  $f$  which yields the desired cardinal properties.

The particular form of  $f$  will depend on the cardinal properties desired as well as the cardinal properties of the preliminary intertemporal poverty measure  $P'$ .

An important cardinality property is that of decomposability, under which the poverty measure is a population-weighted average of the poverties of the components of any disjoint decomposition of the population. This was introduced to the static, unidimensional poverty measure literature by Foster, Greer, and Thorbecke (1984), and may be stated for an intertemporal poverty measure as follows:

**Definition 34.** [POPULATION SUBGROUP DECOMPOSABILITY] *A poverty index  $P$  is POPULATION SUBGROUP DECOMPOSABLE if, for any  $T \in \mathbb{N}$ ,  $N_1, N_2 \in \mathbb{N}$ ,  $X \in \mathcal{X}_{N_1 T}$  and  $Y \in \mathcal{X}_{N_2 T}$ ,*

$$P(X, Y) = \frac{N_1}{N_1 + N_2} P(X) + \frac{N_2}{N_1 + N_2} P(Y). \quad (2.18)$$

(This definition may, by repeated application, be shown to be equivalent to the definition of *decomposability* used by Foster and Shorrocks (1991).)

POPULATION SUBGROUP DECOMPOSABILITY entails POPULATION SUBGROUP CONSISTENCY and yields a very simple form of population-aggregation for the intertemporal poverty measure. In particular, maintaining the properties taken as axiomatic above, Corollary 1 of Foster and Shorrocks (1991) gives us:

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<sup>24</sup>Compare Calvo and Dercon (2007)'s *increasing cost of hardship* (second definition). Note that there is in fact no conflict with intertemporal transfer which is established independently as a property of the trajectory ordering.

**Corollary 17.** *Let  $P : \mathcal{X} \rightarrow \mathbb{R}$  satisfy TRAJECTORY-ORDERING, WEAK MONOTONICITY, CONTINUITY and TIMESPAN INVARIANCE. Then  $P$  satisfies POPULATION SYMMETRY, POPULATION-SIZE INVARIANCE and POPULATION SUBGROUP DECOMPOSABILITY if and only if there exists a function  $\phi : D \rightarrow \mathbb{R}$  such that*

$$P(X) = \frac{1}{N(X)} \sum_{i=1}^{N(X)} \phi(c_i) \quad (2.19)$$

where  $\phi$  is continuous and non-increasing and  $c_i = c(\mathbf{x}_i)$  where  $c$  is the equivalent constant wellbeing from Lemma ?? above.

(Without loss of generality we may drop their constant  $c$ , as we have not imposed FOCUS.)

In some applications the poverty analyst may wish to measure the total rather than per-capita burden of intertemporal poverty; in that case given the distribution of trajectories, the measure should be homogeneous of degree one in population size.

**Definition 35.** [POPULATION-SIZE HOMOGENEITY]  *$P$  satisfies POPULATION-SIZE HOMOGENEITY if for any  $N_1, N_2$  and  $T$  and for any  $X \in \mathcal{X}_{N_1 T}, Y \in \mathcal{X}_{N_2 T}$  such that  $X$  and  $Y$  represent an identical distribution of trajectories,  $P(X) = \frac{N_1}{N_2} P(Y)$ .*

Any POPULATION SUBGROUP DECOMPOSABLE measure may be converted into a POPULATION-SIZE HOMOGENEOUS measure simply by multiplying through by  $N(X)$ .

We finally briefly consider normalisation. Some authors have sought to normalise poverty measures such that, for example, if  $Y$  is a matrix of zeros,  $P(Y) = 1$ . In general given a measure  $P'$  with desired ordinal properties, it will be possible to find a transformation function  $f$  such that  $P(Y) = f(P'(Y)) = 1$ . However in some cases it will not be possible to achieve normalisation together with other desired cardinal properties; for example, if  $P'(Y)$  is not finite it may not be possible to achieve decomposability as well as normalisation. This is the case with the poverty measure introduced in the subsequent section, which we do not attempt to normalise.

### 3 A New Family of Intertemporal Poverty Measures with Increasing Compensation Property

We construct in this section a new family of intertemporal poverty measures which possess the INCREASING COMPENSATION property, that is, they allow a lower degree of compensation for periods of extreme poverty than for mild poverty.

This reflects the idea that it is less easy to compensate for periods of extremely low wellbeing than for periods of less low wellbeing. That is, persistence or path-dependence. In practical terms, this may arise through long-term effects of severe malnutrition, for example.

#### 3.1 Construction

In section 2.4 we suggested a new property of the trajectory ordering, INCREASING COMPENSATION. This reflects the idea that it is very hard to compensate for periods of extremely low wellbeing, but that it may be easier to compensate for periods of less low wellbeing. Alternatively and equivalently, fluctuations in wellbeing have a greater negative impact, the poorer the individual.

None of the intertemporal poverty measures suggested in the literature have this property; Bourguignon and Chakravarty (2003) attempt to construct a multidimensional poverty measure (their equation 22) with a similar characteristic, but are hampered by their commitment to STRONG FOCUS. The properties possessed by their suggested measure, which is expressed only in implicit form, are not clear; it certainly does not satisfy INCREASING COMPENSATION throughout the space of trajectories identified as poor.

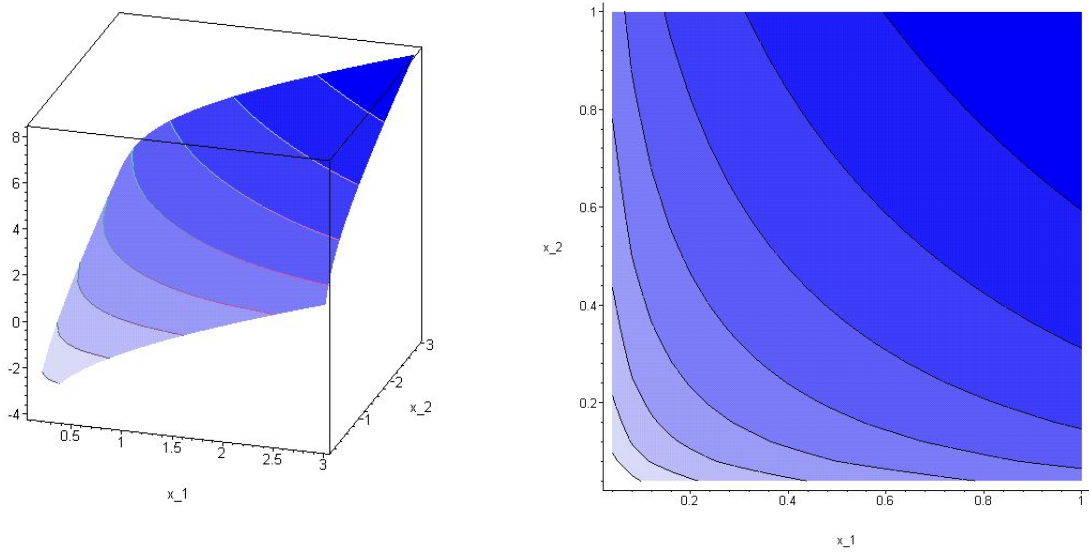
To construct our poverty measure, we follow the steps of the recipe suggested in section 2.

- **Step 1: Choose a function  $p$  which induces a trajectory ordering with the desired properties.**

We have seen (proposition 11) that such a function will have (in the poor domain), increasingly diverging lines of constant marginal rate of compensation (MRC). This rules out any homothetic function; however, we observe that a linear combination of CES functions will yield the required increasing elasticity of substitution if the lower-elasticity function dominates for poorer trajectories, and the higher-elasticity function dominates for less-poor trajectories. For example, the function

$$f(\mathbf{x}) = \frac{1}{T} \sum_{t=1}^T (x_t + \ln(x_t)) \quad (3.1)$$

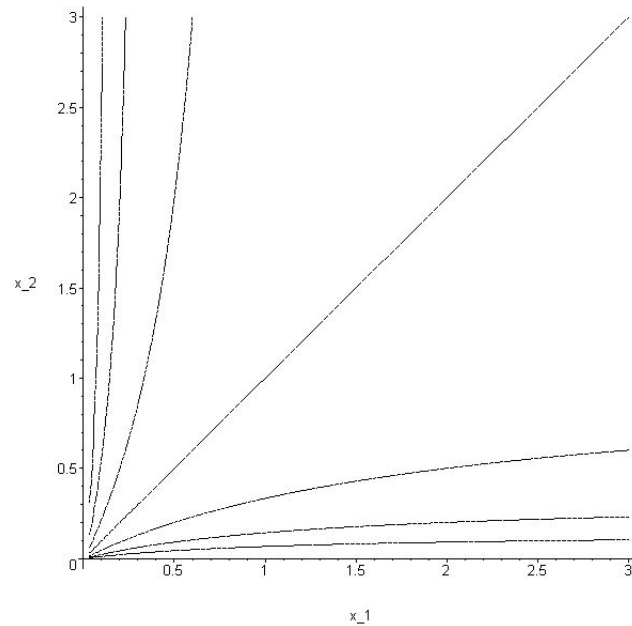
illustrated here for the  $T = 2, z = 1$  case:



has marginal rate of compensation

$$MRC_{ts} = -\frac{x_s + x_t x_s}{x_t + x_t x_s} \quad (3.2)$$

whose isoMRCs diverge as required:



This function is strictly increasing and has no focus property, so it must be transformed to achieve other desirable properties of the trajectory ordering.

**Proposition 18.** *The function*

$$p(\mathbf{x}) = \max \left[ 0, 1 - f \left( \frac{\mathbf{x}}{z} \right) \right] = \max \left[ 0, \frac{1}{T} \sum_{t=1}^T \left( 1 - \frac{x_t}{z} - \ln \left( \frac{x_t}{z} \right) \right) \right] \quad (3.3)$$

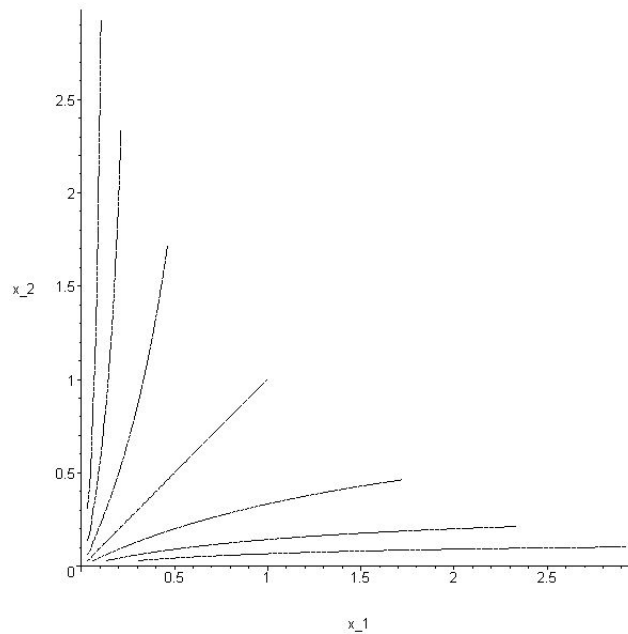
*yields, for all  $T > 1$ , a trajectory ordering with the properties CONTINUITY, WEAK MONOTONICITY, TIMESPAN INVARIANCE, FOCUS, STRONG MONOTONICITY, INTERTEMPORAL TRANSFER and INCREASING COMPENSATION.*

*Proof.* 1. CONTINUITY  $p$  is a continuous function on  $\mathbb{R}_+^T$  for each  $T$ .

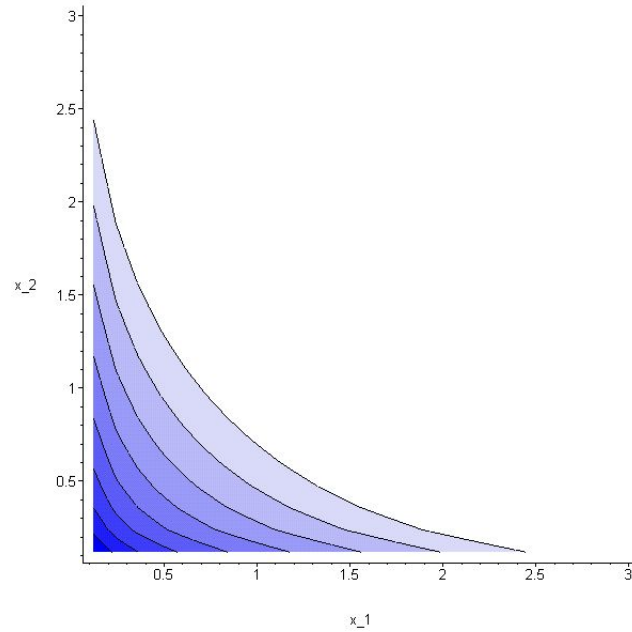
2. WEAK and STRONG MONOTONICITY  $p$  is weakly decreasing in each  $x_t$  and strictly decreasing wherever  $p > 0$ .
3. TIMESPAN INVARIANCE If  $\mathbf{c}$  is a constant-wellbeing trajectory of any duration  $T$  then  $p(\mathbf{c}) = \max \left[ 0, \left( 1 - \frac{c}{z} - \ln \left( \frac{c}{z} \right) \right) \right]$  which is independent of  $T$ .
4. FOCUS  $p = 0$  for all trajectories such that  $\frac{1}{T} \sum_{t=1}^T \left( 1 - \frac{x_t}{z} - \ln \left( \frac{x_t}{z} \right) \right) < 0$  and  $p > 0$  for all others.
5. INTERTEMPORAL TRANSFER Elasticity of compensation is finite for all  $x$  identified as poor.
6. INCREASING COMPENSATION Lines of constant MRC diverge relative to homothetic case.

□

We may illustrate  $p$  in the case of two time periods. As required for increasing compensation, the lines of constant marginal rate of compensation diverge:



Isoquants of  $p$ :

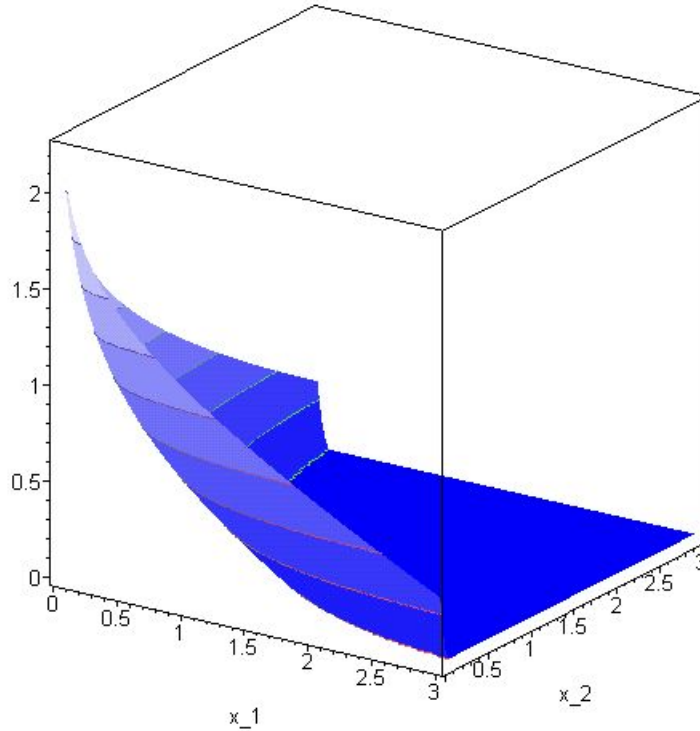


- **Step 2: Choose a social aggregation function  $S$  to aggregate over individuals in a way that yields an ordering of distributions of trajectories with desired properties.**

We do not introduce any innovations here, but maintain the conventional properties INTERPERSONAL TRANSFER, POPULATION SYMMETRY and POPULATION-SIZE INVARIANCE which emerge naturally from the properties of  $p$  under symmetric linear aggregation:

$$S(p(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)) = \frac{1}{N} \sum_{i=1}^N \max \left[ 0, \frac{1}{T} \sum_{t=1}^T \left( 1 - \frac{x_{it}}{z} - \ln \left( \frac{x_{it}}{z} \right) \right) \right] \quad (3.4)$$

The cardinal properties of  $p$ , which directly determine the ordinal properties of  $P$ , may be visualised in three dimensions:



- **Step 3: If necessary, choose a transformation function  $G$  to yield a poverty measure with the desired cardinal properties.**

The important cardinal properties are already achieved; we simply introduce a normalisation  $\frac{1}{2}$  to yield, to some extent, cardinal comparability with the measures proposed by Foster and Santos (2006). The poverty measure is thus

$$P(X) = G(S(p(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N))) = \frac{1}{N} \sum_{i=1}^N \max \left[ 0, \frac{1}{2T} \sum_{t=1}^T \left( 1 - \frac{x_{it}}{z} - \ln \left( \frac{x_{it}}{z} \right) \right) \right]. \quad (3.5)$$

For distributions of constant-wellbeing trajectories (abstracting from the different transfer properties),  $P$  converges to the Foster-Santos measure as  $x \rightarrow z$ .

We may summarise the properties of the aggregate measure  $P(X)$  by noting that it satisfies all the conditions for Corollary 17 and therefore has the properties POPULATION SYMMETRY, POPULATION-SIZE INVARIANCE and POPULATION SUBGROUP DECOMPOSABILITY.



### 3.2 Generalisation

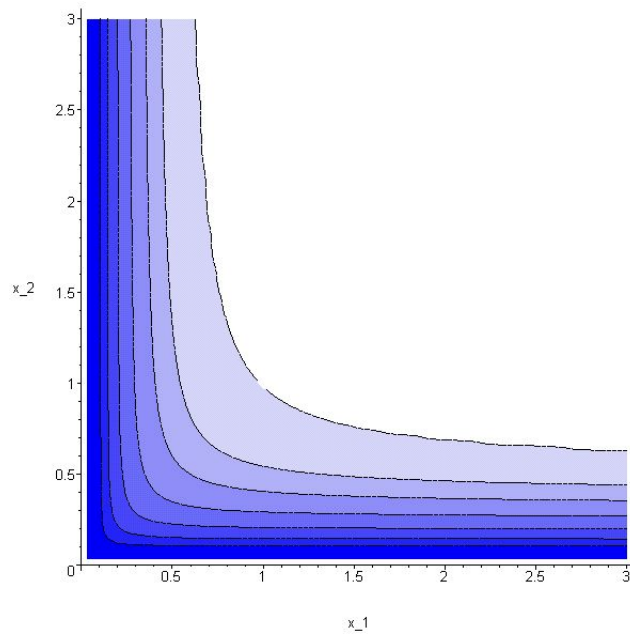
The analysis above may be generalised to a whole family of poverty measures by taking linear combinations of different CES functions, for example:

$$P(X) = \frac{1}{N} \sum_{i=1}^N \max \left[ 0, \frac{1}{2T} \sum_{t=1}^T \left( \frac{z}{x_{it}} + \ln \left( \frac{z}{x_{it}} \right) - 1 \right) \right] \quad (3.6)$$

and

$$P(X) = \frac{1}{N} \sum_{i=1}^N \max \left[ 0, \frac{1}{3T} \sum_{t=1}^T \left( \left( \frac{z}{x_{it}} \right)^2 + \ln \left( \frac{z}{x_{it}} \right) - 1 \right) \right]. \quad (3.7)$$

These measures have the same properties as (3.5) but in fact allow for a lower degree of intertemporal compensation, as illustrated by the isoquants of (3.7):



More generally, for some  $k \in \mathbb{N}$ ,

$$P(X) = \frac{1}{N} \sum_{i=1}^N \max \left[ 0, \frac{1}{(k+1)T} \sum_{t=1}^T \left( \left( \frac{z}{x_{it}} \right)^k + \ln \left( \frac{z}{x_{it}} \right) - 1 \right) \right]. \quad (3.8)$$

The degree of intertemporal compensation decreases as  $k$  increases.

## 4 An empirical application: poverty in rural Ethiopia

In this section we provide an empirical application using data from the Ethiopian Rural Household Survey (ERHS). We firstly provide static measures of consumption based poverty, followed by transition matrices and finally the set of duration-adjusted poverty measures proposed in the literature and discussed in the analytical section above. We examine which households remain classified as poor and the proportion of the households that are classified as poor when the methodology changes. We shall use real household (per adult equivalent) average consumption (defined below) as our measure of welfare.

### 4.1 Data

Data are from the Ethiopian Rural Household Survey (ERHS) collected by the University of Addis Ababa and the Centre for the Study of African Economies (CSAE) at the University of Oxford, as well as the International Food Policy Research Institute (IFPRI). Data are available from fifteen districts<sup>25</sup> in several regions. Seven villages were originally included in IFPRI's survey of 1989, which were chosen primarily because they had suffered hardships in the period 1984–89 (the 1984–85 famine). For a detailed description see Webb, von Braun, and Yohannes (1992). In 1994, 360 of the households in six villages were retraced and the sample frame was expanded to 1477 households. The nine additional communities were selected to account for the diversity in the farming systems in the country. Within each village, random sampling was used. The households were resurveyed again in 1994 and 1995, and subsequently in 1997 and 1999. The sixth and latest round of the survey was completed in 2004. The attrition rate is low, less than one per cent per annum (annualised, or 12.1% in total between 1994 and 2004). Since the three surveys in 1994–1995 are within eighteen months of each other, we drop the second round of 1994, in order to use five rounds of the data for our subsequent analysis.

The dependent variable or welfare measure chosen is real household monthly consumption per adult-equivalent. This is comparable with other studies of consumption and poverty that have been conducted on the dataset, and other studies of poverty. In the ERHS, detailed information is also available on household income and assets. At the individual level, there is information on height and weight though not for all individuals and not for all rounds. Data on monthly consumption of food, purchased food and non-investment non-food items (excluding durables, as well as health and education expenditure) based on purchased items, gifts in cash and in kind, and a diary of consumption from own production from a two-week recall period was divided by adult equivalent units based on World Health Organisation (WHO) guidelines. This was deflated by a food price index constructed from data collected for each village at the same time as the household survey. For a detailed discussion on the construction of the consumption indicator, see Dercon and Krishnan (1998). Food represents around eighty per cent of the consumption basket. We use a consumption poverty line calculated by Dercon and Krishnan (1998) which is village specific (according to local prices) and averages 44.3 Birr per adult equivalent per

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<sup>25</sup> These communities are called Woredas, the equivalent of a county in the UK. They are further divided into Peasant Associations (PAs), the equivalent of a village, and consist of up to several villages (e.g. the ERHS comprises 15 Woredas, and 18 PAs). The administrative system of the PAs was created in 1974 after the revolution.

month (all prices are specified in 1994 real terms). It is based on the monthly cost of the diet to achieve 2100 Kcal per day per adult, using the food consumed by the poorer half of the sample, and minimum ‘essential non-food’ expenditure.<sup>26</sup> For analytical convenience, we include only households in the analysis that have nonmissing values of consumption for each available round, restricting the sample to 1168 households. We also ignore changes in household composition, apart from in the calculation of consumption per adult equivalent, and therefore abstract somewhat from measuring individual welfare. In the ERHS, the only possibility to examine individual wellbeing would be through an anthropometric measure, which is a focus for future work.

## 4.2 Annual poverty measures and transition matrices

Table 2 shows the standard Foster-Greer-Thorbecke (FGT) poverty indices for the five years. We observe a decline in poverty as measured by all three indices, by around similar magnitudes – the percentage of poor falling from almost 40% poor to 26%. Although the headcount falls, the mean consumption of the poor only grows by around 1.5% per annum (or 12.5% in total between the first and last rounds). Table 3 breaks this down by village, and shows that whilst a number of villages have had relatively low poverty in all time periods (e.g. Debre Birhan villages), some villages have experienced high poverty rates for most of the time (e.g. Gara Godo, Imdibir, Adado). Other villages had very high poverty in 1994, but have shown a substantial reduction in poverty rates by 2004 (e.g. Geblen, Domaa, Korodegaga).

Table 4 shows that of the households who were classified as poor in 1994, a third were still poor in 2004, 10% of all households. Table 5 breaks this down further and looks at relative positions in the consumption distribution keeping quintile cutoffs fixed at the 1994 level. Whilst the diagonal (no movement between quintiles) is the most populous quadrant, the table does show that upward mobility is happening, with more than proportional numbers ending up in the top three quintiles. However, just over 22% of households do remain in the two bottom quintiles in both time periods.

## 4.3 Introducing a time dimension to poverty measures

### Poverty duration

Table 2 showed that 24–39% of the households were below the poverty line in each round, and in cross-sectional analysis the aggregate rate is all that can be shown. It also calculated the average consumption of the households under the poverty line, but it was not possible to say anything about the poverty experiences of individual households. Table 6

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<sup>26</sup>Dercon and Krishnan (1998) show the calculations in depth. Also note that the household was considered the same between rounds if the head of the household was unchanged, while if the head had died or left the household, the household was considered the same if the current household head acknowledged that the household (in the local meaning of the term) was the same as in the previous round. In 1994 the exchange rate of the Ethiopian Birr (ETB) with the USD, corrected for purchasing power parity was 0.21875. One PPP dollar is worth 1.75 its equivalent in USD [World Bank (2000)], and the exchange rate was 8 ETB to 1 USD. Therefore the Dollar a Day poverty line (1.08) would be calculated as approximately 148 Birr per adult per month, around three times that which we use here.

shows in fact that only very few (twenty four, or just over 2%) of households are actually classified as poor in all five time periods. However, nine per cent are poor in at least four out of five periods. A substantial majority (just under 65%) of households have fallen into poverty at least once in the ten years, showing that many are vulnerable to poverty despite having consumption above the poverty line.

## Comparing intertemporal poverty measures

In this section, we compare the measures found in the recent intertemporal poverty measurement literature that were analysed above. All of these measures satisfy different properties, and we discussed table 1 showing the variety of properties satisfied by the measures. We calculate the measures of Calvo and Dercon (2007); Jalan and Ravallion (2000); Foster and Santos (2006); Foster (2007) as well as the new measure proposed above and show how altering the policy parameters can affect the classification of which households are poor, and how the aggregates change, firstly by village, and then by some other characteristics that are often cited as correlates of poverty. As noted above, we are using average household consumption per adult equivalent as the welfare measure. A large literature (see Fafchamps (2003) or Dercon (2004) for an overview) shows that incomes in developing countries can be highly erratic and that risk is central to economic decision making. There is also considerable evidence that households who are faced with risk use all available methods to ‘smooth’ their consumption, on a daily and seasonal basis, and also in the face of a negative shock (such as crop failure, or household illness). We therefore assume that households have smoothed their consumption to the best of their ability (for other work on evidence for this in the ERHS dataset see Dercon and Krishnan (2000), Dercon, Hoddinott, and Woldehanna (2005)). The measure by Foster and Santos (2006) allows us to choose a (constant) degree of intertemporal substitution, and therefore we make it relatively low, setting  $\beta = -1$  in equation 2.10 above. We choose the measure outlined above that has a similar elasticity of intertemporal welfare to that of Foster and Santos (2006) close to the poverty line, but reduces the elasticity of substitution as the poverty gap gets larger (to extremely low levels at extremely high poverty), or setting  $k = 2$ .

Table 7 shows what proportion of households are classified as ‘poor’ by the measures proposed in the literature.<sup>27</sup> Foster (2007) has a flexible duration poverty line, which alters the proportion of poor households considerably, as can be understood from table 6: just under ten percent of households are poor for four or five periods, rising to 21.5% if we include households that are poor three or more periods. Calvo and Dercon (2007) generalise Foster (2007), and propose no duration cutoff. Their measure is therefore non-zero for any household experiencing any poverty during the decade. Hence, almost 67% of households are classified as poor by this definition. Foster and Santos (2006) define as chronic poor those households with a generalised mean below the poverty line. In this case we choose  $\beta = -1$  for the choice parameter, the harmonic mean.<sup>28</sup> Just under 30% are classified poor by this method. Jalan and Ravallion (2000) set the chronic poverty identification step as those households whose arithmetic mean consumption is

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<sup>27</sup>None of the authors of the papers recommend using this quasi-headcount for policy or analysis, however we consider it useful for illustration to compare the populations covered by each measure.

<sup>28</sup>The authors (Foster and Santos, 2006) provide an illustration of choice of  $\beta$ .

below the poverty line, which is 13.4% of households. Finally the measure as proposed above classifies 31.2% of households as below the poverty line.<sup>29</sup>

In table 8 the correlations between the calculated poverty measures is shown. As expected, all are positive and significant at 1%. The highest correlation is between the two measures with similar structure, those of Calvo and Dercon, and Foster (2007). The Porter-Quinn measure and Foster-Santos measure are also highly correlated, as the two have very similar properties (diverging mainly at extreme levels of poverty and very close to the poverty line).

In table 9 we compare the measures of duration adjusted poverty discussed above for the fifteen ERHS villages, by showing the share of total poverty as defined by the measure,<sup>30</sup> allowing us to make comparisons despite the measures having different scales and interpretations. One village, Gara Godo dominates with a share of poverty from 29% (Calvo-Dercon) to 50% (Jalan-Ravallion), with a population share of only 8%. This is not necessarily easily predicted from the static poverty estimates (as shown in table 3): Gara Godo is amongst the poorer villages, but Imdibir for example appears to have similar poverty rates taking all periods together. The measures appear to rank similarly, though there are a number of differences. Poverty is more evenly spread using the Calvo-Dercon measure, due to the higher proportion of the population that enter the reference group. Porter-Quinn weights Trurufe Ketchema more heavily than other measures, due to some extremely poor households.

Table 10 provides a more general poverty profile based on household and head of household characteristics in 1994. Given these characteristics, would any of them correlate with future higher poverty in the next decade? None of the measures shows differential in poverty between male and female headed households. All of the measures however show that households with heads that are unable to read make up a more than proportionate share of intertemporal poverty, in particular as measured by Porter-Quinn and Foster-Santos (indicating that these households may suffer episodes of extreme poverty). Asset holdings appear to matter too, with land- and livestock-poor households (defined as having respective asset holdings below the median) accounting more than proportionately for intertemporal poverty. Only Jalan-Ravallion shows that large households are more than proportionately poor.

In table 11 we present some descriptive logits to analyse correlates of poverty as measured with depth and duration over the ten year period and present marginal effects. The numbers classified as poor and non-poor can be taken from tables 2 and 7. We start with correlates of static poverty at the beginning and end of the period (e.g. in 1994 and 2004 respectively). We find a strong correlation between initial assets and initial poverty, with few other significant factors (apart from male children). End-period poverty is also correlated with lower livestock holdings, and also higher household size in 2004. The descriptive logit for the intertemporal measure shows more correlates. Livestock is again a good predictor of poverty experienced over the decade. Similarly a larger household size and uneducated household head increases the probability of being classified

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<sup>29</sup>For ease of reading, we now abbreviate the measures to the names of the lead authors, with Jalan and Ravallion (2000) as Jalan-Ravallion, Foster (2007) as Foster (D3) since we adopt the three period cutoff for that measure, Foster and Santos (2006) as Foster-Santos, Calvo and Dercon (2007) as Calvo-Dercon, and our measure as Porter-Quinn.

<sup>30</sup>Here we take the value of the measure, not the headcount as in table 7.

in poverty over the period. We include measures of shocks in the regressions and find that whilst agricultural shocks are not significant, illness does increase the probability of being classified as poor. Having a high share of crop income in total income in the first period also substantially decreases the probability of being classified as poor, though this may be proxying for land holdings (which we exclude as 100 households have missing values for land).

#### 4.4 Extensions and sensitivity analysis

Measurement error is a problem in many household surveys (see Deaton (1991) for a comprehensive treatment) and has been widely dealt with in the static context. The issue of measurement error could become a more difficult problem if we had reason to suspect serial correlation in the measurement error, or that the direction/magnitude is somehow correlated with the household fixed-effect. Downward measurement error at the lower end of the consumption distribution would probably increase the poverty estimates. We note also that measurement error can cause an over-estimate in mobility between rounds, however, measures that allow a degree of intertemporal compensation may average out some of the measurement error. Sensitivity to extremely low levels of welfare is a very desirable attribute of a poverty measure, but if the low levels are due to downward measurement error then we will over-estimate poverty.

Censoring at both sides of the distribution may cause changes in measures downward that do allow intertemporal compensation of welfare above the poverty line. On the other hand, measures that have a high degree of intertemporal compensation may have less bias with respect to downward measurement error, but may lead to under-counting in the case of upward measurement error. We recalculated the poverty shares by village as shown in table 9 but with consumption per adult censored to the 5th and 95th percentiles, in order to examine the changes. Porter-Quinn and Foster-Santos showed some sensitivity to this change, probably due to the highly sensitive nature of the measures to extreme levels of poverty. Data reliability should be a consideration when choosing the amount of intertemporal compensation that is required in the measure.

We may also wish to examine sensitivity to the choice of poverty line, since it is also fairly arbitrary. We recalculated the village-level poverty rates as in table 9, but with the poverty line augmented by 20%. The distribution did not change for our measure.<sup>31</sup>

The previous analyses have taken real household consumption per adult equivalent as the welfare measure upon which to base calculations of welfare underlying the poverty measures. As outlined in the theory section, it is impossible to have a direct measure of wellbeing. We are assuming in the analysis that the consumption and whatever we mean by ‘wellbeing’ or welfare are monotonically related (Sen, 1976). This will not hold, for example, in the case where public goods provisioning changes over time, and there is an implicit assumption in the measures that when we compare across villages that these things change at a constant rate across all regions. In principle, it would be possible to calculate the intertemporal measures based on some kind of multidimensional welfare measure, though this is beyond the scope of this chapter. Aside from the criticism of

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<sup>31</sup>There were not many major differences in the distribution of poverty for the other measures, with the exception of the Foster(D3) measure, which is perhaps due to the discontinuity inherent in the measure.

consumption as a restrictive welfare measure, we also contend with the argument that a household measure does not capture the welfare of the individuals: it ignores intrahousehold distribution of welfare. In order to be able to compare directly between households we calculate consumption per adult equivalent. Deflating the consumption basket to real 1994 Birr allows us to assert that the welfare measure is directly comparable across time periods. In summary, we are assuming inter-household and intertemporal comparability of the poverty measures.

In summary, the poverty measure should be chosen on the basis of desirable theoretical properties, but also on the basis of empirical considerations that may feed into the theoretical properties. There may be other empirical considerations: do we need to define a sub-group of the poor for policy purposes, or calculate the total burden of poverty in the economy? If the data quality is expected to be very poor, a measure that averages more across time may be theoretically less appealing but may be less sensitive to measurement error. Finally, as noted above, the measure used in practice may depend on intuitive understanding rather than desirable properties.

## 5 Conclusions

The approach to poverty analysis that we have taken in this paper builds firm foundations for the understanding of poverty profiles, and adds considerable depth to the debate on poverty measurement. The empirical section has applied the new measures of chronic poverty to a sample of Ethiopian rural households. We find that the assumptions (both explicit and implicit) in the duration-adjusted poverty measures make a considerable difference to the identification of the poorest households in the sample. We also find that initial characteristics, especially low levels of education, and risk and shocks increase the probability of low welfare over the time period in question.

The analysis in this paper provides a more rigorous foundation for the construction of intertemporal poverty measures that enables clarity regarding their properties and thus, we hope, greater clarity in the interpretation and conclusions that may be drawn when they are applied to empirical or policy analysis.

The method we develop for construction of intertemporal poverty measures requires only mild restrictions on the properties they possess, and the space of measures that may be developed within this framework remains to be further explored. Although a consensus has been reached in the poverty measurement literature regarding desirable properties of the population-distribution ordering, a similar consensus has not been achieved in the intertemporal context for the properties of the trajectory ordering. In fact, as we discuss above, different properties will be necessary in different contexts of application. The measures suggested in the recent literature and the new family of measures proposed in this paper certainly do not exhaust the possibilities. We hope that the analytical framework developed in this paper will prove useful for the development of measures with different trajectory ordering properties. In particular, we believe that there remains a need for a measure that appropriately captures chronicity of poverty.

## A Tables



Table 1: Properties of intertemporal poverty measures

Trajectory ordering	Jalan-Ravallion	Foster	Calvo-Dercon	Foster-Santos	Porter-Quinn
Continuity	✓	✗	✓	✓	✓
Weak monotonicity	✓	✓	✓	✓	✓
Strict monotonicity	✓	✓ <sup>a</sup>	✓ <sup>b</sup>	✓	✓
<b>Focus properties</b>					
Weak focus	✓	✓	✓	✓	✓
Focus	✓	✓	✓	✓	✓
<b>Temporal Properties</b>					
Time symmetry	✓	✓	(✓)	✓	✓
Sensitivity to trend	✗	✗	✓ <sup>c</sup>	✗	✗
Intertemporal transfer	✗	(✓)	(✓)	✓	✓
Non-decreasing compensation	✓	✗	✗	✓	✓
Increasing compensation	✗	✗	✗	✗	✓
Duration Sensitivity (1)	✗	✗	✗ <sup>d</sup>	✗	✗
Duration Sensitivity (2)	✗	✗	✓ <sup>e</sup>	✗	✗

Notes: The table shows properties of the trajectory ordering  $p$  as outlined in analysis section. Formulae for the measures can also be found in this section. ( ) denotes measure has the property below the poverty line only.

<sup>a</sup>If  $\alpha > 0$

<sup>b</sup>If  $\alpha > 0$

<sup>c</sup>If discount rate applied

<sup>d</sup>If  $\alpha > 0$

<sup>e</sup>In one specification

Table 2: Standard poverty measures, by round

Year	Headcount F-G-T (0)	Poverty Gap F-G-T (1)	Sq-Poverty Gap F-G-T (2)	Mean cons. of poor (Birr)
1994	0.39	0.16	0.09	26.84
1995	0.44	0.19	0.10	25.72
1997	0.24	0.08	0.04	30.31
1999	0.28	0.09	0.04	31.13
2004	0.22	0.07	0.03	30.29

Notes: Source is ERHS data, own calculations. Poverty line is 44.3 Birr per adult, per month on average (though varies by village ). Number of observations is 1168. Measures are weighted by household size.

Table 3: Headcount poverty, by village and round

Peasant association	1994	1995	1997	1999	2004
Haresaw	0.34	0.56	0.25	0.17	0.28
Geblen	0.79	0.76	0.24	0.38	0.28
Dinki	0.52	0.62	0.49	0.38	0.24
Yetemen	0.07	0.21	0.08	0.27	0.15
Shumsheha	0.08	0.06	0.09	0.01	0.02
Sirbana Godeti	0.15	0.07	0.11	0	0.02
Adele Keke	0.13	0.07	0.02	0.26	0.24
Korodegaga	0.68	0.56	0.31	0.15	0.26
Trurufe Ketchema	0.23	0.32	0.3	0.15	0.19
Imdibir	0.55	0.85	0.4	0.48	0.61
Aze Deboa	0.3	0.65	0.28	0.76	0.23
Adado	0.32	0.56	0.2	0.31	0.49
Gara Godo	0.89	0.89	0.64	0.54	0.42
Domaa	0.62	0.33	0.35	0.19	0.11
Db-milki	0.31	0.26	0.03	0.04	0.02
Db-kormargefia	0.2	0.24	0.05	0.13	0.03
Db-karafino	0.23	0.29	0.03	0.06	0.14
Db-faji bokafia	0.09	0.26	0	0.12	0

Notes: Poverty line as above, headcount measure.

Table 4: Transition Matrix, Households over ten years

Status in 1994	Status in 2004		
	Poor	Non-poor	
Poor	11.76%	27.22%	38.98%
Non-poor	10.10%	50.92%	61.02%
	21.86%	78.14%	

Table 5: Transition Matrix, consumption quintiles, holding quintile cutoff constant at 1994 level

Quintile in 2004	1994					<i>totals</i>
	1	2	3	4	5	
1	8.53	2.81	1.56	1.27	0.67	<i>14.83</i>
2	3.82	7.12	2.88	2.18	1.57	<i>17.57</i>
3	3.25	3.63	7.76	3.60	3.33	<i>21.57</i>
4	2.17	2.72	3.52	7.69	3.53	<i>19.62</i>
5	2.24	3.72	4.28	5.27	10.91	<i>26.41</i>
	<i>20.00</i>	<i>20.00</i>	<i>20.00</i>	<i>20.00</i>	<i>20.00</i>	<i>100.00</i>

Table shows transition

matrix between quintiles fixed at the 1994 levels as shown in table ??, showing absolute rather than relative movement.

Table 6: Poverty Duration (Nr times hh classified as poor)

Counted poor	Nr households	Percent	Cum. Percent
Never	424	36.30	100
Once	297	25.43	63.70
Twice	202	17.29	38.27
Thrice	138	11.82	20.98
Four times	77	6.59	9.16
In every period	30	2.57	2.57
Total	1,168	100.00	100.00

Notes: Cumulative percent works backwards: i.e. the number of households// poor in *at least* n periods

Table 7: Duration adjusted poverty measures ‘headcount’

Measure	% households ‘poor’	Definition
Foster (2007)	9.7	$x_{ht} < z$ for 4 or more periods
Foster (2007)	21.5	$x_{ht} < z$ for 3 or more periods
Calvo-Dercon (2007)	63.7	$x_{ht} < z$ for more than 0 periods
Foster-Santos (2006)	29.4	general (harmonic) mean $x_h < z$
Jalan-Ravallion (2000)	13.4	arithmetic mean $x_h < z$
Porter-Quinn (2008)	31.2	utility below ‘equivalent constant’ $z$

Notes: Equations showing the poverty measures are outlined in section ?? of the text. The authors of the papers do not recommend using the headcount for policy purposes, this table is simply for illustration so that we can compare what proportion of the population make up the relevant study population for each measure. Note that Foster (2007) and Foster-Santos(2006) have flexible parameters which can change the proportion of households classified as poor. We note as an illustration above the Foster(2007) measure with the duration poverty line set at 3 periods and 2 periods respectively.

Table 8: Correlations between poverty measures

	Foster-Santos	Porter-Quinn	Calvo-Dercon	Foster D(3)	Jalan-Ravallion
Foster-Santos	1				
Porter-Quinn	0.8257*	1			
Calvo-Dercon	0.8462*	0.4695*	1		
Foster (D3)	0.8333*	0.4631*	0.9034*	1	
Jalan-Ravallion	0.6572*	0.3709*	0.6727*	0.7273*	1

Notes: Pearson correlations. \* denotes significant at 1%.

Table 9: Duration adjusted poverty measures by village

Peasant Association	Porter-Quinn	Jalan-Ravallion	Foster (D3)	Calvo-Dercon	Foster-Santos	Population contribution
Haresaw	0.04	0.01	0.03	0.05	0.04	0.06
Geblen	0.09	0.09	0.11	0.10	0.11	0.04
Dinki	0.02	0.06	0.06	0.05	0.04	0.06
Yetemen	0.00	0.00	0.00	0.01	0.00	0.04
Shumsheha	0.00	0.00	0.00	0.01	0.00	0.08
Sirbana Godeti	0.00	0.00	0.00	0.00	0.00	0.06
Adele Keke	0.03	0.00	0.01	0.04	0.02	0.07
Korodegaga	0.05	0.04	0.09	0.09	0.07	0.08
Trurufe Ketchema	0.18	0.03	0.02	0.03	0.06	0.07
Imdibir	0.04	0.11	0.08	0.08	0.07	0.06
Aze Deboa	0.01	0.01	0.05	0.05	0.02	0.05
Adado	0.02	0.03	0.04	0.05	0.03	0.07
Gara Godo	0.34	0.50	0.39	0.29	0.38	0.08
Domaa	0.15	0.13	0.11	0.11	0.14	0.05
Db-milki	0.00	0.00	0.00	0.02	0.01	0.05
Db-kormargefia	0.00	0.00	0.01	0.02	0.00	0.05
Db-karafino	0.00	0.00	0.00	0.01	0.00	0.03
Db-faji bokafia	0.00	0.00	0.00	0.00	0.00	0.02

Notes: Measures defined as in table 7 above. ‘Foster’ refers to Foster’s (2007) chronic poverty measure, with duration cutoff 3/5 time periods. Foster-Santos sets  $\beta = -1$ . Porter-Quinn sets  $k = 2$ .

Table 10: Poverty by household characteristics

Household (Head) Characteristics	Porter -Quinn	Jalan -Ravallion	Foster (D3)	Calvo -Dercon	Foster -Santos	Population contribution
Female	0.26	0.22	0.24	0.27	0.27	0.27
Male	0.72	0.72	0.72	0.71	0.71	0.73
Unable to read	0.84	0.72	0.78	0.76	0.81	0.62
Able to read	0.16	0.28	0.22	0.24	0.19	0.38
Small HH	0.62	0.49	0.53	0.57	0.56	0.62
Large HH	0.38	0.51	0.47	0.43	0.44	0.38
Land Poor 1994	0.62	0.52	0.54	0.53	0.57	0.53
Land Rich 1994	0.29	0.37	0.37	0.37	0.34	0.47
Livestock Poor 1994	0.67	0.59	0.56	0.58	0.60	0.49
Livestock Rich 1994	0.33	0.41	0.44	0.42	0.40	0.51

Notes: In the rows for land poor and land rich, n=999 due to missing data on land holdings, hence percentages do not sum to 100. n for other rows is 1089.

Table 11: Descriptive Poverty Logits

	Pov 94 Marg-FX	Pov 04 Marg-FX	Port-Quinn Marg-FX
Livestock 94	-.028*	-.031***	-.043**
Sex hh head 94 (d)	-0.023	0.001	-0.056
Head schooling (d)	-.109*	-0.027	-.107**
Female adults 94	.047*	.020*	.062***
Male children 94	0.045	0.009	0.029
Female children 94	0.037	0.003	.092***
Male 5-15 94	.082***	.028*	.104***
Female 5-15 94	0.028	-0.003	0.058
Female elderly 94	-0.013	0.014	0.009
Male elderly 94 (d)	-0.026	-0.038	0.003
Crop Share Income	-0.163	-0.071	-.230**
Ill adults 94	0.047		
Crop shock 94	0.105		
Rainfall quintile 94	-0.034		
CV village rainfall	0.003	0.001	0.005
Average ill adults 94-04		0.016	.151*
Average crop shock 94-04		-0.026	-0.357
Rain 94-97		.000**	.000*
Rain 98-04		0	0
HH size 2004		.023***	0.005
Village road improved		0.002	-0.03

Notes: The dependent variable is the poverty headcount in 1994 and 2004 respectively, based on consumption per adult equivalent being below the poverty line, and a binary variable based being poor as defined by the Porter-Quinn measure.

## References

- ATKINSON, A. B. (1987): "On the Measurement of Poverty," *Econometrica*, 55(4), 749–764.
- (2003): "Multidimensional Deprivation: Contrasting Social Welfare and Counting Approaches," *Journal of Economic Inequality*, 1(1), 51.
- BANE, M. J., AND D. T. ELLWOOD (1986): "Slipping Into and Out of Poverty: the Dynamics of Spells," *Journal of Human Resources*, 21(1), 1–23.
- BANK, W. (2001): *World Development Report 2000/2001*.
- BAULCH, B., AND J. HODDINOTT (2000): "Economic Mobility and Poverty Dynamics in Developing Countries," *Journal of Development Studies*, 36(6), 1.
- BOURGUIGNON, F., AND S. R. CHAKRAVARTY (2003): "The Measurement of Multidimensional Poverty," *Journal of Economic Inequality*, 1(1), 25.
- CALVO, C., AND S. DERCON (2007): "Chronic Poverty and All That: the Measurement of Poverty over Time," Discussion paper, Centre for the Study of African Economies, University of Oxford.
- CHAKRAVARTY, S. R. (1983): "A New Index of Poverty," *Mathematical Social Sciences*, 6, 307–313.
- CHAUDHURI, S., AND M. RAVALLION (1994): "How well do static indicators identify the chronically poor?," *Journal of Public Economics*, 53(3), 367–394.
- CPRC (2004): "The Chronic Poverty Report 2004-05," Discussion paper, Chronic Poverty Research Centre.
- DEATON, A. (1991): *Household saving in LDC's : credit markets, insurance and welfare*. Research Program in Development Studies Woodrow Wilson School Princeton University, Princeton, N.J.
- DERCON, S. (2004): *Insurance against poverty*, Studies in development economics. Oxford University Press, Oxford.
- (2006): "Poverty Measurement," in *The Elgar Companion to Development Studies*, ed. by D. A. Clark. Edward Elgar.
- DERCON, S., J. HODDINOTT, AND T. WOLDEHANNA (2005): "Shocks and Consumption in 15 Ethiopian Villages, 1999-2004," *Journal of African Economies*, 14(Number 4).
- DERCON, S., AND P. KRISHNAN (1998): "Changes in Poverty in Rural Ethiopia 1989-1995: Measurement, Robustness Tests and Decomposition," Discussion Paper WPS/98-7, Centre for the Study of African Economies.
- (2000): "In Sickness and in Health: Risk Sharing within Households in Rural Ethiopia," *Journal of Political Economy*, 108(4), 688.
- FAFCHAMPS, M. (2003): *Rural poverty, risk and development*. Edward Elgar, Cheltenham.



- FOSTER, J., J. GREER, AND E. THORBECKE (1984): "A Class of Decomposable Poverty Measures," *Econometrica*, 52(3), 761–766.
- FOSTER, J., AND M. E. SANTOS (2006): "Measuring Chronic Poverty," .
- FOSTER, J. E. (2007): "A class of chronic poverty measures," Discussion paper, Department of Economics, Vanderbilt University.
- FOSTER, J. E., AND A. F. SHORROCKS (1991): "Subgroup Consistent Poverty Indices," *Econometrica*, 59(3), 687–709.
- GORMAN, T. (1968): "The Structure of Utility Functions," *The Review of Economic Studies*, 35(4), 367–390.
- HULME, D., AND A. SHEPHERD (2003): "Conceptualizing Chronic Poverty," *World Development*, 31(3), 403–423.
- JALAN, J., AND M. RAVALLION (1998): "Behavioral responses to risk in rural China," Discussion paper, World Bank Development Research Group Poverty and Human Resources, Jyotsna Jalan and Martin Ravallion. 28 cm. "September 1998"—Cover.
- (2000): "Is Transient Poverty Different? Evidence for Rural China," *Journal of Development Studies*, 36(6), 82.
- MCCULLOCH, N., AND M. CALANDRINO (2003): "Vulnerability and Chronic Poverty in Rural Sichuan," *World Development*, 31(3), 611–628.
- MCKAY, A., AND D. LAWSON (2003): "Assessing the Extent and Nature of Chronic Poverty in Low Income Countries: Issues and Evidence," *World Development*, 31(3), 425–439.
- NARAYAN-PARKER, D., R. PATEL, K. SCHAFF, A. RADEMACHER, AND S. KOCH-SCHULTE (2000): *Voices of the poor : can anyone hear us?*, vol. 1 of *Voices of the poor*. Oxford University Press for the World Bank, Oxford.
- NUSSBAUM, M. C., AND A. K. SEN (1999): *The quality of life*, Studies in development economics. Oxford University Press, New Delhi.
- RAVALLION, M. (1996): "Issues in measuring and modelling poverty," *Economic Journal*, 106(438), 1328–1343.
- ROWNTREE, B. S. (1901): *Poverty : a study of town life*. Macmillan, London.
- SEN, A. K. (1976): "Poverty: An Ordinal Approach to Measurement," *Econometrica*, 44(2), 219–231.
- (1982): *Choice, welfare and measurement*. Basil Blackwell, Oxford.
- (1997): *Choice, welfare and measurement*. Harvard University Press, Cambridge, Mass. ; London.
- (2001): *Development as freedom*. Oxford University Press, Oxford.
- STEVENS, A. H. (1999): "Climbing out of poverty, falling back in: measuring the persistence of poverty over multiple spells," *Journal of Human Resources*, 34, 557–588.

WEBB, P., J. VON BRAUN, AND Y. YOHANNES (1992): "Famine in Ethiopia: Policy Implications of Coping Failure at National and Household Levels," Discussion Paper 92, International Food Policy Research Institute (IFPRI).