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ANALYSIS OF THE TOP OF THE INCOME DISTRIBUTION BY A MULTIRESOLUTION FAMILY OF DENSITY

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Abstract

In this paper, using individual income as a proxy variable of the socioeconomic status, we focused on the social tension generated by the income differences between the top end of the distribution (20% or 10%) and the rest (80% or 90%). With this aim firstly, we pursue a flexible probability model to study the right tail of the income distribution as well as the rest of the income distribution secondly, a measure of social tension is defined.

The top end of the income distribution as well as the rest are modelled using a multiresolution probability density function (MRA pdf). Having estimated the MRA pdf for each group the mixture of both densities is obtained. The estimated coefficients of the MRA pdf of each group as well as the coefficients of the mixture will be used to evaluate social tensions.

A measure that quantifies the level of social tension that may arise between the individuals that belong to the right tail of the distribution and the rest is defined. It takes into account two factors. The first one focuses on the alienation-identification felt by individuals. The second one concentrates on income inequality inter-groups. The measure of social tension used in this paper is obtained multiplying both factors.

The model and the measure defined are applied to the European Community Household panel data (1993 and 2000) for EU-15 countries.

Keywords: multiresolution probability density function (MRA pdf), cubic box-spline, level of resolution, top end of the income distribution, alienation-identification, social tension.

1. Introduction[1]

There is an increasing interest in the study of the high end of the distribution of income induced by the changes that it has experienced. Recent works (see among others Rashid 1994, Piketty and Saez 2006, Murphy et al. 2007, Atkinson and Piketty, 2006) show that income has increased relatively more for those groups at the top of distribution. As a consequence a group has emerged whose income is so large that they can be said to diverge from the rest of the society.

Most publications related to high income focus on the distribution among the top of the income, that is, the share of total income held by the richest groups, using tax data. Beginning with the work by Piketty (2001) on the long-run distribution of top incomes in France, international top incomes database have been developed utilizing tax statistics. The

[1] The authors would like to acknowledge the help given by Nuñez-Velazquez, J. and. Dominguez-Dominguez, J. in the treatment of the data.

research by Atkinson and Piketty (2007) describes the construction of top income series in continental European countries and English-speaking countries using tax data. The rich information provided by these series had been utilized to study the distribution among the top end of the income distribution in France, UK, US, Canada, Australia, New Zealand, Germany, The Netherlands, Switzerland and Ireland.

The aim of this paper is threefold. First, we pursue a flexible probability model to study the income distribution of the top income group and the rest. Second, we provide a measure of the level of social tension that may arise between the individuals that belong to the right tail of the distribution and the rest. In particular in this paper we are interested in the measurement of the social tension that could cause a feeling of discomfort to the majority of the population with respect to the most-favoured income group. Third, we apply the model and the measure to the European Community Household panel data (1993 and 2000) for EU-15 countries.

To model the upper tail and the rest of the income distribution we are going to use a multiresolution probability density function (MRA pdf henceforth). The MRA pdf is based on multiresolution analysis and it can be derived by mixing dilations and translations of a cubic box spline function.

Assume that the distribution is built over a closed interval $[a,b]$ named cuasi-support, that contains the sample data and that it is partitioned at *m* regular segments. Let $\theta(x)$ be a box spline of degree three. The MRA is pdf defined by the expression

$$f_m(x) = \sum_{k=0}^{m} a_k \theta_{mk}(x) \quad (1)$$

where $a_k \geq 0$, $\sum_{k=0}^{m} a_k = 1$ and $\theta_{mk}(x) = s\theta(s(x-a)-k)$, with $s = \frac{m}{b-a}$.

The parameter *s* determines the level of resolution and its inverse s^{-1} is the scale parameter.

First step to estimate the MRA pdf of the right end of the distribution is to draw the high- income line that separates those with high income from those without. There is not a great amount of literature related to the definition of high-income and the selection of an income threshold above which one is considered to have high-income. A number of absolute and relative thresholds have been utilized for separating those with high income from those without (see for instance Murphy et al, 2007). In this paper we utilize relative thresholds. In particular we focus on the top 20% and 10% of the population.

The MRA pdf associated with the 20% (or 10%) of the population with the highest income (f_m^2) as well as the MRA pdf related to the rest of the population (f_m^1) are estimated considering identical cuasi-support of the income distribution and level of resolution in both estimations. Maintaining the cuasi-support of the income distribution and the level of resolution the mixture of both densities is obtained since the MRA pdf can be written as follows:

$$f_m(x) = p_1 f_m^1(x) + p_2 f_m^2(x)$$

where $f_m^j(x) = \sum_{k=0}^{m} a_m^j \theta_{mk}(x)$ with j= 1,2; $p_1 = 0.8$, $p_2 = 0.2$ or $p_1 = 0.9$, $p_2 = 0.1$

The coefficients of $f_m^1(x)$, $f_m^2(x)$ and $f_m(x)$ will be used to calculate the measure of social tension proposed.

In the second part of the paper is defined a measure of the social tension generated by the discomfort of the majority of the population with respect to the most-favoured income group. This measure takes into account two factors. The first factor focuses on the contribution of the alienation-identification to the generation of tension and the second one on the influence that inequality between groups has over the social tension.

The alienation-identification factor is quantified by means of an index, I_W, which is related inversely to the proportion of the variance intra-group over the total variance and directly to the proportion of the variance between groups over the total variance. The smaller (larger) the proportion of the variance intra-group over the total variance (the proportion of the variance between groups over the total variance) the higher the index and the greater the contribution of this factor to the generation of social tension.

The second factor is introduced in the measure using an index, I_μ, that quantifies the inequality between groups. The higher I_μ the greater the discontent felt by individuals and the potential to generate social tension.

Multiplying both factors it is obtained a measure of social tension originated by the feeling of discomfort of the majority of the population with respect to the most-favoured income group.

The measure defined has permitted us to observe that not always the convergence (divergence) between the top income distribution and the rest involves an increase (decrease) of the identification within the groups. It is important to know this fact because we assume that homogeneous groups have more chance of generating social tension and therefore the negative effects of a increasing of the inequality between groups can be amplified when the cohesion within the group is enhanced.

The MRA pdf and the measure proposed are applied to data derived from the ECHP for EU-15 countries. We consider the information of waves 1 and 8 which correspond to years 1993 and 2000 (ECHP 1994 and 2001). Austria, Finland and Sweden are not included in first the wave of the ECHP. To estimate the MRA pdf of Austria and Finland waves 3 (1995) and 8 are used and waves 4 (1996) and 8 are utilized for Sweden. Having calculated the measure defined we compare the tension levels generated by the income differences between the 20 % of the highest income receivers and the rest from 1993 to 2000 in the EU-15. In addition, basing on the social tension evolution given by I_W and I_μ and using a dispersion graph the EU-15 countries are classified into four groups.

This paper is organized as followed. Section 2 develops the statistical approach. Section 3 introduces the measure of social tension. Section 4 describes the empirical application of the model and Section 5 contains final remarks.

2. Statistical approach

2.1 A multiresolution family of density functions

In this Section we are going to define a family of density functions that allows us to study the clustering of the population by means of a local scan of the mass probability.

Assume that the distribution is built over a closed interval[2] $[a,b]$ which is partitioned at *m* regular segments. Let $\theta(x)$ be the box spline of degree three[3] (Mallat,

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[2] In the applications $[a,b]$ will be the sample range and it will be named cuasi-support.

[3] It is a translation of 4 convolutions of $\mathbf{1}_{[0,1]}$ with itself. It is centred at $t=0$. Its Fourier transform is:

$$F_\varphi(\omega) = \left(\frac{\sin(\omega/2)}{\omega/2}\right)^4$$

1999) which is a density function symmetric with compact support $[-2,2]$ and with mean and variance equal to 0 and $1/3$ respectively[4].

A set of density functions can be obtained for each $m \in Z^+$ fixed considering the expression:

$$\theta_{mk}(x) = s\theta(s(x-a)-k) \quad k = 0,..., m$$

where $s = \dfrac{m}{b-a}$.

$\theta_{mk}(x)$ is a density function with compact support, $\left[a + \dfrac{k-2}{s}, a + \dfrac{k+2}{s}\right]$, that results from a translation of $\theta(x)$ toward the pole $a + \dfrac{k}{s}$ and a dilation by the factor $s^{-1} = \dfrac{b-a}{m}$.

Combining $\theta_{mk}(x)$ the following multiresolution family of density functions is obtained:

$$\left\{ f_m(x) = \frac{1}{\sum_{k=0}^{m} b_k} \sum_{k=0}^{m} b_k \theta_{mk}(x) \right\}_{m \in Z^+} \qquad (2)$$

where $b_k \geq 0 \quad \forall k = 0,1,...,m$.

Given $\mathbf{b} = (b_k)_{k=0,1,...,m}$ any vector of coefficients proportional to **b** generates the same density. In particular, for

$$a_k = \frac{b_k}{\sum_{k=0}^{m} b_k} \qquad (3)$$

we have $a_k \geq 0 \; \forall k = 0,2,...,m$; $\sum_{k=0}^{m} a_k = 1$ and therefore (1) it can be written as follows:

$$\left\{ f_m(x) = \sum_{k=0}^{m} a_k \theta_{mk}(x) \right\}_{m \in Z^+} \qquad (4)$$

This is a flexible family of density functions that may be used to model a great variety of distributions which can be asymmetric and multimodal. The *m*+1 points of the partition

[4] These values are obtained using the first and second derivatives in the origin of the characteristic function which by definition is equal to $F_\varphi(-\omega)$ (see footnote 3).

over which are defined the mix of distributions show the location of population around different poles. In particular, each $\theta_{mk}(x)$ is an "atomic-scanner" density located at the "micropole" $x_k = a + \frac{k}{s}$.

The scale parameter m or $s = \frac{m}{b-a}$ determines the level of resolution. Parameter m changes from a minimum value, that is $m=1$ or $s = \frac{1}{b-a}$, to a larger resolution in such a way that each sample element approaches to any pole of the partition as desired. The higher the value of m the greater the number of poles, the narrower the exploring bandwidth (support) of the "atomic-scanner" densities, and the more flexible the family of densities. Thus, for a sufficiently high value of m, the graph of the density function progressively resembles a bar diagram in which there are n bars with the same height over each sample observation.

The coefficients a_k are interpreted as the share of population captured by the micropole $x_k = a + \frac{k}{s}$ and distributed around it following the pdf given by $\theta_{mk}(x)$.

The coefficients of the model are estimated by the maximum likelihood procedure for a given value of m using the EM algorithm (Hartley, 1958; Dempster et al., 1977; McLachlan and Krishman, 1997) and therefore they are consistent, asymptotic unbiased and asymptotic efficient.

Different approximations, to the theoretical distribution, are performed by increasing the resolution level m. Attending to the parsimony principle, the model with minimum m which is non-rejected by the test of Kolmogorov-Smirnov fits well to the pdf and will be used to estimate the MRA pdf.

The mean and the variance of the multiresolution model are calculated considering that $\sum_{k=0}^{m} a_k = 1$ obtaining:

$$\mu = E_f[X] = \sum_{k=0}^{m} a_k E_{\theta_{mk}}[X] = a + \sum_{k=0}^{m} \frac{k a_k}{s} \qquad (5)$$

$$V_f(X)=\sum_{k=0}^{m} a_k E_{\theta_{mk}}\left[(X-\mu)^2\right]=\sum_{k=0}^{m} a_k\left\{V_{\theta_{mk}}(X)+\left[E_{\theta_{mk}}(X)-\mu\right]^2\right\}=$$

$$=\frac{1}{3s^2}+\sum_{k=0}^{m} a_k\left(a+\frac{k}{s}-\mu\right)^2=\frac{1}{3s^2}+\sum_{k=0}^{m} a_k\left(\frac{k}{s}\right)^2-\left(\sum_{k=0}^{m}\frac{ka_k}{s}\right)^2 \quad (6)$$

2.2 The MRA pdf and the right end of the income distribution

The income density function had been modeled frequently by means of convex linear combinations of the density functions of the sub-population whose coefficients represent the share of population in the groups (see, among others, Bakker and Creedy, 1999; Chotikapanich and Griffiths, 2007).

An important feature of the MRA pdf is that for a given level of resolution, the mixture of densities defined in (4) is another density of the same type. For a given value of *m*, let $f_m^j(x)$ be a density function belonging to the family defined by (4):

$$f_m^j(x)=\sum_{k=0}^{m} a_m^j \theta_{mk}(x)$$

Let $p_1,\ldots,p_q$ be non-negative values such as $\sum_{j=1}^{q} p_j = 1$, which can be interpreted as the size of the *q* sub-populations. The mixture of $f_m^j(x)$ provides a density function belonging to family (4), that is:

$$f_m(x)=\sum_{j=1}^{m} p_j f_m^j(x)=\sum_{k=0}^{m} a_k \theta_{mk}(x)$$

with $a_k = \sum_{j=1}^{m} p_j a_m^j$

As we pointed out in the introduction we are going to focus on two income groups: the 20 % and 10% of the population with the highest income, and the rest of population being 80% and 90%. We are going to estimate the MRA pdf associated with the 20% of the population with the highest income as well as the MRA pdf related to the rest of the population that is 80% considering identical cuasi-support of the income distribution and level of resolution in both estimations[5]. Each MRA pdf is estimated by the maximum likelihood procedure using the EM algorithm (Hartley, 1958; Dempster et al., 1977; McLachlan and Krishman, 1997). Having estimated the MRA pdf for each group the mixture of both desnsities is obtained. The estimated MRA pdf of each group and the mixture of both densities will be used to study the social tension between the individuals in the right tail of the distribution and the rest.

3. Measure of social tension

In this Section we are going to define a measure of the social tension generated by the income differences between the 20% of the population with the highest income and the rest of the population being 80%. In particular in this paper we are interested in the measurement of the social tension that could cause a feeling of discomfort to the majority of the population with respect to the most-favoured income group.

The measure proposed is based on the following assumptions:

- The higher the similarity of the income within group the larger the cohesion of the group and the potential to generate social tension.
- The higher the income inequality between groups the greater the potential to generate social tension.
- The smaller the number of groups the higher the potential to originate tension.
- The smaller the size of the groups the lesser the potential to generate tension since minority groups have slight chance of generating social tensions.

Taking into consideration the above assumptions the new measure will be a function of the following factors:

i) The similarity of the income within groups.

ii) The income inequality between groups.

iii) The number of groups.

iv) The distribution of the size of the groups.

In this paper we are going to focus on factors i) and ii). Factors iii) and iv) are not reflected on the measure since the number and the size of the groups considered are the

[5] The MRA pdfs for the 90% and 10% are close to the estimation for the 80% and 20% and they are omitted. Henceforth we refer only to 80% and 20%.

same in all countries. Therefore the contribution of this factor to the generation of tension is identical in each country and for this reason it is omitted.

Let us focus on the first factor. To measure the similarity of the income within the group we are going to use the analysis of the variance. As it is well known the variance of the overall population (*V*) is partitioned as follows:

$$V = V_B + V_W \qquad (9)$$

where V_B is the variance between groups or inter-groups and V_W is the variance within the group or intra-group. Dividing expression (9) by the variance of the overall population we have:

$$1 = \frac{V_B}{V} + \frac{V_W}{V} \qquad (10)$$

According to (10) we have that:

$$1 - \frac{V_W}{V} = \frac{V_B}{V} \qquad (11)$$

Using the terminology introduced by Esteban and Ray (1994) to measure polarization we assume that expressions $1 - \frac{V_W}{V}$ and $\frac{V_B}{V}$ describe the concepts of identification and alienation respectively. We suppose that identification is related to the similarity of the income within the group. An individual feels a sense of identification with the group to which he belongs when his income is closer to the average income of the group. Therefore the smaller the proportion of the variance intra-group over the total variance the higher the identification. Alienation is related to the proportion of the variance between groups over the total variance. The larger the proportion the higher the alienation felt by individuals. Since identification and alienation are linked by expression (10) we are going to define an index based on the share of the variances intra-group and inter-groups over the total variance that represents the contribution of the alienation-identification to the generation of tension. The mentioned index is given by:

$$I_W = 1 - \frac{V_W}{V} = \frac{V_B}{V} \in [0,1]$$

Observe that if $I_w = 1$ the individuals belonging to the same group receive the same income and then V_W is zero. In this situation the identification reaches the maximum. If

$I_W = 0$ the identification is null since the two groups are blended into one located around the same mean and then V_B is equal zero.

The next factor in which we are going to focus on is the inequality between groups. To quantify it we assume that there are two groups of sizes p_1 and p_2. The first one is located around a mean income of μ_1 and the second one is placed around a mean of μ_2. This information is summarized in the following table:

p_1	p_2
μ_1	μ_2

The Gini index associated with the above table can be used to provide a measure of inequality between groups. Let us consider the following expression of the Gini index

$$G = 1 - 2\int_0^1 L(p)dp \qquad (12)$$

where $L(p)$ is the Lorenz curve.

Calculating the Lorenz curve associated with the previous table and applying expression (12) we have:

$$G = p_1\left(1 - \frac{\mu_1}{\mu}\right) \in [0, p_1]$$

where μ represents the mean income of overall population which is given by:

$$\mu = 0{,}8\mu_1 + 0{,}2\mu_2$$

where μ_1 is the mean income associated with the 80% of the population and μ_2 is the mean income of the top 20% of the population.

Rescaling we have the index:

$$I_\mu = 1 - \frac{\mu_1}{\mu} \in [0{,}1]$$

The index I_μ will be used to evaluate the effect that the inequality between groups has on the generation of social tensions. Note that the higher the inequality between groups the larger I_μ and the discontent felt by individuals. If both groups have the same mean income then $I_\mu = 0$. If the total income is concentrated in the right end of the distribution then $I_\mu = 1$.

An easy way of combining these two factors is multiplying both of them, that is

$$T = \left(1 - \frac{V_W}{V}\right)\left(1 - \frac{\mu_1}{\mu}\right) = I_W I_\mu \in [0,1]$$

Nevertheless other manners of combining both factors using non-decreasing functions are not discarded in future research.

To calculate the previous measure we are going to use the expressions of the variance intra-group, the overall variance and the mean associated with the estimated MRA pdf.

The MRA pdf for the total population can be written as follows:

$$f_m(x) = \sum_{j=1}^{2} p_j f_m^j(x)$$

where $f_m^j(x)$ are the groups' MRA pdfs.

The mean and the variance of each group are given by:

$$\mu_j = \int x f_m^j(x)dx \quad j = 1,2$$

$$V_j = \int (x - \mu_j)^2 f_m^j(x)dx \quad j = 1,2$$

The expressions of the mean and the variance of the overall distribution are:

$$\mu = \int x f_m(x)dx$$

$$V = \int (x - \mu)^2 f_m(x)dx$$

The mean and the variance are calculated by means of expressions (5) and (6) using the estimated coefficients of the respective MRA pdfs.

It is not difficult to prove that

$$\mu = \sum_{j=1}^{2} p_j \mu_j$$

and

$$V = \sum_{j=1}^{2} p_j V_j + \sum_{j=1}^{2} p_j (\mu_j - \mu)^2$$

where $V_W = \sum_{j=1}^{2} p_j V_j$ and $V_B = \sum_{j=1}^{2} p_j (\mu_j - \mu)^2$

Therefore I_W is given by:

$$I_W = 1 - \frac{V_W}{V} = 1 - \frac{\sum_{j=1}^{2} p_j V_j}{\sum_{j=1}^{2} p_j V_j + \sum_{j=1}^{2} p_j (\mu_j - \mu)^2}$$

4. Empirical application

In this section the approach defined is applied to empirical data from the European Community Household Panel (ECHP) for the years 1994-2001. The ECHP is conducted by the Statistical Office of the European Communities (EUROSTAT) and provides information about the composition, life conditions and other relevant characteristics of European households (see for instance Nicoletti and Peracchi, 2002 and Nuñez-Velazquez and Dominguez-Dominguez, 2005 for a description of the ECHP dataset).

The variable used to estimate the income distribution is the equivalised disposable income of the European households which were collected the year before making the survey. For the equivalisation the modified OECD equivalence scale is used. The equivalised disposable income is expressed in constant euros at 2005 prices using the Harmonised Index of Consumer Prices (HICPs) for each country.

We consider the information of waves 1 and 8 which correspond to years 1993 and 2000 (ECHP 1994 and 2001). Austria, Finland and Sweden are not included in first the wave of the ECHP. To estimate the MRA pdf of Austria and Finland waves 3 (1995) and 8 are used and waves 4 (1996) and 8 are utilized for Sweden.

The MRA pdf associated with the 20% of the population with the highest income as well as the MRApdf related to the rest of the population being 80% are estimated by the maximum likelihood procedure using the EM algorithm (Hartley, 1958; Dempster et al., 1977; McLachlan and Krishman, 1997). These estimations are made for each country and for the years 1993 and 2000 to establish comparison. Having estimated the MRA pdf for each group the mixture of both densities is obtained. Figures 2-31 show the MRA pdf associated with each group of income and the mixture of both densities.

The mean and the variance of each group and the variance of the mixture are calculated using the estimated coefficients of their respective MRA pdfs by means of expressions (5) and (6). Following the indices I_W and I_μ are obtained and multiplying both of them the measure of tension for the years 1993 and 2000 is obtained (Tables 1 and 2).

Utilizing Table 1 and 2 we can compare the tension levels generated by the income differences between the 20 % of the highest income receivers and the rest from 1993 to 2000 in the EU-15. In general terms, it is observed that inequality between groups decreased in all countries except in Finland, Sweden and Belgium although in the last country it has increased in a small quantity. The identification within the groups increased in Denmark, France, Ireland, Greece, Austria, Finland and Luxembourg. In the rest of countries identification diminished from 1993 to 2000. In the countries where both factors moved in opposite directions the level of tension will depend on the predominant one. Specifically it is observed that the measure of social tension decreased more in United Kingdom, which is followed by Belgium, The Netherlands, Portugal, Spain, Germany, Italy, and Austria. The index of tension increased in the remaining countries. France is the country where the social tension enhanced in greater quantity. The other countries in decreasing order are Finland, Ireland Greece, Denmark, Luxembourg and Sweden.

In addition the MRA pdf related to the 10% of the population with the highest income and the remaining 90 % as well as the measure of tension are estimated. The MRA pdfs for the 90% and 10% are close to the estimations for the 80% and 20%. The measure of tension for 90% and 10% ranks the countries in the same way that if the 80% and the 20% is used. Hence the estimated MRA pdfs and the values of the measure of tension are not including in the paper.

Furthermore, basing on the social tension evolution given by I_W and I_μ from 1993 to 2000 and using the dispersion graph shown by Figure 1 the EU-15 countries are classified into four groups. The first group is formed by those countries in which I_W and I_μ increased. The second group includes the countries in which I_W and I_μ decreased originating a diminishing of the tension. The third and the fourth groups contain the countries in which I_W and I_μ moves in opposite directions. In these cases the tension depends on the predominant factor. Figure 1 shows the classification of the countries. In this Figure each segment represents a country; the origin (year 1993) coincides with the origin of coordinates and the extreme of the segment represents the difference between I_W and I_μ from 1993 to 2000. Therefore the length of the segment shows the magnitude of the change of I_W and I_μ from 1993 to 2000. The directions of the segments indicate in which sense (increasing or decreasing) I_W and I_μ have been modified.

The first group is composed by Finland. The second group includes The Netherlands, Spain, Portugal, Germany, United Kingdom and Italy. The third group is formed by countries in which I_W increased and I_μ decreased. These countries are France, Ireland, Greece, Austria, Denmark and Luxembourg. In these countries, excluding Austria, the contribution of I_W to the generation of tension is higher than the contribution of I_μ to the reduction on tension. Both movements involve an increase of tension in these countries except for Austria in which the social tension decreased slightly. The last group is formed by Sweden and Belgium where I_W and I_μ decreased and increased respectively. These movements reduced tension in Belgium and produced a little increase of tension in Sweden.

5. Final Remarks

In this paper, using individual income as a proxy variable of the socioeconomic status, we focused on the social tension generated by the income differences between the top end of the distribution (20% or 10%) and the rest (80% or 90%). With this aim firstly, we pursue a flexible probability model (MRA pdf) to study the right tail of the income distribution as well as the rest of the income distribution. Secondly, a measure of social tension is defined as a decreasing of the proportion of the intra-group variance over the total variance and the inequality between groups.

Using the household income data provided by ECHP for the years 1993 and 2000 (waves 1 and 8) we estimate the MRA pdf associated with the 20% of the population with the highest income and the MRA pdf related to the rest of the population being 80% by the maximum likelihood procedure using the EM algorithm. A MRA pdf for all the population is obtained as a mixture of the densities associated with the 20% and the rest of the population. The estimated coefficients of the respective MRA pdf are utilized to calculate the measure of social conflict defined in this paper.

The evolution of the social tension from 1993 to 2000 is analyzed. Attending to the empirical results, the measure of tension decreased more in United Kingdom, which is followed by Belgium, The Netherlands, Portugal, Spain, Germany, Italy, and Austria. The index of tension increased in the remaining countries. France is the country where the social tension enhanced in greater quantity. The other countries in decreasing order are Finland, Ireland Greece, Denmark, Luxembourg and Sweden.

Basing on the social tension evolution given by I_W and I_μ from 1993 to 2000 and using the dispersion graph shown by Figure 1 the EU-15 countries are classified into four groups. The first group is formed Finland in which I_W and I_μ increased. The second group includes The Netherlands, Spain, Portugal, Germany, United Kingdom and Italy in which I_W and I_μ decreased originating a diminishing of the tension. The third group is formed by France, Ireland, Greece, Austria, Denmark and Luxembourg. In these countries, excluding Austria, the contribution of I_W to the generation of tension is higher that the contribution of I_μ to the reduction on tension. Both movements involve an increase of tension in these countries except for Austria in which the social tension decreased slightly. The last group is formed by Sweden and Belgium where I_W and I_μ decreased and increased respectively. These movements reduced tension in Belgium and produced a little increase of tension in Sweden.

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Table 1. Measure of tension in 1993

Country	Codes	I_W	I_μ	T
Denmark	DK	0,51046371	0,25799966	0,13173311
Netherland	NL	0,57082152	0,23993392	0,13695944
Belgium	BE	0,30108287	0,26927132	0,08107298
France	FR	0,28042593	0,30898424	0,08664719
Ireland	IE	0,30641547	0,29310726	0,0898126
Italy	IT	0,55438565	0,27410434	0,15195951
Greece	GR	0,39861102	0,30940944	0,12333401
Spain	ES	0,53758303	0,28324373	0,15226702
Portugal	PT	0,56870362	0,36029123	0,20489893
Austria*	AT	0,49382106	0,24185738	0,11943427
Filand*	FI	0,36547278	0,21826994	0,07977172
Sweden**	SE	0,49327656	0,2006776	0,09898956
Germany	DE	0,55258539	0,2415403	0,13347164
Luxembourg	LU	0,52305237	0,27077221	0,14162805
United Kingdom	UK	0,60741689	0,28121164	0,1708127

* Data of 1995
** Data of 1996

Table 2. Measure of tension in 2000

Country	Codes	I_W	I_μ	T
Denmark	DK	0,57981911	0,25196964	0,14609681
Netherland	NL	0,47218117	0,23044232	0,10881052
Belgium	BE	0,18490797	0,27393445	0,05065266
France	FR	0,57360219	0,24174318	0,13866442
Ireland	IE	0,46823466	0,27032873	0,12657728
Italy	IT	0,53165583	0,25170615	0,13382104
Greece	GR	0,50750071	0,28192128	0,14307525
Spain	ES	0,46621288	0,27819539	0,12969827
Portugal	PT	0,56639186	0,31728394	0,17970704
Austria	AT	0,54036033	0,22003657	0,11889903
Filand	FI	0,46861532	0,24881033	0,11659633
Sweden	SE	0,45409171	0,22044831	0,10010375
Germany	DE	0,49808846	0,22442299	0,1117825
Luxembourg	LU	0,62335783	0,23289882	0,1451793
United Kingdom	UK	0,45556706	0,27668024	0,1260464

Figure 1. EU-15 classification according to I_W and I_μ

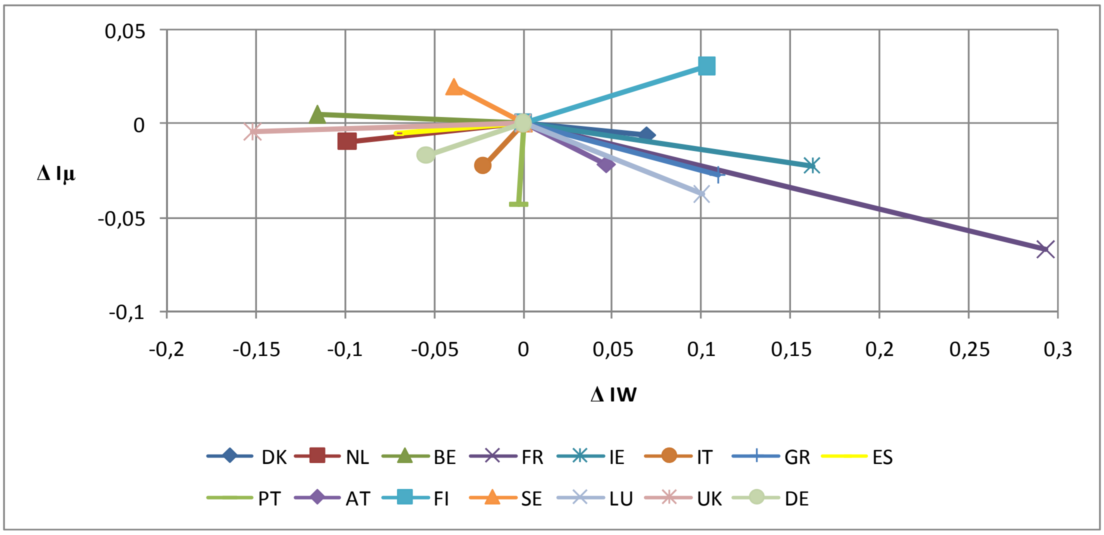
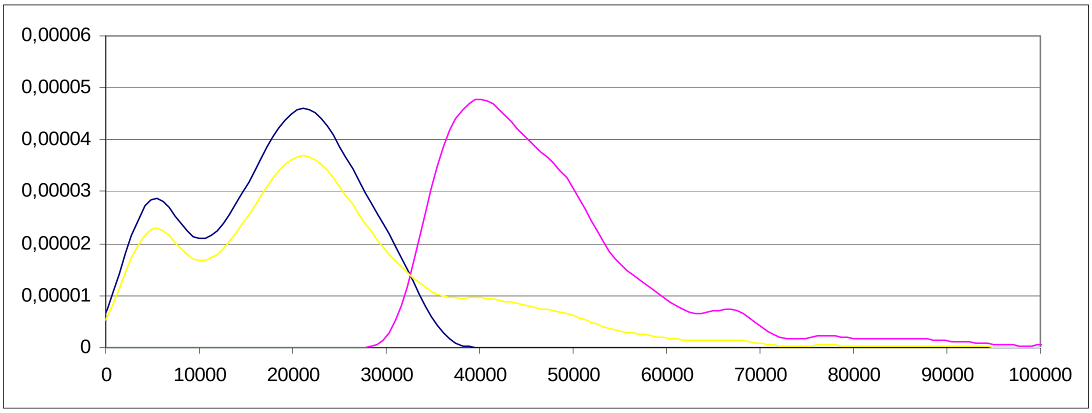


Figure 2. Denmark's estimated pdfs in 1993



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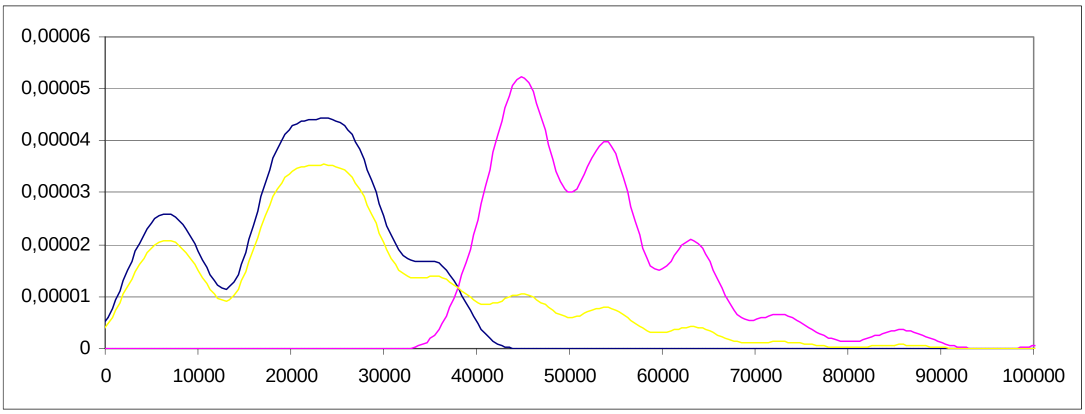


Figure 4. The Netherlands' estimated pdfs in 1993

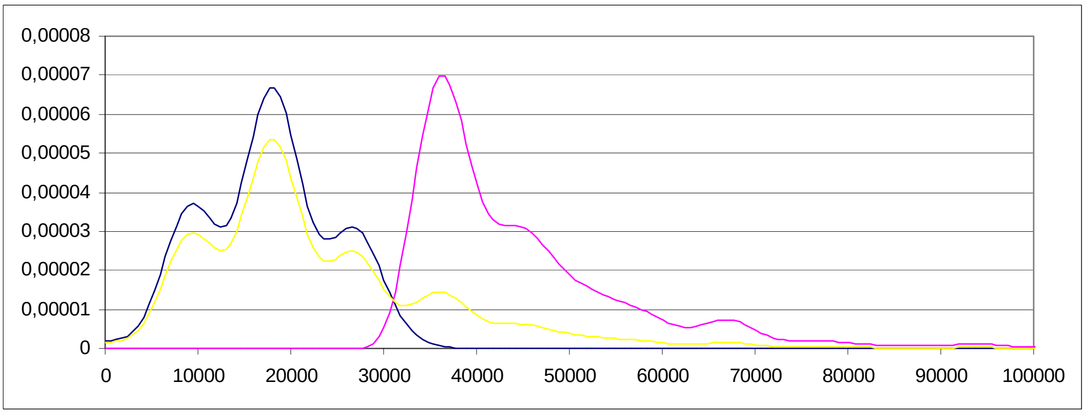


Figure 5. The Netherlands' estimated pdfs in 2000

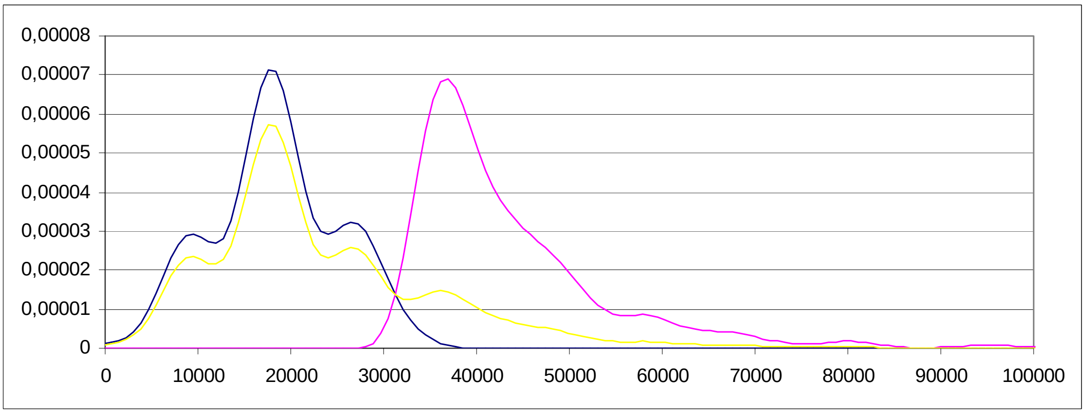


Figure 6. Belgium's estimated pdfs in 1993

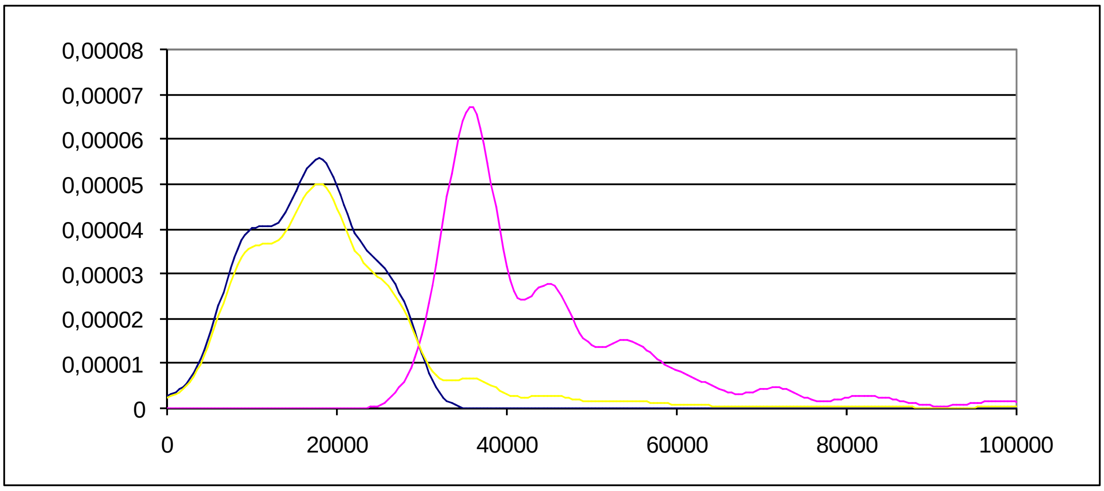
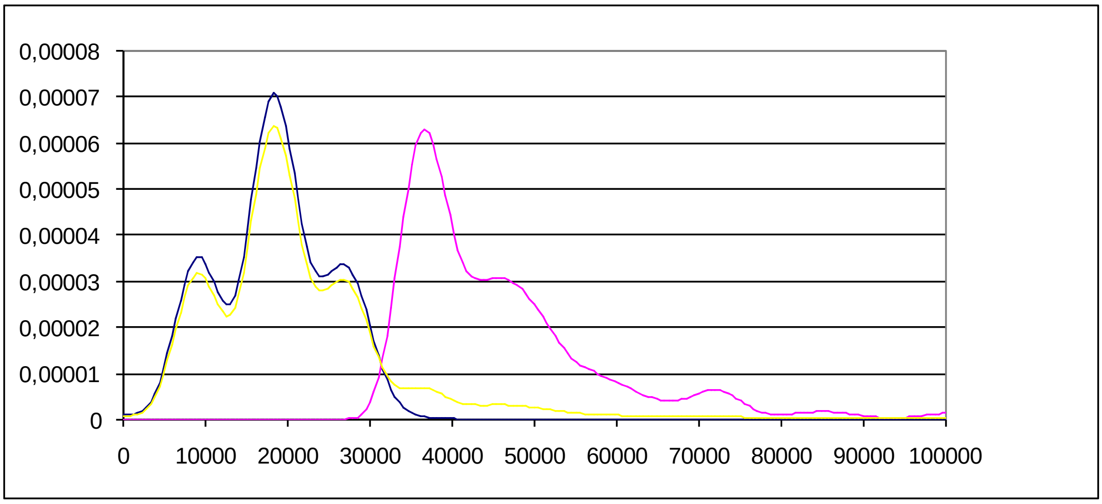


Figure 7. Belgium’s estimated pdfs in 2000



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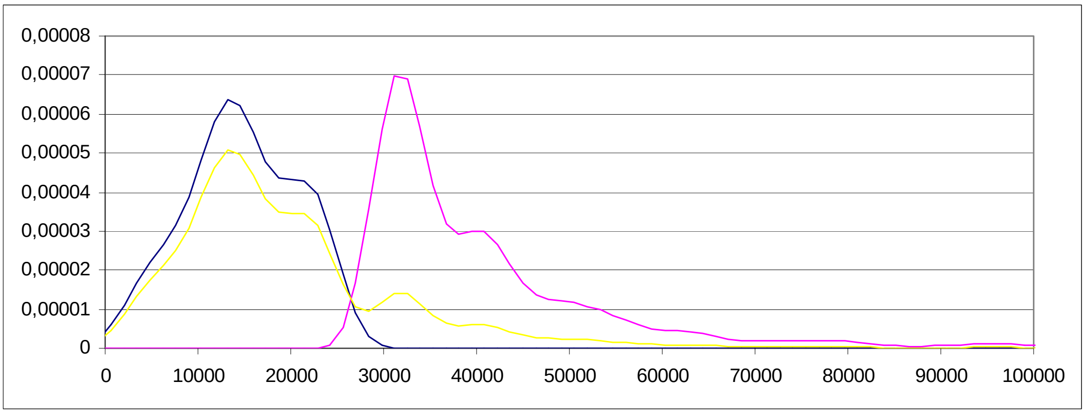


Figure 9. France’s estimated pdfs in 2000

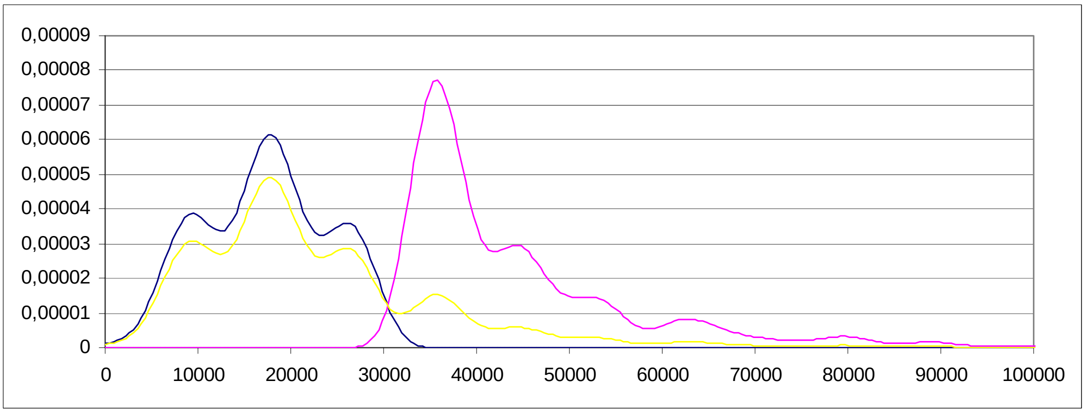


Figure 10. Ireland's estimated pdfs in 1993

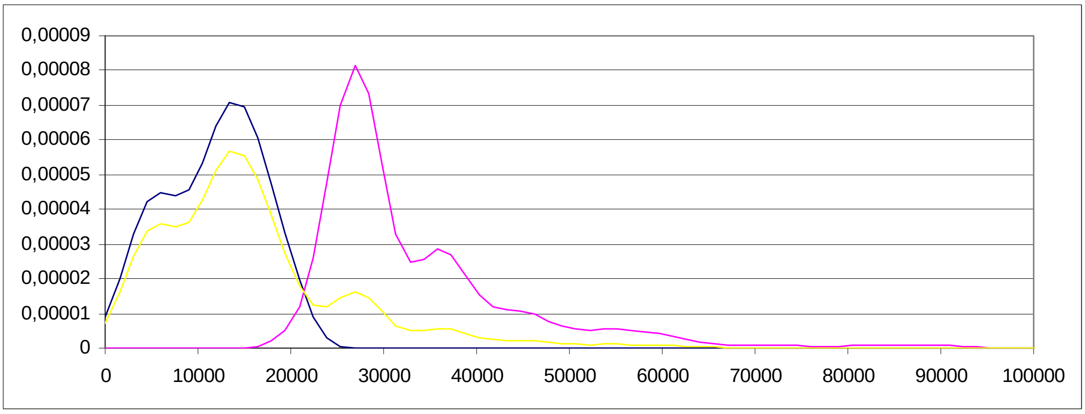


Figure 11. Ireland's estimated pdfs in 2000

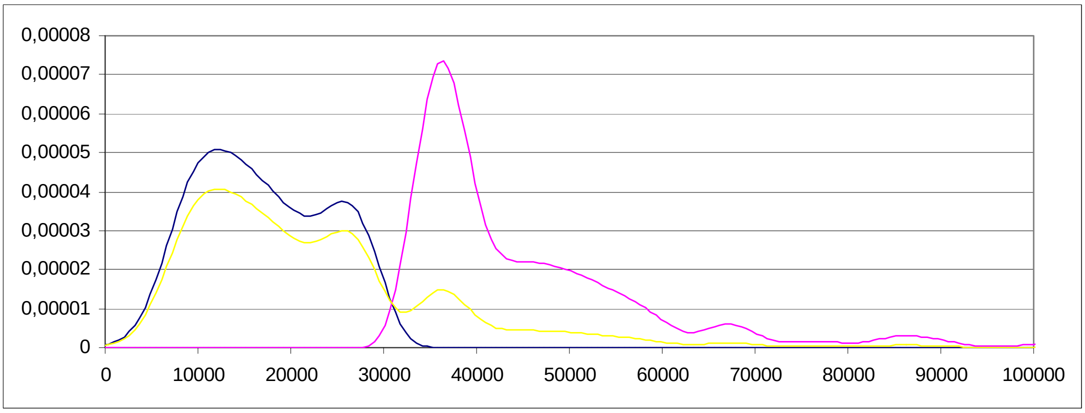


Figure 12. Italy's estimated pdfs in 1993

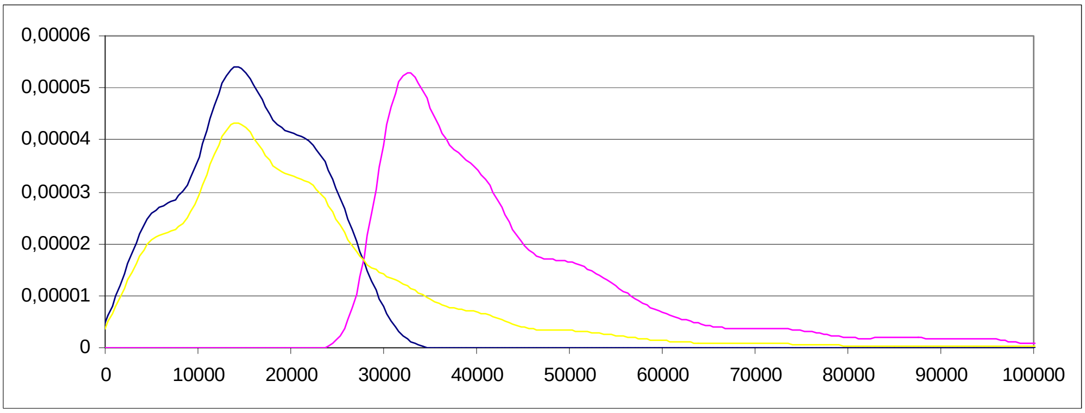


Figure 13. Italy's estimated pdfs in 2000

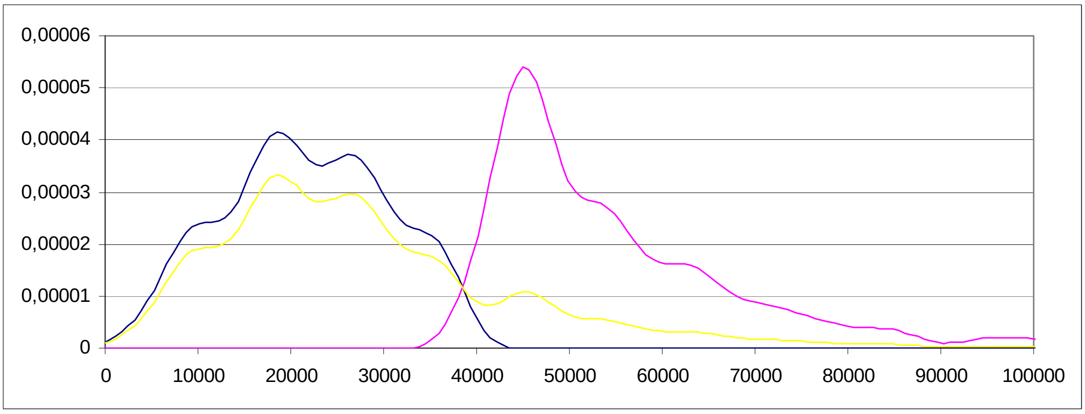


Figure 14. Greece's estimated pdfs in 1993

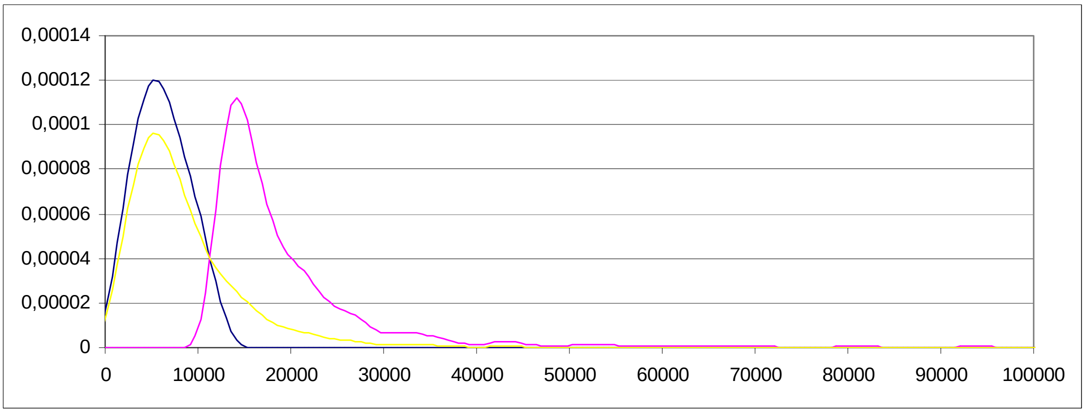


Figure 15. Greece's estimated pdfs in 2000

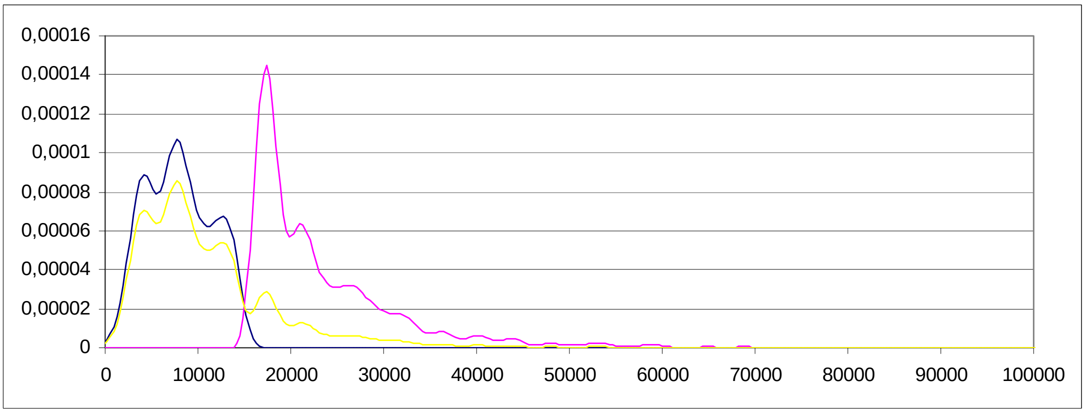
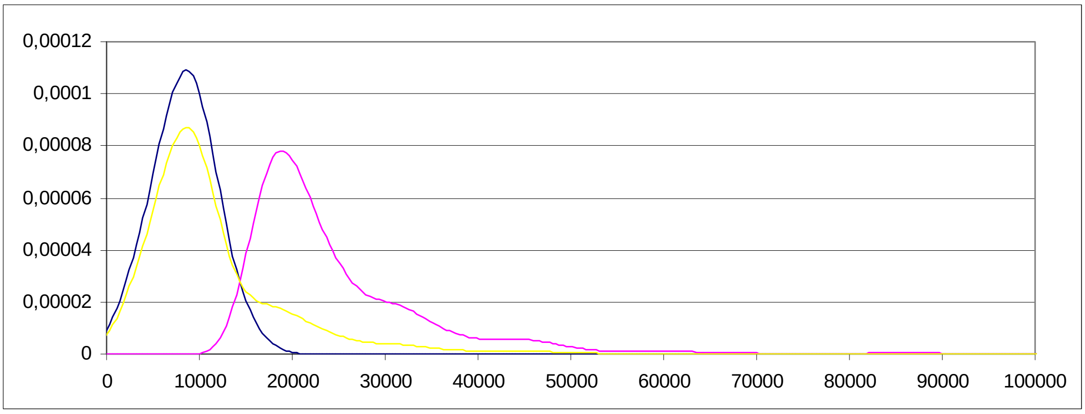


Figure 16. Spain's estimated pdfs in 1993



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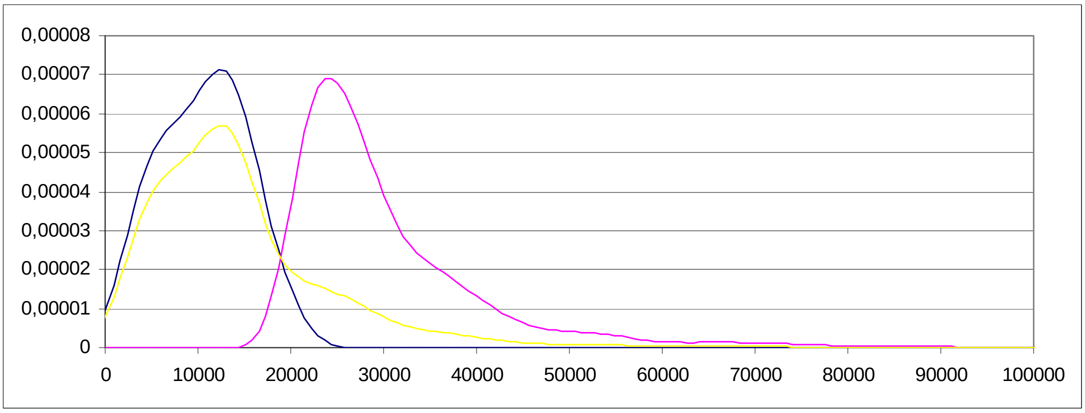

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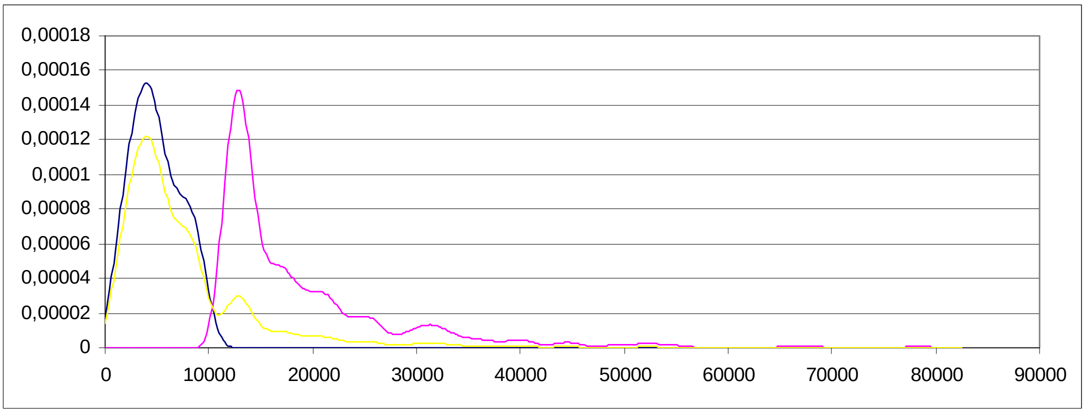

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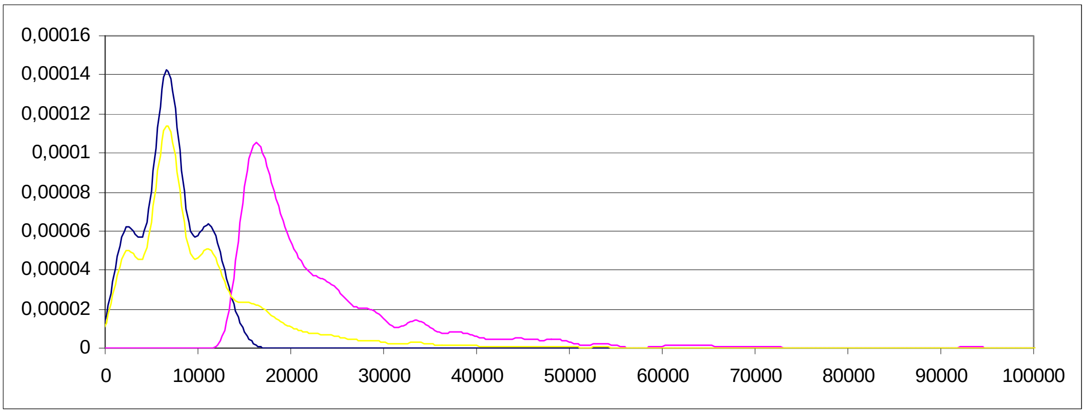
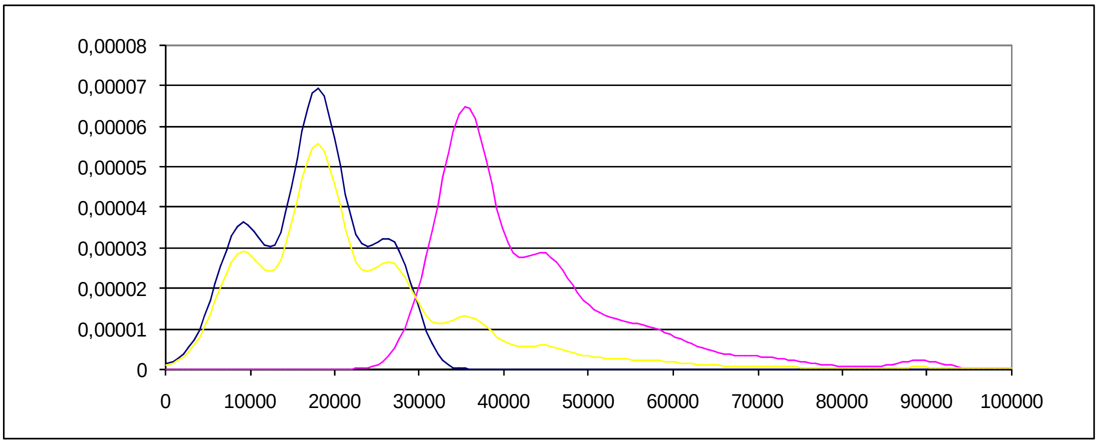


Figure 20. Austria's estimated pdfs in 1995



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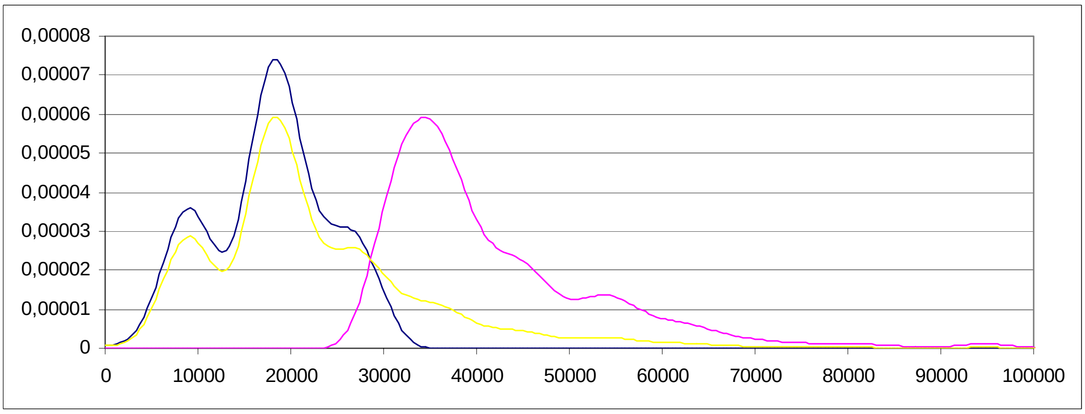

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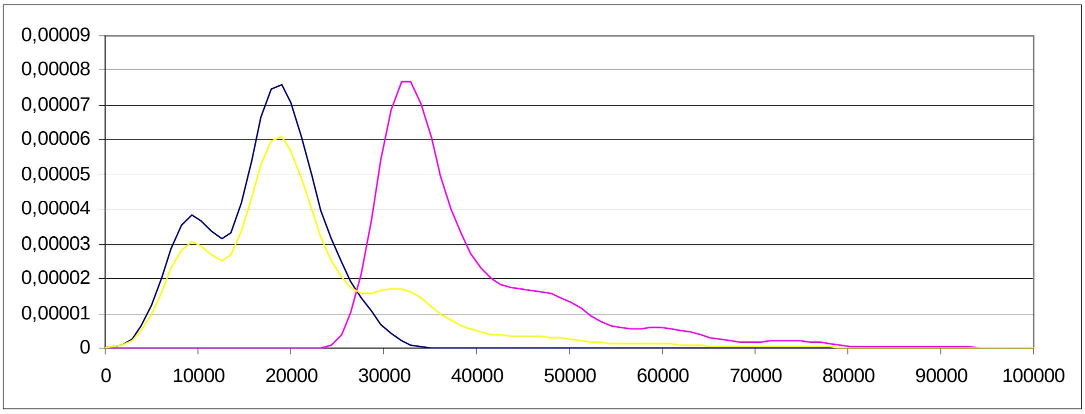

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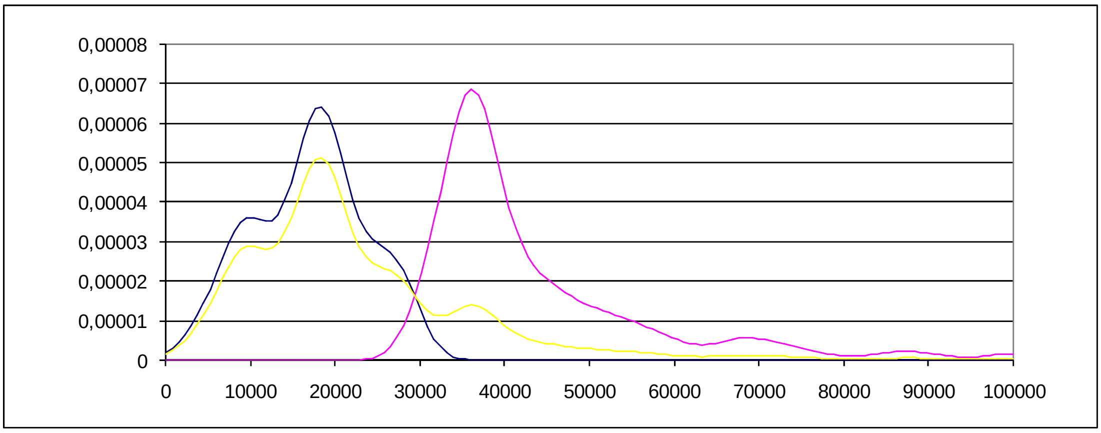
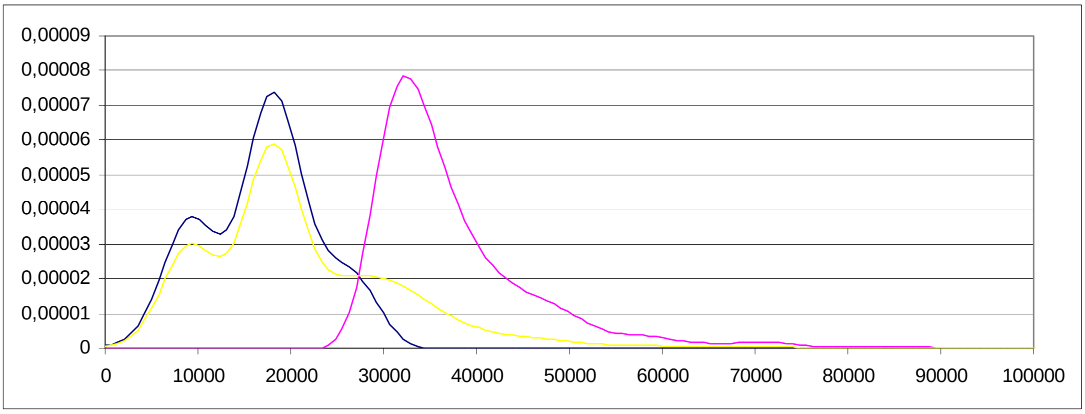


Figure 24. Sweden's estimated pdfs in 1996



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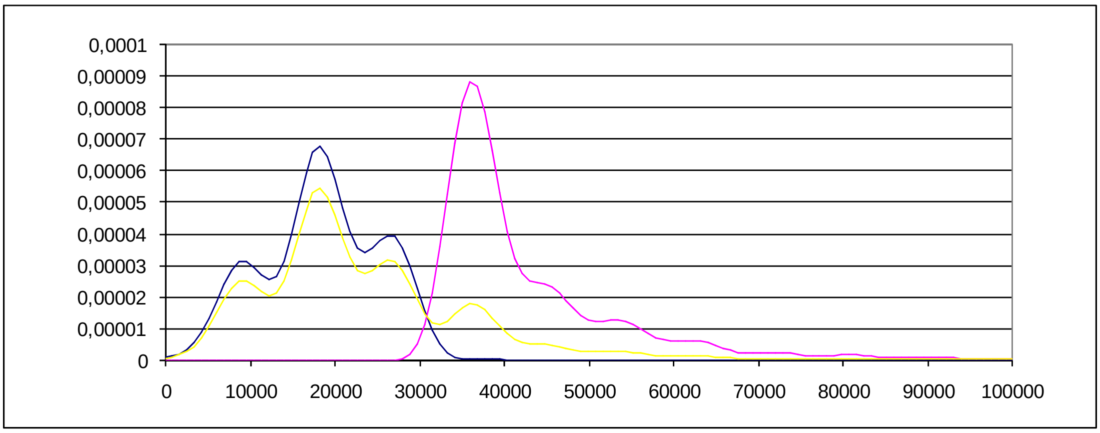

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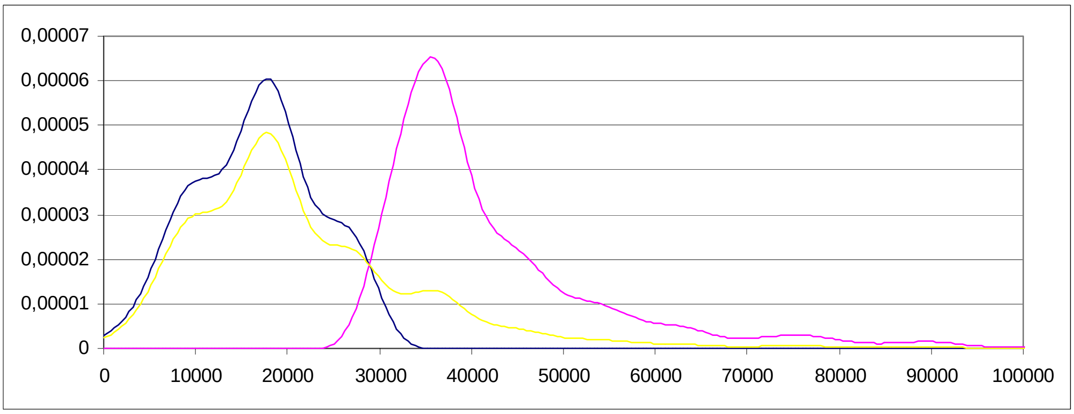

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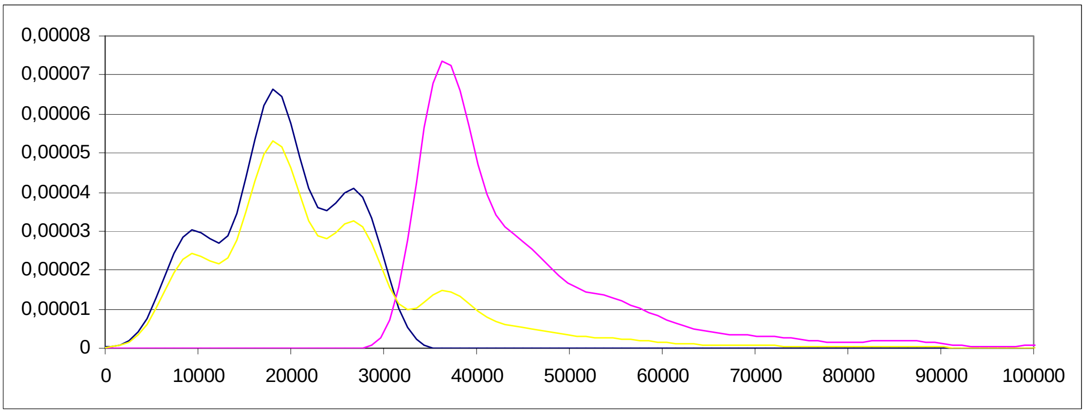

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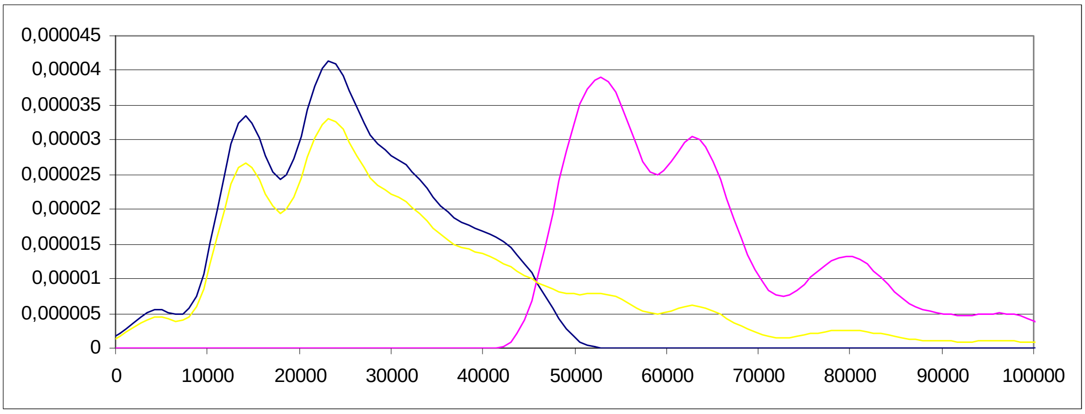

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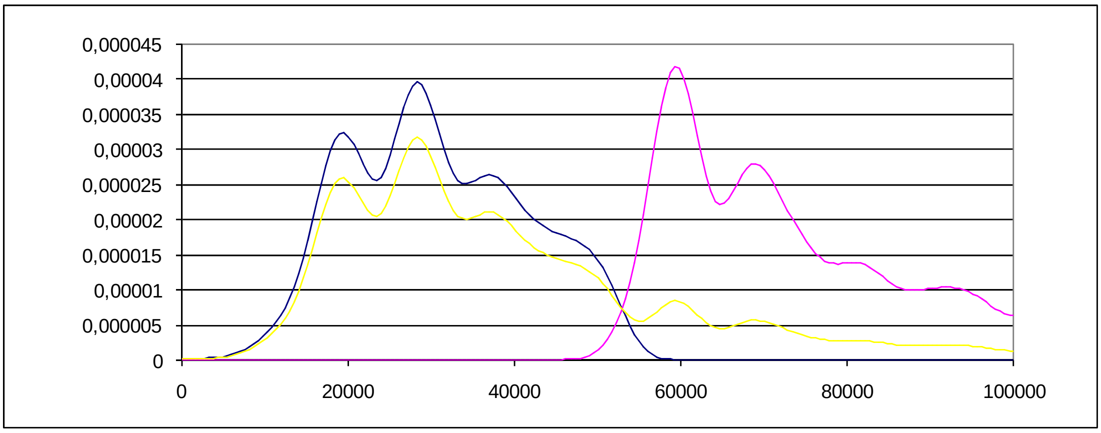

Figure 30. United Kingdom's estimated pdfs in 1993

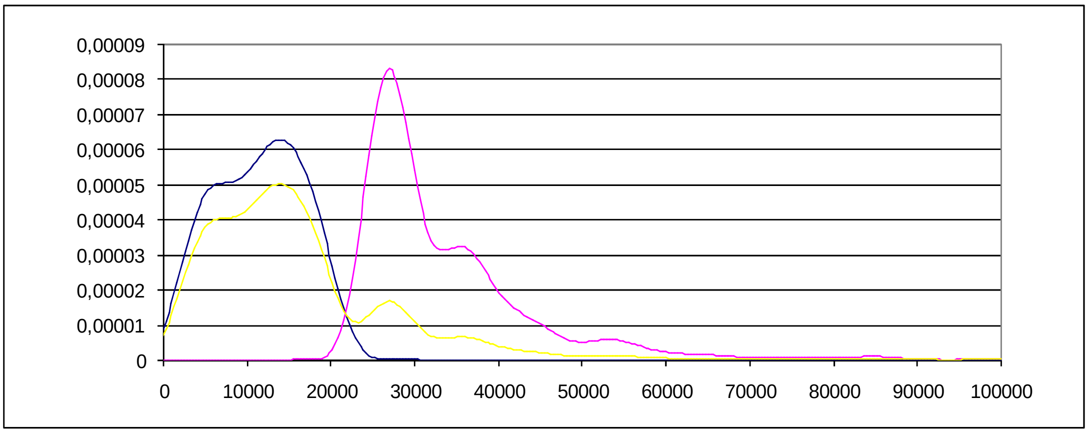

Figure 31. United Kingdom's estimated pdfs in 2000