

Session Number: Session 2A
Time: Monday, August 25, PM

*Paper Prepared for the 30th General Conference of
The International Association for Research in Income and Wealth*

Portoroz, Slovenia, August 24-30, 2008

Assessing implicit pension liabilities for the French pension system:
a micro-founded approach.

Didier BLANCHET and Sylvie LE MINEZ
INSEE - Department of General Economic Studies

For additional information please contact:

Name : Didier BLANCHET
Affiliation : INSEE - Department of general Economic Studies
Full mailing address : 15 Bd Gabriel Péri, BP 100, 92244 MALAKOFF CEDEX, FRANCE
Email address : didier.blanchet@insee.fr

This paper is posted on the following website: <http://www.iariw.org>

Abstract : *An increasing emphasis is currently being put on measures of implicit pension liabilities as instruments that could help monitor pension reform in ageing countries. In France, such measures have been seldom used until now, preference being given to standard projections of replacement rates, equilibrium contribution rates or expected yearly deficits of PAYG schemes. The purpose of the contribution will be to examine whether measures of implicit liabilities for the French system can help us improving our diagnosis on the impact of the two pension reforms that have taken place in 1993 and 2003. The microsimulation model DESTINIE is used for the computing these liabilities. We examine the pros and cons of such measures for assessing the impact of reforms both at the macro level (global financial imbalances) and at the micro-level (differential impact of the reform on various categories of individuals).*

Indicators of implicit liabilities (IL) have been seldom used for the assessment of pension prospects in France, never attracting the attention that they have received in some other countries or from international organisations (Van Der Noord and Herd, 1993; Holzmann et al., 2004). Diagnosis on French pensions remains essentially based on projections of the sequence of potential deficits or chronicles of contribution rates requested, year after year, to maintain the system in financial equilibrium. Such is for instance the line followed by projections that are regularly produced under the *aegis* of the *Conseil d'Orientation des Retraites* (Pensions Advisory Committee). This way to proceed appears more suited to a context where PAYG financing predominates. When synthetic indexes are required instead of full year-to-year projections, e.g. for international comparisons, preference goes to an indicator that remains close to the notion of projected deficit ratios. This indicator is the Tax Gap, also used in projections performed for member states by the Ageing Working Group (AWG) of the European Commission (European Commission, 2006).

As far as IL measures are concerned, only two significant attempts took place in the past, one by Vernière (1992) or, still more remote, by Kessler, Masson and Strauss-Kahn (1980). They had respectively produced orders of magnitudes representing 2.8 or 1.8 years of GDP, both very sensitive to values chosen for the discount rate. Another recent exception has been the Pébureau report (Pébureau, 2005), devoted to the general situation of French public finances. This report used IL indicators as complements to standard measures of explicit debt, at least for pensions due by the State to its current or former employees. However, communication that surrounded this report has reinforced ambient skepticism over the way such indicators can be used in the public debate. In particular, it has proven quite difficult to avoid confusion between different definitions of these ILs that are quite far of having the same meaning, namely the open system (OS) approach on the one side, that focuses on future deficits, and the closed-group (CG) or accrued-to-date liabilities (ADL) approaches that rather focus on commitments towards current contributors or retirees. It is the ADL approach that had been used in the two evaluations by Kessler *et al.* and by Vernière. The Pébureau report rather emphasized the OS concept, with figures comprised between 380 and 490 billion Euros, but also quoted the larger figures of 790 or 1000 billions Euros for the ADL approach and some confusion arose about which of these two figures was the most appropriate for giving the right vision of the magnitude of the pension problem.

Since that episode, interest for these indicators fell off again. Yet, there remains one good reason of looking at them more closely, which is the fact that they are gaining ground under the pressure of international norms both for Public Finance Accounting (see for France Pellé, 2005) and National Accounting. The decision has now been acted to introduce such ADL indicators in the next version of the System of National Accounts. Such an inclusion has two implications : (a) we will have to build these indicators in a more systematic way as we used to do, and (b) we will have to provide their potential users with the right guidelines for their interpretation.

The present paper is a contribution on these two issues. First of all, we shall present how such indicators can be produced relatively easily using the new version of a microsimulation model that exists at INSEE since the mid 1990s, the Destinie Model. Second, we will use this model to analyze the information provided by such indicators, especially concerning the impact of pension reforms. The model allows an easy reconstitution of what this implicit debt would have been today without the two reforms that took place in France in 1993 and 2003. Despite the fact that these reforms have not been marginal at all, it will appear that they impact IL indicators in a very limited way. A similar point has been noticed for Germany by Werding (2006).

Relying on some analytical results presented in the appendix, we will show that such a result is quite easy to understand. The main reason is that ADL or CG indicators are before all indicators of the future size of the system whereas the aim of reforms is not to drastically reduce this size : the aim of these reforms is only to reduce or eliminate expected deficits by containing the growth rate of expenditures.

A simple example can help understand how the two perspectives differ. Assume a pension system that currently represents about 10% of GDP, as it is roughly the case for the French system, after exclusion of survivors' pensions and means tested benefits that will not be covered in the present paper. Assume also that its share in GDP was expected to increase up to 15% without any reform, generating a long-term deficit of 5% of GDP. A rather strong reform that cuts this long run deficit by one half implies a reduction of this future size to 12.5 points of GDP, i.e. a reduction of the size of the system by only one sixth. It is clear that measuring the impact of the reform in terms of such a criterion of size clearly does not give a clear account of how far the reform goes in restoring sustainability. In practice, the image given by IL indicators will be even worse than that, because only one part of this 1/6th reduction will show up in them, due to the fact that most of the reductions of benefits implied by the reform will be borne, in the long run, by cohorts still at the beginning of their active lives, whose entitlements play only a very small role in these indexes. All this means that, if we want to measure the incidence of reforms, we'd rather use indexes that concentrate on these future deficits, such as the Tax Gap or eventually the OS version of implicit liabilities (Blanchet and Ouvrard, 2007).

This does not mean that ADL or CG approaches are of no interest, but our thesis will be that this interest is more from the household's point of view, a kind of application already explored for France by Accardo (1996, 1997) and more specifically at the micro level, rather than at the macro level. Building the IL indicator on the basis of a microsimulation model precisely has this advantage of providing us with information on implicit liabilities at this individual level, in a way whose consistency with macro outcomes is fully warranted. We shall more precisely examine what messages we get on these micro-level impacts of reforms, providing some examples of comparisons across cohorts, genders and social groups.

The paper will be organized as follows. The first section will come back on the features of the French pension system and of its two main reforms that need to be known in order to understand the rest of the paper. Section 2 will briefly present the main characteristics of the new version of the Destinie Model. Section 3 will present in details the two measures of ILs that we are going to examine, the ADL and CG approaches, and will explain how these two indexes have been built with Destinie. Section 4 will discuss results obtained for a reference scenario consisting in the full application of the 2003 reform. This section will use the analytical results of the appendix to explain the orders of magnitude that we have obtained. Section 5 will discuss how the ADL indicator is affected by pension reforms and the reasons why it reacts so little to these reforms. Section 6 will present some illustrations of how ADLs at the micro-level can be used for discussing the redistributive impact of reforms. It will be followed by a short conclusion.

1) The French pension system and its reforms

Understanding the contents and consequences of the French pensions reforms requires a minimal understanding of the organization of the system and of the rules governing eligibility and benefits. The French system is often considered as particularly complex and fragmented. We will not give a full overview of its organization in this paper, but will rather concentrate on the two major schemes that have been concerned by the two reforms that we shall analyze in greater details. These two schemes are the general regime that provides the basic first pillar pension for the vast majority of wage earners working in the private sector (between 60 and 70% of the employed labor force) and the regime that applies to public sector employees (about one fifth of the labor force).

Both regimes are characterized by a legal age at access to retirement that is one of the lowest among developed countries. It is in 1983 that this legal retirement age was lowered to 60. At this age, under normal conditions, a former private sector worker is, or rather used to be, entitled to a pension equal to 50% of a reference wage computed on the basis of his past wages, truncated to a "social security ceiling" roughly equal to the mean wage in the total labor force. At the same age (or sometimes at age 55 for some specific categories) a public sector employee is, or rather used to be, entitled to about 75% of his last treatment. The discrepancy between the two replacement levels is compensated by the

fact that the basic pension for the private sector is combined with one or two additional pensions served by two complementary schemes, ARRCO and AGIRC, the second one being specific to highly qualified white collar workers. In the public sector, there is no such second pillar.

A good understanding of reforms requires some explanations of what is exactly meant by retirement at age 60. The lowering of this retirement age to 60 in 1983 did not establish a fully systematic access to a full rate pension at this age of 60, since there is a supplementary condition on the number of years of contributions to social security schemes at this age. Until 1993, this condition was to have validated at least 37.5 years of contributions, either in the general regime or any other scheme. Not fulfilling this condition did not completely preclude the claiming of benefits, but imposed accepting a very strong reduction of the replacement rate, much stronger than the one implied by the rule of actuarial neutrality, since this penalty consisted in a reduction of the benefit level by more than 10% for each year missing to reach this condition of 37.5 years. Getting a full pension at 60 in the public sector also required the same total number of years of contributions, although the penalty for a lower duration was, in this public sector, much smaller than in the private sector.

On this basis, the 1993 and 2003 reforms included the following elements synthesized in table 1.

Table 1: Pre and post-reform major rules for the major basic French pension schemes

	France, régime general (wage earners in the private sector)			France, public sector	
	Before the 1993 reform	Changes introduced by the 1993 reform	Changes introduced by the 2003 reform	Before the 2003 reform	Changes introduced by the 2003 reform
First age at which retirement is possible	60	No change		55 or 60 years, depending on categories	No change
Age or duration conditions for "normal" (full rate) retirement	Being 60 or more with at least N=37,5 years of contribution, or being 65 without any condition on N	Duration condition raised from 37,5 years to 40 years (in 2003)	Duration condition raised to 41 (between 2008 and 2012), and to be increased to 41,75 years in 2020*.	37,5 years	Duration condition raised to 41 (in 2012), and to be increased to 41,75 years in 2020*.
Pension level at the NRA	If N= 37,5, 50% of the average of wages, truncated to the SS ceiling, over the 10 best years of ones career. If N<37,5, this amount is prorated.	The period over which past wages are averaged is increased from 10 to 25 years . (process to take place between 1993 and 2008).	No change	75% of the last wage	No change
Penalty for retirement before the NRA	Proratization effect plus a reduction of 10% for each missing year	No change	Additional reduction reduced to 5% per missing year	Only the proratisation effect	Proratization effect plus a reduction of 5% for each missing year
Bonus for retirement after the NRA	None	No change	3% for each year of postponement	None	3% for each year of postponement

* Depending on future changes of life expectancy at 60.

The 1993 reform only concerned the private sector. Its most symbolic measure was a first tightening of the 37.5 years condition: it scheduled a shift in this condition by one quarter each year until 2003, in order to reach the new value of 40 years. Although highly symbolical, this measure was not the one that was expected to have the largest effect in the short or medium run. Actually, cohorts that currently leave for retirement started working earlier than 20 on the average, and, thanks to the fact that periods of paid unemployment are validated as full years of contributions, a large majority of new retirees are still able to fulfill this duration condition at age 60, even after it has been raised to 40. More rapid and more decisive reductions in expenditure levels were awaited from three more technical measures:

- One was the shift in the number of years on which past wages are averaged when computing the reference wage to which the replacement rate is applied. From 10 years in pre-1993 conditions, the reform scheduled its increase each year until reaching the value of 25 years.
- The impact of this measure has been amplified by a revision of the rule according to which these past wages are reevaluated before being averaged. Pre-1993 conditions implied a reevaluation on the basis of past changes of average wages. Post-1993 conditions imply that this reevaluation is only based on prices, hence a much weaker reevaluation of wages received several years ago. This new rule considerably amplifies the impact of having shifted from an average of wages over the ten best years to the average over the twenty-five best years of one's career.
- Lastly, but this was rather a confirmation of a policy introduced in the second half of the 80s, the 1993 reform confirmed the principle consisting in indexing on-going pensions on prices rather than on wages.

The second reform that took place in 2003 had the following main features:

- The first measure is to organize a convergence of conditions for accessing to full rate pensions rate between the public and the private sector. The duration condition should be raised to 40 years in the public sector by 2008.
- After realization of this convergence, the duration condition common to the two sectors has already been scheduled to be increased by one more year between 2008 and 2012. It should then follow a path indexed on future life expectancy gains, in order to split these gains between 2/3 of additional length of working life and 1/3 of a remaining increase in the retirement length.
- In addition and partly in compensation for this strengthening of conditions for access to full retirement, more flexibility was introduced around this normal retirement age. We have recalled above the strength of the penalty that was applied by the French general regime in case of a departure before the full rate, and also the complete lack of financial incentives to postpone retirement beyond this age. The 2003 reform corrected this double anomaly by reducing the penalty for early retirement in the private sector (and conversely increasing it in the public sector where it was much weaker than implied by actuarial neutrality) and by introducing an incentive to postpone beyond that age, through a 3% pension bonus for each year of postponement.
- At last, a series of more technical changes should lead to reductions in the levels of full rate pensions, but to a much lesser extent than it had been the case with the 1993 reform.

2) Simulating the impact of pension reforms: the Destinie model.

The preceding section has shown that the French pension system is complex. This complexity has two dimensions. The first one is the coexistence of several different schemes -we have presented only the two most important of these-. The second one is the complexity of rules within each scheme. The pension level in the general regime depends in a highly non-linear way on age at retirement and on the number of years of contribution both to this general regime and to all other existing regimes. It also depends on the profiles of past earnings, here also in a complex way, due to the computation of the reference wage as the average value of wages, truncated to the SS ceiling, over the N best years of one's career. Given such a rule, two persons of the same cohort leaving at the same age with the same numbers of years of contributions and even with the same levels for their last wages or the

same past global average wages will have very different pensions depending on whether their careers have been ascending, flat or irregular.

Precise simulations of the impact of parametric reforms with such complex rules is almost impossible to do with aggregate or semi-aggregate models. Such models can be used as long as the question is to look at the macro-incidence of very global variants such as demographic variants or variants affecting productivity growth or employment rates. But they are of no help anymore when we want to answer more precise questions.

Figures 1a to 1c illustrate this with the example of the impact of the 1993 and 2003 on replacement rates according to age at benefit claiming, in the private sector, for three typical cases of individuals having worked all over their careers at the social security ceiling, but having started these careers at respectively 17, 20 and 23 year. For the first case of an early beginner, the impact of the 1993 reform remains very simple. This person remains entitled to the full rate when the condition at 60 is raised to 40 years of contribution. We assumed that the only impact of the reform is a flat rate reduction of the effective replacement rate, due to the move to the average of the 25 rather than 10 best years of this individual's career. But one first element of complexity appears with the 2003 reform; this reform still offers the full rate at 60 for this individual, but it also gives him the possibility to increase his pension level by leaving later than 60, by 3% per year of postponement, a possibility that he did not had before. How far is he going to use this possibility will depend both on his preferences and on how long his employer will accept to keep him in his job. And the impact of this for public finances will not be neutral at all, given that the 3% progressivity of the pension level beyond the full rate is lower than what would result from actuarial neutrality, even for individuals with relatively high life expectancy.

The picture is still more complicated with the two other cases. For these two cases, the question is not only to know how many people will react to the 2003 reform by postponing beyond the full rate that is respectively obtained at 61.75 years in the case of figure 1.b and 64.75 years in the case of figure 1.c. It is also to know how many people chose or will chose to leave *before* this full rate, either under the strong penalty that existed before the 2003 reform, or under the milder one that exists after the 2003 reform.

On the whole, assessing the impact of such reforms requires a knowledge of

- (a) The full distribution of the number of years of contribution by people reaching the age of 60.
- (b) A knowledge of the dispersion of career profiles, necessary to evaluate the average value of the full replacement rate.
- (c) If possible, some estimations of preference parameters and demand-side constraints that will affect decisions to retire under the different structures of incentives stemming from the successive reforms, especially the 2003 one that tries to enlarge arbitrage opportunities around the full rate.

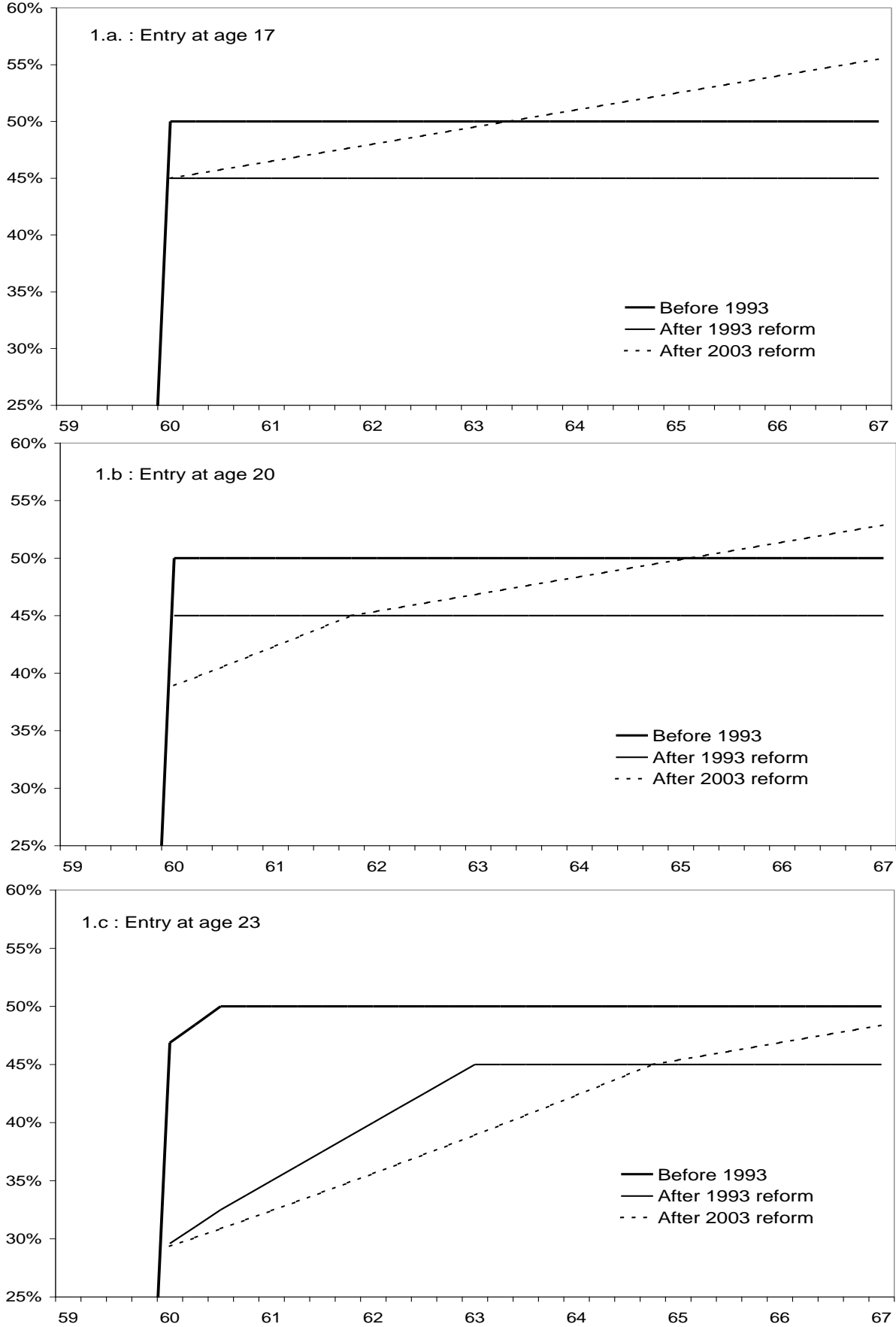
The first element could eventually be made available in a cell-based model projecting the cross-classification of individuals by age and number of years of contributions. But this would solve none of points (b) and (c).

In such a context, there is a strong motivation to rely on the possibilities of microsimulation. This is the strategy that has been followed since the mid 1990s at INSEE with the building of the Destinie microsimulation models. We shall not review here the complete characteristics of this model. Relatively detailed descriptions of its first version are presented in Bardaji, Sédillot and Walraët (2003) and Blanchet and Le Minez (2007). We shall rather concentrate on the main characteristics of the new version of the model, whose construction started in 2005, still not fully validated, but that is the one that has been used for the present paper.

The experience with the Destinie I model has incited to build Destinie II in a highly modular fashion, and with a relatively strong separation between two components.

The first element is a generator of individual demographic and labour market trajectories that fully ignores retirement behavior and the computation of pensions : we project "potential" working histories up to an age that has been fixed at an arbitrarily high level, i.e. 70 years. This first module is relatively time-consuming due to the complexity of events that are simulated and the fact that they are simulated under a certain number of "alignment" constraints that warrant consistency between the microsimulation results and global demographic projections, as well as with projections of labour force for below retirement ages.

Figure 1 : Relationship between age at retirement and the replacement rate provided by the “regime général” for full careers at the social security ceilings, different ages at entry on the labour market and under three successive legislations



Once these biographies have been generated, they are stored in permanent files that are re-read by smaller *ad hoc* programs simulating various pension scenarios and associated pension indicators. To this aim, a library of pension simulation subroutines has been created that allows, among other things :

- The computation of pension benefits given individual profiles and according to various past or present legislations, with possibilities of further fine tuning of the different parameters.
- The simulation of retirement behavior under these legislations using a variety of options, such as systematic retirement at the full rate, retirement at the age that maximizes the discounted stream of social security benefits, or more sophisticated behavioral models such as the Stock and Wise model, already widely used in Destinie I, that takes into account preference for leisure, time preference, and intertemporal substitutability of consumption.
- The computation of derived indicators, such as social security wealth, accruals (increases in SSW resulting from changes in retirement ages), rates of return, and so on...

A standard pension simulation with Destinie takes the following form. The first step consists in rereading results from the biography generator concerning people alive at the initial period. These results includes full retrospective biographies. Once these retrospective biographies have been reread, the status of pensioner and the associated level of benefits is imputed for people in retirement ages. This procedure has been preferred to the fact of using direct information on current pensions, for two reasons. The first one of these two reasons is that it ensures continuity in pensions figures at the beginning of the projection, because the initial stock of entitlements and the flow of new entitlements are computed in a fully homogenous way. If we had used actual pensions for the initial stock, a discontinuity could have resulted due to the shift from self-declared to fully imputed benefits. The second reason is that this way to proceed makes it quite easy to simulate counter-factual scenarios without past reforms or with variants of these past reforms. This possibility will be used in the present paper.

After rereading of past biographies and the imputation of initial pensions, the second step of a projection consists in reading, year after year, individual changes predicted by the biography generator for events other than retirement as well as the information on new entrants into the sample, be it by birth or by in-migration. Once these changes have been read, we simulate new transitions to retirement, and also the updating of pension benefits for people already retired, or the conversion of direct benefits to survivor's benefits in case of death of a married pensioner.

The library of computer tools that is provided with the model allows to produce various tabulations of these simulated micro-variables, without any *a priori* restriction to a closed set of outputs. On the whole, the system is intended to be as flexible as possible. This makes the model quite easy to use for the production of non-standard projection results, and the remaining of the paper makes an intensive use of this versatility. The next section will first examine what we tried to compute exactly, and how this has been done.

3) Computing two notions of implicit liabilities

As it is well-known, there is not one, but there are at least three possible definitions of a pension system's liabilities, that have been rapidly mentioned in the introduction to this paper.

We shall mention the first one only for the sake of completeness, since it will not be used in this paper. This first approach is the open system (OS) approach, that takes into account all future benefits to be paid to current or future pensioners, as well as all future contributions by wage earners, including future workers not yet born or not yet in the labour force. This approach is equivalent to computing a discounted sum of future deficits (or surpluses) for this pension system. It represents the amount of reserves that would be required from this system in order to meet all its future commitments, without any need to increase future contribution rates. It will be zero in balanced steady state and positive if the system is expected to be in disequilibrium. In other words, this indicator is fully focussed on the issue of sustainability. One of its drawbacks is that its value is highly sensitive to the choice of the time horizon until which deficits are discounted. An infinite horizon is conceptually preferable, but difficult to legitimate for non experts. Another drawback is the sensitivity to the discount rate that is retained. In this respect, the related concept of Tax Gap is more satisfactory (Blanchet and Ouvrard, 2007).

Anyway, this OS indicator is not the one that has been favored by the next SNA, probably because its evaluation entails full projections of pension entitlements and contributions: such projection exercises are not and cannot be in the core business of national accountants. The next SNA will rather favour a

very different approach, the accrued-to-date liabilities (ADL) approach, inspired by accounting rules that already apply to private pension schemes, for which full funding is the normal rule. The underlying logic is a simple one. Purely private and non mandatory systems are confronted to the risk of sudden closure, for instance if the system works at the level of the firm and if the firm disappears. Funding is necessary if we want to be sure that former employees by the firm will not see their pensions disappear with the closure of that firm. Even if such a risk was negligible, the associated commitments have to be taken into account when assessing the global value of that firm, since a potential buyer is expected to endorse these commitments of the firm towards its former or current employees. It is for this reason that the IAS19 norm enforces the evaluation of these commitments for all firms that are present on stock markets.

The way to do so consists in trying to figure out what would remain to be paid to these current or former employees in case of closure. For former employees that are already receiving pensions, this amount is equal to the discounted sum of benefits until death, i.e. a discounted sum using survival rates as weights. For current employees of former employees that have left the firm but are not yet retired, the same amount will be a discounted sum of benefits over their entire retirement period, but taking into account the fact that these benefits will be reduced ones, based only on the past flow of contributions of these workers. In the limit, these entitlements are equal to zero for individuals that are just at the beginning of their working lives.

This is the indicator on which we shall put the highest emphasis in this paper, given its role in the next SNA. But we shall also make use in the next section of an alternative indicator, that is relatively comparable to this ADL indicator, and which is interesting to evaluate because, as recalled in the appendix, it is easier to benchmark against theoretical steady-state values. This alternative indicator is the indicator one gets with the closed group (CG) methodology. As with the ADL methodology, this approach ignores all future contributions and associated benefits for potential newcomers for the system. Its treatment of current pensioners is also the same. It is only for current workers that the treatment differs. This approach assumes that these workers are allowed to go on contributing until their normal retirement. It therefore takes into account full entitlements for these workers. This is compensated by incorporating, on the asset side, the discounted value of all contributions by these workers from now on until their retirement. Globally, one can expect some correspondance between these contributions and the extra flow of benefits they generate. But the correspondance need not be perfect, and this is the reason why the ADL and CG methodology are only similar but not completely equivalent.

How did we implement the computation of these two ADL and CG indexes in Destinie ?

For the CG approach, we perform a standard pension projection. The projection being made until 2050, we observe pension liquidation for all individuals currently working. On the other hand we do not observe full retirement periods for these individuals. As a result, the discounted flow of benefits is not obtained by summing up the sequence of benefits simulated year after year by the model, but rather an actuarial equivalent computed at entry into retirement, consisting in multiplying the current benefit level by a discounted sum of survival probabilities derived from the period mortality table that prevails at the time of retirement.

For the ADL approach, the computation is about the same. It is exactly the same for current retirees. For people who are still working, we first project the end of their careers and let them retire according to normal rules, but we then proritize the corresponding benefits with the ratio between the number of years of contributions reached at the current age and the total number of years of contributions reached upon retirement. We then convert these reduced pensions in a discounted flow of benefits until death using the same procedure as for the CG approach.

A few points deserve particular notice at this stage.

The first point is that our method for computing liabilities automatically relies on “realistic” profiles for the future careers of people not yet retired, i.e. careers that take into account labour mobility, eventual transitions through unemployment or inactivity, and wage progressions based on the wage equations that are implemented in Destinie. This was indeed the most direct way to proceed in our context, such careers being the ones routinely provided by the model. But standard applications of the ADL methodology generally make much simpler hypotheses, such as full continuation of contributors’ careers in their current status with very simple rules for wage progression. We shall consider here that the two methods are roughly equivalent on the average, but more precise comparisons could be considered for the future.

A second point is the modelling of retirement age. We have seen, especially with figures 1.a to 1.c., that the French system has moved since the 2003 reform in a direction that makes it nearer to the actuarial neutrality principle. This makes the value of liabilities less sensitive to assumptions concerning ages at retirement, higher ages being compensated by higher pensions amounts. But some dependency remains however. For instance, the bonus per year of postponement beyond the full rate is only of 3% instead of the 5 to 6% that would be consistent with average life expectancy at retirement ages. This means that liabilities decrease if a significant number of people choose to postpone beyond this age. Here, we have assumed systematic departure at the full rate, both because it remains the reference age for most of the population and because, in general, this age should also be the one that maximizes social security wealth. As a consequence, estimations that are provided are a majorant of real liabilities.

Concerning the ADL methodology, another way to proceed would have been to take in a literal way the assumption of an immediate termination of the PAYG schemes. This would have meant fully interrupting the careers of current contributors, and simulating their decisions to retire 10, 20 or 30 years from now with these very partial records of contributions. In general, this would have led to

- (a) simulating a retirement age of 65, to avoid the penalty for retiring before 65 with less than 41.75 years of contribution,
- (b) with a reference wage based on the “25 best years” of these truncated careers, i.e. on wages pertaining only to years before the beginning of the projection, and poorly reevaluated at time of retirement (only reevaluated on the basis of CPI evolutions)
- (c) plus the normal application of the rule that makes the pension proportionnal to the number of years of contributions.

Such a way to implement the ADL methodology would have led to much lower figures than the one provided here, but with very little meaning. In particular, in such a setting where everybody leaves at 65, any reform that intends to change actual retirement ages between 60 and 65 becomes fully neutral for the computation of implicit liabilities, and this strongly limits the value of the indicator for analyzing financial consequences of pension reforms.

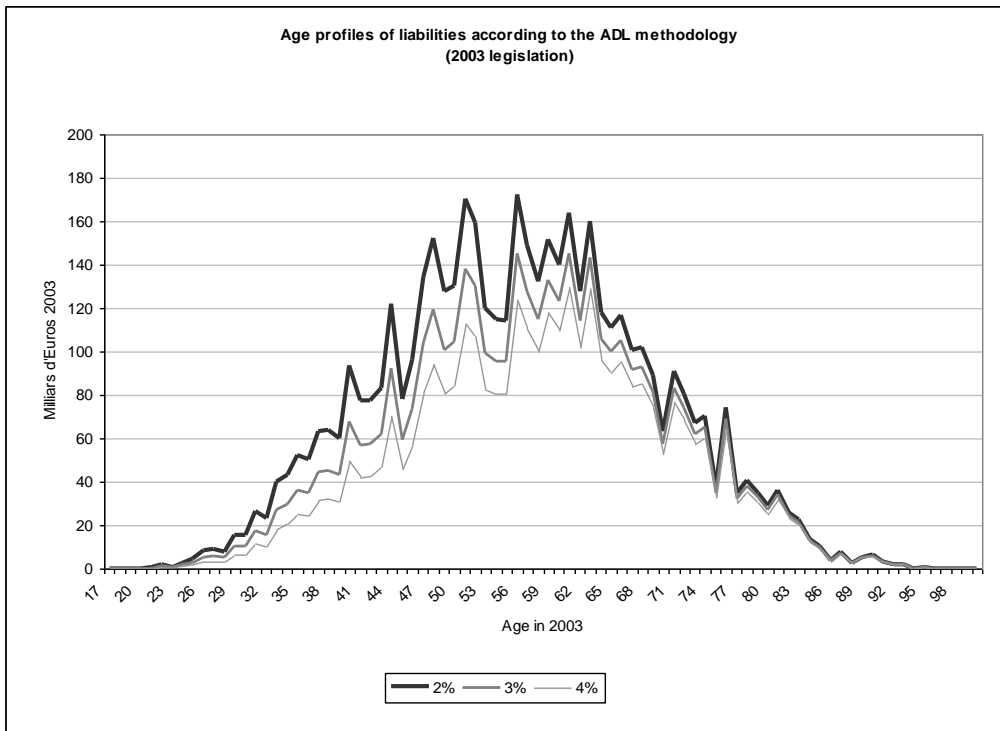
4) First results : age specific profiles and global implicit liabilities for a reference scenario.

Throughout the paper our reference scenario will be the one resulting from the application of the 1993 reform. Figures 2a and 2b give the profiles of liabilities by single years of age for the ADL and CG approaches and table 3 gives corresponding results both in billions of euros and as a share of GDP.

Figure 2.a. shows the typical age-profile that is expected for ADLs. The profile is close to zero for young contributors because they only have a few years of contributions behind them, and because the limited flow of benefits stemming from these few years of contributions is discounted to a very distant future. Liabilities then increase with age and peak around 60, i.e. at an age where people have had the time to accumulate full entitlements, whose actuarial value will be discounted over their entire retirement period. Liabilities decline after that peak because we are considering a decreasing number of survivors, to whom pensions remain to be paid on a number of years that itself declines with age.

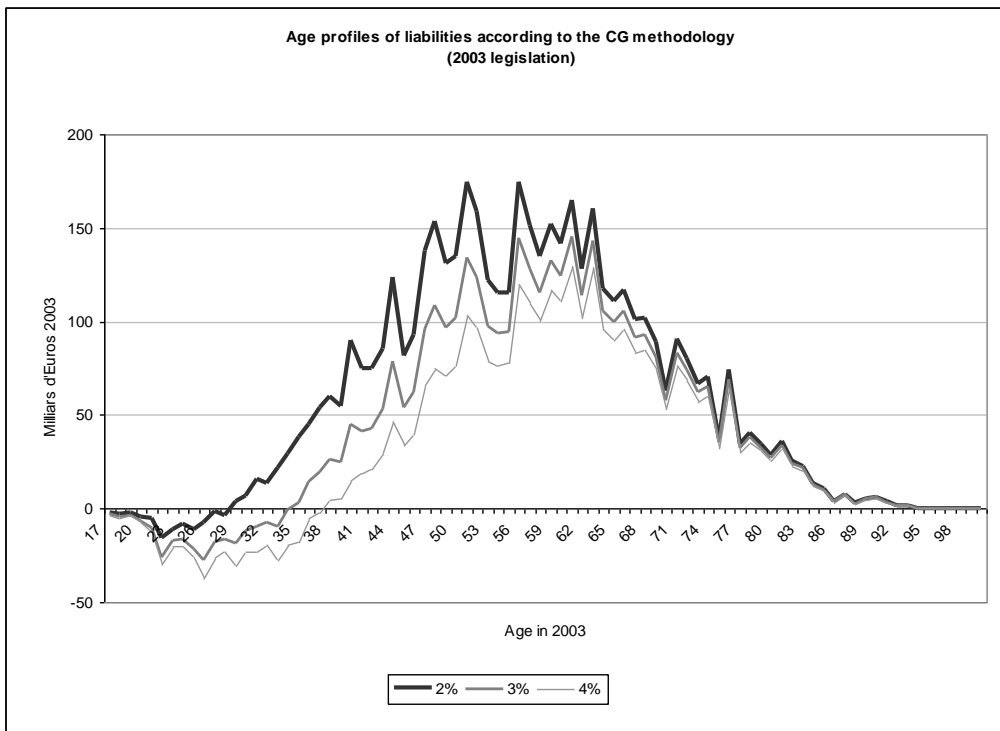
Figure 2.b. tells essentially the same story. We recall that this method takes into account full entitlements for people still in activity, but compensate for this by subtracting the discounted sum of contributions to be paid by these people over the rest of their careers. It is essentially for young cohorts that this alternative computation makes a difference, that progressively disappears when we approach retirement age. Two factors have to be considered here. First, the CG net liability is equal to zero for beginners only for the value of the discount rate that equals the internal rate of return on contributions. For a higher discounted rate, the net balance is negative, and conversely for a smaller discount rate. This explains the ordering of the three lines for $r=2, 3$ and 4% . In principle, the zero value should be attained for r approximately equal to 2% , since, in steady state, the internal rate of return on pensions is equal to the global rate of economic growth, and the French pension system is approximately tailored to be in equilibrium for such a rate of growth. But, and this is the second point, the computation is biased by other elements, playing in both directions: on the one side, the computation does not take into account all pension benefits, e.g. survivor's contributions were not introduced at this stage; on the other side, it doesn't count all contributions, in particular employer's contributions for civil servants.

Figure 2a : Age profiles of liabilities according to the ADL methodology.



Source : Destinie model, authors' computations

Figure 2b : Age profiles of liabilities according to the CG methodology.



Source : Destinie model, authors' computations

Table 2 : Total value of implicit liabilities according to the ADL and CG methodologies.

(according to the 2003 legislation)

	Discount rate		
	2%	3%	4%
ADL methodology			
Total	4 940	4 126	3 502
% of GDP	310%	259%	220%
CG methodology			
Total	4 748	3 522	2 661
% of GDP	298%	221%	167%

Source : Destinie Model, authors' computations

In practice, the two elements almost compensate for each other. Anyhow, this discrepancy between the two approaches lead to the same main message at the global level, which is very large importance of liabilities compared to GDP, her evaluated to about 2.5 years of GDP (between 1.6 and 3 years) (table 2).

Do such results make sense and what is their meaning ? Here, the help of some formalization is useful. We propose in Appendix 1 some analytical computations of what should be the values of these indicators in balanced steady states. It is for the CG approach that computing a general analytical formula is the easiest. In the special case where the discount rate is equal to the economic growth rate, this formula is that, in balanced steady state, the value of liabilities as a fraction of GDP is equal to the share of GDP that the system collects and repays to retirees every year, multiplied by the average length of the period between the paiement of contributions and the receipt of benefits, from the point of view of contributors/pensioners. This formula is relatively well-known and has been applied in other contexts (e.g. Settergren and Mikula, 2005). It can be easily understood if we consider ILs as the amount of reserves that should be held at each period by an equivalent funded system. In such a system, the total amount of these reserves would correspond to amounts collected each year, multiplied by their average duration of conservation into the fund, i.e. the difference between the average age at which the worker pays contributions to the fund, and the average age at which he is paid back by the fund under the form of annuities.

This comforts the evaluation provided by Destinie and displayed on table 2. Excluding survivors' pensions and means tested benefits, a rough order of magnitude for the current size of the French system is 10% and a rough order of magnitude of what could be its future size after the 2003 reform is 12 % of GDP. Assume that the gap $A_p - A_w$ is approximately equal to 30 years (i.e. 70 years - 40 years). For a discount rate equal to the expected growth rate -i.e. about 2%-, this means an IL indicator comprised between 300% and 360% of GDP, since the current value of the IL mixes information on the current and the future size of the system. The figures given on table 2 are consistent with this very rough computation, and it should be clear that such high values are not, in themselves, sufficient to characterize a situation of disequilibrium for this PAYG scheme. Such a high order of magnitude is in no way specific of a situation of unsustainability : it also shows up in situations of perfect sustainability where inflows of contributions permanently match outflows of benefits.

Analytical computations provided in the Appendix also comfort the sensitivity that we observe on table 2 with respect to the discount rate. Formulas are a bit more complicated: they involve both the *mean values* and the *variances* of ages at paiement of contributions and at the receipt of benefits. But these formula lead to a semi-elasticity of liabilities with respect to r of about -30, i.e. a one percentage point increase of r leads to a 30% decline of ILs, at least in the CG sense. This is what is observed on the last two lines of table 2. An equivalent formula is unfortunately unavailable, to our knowledge, for the ADL approach. But figure 2.a. showed the reason for the lower sensitivity to r in this ADL approach, which is that these effects essentially play in the middle of the distribution of liabilities by age, the liability at younger ages being constrained to be close to zero (and always positive) while we had non marginal effect of r on the *net* prospective balance for newcomers in the CG approach.

5) Global and age specific effects of the 1993 and 2003 reforms.

Once the results of our baseline simulation have been comforted by the comparison with analytical results, we can move on to analyzing the messages of these IL indicators concerning the financial impacts of pension reforms. As announced above, we now restrict ourselves to the ADL version of these IL computations.

Table 3 and figure 3 are equivalent to table 2 and figure 2.a, but with variants on r replaced by variants on legislation. We compute ILs in the ADL sense of the term for (a) the reference scenario corresponding to the 2003 legislation, (b) a scenario where the legislation is the one that resulted from the 1993 legislation and (c) a scenario corresponding to pre-1993 conditions. Figure 3 is complemented by figure 4 that gives relative variations by age for scenarios (a) and (b) compared to the no reform scenario (c). Scenarios (b) and (c) are counterfactual in the sense that they not only change pension prospects for people still at work but also current pension levels –and past ages at benefit claiming- for people already retired under the 2003 legislation.

What do these computations show ? We know from other sources that the two reforms of 1993 and 2003 have not been marginal. They do not entirely solve the French pension problem, but, according to projections by the pensions advisory committee, they have gone a significant way in that direction, reducing by roughly one half prospective deficits, at least at the 2020 horizon. It is therefore all the more striking to see how these reforms only slightly impact total ADLs, by only 4,0% for the 1993 reform and 7,1% for the combination of 1993 and 2003 reforms.

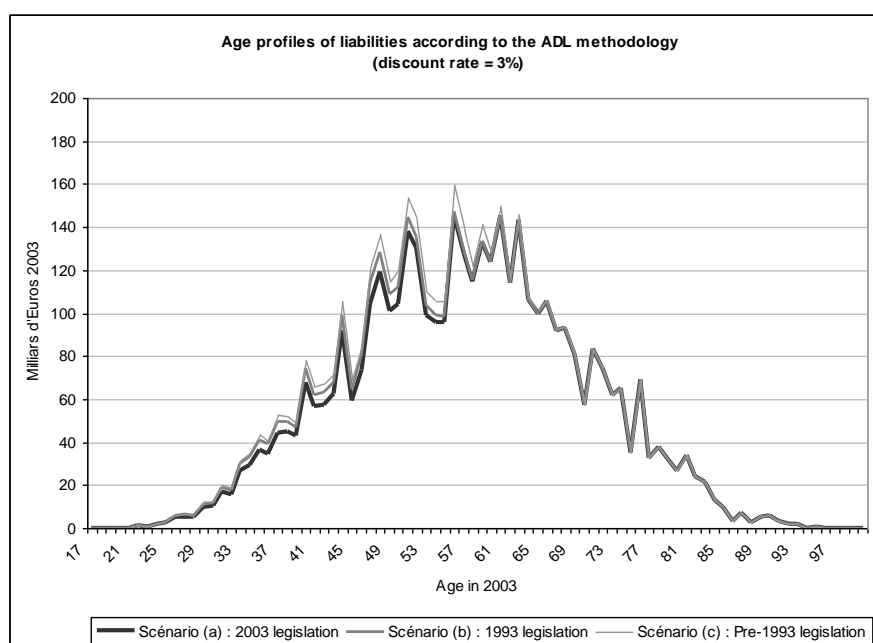
Table 3 : Total value of ADLs according to legislation.

(according to a discount rate of 3%)

	Scenario (a) 2003 legislation	Scenario (b) 1993 legislation	Scenario (c) Pre-1993 legislation
Total	4 126	4 266	4 443
% of GDP	259%	267%	279%
Relative change/scenario (c)	-7,1%	-4,0%	0,0%

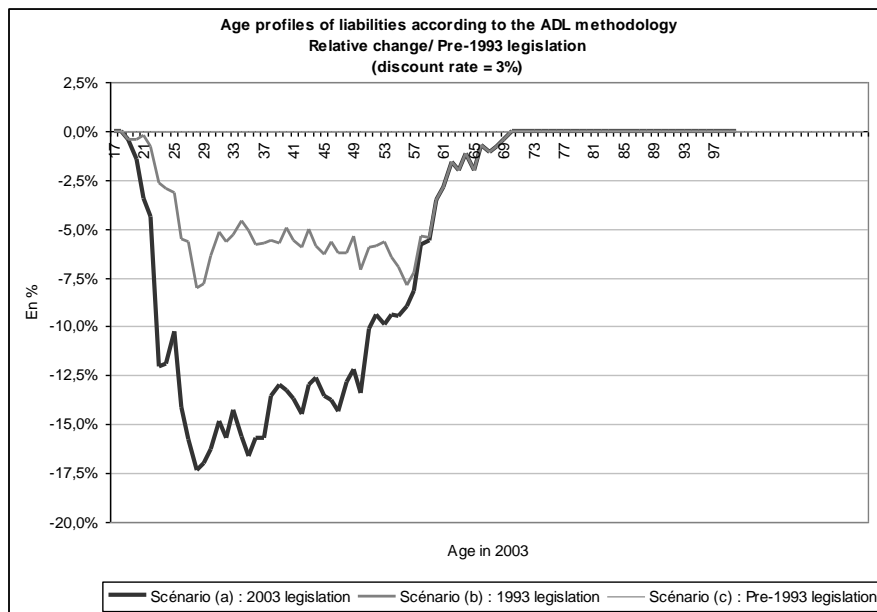
Source : Destinie Model, authors' computations

Figure 3a : Age profiles of liabilities according to the ADL methodology.



Source : Destinie model, authors' computations

Figure 3b : Age profiles of liabilities according to the ADL methodology : relative change / Pre-1993 legislation



Source : Destinie model, authors' computations

Is this surprising ? Not at all in view of analytical results that have been considered in the last section. Interpreting reference values of ADLs as a product of the flow of contributions paid each year to the system by an age interval of about 30 years leads to interpret ADLs as being primarily an indicator of the *size* of the pension system. Under a no-reform scenario, this size, for the French system, was expected to increase to about 18% of GDP, hence a projected deficit of 6% of GDP with unchanged contributions. A reform that cuts benefits or raises retirement age enough to divide by two this expected deficit only has to reduce from 18% of GDP to 15% of GDP this long run size of the system, hence a long run reduction of 1/6th, which is indeed not marginal from the point of view of future retirees, but much less than the impact of the reform on future deficits.

In other words, the impact of the reform on the ADL is dampened by the fact that the ADL is before all an indicator of the systems' size, and only secondarily an indicator of its prospective disequilibrium. The only reforms that would very significantly reduce this size of the system and the ADLs would be reforms that would *substitute* funding to existing PAYG transfers. Reforms that only reduce future imbalances in these transfers have a much less significant impact on this ADL indicator.

In addition, the impact of reforms on the ADL indicator is dampened one second time, due to the fact that the incidence of reduced benefits and increased retirement age essentially affects younger cohorts, reforms generally avoiding strongly reducing benefits for individual already retired or close to retirement. This stronger impact of reforms on young workers is very apparent on figure 4 that gives *relative* changes, but strongly dampened on figure 3, this dampening being due both to the fact that we only considered accrued-to-date entitlements for these young workers, and to discounting. All this contributes to the very low final impact of reforms on ADLs as evaluated in 2003.

This analysis orients us towards a different and probably more relevant utilization of ADLs: this indicator is not very informative of the degree to which pension reforms improve the sustainability of the pension system, because ADLs remain high by nature even in a system that is perfectly sustainable. But its micro components consisting in ADLs for various categories of individuals provide a simple way to evaluate how the reform affects these various categories of individuals. Figure 4 considered the differential impact of the reform according to cohort. The fact of having derived ADLs from a microsimulation model allows looking more systematically at changes in these ADLs across some other directions. This will be the purpose of the next and last section.

6) How do reforms affect ADLs across socio-demographic groups ?

Results according to various socio-economic criteria are given on table 4 and figures 4.a and 4.b.

The first two columns of table 4 are equivalent to the last line of table 3. They give relative variations of ADLs implied by the 1993 and 2003 reform but broken down according to gender, age at leaving school and the fact of having worked in the private or in the public sector. Age at leaving school is the main social marker which is routinely used in the Destinie model. The other columns of table 4 also give measures of what the model predicts on other related variables : pension levels and ages at retirement. It must be noted that, as for the ADL, we here mix information on *all cohorts* currently alive, including those already retired for whom reforms have had or have no impact or only a small one.

Despite this limit, the table already gives some interesting elements of comparisons. Implicit liabilities towards skilled workers are more impacted than those towards the less skilled ones (-9,7% vs -3,9%). The main reason is that skilled workers start their active lives later and will generally be obliged to postpone their retirement more strongly than less skilled one. The average change in age at retirement is +1.0 year after the 1993 and 2003 reforms for these highly skilled workers, this value mixing zero changes for those already retired and much higher changes for those who will be really affected by the reforms. This can be reinforced by the fact that careers for more qualified workers are generally ascending, with the consequence that they are more strongly affected by the shift to a pension based on the 25 instead of the 10 best years of their careers.

Comparisons between genders and sectors are also informative. By construction, the 1993 reform had no impact on liabilities towards public sector employees, while it reduced liabilities toward private sector employees by already 5.6%. The 2003 reform has an impact on public sector employees roughly equivalent to the first figure (-5.3%), but this reform goes on impacting private sector employees, even if the impact of this new reform is lower for them than the impact of the first reform. On the whole, the combination of the two reforms has a stronger impact on private sector employees than on those belonging to the public sector (-7.9% compared to -5.3%).

Table 4 : Impact of the 1993 and 2003 reforms on the ADL and associated indicators, by socio-economic group (all cohorts included, with a discount rate of 3%)

	Impact on the ADL (%)		Impact on ages at retirement (years)		Impact on replacement rates (in percentage points)	
	1993 and 2003 reforms	1993 reform only	1993 and 2003 reforms	1993 reform only	1993 and 2003 reforms	1993 reform only
Gender						
Female	-6.7	-3.3	+0.7	+0.3	-1.0	-0.8
Male	-7.6	-4.6	+1.0	+0.4	-0.6	-0.9
School leaving age						
Quartile 1	-3.9	-3.0	+0.3	+0.1	-1.0	-1.1
Quartile 2	-5.5	-3.6	+0.8	+0.2	-0.9	-1.1
Quartile 3	-7.8	-3.8	+1.1	+0.4	-0.5	-0.9
Quartile 4	-9.7	-5.0	+1.0	+0.5	-0.8	-0.7
Sector						
private sector	-7.9	-5.6	+1.0	+0.4	-0.5	-0.4
public sector	-5.3	0.0	+0.8	+0.0	-1.2	0.0

Source : Destinie model, authors' computations

Note : Replacement rates are computed only for people in employment at least one year during the three years before retirement

Figure 4.a : Age profiles of liabilities according to the ADL methodology : relative change / Pre-1993 legislation for women and men

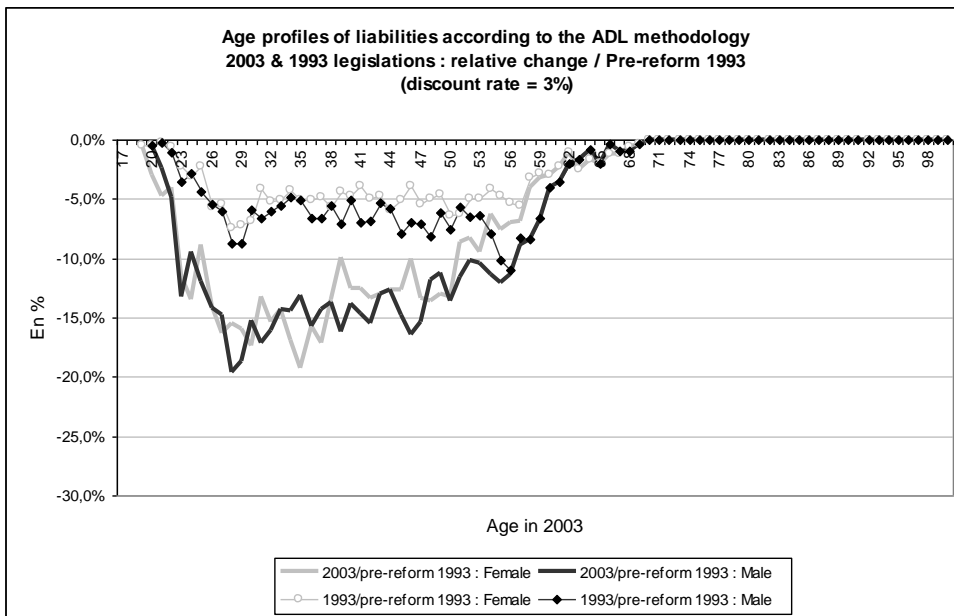
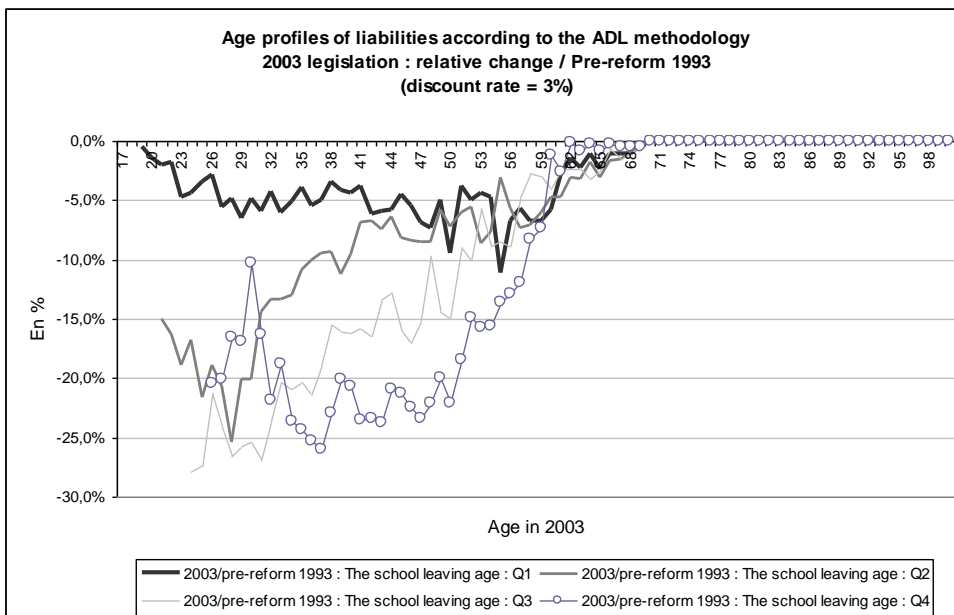


Figure 4.b : Age profiles of liabilities according to the ADL methodology : relative change / Pre-1993 legislation according to the relative school leaving age



Source : Destinie model, authors' computations

Concerning gender differences, both reforms have a slightly stronger impact on men than on women, a result that was not completely expected in view of some earlier results from the Destinie I model (Bonnet, Buffeteau and Godefroy, 2006). One explanation can be the higher relative weight of pensioners in the female population, due to their higher life expectancy : these persons, by definition, are not affected by reforms that change pensions of future pensioners.

This shows the importance of moving beyond these aggregates over all existing cohorts. This is what is done for the gender decomposition on figure 4.a. and for the decomposition by age at leaving school on figure 4.b.

The reading of these graphs is the same as for figure 3.b. Figure 4.a. separates the impacts of the 1993 reform only and of the cumulated 1993 and 2003 reforms. The figures appear now much more similar for men and women. In all cases, most of the adjustment is borne by people not yet retired. For these people, the cost of the 1993 reform has been equal to about 5% to 7.5% of their liabilities, and relatively flat according to cohorts. For the cumulated 1993 and 2003 reform, the cost lies between 17.5% of global entitlements for the younger cohorts and 10% for those currently approaching retirement.

Figure 4.b only gives the impact of the cumulated 1993 and 2003 reforms. The profiles of the impact appear very contrasted according to the age at leaving school. Actually, the strengthening of conditions required to get a full rate pension is much less constraining for people having started working early, as was already apparent from figures 1.a to 1.c.

7) Discussion and conclusions

The main following messages emerge from this tentative application of the microsimulation model Destinie to the evaluation of pension liabilities in France. This model is still in its validation phase so the detailed values of results that have been presented must be interpreted with caution, and could be revised in the future. But some lessons nevertheless emerge.

First, from a technical point of view, it appears that deriving these liabilities from a very flexible microsimulation model is technically easy. It allows the computation of these liabilities under a variety of scenarios. It also allows a consistent estimation of these liabilities at the macro and the micro level.

From the point of view of interpreting outputs, the conclusion concerning IL indicators at the macro level is mixed, at least from the point of view of the follow-up of reforms aiming a restoring sustainability of PAYG pension schemes. These indicators react very little to such reforms, even when they are of a significant importance. The reason for this low relevance of ILs as indicators of reforms' impacts is double. The first point is that they are before all indicators of the size of pension systems, whereas reforms generally only intend to limit the future growth of these systems, not to shrink their current sizes. Second, reforms generally limits the future growth of pension expenditures by reducing entitlement of younger cohorts, whose entitlements are strongly underweighted in these IL indicators.

It looks therefore more interesting to look at these indicators from a more micro-point of view, and section 6 provided some illustrations of such an application of the ADL methodology. However, here also, IL indicators are only one part of the story. For instance, reductions in entitlements are generally made with the purpose of avoiding increases in contribution efforts. Balancing the two effects imply looking either at life-cycle contributions/benefit balances or to rates of return on contributions. And a full picture of social consequences of reforms also involves looking at more at more standard indicators such as replacement rates, retirees' relative incomes or exposures to poverty risks.

Références

- Accardo, J. (1996) « Mesure de l'équivalent patrimonial des droits à la retraite en 1992 », Synthèses n° 5, pages 109-124.
- Accardo, J. (1997) « L'évaluation d'équivalents patrimoniaux des droits à la retraite », in *Comptabilité Nationale, développements récents*, Actes du sixième colloque de l'ACN, E. Archambault et M. Boëda., Eds, Economica.
- Aubert, P., Blanchet, D. and Blau, D. (2007) « Social contract and age at retirement : some elements of a franco-american comparison », in *Ages, generations and the social contract : the demographic challenges facing the welfare state*, J. Veron, S. Penneç et J. Légaré, Eds, Springer
- Bardaji J., Sédillot B. et Walraët E. (2003), « Un outil de prospective des retraites : le modèle de microsimulation Destinie », *Economie et Prévision*, n° 160-161, pp. 193-214.
- Blanchard, O. (1990) "Suggestions for a new set of fiscal indicators", OECD working paper, n° 79.
- Blanchet, D. et Ouvrard, J.F. (2006) « Mesurer les engagements implicites des systèmes de retraite ? », in *L'Economie Française : comptes et dossiers*, INSEE-Références, 2006
- Blanchet, D. et Ouvrard, J.F. (2007) « Indicateurs d'engagements implicite des systèmes de retraite : que mesurent-ils exactement ? », *Revue Française d'Economie*, XXII, pp 3-47
- Bonnet, C., Buffeteau, S. and Godefroy, P. (2006) « Les effets des réformes des retraites sur les inégalités de genre en France », *Population*, vol 61, n°1-2
- Bourgeois-Pichat, J. (1978) « Le financement des retraites par capitalisation », *Population*, n°6, pp 1115-1136.
- Bourgeois-Pichat, J. E. Chapron (1979) « Répartition du revenu national entre capital et travail : Application des systèmes de retraite », *Population*, n° 1, p. 43-63
- Conseil d'Orientation des Retraites (2001) *Retraites: renouveler le contrat social entre les générations*, Premier rapport, La Documentation Française.
- Conseil d'Orientation des Retraites (2004) *Retraites : les réformes en France et à l'étranger ; le droit à l'information*, Deuxième Rapport, La Documentation Française.
- Conseil d'Orientation des Retraites (2006) Troisième Rapport, La Documentation Française.
- European Commission (2005) « The 2005 EPC projections of age-related expenditure (2004-2050) for the EU-25 Member States: underlying assumptions and projection methodologies », EPC/DG ECFIN special report, n°4.
- European Commission (2006) "The impact of ageing on public expenditure: projections for the EU25 member states on pensions, health care, long-term care, education and unemployment transfers (2004-2005)", EPC/DG ECFIN special report, n°1.
- Feldstein, M. (1974) « Social security, induced retirement and aggregate capital accumulation », *Journal of Political Economy*, 82, 905-26.
- Franco, D., Mariono, M.-R. et Zotteri, S. (2004) « Pension expenditures projections, pension liabilities and European union fiscal rules », *International Workshop on the Balance Sheet of Social Security Pensions*, Hitotsubashi University, Tokyo.
- Gautron, N. (2006) « La mise en place dans les entreprises de la norme IAS 19 sur l'évaluation des dettes de retraite », 11eme colloque de Comptabilité Nationale, Paris.
- Heller, P. (2003) *Who will pay? Coping with aging societies, climate change and other long term fiscal challenges*, International Monetary Fund.
- Holzmann, R., Palacios, R. et Zviniené, A. (2004) "Implicit pension debt : issues, measurement and scope in international perspective", *Social Protection Discussion Papers Series*, n° 0403, The World Bank.

- Kessler, D., Masson, A. et Strauss-Kahn, D. (1980) *Systèmes de retraite et accumulation du capital*, Rapport pour le Commissariat Général du Plan.
- Kune, J. (1996) "The hidden liabilities : meaning and consequences", CBP seminar series, La Haye.
- Lequiller, F. (2006) "L'enregistrement des systèmes de retraite dans les comptes nationaux : le point sur les révisions du SCN", Communication au 11eme colloque de l'Association de Comptabilité Nationale, 18-20 janvier.
- Pellé, T. (2006) "Prise en compte des dépenses de pensions et évaluation des engagements de l'état en matière de retraite", Communication au 11eme colloque de l'Association de Comptabilité Nationale, 18-20 janvier.
- Pébereau, M. (2005) *Rompre avec la facilité de la dette publique : pour des finances publiques au service de notre croissance économique et de notre cohésion sociale*. Rapport Officiel de la Commission présidée par M. Pébèreau, La documentation Française
- Settergren, O. et Mikula, B.D. (2005) « The rate of return of pay-as-you-go pension systems : a more exact consumption-loan model of interest », *Journal of Pension Economics and Finance*, 4(2), 115-138, World Bank.
- Swedish Social Insurance Agency (2005) *The Swedish Pension System Annual Report 2004*, Stockholm.
- Van der Noord, P. et Herd, R. (1993) « Pension liabilities in the seven major economies », Working paper n° 142 :1-64, Oecd Economic Department, Paris.
- Vernière, L. (1992) « Une évaluation de l' équivalent patrimonial des droits à la retraite détenus par les ménages », *Economie et Prévision*, n° 105.
- Vernière, L. (2002) « Fonds de réserves pour les retraites et engagements des régimes de retraite par répartition », *Questions Retraite*, n° 02-54.
- Vernière, L. (2005) « *Unfunded obligations* : les mesures des engagements et de la solvabilité de la *Social Security* aux Etats-Unis », Miméo.
- Werding, M. (2006) « Implicit pension debt and the role of public pensions for human capital accumulation : an assessment for Germany », Working Paper, Ifo Institute for Economic Research.

Appendix: Analytical derivation of IL indicators under steady-state conditions

A.1. Notations

We use a continuous description for age. We are interested in the value of pension liabilities in a steady state that is characterized by constant growth rates of n for population and g for productivity and real wages. Age is denoted by a . We note $s(a)$ the survival function and ω the limit age beyond which $s(a)$ is equal to zero.

Under such assumptions, the share of people aged a at time t is equal to :

$$N(a, t) = s(a)e^{n(t-a)}$$

and the wages and pensions by age at time t are :

$$w(a, t) = w(a)e^{gt} \quad \text{and} \quad p(a, t) = p(a)e^{gt}$$

This formalization encompasses all possible assumptions concerning the indexation rule for pensions. The special case of pensions indexed on inflation will lead to a profile $p(a)$ exponentially decreasing at rate g with age. In such a case, we get :

$$p(a, t) = pe^{g(t-a)}$$

meaning that the pension level only depends on cohort $t-a$ and not on the age of the pensioner. On the opposite, pensions fully indexed on productivity will correspond to a profile $p(a)$ independent of age. In that case, all pensioners living at time t share the same pension level irrespective of their age, and this level writes down :

$$p(a, t) = pe^{gt}$$

We shall also observe that these notations avoid specifying where is the frontier between the working period and the retirement period. Periods of pure retirement will correspond to ages where $w(a)=0$ and $p(a)>0$. Periods of pure activity will be those where $w(a)>0$ and $p(a)=0$, but we can very well have periods of transition where both functions are non zero.

We note at last τ the contribution rate to the PAYG scheme and we shall note $x(a, t)$ the balance between current benefits and contributions for age group a at time t , i.e. :

$$x(a, t) = p(a, t) - \tau w(a, t) = e^{gt} [p(a) - \tau w(a)] = e^{gt} x(a)$$

After elimination of terms in e^{gt} , the period budget constraint for the system boils down to:

$$\int_{a=0}^{\omega} e^{-na} s(a) [p(a) - \tau w(a)] da = \int_{a=0}^{\omega} e^{-na} s(a) x(a) da = 0 \quad (1)$$

A.2. Evaluating implicit debt under the Closed Group approach.

The Closed Group approach consists in evaluating, for each cohort in the system at time t , the balance between benefits and contributions that remains to be paid to or by this cohort until its extinction. We shall assume that the evaluation is done at time 0, and we assume a discount rate of r . In $t=0$, the number of individuals aged a is in $s(a)e^{-na}$. For these individuals, the total balance between benefits and contributions that remain to be paid is ::

$$D(a) = s(a)e^{-na} \int_{u=a}^{\omega} e^{-r(u-a)} \frac{s(u)}{s(a)} x(u, u-a) du = e^{-na} \int_{u=a}^{\omega} e^{-(r-g)(u-a)} s(u) x(u) du$$

hence :

$$D(a) = e^{(r-g-n)a} \int_{u=a}^{\omega} e^{-(r-g)u} s(u)x(u)du$$

We get :

$$D_{tot} = \int_{a=0}^{\omega} e^{(r-g-n)a} \left[\int_{u=a}^{\omega} e^{-(r-g)u} s(u)x(u)du \right] da \quad (2)$$

This expression can be integrated by parts. We get :

$$D_{tot} = \left\{ \frac{e^{(r-g-n)a}}{r-g-n} \left[\int_{u=a}^{\omega} e^{-(r-g)u} s(u)x(u)du \right] \right\}_{a=0}^{a=\omega} + \int_{a=0}^{\omega} \frac{e^{(r-g-n)a}}{r-g-n} e^{-(r-g)a} s(a)x(a)da$$

hence :

$$D_{tot} = -\frac{1}{r-g-n} \left[\int_{a=0}^{\omega} e^{-(r-g)a} s(a)x(a)da \right] + \frac{1}{r-g-n} \left[\int_{a=0}^{\omega} e^{-na} s(a)x(a)da \right]$$

The second integral is nil, given the equilibrium condition (1), so that D_{tot} boils down to :

$$D_{tot} = -\frac{1}{r-g-n} \left[\int_{a=0}^{\omega} e^{-(r-g)a} s(a)x(a)da \right] \quad (3)$$

In the general case, the sum of implicit liabilities in the CG sense of the term is therefore equivalent to the discounted sum of the difference between benefits and contributions, with a discount rate of $r-g$, divides by the difference between r and the global economic growth rate $g+n$. One can check that this expression is always positive : if $r > g+n$, the integral between squared brackets is negative. In that case, $r-g > n$, meaning that the discount factor used in that integral weighs more heavily younger age groups than it is the case in the integral (1) where the discount factor is in e^{-na} . Given that these younger ages are those where $x(a)$ is negative, this implies that this integral in (3) is negative, hence a positive sign for D_{tot} and conversely in the case where r is lower than $g+n$.

But expression (3) gives no intuition of what can be the order of magnitude of these implicit liabilities. A more directly interpretable formula can be got if we consider the particular case $r=g+n$ where expression (3) is no more defined, and for which computations need to be redone from the beginning.

Starting back from definition (2), we see that it becomes :

$$D_{tot} = \int_{a=0}^{\omega} \left[\int_{u=a}^{\omega} e^{-nu} s(u)x(u)du \right] da$$

Integration by parts now leads to :

$$D_{tot} = \left\{ a \left[\int_{u=a}^{\omega} e^{-nu} s(u)x(u)du \right] \right\}_{a=0}^{a=\omega} + \int_{a=0}^{\omega} a e^{-na} s(a)x(a)da$$

or :

$$D_{tot} = \int_{a=0}^{\omega} a e^{-na} s(a)x(a)da \quad (4)$$

Contrarily to expression (3), this new expression is very easy to interpret. It can be interpreted by considering D_{tot} as a sum of discounted elementary elements $e^{-na}s(a)x(a)$ where the element

corresponding to a appears as many times as there are cohorts that have still not reached that age of a , hence the weighting by a .

This expression also leads to a very intuitive expression for the liability/wage bill ratio. Replacing $x(a)$ by its expression in $w(a)$, $p(a)$ and τ , and dividing by the total sum of gross wages at time 0, we get :

$$\frac{D_{tot}}{M_{tot}} = \frac{\int_{a=0}^{\omega} a e^{-na} s(a) p(a) da - \tau \int_{a=0}^{\omega} a e^{-na} s(a) w(a) da}{\int_{a=0}^{\omega} e^{-na} s(a) w(a) da}$$

Given the equilibrium condition (1) this expression can still be written as :

$$\frac{D_{tot}}{M_{tot}} = \frac{\tau \int_{a=0}^{\omega} a e^{-na} s(a) p(a) da}{\int_{a=0}^{\omega} e^{-na} s(a) p(a) da} - \frac{\tau \int_{a=0}^{\omega} a e^{-na} s(a) w(a) da}{\int_{a=0}^{\omega} e^{-na} s(a) w(a) da} = \tau (A_p - A_w) \quad (5)$$

where A_p and A_w correspond respectively to the mean ages at perception of pensions and at paiement of pension contributions.

Relation (5) can now be interpreted in the following way. The implicit liability is equivalent to the amount of reserves that the equivalent funded system would need to hold. These reserves are equivalent to the global amount of contributions paid each year by workers, multiplied by the average length of the period where this money will stay in the fund, which is precisely equal to $A_p - A_w$.

A.3. Evaluating the implicit debt under the ADL approach.

If the approach is the approach of accrued-to-date liabilities, the flows of future contributions do not play a role anymore. We must compute the sum of benefits that remain to be paid until death to individuals that are already retired, and of benefits that will be paid to people currently working, after retirement, on the sole basis of entitlement already validated at the current period, i.e. of their past contributions.

We shall now explicitly introduce the age at retirement that will be noted R , and we introduce the new quantity $z(a)$ that indicates the share of its future full entitlements that an individual has already accumulated at age a . By definition, $z(a)$ is equal to 1 for $a > R$.

The amount of accrued-to-date liabilities for an individual aged a is therefore :

$$\bar{D}(a) = e^{-na} s(a) \int_{u=a}^{\omega} e^{-r(u-a)} \frac{s(u)}{s(a)} z(a) p(u, u-a) du = e^{-na} z(a) \int_{u=a}^{\omega} e^{-(r-g)(u-a)} s(u) p(u) du$$

whence :

$$\bar{D}_{tot} = \int_{a=0}^{\omega} e^{(r-g-n)a} z(a) \left[\int_{u=a}^{\omega} e^{-(r-g)u} s(u) p(u) du \right] da \quad (6)$$

Integration by parts is now impossible without a more precise specification of $z(a)$. But we shall rather reason in the opposite direction, and ask to ourselves what must be the form of $z(a)$ for \bar{D}_{tot} to be equal to D_{tot} . The comparison between (2) and (6) directly gives the profile that will be requested for $z(a)$. We need to have :

$$z(a) = \frac{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)x(u)du}{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)p(u)du} = 1 - \tau \frac{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)w(u)du}{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)p(u)du}$$

or, replacing τ by its equilibrium value :

$$z(a) = 1 - \frac{\int_{u=0}^{\omega} e^{-nu} s(u)p(u)du}{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)p(u)du} \cdot \frac{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)w(u)du}{\int_{u=0}^{\omega} e^{-nu} s(u)w(u)du} \quad (7)$$

This expression looks like a formula where entitlements are accumulated in proportion to perceived wages, but is not fully comparable. In particular, we have $z(a)=1$ for $a \geq R$, but we do not necessarily have $z(0)=0$. This formula is therefore a formula that would combine a lump sum component with a component proportional to contributions already paid. But this lump sum component fully disappears if we restrict ourselves to $r=n+g$. In that case, for $a < R$, we have :

$$\frac{\int_{u=0}^{\omega} e^{-nu} s(u)p(u)du}{\int_{u=a}^{\omega} e^{-(r-g)u} s(u)p(u)du} = 1$$

hence :

$$z(a) = 1 - \frac{\int_{u=a}^{\omega} e^{-nu} s(u)w(u)du}{\int_{u=0}^{\omega} e^{-nu} s(u)w(u)du} = \frac{\int_{u=0}^a e^{-nu} s(u)w(u)du}{\int_{u=0}^{\omega} e^{-nu} s(u)w(u)du}$$

In short, for such a profile of accumulation of entitlements with age and with $r=n+g$ the ADL and CG approaches yield exactly the same results. It is roughly what was observed with simulations for $r=2\%$ and a proratisation factor that was close to the one in formula (8). When r and $n+g$ differ, the two methods are no more fully equivalent, but results nevertheless remain of the same order of magnitude.

A.4. Sensitivity to the actualization rate

To conclude this analytical section, we shall precisely discuss this sensitivity to the value of r , but will do so in the case of the CG indicator. We shall consider small variations of r around the value $r=n+g$. We restart from expression (2). We temporarily introduce the following notations :

$$f(r, a) = e^{(r-g-n)a}$$

and

$$h(r, u) = e^{-(r-g)u} s(u)x(u)$$

Expression (2) can thus be rewritten :

$$D_{tot} = \int_{a=0}^{\omega} f(r, a) \left[\int_{u=a}^{\omega} h(r, u) du \right] da$$

and its derivative with respect to r writes down :

$$\frac{d(D_{tot})}{dr} = \int_{a=0}^{\omega} \left[f'_r(r, a) \left(\int_{u=a}^{\omega} h(r, u) du \right) + f(r, a) \left(\int_{u=a}^{\omega} h'_r(r, u) du \right) \right] da$$

Redeveloping, we get :

$$\frac{d(D_{tot})}{dr} = \int_{a=0}^{\omega} \left[a e^{(r-g-n)a} \left(\int_{u=a}^{\omega} e^{-(r-g)u} s(u) x(u) du \right) - e^{(r-g-n)a} \left(\int_{u=a}^{\omega} u e^{-(r-g)u} s(u) x(u) du \right) \right] da$$

At point $r=n+g$, this derivative becomest :

$$\frac{d(D_{tot})}{dr} = \int_{a=0}^{\omega} \left[a \left(\int_{u=a}^{\omega} e^{-nu} s(u) x(u) du \right) - \int_{u=a}^{\omega} u e^{-nu} s(u) x(u) du \right] da$$

We integrate by parts the two components of the right of the equal sign. The first one writes down :

$$\left[\frac{a^2}{2} \int_{u=a}^{\omega} e^{-nu} s(u) x(u) du \right]_{a=0}^{\omega} + \int_{a=0}^{\omega} \frac{a^2}{2} e^{-na} s(a) x(a) da$$

and the second one writes down :

$$- \left[a \int_{u=a}^{\omega} u e^{-nu} s(u) x(u) du \right]_{a=0}^{\omega} - \int_{a=0}^{\omega} a^2 e^{-na} s(a) x(a) da$$

The sum of this two terms boils down to :

$$\frac{d(D_{tot})}{dr} = - \int_{u=a}^{\omega} \frac{a^2}{2} e^{-na} s(a) x(a) da$$

Separating the « benefit » and « contributions » elements of $x(a)$, we get :

$$\frac{d(D_{tot})}{dr} = \tau \int_{u=a}^{\omega} \frac{a^2}{2} e^{-na} s(a) w(a) da - \int_{u=a}^{\omega} \frac{a^2}{2} e^{-na} s(a) p(a) da$$

that can be rewritten as :

$$\frac{d(D_{tot})}{dr} = \frac{M}{2} (V_w + A_w^2 - V_p - A_p^2)$$

where M , A_w and A_p are, as previously defined, the mass of pensions and the mean ages of contributors and pensioners, and where V_w and V_p are the associated variances. Using the fact that $D_{tot}/M=(A_p-A_w)$, this expression can still be rewritten as :

$$\frac{d \ln(D_{tot})}{dr} = \frac{1}{2} \left(\frac{V_w - V_p}{A_p - A_w} - A_w - A_p \right)$$

We can give an order of magnitude for this derivative assuming that entry into activity takes place at 20, that retirement takes place at 60 and death takes place at 80. Counting age from the beginning of active life, as we have done all along, this gives $A_w=20$ and $A_p=50$. Corresponding variances are respectively 140 and 36 years. This implies a semi elasticity of D_{tot} with respect to r of $0.5(104/30-70)\approx-33$. Increasing r from $n+g$ to $n+g+1\%$ therefore leads to a diminution of the mass of implicit liabilities by 33%. This is consistent with orders of magnitudes obtained with full simulations.