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A Graph-Theoretical Approach to the Axiomatisation of National Accounting

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1. Introduction

Some 50 years ago, Karl Polanyi claimed in a famous article ²that the term economic is a compound of two meanings. He called them the substantive and the formal meanings. He wrote: “The substantive meaning of economic derives from man’s dependence for his living upon nature and his fellows. It refers to the interchange with his natural and social environment, in so far as this results in supplying him with the means of material want satisfaction.”

On the other hand, the formal meaning of economic is what he (critically) derived from Lionel Robbins’ famous definition of economics. He wrote: “The formal meaning of economic derives from the logical character of the means-ends relationship, as apparent in such words as ‘economical’ or ‘economizing.’ It refers to a definite situation of choice, namely, that between the different uses of means induced by an insufficiency of those means.” ³

He continued: “The two root meaning of ‘economic,’ the substantive and the formal, have nothing in common. (...) The formal meaning implies a set of rules referring to choice between the alternative uses of insufficient means. The substantive meaning implies neither choice nor insufficiency of means; man’s livelihood may or may not involve the necessity of choice.” ⁴

It is, he claimed, only the substantive meaning of “economic” that is “capable of yielding the concepts that are required by the social sciences for an investigation of all the empirical economies of the past and present.”⁵ In the same time, he stressed that the current concept of economic *fuses* the “subsistence” and the “scarcity” meanings of economic without a sufficient awareness of the dangers to clear thinking inherent in

¹ Thanks are due to Professors Yoshimasa Kurabayashi (Hitotsubashi University), Hiroshi Deguchi (Tokyo Institute of Technology), Yoshiak Koguchi (Chuo University), and Masaru Akiyama (Kyushu Sangyo University) as well as Mr Derek Blades (OECD) for their valuable comments. A special acknowledgment should be made of the suggestions given by Prof.ssa Rita Capodaglio Di Cocco (University of Bologna) concerning some of the axioms included in an earlier version.

² Polanyi (1957b) p. 243.

³ Ibid.

⁴ Ibid.

⁵ Ibid., p. 244.

that merger. ⁶It goes without saying that the *fusion* mentioned above was caused by “the Great Transformation” ⁷ using a well-known term of his.

Then he proceeded to make a proposal on his own definition of an economy: “The fount of the substantive concept is the empirical economy. It can be briefly (if not engagingly) defined as an instituted process of interaction between man and his environment, which results in a continuous supply of want satisfying material means”.

⁸

Thus, an economy in his sense of the term is an “instituted process.” He gave some remarks on the two words included in this definition. First, he wrote: “Process suggests analysis in terms of motion. The movements refer either to changes in location, or in appropriation, or both. In other words, the material elements may alter their position either by place or by changing ‘hands.’” ⁹He added: “These otherwise very different shifts of position may go together or not.” ¹⁰

He made a remark here: “Locational movements include production, alongside of transportation, to which the spatial shifting of objects is equally essential.” Another remark is about “appropriative movements.” According to his remark, they include those resulting from “transactions” (also referred to as “circulation of goods”) and those resulting from “dispositions” that is one-sided act of the hand.¹¹

However, it is “instituting of the economic process,” that “vests that process with unity and stability,” he wrote. ¹² Without “societal conditions from which the motives of the individuals spring,” in other words, if it were “reduced to the bare bones,” he stressed, “there would be little, if anything, to sustain the interdependence of the movements and their recurrence on which the unity and the stability of the process depends.” ¹³ It may be understood that the mere introduction of “money,” without which any accounting system could not be constructed, would make a bare-bone process into what could be called an instituted process. Furthermore, instituting of the process,

⁶ Ibid., p.244.

⁷ Polanyi (1944).

⁸ Polanyi (1957b). p.248.

⁹ Ibid.

¹⁰ Ibid.

¹¹ Ibid. Recently, Utz-Peter Reich, in his contribution to axiomatic reformulation of national accounting, defined an economy as “a set of value transactions between economic units in a currency area.” There is a rather striking resemblance between Reich’s definition and Polanyi’s as we note Polanyi’s economic process may be considered to be a set of movements. See Reich (2001), p.12 in particular. Concerning why he confined it to “a currency area,” a plausible presumption may be that a currency area is a natural scope for people’s cooperation.

¹² Ibid., p.249.

¹³ Ibid.

he expressed, “produces a structure.”¹⁴

In line with his thought sketched above, a graph-theoretical axiomatisation of national accounting will be pursued in what follows, although, as a matter of course, we do not mean to incorporate all the aspects of instituting into the framework.

Following an introductory section on graph theory, we attempt to present a formulation of Polanyi’s “process” or more or less “instituted process” as a (*weighted digraph*). Namely, “man’s dependence for his living upon nature and his fellows,” using his expression, will be described by means of a graph. It seems to us that a graph is a suitable means to express *cooperation* among people as well as people’s dependence upon nature.

In sections that follow, some conditions for an (instituted) process represented by a digraph to be represented by an *accounting system* as well will be considered. In some special cases, the process digraph may be represented by an accounting system by introducing only some additional entries called “balancing items.” However, augmenting additional entries other than “balancing items” are needed for the process to be represented by a set of accounts in general. Alternative augmentation methods for that purpose will be considered and by choosing one of them we attempt to construct a series of accounts for an economic unit like an enterprise as well as a nation.

2. A minimum introduction of graph theory¹⁵

In this section, a text-book-type introduction of graph theory will be given.

It is often said that the theory of graphs has a definite birth date. It is when Leonhard Euler (1707-1783), one of the most distinguished mathematicians of all the time investigated what is now well-known as the “Königsberg Bridges Problem” and he published a paper which contained the solution of the problem in 1736¹⁶.

¹⁴ Ibid.

¹⁵ For further information on graph theory, see, for example, Wilson and Watkins(1990).

¹⁶ The problem solved by him is as follows. The medieval city of Königsberg in Eastern Prussia was built near the mouth of the river Pregel. The river divided the city into four parts (A, B, C, and D), and they were interconnected by seven bridges (a, b, c, d, e, f, and g) as shown in Fig.2-1.a. The question under discussion was whether it is possible to find a route crossing each bridge exactly once, and returning to the starting point. Using graph-theoretic terms, this is equivalent to finding an Eulerian trail in the graph shown above as Fig.2-1.b.

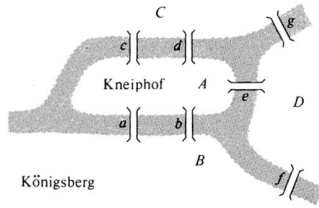


Fig.2-1.a

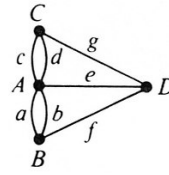


Fig.2-1.b

His finding is of vital importance for us because what we attempt to do in this paper relies upon a theorem about “Eulerian digraphs” that will appear later in this section as theorem 2-1.

Definitions: Graphs and Digraphs

A graph is, simply, a diagram consisting of points joined together by lines. See Fig.2-1.b. More formally, let V and E be two disjoint sets, with V being non-empty, φ a function which assigns to each element e of E an unordered pair of not necessarily distinct elements of V of the form $(u \& v)$.¹⁷ Then a graph is the ordered triplet of V , E , and φ . Thus, $G=(V, E, \varphi)$. For example, in Fig.2-1.b, a graph can be defined by setting

$V= \{A, B, C, D\}$, $E= \{a, b, c, d, e, f, g\}$, and

$$\varphi = \left(\begin{array}{cccccc} a & b & c & d & e & f & g \\ (A \& B) & (A \& B) & (A \& C) & (A \& C) & (A \& D) & (B \& D) & (C \& D) \end{array} \right).$$

Elements of V are called points or vertices and elements of E considered as unordered pairs of these elements selected according to the rule set by function φ are called lines or edges. The set V of the graph G is called the vertex-set of G , denoted by $V(G)$ and the list of edges is called the edge-list of G , denoted by $E(G)$.

The function φ is sometimes called the incidence function of the graph. If $\varphi(e) = (u \& v)$, u and v are said to be joined by e , or u is said to be adjacent to v , or v is said to be adjacent to u , or u and v are said to be adjacent vertices. We also say that e is incident with u and v (incident on u or v), or that u and v are incident with e . Two or more edges joining the same pair of vertices are called multiple edges, and an edge joining a vertex to itself is called a loop. A graph with no loops or multiple edges is called a simple graph.

A complete graph is a graph in which every two vertices are joined by exactly one edge. The complete graph with n vertices is denoted by K_n .

If, for each edge, φ assigns an ordered pair of vertices, instead of an unordered pair

¹⁷ While an element of Cartesian product $V \times V$ will be denoted by (u, v) , an element of unordered product $V \& V$ will be denoted by $(u \& v)$ or $(v \& u)$. Note that u may be equal to v .

of vertices, a directed graph or a digraph is obtained. In the case of a digraph $D = (V, A, \varphi)$, the vertex-set of D is denoted by $V(D)$, the *arc-list* rather than the edge-list of D is denoted by $A(D)$.

In the case of a digraph D , the incidence function φ assigns to each arc an ordered pair, so $\varphi(a) = (u, v)$, $a \in A$, $(u, v) \in V \times V$. Then u and v are said to be adjacent or v (or u) is said to be adjacent to u (or v). The arc a is said to be directed from u to v , and the arc a is said to be incident from u and incident to v . In this case, the vertex u is called the tail of a and the vertex v is called the head of a .

One can consider the underlying graph of digraph D , that is, the graph obtained by replacing each arc of D by the corresponding undirected edge.

Definitions: Subgraphs and Subdigraphs

Let $G = (V, E, \varphi)$ be a graph. A graph $G' = (V', E', \varphi')$ is a subgraph of G if and only if

- 1) $V' \subseteq V, E' \subseteq E$;
- 2) for all $e \in E', \varphi'(e) = \varphi(e)$;
- 3) if $e \in E'$ and $\varphi(e) = (v \& w)$, $v \in V'$ and $w \in V'$.

It is easy to define subdigraph by replacing $G = (V, E, \varphi)$ by $D = (V, A, \varphi)$.

Definitions: Degree, In-degree, and Out-degree

The degree of a vertex v is the number of $e \in E$ such that e is incident on v . In other words, it is the number of edges meeting at v , and the degree of a vertex v is denoted by $\deg v$. As a matter of course, the sum of all the vertex-degrees is equal to twice the number of edges (the handshaking lemma). Consequently, the sum of all the vertex-degrees is an even number and the number of vertices of odd degrees is even.

Let D be a digraph and let v be a vertex of D . The out-degree of v is the number of arcs incident from v , and is denoted by $\text{outdeg } v$. Similarly, the in-degree of v is the number of arcs incident to v is denoted by $\text{indeg } v$.

In any digraph, the sum of all the out-degrees and the sum of all the in-degrees are each equal to the number of arcs (the handshaking di-lemma).

Definitions: Adjacency matrices

Let G be a graph with n vertices labeled $1, 2, 3, \dots, n$. The adjacency matrix $M(G)$ is the $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j . If, in Fig.2-1.b, the vertices A, B, C, D are relabeled $1, 2, 3, 4$, the adjacency matrix is as follows:

$$\begin{pmatrix} 0 & 2 & 2 & 1 \\ 2 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

Let D be a digraph with n vertices labelled $1, 2, 3, \dots, n$. The adjacency matrix $M(D)$ is the $n \times n$ matrix in which the entry in row i and column j is the number of arcs from vertex i to vertex j . The following adjacency matrix corresponds to the digraph shown below right in Fig.2-2;

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

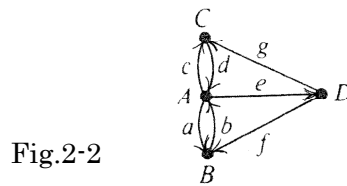


Fig.2-2

Definitions: Walks, Trails, Paths, and Cycles

A succession of k edges e_1, e_2, \dots, e_k in a graph G is said to form a walk of length k if

$$\varphi(e_i) = (v_{i-1}, v_i) \text{ for } i = 1, 2, \dots, k.$$

This walk is referred to as a walk between v_0 and v_n .

If all the edges of a walk are different, then the walk is called a trail. If, in addition, all the vertices are different, then the trail is called a path. If $v_0 = v_n$, then the walk or trail is said to be closed. A path may be closed but it is said to be a cycle.

A succession of k arcs a_1, a_2, \dots, a_k in a digraph D is said to form a walk of length k if

$$\varphi(a_i) = (v_{i-1}, v_i) \text{ for } i = 1, 2, \dots, k.$$

If all the arcs of a walk are different, then the walk is called a trail. If, in addition, all the vertices are different, then the trail is called a path. A closed walk, trail, and cycle in a digraph are defined as in a graph.

Definition: Connectedness

Using the term just introduced, a graph G is said to be connected if there is a path (or walk or trail¹⁸) in G between any given pair of vertices, and disconnected otherwise. Every disconnected graph can be split up into a number of connected subgraphs, called components.

A connected graph which contains no cycle is called a tree. If G is connected, then a spanning tree in G is a subgraph of G which includes every vertex of G and is also a tree.

¹⁸ It is easy to show the following three propositions are equivalent: (1) There is a walk from u to v ; (2) There is a trail from u to v ; (3) There is a path from u to v .

An edge in a connected graph is a bridge if its removal leaves a disconnected graph.

A digraph D is connected if its underlying graph is a connected graph, and disconnected otherwise. It is strongly connected if there is a path in D from any vertex to any other.

Definition: Weighted graph and Weighted digraphs

If each edge (arc) of a graph (digraph) has been assigned a positive number called its weight, the graph (digraph) is called a weighted graph (digraph).

Definition: Eulerian graphs and Eulerian digraphs

A connected graph G is Eulerian if there is a closed trail which includes every edge of G ; such a trail is called an Eulerian trail. On the other hand, a connected graph G is edge-traceable (semi-Eulerian) if there is an open trail (a trail that is not closed) which includes every edge G .

A connected digraph D is Eulerian if there is a closed trail which includes every arc of D ; such a trail is called an Eulerian trail in D . A connected digraph D is arc-traceable (semi-Eulerian) if there is an open trail which includes every arc of D .

Theorem2-1: Let G be a connected graph. The following three propositions are equivalent.

- (a) G is Eulerian;
- (b) Every vertex has even degree;
- (c) G can be split into disjoint cycles.¹⁹

A digraph version of the theorem is as follows:

Let D be a connected digraph. The following three propositions are equivalent.

- (a) D is Eulerian;
- (b) The out-degree of each vertex equals its in-degree;
- (c) D can be split into disjoint cycles.

Proof (for the digraph version):

We will prove in the order of (a) \rightarrow (b) \rightarrow (c) \rightarrow (a). The proof of (a) \rightarrow (b) part is obvious. The (b) \rightarrow (c) part is an induction proof involving arcs. The result clearly holds when the number of arcs is zero. Now assume that the result holds for digraphs with less than m arcs—that is, that any digraph with k arcs in which the out-degree of

¹⁹ By “disjoint” we mean that no two cycles have any edges in common.

each vertex equals its in-degree can be split into disjoint cycles whenever $k < m$.

Let D be a digraph with m arcs in which the out-degree of each vertex equals its in-degree, and let v_0, \dots, v_t be the vertices of a path P of the greatest length in D . Since $\text{out deg } v_t = \text{in deg } v_t$, there must be some arc that is incident from v_t . For example, the arc is incident to v . Because P is a path of the longest length, this v must be one of the vertices v_0, \dots, v_{t-1} , say, v_i . So, $v_i, v_{i+1}, \dots, v_t, v_i$ are the vertices of a cycle C in D . Removing the arcs of C from D yields a digraph D_1 with fewer than m arcs, in which the out-degree of each vertex equals its in-degree. By considering the connected components of it if necessary, D_1 or of its connected components, can be split into disjoint cycles. Together with C , they give the cycles as required.

Finally, for the proof of (c) \rightarrow (a) part, we will use mathematical induction again on the number of arcs m . For $m=0$, the only connected digraph is K_1 , which is clearly Eulerian. Assume (c) \rightarrow (a) is true for any connected digraph with fewer than m arcs, Delete one of the disjoint cycles, say C , of D . The resulting digraph D_1 may not be connected but each component of it is connected and composed of disjoint cycles. Therefore, by the induction hypothesis, each component is Eulerian. So, we can find an Eulerian trail for D as a whole by starting at any vertex of C . Thus, we traverse the arcs of C until we come to one of the components referred above, and then take the Eulerian trail for this component, eventually returning to the cycle C . We continue along C , eventually returning to the starting vertex.

The theorem shown below without proof is a well-known extension to the theorem 2-1.

Theorem 2-2:

Let G be a connected graph. Then G is edge-traceable if and only if G has exactly two vertices of odd degree.

Let D be a connected digraph. Then D is arc-traceable if and only if there are two vertices x and y of D such that

$$\text{outdeg } x - \text{indeg } x = 1, \text{indeg } y - \text{outdeg } y = 1$$

and

$$\text{indeg } v = \text{outdeg } v \text{ for all vertices } v \text{ other than } x \text{ and } y.$$

To close this section, we introduce “T-form” representation of digraphs. For each vertex, a “T-form” is assigned. On the right-hand side of the form are arcs

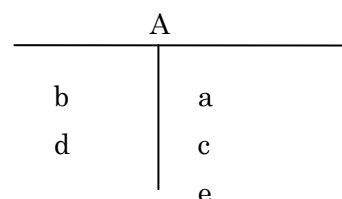


Fig.2-3

that are incident from the vertex, while arcs incident to the vertex are placed on the left side of the form as depicted in Fig.2-3.

Fig. 2-3 describes the situation at vertex A in Fig.2-2.

Of course, it is premature at this stage to say T-forms are accounts. First of all, there is no sense in talking about whether these T-forms are balanced or not. However, at least, we can count and compare the number of arcs on each side. If we consider weighted digraphs and put the weight accompanying rather than arcs in the T-forms, we can put a meaning to the statement by comparing the sum of weight attached to the arcs which lie on the left-hand side of the T-form and the sum of weight attached to the arcs on the right-hand side. Note that by assigning unit weight to each arc, we can form a weighted digraph from any digraph.

Of course, even if we do so, these T-forms are not necessarily balanced. But, because of handshaking di-lemma, the total number of arcs which lie on the left-hand sides of the T-forms equals the total number of arcs which lie on the right-hand sides of the T-forms. So, if we attach to each arc unit weight, we have a chance to construct a system of balanced accounts, so to speak, to represent a digraph.

Following the accounting convention, we will call the left-hand side of T-forms “debit” and the right-hand side “credit” in what follows.

3. Digraphic representation of Polanyian process

In what follows, we attempt to show a digraphic representation of Polanyian process or Polanyian process.

People cooperate with each other to live their lives. Digraphs seem to be very useful tools to describe how they do so.

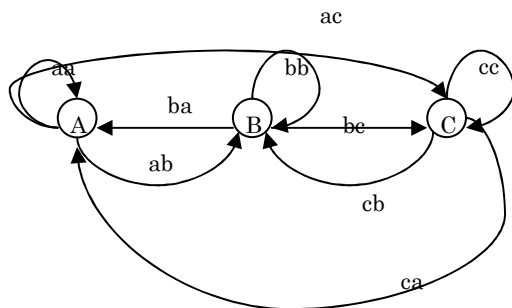


Fig. 3-1

In Fig.3-1, circles A , B , C represent economic agents and their places for production and consumption, i.e., fields and their houses. The figure depicts the situation in which the agents A , B , and C have their own fields but render their labour services to each other.

In market economies, people’s direct cooperation is replaced by rather indirect

cooperation,²⁰ typically through transactions in (or distribution of) “commodities.” This is why economics, as an independent discipline, emerged from one of many areas of ethics in the 17th century. It should be noted that Polanyi’s definition of an economy as an instituted process clearly takes into account direct and indirect cooperation (direct and indirect dependence upon their fellows) alike.²¹ Exchange of goods is exchange of services after all as Polanyi put it. ²²

At the same time, economic process as Polanyi defined it formulates “man’s dependence upon nature” as well as “his fellows.” It may be claimed that digraphs are very useful tools to represent economic process as includes “the interchange with his natural environment” as well.

A special convention is introduced to represent by using a digraph the interchange between the human beings and the nature, that is, production and consumption.

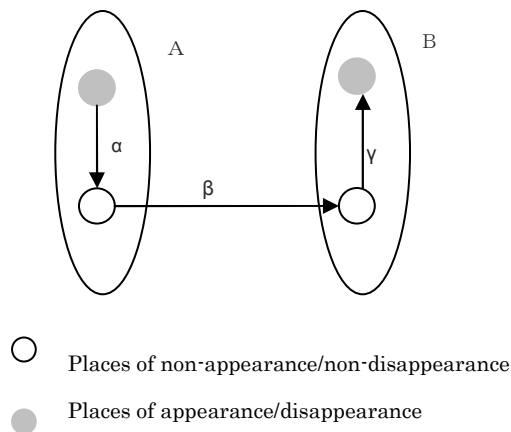


Fig.3-2

For that purpose, we introduce two kinds of vertices or places. One type of places is called places of appearance/disappearance. Arcs incident from or to this kind of vertices represent production or consumption, that is, output flows and consumption (intermediate and final) flows. Production flows as well as consumption flows may change the count of the goods existing in the system. Service flows are arcs incident from the places where they are produced and incident to the places where they are consumed.

²⁰ It should be noticed that the word “cooperation” is used here in a somewhat broader sense in that even profit-driven motives may lead to cooperation among people as a matter of fact.

²¹ See, in particular, Polanyi (1957a), pp.64-94.

²² Ibid., p.80.

The other type of places may be called *places of non-appearance/non-disappearance*, which are stores-like places and the arcs incident from or to this type of places indicate the move between places of goods or other economic objects. Arcs incident from or to these places cannot change the count of existing objects in the system. Although the services do not have their count, the same counting rule applies because arcs representing service flows, logically, only reach to places of appearance/disappearance.

There are sets of vertices or places that are under the control of a particular *economic unit (economic entity)*. We assume that every vertex is under the control of some economic unit including a special *central unit* (the government). Economic objects are produced goods or natural objects which have entered in the control of some agent already, conceptually without any process of production.

Black circles in the Fig.3-2 indicate places or vertices of appearance/disappearance, while white circles indicate the other type of, that is, stores-like, places or vertices. A black circle is a gateway to nature, so to speak. For, production is considered to be a human-controlled natural process after all, which is restricted with environmental conditions as well as human knowledge. Consumption as well is also a natural process noting human bodies are part of nature. ²³

In the figure, an economic unit, *A*, produced something without using any input and put it into his/her stores for the moment, and then gave it to another economic unit, *B*. Then *B* put it into his/her stores for some time and then consumed it.

If economic objects or services have monetary values, arcs can be weighted.²⁴ Then a weighted digraph emerges. And as we saw in the last section, we can consider this digraph by using T-forms. By attaching monetary weight to arcs, we can consider T-forms in monetary terms. For simplicity we assume that each arc is attached with equal (unit) weight. However, T-forms do not necessarily balance.

Thus, *A*'s situation in Fig.3-2 is described in a system of T-forms found in Fig.3-3a while *B*'s situation is depicted in the Fig.3-3b.

²³ It is not so clear whether Polanyi's process in his original formulation include consumption in addition to production. However, the both sides should be considered to be the interchange between human beings and the nature.

²⁴ Here, money as weight not as an economic object is introduced. In other words, the standard use of money, as contrasted with the payment and the exchange uses of money is considered here. Note that according to Polanyi's account of early moneys, the uses are instituted independently of one another. See Polanyi, Ibid, pp.264-66.

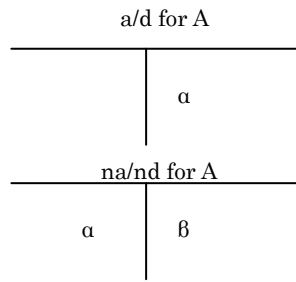


Fig. 3-3 a

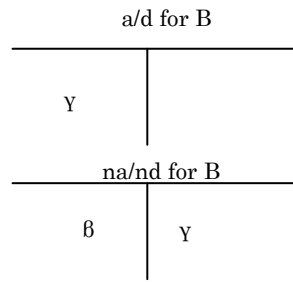


Fig. 3-3 b

It is easily seen that the T-forms above (in Fig.3-3a and b) representing Polanyi's or *Polanyian* process do *not* constitute "an accounting system."

By contrast, because of handshaking di-lemma, if there is no distinction between economic units, the total weight of the arcs on the left-hand sides of the T-forms equals to the total weight on the right-hand sides under the assumption of equal weight. Thus, by introducing some additional arcs called "balancing items" ²⁵ into the digraph, T-forms representing a Polanyian process *augmented* with arcs corresponding balancing items can be considered to constitute "an accounting system."

By the set of T-forms being an accounting system, we mean that *for each economic unit, the total weight of the arcs (flows) in the left-hand sides of the T-forms (accounts) equals to the total weight in the right-hand sides of the T-forms (accounts)*. If this condition is met, by introducing balancing items if necessary, all the T-forms for each economic unit could be balanced.

If the distinction between economic units is introduced as in Fig.3-2, the digraph is not necessarily able to be represented as an accounting system. Thus, the digraph in the figure cannot be represented as an accounting system in that the total weight of the arcs in the left-hand sides of the T-forms for $A(B)$ does not equal to the total weight in the right-hand sides of the T-forms for $A(B)$, assuming equal weight for arcs (flows).

²⁵ An ordinary definition of the term says that a balancing item is obtained by subtracting the total value of entries on one side of an account from the total value for the other side (93SNA, namely United Nations et al., 1993, para.3.64). In the context of the axiomatisation being attempted here, it should be noted that a balancing item relates one place with another, either one being controlled by the same economic unit. It is easily understood that by introducing balancing items, the T-forms for each economic unit can be balanced except the last one. Then, if there is only one economic unit in the system or if for a given economic unit, the total in-degree for all the places controlled by the entity equals to the total out-degree for all the places controlled by the same entity, all the T-forms can be balanced by adequately introducing balancing items if necessary given an equal weight is assigned to each arc. This could be called "balancing items lemma."

In some special cases including the case where there is no distinction between economic units, the digraph describing a Polanyian process can be represented as an accounting system. This may be the case if the economy is a centrally planned one.

It is a straightforward corollary to Theorem 2-1 that if the digraph describing Polanyian process is *Eulerian*, it can be represented as an accounting system without using any balancing item. Furthermore, if balancing items are introduced, the theorem can be extended. For that purpose, we define the term “external” (flows or arcs). An arc (or a flow) is *external* if the vertex it is incident to is under the control of an economic unit that is not the same as the economic unit which it is incident from. And it is “internal” otherwise.

Theorem 3-1:

Polanyian digraph that is, a digraph that represents Polanyian process can be represented by an accounting system if and only if for each economic unit, the total number of external arcs (flows) incident from the vertices under its control is equal to the total number of external arcs (flows) incident to the vertices under its control.

Although there are some exceptional cases where the condition of the above theorem is met, Polanyian digraphs *cannot* generally be represented by an accounting system.

The exceptional cases include fully aggregated systems where no distinction is made between economic units. In addition to this case, the market economies case, in which all the external flows are the results of market transactions, is considered to be another example. Thus, remembering our assumption that equal weight is assigned to each arc (flow), for any pair of entities, say $\{A, B\}$, any external flows incident from a vertex under the control of A and to a vertex under the control of B can be paired with an external flow incident from a vertex under the control of B and to a vertex under the control of A . Each external flow of the system is included in one and only one of these pairs and all these pairs constitute the set of all the external flows. See (i) in the figure below.

The case where there is a central unit and it provides both parties to any external flow with a flow of an economic object, say, “money” as in Fig.3-1 (ii) below may be another possibility. Note this economy needs money supply from outside of the process described. This economy need not be a typical modern monetary economy. First of all, this money must be a real object like cowries. Rather, it might be a redistribution

economy where the central unit (C in the Fig.3-1(ii)) intervenes to facilitate the redistribution process with supplying money. In the figure, the flow a of a real non-monetary object is facilitated by two monetary flows m_1 and m_2 . If you neglect the monetary flows and reroute the flow a through the central unit C, you will find a redistribution process.

Reciprocity as Polanyi described it as a form of economic integration,²⁶ may be still another example, because every external flow in the system is deemed to constitute a cycle and the set of all the external flows are the union of the disjoint cycles, remembering our equal-weight assumption again. So, a reciprocity system satisfies the condition of theorem 3-1 for the system's digraph to be Eulerian, Thus, reciprocity systems can be represented by accounting systems. See (iii) in the figure below.

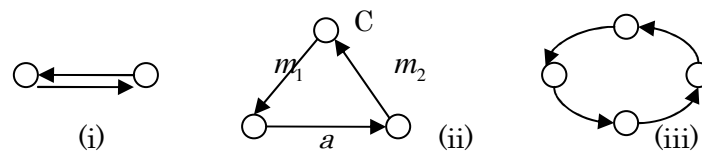


Fig.3-1

A Formal Presentation of the Definitions and Axioms (1):

Definition I: Polanyian Process, Polanyian Process Prime, and Polanyian Process Double Prime

A Polanyian Process for an accounting period can be defined as a weighted digraph $PP = (P, F, \varphi, w)$, where P , a *vertex-set*, is the set of the places in the system and F , an *arc-set*, and φ , an *incidence function*, determine all the flows in the accounting period and w is a *weight function* that assigns monetary weight to each element of F .

When the weight function w is a simplified weight function that assigns a unit weight to each element of F , the process is called A Polanyian Process Prime PP' . By disregarding the weight function involved, we get A Polanyian Process Double Prime $PP'' = (P, F, \varphi)$.

Interpretation I: An accounting period

A Polanyian process takes place in a time interval called *an accounting period*. It is

²⁶ Polanyi (1957b), pp. 250-56.

implicitly presumed that this period is so short that we can assume prices are constant.

Interpretation II: Places

The vertices in the Processes defined are interpreted as places where “real” economic processes such as production, consumption, and accumulation take place.²⁷ Some of the places are often called “establishments” in statistics.

Interpretation III: Flows

The arcs in the Processes defined are interpreted as flows in national accounting. Although flows in national accounting are what are sometimes called “transactions,” it is to be remembered that the entire flows as a system should represent the framework of co-operation (inter-dependence) among people as well as interchange between people and nature. At this stage, flows are confined to “real” flows.

Interpretation IV: Valuation (weight).

Each flow has a weight of a positive real number. This is deemed to be its monetary valuation.

Axiom I: The number of flows is finite.

Axiom II: The number of places is also finite.

Namely, Polanyian processes (or Polanyian processes prime or Polanyian processes double prime) are finite.

Because we can think about, say, 100 flows each of which has a weight of one instead of the flow weight of which is 100, we can consider any Polanyian process $PP = (P, F, \phi, w)$ as a corresponding Polanyian process prime $PP' = (P, F', \phi', w')$ although we have to say they are different mathematically. Then the weight that each flow has is its value as well as its quantity.

In addition, we can identify Polanyian process prime $PP' = (P, F', \phi', w')$ with Polanyian process double prime $PP'' = (P, F', \phi')$. Instead of summing the weight of the flows, say, on the right-hand side of the T-form, we can just count the number of the arcs incident to the place. In mathematical terms, we can think about a *digraph* instead of a *weighted digraph*. If 100 flows start from a place, it can be said that the place's out-degree is 100, while 100 flows reach to a place, it can be said that the place's

²⁷ In the case of services, production and consumption may be considered to take place in a single place in fact.

in-degree is 100.

Axiom III: Places of appearance/disappearance and Places of non-appearance/non-disappearance

There are two kinds of places: *places of appearance/disappearance* and *places of non-appearance/non-disappearance*. P_N denotes the set of the former type of places and P_S denotes the set of the latter type of places;

$$P = P_N + P_S.^{28}$$

For the latter type of places, a function μ is defined. This function measures the stock of economic objects found in the place of non-appearance/non-disappearance. Thus, given an initial value for the place, $\mu(p) = \mu_0(p) + [\text{in-degree}] - [\text{out-degree}]$ for any $p \in P_S$ in the case of PP'' . In the case of PP or PP' , the weight total of the arcs incident to the place instead of the in-degree, that of the arcs incident from the place instead of the out-degree should be used. For the sake of convenience, we define μ for the former type of places as well such that always $\mu(p) = 0$ for any $p \in P_N$.

Definition II: Economic units (economic entities or economic agents)

An economic unit is a subset P_i ($i = 1, 2, \dots, n$) of the vertex-set of the Process assuming there are n economic units.

Interpretation V: Economic units

Each place is controlled by an economic unit which operates the economic processes in question. An economic unit is conceived to be an institutional unit rather than an establishment-type unit in national accounting terminology. Grouping of economic units is often called sectoring.

Axiom IV: If the suffix i denotes economic units and the number of economic units is n ,

$$P = \sum_{i=1}^n P_i$$

Axiom V: There is a special economic unit called "central economic unit" or simply "central unit."

Interpretation VI: Central Unit

The central unit plays the role of the government or the central bank. The functions that should be played by the unit will be introduced later.

²⁸ Here, " $A+B$ " indicates the union of two disjoint sets A and B .

Definition III: Production, Consumption, Services, and Economic Objects

If φ assigns to an element of F an ordered pair (a, b) , “a” being a member of P_N and “b” being a member of P_S , then a flow ($\in F$) of the form “a \rightarrow b” indicates production of economic objects called goods and if “b” is a member of P_N and “a” is a member of P_S , then a flow of the form “b \rightarrow a” indicates consumption (final or intermediate) of economic objects.²⁹ And if both of “a” and “b” are members of P_N , a flow of the form “b \rightarrow a” indicates services rendered by the unit of which b is a member to the unit of which a is a member. ³⁰Rendering services is also deemed to be production. Receiving services is consumption. Finally, if both of “a” and “b” are members of P_S , a flow of the form “a \rightarrow b” indicates a move (physical and/or appropriative) of an economic object.

Interpretation IV: Economic Objects

Flows may be those of economic objects. An economic object is an undefined term and we presume it may be something that has started to exist as a result of production process in the accounting period or some earlier accounting period or that an economic object may be a natural object like land that had entered under the control of an economic unit without production process.

Interpretation V: Services

Flows may be those of services. The concept of services has been given by a famous article by T. P. Hill³¹ who defined services as “changes,” caused by one economic unit, to the condition surrounding another economic unit including human beings’ physical bodies and its belongings as well as its economic assets under the prior consent between the units involved. Services may be factor services as well as non-factor services. Incidentally, so-called capital services are often not services but the use of some economic objects called capital in the production processes.³² Factor services, for example, labour services, appear at a place without any input, that is, a flow incident to the place and non-factor services are produced with input(s) normally.

²⁹ The rule is followed that the output flows which carry economic objects reach only to places of non-appearance/non-disappearance. In reality, there may be the case where the economic objects in question cease to exist without being stocked in any stores-like places.

³⁰ Actually, transfer flows, which follow the same accounting rule as services, will be introduced at a later stage.

³¹ Hill (1977).

³² Ibid.,pp.327-28.

Definition IV: External flows and internal flows

A flow (or an arc) is *external* if the place it is incident to is under the control of an economic unit that is not the same as the economic unit which it is incident from. Otherwise, a flow is internal.

An accounting system may be constructed for the Polanyian digraphs formulated above. To start with, the following definition is introduced.

Definition V:

The Polanyian Process Double Prime PP'' is said to be *representable by an accounting system* if for every economic unit i , the number of the external flows that are incident from the places included in the set P_i is equal to the number of the external flows that are incident to the places in the set P_i . In the case of PP' and PP , the numbers of the external flows should be replaced with their weight totals.

Remark: In what follows, for simplicity, axioms, definitions and propositions will be stated in terms of Polanyian Processes Double Prime (PP''). Sometimes, we will call them Polanyian digraphs or process digraphs simply.

Remark:

The Polanyian digraph PP'' corresponding to one of the following cases is *representable by an accounting system*:

- (i) Fully aggregated systems;
- (ii) Full market economies;
- (iii) Redistribution economies with money supplied from outside;
- (iv) Reciprocity systems.

4. The Construction of Accounting Systems: the first stage

In general, Polanyian digraphs, that is, digraphs representing Polanyian processes are not representable by an accounting system. However, by adding some artificial flows, the digraphs come to be *representable by an accounting system*.

Thus, there are at least four ways of adding required artificial flows, which are shown below without proof:³³

³³ It is easy to find that for (i) – (iv), in the “augmented” digraph, the total in-degree of the places controlled by each economic unit equals the total out-degree of the places controlled by the same economic unit.

- (i) for each external flow in the system incident from a place in P_i to a place in P_j , with $i \neq j$, adding a new external flow incident from a place in $P_j \cap P_N$ to a place in $P_i \cap P_N$;³⁴
- (ii) entering so-called transfer entries, that is, for each external flow in the system incident from a place in P_i to a place in P_j , with $i \neq j$, which is not able to be paired with any external flow incident from a place in P_j to a place in P_i , adding a new external flow (called a transfer flow in the ordinary sense) incident from a place $P_j \cap P_N$ to a place in $P_i \cap P_N$;
- (iii) recording transfer flows between the units concerned and the central unit, that is, for each external flow in the system incident from a place in P_i to a place in P_j , with $i \neq j$, adding a flow incident from a place in $P_c \cap P_N$ with c being the special central unit c , to a place in $P_i \cap P_N$, and adding a flow incident from a place in $P_i \cap P_N$ to a place in $P_c \cap P_N$;
- (iv) completion of (reciprocity) cycles, that is, if there is a sequence of flows of the form $a_0 a_1, a_1 a_2, \dots, a_{n-1} a_n$ in which $a_i a_{i+1}$ is a flow incident from $a_i \in P_i$ and incident to $a_{i+1} \in P_{i+1}$, $P_i \neq P_j$, $j = 1, \dots, n$, adding a flow of the form $a_n a_e$ in which $a_n \in P_n \cap P_N$, and $a_e \in P_0 \cap P_N$ so that all the external flows in the system might be included in one and only one of the “closed” and disjoint sequences of flows.

³⁴ It is assumed that $P_i \cap P_N$ is not empty for any i .

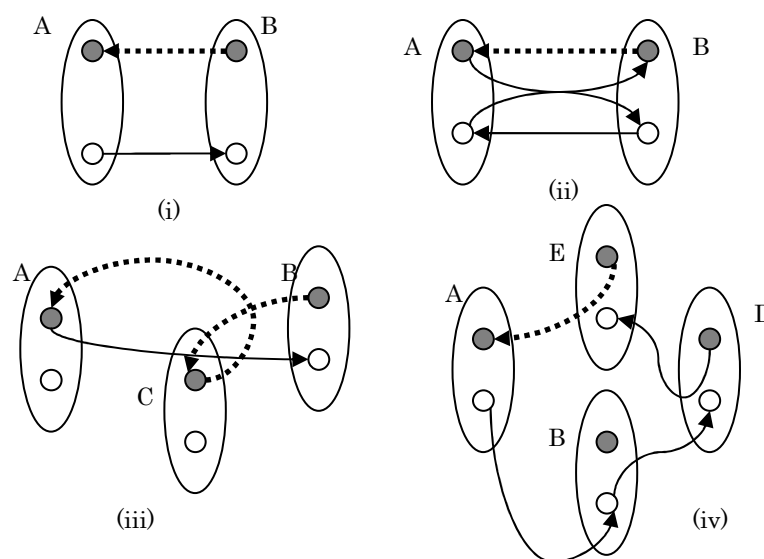


Fig. 4-1

The figure 4-1 shows these four cases. It may be interesting to note that (i) and (ii) grouped, (iii), and (iv) correspond respectively to cases (ii), (iii) and (iv) in the previous section where a process digraph is representable by an accounting system without added arcs.³⁵

By choosing one of the sufficient conditions listed above, we can get an “augmented” Polanyian digraph (APP''), which may be transformed to an Eulerian digraph by augmenting balancing items in addition if necessary. We might call the flows that are stated in the above conditions transfers in an extended sense. For example, we can choose (ii), that is, the introduction of transfers in the ordinary sense to get an accounting system that represents the original Polanyian process. Note that added flows follow the rule that is the same as services.

Summing up,

Theorem 4-1:

There is a way (normally more than one way) of adding arcs, by the addition of which the original Polanyian Process becomes representable by an accounting system.

³⁵ It may be also interesting to note that these cases correspond roughly to the three main patterns of economic integration that Polanyi considered, that is, *exchange*, *redistribution*, and *reciprocity*. See Polanyi (1957b) p.250.

These ways of addition will be called “augmentation rules” in what follows.

Because in this system the concept of economic objects as well as that of services is confined to that of physical objects and processes (in the World I using Karl Popper’s term³⁶), transfer flows in this system include financial transactions and transactions in other intangible assets as well as transfer flows in the usual sense whence in that respect, it resembles Material Product System (MPS).³⁷

In the next section, we are going to proceed to the introduction of financial or non-real objects. In the rest of this section, however, we modify slightly or reformulate the process digraph upon which an accounting system is constructed.

On the one hand, places of appearance/disappearance are naturally divided into

- (i) production places,
- (ii) places of production factors
- (iii) consumption places,

and in addition, for augmented digraphs,

- (iv) transfer places.

A production place is created through grouping the flows incident to or from a given place. For a given place of appearance/disappearance, a set (or an ‘activity’) of “input” flows (incident to the place) and “output” flows (incident from the place) may be picked up to form a production place. Input flows may be, say, those of materials for the production or labour input, while the output flows may be those of goods or services produced or rendered. Note that more than one production places can be created for a given place of appearance/disappearance.

There may be a type of flows that is incident from the place and never grouped with other flows incident to the place. They are called “production factors.” Labour services are typical examples. However, as noted before we do not admit the concept of capital services following the thought of Hill in his 1977 paper. The rest of the flows are consumption flows and possibly transfer flows.

In line with the grouping as described above of the flows incident from or to a place of appearance/disappearance, it may be possible that a place is divided into two or more places each with purer characteristics.

On the other hand, places of non-appearance/non-disappearance are not only classified (divided) into those for different categories of economic objects but also into

³⁶ Popper (1994).

³⁷ Árvay (1994) pp.218-36.

capital places and non-capital stores-type places.

A capital place may be defined as a stores-type place which is, at least potentially, adjacent to one or more production places controlled by the same economic unit. Capital places may be those for inventories as well as for fixed capitals. Input flows like materials flows are incident from capital places and incident to production places and output flows like those of finished goods are incident from production places and incident to capital places. Sometimes capital places are bi-directionally adjacent to production places like work-in-progress or fixed capitals. As stated earlier, a convention is followed that economic objects produced are put in stores-type (namely capital) places first and then directed toward other places and another convention might be followed that economic objects to be used as input to production are put in stores-type places first and then directed toward production places controlled by the unit.

As for the treatment of fixed capitals, we adopt a kind of joint production view of capitals, that is, we assume as if a t -year-old capital good is used up in a certain production process in an accounting period t while a $t+1$ -year-old capital good is jointly produced with the output in the ordinary sense in the period. Fixed capital consumption can be defined consistently with this view of fixed capitals. According to this definition, valuables are not necessarily capitals. Although they are capitals only when they are used in the production processes in museums, for example, their places are not generally adjacent to production places controlled by the same economic unit. Land and other natural objects are part of environment and may be controlled by economic units at the same time. Potentially it might be better to treat land as fixed capital.

A Formal Presentation of the Definitions and Axioms (2):

Definition VI: A Balancing Item Tree

A balancing item tree is defined for each economic unit as a tree which contains all the vertices in the economic unit.

Definition VII: An Accounting System

Given a Polanyian Digraph (namely) Polanyian Process Double Prime PP'' , “an accounting system” may be defined as a triplet of (1) the digraph combined with (2) an augmentation rules selected and (3) a balancing items tree.

Remark: Among the augmentation rules listed in section 4, the most familiar one may be the addition of transfer entries in the ordinary national accounting sense. That is, for

any pair of economic unit, the flows between them are investigated and the flows are divided into pairs of flows, one of which is directed from one economic unit to another and the other of which has an opposite direction. Then wherever there are un-paired flows, new flows between the units which have reversed directions are to be created. They are called “transfers” in the ordinary sense. Transfer flows are to be put between two places of appearance/disappearance.

Remark: For any Polanyian digraph after applying one of the augmentation rules and setting a balancing item tree, replacing each edge of the tree by a non-minus integer number of arcs so that the in-degree of each place be equal to the out-degree of the place, the digraph can be transformed into an Eulerian digraph due to theorem 4-1. The Eulerian digraph thus constructed might also be called “an accounting system.”

Definition VIII:

An Eulerian digraph created in the way shown in Definition VII above, namely an accounting system, may be called an augmented Polanyian digraph or an Augmented Polanyian Process Double Prime *APP''*. Related weighted digraphs may be called an Augmented Polanyian Process *APP* and an Augmented Polanyian Processes Prime *APP'*.

Remark: Augmented Polanyian Processes can be represented by a series of T-forms and (adjacency) matrices called NAMs or SAMs as well as typical di-graphical figures known as “eco-circs.”³⁸ It will be not so confusing that not only Eulerian digraphs but also these related weighted digraphs are called accounting systems.

Definition IX: Capital places

A capital place may be defined as a place of non-appearance/non-disappearance which is adjacent to one or more production places (places where production takes place) contained in the same economic unit set provided that the adjacency is *not* established by a balancing item flow.

Remark: The use of capital in production processes may be described as a kind of joint production. It may be represented by a set of capital input and capital output so to speak. The latter is of course the economic objects one year (period) older than those

³⁸ Aukrust (2003) includes a concise account of Ragnar Frisch’s concept of eco-circ. See Appendix A of the paper referred in particular.

used as capital input. Note that there is no reason to treat the use of capital as services.

5. The Construction of Accounting Systems: the second stage

In this section we will conduct the construction of an accounting system at the second stage. As we noted, so far in this paper, there are no financial assets or liabilities. Transfer flows are those of MPS- type that include financial flows as well. Here, we attempt to introduce non-real economic objects that include financial objects (financial claims and liabilities) and non-financial non-real objects like patents. Economic objects are said to be non-real when though it is necessary to be evidenced or represented by some physical objects (or processes namely habitants in World 1 in Popper's term), something represented is not in world 1. For example, they may be commitments or promises, which as such should be considered to be *mental* (say, "Someone thinks he/she will repay a certain sum at (a) certain time point(s) in the future"), so their habitat should be considered to be originally World 2 of mental states and processes in Popper's term and then move to World 3 of products of human minds again in his term once recorded as claims and liabilities.

As for financial objects, we define financial places of appearance/disappearance and financial places of non-appearance/non-disappearance. They are like corresponding places for real objects, but there is one thing different. Although financial objects appear at financial places of appearance/disappearance, they disappear only at financial places of appearance/disappearance at which they first appeared.

Financial objects are *bilaterally* created between the two parties involved. The flow of a newly created financial object is incident from a place of appearance/disappearance controlled by the one party to the creation, the borrower, and incident to a place of non-appearance/non-disappearance controlled by the other party to it, the lender. The lender is said to have a financial claim and the borrower is said to have a liability. Although financial objects are bilaterally created, they are recognised by the community as a whole. For example, the holders of the claims are often protected by law and social rules. The bankruptcy rules, of course, take into account the liabilities thus created.

When financial objects are created, borrowers make some commitments about what they will do for the lenders in the future. They may be reflected in a series of transfer flows in the future. Sometimes financial objects are considered to have the value that equal the capitalised value of these transfer flows.

Most of "financial assets" are financial claims created in this manner though there are several exceptions to this description. Most notable exception may be gold, for which

some special treatments may be needed that is not specified here.

Some financial objects may be used as money, in particular, media of exchange or payment. The flow of financial objects that exits for settlement purposes including some of money flows is often called “below the line.”³⁹ If the settlement flows are “balanced” for each economic unit and therefore the entire settlement flows are considered to form another Eulerian (sub-digraph of the main Eulerian), it may be considered to be reasonable that the settlement flows should be omitted and the remaining (main) system should be highlighted.

Apart from financial objects, there are quite a few kinds of non-real, non-financial objects. They are *socially* created non-real economic objects. Thus, annex table of chapter 13 of the 93SNA manual (United Nations *et al.*, 1993), gave a description that they are “non-produced assets that are constructs of society.” Then, a convention is followed that the central unit is involved in the creation of non-real non-financial objects as one of the parties involved. As in the case of financial places of appearance/disappearance, we can introduce non-financial non-real places of appearance/disappearance. Appearance of patents, for example, may be recorded as flows starting from the non-financial non-real places of appearance/disappearance controlled by the central unit and directed to the right holder’s non-financial non-real places of non-appearance/non-disappearance.

This treatment is different from that prescribed in the 93SNA manual. However, it should be stressed that distributive implications of the *social* creation of these assets are *not* adequately described by the rule set by the SNA. If a developing country’s government granted a pharmaceutical company of a developed country a patent on a new medicine, we can consider that the government helps the company collect a sum (rent) added on the price of it from the users.

Often, the value of non-financial non-real objects may be the capitalised sum of the values of the future payment flows, sometimes called royalties, expected that are due because of social creation of the asset in question, say patents.⁴⁰

³⁹ Although gold as such is a real object, a certain kind of gold may be used as a means of payments, especially in international settlements and so it is thought that such gold is money and should be included in “below the line,” and because of this, it is taken for granted that (monetary) gold should be treated as financial objects. Similarly, it might be better to treat cowries in the kingdom of Dahomey as described in Polanyi (1966), as if they were financial items rather than real objects.

⁴⁰ In 93SNA, an inadequate convention about royalties was introduced, unfortunately, in which they are treated as if they are services. Incidentally, the treatment of “purchased goodwill,” which SNA recognises as another type of non-financial non-real objects, may be questionable. It seems that it lacks the important feature that is characteristic of non-financial non-real objects. That is, this type of asset items in the

Transfer flows must be reintroduced in order for the process to be represented by an accounting system. Transfer flows may be classified into (i) current and (ii) capital, and (i) current transfers may be classified into (ii) primary and (ii) secondary. However, this classification is not introduced here to simplify the system.

A series of accounts may be formulated by selecting a set of all the accounts (places) for an economic unit and specifying the flows of balancing items, each connecting two of them. The set of all accounts of the unit (as “vertices”) and balancing items (as “edges”) will make a “tree” (in a graph-theoretical term). It is shown below as Fig. 5-1a. Note that (undirected) edges rather than (directed) arcs are used to represent balancing items because their signs are unknown. Also note in the figures Fig.5-1a and b below, places of appearance/disappearance and places of non-appearance/non-disappearance are combined for the sake of simplicity as far as non-real objects, financial as well as non-financial are concerned. In the figures, such combined places are shown as striped-patterned circles.

It may be advisable to note that the consumption account (place) in Fig.5-1a plays the role of an income and outlay account as well and that the non-financial non-real account (place) plays the role of an accumulation account as well. As for the latter case, to disburden the double role we can put into the system a new artificial account (place) shown as a checked-patterned circle in Fig.5-1b. It is seen from the figure that no flows other than those representing balancing items are incident with the place.⁴¹

The T-form representation of the digraphs will be found below. Fig.5-1a and b correspond to accounts 1-7 and 8a and accounts 1-7, 8b and 9 respectively. These systems are so simplified that you can understand without any detailed description about the accounts and the items shown. But one specific comment is in order. That is,

business accounting convention is NOT something socially created. It seems difficult to answer why national accountants should follow the same rule as business accountants. In fact, if you compare the pooled account before and after the acquisition, you will find the new asset appear in the latter rather abruptly. Thus, the concept of “purchased goodwill” is a business accounting concept rather than a national accounting one. The concept of “independent net worth” will work well in the situation involved with related entries for valuation change in shares and other equities. In the case of the acquisition of an un-incorporated business, it seems reasonable to deal with the business as quasi-corporate. About the current SNA’s treatment of purchased goodwill and its amortisation, see paragraphs 12.22 and 12.34 in the 93SNA.

⁴¹ Another way of formulating a tree of accounts may be making a new artificial account called “trial balance” as in business accounting practices.

in this system, the internal factor flows are shown explicitly. In 93SNA⁴², the concept of mixed income is introduced to advise the users to direct their attention to the own-account factor flows involved in the item “operating surplus.”

Once a series of accounts is formulated for an individual economic unit, it is quite easy to construct a macro-economic system of accounts by defining sectors (groups of economic units) and placing so-called screen accounts for selected categories.

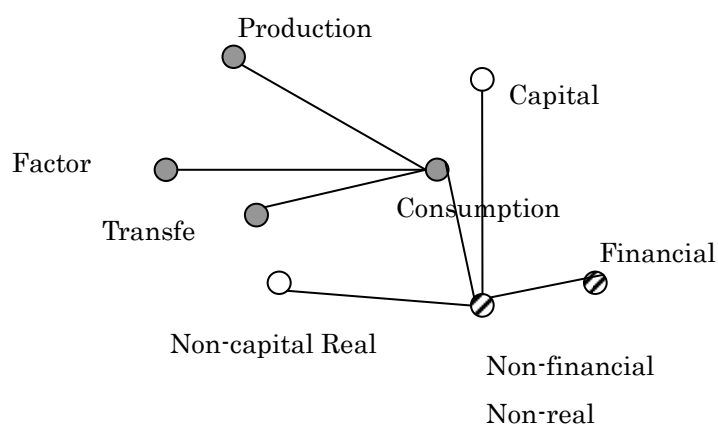


Fig.5-1 a

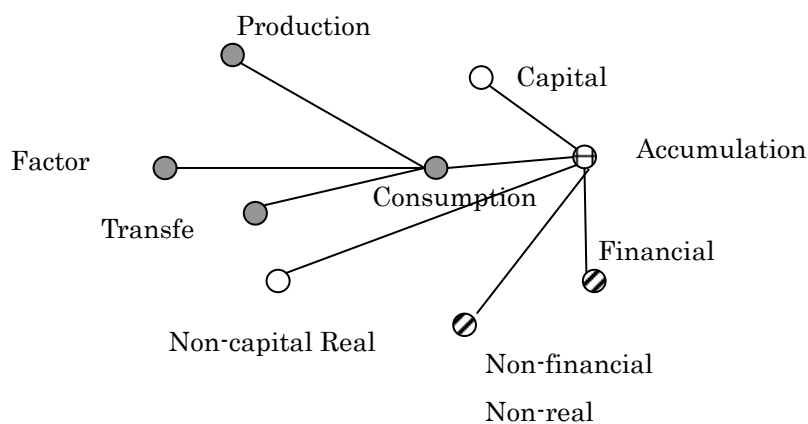


Fig.5-1 b

⁴² United Nations *et al.*(1993).

Table 5-1 A Simplified System of Accounts

	Account/Place	Debit	Credit
1	Production	Intermediate Consumption Factor input Fixed Capital Consumption <i>Operating Surplus</i>	Gross Output
2	Factor	<i>Balance of Factor Account</i>	Factor Outflows, external and internal
3	Transfer	Transfer Paid <i>Balance of Transfer Account</i>	Transfer Received
4	Consumption/ Income and Outlay	Final Consumption <i>Saving</i>	Operating Surplus Balance] of Factor Account Balance of Transfer Account
5	Capital	Net Increase in Inventories Gross Fixed Capital Formation	Fixed Capital Consumption <i>Net Capital Formation</i>
6	Non-Capital Real	Acquisition of Non-Capital Real Assets	Disposition of Non-Capital Real Assets <i>Net Acquisition of Non-Capital Real Assets</i>
7	Financial	Acquisition of Financial Assets Redemption of Liabilities	Disposition of Financial Assets Incurrence of Liabilities <i>Net Lending</i>
8a	Non-Financial Non-Real/ Accumulation	Acquisition of Non-financial Non-Real Assets Net Acquisition of Non-Capital Real Assets Net Acquisition of Non-Financial Non-Real Assets Net Capital Formation Net Lending	Disposition of Non-financial Non-Real Assets Saving
8b	Non-Financial Non-Real	Acquisition of Non-financial Non-Real Assets	Disposition of Non-financial Non-Real Assets <i>Net Acquisition of Non-financial Non-real Assets</i>

	Accumulation	Net Capital Formation	Saving
9		Net Acquisition of Non-Capital Real Assets	
		Net Acquisition of Non-financial Non-Real Assets	
		Net Lending	

A Formal Presentation of the Definitions and Axioms (3):

Axiom VI:

The places in an augmented Polanyian digraph APP'' comprises places of appearance/disappearance P_N and places of non-appearance/non-disappearance P_S , both of which may be further subdivided, that is,

$$P = P_N + P_S = \sum P_P + \sum P_F + \sum P_T + \sum P_C + \sum P_K + \sum P_{NK} \\ + \sum NRFP_N + \sum NRNFP_N + \sum NRFP_S + \sum NRNFP_S,$$

where summations are over economic units. The following notations are used in this expression:

- P_P Production places;
- P_F Factor places;
- P_T Transfer places;
- P_C Consumption places;
- P_K Capital places;
- P_{NK} Non-capital real places;
- $NRFP_N$ Non-real financial places of appearance/disappearance;
- $NRNFP_N$ Non-real non-financial places of appearance/disappearance;
- $NRFP_S$ Non-real financial places non-appearance/non-disappearance;
- $NRNFP_S$ Non-real non-financial places of non-appearance/non-disappearance.

Definition X: Production places

Production places in are places of appearance/disappearance which there is a flow other than transfer and a balancing item incident from and a flow other than transfer and a balancing item incident to.

Remark: This definition is related to so-called “impossibility of the land of Cockaigne.”

Definition XI: Consumption places

Consumption places are places of appearance/disappearance which there is a flow other than transfer and a balancing item incident to and are not production places.

Definition XII: Factor places

Factor places are places of appearance/disappearance which there can be only flows incident from.

Definition XIII: Transfer places

Transfer places are places of appearance/disappearance which only transfer flows can be incident from or to.

Interpretation VI: Production places, etc.

The places of appearance/disappearance in Polanyian digraphs may be divided into places of purer characteristics. In order to form production places (accounts) in practice, you must combine input flows and output flows meaningfully. The input flows may be those of intermediate or capital inputs as well as factor inputs. Capital inputs are flows incident from capital places to production places which represent the use of capital in production processes as already remarked. Therefore, the use of capital is not considered to be factor input (or capital services). Factor inputs are primary in that they are not something produced in the system. A typical example may be labour. Land might be treated as if it is also capital of a sort, but in the interpretation of the axiomatic system, it is non-produced, non-capital real economic objects like in SNA.

Definition XIV: Economic objects, real and non-real

Economic objects which have been created by means of production in the present accounting period or any one of the past periods as something arcs incident (output flows) from production places carry and will be stored (counted) at places of non-appearance/non-disappearance are called real economic objects. Other than produced economic objects described above, there may be real economic objects that are neither produced nor consumed and that are stored in places of non-appearance/non-disappearance. On top of these real economic objects produced or non-produced, there may be non-real economic objects. Non-real economic objects may be mutually created between two distinct economic units or between an economic unit and the central economic unit. In the former type of creation, the non-real objects may

be called financial and in the latter type of creation, the non-real objects are called non-financial.

Definition XIV: Places of appearance/disappearance for financial economic objects. Places of appearance/disappearance for financial economic objects are places where financial economic objects are created or cease to exist. A flow starting from such a place means that some financial economic object is created. Note that financial economic objects are created *bilaterally* between two economic units, a creditor and a debtor. For the debtor, liability is created and for the creditor, financial asset is created. A flow ending at such a place means that some liability (or a financial claim) ceases to exist. Concerning disappearance, a special rule exists. In contrast with the case of real economic objects, financial economic objects only cease to exist at the places where they were created in the past. See the axiom below.

Axiom VII: If flows representing the move of financial objects reach places of appearance/disappearance, they are those in the same economic unit where the objects were created.

Definition XV: Places of non-appearance/non-disappearance for financial economic objects.

Places of non-appearance/non-disappearance for financial economic objects are places where flows representing the creation or move (change of ownership) of financial economic objects reach and they do not cease to exist when they reach.

Definition XVI: Places of appearance/disappearance for non-financial non-real economic objects.

Places of appearance/disappearance for non-financial non-real economic objects are places where non-financial non-real economic objects are created or cease to exist. A flow starting at such a place means that some non-financial non-real economic object is created. Because in contrast with the case of financial objects, non-financial non-real economic objects are created *socially* between an economic unit and *the central unit*, this type of places can exist only in the central unit.

Axiom VIII: If flows representing the move of non-financial non-real objects reach places of appearance/disappearance, they are those in the same economic unit where the objects were created, namely the central unit.

Definition XVII: Places of non-appearance/non-disappearance for non-financial non-real economic objects.

Places of non-appearance/non-disappearance for non-financial non-real economic objects are places where non-financial non-real economic objects are not created nor cease to exist. When they are created at places of appearance/disappearance for non-financial non-real economic objects, the flow starts at the central economic unit's place of appearance/disappearance and reaches to a right holder's place of non-appearance/non-disappearance.

Interpretation VII: Non-financial non-real economic objects

Typical example of non-financial non-real economic objects may be copy rights, patents, and trademark rights. As was already discussed in a footnote, purchased goodwill is not counted among non-financial non-real economic objects.

Axiom VII: Flow Rules

Rules concerning flows are as shown in the following figure. That is, admissible incidence between places except for the flows representing balancing items is as shown in the figure.

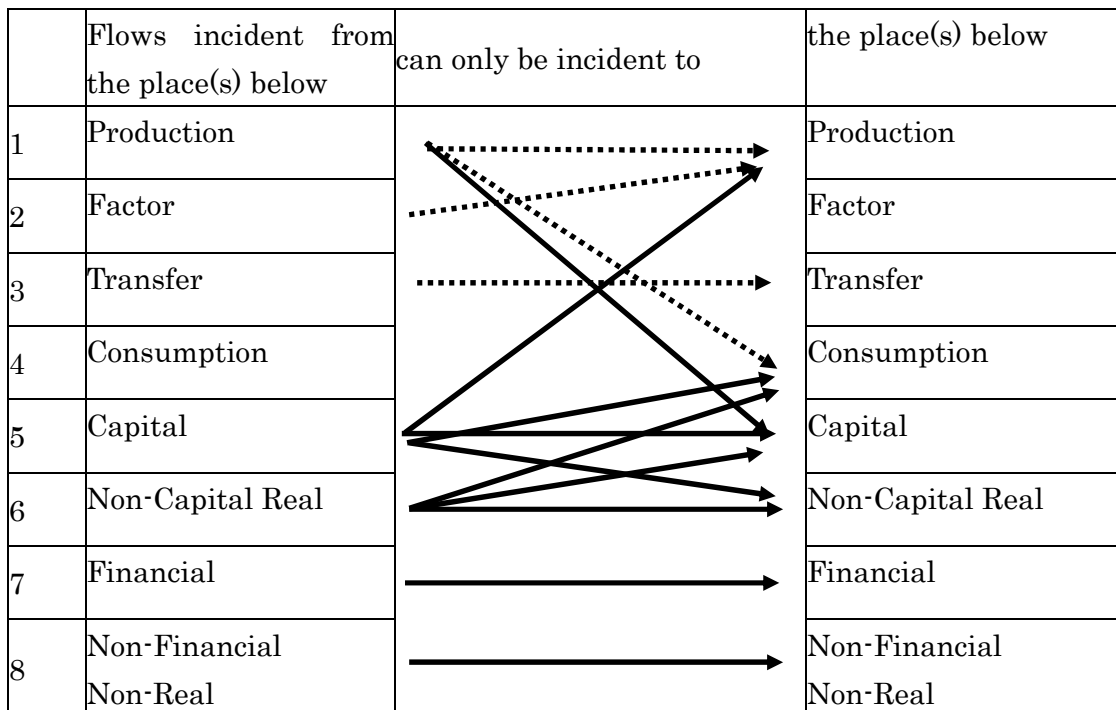


Figure 5-1 Flow Rules

Note to Figure 5-1: Solid arrows denote flows along which objects (real, non-real) are moving while broken arrows denote flows with which no movement of objects is connected like services or transfers. The rows 7 and 8 are somewhat simplified in that places of appearance/disappearance and places of non-appearance/non-disappearance are combined. Balancing items flows are not considered in the above table.

Remark: More detailed flow rules might be needed if places are further subdivided, say, by the type of goods stored.

5. Closing remarks

Axiomatic approaches to national accounting are useful in that national accountants are invited to rethink about what they are doing.

As is well-known, Odd Aukrust, a distinguished Norwegian statistician, made an enormous step forward in this direction some 50 years ago.⁴³ His work was influential not only in the field of national accounting but also in the field of business accounting, among others, Richard Mattessich's axiomatic approach to it.⁴⁴ In this paper we have attempted another axiomatisation of national accounting by using a graph-theoretical framework.

While Aukrust's primary focus was on the objects, real or financial, our approach is rather like Mattessich's in that both of them are "flow system" oriented.⁴⁵

However, in Mattessich's formulation as well as Aukrust's, "duality" requirement seems to be too demanding. Thus, Aukrust's Axiom XVI writes: "A real flow from one sector to another is always associated with a financial contribution in the opposite direction." Our approach is less demanding in that we assume only horizontal double entry is feasible. In other words, our assumption is that the set of places (vertices) and flows (arcs), namely the components of a process digraph can be defined. It does not have any implication on "vertical double entry," the feasibility of which is derived from the Eulerian property we place on the original process digraph through adding to it so called transfer flows.

It should be noted that the meanings of the two kinds of double entry are totally

⁴³ See Aukrust (1966). Prior to the publication in *Review of Income and Wealth*, the original version of his 1966 paper was published in Norwegian as early as 1955 as an appendix to his doctoral thesis.

⁴⁴ Mattessich (1964). His work on the axiomatisation of business accounting had started earlier. In fact, in 1957, he published an article titled "Towards a General and Axiomatic Foundation of Accountancy" (Mattessich, 1957) in *Accounting Research*.

⁴⁵ Also see Stuvell (1966).

different. That is, horizontal double entry helps identify how different economic units (sectors) co-operate, and vertical double entry provides a framework of observing and analysing rational behaviour of different economic units (sectors).⁴⁶

Both types of double entry, vertical as well as horizontal, are necessary because economics is a subject addressed to the analyses of *institutions* surrounding people's lives and they are useful only when people's rational behaviour underpinning the institutions is analysed appropriately in the framework well fit for that purpose.

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⁴⁶ The terms "horizontal double entry" and "vertical double entry" were used in paras.45-47 in the third edition of IMF's *Balance of Payments Manual* released in 1961.

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