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Author(s)
Magne Mogstad and Rolf Aaberge

For additional information please contact:

Author Name(s) : Magne Mogstad
Author Address(es) : Statistics Norway, Kongens gate 6, Postboks 8131 Dep, 0033 Oslo, Norway
Author E-Mail(s): mmo@ssb.no
Author FAX(es): +4721090040
Author Telephone(s): +4721094270

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Restricted Interpersonal Comparability and Measurement of Income Inequality

Magne Mogstad* and Rolf Aaberge
Research Department, Statistics Norway

Abstract:
The standard approach in the applied literature on income distribution is to estimate income inequality under the assumption of full interpersonal comparability of income. In order to be meaningful, this approach requires identical prices and qualities of goods as well as uniform norms and consumption habits across regions. To account for relevant non-income heterogeneity between individuals, we propose to partition the municipalities into subgroups determined by geographic locations and prices of basic goods. Based on income data for each of the subgroups a set of subgroup-specific Lorenz curves and corresponding measures of inequality can be estimated. Moreover, when incomes cannot be justified to be comparable across subgroups a weighted average of the subgroup-specific inequality measures can bee used as an alternative to the conventional overall measure of inequality. Applying Norwegian household register data for the period 1993- 2001, it is demonstrated that the level of and trend in overall inequality as well as the inequality contributions of various income factors are insensitive to whether subgroup-specific Lorenz curves or a country-specific Lorenz curve form the basis of the inequality analysis.

Keywords: Measurement of income inequality, restricted interpersonal comparability, decomposition of inequality by income factors

*Corresponding author:
Magne Mogstad
Research Department, Statistics Norway
Postboks 8131 Dep, 0033 Oslo, Norway
E-mail: magne.mogstad@ssb.no

1. Introduction

An underlying assumption for the meaningfulness of comparing and ranking a set of income distributions according to the degree of inequality is that the assessment carries over to the distributions of well-being. This requires that there must be insignificant interpersonal variations in the conversion of individual incomes into individual well-beings. Otherwise, an equal distribution of income may yield significantly unequal well-being levels, and it becomes hard to justify equality in the income space in terms of distributional justice; the reason is that income is a good that does not have intrinsic value but is important merely as an instrument for individuals to pursue well-being.[1] By and large, the literature on the measurement of income inequality does not, however, consider the implications of non-income differences between individuals beyond accounting for resource sharing and scale economies in the households.

To be meaningful, measurement of income inequality requires pattern of prices of goods that do not vary across regions. Since empirical evidence suggests that prices of basic goods, such as houses, differ significantly between urban and rural areas, the conventional analysis of income inequality based on the distribution of equivalent income within a country might be biased. Accounting for regional variation in consumer prices could be achieved in countries where region-specific price indices are produced. Unfortunately, this type of information is normally not available in the OECD-countries. Furthermore, it seems plausible that differences in observed prices, at least partly, reflect unobserved, and possibly also inherently unobservable, heterogeneity in the quality of goods. If this is the case, an equal distribution of income may yield significantly unequal well-being levels even after adjusting for differences in observed prices. In addition, even in cases where neither the pattern of prices nor the quality of goods varies across regions, norms and consumption habits might turn out to be region-specific. Thus, there might be no perfectly egalitarian income distribution after controlling for heterogeneity in pattern of prices and quality of goods at which the individuals are equally well off. As pointed out in Coulter et al. (1992) and Cowell (1995), there are two possible strategies available for coping with problems of comparability to achieve distributions of income that mirror the distributions of unobserved well-being; either one transforms the income measure by incorporating the relevant non-income heterogeneity and aggregates across individuals, or one uses the observed income data and accounts for non-income heterogeneity at the aggregation stage. In practice, however, the first strategy seems infeasible, since the data requirements are far beyond what is available. It is, therefore, necessary to reconsider the standard approaches conventionally applied at the aggregation state of measuring inequality.

The objective of this paper is to introduce a method that enables us to measure income inequality in a meaningful way when comparability of income between subgroups cannot be justified. Specifically, we propose to partition the municipalities into subgroups determined by geographic location and prices of basic goods. On the basis of the distribution of individual equivalent income for each of the subgroups we estimate a set of subgroup-specific Lorenz curves and corresponding summary measures of inequality. By aggregating inequality measures across subgroups, we get an estimate of overall inequality where the between-group inequality term is excluded. The purpose of this approach is to achieve meaningfulness in the measurement of inequality in a heterogeneous population by restricting the comparison of incomes to individuals who face similar price pattern and quality of goods and share norms and consumption habits, when income is supposed to capture individuals' abilities to achieve well-being. In end effect, the proposed method may contribute to abridging the wide gap between theoretical work on the measurement of inequality presupposing a homogenous population, and their

[1] See Sen (1992) and Roemer (1996) for an assessment of various evaluative criteria that may be employed as measures for judging between alternative social states.

empirical counterparts forced to deal with a population of heterogeneous individuals inhabiting a heterogeneous environment.[2]

The paper is organized as follows. In Section 2, we provide a theoretical justification of the method for measuring inequality on the basis of a set of subgroup-specific Lorenz curves by demonstrating that it may be considered as the second-best solution to the problem of measuring inequality in a heterogeneous population. In Section 3, the method is applied on Norwegian household register data, and estimates of income inequality based on a set of subgroup-specific Lorenz curves as well as a joint country-specific Lorenz curve are compared. In Section 4, policy implications are discussed.

2. Restricted interpersonal comparability and the choice of method for measuring income inequality

When making a distributional assessment on the basis of income we implicitly assume that this assessment mirrors the distribution of well-being. The reason for focusing on the distribution of income is that the distribution of well-being is unobservable, and income is an important and observable mean to achieve well-being. Suppose that there are considerable non-income differences between individuals causing interpersonal variation in the welfare-equivalents of a given amount of income, i.e. the informational basis does not allow full interpersonal comparability of income. Suppose that we are able to partition the population into subgroups by relevant non-income heterogeneity such that the incomes can be considered to be comparable within but not between the subgroups. Relaxing the controversial assumption of homogenous individuals inhabiting a homogenous environment raises two distinct yet interdependent questions. Firstly, is it possible to say something meaningful about the degree of income inequality in the population as a whole when incomes are not comparable between subgroups? Secondly, are the conventional methods for measuring inequality, which presupposes a homogenous population, appropriate when the population consists of heterogeneous individuals inhabiting a heterogeneous environment or is it necessary to develop alternative approaches?

Below, we show that the meaningfulness of a given method for measuring income inequality depends critically on the underlying informational basis. Furthermore, we demonstrate that the method proposed in this paper can be considered as the second-best solution to the problem of measuring inequality in a heterogeneous population. Essentially, the theory of second-best tells us that we cannot blindly apply lessons of first-best economics to real-world problems. Applied to public economics, for example, second-best theory has dramatically altered how we analyse government policies in situations where some of the conditions for Pareto efficiency are not satisfied, typically because relevant household characteristics or behavioural outcomes are unobservable. By contrast, second best considerations have not affected the literature on the measurement of income inequality, despite being haunted by severe problems of measurability and comparability.

Analytical framework

Consider a population of n individuals and define for each person i = 1,2,…, n:

y_i - the equivalent income of person i after adjusting for household size and composition

p_i - the vector of prices facing person i after adjusting for differences in the quality of goods

z_i - the vector of characteristics of person i

Let the distribution of personal well-being be given by the function

(2.1) $$W = W(u_1, u_2, \ldots, u_n),$$

[2] See Foster (1984) for a discussion of the divergence between theoretical and applied work in the measurement of inequality, and Mogstad et al. (2006) for a discussion of the measurement of poverty and problems of comparability.

where $u_i = v(y_i; p_i, z_i)$ is an increasing function of y_i that measures the well-being of individual i for equivalent income y_i conditioned on the prices of goods p_i he is faced with and individual characteristics such as his norms and consumption habits z_i. Inverting u_i yields $y_i = g(u_i; p_i, z_i)$, which measures how much income individual i would need to achieve well-being level u_i at the prices p_i given his individual characteristics z_i. We partition the population into r exhaustive and mutually exclusive subgroups, in which each member has the same z and faces the same p. Let the proportion of the population that belongs to subgroup j be $a_j = \frac{n_j}{n}$, where n_j represents the number of individuals in subgroup j, $j = 1, 2, .., r$. Thus, $\sum_{j=1}^{r} a_j = 1$. If the entire population has the same p and z then $r = 1$. By contrast, if each individual differs in terms of z and/or p, then $r = n$.

2.1. Measuring income inequality in a homogenous population: The conventional approach

Suppose that y is perfectly measurable and fully comparable between each individual of the population. Furthermore, assume that z and p are homogenous across the population. In this benchmark case, a distributional assessment on the basis of y is cardinally equivalent to the distribution of well-being.[3] Let F be the cumulative distribution function of y whose mean is μ, and let L be the Lorenz curve. Provided that one chooses an inequality measure that is decomposable, the total inequality in the population can mechanically be expressed as a function of inequality within and between/across subgroups

(2.2) $$I = \sum_{j=1}^{r} w_j I_j + R$$

where I is a measure of overall inequality, I_j is a measure of inequality in subgroup j, w_j is the weight attributed to subgroup j, and the term R is supposed to capture inequality between/across subgroups.[4] In the underlying assumption of full interpersonal comparability of income is satisfied, (2.2) will not only provide a meaningful summary measure of overall inequality in a population, but also enable us to study the relationship between overall inequality and inequality within and between/across subgroups of the population formed by e.g. gender or region of residence. By and large, the theoretical as well as the empirical literature on the measurement of inequality is based on this approach. This implies that unless the population of study consists of homogenous individuals inhabiting a homogenous environment, the conventional analyses of income inequality based on the distribution of equivalent income within a country might be biased.

2.2. Measuring income inequality in a heterogeneous population: A first-best approach

[3] Formally, two measures are cardinally equivalent if the value of one measure can be obtained from the other by multiplying a positive constant and adding or subtracting another constant (Cowell, 1995).

[4] For a strict statistical decomposition, the between-group inequality depends only on group means and the within-group inequality depends only on group inequality measures (Das and Parikh, 1982). As opposed to the inequality measures that are additively decomposable, the so-called generalised entropy family of inequality measures, the Gini-coefficient does not admit strict statistical decomposition into within- and between-group components but does also require an overlapping term. More on the subgroup decomposition issue, see e.g. Rao (1969), Shorrocks (1980, 1984), Cowell (1980, 1988), Das and Parikh (1982), and Foster and Shneyerov (1999).

Empirical evidence suggests that the prices of basic goods, such as houses, differ significantly between urban and rural areas. Thus, a given amount of income will give greater consumption possibilities in areas with low housing prices than in areas with high housing prices. Furthermore, one could also question whether the norms and consumption habits of individuals apply broadly to the entire country or differ according to region of residence. Arguably, individuals' commodity requirements depend on the local environment they inhabit as well the reference group's circumstances, which presumably are heavily influenced by the community to which one belongs. If one agrees with Sen (1984) that there is significant variability in the commodity requirements within a given country, then the levels of well-being individuals may achieve for a given amount of income may vary depending on their region of residence even when price patterns and quality of goods across regions are identical. This implies that $y_i = g(u_i; p_i, z_i)$ may differ from $y_k = g(u_k; p_k, z_k)$ even if $u_i = u_k$.

Suppose that *y*, *z* and *p* are perfectly measurable and fully comparable though heterogeneous across the population. If we know *g*, we may transform the incomes by incorporating the relevant non-income heterogeneity. Let $y^*_i = m(y_i; p_i, z_i)$ represent the transformed income of individual *i*. On the basis of the distribution of y^*, an assessment of the overall inequality and inequality within and between subgroups of the population based on (2.2) would carry over to the distribution of well-being. In fact, $u_i \geq u_k$ if and only if $y^*_i \geq y^*_k$. The transformation of the incomes so that they are representative of well-beings may be considered as the first-best solution to the problem of measuring inequality in a heterogeneous population. Unfortunately, this first-best solution/approach appears to have as modest practical relevance to research in the field of income distribution of as first-best considerations have in public economics; just as the conditions for Pareto efficiency hardly ever are satisfied, the informational requirements necessary to incorporate non-income heterogeneity by transforming incomes are far beyond what usually is available. The lesson is the same; we have to rely on second-best analysis.

2.3. Measuring income inequality in a heterogeneous population: A second-best approach

In practice, we will typically have micro data on *y*, but not on *p* and *z*. However, we may have access to aggregate data on *p* and *z* for subgroups of the population. For example, we may know that the prices of key goods differ substantially between two regions, but the exact price differentials and the qualities of the goods are unobservable. On the basis of these informational assumptions, we are unable to make a transformation of the incomes so that they are representative of levels of well-being. Nevertheless, we might be able to divide the population into r subgroups, in which each individual has approximately the same *z* and *p*. Let F_j be the cumulative distribution function of equivalent income in subgroup j whose mean is μ_j, and let L_j be the Lorenz curve of subgroup j, $j = 1,2,\ldots,r$. On the basis of the subgroup-specific Lorenz curves one may compare and rank alternative distributions of income for each of the subgroups. The reason is that the population of each subgroup consists of identical individuals in every relevant aspect other than income. If the curves belonging to each subgroup do not cross, the distributions can be ranked without ambiguity. By introducing subgroup-specific inequality measures, and specifying the sensitivity to different parts of the income distribution, one may obtain a complete ordering over the set of possible distributions of equivalent income for each subgroup. Since there is non-comparability of incomes between the subgroups, the subgroup aggregation in (2.2) is, as pointed out in Coulter et al. (1992) and Cowell (1995), contentious. The problem is twofold:

- The measure of between-group inequality is intrinsically meaningless.

- The weights of the within-group inequalities, which conventionally depend on the shares of each subgroup in total income, are no longer appropriate.

Although measured inequality within a subgroup can be seen as reflecting a genuine disparity among individuals' abilities to achieve well-being, the between/across-group measure may simply reflect the fact that prices and individual characteristics vary between the subgroups. The reason is that non-comparability between subgroups implies that inequality in incomes between/across subgroups has no informational value for assessing total inequality; we risk comparing apples with pears. Thus, R cannot be interpreted as a measure of inequality in well-being between/across groups. Furthermore, the weighting scheme of within-group inequalities cannot be based on shares of total income, simply because income are not comparable across subgroups. Thus, it is necessary to introduce an alternative weighting scheme that does not involve subgroup-specific average incomes in order to aggregate the inequality across the subgroups. As an alternative, we propose to determine the weights according to the population shares of the subgroups. By inserting a_j for w_j and dropping R in (2.2), the inequality in the population as a whole can be expressed exclusively as a weighted sum of inequality within the subgroups

(2.3) $$\tilde{I} = \sum_{j=1}^{r} a_j I_j$$

Arguably, the primary problem of measuring income inequality is not of the kind of constructing inequality measures from fundamental properties they presumably ought to have in the conventional context of a population of homogenous individuals inhabiting a homogenous environment; it is a problem of the meaning that we can give to the measures that we choose to employ depending on the underlying informational assumptions.[5] This relates to Rawls (1971), who argues that methods for evaluating alternative social states should be judged by a principle referred to as reflective equilibrium. This principle postulates that the soundness of a given method for social evaluation should not only be evaluated by the ethical and technical conditions on which it is built, but also on its consequences in specific environments. Drawing on the principle of reflective equilibrium it follows that one needs to move away from sole focus of scrutinizing the conditions on which the different inequality measures are built on to addressing the question of 'what are the actual implications in specific environments of employing a given inequality measure and how does these implications connote to our intuitive notion of justice?'. This paper argues that if incomes are comparable within but not between certain subgroups of the population then the overall inequality measure cannot include between/across-group terms. Consequently, the measure of total inequality in (2.3) is bound to violate fundamental properties conventionally used to solve the choice-of-index problem. However, the relevance of axiomatic results depends entirely on the meaningfulness of its constituent properties. Thus, the choice-of-index problem should be solved subject to realistic informational conditions, which presumably involve heterogeneous individuals inhabiting a heterogeneous environment. For example, an underlying assumption for the meaningfulness of imposing the Pigou-Dalton transfer principle, which requires an inequality index to increase when income is transferred from poor to rich, is that we can distinguish the rich from the poor in the population of study. In the presence of non-comparability between subgroups it does not make sense to impose that $\tilde{I}$ should comply with this transfer principle. By contrast, such an axiom should be satisfied for the within-inequality indices. In comparison, requiring that $\tilde{I}$ satisfies the axioms of invariance of scale and population size does not call for income being fully comparable between each individual of the population.

[5] For axiomatic characterisation of the Theil index, the Generalised Entropy family, and the Gini-coefficient see Foster (1983), Shorrocks (1984), and Thon (1982) and Aaberge (2001), respectively.

3. The sensitivity of income inequality estimates to the choice between subgroup-specific and country-specific Lorenz curves

The objective of this section is to explore the effects on income inequality estimates when using a set of subgroup-specific Lorenz curves rather than a joint country-specific Lorenz curve. The informational basis for the empirical analysis is a register household panel data set covering the entire resident population of Norway from 1993-2001, which is supplemented with detailed income data from the Tax Assessment Files. Furthermore, we use income after tax as the focal variable for the empirical analysis of income inequality. Income after tax, which is defined in close agreement with international recommendations (e.g. Expert Group on Household Income Statistics, 2001), incorporates earnings, self-employment income, capital income, public cash transfers and taxes. To enable the comparison of incomes across individuals belonging to households of varying size and composition the standard OECD equivalence scale is applied, for which the weight of the first adult is set to 1, additional adults are given weights of 0.7, and each child gets a weight equal to 0.5.

3.1. Partitioning the population into subgroups

In order to make the proposed method for measuring inequality on the basis of a set of subgroup-specific Lorenz curves operational, it is necessary to partition the population into subgroups such that the incomes within each subgroup can be justified as comparable. To account for non-income heterogeneity in the population of Norway, it appears relevant to classify its 435 municipalities according to their regional location. Since the level of housing costs is the main expenditure for most households, we will use housing prices as a second classifying variable. Specifically, we partition the municipalities into housing price quartiles according to their average housing price per square meter.[6] This is possible since data on prices per square meter for houses sold in each municipality are available.[7] Next, we partition the municipalities into three subgroups corresponding to the quartiles they belong to; the 1st quartile is labelled *low housing prices*, the 2nd and 3rd quartiles *medium housing prices,* and the 4th quartile *high housing prices*.

By combining the three housing price categories with seven regions the municipalities are, as illustrated in Table 1 for the years 1993, 1997, and 2001 (in the Appendix), partitioned into 21 subgroups. On the basis of the distribution of individual equivalent income for each of the subgroups a set of subgroup-specific Lorenz curves and corresponding Gini-coefficients can be estimated. As expected we find a positive association between the mean income level and housing prices of the municipalities. This relationship arises because the capacity of individuals to purchase goods, such as housing, depends on the level of resources of the other individuals around them through the geographic pattern of competition. This makes it likely that housing prices, just as prices of other normal goods, increase with the general income level in a municipality. The implication is that the consumption potential of a given amount of income differs systematically within the country, implying non-comparability of incomes between the subgroups.

Insert Table 1 here

The argument for relying on a set subgroup-specific Lorenz curves is to ensure meaningfulness in the measurement of income inequality by restricting the comparison of equivalent incomes to individuals

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[6] In this paper, we will group the municipalities according to real estate prices. One could argue that rental prices would be a more appropriate classifying variable for identifying poverty thresholds. However, detailed data on local levels of rental prices are not available on a regular basis in Norway. Moreover, most people in Norway are owners rather than renters. Furthermore, Norwegian data show that the geographic pattern for real estate prices is relatively stable and remarkably similar to the geographic pattern for rental prices (Langsether and Medby, 2004).

[7] Source: Statistics Norway, Division for Construction and Service Statistics.

belonging to the same subgroup of municipalities. Hence, one avoids comparing the incomes of individuals from municipalities with high housing prices with that of individuals from municipalities with relatively low housing prices, even if these municipalities are neighbours. For example, the urban municipality of Trondheim with high housing prices will not belong to the same subgroup of municipalities as its rural neighbouring municipality Agdenes where housing prices are relatively low. By contrast, the standard approach ignores the comparability problems by measuring inequality on the basis of the distribution of equivalent for the entire population. Hence, one implicitly makes the contentious assumption that all individuals within a country face the same price patterns and qualities of goods and have identical norms and consumption habits (once differences in scale economies in consumption are taken into account by the equivalence scale that is selected).

Obviously, there are price differentials on other goods than housing that matter when we compare the consumption potential of the income of different individuals. However, this will only be an argument against the proposed classifying procedure if there is greater variation in the price of the respective good within a subgroup of municipalities than across the subgroups. Furthermore, even if one suspects that there are certain price differentials within a country which are incompatible with the pattern of housing prices, it is necessary to keep in mind the serious drawback of the conventional method of measurement where non-income heterogeneity in the population of study is simply ignored.

3.2. Choice of Inequality Measure and Decomposition by factor components

To summarize the informational content of the Lorenz curve and to achieve rankings of intersecting Lorenz curve, the conventional approach is to employ the Gini-coefficient in combination with measures from the Atkinson or Theil family. However, the Gini-coefficient and inequality measures from the Atkinson or the Theil family have distinct theoretical foundations making it inherently difficult to evaluate their capacities as complimentary measures of inequality. Thus, we have examined the sensitivity of the empirical results to the choice of inequality measure by using two close relatives to the Gini-coefficient.[8] The results show that the qualitative implications of this paper are robust to the choice of inequality index within the Lorenz family of inequality measures.[9] On the basis of (2.3), it is clear that the Gini-coefficient for overall inequality $\tilde{G}$ in the population based on the set of subgroup-specific Lorenz curves is defined as the weighted average of the subgroup-specific Gini-coefficients

(3.1) $$\tilde{G} = \sum_{j=1}^{r} a_j G_j$$

where a_j is group *j's* population share and $G_j = \frac{1}{2n_j \mu_j} \sum_{i=1}^{n_j} \sum_{k=1}^{n_j} | y_i - y_k |$ is the measure of inequality within subgroup j = 1, 2, …, r . By contrast, when full interpersonal comparability can be justified the Gini-coefficient for overall inequality is given by

(3.2) $$G = \frac{1}{2n\mu} \sum_{i=1}^{n} \sum_{k=1}^{n} | y_i - y_k | .$$

Note that G admits the following decomposition

(3.3) $$G = \tilde{G} + R$$

[8] See Aaberge (1999, 2000).

[9] The estimates of inequalities based on two close relatives of the Gini-coefficient may be provided by the authors upon request.

where $R = G - \tilde{G}$ is a term that captures inequality between as well as across subgroups.[10] The above decomposition is attractive since it provides a direct link between $\tilde{G}$ and G, and allows us to straightforwardly determine the contribution of $\tilde{G}$ to G.[11]

A major topic of interest in the literature on income distribution concerns the way various income factors contribute to the inequality in total income. For example, the policymaker may be interested in assessing the extent to which overall inequality is due to earnings or capital income, or the redistributive nature of taxes and transfers. An objective of this paper, is to examine to what extent results from a decomposition analysis by income factors depend on the choice between using a set of subgroup-specific Lorenz curves or a joint country-specific Lorenz curve.

Let there be *k = 1,2, … s* mutually exclusive and exhaustive sources of income. The Gini-coefficient for overall inequality in the population of income after tax based on a country-specific Lorenz curve may then, following Rao (1969), be expressed as the weighted average of the factor concentration coefficients with factor income shares as weights

$$G = \sum_{k=1}^{s} \beta_k \tag{3.4}$$

and the inequality share β_k of income factor k is given by

$$\beta_k = \frac{\mu_k}{\mu} \alpha_k \tag{3.5}$$

where the income share μ_k/μ is the ratio between the means of the cumulative distribution function of income after tax and income factor k and the factor concentration coefficient α_k can be interpreted as the conditional Gini-coefficient of factor k given the rank order in income after tax. Thus, a negative value of α_k implies that income factor k gives an equalizing contribution to overall inequality in the population of income after tax. If $\alpha_k = 0$ each individual receives an equal amount of income factor k. The product of the income share and the concentration coefficients is denoted the inequality contribution of factor k to overall inequality in the population of income after tax based on a joint country-specific Lorenz curve, β_k.[12]

A similar factor decomposition of the Gini-coefficient $\tilde{G}$ defined by (3.1) is obtained by employing (3.4) for each of the subgroup-specific Gini coefficients,

$$\tilde{G} = \sum_{k=1}^{s} \tilde{\beta}_k \tag{3.6}$$

and

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[10] See Rao (1969) for an alternative expression for *R*.

[11] Alternative subgroup decompositions of the Gini-coefficient are proposed in Bhattacharya and Mahalanobis (1967), Pyatt (1976) and Aaberge et al (2005). More on the derivation and interpretation of the subgroup decomposition of the Gini-coefficient, see Das and Parikh (1982), Silber (1989), Yitzhaki and Lerman (1991), Lambert and Aronson (1993), Yitzhaki (1994) and Dagum (1997).

[12] Note that the decomposition method defined in (3.4) provides a simultaneous treatment of the income factors in question. This implies that we focus on the contributions from the various income factors to the observed overall income inequality. By contrast, the elasticity approach for decomposing the Gini-coefficient by income sources proposed by Lerman and Yitzhaki (1985) concerns the effect of a marginal change in an income component. See also Fei et al. (1978), Pyatt et al. (1980), Kakwani (1980), Silber (1989, 1993), and Yao (1997) for alternative decomposing methods of the Gini-coefficient.

(3.7) $$\tilde{\beta}_k = \sum_{j=1}^{r} \frac{\mu_{jk}}{\mu_{j+}} \alpha_{jk}$$

where μ_{jk}/μ_{j+} is the ratio between the means of the cumulative distribution function in subgroup j of income after tax and income source k respectively, which is denoted the income share of factor k in subgroup j. The subgroup-specific factor concentration coefficient α_{jk} can be considered as a measure of the interaction between income factor k and income after tax in subgroup j. By taking the sum of the products of the subgroup-specific income shares and concentration coefficients, we get the inequality contribution of factor k to overall inequality in the population of income after tax, $\tilde{\beta}_k$.

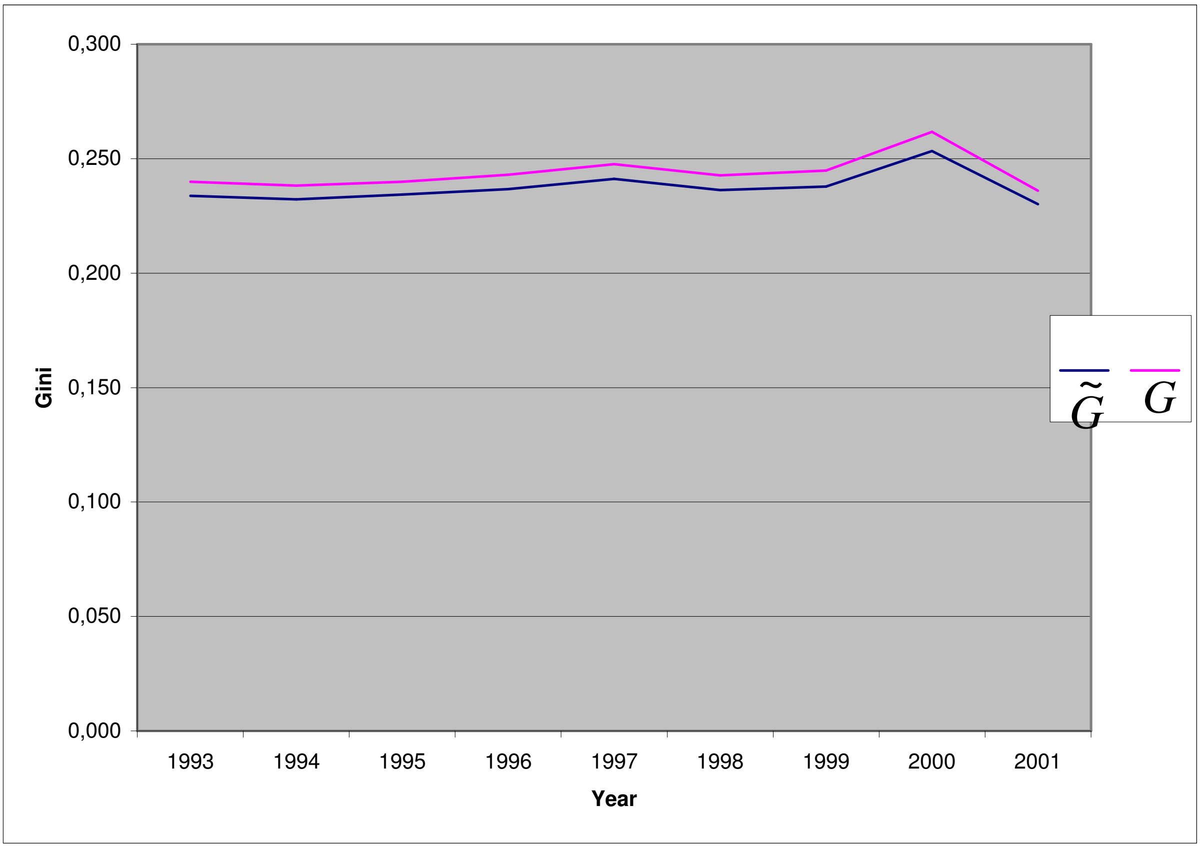
3.3. Empirical results

Below, we explore the effects on income inequality estimates when using a set of subgroup-specific Lorenz curves rather than a joint country-specific Lorenz curve. Figure 1 shows Gini-coefficients based on subgroup-specific Lorenz curves as well as a joint country-specific Lorenz curve. The results demonstrate that the level of and trend in overall inequality in Norway for the period 1993- 2001 is remarkably robust to the choice of method for measuring income inequality.[13] The reason is that the contribution of $\tilde{G}$ to G, both cross-sectionally and to its trend over time, is substantially more important than the contribution of between-group inequalities. In fact, the percentage difference between G and $\tilde{G}$ is no more than about 3 percent.

Measures of inequality, such as the Gini-coefficient, do not, however, offer an immediate interpretation that clarifies the significance of a certain change in inequality. In general, neither the numerical values of inequality measures nor the numerical values of changes in inequality measures have any straightforward meaningful interpretation themselves, and are primarily used as means to compare and order distributions by degree of inequality. Thus, a method for quantifying the economic implications of the observed difference in inequality is required to draw conclusions about the actual impact of letting subgroup-specific Lorenz curves rather than a joint country-specific Lorenz curve form the basis for the analysis. To this end, we apply the method outlined in Aaberge (1997), which introduces a hypothetical tax/transfer intervention that redistributes a constant amount of income so as to make the value of the Gini coefficient for a specific distribution of income equal to the Gini coefficient of a different distribution of income. Specifically, the 3 percent difference between G and $\tilde{G}$ corresponds to introducing a proportional tax with a tax rate equal to 3 percent followed by redistributing the collected tax revenue as equal-sized amounts equal to 3 percent of the mean income. Such an intervention would, of course, leave the mean income unchanged. This hypothetical intervention method enables us to interpret the impact of using subgroup-specific Lorenz curves rather than a joint country-specific Lorenz curve as economically insignificant on the level of and trend in income inequality in Norway.

[13] The level of and trend in income inequality presented in Figure 1 correspond, by and large, with results from other studies of income inequality in Norway in the 90s, such as Fjærli and Aaberge (1999) and Aaberge et al. (2000). Notice that the rise in income inequality in 2000 coincides with a major tax reform that affected the financial incentives in the corporate sector and the income shifting incentives in small enterprises.

Figure 1. Gini-coefficients based on a country-specific Lorenz curve (G) and a set of subgroup-specific Lorenz curves ($\tilde{G}$), 1993-2001



Although the level of and trend in inequality in Norway appear to be robust to whether estimates of income inequality are based on subgroup-specific or country-specific Lorenz curves, it is far from obvious how a decomposition analysis by income factors will be affected by this methodological choice. Figure 2 shows the contribution of various income factors to overall inequality based on a set of subgroup-specific Lorenz curves. As expected, earnings are shown to be the dominating income component with a clear disequalising effect on the distribution of income after tax. Taxes on the other hand have a strong equalising contribution. From Figure 3, it is clear that the inequality contributions of various income factors are remarkably robust to whether subgroup-specific or country-specific Lorenz curves form the basis for the analysis. Specifically, the results suggest that decomposition analysis based on a country-specific Lorenz curve rather than a set of subgroup-specific Lorenz curves will slightly overestimate the impact of capital income, transfers, and earnings on overall inequality. In comparison, the contributions from taxes and income from self-employment will be somewhat underestimated.

Figure 2. Decomposition of Gini-coefficients based on a set of subgroup-specific Lorenz curves by income factors, 1993-2001

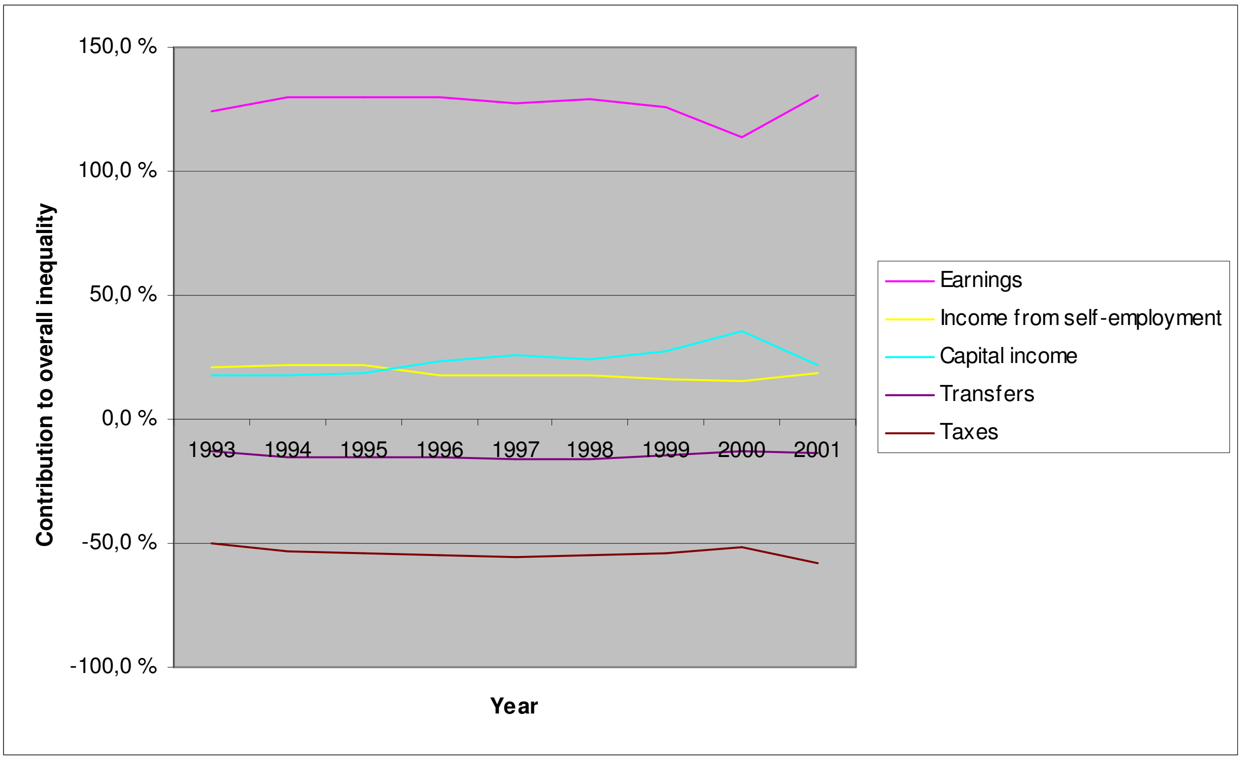
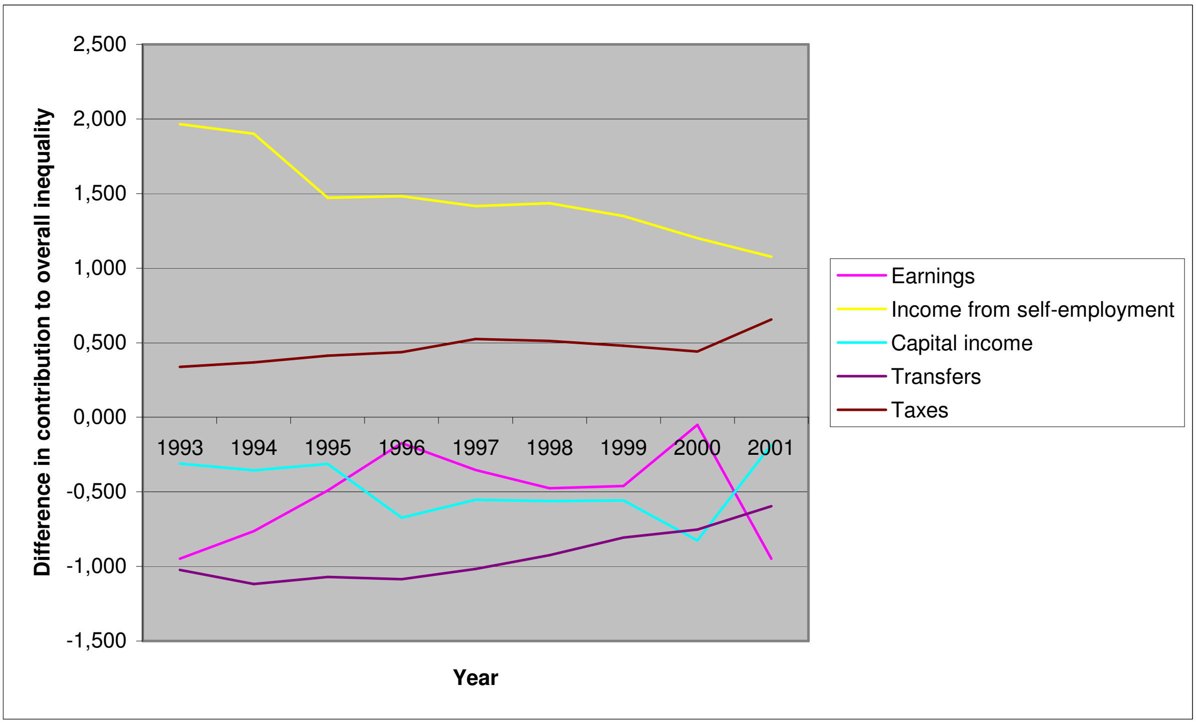


Figure 3. Difference in contribution from various income factors to Gini-coefficients based on a set of subgroup-specific Lorenz curves and a country-specific Lorenz curve, percentage points, 1993-2001

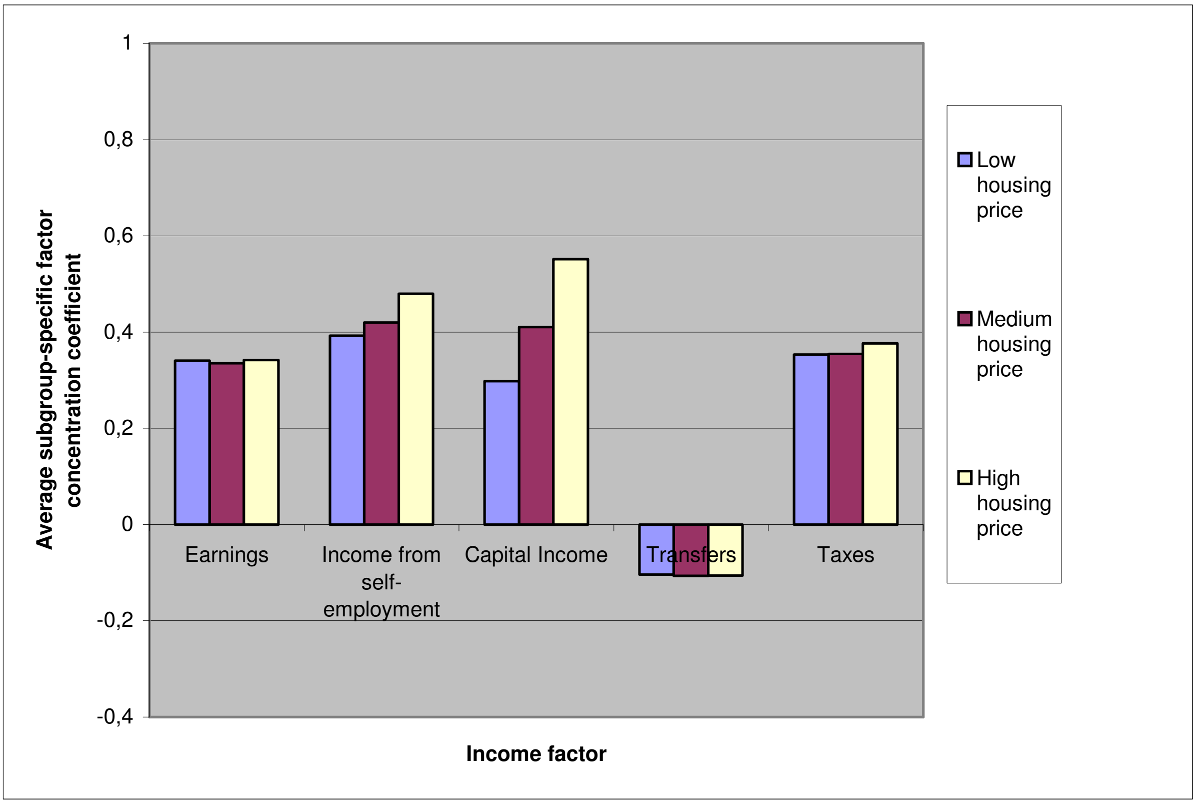


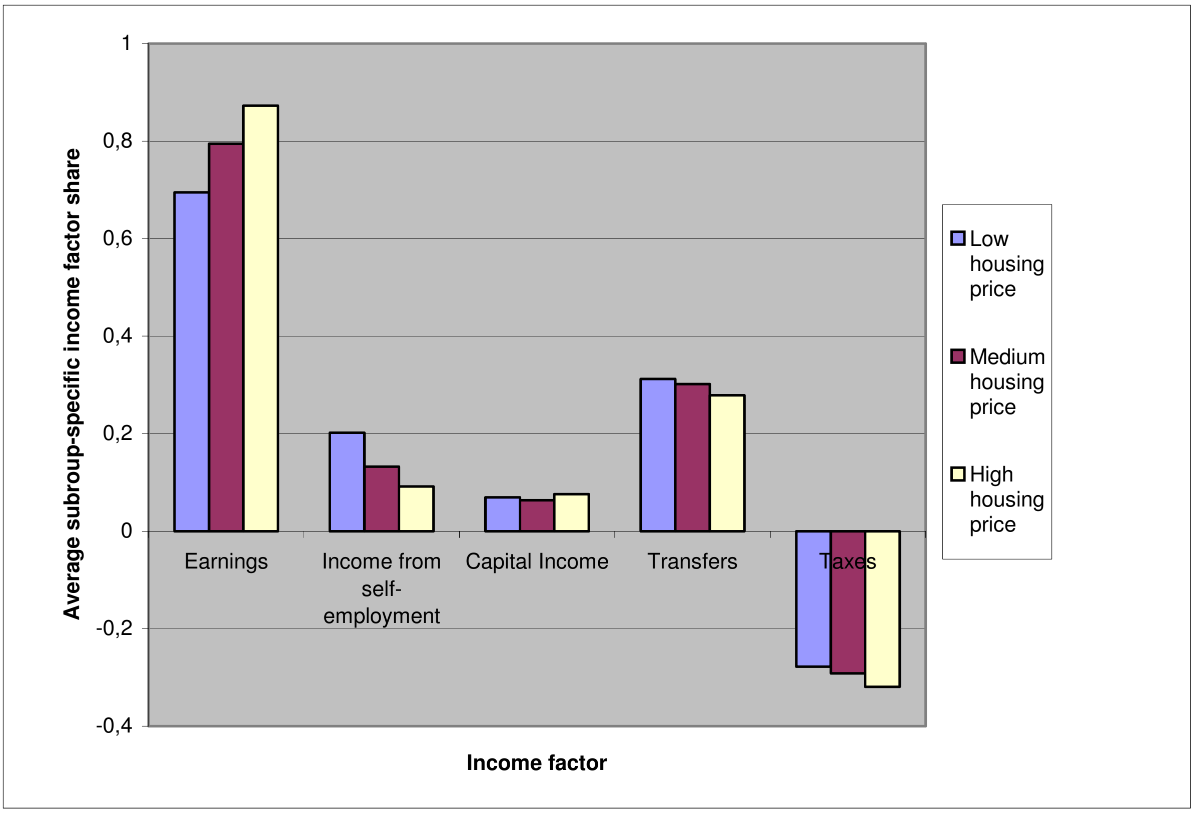
The Gini-coefficients for overall inequality in the population are functions of inequality within the subgroups. Thus, an inspection of inequality in the subgroups of the population, and not only the aggregate, may complement the picture of income distribution drawn above. The results presented in Table 2 (in the Appendix) show that the subgroup consisting of Oslo, the capital, and most of its surrounding municipalities has, in each year, the highest income inequality. In particular, earnings and capital income appear to be particularly unequally distributed in this subgroup compared to the other subgroups, while taxes appears to have a relatively weak equalising contribution.

Insert Table 2 here

The results presented in Table 2 (in the Appendix) also reveal a strong relationship between average housing prices and the degree of inequality in the subgroups. A plausible explanation is that favourable labour market conditions in a subgroup may not only inflate its general income level and housing prices through the geographic pattern of competition but also increase inequality within the subgroup. The reason is that the incomes of individuals at the upper parts of the income distributions are likely to be relatively sensitive to geographic variation in economic activity compared to the incomes of individuals at the lower parts of the income distributions; while benefit rates are, by and large, uniform across regions, market incomes will be relatively sensitive to geographic variation in economic activity. In fact, the evidence presented in Figure 4, which shows the relationship between housing prices and subgroup-specific income factor shares and factors concentration curves in the year 1993, supports this explanation. Specifically, Table 3 demonstrates that capital income and income from self-employment are, on average, relatively unequally distributed in subgroups with high housing prices. Furthermore, the results illustrate that although earnings are not particularly unequally distributed when housing prices are high, its income share is, on average, relatively high. Thus, earnings will have a strong disequalising contribution in subgroups with high housing prices. By contrast, the income share of transfers and its concentration curve appear to be especially stable across subgroups with different prices on housing.

Figure 3. Average subgroup-specific income factor shares and factor concentration coefficients by housing prices, 1993





4. Policy implications

While theoretical work on the measurement of income by and large presupposes full comparability of incomes, their empirical counterparts are forced to deal with comparability problems along several dimensions such as time, space, and income receiving unit. To design and evaluate redistribution programs it is necessary for practitioners to provide an understandable picture of income distributions even when comparability of incomes is restricted. The widespread use of more or less justifiable equivalence scales - to enable comparison of incomes across individuals belonging to households of varying size and composition - illustrates the strive for interpersonal comparability in empirical assessments of income distributions. Since the conditions of identical prices and qualities on goods as well as uniform norms and consumption habits across regions are usually not fulfilled, the conventional analysis of income inequality based on the distribution of equivalent incomes for the entire population of a country may nevertheless be biased. It is thus important to go beyond imposing equivalence scales and introduce methods for measuring inequality accounting for heterogeneity also in the spatial dimension.

In this paper, we introduce a method that enables us to measure income inequality in a meaningful way when comparability of income between subgroups cannot be justified. This method relies on the distributions of individual equivalent incomes for a set of subgroups determined by geographic locations and prices of basic goods, as a basis for specifying subgroup-specific Lorenz curves. Applying Norwegian household register data for the period 1993- 2001, we find that the level of and trend in overall inequality as well as the inequality contributions of various income factors are remarkably robust to whether subgroup-specific or conventional country-specific Lorenz curves form the basis for the analysis. Consequently, estimates of inequality based on a country-specific Lorenz curve can be argued to provide reliable guidelines for economic policy, at least in the setting of a Scandinavian welfare state characterised by generous cash benefits, comprehensive public sector, and centralised wage setting which presumably contributes to equalising the distribution of incomes across subgroups. A next step would thus be to examine the sensitivity of inequality estimates in other OECD-countries to the choice between subgroup-specific and country-specific Lorenz curves.

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Table 1. Allocation of municipalities in Norway by housing prices and region, 1993, 1997, and 2001

Region	Housing prices	1993		1997		2001	
		No. of municipalities	Mean equivalent income (NOK)	No. of municipalities	Mean equivalent income (NOK)	No. of municipalities	Mean equivalent income (NOK)
Oslo and its surrounding municipalities	Low	-	-	-	-	-	-
Oslo and its surrounding municipalities	Medium	4	120312	2	136322	1	171392
Oslo and its surrounding municipalities	High	19	144708	21	170541	22	210557
Eastern Norway	Low	12	106789	15	122389	9	155453
Eastern Norway	Medium	30	111974	29	130749	33	165202
Eastern Norway	High	6	117612	4	139991	6	175793
South Eastern Norway	Low	15	111174	15	132397	7	162711
South Eastern Norway	Medium	27	117630	23	137049	29	172876
South Eastern Norway	High	34	122721	34	145506	36	181628
South Western Norway	Low	20	111777	19	129031	15	164737
South Western Norway	Medium	16	113066	15	131329	22	167820
South Western Norway	High	20	127242	22	147877	19	184397
Western Norway	Low	40	111963	44	132840	31	166016
Western Norway	Medium	40	114161	38	136099	50	171581
Western Norway	High	18	125465	16	147242	17	182380
Mid Norway	Low	19	107200	21	126311	16	153984
Mid Norway	Medium	22	111115	24	128836	28	161634
Mid Norway	High	8	123286	4	146788	5	180378
Northern Norway	Low	51	112750	54	129805	54	162940
Northern Norway	Medium	30	116109	27	133055	31	168065
Northern Norway	High	8	127131	8	143522	4	181759
Norway		*439*	*124499*	*435*	*146171*	*435*	*183081*

Table 2. Gini-coefficients based on a set of subgroup-specific Lorenz curves by housing prices and region, 1993-2001

		Gini-coefficient								
Region	**Housing prices**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**
Oslo and its surrounding municipalities	Low	-	-	-	-	-	-	-	-	-
Oslo and its surrounding municipalities	Medium	0,219	0,210	0,220	0,214	0,213	0,210	0,205	0,195	0,195
Oslo and its surrounding municipalities	High	0,280	0,276	0,279	0,289	0,291	0,285	0,290	0,320	0,270
Eastern Norway	Low	0,210	0,212	0,206	0,207	0,214	0,202	0,207	0,207	0,204
Eastern Norway	Medium	0,217	0,214	0,212	0,213	0,216	0,211	0,208	0,218	0,209
Eastern Norway	High	0,219	0,220	0,219	0,217	0,218	0,213	0,215	0,227	0,212
South Eastern Norway	Low	0,206	0,212	0,222	0,225	0,230	0,226	0,263	0,239	0,204
South Eastern Norway	Medium	0,220	0,219	0,221	0,218	0,222	0,218	0,216	0,232	0,215
South Eastern Norway	High	0,232	0,231	0,236	0,234	0,238	0,232	0,236	0,249	0,228
South Western Norway	Low	0,206	0,211	0,207	0,205	0,214	0,221	0,213	0,215	0,209
South Western Norway	Medium	0,210	0,210	0,211	0,212	0,209	0,208	0,210	0,218	0,211
South Western Norway	High	0,244	0,244	0,244	0,242	0,249	0,249	0,243	0,259	0,234
Western Norway	Low	0,207	0,203	0,211	0,213	0,220	0,210	0,206	0,223	0,207
Western Norway	Medium	0,208	0,211	0,213	0,216	0,219	0,210	0,217	0,221	0,211
Western Norway	High	0,234	0,231	0,235	0,238	0,239	0,236	0,235	0,253	0,228
Mid Norway	Low	0,197	0,193	0,196	0,196	0,211	0,194	0,190	0,199	0,193
Mid Norway	Medium	0,201	0,202	0,203	0,208	0,207	0,202	0,201	0,205	0,202
Mid Norway	High	0,223	0,220	0,222	0,222	0,235	0,227	0,226	0,236	0,222
Northern Norway	Low	0,209	0,211	0,210	0,212	0,211	0,209	0,207	0,219	0,209
Northern Norway	Medium	0,212	0,211	0,207	0,208	0,209	0,208	0,206	0,218	0,206
Northern Norway	High	0,216	0,211	0,216	0,215	0,217	0,212	0,213	0,223	0,222
Norway		*0,234*	*0,232*	*0,234*	*0,237*	*0,241*	*0,236*	*0,238*	*0,253*	*0,230*