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**Intergenerational Educational Attainment Mobility and
Family Structure.**

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Intergenerational Educational Attainment Mobility and Family Structure.

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Very Preliminary

Intergenerational mobility is addressed through an examination of the role of family structure in the transmission of educational attainment within a family using the one percent Integrated Public Use Microsample Series (IPUMS) of the decennial Census for the decades 1970 and 1990. Two alienation measures, which test the degree to which children from different family structures differ in attainment, indicate that although children from intact family consistently outperform their peers from single parent families, this differential has shrunk between 1970 and 1990. They also indicate improvements in attainment for endogenously (divorced and separated) relative to exogenously (widowed) single parent children, suggesting changes in custody laws over the 1980s had a desirable impact on children from endogenously single parent families. Welfare implications based upon Stochastic Dominance tests accord with the above findings. Assuming a quadratic human capital production technology, we found that income effects are smaller in 1990 than they were in 1970, but the returns to parental educational status increased over the same period. These observations are similar for both intact and endogenously single parent families. Further, there seems to be a convergence in technology between intact and single parent families, evidence of a trend towards equal opportunity for all children. Finally mobility indices are developed which indicate that mobility in the generational transition has improved for all but exogenously single parent family types.

Introduction.

Interest in the relationship between economic growth and inequality has spawned a considerable literature on the issue of intergenerational income mobility (for a survey and recent literature see Corak (2004) and references therein). At the extreme the question is, do agents in subsequent generations match the location of their forebears in their respective distributions of wellbeing, or are the respective generational wellbeing distributions independent? When viewing children as investments in the future, the degree of intergenerational mobility can be seen as a long term consequence of the levels of human capital inherited and augmented in childhood. This makes the family an integral component of the technology that transmits the capacity for wealth creation between generations. If current generations do influence future generations what is the role, if any, of the family type in this process. The issue speaks to the existence or otherwise of “dynastically” rich and poor segments in a society and is of great interest to policy makers concerned with childhood poverty (Brewer et. al (2004)), notions of equal opportunity (Roemer (1998, 2002, 2004)) and, as we hope to convince the reader, family and custodial law issues (Leo (2005)).

Practically, intergenerational relationships have been examined empirically via correlations between parent and child incomes and by looking at the child’s empirical income distribution and its parent’s empirical income distribution as subsequent realizations of a Markov chain process. In the former case Solon (2004) has modified the Becker-Tomes (1979) model in order to present a theoretical framework for interpreting log – linear intergenerational income regressions. He develops a one period child - one period parent model with a parental utility function separable in parent’s consumption and child’s future income and a technology which translates the parent’s and governments investment in the child and the child’s endowed human capital into future income (also in a fashion separable in investments and endowments). The model produces a rich set of predictions. Steady state intergenerational income elasticity increases with the heritability of income related traits, the efficacy of human capital investment and the earnings return to human capital and it decreases with the progressivity of public investment in human capital. In the latter case, the transition matrix of the Markov Chain can be used to interpret the degree of mobility, with complete mobility the columns would be identical with complete immobility the leading diagonal would equal 1 (see for example Blanden et.al.(2004)).

Here the influence of different types of parental arrangement on the structure of this relationship is considered, in particular we examine the relative importance of endowed traits and parental incomes in the human capital augmenting process and whether or not different parental arrangements imply different types of transmission process. Ideally in exploring this relationship empirically, following Solon’s model, one would wish to link the child’s permanent income at a particular age with that of its parents at the same age. Since permanent income is a fundamentally empirically unobservable concept, this is not directly possible so that instruments have to be used in its place, typically current income or earnings of the parent and child sometimes adjusted for the point in the life-cycle at which it is observed are employed. One interpretation of the results presented here is that

the instrument used is the educational attainment of the child (in terms of the grade it has attained at a given age) which is related to the educational attainment and income of its parent(s). An argument for this formulation emerges from the returns to education literature (see Heckman and Krueger (2003) and Blundell et.al.(2004)) emphasizing the extent to which education influences the permanent income (and mobility thereof) of future generations. Here academic attainment is construed as a proxy for the permanent income of both parents and children. It is one of the channels through which current generations influence future generations in a dynastic sense via the within family endowment and enhancement of human capital which is heir specific (mobility reducing) as opposed to publicly provided education which is to a greater degree heir indifferent (mobility increasing).

The role of the family type¹ is investigated in this process, identifying three broad categories of family, two parent (referred to as “Intact”) families and two types of single parent family “Exogenously” single and “Endogenously” single. An exogenously single parent is one whose spouse has deceased whereas an endogenously single parent is one where the parents have separated or divorced, the latter being a consequence of choice whereas the former is (hopefully!) not. In the United States throughout the 80's a trend in child custody law and dispute resolution emerged where previous societal preference (as expressed by its laws) for maternal custody changed toward one with less gender based bias, this was coupled with a gradual trend toward joint custody awards in dispute resolution². Following Rasul (2006) Leo (2005) argues, and finds evidence that, such changes will not only affect the investments in children in endogenously single families but it will also influence the corresponding behavior of intact families. Here the idea that different family types may correspond to different blueprints by which the location in the permanent income distribution of its offspring can be influenced is examined. The nature of this influence is explored in the context of parent-child educational achievement data from the United States for cohorts of 15, 16, 17 and 18 year olds in 1970 and 1990. Section 1 examines whether or not the academic achievement of children from different family types is different and if so how it progresses with cohort age. The relative welfare of the cohorts is compared empirically in section 2. Section 3 studies differences by family type in the structure of the transmission technology using intergenerational regressions and section 4 looks at the transmission mechanisms via the transmission matrix methodology and proposes a new mobility index for non-standard transition structures. To anticipate the conclusions drawn in section 5, significant divergence and welfare differences are observed between the children of single parent and intact families and between the children of exogenously and endogenously single parent families. Changes in custodial law and practice over the observation period appear to have ameliorated these differences. The nature of the parent to child transition matrices, which

¹ Bjorklund et. al. (2004) consider family structure in terms of family size, gender mix and birth order of children, here we focus on the parental structure of the family and leave family size, gender, and ethnicity issues for further research.

² This is exemplified by the fact that before 1980, only 4 states acknowledged joint custody as a possible arrangement in custody awards. However, by 1990, only 14 states had not incorporated joint custody. The force of this statutory amendment may be noted from the surge in joint custody awards in California (from 2.2% in 1979 to 13% in 1981 (Maccoby & Mnookin, 1994)), and Wisconsin (from 2.2% in 1980-81 to 14.2% in 1991-92 (Brown et. al., 1997)).

exhibit strong child-parent dependency appears to have changed substantially over the 1970-1990 period and reflect increased mobility.

1. The Divergence in Academic Achievement of children from different family types.

Human capital development will be measured by educational attainment at a given age where attainment is measured by the completed grade level. Letting p be the probability of transiting to the next grade, we posit a model $p = p(y,e,x)$ where y is family income, e refers to the educational attainment of the parents and x to a set of variables describing the family type (e.g. two parent family, single parent family {widowed, divorced}, non-working mother etc.). Essentially p corresponds to a production function blueprint for each family type which converts a family's genetic endowments and current income into the child's academic achievement as measured by grade attainment. In our empirical analysis the academic achievement of 15, 16, 17 and 18 year old cohorts will be the subject of investigation. Clearly not everyone starts school in the year that they were 14, unfortunately the admission birth month cutoff varies across jurisdictions and information is not available for specific students. However assuming births to be uniformly distributed throughout the year and that everyone starts in grade 1 who were age 14 before θ of the year had elapsed. The corresponding cohort grade attainment distributions would be:

	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Age 15	$1-p$	p			
Age 16	$\theta(1-p)^2 + (1-\theta)(1-p)$	$\theta 2p(1-p) + (1-\theta)p$	θp^2		
Age 17	$\theta(1-p)^3 + (1-\theta)(1-p)^2$	$\theta 3p(1-p)^2 + (1-\theta) 2p(1-p)$	$\theta 3p^2(1-p) + (1-\theta)p^2$	θp^3	
Age 18	$\theta(1-p)^4 + (1-\theta)(1-p)^3$	$\theta 4p(1-p)^3 + (1-\theta)3p(1-p)^2$	$\theta 6p^2(1-p)^2 + (1-\theta) 3p^2(1-p)$	$\theta 4p^3(1-p) + (1-\theta)p^3$	θp^4

For cohorts 16 and above this model projects an expected cohort age grade attainment of $1+p(\theta(\text{age}-14)+(1-\theta)(\text{age}-15))$ with a variance of $p(1-p)(\theta(\text{age}-14)+(1-\theta)(\text{age}-15))$, for the age = 15 cohort the expected cohort age grade attainment is $1+p$ with a variance of $p(1-p)$. The essential point being that expected values and variances both grow with cohort age. The issue is, if p varies by family type, are children from different family types converging or diverging with cohort age? This is really a question of whether or not children of different family types are alienating in terms of their academic attainments which may be examined by employing various types of polarization or alienation measures (Anderson (2005)). Here we consider two types of alienation measure, one based upon the extent of distributional overlap (Anderson, Ge and Leo 2005) and one based upon Gini type measures (Anderson (2005)) which amount to overall mean normalized subgroup mean differences (the latter index ignores the effects of increasing variances in the subgroups and ostensibly provides different information). Appendix 1 outlines the nature of these indices.

Alienation Indices measure the degree to which two distributions fail to accord and are thus a measure of the lack of similarity between two distributions. Most importantly they reflect differences across the whole distribution, not just differences in location or scale etc. In the present context the indices will indicate the degree to which the high school attainment levels of the children of a given age cohort from single parent families differ from those of two parent families. Further within single parent families they will reveal whether or not there are differences between the children of widowed versus divorced parents. Theoretically it is expected that attainment levels will be higher in intact (two parent) household situations than in single parent environments for obvious reasons (economies of scale and specialization in household production of human capital in the offspring) and that the diversity of attainments will increase with cohort age (essentially the progress of time presents a greater opportunity for students to fall behind). Thus if the environment of a 2 parent situation is more conducive to human capital accumulation than that of the single parent situation it is to be expected that alienation of the two attainment distributions should increase with cohort age but with increasing diversity this may not be so. With regard to widowed versus divorced parents our conjecture is that prior to changes in custody laws in the US in the 1980's there would be substantial alienation but this would not be observed in the post 1980 observation group.

Table 1. Alienation Measures 2 Parent versus Single Parent families

Age Cohort	1970		1990	
	AOV	AGINI	AOV	AGINI
15 year olds	0.0608	0.0041	0.0283	-0.0008
16 year olds	0.0847	0.0267	0.0495	0.0094
17 year olds	0.0912	0.0324	0.0681	0.0162
18 year olds	0.1076	0.0428	0.0685	0.0275

As may be seen from Table 1 the degree of alienation between the educational attainment distributions of the children of intact and single parent families appears to increase with the cohort age but the extent of the alienation appears to diminish between 1970 and 1990 which accords closely with expectations. Generally the Overlap and Gini based measures maintain the same monotonic ordering. Note that AGINI has a smaller range and is somewhat less variable than the overlap based measure which quite possibly reflects the fact that AGINI is focused on subgroup mean differences whereas the overlap measure reflects a wider range of distributional differences.

Alienation Tests provide evidence on the question have the attainment distributions of single parent versus intact families children of a given age cohort converged or diverged between cohorts in a statistically significant fashion. That is to say, are the differences observed in Table 1 significant, or are they simply a matter of sampling variability. Differences between alienation indices can be shown to be asymptotically normal thus providing a standard normal test statistic for their comparison. Table 2 indicates that alienation develops significantly over the cohort years though it tends not to be substantial over the 17 to 18 year old transition. The transitions recorded in terms of the overlap measure tend to be statistically significant whereas those recorded in terms of the

Gini based alienation statistic do not, probably reflecting the greater power of the overlap based statistic, a function of it reflecting a greater variety of distributional differences.

Table 2. Standard Normal, Between Cohort, Overlap and AGini Alienation Tests (Upper tail probabilities in brackets)
1970

Age Cohort	15 year olds	16 year olds	17 year olds
16 year olds	3.8697 (0.0001) 1.2509 (0.1055)		
17 year olds	4.7431 (0.0000) 1.3821 (0.0835)	0.9599 (0.1686) 0.2609 (0.3971)	
18 year olds	6.3035 (0.0000) 1.4885 (0.0683)	2.9377 (0.0017) 0.5923 (0.2768)	2.0418 (0.0206) 0.3592 (0.3597)

1990

Age Cohort	15 year olds	16 year olds	17 year olds
16 year olds	5.1090 (0.0000) 0.6379 (0.2618)		
17 year olds	8.7346 (0.0000) 1.0091 (0.1565)	3.6948 (0.0001) 0.3772 (0.3530)	
18 year olds	8.3701 (0.0000) 1.4688 (0.0710)	3.6184 (0.0001) 0.8903 (0.1866)	0.0793 (0.4684) 0.5371 (0.2966)

Attention is now focused on the two types of single parent families. Two types of family are distinguished, Exogenously Single and Endogenously Single. It may be argued that children under the latter regime have been reared in a conflict situation whereas those in the former regime have not, thus one may well suspect that children from the former regime may be expected to perform better than children from the latter regime. Furthermore the changes in the nature of custody arrangements over time will affect the latter but not the former. For Endogenously Single parents family maintenance payment arrangements were less enforceable prior to the changes in custody law than after, endogenous single parents would generally be more income poor and income uncertain relative to exogenous single parents (who would have been the recipients of death benefits) in 1970 than would have been the case in 1990.

Table 3 reports the Alienation measures and Table 4 present the alienation tests. Note that Alienation measures are smaller between Exogenous and Endogenous single households than they are between intact and single parent families. Exogenous single and Endogenous single head of household families have much more in common with each other than do 2 Parent and Single parent families. A fortiori differences between the two types of single family will be harder to detect. With the Overlap measure a similar effect to the Two versus Single parent comparison (that is increasing alienation with cohort age) prevails here for the 1970 period (though the effect does not carry through to the 18 year old cohort, indeed there are some cohesion as opposed to alienation effects here). The degrees of alienation are much less pronounced than in the single parent versus two parent comparison. The 1990 data present a completely different story with very little discernable alienation movement with very few of the comparisons significant in a

statistical sense. The Gini based measure no longer tracks the overlap measure in any discernable way, indeed in the 1990 data the Gini based measure becomes negative reflecting the fact that children from Endogenously single parent families now have higher average attainment levels than those from Exogenously single parent families.

Table 3. Alienation Measures Exogenous versus Endogenous Single Parent Families.

Age Cohort	1970		1990	
	OV	AGINI	OV	AGINI
15	0.0527	0.0223	0.0327	-0.0103
16	0.0682	0.0205	0.0429	-0.0150
17	0.0819	0.0190	0.0373	-0.0154
18	0.0545	0.0161	0.0456	-0.0085

Table 4 indicates that significant changes in the degree of alienation are much harder to come by in the Endogenous-Exogenous single parent family comparison. The Gini based measure is again never significant and is frequently of a different sign than the overlap measure. Of great interest is the sign reversal in the Agini measure in the 1990's probably indicating the success of changes in the custody rules (which only affect the Endogenous single parent families). Interestingly here is an example of where the Gini based measure may be more effective in reflecting a change than the overlap based measure. Suppose the two family types retained their attainment distribution structures, they just suffered mean shifts such that their average attainment levels were exchanged. Then the overlap measure would record no change whereas the Gini measure would reflect the change in the means.

Table 4. Standard Normal, Between Cohort, Overlap and AGini Alienation Tests (Upper tail probabilities in brackets)

1970

Age Cohort	15 year olds	16 year olds	17 year olds
16 year olds	1.4592 (0.0723) -0.0714 (0.5674)		
17 year olds	2.5782 (0.0050) -0.1146 (0.5456)	1.1578 (0.1235) -0.0489 (0.5105)	
18 year olds	0.1644 (0.4370) -0.1709 (0.5678)	-1.1742 (0.8798) -0.1171 (0.5466)	-2.2288 (0.9871) -.07310 (0.5291)

1990

Age Cohort	15 year olds	16 year olds	17 year olds
16 year olds	1.0538 (0.1460) -0.2365 (0.4065)		
17 year olds	0.4970 (0.3096) -0.2470 (0.5975)	-0.5810 (0.7194) -0.0174 (0.5069)	
18 year olds	1.3081 (0.0954) 0.0779 (0.4689)	0.2596 (0.3976) 0.2709 (0.6067)	0.8428 (0.1997) 0.2798 (0.3908)

3. Grade Attainment Differences, a Welfare Comparison.

If the welfare of a person in a particular cohort is measured by her discretely ordered educational attainment index, the i 'th Order of Stochastic Dominance for discrete attainment distributions (see Dardanoni and Forcina (1998)) f and g (written $f \succ D(i) g$) each defined over integers $k = 0, \dots, K$ will provide welfare rankings of attainment distributions in the following forms:

$$f \succ D(1) g \text{ when } F(j) - G(j) = \sum_{k=0}^j (f(k) - g(k)) \leq 0 \quad \forall j \in 0, K$$

is a necessary and sufficient condition for $E_f(U(k)) \geq E_g(U(k))$ when $U(k)$ is a function expressing a preference for higher educational attainment. First order dominance simply means that the cumulative density of f is everywhere to the right of the cumulative density of g . In the present context if f corresponds to the educational attainment distribution of children of intact families and g corresponds to that of the children of single parent families then $f \succ D(1) g$ implies that the proportion of single parent children at or below attainment level j is always greater than or equal to the corresponding proportion of intact family children. Endowing the preference function with slightly richer properties facilitates other dominance comparisons, for example second order dominance is defined as:

$$f \succ D(2) g \text{ when } F^2(j) - G^2(j) = \sum_{k=0}^j (F(k) - G(k)) \leq 0 \quad \forall j \in 0, K$$

It implies that $E_f(k|k \leq j)F(j) \geq E_g(k|k \leq j)G(j)$ for all j and is based upon a preference function for successively higher levels of attainment but at a diminishing rate. In a similar fashion higher orders of dominance can be contemplated as:

$$f \succ D(i) g \text{ when } F^i(j) - G^i(j) \leq 0 \quad \forall j \in 0, K$$

Unlike comparisons of means an essential feature of dominance comparisons is that they compare the whole distribution of attainments, establishing whether or not the weakest of the children from intact families perform better than the weaker students of the single parent families. Indeed infinite order dominance can be shown in the continuous environment to compare just the weakest agent in each of the two groups. However unlike indices, which provide complete orderings, stochastic dominance orderings are not complete in that distributions cannot always be ranked at a particular order (because the inequality cannot be maintained for all j or the distributions are not significantly different from one another). One feature of stochastic dominance rankings is that $f \succ D(i) g$ implies $f \succ D(j) g$ for all $j > i$.

Sometimes the issue is not whether or not one group is worse off than another in a welfare sense but whether the extent to which they are worse off has diminished or increased over time. Thinking of $E_f(U(k)) - E_g(U(k))$ as a measure of the welfare gap, the change in the gap over periods 1 and 2, $(E_{f2}(U(k)) - E_{g2}(U(k))) - (E_{f1}(U(k)) - E_{g1}(U(k)))$, suggests the dominance of differences (DD(i)) as a test of whether the gap between two groups is closing or widening. Alternatively put, it is a test of whether the two groups are converging or polarizing (Anderson (2004)) and may be contemplated as:

$$f \text{ DD}(i) g \text{ when } (F_2^i(j) - G_2^i(j)) - (F_1^i(j) - G_1^i(j)) \leq 0 \forall j \in 0, K$$

Test statistics and their distributions for these comparisons are readily available and easily calculated (Anderson (2004), Dardanoni and Forcina (1998)). In the present context such comparisons ask if the gap between the attainment distributions of children from different family types has closed by comparison over the complete range of the distributions rather than just at the means for example.

Sometimes distributions cannot be ranked (because the inequality cannot be maintained for all j or the distributions are not significantly different from one another) and the comparison is indeterminate in which case a higher order comparison can be made. One feature of stochastic dominance rankings is that $f \text{ D}(i) g$ implies $f \text{ D}(j) g$ for all $j > i$. Here first and second order dominance relationships will be examined. For comparison the average grade attainments, the standard normal test of their difference and the stochastic dominance tests are reported in Table 5. As may be seen the differences, both in terms of average attainments and stochastic dominance orderings become more substantial with the cohort age. This is of course a reflection of the increasing alienation reported in tables 1 and 2 and accords with expectations. That is, with the exception of the 15 year old cohort in 1970 and 1990, all the grade attainment distributions are significantly different and stochastically ordered in favor of the children from married couples and the extent of the differences increase with cohort age. Average family income and adult academic attainment (neither of which are reported for space reasons) was higher for in tact couples than for singles in every comparison (the former because there are frequently two earners in the in tact family situation and the latter because the maximum of the two parents attainments was used in the case of in tact families).

Table 5. 2 Parent versus Single Parent Family Children's Average Grade Attainments and Stochastic Order Tests*

Cohort	1970	1990
15	3.5741 v 3.5595 (1.2269) Indeterminate	3.6935 v 3.6966 (0.2767) Indeterminate
16	4.4679 v 4.3491 (8.0883) 2 nd Order	4.6040 v 4.5608 (3.2530) 2 nd Order
17	5.4045 v 5.2301 (9.7383) 2 nd Order	5.5541 v 5.4643 (6.2094) 2 nd Order
18	6.1878 v 5.9246 (10.6920) 1 st Order	6.4358 v 6.2600 (9.9108) 1 st Order

The corresponding comparison for Endogenous and Exogenous Single Parent Families is reported in Table 6. The detailed comparisons are quite different in nature from those found in Table 5. The 1990 ordering reversal has already been alluded to as has the fact that the differences between types of family are now much less distinct with a failure to determine an ordering in 4 of the 8 comparisons. One very interesting feature of these results is that on average in 1970 Exogenous Parental Incomes were significantly higher than Endogenous family incomes whereas Exogenous Parental Achievement was

significantly lower than Endogenous Parental Achievement. In 1990 the income relationship was reversed (though the difference was no longer significant) while the parental educational attainment gap remained and indeed strengthened. The change in the income status between the two observation periods had no doubt much to do with changes in custodial law and dispute resolution which saw an increased participation by fathers both materially and in terms of time in the child's development.

Table 6 Exogenous versus Endogenous Single Parent Families Children's Attainment and Stochastic Order Tests together with Parental Income and Attainment Mean Differences*

Cohort	1970	1990
15	3.6041 v 3.5249 (3.3775) Indeterminate 18.7924 v 16.0469 (5.0008) 5.5442 v. 5.7337 (-2.8918)	3.6652 v 3.7033 (-1.3515) -ve 1 st Order 19.3496 v 20.0323 (-0.9596) 6.5590 v 7.0877 (-7.9436)
16	4.3972 v 4.3079 (3.1908) 1 st Order 17.5877 v 16.1007 (3.0902) 5.4783 v 5.7460 (-4.1273)	4.5047 v 4.5732 (-2.1223) -ve 2 nd Order 19.3913 v 20.6720 (-1.9380) 6.5785 v 7.1002 (-7.8103)
17	5.2802 v 5.1807 (2.9404) Indeterminate 17.4957 v 16.6762 (1.5916) 5.5827 v 5.7980 (-3.2393)	5.3964 v 5.4805 (-2.4736) Indeterminate 20.5137 v 20.9146 (-0.5079) 6.5314 v 7.0970 (-8.9582)
18	5.9721 v 5.8768 (2.0462) 1 st Order 17.4245 v 16.4140 (1.6735) 5.3953 v 5.6819 (-3.7019)	6.2199 v 6.2709 (-1.3210) Indeterminate 20.1967 v 21.5534 (-1.6831) 6.5473 v 7.1240 (-8.7831)

*It is interesting to note (though not germane to the present discussion) that average attainment levels have generally drifted upwards over the two decades for all family types, all cohorts and both children and parents.

To establish a significant dominance of differences relationship for a given cohort (in this case with single parent children closing the welfare gap on children from intact families) at least one significant positive cell and no significant negative cells is required. Table 7 compares intact families and single parent households, choosing a significance level of 5% or less for individual cells indicates that the welfare gap has been closed for 16 and 17 year old cohorts. It also appears to have been closed for all but the highest grade category in the comparison for 18 year olds. All in all, the welfare gap does appear to have been closed for all but the 15 year old cohort the conclusions for which appear to be indeterminate. With regard to the comparison between single parent types the results in Table 8 indicate a significant change in the endogenous single family distribution relative to the exogenous family distribution change (at least one significantly negative and no significantly positive cells) with all but the 17 year old cohort.

Table 7. First order Dominance of Differences test Intact versus Single Parent Families. {Test statistic (standard error) [upper tail probability]}

Grade Category	15 year olds	16 year olds	17 year olds	18 year olds
1	0.0040 (0.0017) [0.9909]	0.0034 (0.0015) [0.9893]	0.0015 (0.0015) [0.8503]	0.0012 (0.0019) [0.7365]
2	0.0059 (0.0023) [0.9948]	0.0053 (0.0019) [0.9976]	0.0021 (0.0019) [0.8721]	0.0027 (0.0025) [0.8636]
3	0.0278 (0.0113) [0.9931]	0.0361 (0.0076) [0.9999]	0.0207 (0.0054) [0.9999]	0.0116 (0.0054) [0.9845]
4	-0.0055 (0.0062) [0.1864]	0.0342 (0.0113) [0.9987]	0.0259 (0.0084) [0.9999]	0.0304 (0.0077) [0.9999]
5	-0.0059 (0.0025) [0.0092]	0.0008 (0.0060) [0.5524]	0.0315 (0.0115) [0.9969]	0.0387 (0.0104) [0.9999]
6	-0.0076 (0.0018) [0.0000]	-0.0039 (0.0024) [0.0515]	0.0027 (0.0059) [0.6744]	0.0313 (0.0127) [0.9932]
7	-0.0008 (0.0003) [0.0111]	-0.0003 (0.0007) [0.3181]	0.0002 (0.0013) [0.5653]	-0.0284 (0.0058) [0.0000]

The 8th category dominance of differences would be uniformly 0 since it represents the upper bound of the distribution.

Table 8. First order Dominance of Differences test Endogenous versus Exogenous Single Parent Families. {Test statistic (standard error) [upper tail probability]}

Grade Category	15 year olds	16 year olds	17 year olds	18 year olds
1	0.0004	0.0015	0.0081	0.0037
	0.0035	0.0032	0.0028	0.0038
	0.5504	0.6817	0.9980	0.8360
2	0.0005	0.0036	0.0101	0.0044
	0.0055	0.0039	0.0035	0.0048
	0.5330	0.8166	0.9979	0.8211
3	-0.0684	-0.0374	-0.0155	-0.0021
	0.0239	0.0165	0.0115	0.0110
	0.0021	0.0117	0.0894	0.4251
4	-0.0329	-0.0979	-0.0555	-0.0298
	0.0137	0.0237	0.0177	0.0158
	0.0080	0.0000	0.0009	0.0297
5	-0.0079	-0.0237	-0.1092	-0.0454
	0.0062	0.0131	0.0235	0.0213
	0.1001	0.0352	0.0000	0.0167
6	-0.0098	-0.0020	-0.0184	-0.0452
	0.0045	0.0057	0.0124	0.0253
	0.0141	0.3612	0.0692	0.0369
7	0.0008	-0.0019	-0.0033	-0.0340
	0.0011	0.0014	0.0024	0.0114
	0.7792	0.0984	0.0844	0.0014

* The 8th category dominance of differences would be uniformly 0 since it represents the upper bound of the distribution.

4. Differences in the Human Capital Production Functions by Family Type.

To examine the nature of the Human Capital Production Technology by family type we posit a production function that is quadratic in family income and parental educational attainment (we take the maximum educational attainment of the parents in the two parent case) together with an interaction effect. The regression results for two comparisons, one between two parent and single parent households and one between exogenously and endogenously single parents³ for each of the 15, 16, 17 and 18 year old cohorts are reported in tables 13 and 14. Tables 9 through 12 report some specification tests.

Table 9 Tests for Quadratic Form (χ^2 test and (upper tail probability) of linear null)

Cohort	1970		1990	
	Couples/Singles	Endog/Exog	Couples/Singles	Endog/Exog
15	89.45 (0.0000)	29.59 (0.0000)	49.21 (0.0000)	5.61 (0.4687)
16	135.99 (0.0000)	26.10 (0.0002)	67.40 (0.0000)	5.55 (0.4753)
17	235.94 (0.0000)	37.94 (0.0000)	65.69 (0.0000)	10.21 (0.1160)
18	347.64 (0.0000)	55.97 (0.0000)	122.53 (0.0000)	23.84 (0.0006)

Table 10 Tests for Family type (χ^2 test and (upper tail probability) of no difference null)

Cohort	1970		1990	
	Couples/Singles	Endog/Exog	Couples/Singles	Endog/Exog
15	86.70 (0.0000)	21.70 (0.0013)	16.99 (0.0093)	9.23 (0.1612)
16	23.76 (0.0006)	28.46 (0.0001)	9.51 (0.1469)	10.29 (0.1128)
17	27.07 (0.0001)	23.96 (0.0005)	11.80 (0.0666)	8.10 (0.2309)
18	9.09 (0.1686)	34.04 (0.0000)	11.12 (0.0848)	20.33 (0.0024)

Table 11 Tests for Parent Attainment (χ^2 test and (upper tail probability) of no affect null)

Cohort	1970		1990	
	Couples/Singles	Endog/Exog	Couples/Singles	Endog/Exog
15	301.79 (0.0000)	59.54 (0.0000)	161.98 (0.0000)	49.39 (0.0000)
16	929.09 (0.0000)	187.02 (0.0000)	397.67 (0.0000)	117.70 (0.0000)
17	986.66 (0.0000)	227.60 (0.0000)	727.58 (0.0000)	237.42 (0.0000)
18	1005.25 (0.0000)	250.00 (0.0000)	753.89 (0.0000)	220.99 (0.0000)

Table 12 Test for Parental Income (χ^2 test and (upper tail probability) of no affect null)

Cohort	1970		1990	
	Couples/Singles	Endog/Exog	Couples/Singles	Endog/Exog
15	157.85 (0.0000)	62.28 (0.0000)	17.50 (0.0076)	7.17 (0.3051)
16	157.76 (0.0000)	40.82 (0.0000)	10.29 (0.0000)	12.44 (0.0529)
17	280.76 (0.0000)	62.15 (0.0000)	69.27 (0.0000)	16.65 (0.0107)
18	254.17 (0.0000)	38.84 (0.0000)	148.84 (0.0000)	46.70 (0.0000)

With the exception of 15, 16 and 17 year old single parent family cohorts in 1990 the quadratic specification and difference in family type hypotheses are never rejected at the 5% critical value. As for the importance of parental educational attainment and income neither can be excluded from the analysis except in the case of parental income in 15 and 16 year old single parent family cohorts in 1990 at the 5% critical value.

³ Of peripheral interest in the case of intact families is the comparison of the one parent staying at home versus both parents participating in the work force. This is reported in Appendix 2.

Table 13 Couples versus Singles (Standard errors in brackets)

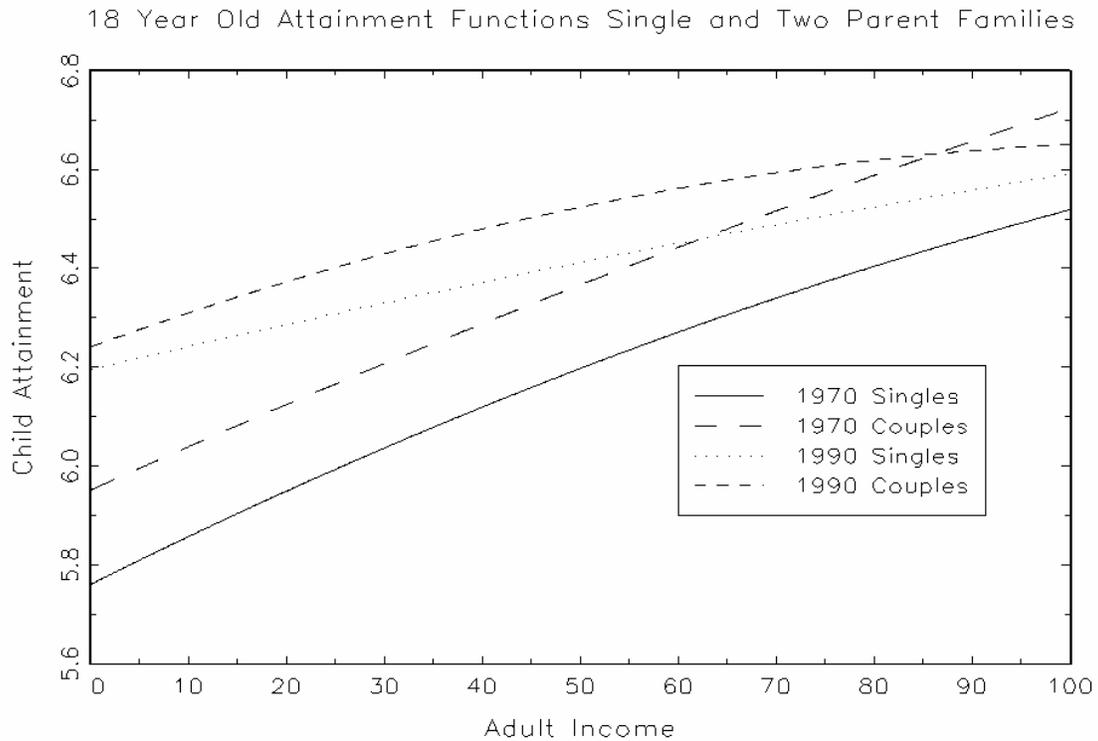
	1970		1990	
	2 Parent	Δ Single	2 Parent	Δ Single
[15] Constant	3.1232 (0.003504)	0.001393 (0.009689)	3.0208 (0.004275)	0.1769 (0.01004)
Parental Income	0.005205 (0.00007357)	0.009502 (0.0004032)	0.003679 (0.00007738)	-0.003581 (0.0003733)
Parental Achievement	0.06551 (0.0005117)	-0.009227 (0.001611)	0.1414 (0.0005568)	-0.02387 (0.001399)
(Parental Income) ²	-0.000006957 (0.0000007311)	-0.00003673 (0.000005200)	-0.000003159 (0.0000005479)	-0.0000009097 (0.000004740)
(Parental Achievement) ²	-0.001418 (0.00006756)	0.001154 (0.0002275)	-0.007446 (0.00006924)	0.001090 (0.0001823)
Income x Achievement	-0.0004023 (0.000009413)	-0.0005227 (0.00005453)	-0.0003270 (0.000009223)	0.0003640 (0.00004712)
Sigma		0.3844		0.5341
R ²		0.02377		0.007161
[16] Constant	3.5300 (0.004368)	-0.09322 (0.01175)	3.6009 (0.004869)	0.2555 (0.01126)
Parental Income	0.007575 (0.00009112)	0.006380 (0.0005204)	0.007489 (0.00008809)	-0.0006628 (0.0004116)
Parental Achievement	0.1641 (0.0006390)	0.03267 (0.001963)	0.1808 (0.0006365)	-0.07823 (0.001564)
(Parental Income) ²	-0.000009583 (0.0000009105)	-0.00003534 (0.000008037)	-0.000001621 (0.0000006186)	0.000009945 (0.000005076)
(Parental Achievement) ²	-0.005748 (0.00008435)	-0.003084 (0.0002778)	-0.007196 (0.00007930)	0.006833 (0.0002030)
Income x Achievement	-0.0005972 (0.00001165)	-0.0002419 (0.00007252)	-0.0007748 (0.00001050)	-0.0001433 (0.00005163)
Sigma		0.5653		0.6743
R ²		0.05804		0.01948
[17] Constant	4.1611 (0.005317)	-0.2900 (0.01430)	4.2839 (0.005231)	0.1839 (0.01180)
Parental Income	0.01270 (0.0001092)	0.005552 (0.0006128)	0.007958 (0.00009381)	0.004305 (0.0004160)
Parental Achievement	0.2086 (0.0007790)	0.1100 (0.002363)	0.2062 (0.0006840)	-0.09084 (0.001644)
(Parental Income) ²	-0.00001881 (0.000001056)	-0.00002952 (0.000008689)	-0.000003706 (0.0000006668)	0.000001911 (0.000004239)
(Parental Achievement) ²	-0.007247 (0.0001030)	-0.01004 (0.0003326)	-0.006494 (0.00008527)	0.009774 (0.0002141)
Income x Achievement	-0.0009699 (0.00001397)	-0.00001058 (0.00008402)	-0.0007427 (0.00001122)	-0.0007428 (0.00005225)
Sigma		0.7822		0.7640
R ²		0.07370		0.03902
[18] Constant	4.3839 (0.007267)	-0.3287 (0.01946)	4.7883 (0.006523)	0.2151 (0.01444)
Parental Income	0.01936 (0.0001538)	0.002668 (0.0008322)	0.01444 (0.0001167)	0.009252 (0.0004990)
Parental Achievement	0.3434 (0.001082)	0.1420 (0.003291)	0.2377 (0.0008566)	-0.1121 (0.002009)
(Parental Income) ²	-0.00002315 (0.000001516)	0.000011633 (0.00001056)	-0.0000007506 (0.0000007714)	-0.00002369 (0.000005371)
(Parental Achievement) ²	-0.01502 (0.0001446)	-0.01284 (0.0004688)	-0.005004 (0.0001072)	0.009726 (0.0002617)
Income x Achievement	-0.001708 (0.00001995)	-0.0002858 (0.0001155)	-0.001388 (0.00001406)	-0.0007769 (0.00006331)
Sigma		1.08042		0.9986
R ²		0.08952		0.05561

Table 14 Endogenous vs Exogenous Singles

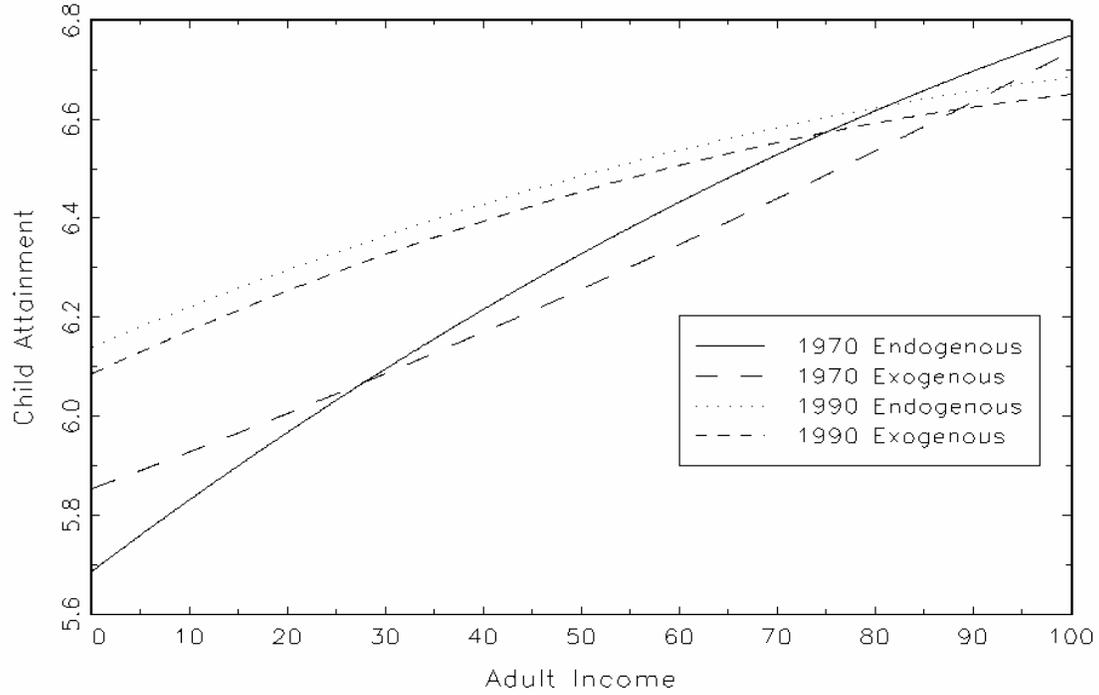
	1970		1990	
	Endogenous	Δ Exogenous	Endogenous	Δ Exogenous
[15] Constant	3.1273 (0.01116)	-0.04510 (0.01690)	3.1860 (0.01034)	-0.03116 (0.02466)
Parental Income	0.01083 (0.0004644)	0.004800 (0.0006203)	0.0005461 (0.0003845)	-0.003833 (0.0008832)
Parental Achievement	0.05229 (0.001856)	0.03662 (0.002839)	0.1148 (0.001441)	0.04911 (0.003610)
(Parental Income) ²	-0.00002140 (0.000005990)	-0.00003079 (0.000007003)	-0.0000005873 (0.000004882)	-0.00002110 (0.00001041)
(Parental Achievement) ²	-0.0002644 (0.0002621)	-0.002698 (0.0003967)	-0.005741 (0.0001877)	-0.006518 (0.0004781)
Income x Achievement	-0.0005508 (0.00006282)	-0.0004594 (0.00008259)	-0.00003494 (0.00004854)	0.0006566 (0.0001127)
Sigma		0.5101		0.5666
R ²		0.1979		0.08666
[16] Constant	3.2056 (0.01340)	0.4015 (0.01973)	3.8472 (0.01200)	0.06982 (0.02824)
Parental Income	0.009569 (0.0005933)	0.004266 (0.0008117)	0.003170 (0.0004388)	0.009999 (0.001050)
Parental Achievement	0.2742 (0.002238)	-0.1165 (0.003352)	0.1212 (0.001668)	-0.07660 (0.004118)
(Parental Income) ²	-0.00007590 (0.000009162)	0.00004728 (0.00001143)	0.00001333 (0.00005411)	-0.00003077 (0.00001252)
(Parental Achievement) ²	-0.01658 (0.0003168)	0.01150 (0.0004721)	-0.002349 (0.0002165)	0.007311 (0.00054271)
Income x Achievement	0.0001760 (0.00008268)	-0.001315 (0.0001136)	-0.0005480 (0.00005505)	-0.0007516 (0.0001366)
Sigma		0.7348		0.7664
R ²		0.1225		0.07600
[17] Constant	3.7644 (0.01611)	0.1491 (0.02287)	4.4455 (0.01290)	0.1334 (0.02945)
Parental Income	0.01929 (0.0006904)	-0.002298 (0.0009311)	0.01240 (0.0004546)	0.0002945 (0.0009313)
Parental Achievement	0.3338 (0.002660)	-0.005368 (0.003830)	0.1300 (0.001797)	-0.08238 (0.004328)
(Parental Income) ²	-0.00002373 (0.000009789)	-0.00004078 (0.00001227)	0.000001294 (0.000004632)	-0.00002806 (0.000007055)
(Parental Achievement) ²	-0.01856 (0.0003747)	0.0005535 (0.0005385)	0.001679 (0.0002340)	0.009042 (0.0005759)
Income x Achievement	-0.001226 (0.00009467)	0.0004728 (0.0001281)	-0.001493 (0.00005710)	-0.0001794 (0.0001186)
Sigma		0.9928		0.9123
R ²		0.1340		0.07779
[18] Constant	4.2415 (0.02190)	-0.4621 (0.03091)	4.7372 (0.01558)	0.5629 (0.03439)
Parental Income	0.03296 (0.0009364)	-0.01868 (0.001248)	0.02724 (0.0005384)	-0.009456 (0.001111)
Parental Achievement	0.3035 (0.003703)	0.3916 (0.005322)	0.1705 (0.002168)	-0.04645 (0.005041)
(Parental Income) ²	-0.00004013 (0.00001189)	0.00005553 (0.00001510)	-0.00002928 (0.000005795)	-0.000005112 (0.000009029)
(Parental Achievement) ²	-0.007728 (0.0005275)	-0.04200 (0.0007574)	0.003537 (0.0002824)	-0.004115 (0.0006681)
Income x Achievement	-0.003187 (0.0001300)	0.001895 (0.0001717)	-0.002647 (0.00006831)	0.001318 (0.0001425)
Sigma		1.3678		1.1624
R ²		0.1373		0.08637

With respect to the 1970 results observe that the influence of parental income and attainment grows with the age cohort (but much less so for income), that the divergence between endogenously and exogenously single parent families with respect to these effects also grows (though the income effects are not significant) and that income and achievement effects are steeper for endogenous single parent families. The 1990 effects are similar but much more muted especially with respect to the income effect. The invariably significantly negative parental ability / income interaction effect indicates substitutability between the income and parental attainment inputs.

The implications of the equations are best illustrated diagrammatically, the following show the relationship between a child's educational attainment and its family income conditional on average educational attainment of the parents for the 18 year old cohort.



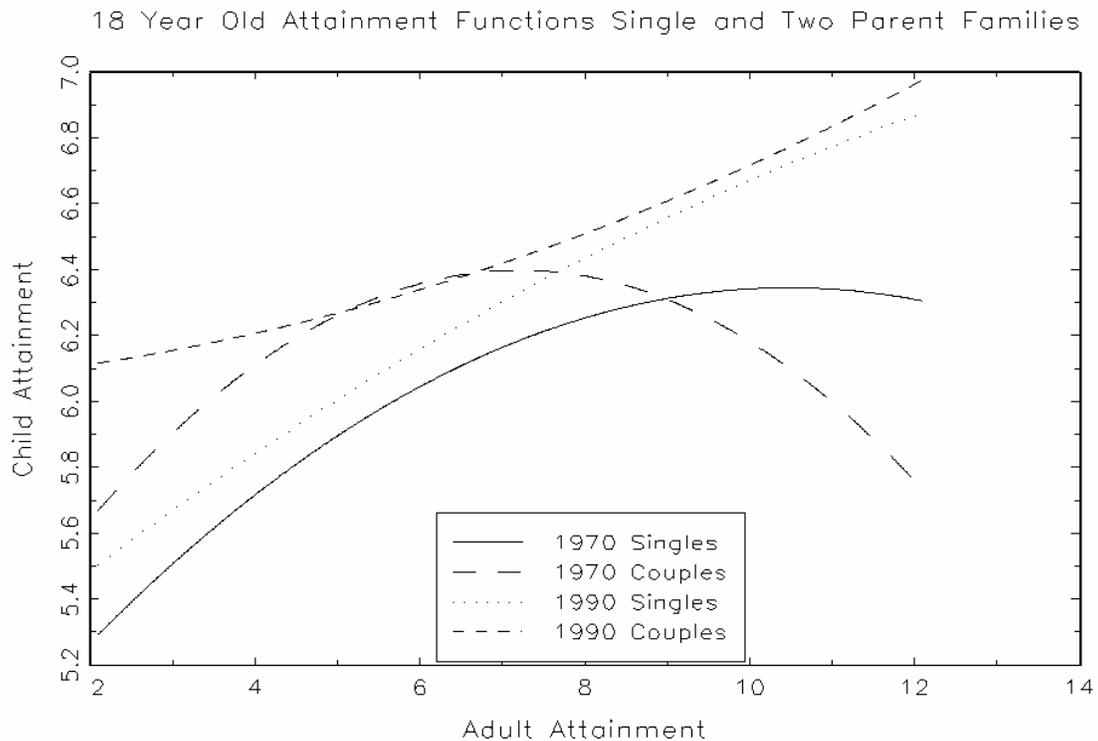
18 Year Old Attainment Functions Single Parent Families



Note that the attainment income profiles are much closer together than in the Intact-Single Parent family comparisons and that the income response is higher for the endogenous than it is for the exogenous families.

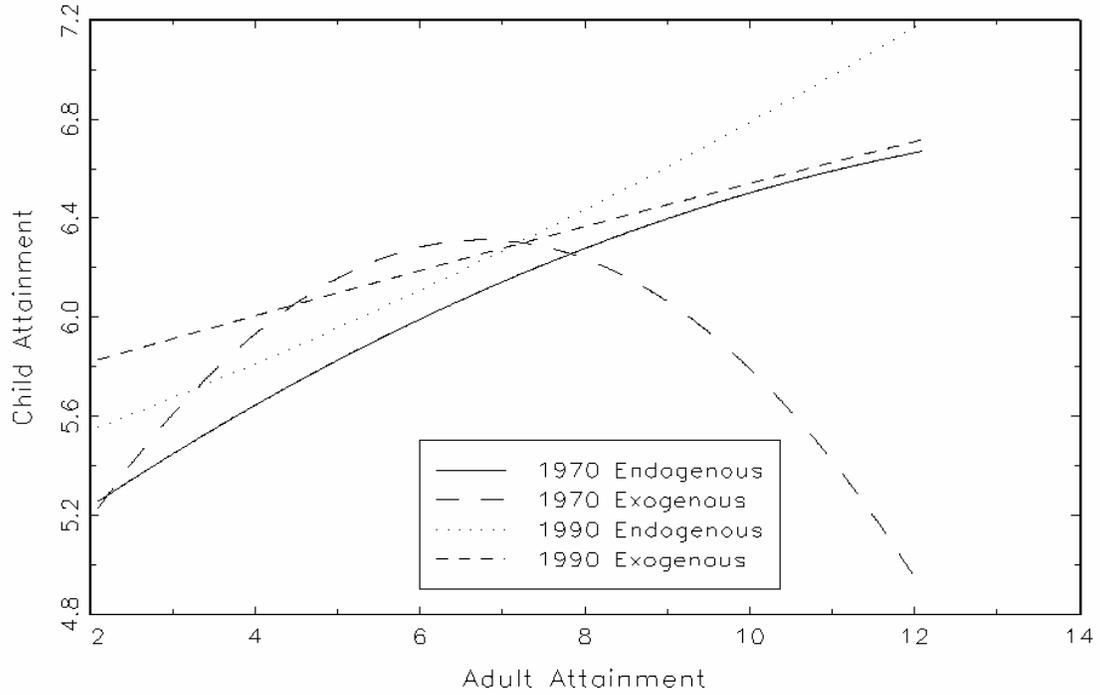
The overall impact levels are higher in the 1990's than they are in the 1970's for all family types which, if school standards are assumed to have remained constant, may be interpreted as reflecting technological progress. In both comparisons the income slopes are steeper in the 1970's than in the 1990's though the overall levels are higher in the 1990's than in the 1970's. The flatter profiles can be interpreted as "more equality of opportunity" in the sense that income has a smaller marginal effect on outcomes. The differences between the income effects of single parent versus two parent families are much more substantive than between the two types of single parent family (indeed the differences in the second comparison are negligible) and the differences appear to have attenuated between 1970 and 1990 suggesting that the children of single parent families are less disadvantaged in the 90's than they were in the 70's.

The relationship between a child's attainment and their parent's attainment conditional on average income levels are illustrated in the following diagrams.



Similar to the income relationship the results of technological progress may be observed here though as may be seen the structure of the impact of adult attainment has changed substantially over the 20 years with increasing returns to parental educational status emerging for married couples in the 1990's. In the relevant range of adult achievement (5-10) the gap between single parent and two parent technologies seems to have narrowed substantially.

18 Year Old Attainment Functions Single Parent Families



The marginal effect of parental achievement is greater in the endogenous environment than in the exogenous environment with the average affect being roughly the same at the sample means.

5. The Transmission Mechanism and Mobility.

Let $f_c(y)$ be the distribution of a child's permanent income y and let $f_p(x)$ be the distribution of the parents permanent income x the issue to be addressed is the relationship between the two distributions, i.e. the extent to, and manner in, which y and x are related. More specifically is there a sense of causality whereby x to some degree predetermines y ? Thinking for the moment of x and y having the joint distribution $f(y,x)$ so that $f(y)$ and $f(x)$ are the respective marginal distributions, at one extreme there is a sense of no relationship when $f(y,x) = f(y)f(x)$ at the other there is the completely deterministic environment whereby $y = a + bx$. Partitioning y and x into k mutually exclusive and exhaustive regions where $p(y)$ and $p(x)$ are respectively the marginal probabilities of falling into those regions such that $p(y)=p(x)$, we are interested in the elements of the square matrix T defined by $p(y) = T(y,x)p(x) = J(y,x)M(x)^{-1}p(x)$. T is the matrix of conditional probabilities formed by the product of the two square matrices in the equation:

$$\begin{pmatrix} p_1(y) \\ p_2(y) \\ \vdots \\ p_k(y) \end{pmatrix} = \begin{pmatrix} p_{11}(y,x) & p_{12}(y,x) & \dots & p_{1k}(y,x) \\ p_{21}(y,x) & p_{22}(y,x) & \dots & p_{2k}(y,x) \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1}(y,x) & p_{k2}(y,x) & \dots & p_{kk}(y,x) \end{pmatrix} \begin{pmatrix} p_1(x) & 0 & 0 \\ 0 & p_2(x) & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & p_k(x) \end{pmatrix}^{-1} \begin{pmatrix} p_1(x) \\ p_2(x) \\ \vdots \\ p_k(x) \end{pmatrix}$$

$T = \|p_{ij}(y,x)/p_j(x)\|$ $i, j = 1, \dots, k$ is familiar in the convergence literature made popular by Quah (1996) when the ranges of y and x are partitioned into common quantiles. As such its properties are well known as are the techniques for its estimation, its columns each sum to one etc.

Interest centers on the properties of the elements of T both of themselves and as functions of household characteristics however difficulties arise here in that for the analysis at hand y and x cannot be conveniently partitioned into common quantiles. At one extreme when x and y are independent (parent's outcome does not affect child's outcome) $p_{ij}(y,x) = p_i(y)p_j(x)$ and T will be of the form:

$$T = \begin{pmatrix} p_1(y) & p_1(y) & p_1(y) \\ p_2(y) & p_2(y) & p_2(y) \\ \vdots & \vdots & \vdots \\ p_k(y) & p_k(y) & p_k(y) \end{pmatrix}$$

It is quite simple to test whether or not T has identical columns whether or not T is square, it simply corresponds to the contingency table test based on the joint density. At the other extreme when $y = a + bx$, that is to say the relationship becomes deterministic, T becomes the identity matrix and the joint density is essentially degenerate. For technical reasons (the discrete nature of the data and different ranges of variability of child and adult attainment) it is not possible to partition our data in such a way that $p(x) =$

$p(y)$ so that the extent to which T corresponds to an identity matrix is no longer an issue, however we can examine Complete dependence in the $p(y) \neq p(x)$ case by inferring what the T matrix would be from the marginals just as we do in the case of independence⁴.

Several mobility indices have been proposed for the standard aligned transition matrix case⁵. $\text{Trace}(T)$ (which is criticized for ignoring the off-diagonal elements of T), $|T|/(n-1)$ (criticized for attaining perfect mobility with just 2 rather than all common columns), the second largest eigenvalue of T , and $\sum_i p_i(x) \ln(p_i(x)/p_i(y))$ (where common states rather than common quantiles are used) have all been used as mobility indices. Note that they all correspond to measures of the extent to which J represents independence between y and x , all depend upon square transition matrices and none could be used in the miss-aligned case we consider. The χ^2 test statistics introduced above could be used as indices in the miss-aligned case, the problem with them is that they do not fit conveniently into the unit interval, one of the desirable properties for mobility indices outlined in Shorrocks (1978). However the extent to which dependence or independence accords with the data can be just as well indexed by an overlap measure given by:

$$OV = \sum_i \sum_j \min(p_{ij}^o, p_{ij}^e)$$

⁴ The resultant T for such a case can be computed given the marginal values $p(y)$ and $p(x)$. Imagine that x is partitioned at x_1 and x_2 (where $x_2 > x_1$) and y is partitioned at y_1 , y_2 and y_3 (where $y_1 < y_2 < y_3$) further more suppose $F(y_1) < F(x_1) < F(y_2)$ and $F(y_2) < F(x_2) < F(y_3)$ (where $F(\cdot)$ are the corresponding cumulative densities). Then T can be shown to be of the form:

$$T = \begin{array}{ccc} \frac{F(y_1)}{F(x_1)} & 0 & 0 \\ \frac{F(x_1) - F(y_1)}{F(x_1)} & \frac{F(y_2) - F(x_1)}{F(x_2) - F(x_1)} & 0 \\ 0 & \frac{F(x_2) - F(y_2)}{F(x_2) - F(x_1)} & \frac{F(y_3) - F(x_2)}{1 - F(x_2)} \\ 0 & 0 & \frac{1 - F(y_3)}{1 - F(x_2)} \end{array}$$

Just as T with identical columns corresponds to the null of complete independence, a T of this form corresponds to the null of complete dependence (notice again the columns sum to one).

⁵ Bartholemew (1982), Blanden et. al. (2004), Chakravaty (1995), Dearden et. al. (1997), Hart (1983), Maasoumi (1986), Maasoumi and Zandvakili (1986), Prais (1955), Shorrocks (1978), have all produced mobility indices all of which are discussed in Maasoumi (1995)).

Where p^o corresponds to the observed cell probability and p^e corresponds to the expected cell probability under the null hypothesis (be it independence or dependence). This measure forms a very natural index since it reflects the proximity of the data to the mobility (immobility) hypothesis of interest (furthermore it can be shown to be normally distributed asymptotically thus facilitating inference). When the data completely conform to the hypothesis of interest $OV = 1$, otherwise $0 \leq OV < 1$.

Shorrocks (1978) and Maasoumi (1996) discuss “desirable” properties for mobility indices. In this context using OV_{Ind} as a mobility index would not satisfy the Immobility axiom discussed in Shorrocks (1978) whereas OV_{Dep} would. On the other hand if independence is to be construed of as perfect mobility $1 - OV_{Dep}$ would not satisfy the Perfect Mobility axiom. Population Symmetry (permutations of the agent outcome vectors yield the same Mobility index), Population Replication Invariance (a replication of an outcome vector yields the same mobility index) and Scale Invariance (a scale transformation of the outcome vectors yields the same mobility index) are all satisfied by the indices presented here. Continuity (the degree of mobility varies continuously with continuously variable outcome vectors) is only satisfied for a sufficiently fine partition of the outcome space. Decomposability is not satisfied by this index (the weighted sum of the min function of the sub distributions is not generally the min function of the weighted sum of the sub distributions).

The most attractive feature of these mobility indices is that they can be readily applied when the transition matrices are not square, in addition they appear to have asymptotically normal sampling distributions (Anderson Ge and Leo (2005)), conveniently facilitating inferences about trends toward independence or dependence over time. Furthermore they are readily implemented in the multivariate domain, facilitating multivariate mobility analysis and, with some small adaptation, can be modified to focus on a particular subgroup for example the mobility of the i 'th subgroup can be considered in terms of:

$$OV = \sum_{j=1}^k \min\left(\frac{p_{ij}}{p_i}, p_{.j}\right)$$

Where p_i and p_j are marginal row and column probabilities respectively.

Mobility Results.

The specification tests reported in the previous section examine the linear and quadratic dependencies represented by the regression equation specifications. More general types of dependencies can be explored via Contingency Table Tests. These tests for the transmission of parental attainment into child attainment and for the transmission of parental income into child attainment were performed based upon a 4 x 4 partition in each case. The results are reported in Tables 15 and 16 and essentially indicate some degree of independence for 15 year olds (in the single parent and two parent Child Grade – Parent Income comparisons in 1990 and in all the Exogenous Endogenous Single Parent comparisons except for the Child Grade – Parent Grade Endogenous Single parent

1990 comparison) but unlike the regression models independence is rejected for all other age cohorts in all comparisons.

Table 15 Independence Tests.

Year	Age	Single Parent Families		Two Parent Families	
		Grade-Grade	Grade-Income	Grade-Grade	Grade-Income
1970	15	29.63 (0.0005)	31.28 (0.0003)	129.99 (0.0000)	99.57 (0.0000)
	16	301.46 (0.0000)	146.68 (0.0000)	2448.89 (0.0000)	944.32 (0.0000)
	17	355.26 (0.0000)	190.00 (0.0000)	2013.35 (0.0000)	1159.26 (0.0000)
	18	275.20 (0.0000)	105.76 (0.0000)	1442.91 (0.0000)	908.10 (0.0000)
1990	15	43.76 (0.0000)	19.68 (0.0200)	77.49 (0.0000)	17.91 (0.0362)
	16	208.88 (0.0000)	105.59 (0.0000)	815.94 (0.0000)	481.36 (0.0000)
	17	379.87 (0.0000)	140.58 (0.0000)	992.04 (0.0000)	557.17 (0.0000)
	18	307.35 (0.0000)	184.32 (0.0000)	1059.11 (0.0000)	489.70 (0.0000)

Table 16 Independence Tests

Year	Age	Endogenously Single		Exogenously Single	
		Grade-Grade	Grade-Income	Grade-Grade	Grade-Income
1970	15	17.28 (0.0445)	12.77 (0.1732)	18.63 (0.0286)	19.76 (0.0195)
	16	185.16 (0.0000)	88.66 (0.0000)	131.05 (0.0000)	65.44 (0.0000)
	17	170.32 (0.0000)	87.51 (0.0000)	205.12 (0.0000)	91.41 (0.0000)
	18	153.99 (0.0000)	80.43 (0.0000)	136.67 (0.0000)	39.12 (0.0000)
1990	15	29.50 (0.0005)	12.08 (0.2087)	20.74 (0.0139)	9.49 (0.3930)
	16	159.63 (0.0000)	83.60 (0.0000)	50.15 (0.0000)	30.09 (0.0004)
	17	269.76 (0.0000)	97.47 (0.0000)	107.23 (0.0000)	53.42 (0.0000)
	18	293.95 (0.0000)	143.52 (0.0000)	46.45 (0.0000)	39.50 (0.0000)

This clear dependence may have been generated because of dependent changes in the underlying conditioning variables, investigating the structure of $J(y,x)M(x)^{-1}$ is thus of interest. Elements in the columns of T may be investigated via ordered probit techniques since they correspond to conditional probabilities of y given x where y is ordered. Two questions are of interest, has the nature of the transmission matrix structurally changed over the 1970-1990 period (reflecting changes in custody law and practice), and to what extent are the columns of the matrix identical (parental outcomes and child outcomes are independent)?

Table 17. Structural Relationships underlying the Transition Matrices

H ₀	Number of Restrictions	Likelihood Ratio	P value.
1970 and 1990 transmission matrices identical	272	2106.36	0.0000
1970 columns are identical	204	1251.38	0.0000
1990 columns are identical	204	1004.80	0.0000

Table 17 reports the results from ordered probit equations for 18 year olds which characterize the transition from adult academic attainment to child's academic attainment. The parent attainments were grouped into 4 categories and several versions of

equations suggested by the structures explored in section 4 were investigated. All yielded very similar results of which the following are an example (where income and family type variables were augmented by state and custodial jurisdictional dummies). The short version of the story is that there have been substantive structural changes in the nature of the adult attainment-child attainment transmission mechanisms over the 1970- 1990 period and that there is strong structural child attainment dependence on parental and family type characteristics in both periods. With regard to the mobility indices the reference group may be the population or the family type itself so we may examine mobility changes relative to the rest of the population or within the family type itself.

Table 18. OV Mobility Indices (relative to population subgroup), Standard Normal Difference Tests and lower tail probabilities for $H_0: OV_{1970} - OV_{1990} \geq 0$

Family Type	Relation	Cohort	1970	1990	"z"	F(z)
Single Parent	Education-Education	15	0.9893	0.9864	1.2688	0.8978
		16	0.9085	0.9485	-7.3529	0.0000
		17	0.8922	0.9348	-7.0749	0.0000
		18	0.8866	0.9316	-6.4686	0.0000
	Education-Income	15	0.9852	0.9865	-0.5080	0.3057
		16	0.9289	0.9568	-5.7123	0.0000
		17	0.9144	0.9556	-7.7571	0.0000
		18	0.9267	0.9349	-1.3574	0.0873
Intact Family	Education-Education	15	0.9942	0.9919	3.1614	0.9992
		16	0.9278	0.9676	-19.9637	0.0000
		17	0.9287	0.9501	-9.7002	0.0000
		18	0.9197	0.9548	-13.7797	0.0000
	Education-Income	15	0.9933	0.9967	-5.5889	0.0000
		16	0.9485	0.9645	-8.6658	0.0000
		17	0.9433	0.9555	-6.0106	0.0000
		18	0.9360	0.9605	-10.516	0.0000
Single Parent Endogenous	Education-Education	15	0.9896	0.9855	1.4522	0.9268
		16	0.9087	0.9541	-6.5628	0.0000
		17	0.9032	0.9379	-4.5260	0.0000
		18	0.8724	0.9311	-6.0182	0.0000
	Education-Income	15	0.9893	0.9853	1.4474	0.9261
		16	0.9285	0.9546	-4.1295	0.0000
		17	0.9292	0.9562	-4.0809	0.0000
		18	0.9127	0.9374	-2.9284	0.0017
Single Parent Exogenous	Education-Education	15	0.9839	0.9844	-0.1056	0.4580
		16	0.9087	0.9205	-1.0765	0.1409
		17	0.8781	0.9021	-2.0239	0.0215
		18	0.8868	0.9364	-4.3375	0.0000
	Education-Income	15	0.9783	0.9755	0.4701	0.6809
		16	0.9248	0.9327	-0.7779	0.2183
		17	0.9123	0.8790	2.7771	0.9973
		18	0.9358	0.9350	0.08169	0.5326

Table 18 presents a “within family type” analysis addressing the question of whether mobility has increased relative to the norm for that family type. As is evident, all family types with the exception of exogenously single households experienced an increase in mobility (equality of opportunity) for age cohorts 16 through 18 though there is little evidence of mobility changes for 15 year olds. The latter result for 15 year olds is not surprising given the measure is progress through high school, there is little variability across 15 year olds attainments and hence little opportunity for substantive change.

Table 19. OV Mobility Indices (Relative to Population Independence Structure), Standard Normal Difference Tests and lower tail probabilities for $H_0: OV_{1970} - OV_{1990} \geq 0$.

Family Type	Relation	Cohort	1970	1990	“z”	F(z)
Single Parent	Education-Education	15	0.7878	0.8421	-6.6932	0.0000
		16	0.7514	0.8428	-10.8878	0.0000
		17	0.7609	0.8294	-7.9953	0.0000
		18	0.7486	0.8395	-9.3721	0.0000
	Education-Income	15	0.9672	0.9838	-5.0744	0.0000
		16	0.9007	0.9458	-8.0426	0.0000
		17	0.9026	0.9376	-6.0179	0.0000
		18	0.8858	0.9264	-5.7645	0.0000
Intact Family	Education-Education	15	0.9675	0.9635	2.4233	0.9923
		16	0.9286	0.9628	-16.7910	0.0000
		17	0.9308	0.9517	-9.5941	0.0000
		18	0.9363	0.9636	-11.8873	0.0000
	Education-Income	15	0.9909	0.9914	-0.6061	0.2722
		16	0.9420	0.9637	-11.3605	0.0000
		17	0.9392	0.9527	-6.4489	0.0000
		18	0.9343	0.9531	-7.7538	0.0000
Single Parent Endogenous	Education-Education	15	0.7982	0.8591	-6.1680	0.0000
		16	0.7644	0.8576	-8.8979	0.0000
		17	0.7623	0.8490	-7.8202	0.0000
		18	0.7547	0.8583	-8.1280	0.0000
	Education-Income	15	0.9645	0.9848	-4.7402	0.0000
		16	0.8987	0.9452	-6.3779	0.0000
		17	0.8899	0.9386	-6.0961	0.0000
		18	0.8788	0.9282	-5.1347	0.0000
Single Parent Exogenous	Education-Education	15	0.7727	0.7601	0.7310	0.7676
		16	0.7359	0.7701	-2.0189	0.0217
		17	0.7579	0.7410	1.0109	0.8440
		18	0.7397	0.7619	-1.2397	0.1075
	Education-Income	15	0.9625	0.9869	-4.1709	0.0000
		16	0.9005	0.9424	-4.1083	0.0000
		17	0.9061	0.8671	3.1424	0.9992
		18	0.8864	0.9212	-2.8971	0.0019

Table 19 addresses the question of whether or not mobility has increased relative to the norm for the population. In this instance the overlap measures for all single parent Education-education comparisons are substantially lower than the corresponding entries in the own subgroup comparisons in Table 18. All family types except exogenously single households experienced an increase in mobility for age cohorts 16 through 18.

Table 20. Joint Parental Income and Attainment OV Mobility Indices, Standard Normal Difference Tests and lower tail probabilities for $H_0: OV_{1970} - OV_{1990} \geq 0$.

Family Type	Quantile Basis	Cohort	1970	1990	"z"	F(z)
All Families		15	0.9899	0.9872	3.1668	0.9992
		16	0.9184	0.9547	-18.0598	0.0000
		17	0.9150	0.9427	-12.7097	0.0000
		18	0.9071	0.9403	-12.9996	0.0000
Single Parent	Subgroup	15	0.7783	0.8317	-6.4472	0.0000
		16	0.7644	0.8346	-8.3973	0.0000
		17	0.7726	0.8347	-7.3673	0.0000
		18	0.7690	0.8536	-8.9989	0.0000
	Population	15	0.5089	0.5869	-7.5444	0.0000
		16	0.5113	0.5974	-8.3520	0.0000
		17	0.5118	0.5987	-8.3184	0.0000
		18	0.5040	0.6004	-8.2133	0.0000
Intact Family	Subgroup	15	0.9579	0.9598	-1.0744	0.1413
		16	0.9006	0.9319	-12.522	0.0000
		17	0.8975	0.9201	-8.4497	0.0000
		18	0.8895	0.9197	-9.7627	0.0000
	Population	15	0.9204	0.9052	6.0486	1.0000
		16	0.8701	0.8852	-5.1026	0.0000
		17	0.8662	0.8736	-2.3475	0.0095
		18	0.8610	0.8676	-1.8340	0.0333
Single Parent Endogenous	Subgroup	15	0.7749	0.8488	-7.2131	0.0000
		16	0.7617	0.8512	-8.4803	0.0000
		17	0.7744	0.8583	-7.7152	0.0000
		18	0.7632	0.8702	-8.5472	0.0000
	Population	15	0.4786	0.5857	-8.3765	0.0000
		16	0.4914	0.5955	-8.0159	0.0000
		17	0.5011	0.5971	-7.0729	0.0000
		18	0.4898	0.6032	-7.3393	0.0000
Single Parent Exogenous	Subgroup	15	0.7461	0.7275	1.0401	0.8508
		16	0.7437	0.7295	0.8086	0.7906
		17	0.7547	0.7115	2.5240	0.9942
		18	0.7582	0.7308	1.5137	0.9350
	Population	15	0.5379	0.5614	-1.1657	0.1219
		16	0.5284	0.5856	-2.9145	0.0018
		17	0.5185	0.5716	-2.7797	0.0027
		18	0.5174	0.5546	-1.8028	0.0357

Table 20 presents the results of the analysis of the joint dependence of a child's attainment on its parent's income and attainment. The analysis has been performed both on the basis of subgroup income quartiles and population income quartiles. Overall the overlap measures are much lower than with the partial attainment-attainment and attainment comparisons of Tables 4 and 6. As may be noted, there is little qualitative difference between the two sets of results for intact families however subgroup and population comparisons for the single parent groups vary substantially. Again mobility has significantly increased in all cases under both quartile bases for the 16 to 18 year old cohorts with much weaker evidence for such a change in the Exogenous Single Parent households.

Finally from a "Dynastic Poverty" perspective it is of interest to examine the sources of mobility by income group. Table 19 presents the subgroup mobility measure for all four income quartiles for each year for 18 year olds (16 and 17 year age cohorts yield very similar results though 15 year old cohort yields no discernable differences).

Table 21. Income Subgroup Mobility Index.

	$\sum_{j=1}^k \min\left(\frac{p_{ij}}{p_i}, p_{.j}\right)$	"t" value for the difference.
Quartile i=1 1970/1990	0.8674	5.6863
	0.9080	
Quartile i=2 1970/1990	0.9887	4.2038
	0.9972	
Quartile i=3 1970/1990	0.9486	3.0394
	0.9659	
Quartile i=4 1970/1990	0.9093	5.0048
	0.9404	

In both observation year's mobility is lowest in the 1st and 4th income quartiles which evidently make the biggest contribution to the lack of mobility. In all quartiles mobility has increased over the two decades.

6. Conclusions.

Using data drawn from the one percent Integrated Public Use Microsample Series (IPUMS) of the decennial Census for the decades 1970 and 1990 the role of the family type in the intergenerational income transmission process has been examined from several aspects and several questions have been pursued. Are there differences in the academic attainment of young people from different family types and if so are there discernable welfare implications? Do different family types correspond to different technologies for transmitting and augmenting attainment characteristics from parent to child? Have these technologies changed in response to changes in custodial law and

practice and do the technologies exhibit structural independence or equality of opportunity.

In the 1970's the attainment distributions of children from different family types were found to diverge or alienate through the education years 15-18 with children from single parent families faring worse than children from intact families and children from Endogenous Single Parent families faring worse than those from Exogenous Single Parent families. By the 1990's these differences had been ameliorated somewhat, some alienation between Intact family children and Single Parent children remained but the divergence between the children from Exogenous and Endogenous Single Parent families had all but disappeared. The trend from Maternal Preference to Joint Custody arrangements throughout the 1980's no doubt had much to do with this in ensuring greater participation in the investment process by fathers in the endogenous singles situation and in seeing a relative reduction in investment by Intact families (Leo (2005)).

There is a great deal of evidence for intergenerational dependence both in terms of income and attainment (hereditary) characteristics, that family type does make a difference with intact families being more effective than single parent families in the transmission and augmentation process. Within the single parent family group in 1970 exogenous single parent situations seemed to be more effective than endogenous single parent situations, though the gap had been largely closed by 1990 suggesting that changes in Custodial Laws have had an effect. Though, as has been said, child parent dependencies appear to be strong, the flatter response profiles in 1990 relative to 1970 suggest a trend toward more equal opportunity.

With regard to mobility, indices and tests have been proposed for examining notions of mobility between the quantiles of two distributions which permit analysis when the quantiles are unmatched. The indices have intuitive appeal since they can be directed specifically to the notion of mobility that is of interest in a given context and are easily extended to multivariate environments. They also apparently have well defined distributions which permit the "statistical significance" of changes in the value of the index. While they do not satisfy all of the desirable properties of such indices called for in Shorrocks (1978) and Maasoumi (1996) they satisfy a good many of them and have the added attraction of being readily employable in circumstances in which current transition based mobility indices are not. The indices and tests indicate that mobility significantly increased, both in the population as a whole and within intact parent and single parent sub populations, over the period. Within the single parent group there was much less evidence for significant mobility change for children from exogenous single parent families than for children from endogenously single parent families which was again consistent with theoretical predictions.

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Appendix 1. Overlap and Gini Based, Alienation Measures.

Anderson et al (2005) and Anderson (2005) introduce and discuss Overlap and Gini based Alienation measures, both statistics have asymptotically normal distributions. Here they are briefly outlined.

The Overlap Measure.

The extent to which two distributions $f(x)$ and $g(x)$ overlap is given by:

$$OV = \int_{-\infty}^{\infty} \min(f(x), g(x)) dx$$

Clearly it is a number between 0 and 1 with 0 corresponding to no overlap and 1 to the perfect matching of the two distributions. It follows that $AOVER = 1-OV$ is a measure of the extent to which the distributions do not match or are alienated. When $f(x)$ and $f(y)$ are specified to the extent that all of their parameters can be estimated and the intersection points of $f(x)$ and $g(x)$ calculated OV can be estimated parametrically (see Anderson et. al. (2005)). When $f(\cdot)$ and $g(\cdot)$ are unknown, given independent samples from $f(\cdot)$ (represented by x) and $g(\cdot)$ (represented by y) of sizes n_x and n_y respectively, its empirical counterpart may be implemented by choosing $K + 1$ partitions of the range of x defined by $x_k, k = 1, \dots, K$ and calculating:

$$OVEST = \max\left(\frac{I(x - x_K)}{n_x}, \frac{I(y - x_K)}{n_y}\right) - \sum_{k=1}^K \min\left(\left(\frac{I(x - x_k)}{n_x} - \frac{I(x - x_{k-1})}{n_x}\right), \left(\frac{I(y - x_k)}{n_y} - \frac{I(y - x_{k-1})}{n_y}\right)\right)$$

Where $I(z)$ is an indicator function equal to 1 when $z \leq 0$ and 0 otherwise. The estimator and associated tests are most effective when the intersection points of the unknown distributions correspond with the chosen partition points. Since $f(x)$ and $g(x)$ are unknown so will their intersection points be, however they could be estimated by adapting kernel estimation techniques though Monte Carlo evidence (Anderson et.al. (2005)) suggests there is little to be gained from this.

The Gini Based Measure.

Starting with the classic Gini inequality coefficient which, with x_i being the income of the i 'th agent for agents $i = 1, \dots, n$ and where for convenience and without loss of generality, incomes are arranged in ascending rank order, may be written as:

$$Gini = \frac{1}{2n^2 \mu} \sum_{i=1}^n \sum_{j=1}^n |x_i - x_j| \quad [1]$$

where μ is the mean of the x 's. Suppose the poverty cut-off, which defines the rich and poor clubs, is somewhere between x_p and x_{p+1} where $p < n$, then Gini may be thought of as the average mean normalized differences between agents within the poor club,

between agents within the rich club and between poor and rich club agents. In measuring alienation between rich and poor it is only the last group of comparisons that are relevant, i.e. the average normalized difference between the rich group and poor group agents. In this case the new statistic “AGini” could be written as:

$$AGini = \frac{1}{p(n-p)\mu} \sum_{j=p+1}^n \sum_{i=1}^p (x_j - x_i) = \frac{1}{\mu} \left(\frac{\sum_{j=p+1}^n x_j}{n-p} - \frac{\sum_{i=1}^p x_i}{p} \right) \quad [2]$$

Clearly this is still a number greater than 0 (but it is no longer guaranteed to be less than 1) which reflects the mean normalized average distance between the poor group and the rich group. Indeed the formulae in [2] can be generalized to general group differences where stratification is imperfect, i.e. where the incomes of rich and poor groups overlap. The income distribution is now presumed to be a mixture of two sub-group population distributions (Poor and Non Poor), where relationship to the poverty line is no longer the defining feature of the clubs. Using $I_{poor}(i)$ as an indicator function equaling 1 when the i 'th person is from the Poor club and 0 otherwise AGini becomes:

$$AGini = \frac{1}{\mu} \left(\sum x_i \left(\frac{I_{poor}(i)}{\sum_{j=1}^n I_{poor}(j)} - \frac{1 - I_{poor}(i)}{\sum_{j=1}^n (1 - I_{poor}(j))} \right) \right) \quad [3]$$

which is the mean normalized difference between the sub-group means.

Appendix 2. Mothers at Home versus Mothers at Work.

Of interest is the matter of whether one of the parents (usually the mother) staying at home changes the technology of attainment transformation. The following table presents the sample details for Intact Families. On average children with mothers at work do better than children with mothers at home, not surprisingly on average family income is higher with mothers at work as opposed to the mother at home situation however maximum adult educational attainment is higher in the mother at work situation as opposed to the mother at home case. One significant change between the two sample periods is that in 1970 roughly 45% of intact households had mothers in the home by 1990 that had reduced to 22% reflecting the substantial increase in mothers participation in the labour force.

Table A1. Sample details of Intact Families Mothers at Home versus Mothers at Work*.

1970	Age Cohort			
	15	16	17	18
Child Attainment (Mother at Home)	3.5606	4.4440	5.3866	6.1579
Child Attainment (Mother at Work)	3.5859	4.4878	5.4190	6.2126
“t” test for difference	[-3.4044]	[-4.6085]	[-2.7615]	[-3.3850]
Household Income (Mother at Home)	39.4363 (0.2516)	39.6864 (0.2722)	40.1462 (0.2907)	38.6245 (0.3217)
Household Income (Mother at Work)	44.4577 (0.2180)	44.9187 (0.2225)	45.4051 (0.2348)	44.6242 (0.2646)
Maximum Adult Educational Attainment, (Mother at Home)	6.6263 (0.0165)	6.6016 (0.0176)	6.5401 (0.0186)	6.4338 (0.0215)
Maximum Adult Educational Attainment, (Mother at Work)	6.6824 (0.0143)	6.8225 (0.0146)	6.8321 (0.0148)	6.7227 (0.0177)
Sample Size (Mother at Home)	12666	11575	10631	7985
Sample Size (Mother at Work)	14544	13968	13208	9605

1990	Age Cohort			
	15	16	17	18
Child Attainment (Mother at Home)	3.6628	4.5663	5.4958	6.3344
Child Attainment (Mother at Work)	3.7025	4.6151	5.5707	6.4656
“t” test for difference	[-3.5314]	[-3.6914]	[-5.1163]	[-7.2684]
Household Income (Mother at Home)	42.5216 (0.5132)	41.9603 (0.5420)	41.8376 (0.5221)	42.0025 (0.5945)
Household Income (Mother at Work)	50.1675 (0.2490)	50.1754 (0.2540)	51.3840 (0.2616)	51.4336 (0.2879)
Maximum Adult Educational Attainment, (Mother at Home)	7.3599 (0.0213)	7.2737 (0.0224)	7.3114 (0.0224)	7.2198 (0.0251)
Maximum Adult Educational Attainment, (Mother at Work)	7.7615 (0.0090)	7.7376 (0.0093)	7.7376 (0.0094)	7.7193 (0.0100)
Sample Size (Mother at Home)	5448	5262	4958	4249
Sample Size (Mother at Work)	18484	17856	17473	14428

*[] ~ “t” test for differences () ~ standard error

Table A2 reports estimates of the transformation technology.

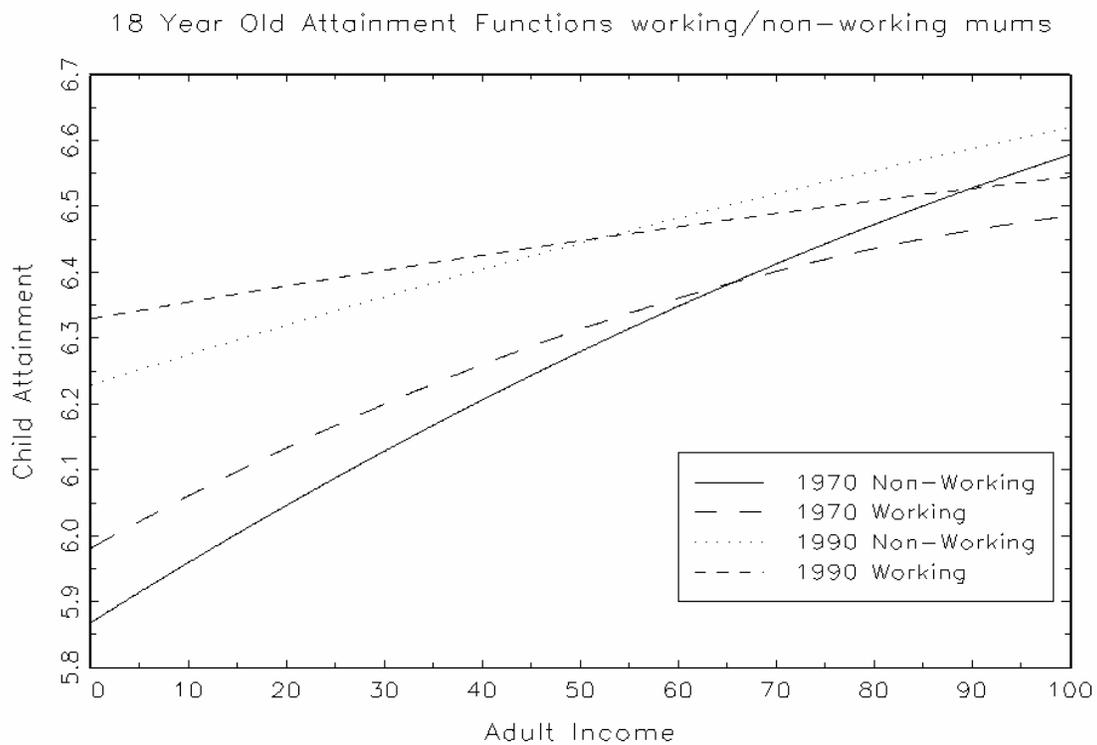
Table A2 Working and Non-Working Mums (Standard errors in brackets)

	1970		1990	
	Working	Δ Working	Working	Δ Working
[15] Constant	3.0629 (0.003664)	0.1117 (0.005370)	3.03855 (0.004691)	-0.01044 (0.009832)
Parental Income	0.005494 (0.00007294)	-0.0005064 (0.0001106)	0.005781 (0.00007854)	-0.006204 (0.0001727)
Parental Achievement	0.08340 (0.0005260)	-0.03408 (0.0007804)	0.1267 (0.0006027)	0.02638 (0.001306)
(Parental Income) ²	-0.000006042 (0.0000007197)	-0.000001222 (0.000001108)	-0.000003330 (0.0000005477)	0.000002425 (0.000001154)
(Parental Achievement) ²	-0.002509 (0.00006898)	0.0021680 (0.0001027)	-0.006053 (0.00007433)	-0.002569 (0.0001639)
Income x Achievement	-0.0004636 (0.000009315)	0.000097585 (0.00001410)	-0.0005519 (0.000009340)	0.0006027 (0.00002059)
Sigma		0.3653		0.5267
R ²		0.02074		0.007162
[16] Constant	3.4927 (0.004588)	0.07386 (0.006815)	3.5545 (0.005315)	0.1272 (0.01114)
Parental Income	0.006785 (0.00009025)	0.001529 (0.0001381)	0.007005 (0.00008874)	0.001108 (0.0001937)
Parental Achievement	0.1835 (0.0006591)	-0.03828 (0.0009924)	0.1996 (0.0006857)	-0.05465 (0.001495)
(Parental Income) ²	-0.000009897 (0.0000008937)	0.000001814 (0.000001354)	-0.000001578 (0.0000006134)	-0.0000001118 (0.000001254)
(Parental Achievement) ²	-0.007525 (0.00008633)	0.003581 (0.0001305)	-0.008922 (0.00008477)	0.005163 (0.0001884)
Income x Achievement	-0.0004973 (0.00001151)	-0.0002191 (0.00001753)	-0.0006962 (0.00001056)	-0.0001888 (0.00002300)
Sigma		0.5376		0.6530
R ²		0.05268		0.01732
[17] Constant	4.0405 (0.005600)	0.2011 (0.008383)	4.2099 (0.005693)	0.2362 (0.01211)
Parental Income	0.01424 (0.0001084)	-0.002087 (0.0001673)	0.01058 (0.00009394)	-0.008253 (0.0002175)
Parental Achievement	0.2394 (0.0008064)	-0.05807 (0.001230)	0.2166 (0.0007337)	-0.04748 (0.001619)
(Parental Income) ²	-0.00001991 (0.000001040)	-0.000003871 (0.000001600)	-0.000005401 (0.0000006587)	0.00001031 (0.000001557)
(Parental Achievement) ²	-0.01032 (0.0001059)	0.005957 (0.0001624)	-0.007080 (0.00009069)	0.004004 (0.0002042)
Income x Achievement	-0.001017 (0.00001385)	0.00009325 (0.00002135)	-0.0009735 (0.00001121)	0.0005858 (0.00002608)
Sigma		0.7471		0.7271
R ²		0.06560		0.03364
[18] Constant	4.4570 (0.007656)	-0.1724 (0.01136)	4.8701 (0.007153)	-0.1491 (0.01500)
Parental Income	0.02261 (0.0001529)	-0.005769 (0.0002360)	0.01549 (0.0001174)	-0.004678 (0.0002624)
Parental Achievement	0.2810 (0.001119)	0.1287 (0.001692)	0.2101 (0.0009264)	0.06456 (0.002026)
(Parental Income) ²	-0.00002250 (0.000001497)	-0.000009578 (0.000002341)	-0.000007962 (0.0000007626)	0.000003584 (0.000001722)
(Parental Achievement) ²	-0.008808 (0.0001485)	-0.01314 (0.0002259)	-0.002893 (0.0001150)	-0.005494 (0.0002568)
Income x Achievement	-0.002061 (0.00001978)	0.0007825 (0.00003042)	-0.001494 (0.00001411)	0.0004284 (0.00003142)
Sigma		1.03105		0.9557
R ²		0.07835		0.04592

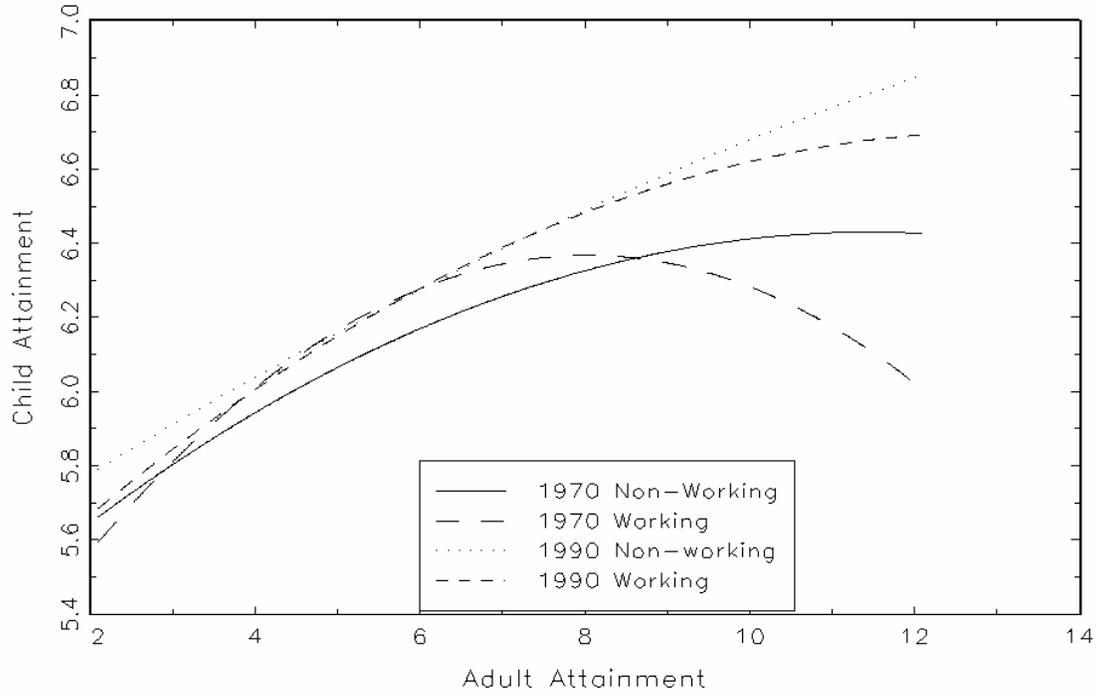
Table A3 Test for Differences in technology (χ^2 statistic and (upper tail probability))

	1970	1990
Cohort	Working/Non-Working	Working/Non-Working
15	4.3601 (0.6281)	11.5626 (0.0725)
16	2.6589 (0.8501)	4.6143 (0.5941)
17	26.5527 (0.0002)	23.0411 (0.0008)
18	17.2107 (0.0085)	5.9328 (0.4308)

The differences in transmission technology between intact families where the mother participates in the labour force and those where she does not are negligible. Even where the differences are significantly different the quantified effects are small as the following diagrams indicate.



18 Year Old Attainment Functions working/non-working mums



As table A1 indicates the child and adult educational attainments and incomes of the two family types are quite different however there is little evidence that the technological structure of the transmission mechanism changes with a parent staying at home at this stage of the child's development. If the child's attainment is the only matter for concern there appears to be a negative return to mothers staying at home if the effect of the foregone income of the mother is also imputed to the child's educational attainment outcome.