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Multidimensional Approaches to Poverty Measurement: An Empirical Analysis of Poverty in Belgium, France, Germany, Italy and Spain, based on the European Panel

Conchita D'Ambrosio

Joseph Deutsch

and

Jacques Silber

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For additional information please contact:
Jacques Silber
Department of Economics, Bar-Ilan University, 52900 Ramat-Gan, Israel.
Email: jsilber@mail.biu.ac.il
Fax: 972 3 53 53 180
Phone: 972 3 53 18 345

I) Introduction:

Recent years have witnessed an enlargement of the attributes analyzed in the studies of poverty in OECD countries and particularly so in the EU member-states. Poverty is interpreted not only as lack of income, but more generally as deprivation in various life domains. These include financial difficulties, basic needs, housing conditions, durables, health, social contacts, participation, and life satisfaction.

On one hand, more detailed information on households has become available thanks to new datasets that allow adopting a wider concept of human well-being. On the other hand, social policy gained a key role in the EU political debate, and since the European Council of Lisbon (March 2000), it was placed at the center of the EU strategy to become "the most competitive and dynamic knowledge-based economy in the world capable of sustainable economic growth with better jobs and greater social cohesion". To monitor social cohesion, multidimensional aspects of well-being were necessary. It was then acknowledged that "the number of people living below the poverty line and in social exclusion in the Union is unacceptable".

Various official reports were produced to extend the analysis of monetary poverty into a dynamic framework and to examine the interaction with non-monetary aspects of deprivation (Eurostat, 2000 and 2002). The present paper goes also in that direction. Its aim is a systematic examination of various multidimensional approaches to poverty measurement on the basis of the same data set by answering the following questions:

a) To what extent are the same households identified as poor by the various approaches?
b) Are there differences between the approaches in the determinants of household poverty?
c) Which explanatory variables have the greatest *marginal impact* as determinants of poverty.

We first review (Section II) the relevant theoretical literature on multidimensional poverty, describing three multidimensional approaches to poverty measurement: the "Fuzzy" approach, an approach derived from Information Theory and the more recent axiomatic approaches to poverty measurement[1]. Then we give (Section III) the informational basis of our analysis (the variables that were selected). In Section IV we check to what extent the different approaches identify the same households as poor while in Section V we analyze, on the basis of Logit regressions, the determinants of poverty. Finally, using the so-called Shapley decomposition procedure, we attempt to determine the *marginal* impact on poverty of the various categories of explanatory variables that were introduced in the Logit regressions (Section VI). Concluding comments are given at the end.

II) Theoretical Background:

A) The "Fuzzy Set" Approach to Poverty Analysis

The theory of "Fuzzy Sets" was developed by Zadeh (1965) on the basis of the idea that certain classes of objects may not be defined by very precise criteria of membership. In other words there are cases where one is unable to determine which elements belong to a given set and which ones do not. Zadeh himself (1965) characterized a fuzzy set (class) as "a class with a continuum of grades of membership".

Let there be a set X and let x be any element of X. A fuzzy set or subset A of X is characterized by a membership function $\mu_A (x)$ that will link any point of X with a real number in the interval [0,1]. $\mu_A (x)$ is called the degree of membership of the element x to the set A. If A were a set in the sense in which this term is usually understood, the membership function which would be associated to this set would take only the values 0 and 1. But if A is a fuzzy subset, we will say that $\mu_A (x) = 0$ if the element x does not belong to A and that $\mu_A (x) = 1$ if x *completely* belongs to A. But if $0< \mu_A (x) < 1$, x belongs only partially to A.

These simple ideas may be easily applied to the concept of poverty. In some cases an individual is in such a state of deprivation that she certainly should be considered as poor while in others her level of welfare is such that she certainly should not be classified as poor. There are however also instances where it is not clear whether a given person is

[1] In another paper, Deutsch and Silber (2003) have also used the so-called efficiency analysis approach,

poor or not. This is especially true when one takes a multidimensional approach to poverty measurement, because according to some criteria one would certainly define her as poor whereas according to others one should not regard her as poor. Such a fuzzy approach to the study of poverty has taken various forms in the literature[2].

1) The Totally Fuzzy Approach (TFA)

Cerioli and Zani (1990) were the first to apply the concept of fuzzy sets to the measurement of poverty. Their approach is called the Totally Fuzzy Approach (TFA) and the idea is to take into account a whole series of variables that are supposed to measure each a particular aspect of poverty. They considered the case of dichotomous, polytomous and continuous variables but to illustrate their approach we consider only the case of continuous variables.

Income or consumption expenditures are good examples of deprivation indicators that are continuous. Cerioli and Zani (1990) have proposed to define two threshold values x_{min} and x_{max} such that if the value x taken by the continuous indicator for a given individual is smaller than x_{min} this person would undoubtedly be defined as poor whereas if it is higher than x_{max} he certainly should be considered as not being poor.

Let X_l be the subset of individuals (households) who are in an unfavorable situation with respect to the l-th variable with $l= 1,...,k_x$. Cerioli and Zani (1990) have then proposed to define the membership function $\mu_{xl}(i)$ for individual i as

$$\mu_{xl}(i) = 1 \text{ if } 0 < x_{il} < x_{l,min}$$
$$\mu_{xl}(i) = ((x_{l,max} - x_{il})/(x_{l,max} - x_{l,min})) \text{ if } x_{il} \in [x_{l,min}, x_{l,max}]$$
$$\mu_{xl}(i) = 0 \text{ if } x_{il} > x_{l,max} \qquad (1)$$

but we decided not to include it in this paper because of space constraints.

[2] In this section we discuss only the so-called Totally Fuzzy Absolute and Relative Approaches. Other "Fuzzy" approaches have been proposed such as that of Vero and Werquin (1997) but because of space constraints this approach will not be presented. Moreover in the empirical section we used only the TFR approach.

Some authors (Cheli et al., 1994, and Cheli and Lemmi, 1995) have proposed to modify Cerioli and Zani's (1990) Totally Fuzzy Approach (TFA) and suggested what they have called the Totally Fuzzy and Relative Approach (TFR).

2) The Totally Fuzzy and Relative Approach (TFR)

As an illustration let ξ_j be the set of polytomous variables ξ_{1j} ,..., ξ_{nj} which measure the state of deprivation of the various n individuals with respect to indicator j and let F_j be the cumulative distribution of this variable. Let $\xi_{j(m)}$ with m=1 to s refer to the various values, ordered by increasing risk of poverty, which the variable ξ_j may take. Thus $\xi_{j(1)}$ represents the lowest risk of poverty and $\xi_{j(s)}$ the highest risk of poverty associated with the deprivation indicator j. The authors propose then to define the degree of poverty of individual (household) i as

$$\mu_{\Xi j}(i) = 0 \text{ if } \xi_{ij} = \xi_{j(1)}$$

and

$$\mu_{\Xi j}(i) = \mu_{\Xi j}(\xi_{j(m-1)}) + ((F_j(\xi_{j(m)}) - F_j(\xi_{j(m-1)}))/(1 - F_j(\xi_{j(1)})) \quad \text{if } \xi_{ij} = \xi_{j(m)}, \; m > 1 \qquad (2)$$

This TFR approach has the double advantage of not requiring defining threshold values and of taking a relative approach to poverty, the one which is taken in most developed countries.

The next step in the analysis is to decide how to aggregate the various deprivation indicators. Let $\mu_{\Xi j}(i)$ refer as before to the value taken by the membership function for indicator j and individual i, with j = 1 to k and i = 1 to n. Let w_j represent the weight one wishes to give to indicator j. The overall (over all indicators j) membership function $\mu_P(i)$ for individual i is then be defined as

$$\mu_P(i) = \sum_{j=1 \text{ to } k} w_j \, \mu_{\Xi j}(i) \qquad (3)$$

For the choice of the weight w_j, Cerioli and Zani (1990) as well as Cheli and Lemmi (1995) have proposed to define w_j as

$$w_j = \ln(1/\mu b_{\Xi j}) / \sum_{j=1 \text{ to } k} \ln(1/\mu b_{\Xi j}) = \ln(\mu b_{\Xi j}) / \sum_{j=1 \text{ to } k} \ln(\mu b_{\Xi j}) \quad (4)$$

where $\mu b_{\Xi j} = (1/n) \sum_{i=1 \text{ to } n} \mu_{\Xi j}(i)$ represents the fuzzy proportion of poor individuals (households) according to the deprivation indicator ξ_j . One may observe that the weight w_j is an inverse function of the average degree of deprivation in the population according to the deprivation indicator ξ_j . Thus the lower the frequency of poverty according to a given deprivation indicator, the greater the weight this indicator will receive. The idea, for example, is that if owning a refrigerator is much more common than owning a dryer, a greater weight should be given to the former indicator so that if an individual does not own a refrigerator, this rare occurrence will be taken much more into account in computing the overall degree of poverty than if some individual does not own a dryer, a case which is assumed to be more frequent.

Having computed for each individual i the value of his membership function $\mu_{\Xi j}(i)$, that is, his "degree of belonging to the set of poor", the Totally Fuzzy and Relative Approach (TFR), following in fact Cerioli and Zani (1990), defines the average value P of the membership function as

$$P = (1/n) \sum_{i=1 \text{ to } n} \mu_P(i) \quad (5)$$

B) The Information Theory Approach

1) Basic concepts:

Information theory was originally developed by engineers in the field of communications. Theil (1967) was probably the first one to apply this theory to economics. Here is a summary of the basic ideas.

Let E be an experience whose result is x_i with i = 1 to n. Let $p_i = \text{Prob}\{x=x_i\}$ be the probability that the result of the experience will be x_i with evidently $0 \leq p_i \leq 1$. When we receive the information that a given event x_i occurred, we will not be surprised if the a priori probability that such an event would occur was high. In other words in such a case the information included in the message is not very important. On the other hand if the a priori probability that an event x_i will occur is very low, knowing that this event did occur, will indeed surprise us and such a message will contain a significant amount of information.

The information included in a message should thus be an inverse function of the probability a priori p that the corresponding result will occur. Let h(p) be such an information function. The most popular form taken by h(p) is

$$H(p) = \log (1/p) = - \log (p) \qquad (6)$$

Let us now define the concept of information expectancy. Since for each event x_i whose a priori probability of occurrence is p_i the information content of a message confirming the occurrence of such an event is $h(p_i)$, the expected information H(p) will be

$$H(p) = \sum_{i=1 \text{ to } n} p_i \, h(p_i) \qquad (7)$$

with $p = (p_1,\ldots,p_n)$.

Often the term entropy is used to refer to this expected information. Note that $H(p) \geq 0$ given the properties of the information function. Combining (18) and (19) we derive

$$H(p) = \sum_{i=1 \text{ to } n} p_i \log(p_i) \qquad (8)$$

where H(p) is often called Shannon's entropy (cf., Shannon, 1948).

Note (see, Maasoumi, 1993) that this entropy may be interpreted as a measure of the uncertainty, the disorder or the volatility associated with a given distribution. It will be minimal (and equal to 0) when a specific result x_i is known to occur with certainty since in such a case a message informing us that the event x_i did indeed occur will not provide us with any information. To derive the maximal value of entropy, we have to maximize H(p) subject to the constraint that $\sum_{i=1 \text{ to } n} p_i = 1$. In such a case uncertainty will be maximal because we have no idea a priori as to which event will occur. Imposing some restrictions on the function h(p), it turns out that entropy will be maximal when all the events have the same probability, that is when $p_i = (1/n)$ for all i=1 to n. We may then derive that

$$0 \leq H(p) \leq \log (n) \qquad (9)$$

2) Measuring the distance or the divergence between distributions

Assume we make a given experiment E which has n potential results $x_1, \ldots, x_n$ with corresponding a priori probabilities $p_1, \ldots, p_n$. It may however happen that we receive some information that implies a modification of these a priori probabilities. In other words assume we have now received a message that transformed the a priori probabilities $p_1, \ldots, p_n$ into a posteriori probabilities $q_1, \ldots, q_n$ with $\sum_{i=1 \text{ to } n} q_i = 1$.

The supplement of information D(q,p) that is obtained when shifting from the distribution of a priori probabilities $\{p_1, \ldots, p_n\}$ to that of the a posteriori probabilities $\{q_1, \ldots, q_n\}$ will be expressed as

$$D(q, p) = \sum_{i=1 \text{ to } n} q_i \log (q_i / p_i) \qquad (10)$$

D(q,p) represents actually the expected information of a message transforming the a priori probabilities $\{p_1, \ldots, p_n\}$ into the a posteriori probabilities $\{q_1, \ldots, q_n\}$. Note that this supplement of information D(p,q) may also be considered as a measure of the divergence between the distributions $\{p_1, \ldots, p_n\}$ and $\{q_1, \ldots, q_n\}$ or as the difference between the entropy corresponding to the distribution $\{p_1, \ldots, p_n\}$ and that relative to the distribution $\{q_1, \ldots, q_n\}$, assuming the weights to be chosen are those corresponding to the latter distribution.

This measure of divergence D(p,q) is generally positive and will be equal to zero only in the very specific case where $p_i = q_i$ for all i (i=1 to n), that is when the new message does not modify any of the a priori probabilities.

$D(p,q)$ will be maximal when there is a result x_i such that $q_i > p_i = 0$ because in such a case the probability a priori that the event x_i would occur was nil whereas now, after reception of the correcting message, the probability that it will occur is not nil any more and thus the degree of surprise may be considered as infinite.

An interesting measure of divergence is the Kullback-Leibler-Jeffreys measure $J(q,p)$ (see, Kullback and Leibler, 1951, and Jeffreys,1967) which is defined as

$$J(q,p) = D(q,p) + D(p,q) = \sum_{i=1 \text{ to } n} (q_I - p_i)\,[\log(q_i) - \log(p_i)] \qquad (11)$$

Maasoumi (1986) generalized this idea and proposed two additional classes of measures. The first one $D_k(q,p)$ is defined as

$$D_k(q,p) = (1/(k-1))[\ \sum_{i=1 \text{ to } n} \{[((q_i)^k)/((p_i)^{k-1})] - 1\} \qquad (12)$$

with $k \neq 1$. Note that when $k \to 1$, $D_k(q,p) \to D(q,p)$.

The other class of generalized divergence measure mentioned by Maasoumi is $D_\gamma(q,p)$ with

$$D_\gamma(q,p) = [1/(\gamma(\gamma+1))]\ \{\ \sum_{i=1 \text{ to } n} q_i\,[((q_i/p_i)^\gamma) - 1\} \qquad (13)$$

with $\gamma \neq 0, -1$. Note that as $\gamma \to 0$, $D_\gamma(q,p) \to D(p,q)$. One may also observe that as
$\gamma \to 0$, $D_\gamma(q,p) \to D(p,q)$.

3) Information Theory and Multidimensional Measures of Inequality:

The idea of using concepts borrowed from information theory to define multidimensional measures of welfare and inequality was originally proposed by Maasoumi (1986). He suggested proceeding in two steps. First a procedure would be defined that would allow aggregating the various indicators of welfare to be taken into account. Second an inequality index would be selected to estimate the degree of multidimensional inequality.

Assume n welfare indicators have been selected, whether they be of a quantitative or qualitative nature. Call x_{ij} the value taken by indicator j for individual (or household) i, with i = 1 to n and j = 1 to m. The various elements x_{ij} may be represented by a matrix X = $[x_{ij}]$ where the i^{th} line will give the welfare level of individual i according to the various m indicators, while the j^{th} column the distribution among the n individuals of the welfare level corresponding to indicator j.

Maasoumi's idea is to replace the m pieces of information on the values of the different indicators for the various individuals by a composite index x_c which will be a vector of n components, one for each individual. In other words the vector $(x_{i1},\ldots x_{im})$ corresponding to individual i will be replaced by the scalar x_{ci}. This scalar may be considered either as representing the utility that individual i derives from the various indicators or as an estimate of the welfare of individual i, as an external social evaluator sees it.

The question then is to select an "aggregation function" that would allow to derive such a composite welfare indicator x_{ci}. Maasoumi (1986) suggested finding a vector x_c that would be closest to the various m vectors x_{ij} giving the welfare level the various individuals derive from these m indicators. To define such a "proximity" Maasoumi proposes a multivariate generalization of the generalized entropy index $D_\gamma(q,p)$ that is expressed as

$$D_\gamma(x_c, X; \alpha) = (1/(\gamma(\gamma+1))) \sum_{j=1 \text{ to } m} \alpha_j \{\sum_{i=1 \text{ to } n} x_{ci} [(x_{ci} / x_{ij})^\gamma - 1]\} \qquad (14)$$

with $\gamma \neq 0, -1$, and where α_j represents the weight to be given to indicator j.

When $\gamma \rightarrow 0$ or -1, one obtains the following indicators

$$D_0(x_c, X; \alpha) = \sum_{j=1 \text{ to } m} \alpha_j [\sum_{i=1 \text{ to } n} x_{ci} \log(x_{ci} / x_{ij})] \qquad (15)$$

and

$$D_{-1}(x_c, X; \alpha) = \sum_{j=1 \text{ to } m} \alpha_j [\sum_{i=1 \text{ to } n} x_{ij} \log(x_{ij} / x_{ci})] \qquad (16)$$

The minimization of the "proximity" defines a composite index x_{ci} in each of the three cases corresponding to expressions (26) to (28).
In the first case x_{ci} is defined as

$$x_{ci} \propto [\sum_{j=1 \text{ to } m} \delta_j \ (x_{ij})^{-\gamma}]^{-(1/\gamma)} \qquad (17)$$

In the second case, when $\gamma \rightarrow 0$, one gets

$$X_{ci} \propto [\Pi_{j=1 \text{ to } m} \ (x_{ij})^{\delta j} \qquad (18)$$

Finally in the case where $\gamma \rightarrow -1$, one obtains

$$x_{ci} \propto [\sum_{j=1 \text{ to } m} \delta_j \ (x_{ij})] \qquad (19)$$

In expressions (29) to (31) δ_j is defined as the normalized weight of indicator j, that is $\delta_j = \alpha_j / \sum_{j=1 \text{ to } m} \alpha_j$.
Thus it turns out that the composite indicator x_c is a weighted average of the different indicators. In the general case (29) it is an harmonic mean; in the case where $\gamma \rightarrow 0$, it is a geometric mean while in that where $\gamma \rightarrow -1$, it is an arithmetic mean. Moreover it is easy to interpret this composite welfare indicator as a utility function of the CES type with an elasticity of substitution $\sigma = 1 / (1+\gamma)$ when $\gamma \neq 0, -1$, as a Cobb-Douglas utility when $\gamma \rightarrow 0$, and as a linear utility function when $\gamma \rightarrow -1$.
Having derived a composite index x_{ci} for each individual i, one may measure inequality by applying generalized entropy inequality indices that were defined by Shorrocks (1980) and applied to the multidimensional case by Maasoumi (1986).

4) Information Theory and a Multidimensional Approach to Poverty Measurement:

Although Information Theory has been applied several times to the analysis of multidimensional inequality (see, the survey by Maasoumi, 1999), it seems to have been used only rarely in the study of multidimensional poverty (see, however, Miceli, 1997). Miceli has suggested deriving the measurement of multidimensional poverty from the distribution of the composite index X_C whose definition is given in expressions (29) to (31). Such a choice implies evidently that a decision has to be made concerning the selection of the weights δ_j to be given to the various indicators x_{ij} (the subindex i referring to the individual while the subindex j denotes the indicator) as well as to the parameter γ. We decided to give an equal weight (1/m) to all the indicators j (where m refers to the total number of indicators) and we assumed that the parameter γ was equal to –1 (the case of an arithmetic mean).

Once the composite indicator X_C is defined, one still has to define a procedure to identify the poor. Here again we will follow Miceli (1997) and adopt the so-called "relative approach" which is commonly used in the uni-dimensional analysis of poverty. In other words we will define the "poverty line" as being equal to some percentage of the median value of the composite indicator X_C. More precisely we have chosen as cutting points a "poverty line" assumed to be equal to 70% the median value of the distribution of the composite index X_C. In other words any household i for which the composite index X_{Ci} will be smaller than the "poverty line" will be identified as poor.

C) Axiomatic Derivations of Multidimensional Poverty Indices

Very few studies have attempted to derive axiomatically multidimensional indices of poverty. Tsui (2002) made recently such an attempt, following his earlier work on axiomatic derivations of multidimensional inequality indices (see, Tsui, 1995 and 1999) but it seems that Chakravarty, Mukherjee and Ranade (1998) were the first to publish an article on the axiomatic derivation of multidimensional poverty indices.

The basic idea behind Chakravarty et al. (1998) as well as Tsui's (2002) approach is as follows. Both studies view a multidimensional index of poverty as an aggregation of shortfalls of all the individuals where the shortfall with respect to a given need reflects the fact that the individual does not have even the minimum level of the basic need. Let z = $(z_1,\ldots,z_k)$ be the k-vector of the minimum levels of the k basic needs and $x^i=(x_{i1},\ldots,x_{ik})$

the vector of the k basic needs of the i^{th} person. Let X be the matrix of the quantities x_{ij} which denote the amount of the j^{th} attribute accruing to individual i.

Chakravarty et al. (1998) defined then a certain number of desirable properties that a multidimensional poverty measure should have, on the basis of which they derived axiomatically two families of multidimensional poverty indices.

The first family of indices may be expressed as

$$P(X;z) = (1/n) \sum_{j=1 \text{ to } k} \sum_{i \in Sj} a_j [1 - (x_{ij}/z_j)^e] \quad (20)$$

where s_j is the set of poor people with respect to attribute j.

This index is a multidimensional extension of the subgroup decomposable index suggested by Chakravarty (1983).

When e=1 we get

$$P(X;z) = (1/n) \sum_{j=1 \text{ to } k} \sum_{i \in Sj} a_j [(z_j - x_{ij})/z_j)] = \sum_{j=1 \text{ to } k} a_j H_j I_j \quad (21)$$

where $H_j = (q_j/n)$ and I_j are respectively the head-count ratio and the poverty-gap ratio for attribute j ($I_j = \sum_{i \in Sj} [(z_j - x_{ij})/(q_j z_j)]$).

The second family of indices is expressed as

$$P_\alpha(X;z) = (1/n) \sum_{j=1 \text{ to } k} \sum_{i \in Sj} a_j [1 - (x_{ij}/z_j)]^\alpha \quad (22)$$

This index is a multidimensional generalization of the Foster-Greer and Thorbecke (1984) subgroup decomposable index (known under the name of FGT index).

In the empirical investigation that will be reported below we used this multidimensional generalization of the FGT index with the parameter α equal to 2. We assumed that for each indicator the “poverty line” was equal to half the mean value of the indicator. We also decided to give an equal weight was given to all the indicators. Finally an individual was considered as poor when the expression $\sum_{j=1 \text{ to } k} a_j [1 - (x_{ij}/z_j)]^2$ was greater than the value of this expression for the 75^{th} percentile (in other words we assumed that 25% of the individuals were poor).

III) The Information Basis for the Derivation of Multidimensional Poverty Indices

The empirical analysis that will be presented below is based essentially on the third wave of the European Panel. The following 18 indicators have been taken into account to derive multidimensional measures of poverty:

1) *Indicators of Income*:
   - total net household income

2) *Indicators of Financial Situation*:
   - ability to make ends meet
   - can the household afford paying for a week's annual holiday away from home
   - can the household afford buying new rather than second-hand clothes?
   - can the household afford eating meat, chicken or fish every second day, if wanted?
   - has the household been unable to pay scheduled rent for the accommodation for the past 12 months?
   - has the household been unable to pay scheduled mortgage payments during the past 12 months?
   - has the household been unable to pay scheduled utility bills, such as electricity, water or gas during the past 12 months?

3) *Indicators of quality of accommodation*:
   - does the dwelling have a bath or shower?
   - does the dwelling have shortage of space?
   - does the accommodation have damp walls, floors, foundations, etc…?

4) *Indicators on ownership of durables:*
   - possession of a car or a van for private use
   - possession of a color TV
   - possession of a telephone

5) *Indicators of health:*

   - how is the individual's health in general?
   - is the individual hampered in his/her daily activities by any physical or mental health problem, illness or disability?

6) *Indicators of social relations:*

   - how often does the individual meet friends or relatives not living with him/her, whether at home or elsewhere?

7) *Indicators of satisfaction:*

   - is the individual satisfied with his/her work or main activity?

Multidimensional measures of poverty have been computed for the following countries:

- Belgium
- France
- Germany
- Italy
- Spain

IV) Do the Different Multidimensional Indices Identify the Same Households as Poor:

To check the degree of overlapping between the various multidimensional poverty indices we have assumed that 25% of the individuals were poor, whatever the index that was selected. We then checked to which degree two indices identified the same households as poor. The results of this analysis are given in Table 1.

It appears that, on average, when comparing two of the three approaches, only 80% (19.8% out of the 25%) of the households defined as poor are the same households. The highest common percentage (20.5% our of 25%) is observed when comparing, for the five countries examined, the TFR with the Information Theory approaches. In the two other cases the common percentages are somehow lower (19.3% when comparing the TFR and FGT approaches and 19.5% when comparing the Information theory and FGT approaches). Note also that the common percentage is highest for Belgium (20.5% out of 25%) and lowest for France (19.1 out of 25%)[3].

In the next section an attempt is made, for each of the three approaches, to determine the impact of the various explanatory variables on the probability that an individual is considered as poor.

[3] Similar results were obtained when computing the correlation coefficients between two approaches. Because of the lack of space, these results are not reported here.

Table 1: Degree of overlapping between the various multidimensional poverty indices (Percentage of households defined as poor by two multidimensional indices, assuming 25% of the households are poor).

	Belgium	France	Germany	Italy	Spain	Average of binary comparisons
TFR index and Information theory based index	19.5	19.0	18.3	19.7	19.9	19.3
TFR index and Generalization of FGT index	21.6	19.4	21.7	19.8	19.8	20.5
Information theory based index and Generalization of FGT index	20.5	18.9	19.2	18.8	20.3	20.3
Average of countries	20.5	19.1	19.7	19.4	20.0	19.8

V) Results of the Logit regressions:

The following exogenous variables have been taken into account: the size of the household and its square, the age of the individual and its square, the gender, the marital status (three dummy variables) and the status at work (two dummy variables) of the individual.

Results of the Logit Regressions

In each Logit regression the dependent variable is the probability that an individual is considered as poor (the variable is equal to 1 if he/she is poor, to 0 otherwise). The results of these estimations are given for Belgium, France, Germany, Italy and Spain in Tables 2-A to 2-E, giving in each case the coefficients of the regression obtained on the basis of the three multidimensional approaches to poverty measurement: the Totally Fuzzy and Relative (TFR), the information theory and the axiomatic approach (generalization of the FGT index).

To have an idea of the goodness of fit of the logit regressions we used a criterion that is similar to the R-square used in linear regressions. The idea is to compute the maximal value of the log-likelihood (ln L) and compare it with the log likelihood obtained when only a constant term is introduced ($\ln L_0$). The likelihood ratio LRI is then defined as

$$LRI = 1 - (\ln L / \ln L_0) \qquad (23)$$

The bounds of this measure are 0 and 1 ((see, Greene, 1993, pages 651-653).

The value of the likelihood ratio LRI is given in Tables 2-A to 2-E.

These tables indicate that in most cases there is, ceteris paribus, a U-shaped relationship between the size of the household to which the individual belongs and the probability that he/she will be considered as poor. Such a link is observed for the five countries, whenever the generalized FGT approach is adopted. The TFR approach does not show such a relationship in the case of Belgium and France. The Information theory approach shows such a U-shaped relationship only in the cases of Germany and Italy.

There seems also to be a U-shaped relationship between the age of the individual and the probability that he/she will be considered as poor. The FGT approach gives such a link for all the five countries examined. The TFR approach shows similar results in four of the five cases, Germany being the only country for which such a relationship is not observed. The Information Theory approach however indicates such a U-shaped link only in the case Italy. Moreover for Germany it curiously gives an inverted-U relationship between the age of the individual and the probability that he/she is considered as poor.

As far as the other explanatory variables are considered we have introduced interaction terms between the gender of the individual and his/her marital status so that we analyze here the joint impact of these variables on the probability that the individual is considered as poor. This impact varies actually from one country to the other and sometimes there are even differences between the approaches adopted. For Belgium (see, Table 2-A) the following observations may be made, assuming the vector of the coefficients of these variables and their interaction is significantly different from zero. First only the generalized FGT approach shows really a higher probability of being poor among single males than among single females. This probability is also higher among married females according to the FGT and information theory approach but the result is the opposite for the FGT approach. The three approaches indicate a higher probability of being poor among divorced men than among divorced women, the same being true when comparing widower and widows. Finally the probability of being considered as poor is the lowest for married individuals and the highest for singles.

For France (see, Table 2-B) the probability of being poor seems to be higher among single males than among single females. The same differences between the genders are observed when comparing married men and married women. For divorced individuals, poverty is higher among women according to the TFR and Information Theory approach but the contrary is true according to the FGT approach. Finally it seems that the probability of being considered as poor is higher among widowers than among widows. It appears also that in France poverty is highest among divorced individuals, whatever their gender, and lowest among married people.

When we look at the results for Germany (see, Table 2-C) we see that for those who are single the probability of being poor is highest among males. This gender difference is also observed when comparing married men and women as well as widowers and widows. Among divorced individuals the TFR and Information theory approach show a higher degree of poverty among females but the contrary is true when using the FGT approach. In Germany the probability of being poor is the lowest among married and the highest among divorced individuals.

The results for Italy (see, Table 2-D) indicate that the probability of being poor is higher among single men than among single women. The contrary is observed among married individuals, whatever the approach that is used. Among divorced individuals the probability of being considered as poor is higher among males, this being also the case when comparing widowers with widows. No clear conclusions however may be drawn as far as the impact of the marital status on the probability of being poor is concerned, the gender playing here an important role.

Finally when looking at the Spanish data (see, Table 2-E) we observe that only the FGT approach seems to show a higher probability of being poor among single males than among single females. All three approaches show however a higher probability of being considered as poor among married males than among married females. Among divorced individuals this probability is higher among females according to the TFR and Information theory approach but the opposite is true when using the FGT approach. Among widowers and widows the impact of the gender depends also on the approach adopted: the probability of being poor is higher among widows according to the TFR approach but the opposite is true when adopting the Information theory or FGT approach. As far as the impact of the marital status is concerned the probability of being poor is highest among divorced and lowest among married individuals.

Concerning the effect of the work status we observe in all countries that the probability of being poor is highest, as expected, among unemployed individuals (the category of reference in the regressions). It is lowest (in most cases) among self-employed.

To better analyze the impact of the explanatory variables on the probability of being poor we apply in the next section the so-called Shapley decomposition procedure, a technique that will allow us determining the exact *marginal* impact on the probability of being poor of each of the five categories of explanatory variables: household size, age, gender, marital status and work status.

Table 2-A: Results of the Logit Regressions for Belgium

Explanatory variables	TFR: coef.	TFR: t-values	Inf.Th.: coef.	Inf.Th. t-val.	F.G.T.: coef.	F.G.T.: t-values
constant	2.05309	2.39	0.77765	0.69	3.87636	6.13
Household size	-0.12100	-0.64	-0.22763	-0.97	-0.34960	-2.33
Square of household size	0.04211	1.71	0.05259	1.76	0.05866	2.87
Age	-0.10416	-3.48	-0.07112	-1.83	-0.15955	-7.36
Square of age	0.00073	2.63	0.00047	1.32	0.00150	7.14
Male	0.08752	0.33	-0.00280	-0.01	0.60513	2.81
Married	-0.58326	-0.73	-0.01044	-0.01	-0.29108	-0.58
Divorced	0.30657	0.27	-1.45477	-0.66	-0.96248	-1.10
Widower	-0.34303	-0.40	-1.33642	-1.08	-0.86668	-1.44
Interaction Married/ Male	-0.23089	-0.34	-0.69015	-0.63	-0.29181	-0.70
Interaction Divorced/ Male	0.37116	0.57	1.40434	1.22	1.00108	1.83
Interaction Widower/ Male	0.49156	1.02	1.29518	1.94	0.79772	2.22
Salaried Worker	-1.80857	-3.70	-3.22800	-4.35	-0.81437	-2.43
Self-employed	-2.52832	-2.80	-4.75431	-2.95	-1.11522	-1.86
Interaction: Salaried/ Male	0.01264	0.04	0.66678	1.46	-0.22716	-0.97
Interaction: Self-employed/ Male	0.79855	1.19	1.92892	1.96	0.14192	0.29
Likelihood Ratio LRI	0.13390		0.20304		0.15808	
Number of Observations	2395		2395		2395	

Table 2-B: Results of the Logit Regressions for France

Explanatory variables	TFR: coef.	TFR: t-values	Inf.Th.: coef.	Inf.Th. t-val.	F.G.T.: coef.	F.G.T.: t-values
constant	-0.29848	-0.70	-2.81969	-5.33	1.08906	3.22
Household size	-0.00055	-0.01	0.26864	2.67	-0.35365	-3.70
Square of household size	0.03323	2.53	0.00565	0.52	0.07375	5.72
Age	-0.04876	-3.09	0.00086	0.05	-0.07042	-5.80
Square of age	0.00039	2.61	0.00004	0.21	0.00066	5.75
Male	0.10311	0.72	0.31901	1.87	0.51112	4.47
Married	-1.75107	-2.80	-1.24785	-1.48	-0.80734	-1.51
Divorced	1.86307	2.61	1.80340	1.97	0.63829	0.97
Widower	0.50634	1.12	0.28871	0.53	0.59320	1.64
Interaction Married/ Male	0.82957	1.43	0.37081	0.47	0.15497	0.31
Interaction Divorced/ Male	-0.66499	-1.37	-0.86732	-1.37	0.02641	0.06
Interaction Widower/ Male	-0.07635	-0.28	0.20379	0.64	-0.09370	-0.42
Salaried Worker	-1.41815	-5.00	-1.65631	-4.53	-1.22752	-5.74
Self-employed	-1.82291	-2.50	-2.06412	-2.12	-1.94644	-3.64
Interaction: Salaried/ Male	0.23432	1.18	0.07176	0.28	-0.04473	-0.30
Interaction: Self-employed/ Male	0.71900	1.19	0.53647	0.66	0.78749	1.74
Likelihood Ratio LRI	0.08293		0.10842		0.14247	
Number of Observations	6284		6284		6284	

Table 2-C: Results of the Logit Regressions for Germany

Explanatory variables	TFR: coef.	TFR: t-values	Inf.Th.: coef.	Inf.Th. t-val.	F.G.T.: coef.	F.G.T.: t-values
constant	-0.55457	-0.73	-5.52145	-4.58	1.81527	3.67
Household size	-0.32875	-1.99	-0.72504	-3.10	-0.62671	-4.60
Square of household size	0.07735	3.48	0.13747	4.51	0.10922	5.48
Age	0.00627	0.19	0.19490	4.04	-0.11097	-5.46
Square of age	-0.00036	-1.06	-0.00221	-4.54	0.00111	5.23
Male	0.16318	0.74	0.45502	1.41	1.19530	7.44
Married	-0.35122	-0.70	0.54989	0.71	0.50780	1.68
Divorced	0.68598	0.69	1.82271	1.31	-0.48413	-0.66
Widower	0.52661	0.80	1.25362	1.40	0.41231	0.89
Interaction Married/ Male	-0.69558	-2.10	-1.28033	-2.55	-0.96564	-5.04
Interaction Divorced/ Male	-0.39932	-0.66	-1.22406	-1.37	0.46450	1.01
Interaction Widower/ Male	-0.11565	-0.31	-0.49800	-1.00	0.04267	0.16
Salaried Worker	-1.85829	-4.64	-3.06154	-4.92	-0.32709	-1.34
Self-employed	-2.49428	-2.76	-2.86997	-3.94	-0.69513	-1.26
Interaction: Salaried/ Male	0.24471	0.97	0.61487	1.59	-0.50270	-3.16
Interaction: Self-employed/ Male	0.95473	1.58	-1.28033	-2.55	-0.33825	-0.82
Likelihood Ratio LRI	0.11926		0.17594		0.16212	
Number of Observations	4396		4396		4396	

Table 2-D: Results of the Logit Regressions for Italy

Explanatory variables	TFR: coef.	TFR: t-values	Inf.Th.: coef.	Inf.Th. t-val.	F.G.T.: coef.	F.G.T.: t-values
constant	0.78306	1.53	-1.15255	-1.85	2.06227	5.28
Household size	-0.55139	-5.00	-0.36531	-3.28	-0.77544	-8.52
Square of household size	0.08990	6.34	0.06668	4.75	0.10476	8.52
Age	-0.07341	-4.42	-0.04762	-2.48	-0.08481	-6.79
Square of age	0.00065	4.41	0.00053	3.23	0.00080	7.16
Male	0.17322	1.11	0.45808	2.62	0.41638	3.38
Married	-0.21050	-0.57	0.48566	1.13	0.38341	1.41
Divorced	0.05216	0.06	1.48376	1.72	-0.07586	-0.12
Widower	-1.13976	-0.72	0.26080	0.16	-0.61068	-0.60
Interaction Married/ Male	-0.20808	-0.81	-0.71432	-2.33	-0.51747	-2.76
Interaction Divorced/ Male	0.34086	0.69	-0.43221	-0.81	0.25481	0.63
Interaction Widower/ Male	0.58028	0.65	-0.29014	-0.30	0.24933	0.42
Salaried Worker	-0.31695	-0.94	-0.76584	-1.84	-0.21291	-0.93
Self-employed	-0.82192	-1.74	-1.20771	-2.01	-0.30616	-0.81
Interaction: Salaried/ Male	-0.46297	-1.76	-0.35997	-1.09	-0.46250	-2.63
Interaction: Self-employed/ Male	0.03371	0.09	0.03428	0.07	-0.59514	-1.82
Likelihood Ratio LRI	0.06820		0.10906		0.10172	
Number of Observations	7063		7063		7063	

Table 2-E: Results of the Logit Regressions for Spain

Explanatory variables	TFR: coef.	TFR: t-values	Inf.Th.: coef.	Inf.Th. t-val.	F.G.T.: coef.	F.G.T.: t-values
constant	-0.09133	-0.19	-1.86868	-3.14	1.82823	5.09
Household size	-0.17296	-1.64	-0.09067	-0.78	-0.24172	-3.00
Square of household size	0.03539	2.97	0.02381	1.88	0.02903	2.98
Age	-0.04396	-2.64	0.00188	0.10	-0.07669	-6.23
Square of age	0.00039	2.49	0.00003	0.17	0.00075	6.29
Male	-0.03969	-0.26	-0.01316	-0.08	0.30671	2.56
Married	-1.12560	-3.12	-1.13306	-2.73	-0.67259	-2.60
Divorced	0.99732	1.35	1.69096	2.08	0.58653	0.94
Widower	0.13689	0.11	-1.07070	-0.68	0.21249	0.24
Interaction Married/ Male	0.68228	2.46	0.77730	2.52	0.38947	1.96
Interaction Divorced/ Male	-0.19864	-0.44	-0.61016	-1.20	0.02060	0.05
Interaction Widower/ Male	-0.08729	-0.12	0.79087	0.91	-0.21452	-0.41
Salaried Worker	-0.83704	-2.53	-1.41602	-3.57	-1.16326	-5.48
Self-employed	-1.47648	-3.17	-2.18810	-3.68	-1.85094	-5.78
Interaction: Salaried/ Male	-0.36464	-1.43	-0.08100	-0.27	-0.17017	-1.08
Interaction: Self-employed/ Male	0.31376	0.89	0.54786	1.27	0.36380	1.49
Likelihood Ratio LRI	0.07360		0.09108		0.14178	
Number of Observations	6004		6004		6004	

VI) The Shapley Approach to Index Decomposition and its Implications for Multidimensional Poverty Analysis:

a) The Concept of Shapley Decomposition:

Let an index I be a function of n variables and let I_{TOT} be the value of I when all the n variables are used to compute I. I could for example be the R-square of a regression using n explanatory variables, any inequality index depending on n income sources or on n population subgroups.

Let now $I_{/k}^{k}$ (i) be the value of the index I when k variables have been dropped so that there are only (n-k) explanatory variables and k is also the rank of variable i among the n possible ranks that variable i may have in the n! sequences corresponding to the n! possible ways of ordering n numbers. We will call $I_{/(k-1)}^{k}$ (i) the value of the index when only (k-1) variables have been dropped and k is the rank of the variable (i).

Thus $I_{/1}^{1}$ (i) gives the value of the index I when this variable is the first one to be dropped. Obviously there are (n-1)! possibilities corresponding to such a case. $I_{/0}^{1}$ (i) gives then the value of the index I, when the variable i has the first rank and no variable has been dropped. This is clearly the case when all the variables are included in the computation of the index I.

Similarly $I_{/2}^{2}$ (i) corresponds to the (n-1)! cases where the variable i is the second one to be dropped and two variables as a whole have been dropped. Clearly $I_{/2}^{2}$ (i) can also take (n-1)! possible values. $I_{/1}^{2}$ (i) gives then the value of the index I when only one variable has been dropped and the variable i has the second rank. Here also there are (n-1)! possible cases.

Obviously $I_{/(n-1)}^{n}$ (i) corresponds to the (n-1)! cases where the variable i is dropped last and is the only one to be taken into account. If I is an inequality index, it will evidently be equal to zero in such a case. But if it is for example the R-square of a regression it would give us the R-square when there is only one explanatory variable, the variable i. Obviously $I_{/n}^{n}$ (i) gives the value of the index I when variable i has rank n and n variables have been dropped, a case where I will always be equal to zero by definition since no variable is left.

Let us now compute the contribution $C_j(i)$ of variable i to the index I, assuming this variable i is dropped when it has rank j. Using the previous notations we define $C_j(i)$ as

$$C_j(i) = (1/n!) \sum_{h=1 \text{ to } (n-1)!} [I_{/(j-1)}^{j} (i) - I_{/j}^{j} (i)]^h \qquad (24)$$

where the superscript h referes to one of the (n-1)! cases where the variable i has rank j. The overall contribution of variable i to the index I may then be defined as

$$C(i) = (1/n!) \sum_{k=1 \text{ to } n} C_k(i) \qquad (25)$$

It is then easy to prove that

$$I = (1/n!) \sum_{i=1 \text{ to } n} C(i) \qquad (26)$$

b) Determining the Marginal Impact of the Different (Categories of) Explanatory Variables in the Logit Regression:

The Shapley decomposition previously described has been applied to the various Logit regressions that were estimated. To simplify the computations, we did not compute the *marginal* impact of each variable but the *marginal* impact of each category of explanatory variables: household size, age, gender, marital status and work status.

As indicated before, the likelihood ratio LRI that was defined previously will serve as indicator of the goodness of fit of the logit regressions. The marginal impact of each category of variables that was estimated using the Shapley decomposition procedure will then give their (marginal) contribution to this Likelihood Ratio and the sums of these contributions will be equal, as was just mentioned, to the Likelihood Ratio itself.

c) The empirical investigation:

Table 3 reports for each country and approach the *marginal* impact of each of the five categories of explanatory variables on the Likelihood Ratio LRI that was defined previously. This marginal impact is given both in absolute value and in percentage terms.

As far as the Likelihood Ratio is concerned we may observe that the best results are obtained for Belgium and Germany with the Information Theory and Generalized FGT approaches. The greatest marginal impacts are those of the work status and of the marital status, the impact of the former category of variables being generally higher than that of the latter. This is not too surprising given that one expects a very important effect of unemployment (one of the dummy variables of the status at work) on poverty. The impact of the marital status is not surprising either, because it is well-known that married individuals have generally a higher level of welfare than singles, divorced or widowers (widows). The relative importance of the other three categories of explanatory variables depends both on the country examined and the approach adopted. Among these three categories of variables, the impact of the gender is generally the weakest and that of the size of the household the strongest but there are many exceptions.

In fact there is one variable, the level of education, that we had planned to introduce as explanatory variable but could not for two reasons. First education is generally measured differently from one country to the other. Second when a common definition was adopted there were too many missing observations so that finally we had to drop this variable. It is in fact very likely that education has an important impact on poverty (see, Deutsch and Silber, 2003). Moreover it is quite possible that its introduction in the Logit regressions would have modified the impact of the gender on poverty. We suspect that, had we been able to introduce this variable, there would have been less cases where the probability of being poor is, ceteris paribus, higher among males. One should not forget that today in many Western countries the average level of education is higher among females.

Table 3: Shapley Decompositions for the Logit Regressions.
Marginal Impact[4] of the Five Categories of Explanatory Variables
on the Likelihood Ratio LRI

Country	Multi-dimensional Poverty Index	Marg. Impact of the Size of Household	Marg. Impact of the Age	Marg. Impact of the Gender	Marg. Impact of the Marital Status	Marg. Impact of the Status at Work	Likeli-hood Ratio LRI
Belgium	TFR	1.1	1.7	1.7	3.7	5.3	13.4
		(8.2)	(12.6)	(12.6)	(27.4)	(39.3)	(100)
Belgium	Inf. Th.	1.3	1.2	3.8	5.1	8.9	20.3
		(6.4)	(5.9)	(18.7)	(25.1)	(43.8)	(100)
Belgium	FGT	1.9	3.5	3.2	3.4	3.8	15.8
		(12.0)	(22.2)	(20.3)	(21.5)	(24.1)	(100)
France	TFR	1.3	0.7	0.9	2.9	2.5	8.3
		(15.7)	(8.4)	(10.8)	(34.9)	(30.1)	(100)
France	Inf. Th.	1.2	1.0	1.3	2.4	4.9	10.8
		(11.1)	(9.3)	(12.0)	(22.2)	(45.4)	(100)
France	FGT	2.4	2.1	2.1	2.9	4.7	14.2
		(16.9)	(14.8)	(14.8)	(20.4)	(33.1)	(100)
Germany	TFR	2.0	1.2	1.0	4.4	3.3	11.9
		(16.8)	(10.1)	(8.4)	(37.0)	(27.7)	(100)
Germany	Inf. Th.	3.8	1.4	1.2	4.2	7.0	17.6
		(21.6)	(8.0)	(6.8)	(23.9)	(39.8)	(100)
Germany	FGT	2.9	2.3	2.9	4.5	3.6	16.2
		(17.9)	(14.2)	(17.9)	(27.8)	(22.2)	(100)
Italy	TFR	1.9	1.1	0.6	1.3	1.9	6.8
		(27.9)	(16.2)	(8.8)	(19.1)	(27.9)	(100)
Italy	Inf. Th.	1.6	2.7	0.9	1.7	4.0	
		(14.7)	(24.8)	(8.3)	(15.6)	(36.7)	
Italy	FGT	2.5	2.3	0.8	1.6	2.9	
		(24.8)	(22.8)	(7.9)	(15.8)	(28.7)	
Spain	TFR	0.9	0.9	0.4	1.0	4.1	
		(12.3)	(12.3)	(5.5)	(13.7)	(56.2)	
Spain	Inf. Th.	0.8	1.2	0.6	0.9	5.6	
		(8.8)	(13.2)	(6.6)	(9.9)	(61.5)	
Spain	FGT	1.6	3.1	1.2	1.3	7.0	
		(11.3)	(21.8)	(8.5)	(9.2)	(49.3)	

[4] The numbers in parenthesis on the separate lines give the marginal impact in relative terms.

VII) Concluding comments

This paper had three goals. First we wanted to compare three multidimensional approaches to poverty and check to what extent they identified the same households as poor. Second we planned to better understand the determinants of poverty by estimating Logit regressions with five categories of explanatory variables: size of the household, age of the head of the household, his/her gender, marital status and status at work. Third we wished to introduce a decomposition procedure introduced recently in the literature, the so-called Shapley decomposition, in order to determine the exact marginal impact of each of the categories of explanatory variables. Our empirical analysis was based on data made available by the European panel. We used its third wave and selected five countries: Belgium, France, Germany, Italy and Spain.

The following conclusions may be drawn. First the three multidimensional approaches adopted (the Totally Fuzzy and Relative Approach, that based on Information Theory and the axiomatically derived approach using the generalized FGT index) indicate that, on average, 80% of the households defined as poor by two approaches are identical.

Second the impact of the explanatory variables introduced in the Logit regressions may be summarized as follows. There seems generally to be a U-shaped relationship between poverty and the size of the household as well as between poverty and the age of the individual. Unemployed individuals have a much higher probability, ceteris paribus, of being poor while the probability of being poor seems to be lower among self-employed than among salaried workers. Finally, ceteris paribus, married individuals, whatever their gender, have a lower probability of being poor than singles, divorced or widowers (widows). Differences between the three other categories of marital status seem to depend both on the country examined and on the approach adopted.

Finally the Shapley decomposition procedure indicates clearly that the work and marital status have the greatest marginal impact on poverty, this being true generally for all the five countries and for the three approaches examined.

In future work we plan to increase the number of indicators used in measuring multidimensional poverty, adopting thus recent recommendations of the European Union. We also plan to include additional approaches in our analysis and take a closer look at the marginal impact of each category of indicators on the value taken by the multidimensional indices of poverty.

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