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**What numerically determines the difference between catching up and  
endless poverty in African countries?<sup>1</sup>**

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## What numerically determines the difference between catching up and endless poverty in African countries?

Hideyuki Kamiryo

### Key words:

Capital estimation using IFS/IMF data, investment in quality and quantity, an endogenous rate of technological progress, conditional convergence, twin-peaks, the four initial parameters, the relationship between structural reform/the limit of growth and the capital-output ratio under convergence, neutrality of financial assets, the Central Bank interest rate

### Abstract:

This paper clarifies real interrelationships among key values by country in both the real and theoretical/convergence situations: using (1) four initial parameters, instead of initial income and time-averaged growth rates and adding (2) a new structural reform/limit of growth parameter, *beta*, calibrated in recursive programming and also measured by an equation under convergence. For this, I use an endogenous growth model by country and related methodology, instead of such dynamic panel techniques as Dowrick and Rogers [2002] used. A common conditional convergence is found by country if I use the same four initial parameters. However, in African economies, influenced by extremely low rates of saving/net investment and high growth rates of population, each conditional convergence by country differs from those in other countries. I find that there are “twin” conditional convergences among countries as shown in Quah [1993, 1996], each characterized by the level of capital-output ratio. I classify countries into four convergence clubs. When the capital-output ratio is below 1.0, it seldom increases as in African countries, while it has a rigorous upper limit in advanced countries (e.g., 3.5 to 4.0 as seen in Japan). For data, I present a new method to estimate capital using three values from national accounts and the Central Bank interest rate both available in IFS/IMF.

## 1. Introduction

This paper aims to clarify the true causes of growth differences among countries by numerically clarifying the relationship between key ratios by country, estimating capital stock (hereafter, capital) and using an endogenous growth model in Kamiryo [2004a]. For data, I use International Financial Statistics, IMF [2003], where I estimate the capital-labor ratio and capital using a proposed method in this paper. Kamiryo [2004a] was extended from Kamiryo [2003] and integrated with Kamiryo [2004b] that clarified an endogenous Penrose curve/the limit of growth.

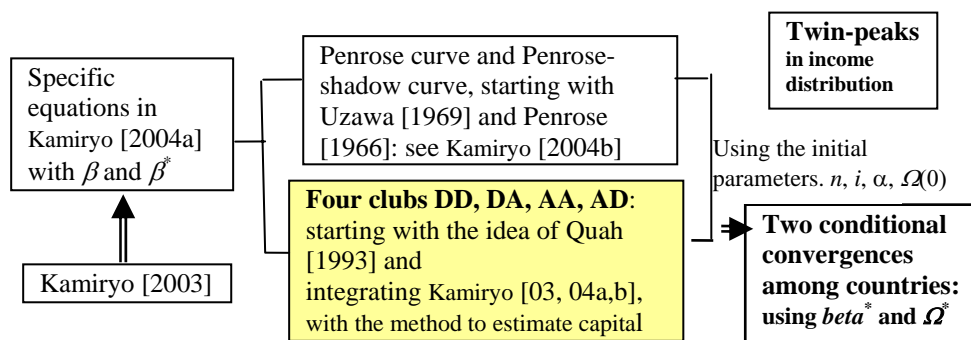


Figure 1 Framework for discussing issues of this paper

I distinguish four clubs among countries that show different economic stages by country: Clubs DD, DA, AA, and AD, where D indicates poor or developing, and A indicates rich or advanced. DA, for example, indicates a shift from D to A, and AD indicates degradation from stable AA. My research question is: Why do African countries typically remain Club DD while some other countries take off towards Club DA? For this question, however, I also need to find and measure the real causes and factors that may explain the differences between Club DA and Club AD.

Now, I will briefly explain my model and related methodology:

For an endogenous growth model [2003, 2004a]: I do not directly use Kamiryo [2003] that divides investment into quantity and quality (technology) by introducing “three” and *delta* decision-making financial parameters. Instead, I use Kamiryo [2004a] that integrates the above “three” financial parameters with “*beta*.” Kamiryo [2004a] formulates equations under convergence, where *beta*<sup>\*</sup> measure the level of technological progress and *delta*<sup>\*</sup> works for neutralizing diminishing returns. This paper, for simplicity, basically uses *beta*<sup>\*</sup> under *delta*<sup>\*</sup>=0. Thus, an endogenous rate of technological progress is driven using the Cobb-Douglas production function under constant returns to scale, as a reform of Solow [1956]. One of characteristics of my model is that the variables in the real world are compared in parallel with those under convergence. These variables by country can be averaged by using the sum of each country’s data. Thus, for the test of convergence, I do not use the initial output per capita and time-averaged growth rate in panel data as typically shown in Barro and Sala-i-Martin [1995].<sup>2</sup>

For the estimation of capital: I propose a new method, where I set two equations, each for the capital-labor ratio and the capital-output ratio. Capital is estimated so that the results respectively derived from both equations are wholly consistent in my model. The capital-labor ratio continues to increase under both the current and convergence situations, but the capital-effective labor ratio becomes constant under convergence since it is directly related to a constant capital-output ratio under convergence.

For related methodology: As a base I use Kamiryo [2004b]. Kamiryo [2004b] modifies, by introducing the role of financial assets by Friedman [1968], the Penrose curve (as a limit of economic growth) that was first formulated by Uzawa [1969] using the utility function. This paper does not aim to explain this in detail but shows the framework, particularly in relation to the estimation of capital.

For the classification of four clubs: Each country has its own characteristic of growth and convergence, and any country may be classified into one of the four clubs. Yet, among countries I agree with such results as shown by Quah’s [1993, 1996] twin-peaks in distribution dynamics. The literature does not specify the differences between clubs. This is partly because the rate of technological progress is not measured endogenously and partly because the relationships between the initial

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<sup>2</sup> My analysis does not use panel data approaches such as Mankiw, Romer, and Weil [1992], Durlauf, Kourtellos, and Minkin [2001] using initial income and a scalar index variables, and Dowrick and Rogers [2002] using dynamic panel techniques (GMM). However, these results can be compared with the result of this paper. For example, my conclusion is similar to that in Dowrick and Rogers [2002, p. 381], where it was suggested that poor countries with a low capacity to save and invest are doubly disadvantaged, losing out not only in terms of factor accumulation but also in building the educational capacity that would facilitate technological catch-up. Nevertheless, my model directly by country measures the endogenous rate of technological progress compared with the actual rate of technological progress. And also, if the value-data of countries in the same club are aggregated, averaged variables are obtained, whose value is useful to decide a peak of each club convergence.

parameters (in particular, the growth rate of population and the rate of saving/net investment) and the capital-output ratio under convergence are not yet clarified.

## 2. The method to estimate capital under the neutrality of financial assets

As a preliminary step to answering my research question, I propose a method for estimating capital. This method is unique in that I do not apply the perpetual inventory method and, instead, I estimate capital using the relationship between the capital-labor ratio and the capital-output ratio under some assumptions. For preparation of capital estimation, I pick up three values (GDP, population/employed persons, and gross investment), and the Central Bank interest rate/market interest rate) that are available in International Financial Statistics, IMF, by country and year. Underlying notion is that the real rate of profit directly related to the capital-output ratio and, accordingly, technology should be neutralized from financial assets: this was discussed in Kamiryo [2004b] using the Friedman's [1968] role of financial assets. Then, I estimate capital together with other ten values<sup>3</sup> that are needed for my endogenous growth model and recursive programming. And, more importantly, through the process to estimate capital, I will be able to clarify the essential differences lying between four convergence clubs and answer my research question.

The relationship between the capital-labor ratio and the capital-output ratio is significantly meaningful in the reply to my research question. Under both current and convergence situations (in both the real and theoretical world), the capital-labor ratio increases but the capital-output ratio will stop increasing when it hits a limit (e.g., 3.0-3.5). What factor connects both ratios? It may be the level of technology,  $A(t)$ :  $A(t) \cdot \Omega(t) = k(t)^{1-\alpha}$  as shown using Hicks-neutral. Under optimum convergence<sup>\*</sup>, its capital-output ratio,  $\Omega^*$ , will stop at the level of the initial capital-output ratio,  $\Omega(0)$ , where profit is maximized. And, when Hicks-neutral,  $A(t)$ , is replaced by Harrod's neutral,  $B(t)$ ,  $A(t) = B(t)^{1-\alpha}$  holds. Then, the capital-effective labor ratio becomes constant under convergence<sup>\*</sup>.

$$(1) \quad k_e^* = \Omega^* \frac{1}{1-\alpha} = \Omega(0) \frac{1}{1-\alpha} \quad 4$$

Eq. 1 is not used for estimation of the initial capital-labor ratio and, accordingly, capital, but indirectly suggestive (see below). We must stay at the current situation. If we separately use (1) an equation for the capital-labor ratio and (2) an equation for the capital-output ratio, the capital-labor ratio will be estimated. First, for the current capital-labor ratio, I use  $k(0) = \alpha / x_0$ , where  $\alpha$  is the relative share of profit and  $x_0$  is an arbitrary parameter.<sup>5</sup> Second, for the current capital-output ratio, I

<sup>3</sup> Corporate saving/undistributed profit, dividends, profit, wages, net output, saving, the growth rate of population/employed persons, the actual growth rate of output, the actual growth rate of capital, and population/employed persons in IFS (see Appendixes Data 1 and 2). I assume that the saving as the sum of corporate and household saving is equal to gross fixed capital formation in IFS less depreciation whose depreciation rate is estimated at 15 to 12 % of gross fixed capital formation. Also I assume that net investment is equal to the above saving less banking costs, where I use a financial parameter,  $\theta_l$ , and banking costs are expressed as  $(1 - \theta_l)$  multiplied by net output. The reason why I use net investment after depreciation is that my model clarifies the relationship between the growth rate of capital (after depreciation) and the depreciation rate under convergence.

<sup>4</sup> In other way: the same result is obtained if  $A(0)=1$  is assumed.

<sup>5</sup> If  $\alpha/(1-\alpha)$ , is used instead of  $\alpha$ ,  $x_0$  is equal to  $r/w$ , where  $r$  is the rate of profit and  $w$  is the wage rate in the real world. In this case, however, equations under convergence do not hold slightly over years: for example, the optimum rate of profit under convergence is not exactly equal to the initial rate of profit in

use  $\Omega(0) = \alpha / r(0)$ . Then, I find a relative share of profit,  $\alpha$ , common to the two equations:

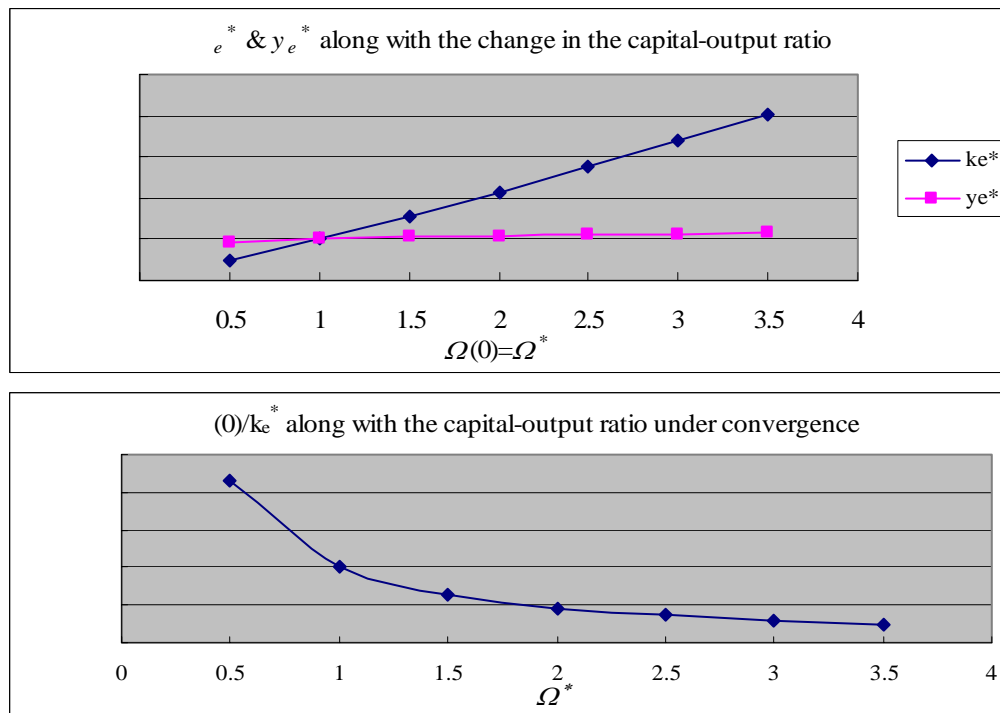
$$(2) \quad \alpha = k(0) \cdot x_0 = \Omega(0) \cdot (c_{CB} \cdot r_{CB}), \text{ where } r(0) = r^* = c_{CB} \cdot r_{CB}.$$

The current rate of profit,  $r(0)$ , is equal to the optimum rate of profit under convergence,  $r^*$ .<sup>6</sup> The Central Bank discount rate (or, market rate),  $r_{CB}$ , is set by year so that the coefficient,  $c_{CB}$ , remains within a certain range, aiming at a neutrality of financial assets.<sup>7</sup> Now using  $k_e^*$  similarly to Eq. 2,

$$(3) \quad k_e^* = \alpha / x_e^*, \text{ under convergence.}$$

$$(4) \quad \text{If } k(0)/k_e^* \text{ is compared with } x_0/x_e^*, \frac{k(0)}{k_e^*} \cdot \frac{x_0}{x_e^*} = 1 \text{ or } \frac{k(0)}{k_e^*} = \frac{x_e^*}{x_0} \text{ holds.}$$

The above  $k(0)/k_e^*$  is suggestive for capital estimation in that the changes in both the capital-output ratio and the interest rate are implicitly related to the capital-output ratio. As shown in Figure 2, when the capital-output ratio under convergence is below 1.0,  $k(0)/k_e^*$  becomes significantly high. Thus,  $k(0)/k_e^*$  will be a good indicator for estimation of capital and also for convergence clubs.



Note: The higher the capital-output ratio the higher the effective-labor ratio and lower the  $k(0)/k_e^*$ . If  $k(0)/k_e^*$  is very low at a low capital-output ratio under convergence, this will be a big issue.

Figure 2 The relationship between the capital-output ratio and the capital-effective labor ratio under convergence

the real world. Furthermore, the value of  $r/w$  or  $w/r$  should be given independently from the data. I stress here that Eq.1 and Eq.2 be used under a certain level of neutrality of financial assets.

<sup>6</sup> This equality between  $r(0)$  and  $r^*$  corresponds with the equality between  $\Omega(0)$  and  $\Omega^*$ .

<sup>7</sup> It is true that if  $r_{CB}=r^*$  or  $c_{CB}=1$ , the neutrality of financial assets is perfect and economic growth is most stable over years. The Central Bank of each country can have its own policy with a certain range of  $c_{CB}$ .

Once the capital-labor ratio is estimated, capital is obtained by multiplying population/employed persons. Estimated values of capital are carefully examined and adjusted at least for several years, using Eq. 2 and comparing the decreasing values of  $x_0$  with the changes in the coefficient of neutrality,  $c_{CB}$ . If  $c_{CB}$  fluctuates by year, the policy of the Central Bank must be unstable, neglecting the neutrality of financial assets. In this case, the value of  $x_0$  should be carefully readjusted (decreased or increased) so as to have a constant  $c_{CB}$  over years. As a result, capital is finally estimated to guarantee a stable growth. Since the relative share of profit,  $\alpha$ , is not given in IFS, this value also must be arbitrarily set when we use IFS data: e.g., 0.07 or 0.05, by country.<sup>8</sup>

In the case of Club DA countries (such as China and India), the value of  $x_0$  in 1995, is much higher than that in Club DD countries and thus can decrease year by year, which implies stable continuous economic growth. In the case of Club DD, the value of  $x_0$  in 1995 is already extremely low and cannot decrease by year, which implies that the capital-labor ratio cannot easily increase. Table 1 shows these two values after several adjustments by country and year.

### 3. Classification of clubs: corresponding with Quah's [1993] twin-peaks

#### 3.1 Four clubs and each characteristic

What distinguishes “catching up” with “falling back” among countries? To solve this question, I adopt the notion of the “twin-peaks” in distribution dynamics advocated by Quah [1993, 1996]. My approach is based on data by country, yet I confirm the existence of Quah's twin-peaks. Furthermore, my model clarifies and measures the true causes and factors that result in twin-peaks among countries.

Table 1 The relationship between the two parameters,  $x_0$  and  $c_{CB}$

	1995	1996	1997	1998	1999	2000	2001	2002
1. $x_0$ for $k(0)$	For the estimation of $k(0)$ : $k(0)=\alpha/x_0$							
1. China	0.041867	0.0305	0.026115	0.0145	0.011	0.0095	0.0081	0.0072
2. India	0.01625	0.01432	0.01132	0.0092	0.0072	0.0061	0.0051	
3. Algeria	0.0047705	0.0034656	0.0029183	0.0023535	0.001892	0.0013	0.00111	
4. Egypt	0.000116	0.000105	0.000088	0.000079	0.000074	0.000068	0.000063	0.000058
5. Ethiopia	0.000532	0.00051	0.00042	0.000414	0.00038	0.000367	0.000356	
6. Kenya	0.000039	0.000033	0.00003	0.000023	0.000021	0.000019	0.000016	
7. Mali	0.0073	0.004	0.0029	0.0044	0.005	0.0054	0.0049	0.0039
8. Nigeria	0.0127	0.00894	0.00907	0.0097	0.0105	0.00685	0.0063	0.0061
9. S.Africa	0.000023	0.000020	0.000018	0.000018	0.000014	0.000012	0.000010	0.000009
10. Tanzania	0.005	0.00365	0.00263	0.00238	0.002	0.0013	0.001	
2. $c_{CB}=r^*/r_{CB}$	If $c_{CB}=\text{constant}$ , $B_K/A_{CB}=1/c_{CB}$ , where financial assets are neutral [Friedman, 1968].							
1. China	1.1384	1.1149	1.1044	1.2071	1.3215	1.2285	1.1510	1.2652
2. India	1.1275	1.1122	1.2811	1.1697	1.1220	1.0195	1.1264	
3. Algeria	1.1238	1.1142	1.1800	1.0931	1.1095	1.3488	1.1723	
4. Egypt	1.1430	1.1060	1.0823	1.0876	1.0698	1.0663	1.1146	1.1774
5. Ethiopia	1.1781	1.0093	0.1278	1.2162	1.1721	1.1467	1.1061	
6. Kenya	1.2144	1.1554	1.0746	0.9167	0.9820	1.1080	1.1723	
7. Mali	1.2034	1.2665	1.3194	1.2256	0.1301	1.1321	1.1753	1.1648
8. Nigeria	1.1245	1.0932	1.1221	1.1434	1.0411	1.2544	0.8925	1.0828
9. S.Africa	1.2227	1.0680	1.0935	0.9615	1.2882	1.2110	1.3976	0.9819
10. Tanzania	0.5574	1.1409	1.1729	1.1299	0.9342	1.2599	1.3231	

<sup>8</sup> I confirm, using Eq. 23 [Kamiryo, 2004a], that the relative share of profit does not so strongly influence conditional convergence, as the growth rate of population and the rate of net investment. Solow [1958] is suspicious about the constancy of  $\alpha$ . I fix  $\alpha$  for the estimation of capital.

As described above, I set up four clubs: Clubs DD, DA, AA, and AD. The classification of four clubs is fully justified by the following two ratios among countries: the capital-output ratio,  $\Omega^*$ , and  $\beta^*$  that changes  $\Omega^*$  and determines the level of technology through the endogenous rate of technological progress. And, the four clubs, I find, are empirically reduced to two clubs: Club DD and Club AA. Under convergence,  $\beta^*$  is related to the level of rate of technological progress and  $\delta^*$  neutralizes diminishing returns and changes the speed of convergence, although both  $\beta^*$  and  $\delta^*$  are interrelated under convergence.

The characteristics of Clubs DD, DA, AA, and AD are now explained, using the illustration in Figure 3.

### 1. Club DD:

Club DD is the least hopeful in that both investment and human capital are too weak to support competition. A poor country must first increase the amount of investment and then steadily pay attention to education and R&D so that the capital-effective labor ratio begins to rise and stop further inequality among people. As a base, it is essential for poor countries to maintain and steadily improve “the human development indices” as published by the United Nations [2002]. It is also important for poor countries to maintain the neutrality of financial assets (as proved by Friedman [1968]) through the manipulation of the market rate of interest by the Central Bank. It is not wise to rely on an extremely high market rate of interest.

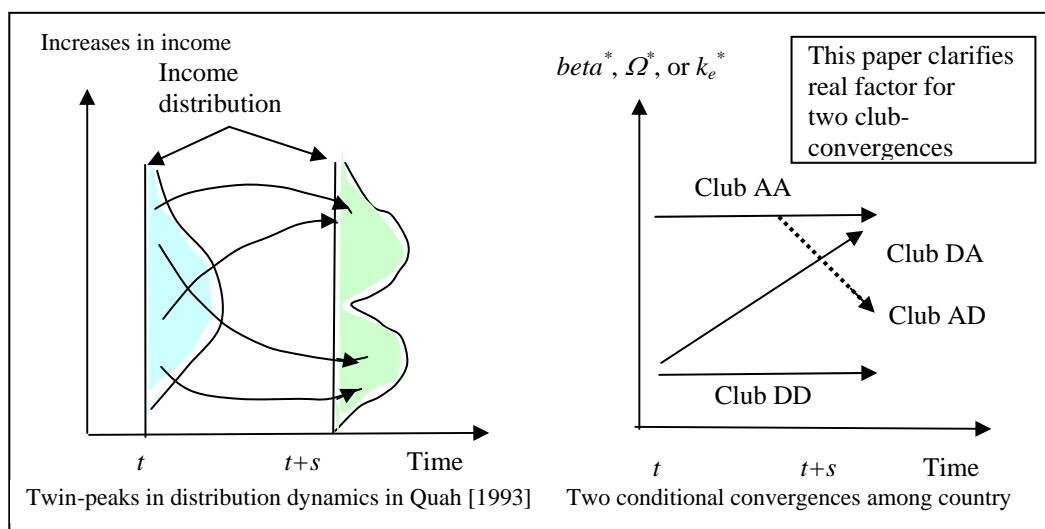


Figure 3 The relationship between Quah [1993] and four clubs classified by each country's characteristics

### 2. Club DA:

When the ratio of net investment to output or the rate of saving increases year by year, the capital-output ratio will increase, together with the growth rate of output. These increases will not continue unless the central government takes continuous action to ensure towards improvement in technology and human capital. Before reaching an upper limit of the capital-output ratio prevailing in Club AA, Club DA countries must, step-by-step, shift investment from quantity to quality, together with mitigation of inequality among people. A most successful case of this in the past is China. Future of China depends on current policy to stop the rapid increase in the capital-output

ratio. Note that the difference between China and India partly comes from a significant difference in the growth rate of population.

### 3. Club AA:

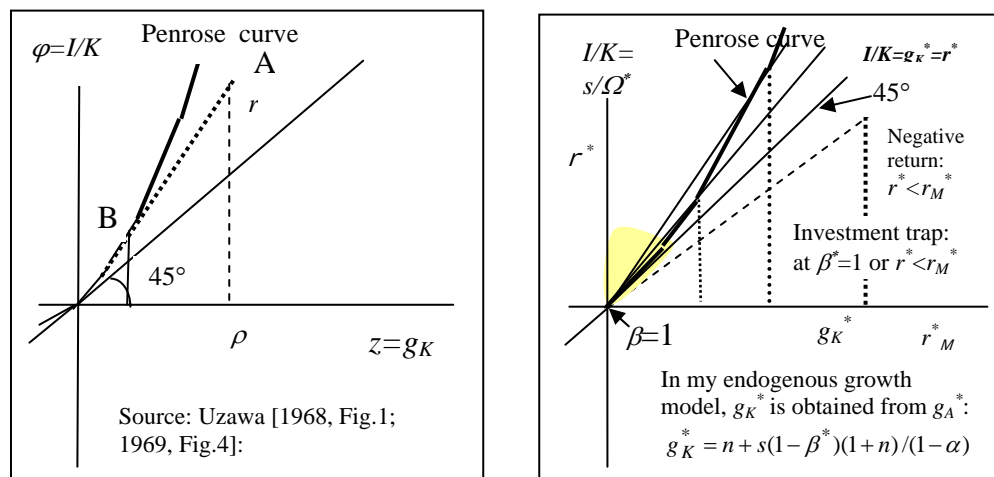
A country can move to Club AA after it continuously maintains Club DA. Countries in Club AA need to intentionally and urgently shift the investment in quantity to the investment in quality (from physical investment to human investment). Any mature country must redistribute resources and investment between quantity and quality (through structural reform) in order to avoid the limit of growth and maintain a certain rate of technological progress.

### 4. Club AD:

Club AD does not mean a decline from Club AA to Club DD, but once a country falls into Club AD, it is difficult to return to Club AA, unless its national debt is reduced. Any country will fall into Club AD when the capital-output ratio overrides its limit. When a Club AA country continues to excessively invest in public physical capital, the country will lose its continuous growth. The Japanese huge public investment has offset the robustness of the economy as a whole, resulting in severe assets-deflation. If national debt is moderate,  $\beta^*$  cannot be above 1.0. National debt implicitly accelerates the limit and instability of growth once the output share of the government sector overrides a moderate range (in detail, see Kamiryō [2004b]).

## 3.2 Propositions relating to shift between clubs

With the results using the method to estimate capital in Section 2, some propositions related to four clubs are suggested. These propositions are consistent with the characteristic of each club above. First of all, basic observations for the capital-labor ratio,  $k(t)$  and  $k_e^*$ , and the capital-output ratio,  $\Omega(t)$  and  $\Omega^*$ , under current and convergence situations are summarized as follows:



Note: Uzawa [1969] uses the utility function while Kamiryō [2004b] uses  $\beta^*$  in Kamiryō [2004a]

Figure 4 From the Penrose curve [Uzawa] to the Penrose curve [Kamiryō]

1. Under the current situation, if both  $k(t)$  and  $\Omega(t)$  increase over years, the economy grows. If both  $k(t)$  and  $\Omega(t)$  do not increase over years, the economy does not grow. If  $\Omega(t)$  increases continuously beyond its limit (at somewhere close to 4.0), the economy falls into difficult times.
2. Under convergence,  $k_e^*$  and  $\Omega^* (= \Omega(0))$  become constant.



3. To maintain economic growth,  $k(t)$  can slightly increase over years at a constant  $\Omega(0)$ ,<sup>9</sup> by neutralizing diminishing returns to capital (DRC)<sup>10</sup> under the neutrality of financial assets.

The above results are summarized using four clubs as in Figure 5.

For the estimation of capital, see Section 2		
	$k(t)$	$\Omega(t)$
Club DD	Slightly increasing $x_0$ is extremely low and $r(0)$ is extremely high	Slightly increasing
Club DA	Significantly increasing High $x_0$ and $r(0)$ at the beginning become lower over years	Significantly increasing
Club AA	Stably increasing Both $x_0$ and $r(0)$ are relatively low and stable	At a constant level
Club AD	Slightly increasing $x_0$ is extremely low and $r(0)$ approaches zero, violating the role of financial assets	Increasing beyond a limit

Figure 5 Four clubs and findings with the estimation of capital

Why are these differences between clubs brought about? What mechanism works equally in these clubs? Vital keys for understanding this mechanism are (1) an equation in Kamiryo [2004a]<sup>11</sup> under convergence and (2) the difference in the combinations of four parameters (soon below). First, for the equation, I introduce Eq.

23 of Kamiryo [2004a] under convergence:  $\Omega^* = \frac{\beta_{\delta=0}^* \cdot i (1-\alpha)}{i (1-\beta_{\delta=0}^*)(1+n) + n(1-\alpha)}$ , where five

ratios,  $\beta^*$ ,  $n$ ,  $i$ ,  $\alpha$ , and  $\Omega^*$ , are used and  $\beta^*$  is measured from the four parameters,  $n$ ,  $i$  (equals the rate of saving,  $s$ , if no banking costs),  $\alpha$ , and  $\Omega^* = \Omega(0)$ .<sup>12</sup> Eq. 23 expresses the following relationships (see Figure 6):

1.  $\Omega^*$  becomes lower if  $i$  is lower under  $n > 0$ , but  $\Omega^*$  becomes much higher if  $i$  is lower under  $n < 0$ .
2.  $\Omega^*$  becomes linearly higher if  $\beta^*$  is higher under  $n > 0$ , but  $\Omega^*$  becomes much higher if  $\beta^*$  is higher under  $n < 0$ .

Or, if  $n$  is positive,  $\Omega^*$  is lower under a lower  $i$ , and  $\Omega^*$  is lower under a lower  $\beta^*$ , as in Club DD. And, if  $n$  is negative,  $\Omega^*$  is much higher under a lower  $i$ , and  $\Omega^*$  is much higher under a higher  $\beta^*$ , as in Club DA. Thus, it is noted that a high growth rate of population or employed persons does not increase the capital-output ratio.

Therefore, I state the above relationships as propositions (see Figure 7):

**Proposition 1.** If an economy is stably supported by a high rate of net investment with improving education and R&D, this economy can get into Club DA from Club DD under a moderate growth rate of population. The capital-output ratio steadily increases from 1.0 to 2.0.

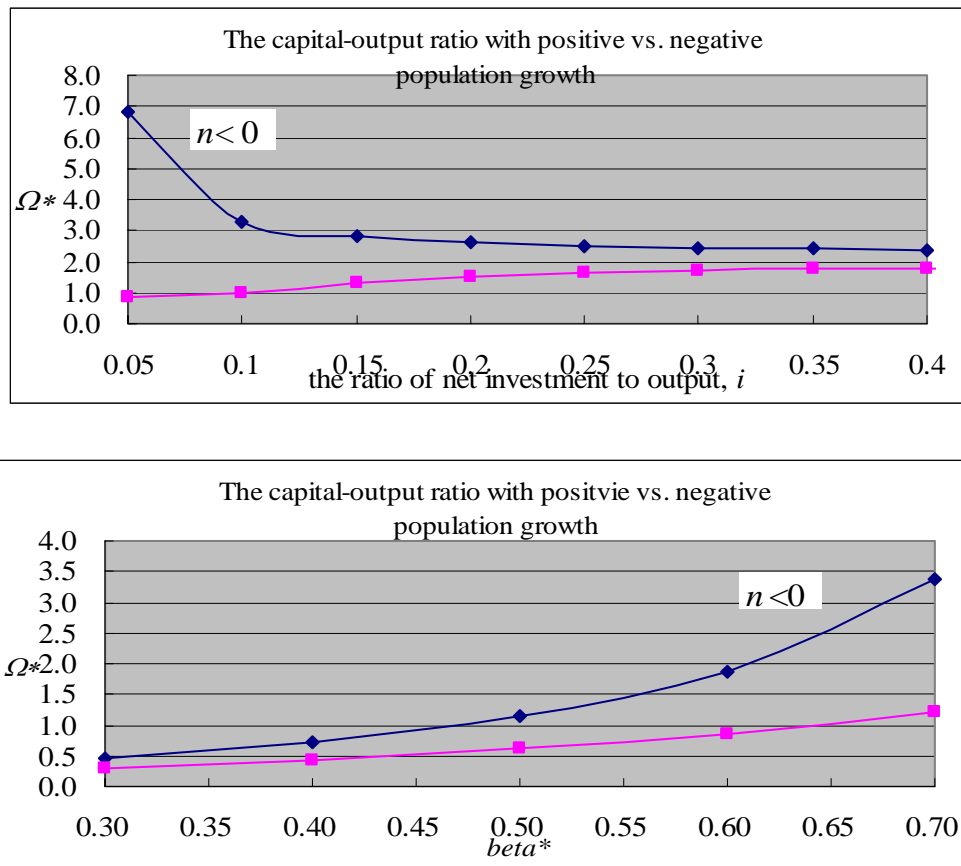
<sup>9</sup> The capital-output ratio usually remains unchanged as observed by Kaldor [1957] and Jones [1998].

This paper intends to clarify its foundation using Club AD, where this ratio is beyond its limit.

<sup>10</sup> Kamiryo [2003] classify the rate of profit into diminishing, constant, and increasing returns to capital (DRC, CRC, and IRC). Under the current situation, we observe only DRC. Under convergence, the situation is under CRC. Kamiryo [2004a] divides DRC into weak DRC and strong DRC, where diminishing returns are neutralized using  $\delta$ . For the speed of convergence, see Kamiryo and Fujimoto [2005]).

<sup>11</sup> For the comparison with the literature, see Kamiryo [2003] and Kamiryo [2004a].

<sup>12</sup> Under the current situation,  $\beta^*$  is easily calibrated in recursive programming. Under convergence,  $\beta^*$  is obtained by using either Eq. 23 or calibration.



Note: In both cases, if  $n=0$ , the curve is between the two curves,  $n < 0$  and  $n > 0$ . This implies that even if African countries reduce population growth, it is still difficult to raise low  $\beta^*$  and  $\Omega^*$ .

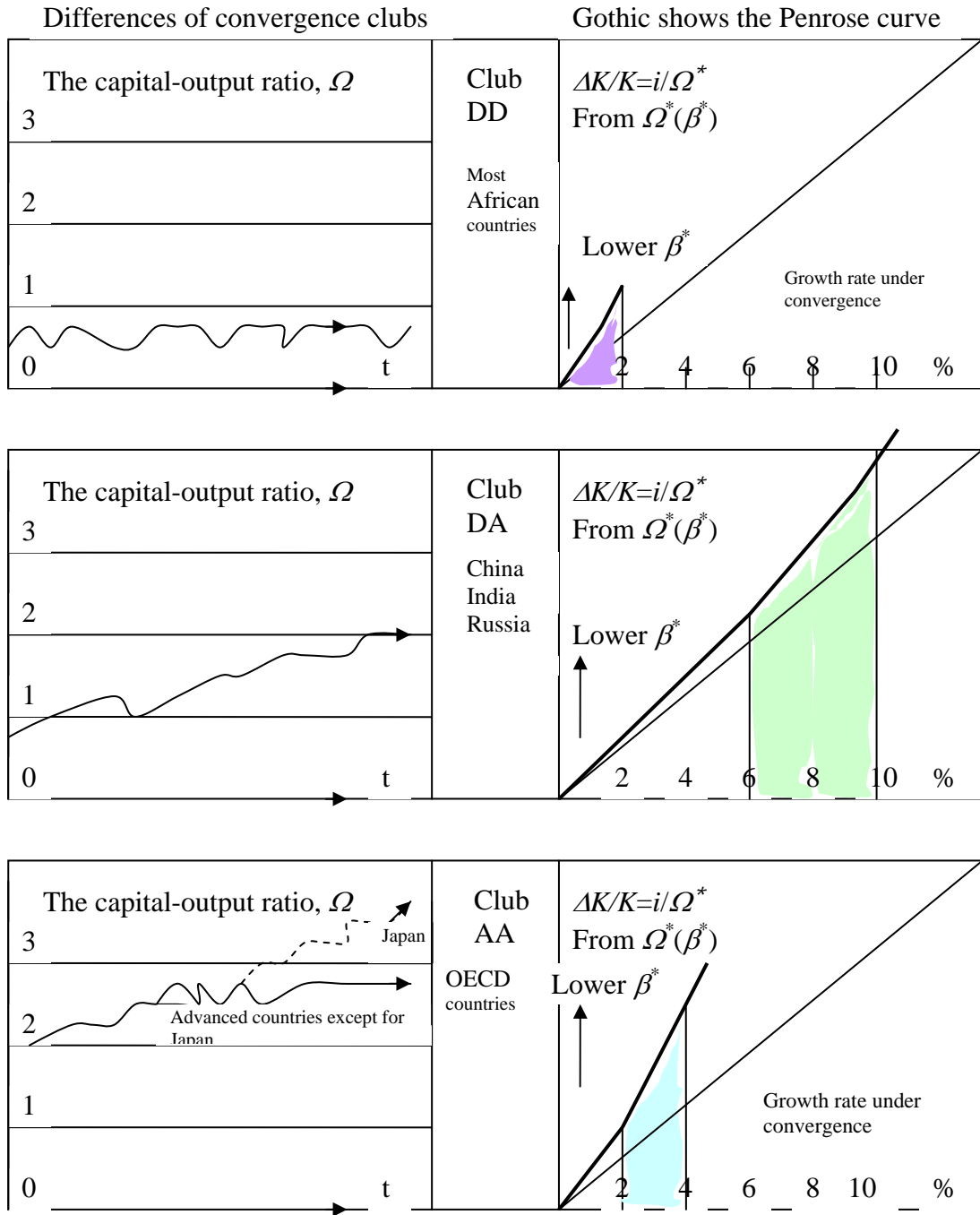
Figure 6 The capital-output ratio,  $\Omega^*$ , functions of the ratio of net investment to output,  $i$ , and  $\beta^*$

**Proposition 2.** If an economy in Club AA fails to maintain the capital-output ratio within a range between 2.0 and 3.0, this economy falls into Club AD, where economic growth is not guaranteed unless an extremely high capital-output ratio of the government sector decreases and the share of output in the corporate sector increases.

Second, for the difference in the combinations of four parameters, I stress that if the four parameters are the same then all the countries converge into a unique club, with the same value of  $\beta^*$ . Why, then, do two conditional convergences exist? I indicate that a combination such as an extremely high growth rate of population with an extremely low rate of saving/net investment will bring about Club DD, where the capital-output ratio cannot increase beyond 1.0 under political instability, inequality, and corruption. Thus,

**Proposition 3.** The difference between Club DD and Club AA comes from the combinations of four parameters, in particular due to the difference of the level of the capital-output ratio that is most closely related to technology.

We must accept the fact that both  $i$  (or  $s$ ) and  $\beta^*$  are significantly low in Club DD. Why is  $\beta^*$  extremely low in Club DD? Too low  $\beta^*$  implies that investment in quantity should be ironically accelerated to realize convergence under extremely diminishing returns. Or, with high population growth and low education, there is a first priority to the investment in quantity.



Note: For Japan,  $\Omega=2$  in the corporate sector but  $\Omega=15$  in the public sector, falling into Club AD.

Figure 7 The capital-output ratio via  $\beta^*$  determines the difference of convergence clubs

The above Propositions 1, 2, and 3 are typically expressed using the capital-output ratio as shown in Figure 7.

The capital-output ratio shows the limit of economic growth much more definitely than the capital-labor ratio. The level of capital-labor ratio differs significantly by country and increases even under convergence. The capital-output ratio, on the other hand, requires a certain range, say, between 1.0 to 3.0. The capital-output ratio of the corporate sector is almost 2.0 in moderate countries under worldwide competition.

## 4. Empirical results and policies to catching up

### 4.1 Empirical results in Clubs DD and DA

This section first shows the results of the relationship between  $\beta^*$  and  $\Omega^*$ , or  $k_e^*$ , to distinguish between catching up and falling back. Secondly, it shows policies necessary for shifting Club DD to Clubs DA and AA. These policies eventually include how to prevent Clubs DA and AA countries from falling into Clubs DD and AD. I select ten countries for my research question including China and India for comparison, by using data in IFS/IMF [2003], and also my method to estimate capital.

Before starting, I will accept the human development indices of United Nations [2002].<sup>13</sup> These indices, both subjective and objective, are extremely low in African countries and consistent with my findings.

The indices and ranks in Ethiopia, Kenya, Mali, Nigeria, and Tanzania differ significantly but are all extremely low. Those in Algeria, South Africa, and Egypt each show their own individual characteristics. These results are clearer when compared with those for China and India. And, these results to some extent correspond with my numerical results in  $\beta^*$ ,  $\Omega^*$ , and  $k_e^*$ . It is true that most African countries do not have high levels of education, not due to the lack of money/assistance, but to political instability and the lack of real humanitarian leadership, in addition to a too high growth rate of population.

The above ten countries each has convergence except for Kenya 2002. Each growth rate under convergence is not extremely low, but is unrealizable because their current  $\beta$  under DRC and their  $\beta^*$  under convergence differ widely. The capital-labor ratio and the capital-output ratio are both extremely low compared with those in China and India. These African countries doubtlessly are in Club DD.

Let me interpret the results found in ten countries, using Table 2. Key ratios determine club-stages: Algeria and Egypt had a chance to shift to Club DA but not any more in 2002. Club DA countries need urgent policies to raise both the capital-output ratio and the capital-effective labor ratio. Interestingly,  $k(0)/k_e^*$  delicately indicates the difference of club-stages. African countries sometimes improve and sometimes aggravate these ratios in unstable and widely varying ways. This corresponds with the finding by Baumol and Wolff [1988, p. 1158]: “the coefficient of variance of per capita GDP between 1950 and 1980 of the upper-income group is 0.30 and that of the lower-income group is 0.03.” This implies that the upper-income group is much more stable in technology than the lower-income group. I assert that Club DD cannot easily shift to Club DA. Successful cases are China and India. Note the relationship between the investment ratio and population growth (see Figure 6). Social infrastructure summarized in Hall and Jones [1999, p. 114] will influence both Clubs DD and DA, although their approach uses ‘the Solow residuals.’ In short, African countries are not continuously able to raise their level of technology.

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<sup>13</sup> According to the Human Development Report 2002 [the United Nations Development Programme: Oxford University Press, 2002], the human development indices (HDI) are divided into two: subjective and objective indicators of governance. In the subjective indicators of governance, HDI ranks in “medium human development” are: China 96, Algeria 106, South Africa 107, Egypt 115, India 124, and Kenya 134. And HDI ranks in “low human development” are: Nigeria 148, Tanzania 151, Mali 164, and Ethiopia 168. In the similar way, in objective indicators of governance, HDI ranks in “medium human development” are: China 96, 106 Algeria 106, South Africa 107, Egypt 115, India 124, and Kenya 134. And, HDI ranks in “low human development” are: Nigeria 148, Tanzania 151, Mali 164, and Ethiopia 168.

Table 2 What determines the shift to Club DA from Club DD in African countries?

	$\beta_{(\delta=0)}$	$\Omega^*$	$r^*$	$k(0)/k_e^*$	$k_e^*$	$n$	
China 1996	0.4785	0.7309	0.1003	3.36	0.7817	0.0077	Club DA
2002	0.7345	2.2891	0.0342	4.41	2.5217	0.0076	catching up
India 1996	0.4206	0.5178	0.1335	9.08	0.4996	0.0289	Club DA
2001	0.5551	0.9553	0.0732	13.38	0.9528	0.0162	catching up
Algeria 1996	0.2896	0.3472	0.1448	57.46	0.3264	0.0114	Club DD
2002	0.5004	0.6551	0.0703	83.87	0.6982	0.0169	catching up
Egypt 1996	1.4524	0.3483	0.1438	2316.28	0.3289	0.0923	Club DD
2002	0.4535	0.4243	0.1177	3397.81	0.4059	0.0201	pushing back
Ethiopia 1996	0.6220	0.3466	0.1405	407.23	0.3370	0.0800	Club DD
2001	0.5925	0.4176	0.1202	495.17	0.3971	0.0256	unchanged
Kenya 1996	(0.0247)	0.1641	0.3106	12433.38	0.1462	(0.0796)	Club DD
2001	(0.1402)	0.2168	0.2306	18743.41	0.2001	0.0294	pushing back
Mali 1996	0.2394	0.1680	0.2976	81.72	0.1530	0.0715	Club DD
2002	0.1658	0.1143	0.4375	125.73	0.1020	0.0294	unchanged
Nigeria 1996	0.3781	0.3286	0.1476	17.48	0.3200	0.0337	Club DD
2002	0.3244	0.2808	0.1787	31.32	0.2617	0.0262	unchanged
S.Africa 1996	0.2674	0.2709	0.1816	9524.99	0.2573	0.0102	Club DD
2002	0.3400	0.3755	0.1326	15503.54	0.3583	0.0077	unchanged
Tanzania 1996	1.0746	0.2359	0.2168	76.99	0.2135	0.1216	Club DD
2002	0.5216	0.4325	0.1151	144.33	0.4157	0.0207	unchanged

Note: (1) If the key values,  $\beta_{\delta=0}^*$ ,  $i$ ,  $\Omega^*$ , or  $k_e^*$ , each improve to a certain level, then catching up is possible. (2) Both the value of  $k(0)/k_e^*$  and the ratio of net investment to capital to the rate of profit under convergence,  $(I/K)/r^*$ , vividly reflect club-stages.

#### 4.2 Policies necessary for catching up

I found in the previous section that it was very difficult for a poor country to catch up. What policies are required for a poor country in Club DD to shift to Club DA? I can partly answer this question as follows (see Table 2).

1. To avoid an unfavourable combination of four parameters as much as possible. For this, first of all to maintain a moderate rate of net investment to output year by year. This policy will steadily pull up the extremely low level of the capital-labor ratio (or steadily decrease the high rate of profit at  $\beta^*$ ). Recall that an unfavourable combination of high population growth and low rate of net investment will significantly aggravate the relationship between  $\beta^*$  and  $\Omega^*$ .
2. To maintain the neutrality of financial assets by manipulating the Central Bank interest rate so that the coefficient,  $c_{CB}$ , becomes as constant as possible over years. To fix a high interest rate does not induce capital transfer in the long-term. The above neutral policy will adversely affect the continuation of steady investment, with sound “exchange rates as national currency units per SDR” in IFS/IMF.
3. Of course, basically, to avoid unstable social and political circumstances (see indicators in UN [2003]) and provide a certain amount for education and R&D in the government’s budget. And, to check the trends of  $\beta^*$  and  $\beta$ . This policy is essential for establishing a long-term base of the steady increase in the capital-labor ratio and a sound relationship between  $\beta^*$  and  $\Omega^*$ .

#### 4. Conclusions

My findings in this paper are consistent with Baumol and Wolff [1988], Quah [1993, 1996], and Lawrence and Thirtle [2003]. In particular, this paper numerically measured real causes and factors that would result in two conditional convergences, by using my endogenous growth model and related methodology. My questions are:

1. Why do economies in poor countries not realize catching up?

2. Why does an economy in rich countries happen to fall into difficulties (as in Japan that has suffered from long assets-deflation)?

Table 3 Summary of each characteristic of four clubs

	$\beta_{\delta=0}^*$	$\Omega^*$	$ke^*$	$s$ or $i$	situations	conv.speed
<b>Club DD</b>	lowest	lowest	lowest	lowest	under extreme DRC	much slow
Club DA	higher	taking off	taking off	highest	under weak DRC	faster
<b>Club AA</b>	high	high	high	moderate	under DRC	intermediate
Club AD	beyond limit	beyond limit	lower	biased	falling into severe DRC	slower

Note: In Kamiryo [2004a], the speed of convergence under  $\delta=\alpha$  is defined and measured as the number of times when the capital-output ratio becomes horizontal over time in recursive programming (see above speed). In Kamiryo and Fujimoto [2005], the endogenous speed of convergence is measured under  $\delta>\alpha$ , using  $(\delta-\alpha)n$ , which corresponds to the exogenous speed of convergence of Barro and Sala-i-Martin [1995, p.36]:  $(1-\alpha)(x+n+\delta)$ , where  $x$  is the exogenous rate of technological progress and  $\delta$  is the depreciation rate. A large difference comes from the depreciation rate. In my model, the depreciation rate is measured using the growth rate of capital under convergence.

The literature usually addresses Club DA: catching up by developing countries. I indicate that without clarifying the characteristics of Club AD<sup>14</sup> we cannot more accurately answer the characteristic of Club DA. In both Clubs DD and AD, productivity enhancement does not mitigate diminishing returns to capital (DRC), where Club DD suffers from an extremely low capital-output ratio and Club AD from an extremely high capital-output ratio beyond its limit for growth (caused by an extremely high levels of public investment and national debt). Then we generally observe the two conditional convergences together with Clubs DD and AA among countries.

We know some reasons including political corruption that hold back African countries in Club DD, but this paper concentrated on economic measurements and clarified both the causes and factors in African economies mainly using the relationship between  $\beta^*$  and  $\Omega^*$ . The variety of  $\beta^*$  and  $\Omega^*$  depend on the combinations of the four initial parameters of the growth rate of population, the rate of net investment, the relative share of profit, and the capital-output ratio.

An economy in Club AD indicates that when the rate of saving is high,  $\beta^*$  and  $\beta$  should be low (i.e., with a decisive shift of investment from quantity to quality in public sector<sup>15</sup>). This economy continuously enjoys economic growth only if the capital-output ratio is controlled below a certain level. The adverse is the case of Club DD. This economy must be controlled so as to increase the capital-output ratio above 1.0. This economy must increase the rate of saving and the investment ratio under a moderate growth rate by promoting technology by their own way.

An economy in Club DA must be conscious of the above fact before entering into Club AA. It takes a long time for an economy to shift investment from quantity

<sup>14</sup> This is because researchers have not found the true causes for assets-deflation in Japan. One of the causes is that the capital-output ratio in the corporate sector is 2.0 supported by severe international competition, but that in the public sector is beyond 15.0 due to extremely high levels of public investment. Kamiryo [2004b] clarified the structure of the valuation ratio, using the relationship between the Penrose curve and the neutrality of financial assets, and solving the Petersburg paradox.

<sup>15</sup> The Japanese actual growth rate of output increases in 2003 and 2004 significantly. However, this comes only from the quality-shift (the decrease in  $\beta$ ) in the corporate sector. The government/public sector becomes worse year by year, increasing national debt unbelievably. The Japanese growth rate as a whole will soon be influenced strongly by the inefficiency of the government sector and by the difficulty of maintaining the role of financial assets that stems from huge national debt. An expected policy is to decrease the share of the government sector and not to increase national debt.

to quality. Already, the capital-output ratio in China approaches the limit of catching-up or almost enters into Club AA.

Assuming that the above four parameters are the same, we observe in my model one conditional convergence among countries. The fact is that some countries suffer from unfavourable combinations of the four parameters, which leads to their staying at Club DD. In other words, the level of technology promoted by education and R & D does not work similarly among countries even if the investment ratio is moderate and the same. Club DD countries first require more investment in quantity or rather a higher  $\beta^*$  and  $\Omega^*$  due to an unfortunate combination of the four parameters. To improve the condition of Club DD, they need continuous improvement of education, R & D, and political stability that aims at equality among people and the extermination of corruption. These improvements are tested measuring the values of  $\beta^*$  and  $\Omega^*$ .

In this paper, to estimate the capital-labor ratio and, accordingly, capital stock, I used an arbitrary parameter,  $x_0$  (see Eqs. 1 to 4). If we can get the data of net profit by country (apart from IFS data), then we can more precisely get the relative share of profit. In this case, we can replace  $x_0$  by  $r/w$  (the rate of profit divided by the wage rate in the real assets), where  $k(t) = \frac{\alpha}{1-\alpha} \cdot \frac{r(t)}{w(t)}$  or  $k(t) = \frac{\alpha}{1-\alpha} \cdot \frac{w(t)}{r(t)}$  as shown in the literature.<sup>16</sup> Then we can compare this wage rate with the wage rate in the statistics by country. As a result, the capital-labor ratio will more precisely be estimated.

Finally, I stress in Eqs. 1 and 2 that the rate of profit and the capital-output ratio both cannot increase over years but the wage rate and the capital-labor ratio can increase in both the actual and theoretical/convergence world. Thus, I fix  $\alpha$ .

For my future study, I will continue to estimate capital by country using IFS/IMF data and compare the results with those in PWT and OECD when available in the near future. I found that the values of capital by country using my method were almost (except some countries) similar to those that I calculated from “KapW” in PWT 5.0 [1991].<sup>17</sup> If capital is estimated analytically in SNA data as in my approach based on an endogenous growth model and is compared with the results using perpetual inventory method, we may get more confidence in the estimation of capital. As a result, my propositions in this paper will be justified and robust (see Figure 7). Also, for longer periods (as shown by Maddison Angus [1982] and Baumol [1986]), I intend to estimate capital and compare each endogenous convergence by country.

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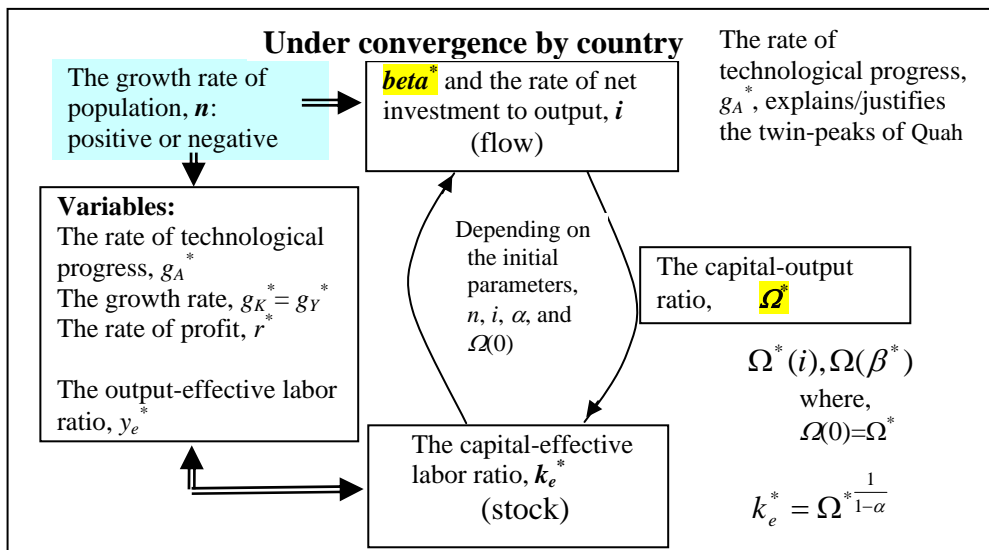
<sup>16</sup> If I replace  $r/w$  by  $\omega=w/r$ , and set  $\omega$  on the Y axis and  $k$  on the X axis, this was used in Uzawa [1964]. For capital estimation, however, this replacement is not convenient since the value of  $\omega$  becomes too large for adjustment of  $k$ . This notion was earlier discussed in Findlay with Kaldor [1960].

<sup>17</sup> In the recent Penn World Tables 5.6 in 1994 and 6.1 published on Oct 18, 2002 on URL, capital stock per worker “KapW” is not available. This shows how it is difficult to estimate the capital-labor ratio and capital using perpetual inventory method alone. I am thankful to Robert Heston for his advice and future cooperation, at Cork, IARIW in Aug 2004.

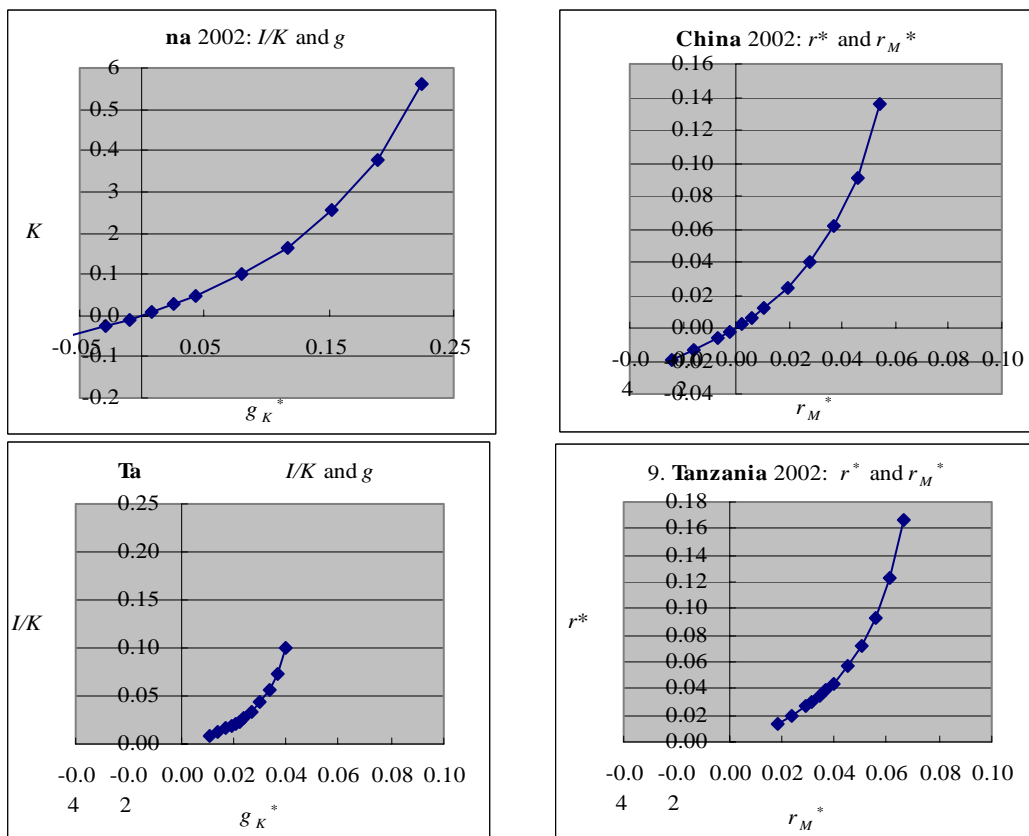
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Appendixes



Appendix A.1 A framework that determines technological change by country



Note: The Penrose curve using real assets is shown on the LHS and the Penrose-shadow curve using both real and financial assets is shown on the RHS. The ratio with \* is the ratio under convergence. Each figure of these figures shows the first quadrant and at its origin  $\beta^*=1$  and as the curve goes up,  $\beta^*$  decreases. The limit of growth is set at the origin. I reformulated the Penrose curve of Uzawa [1969] using  $\beta^*$  in my endogenous growth model (see Appendix B).

Appendix A.2 The Penrose curve,  $B_K=(I/K)/g_K^*$ , and the Penrose-shadow curve,  $A=r^*/r_M^*$

**Appendix B** An example: the procedure to totally estimate  $K(0)$  or  $k(0)$

For this procedure, I use the method to estimate capital in Section 2, together with Kamiryo [2004a, 2004b]. I omit the explanation for the Penrose curve,  $B_K=(I/K)/g_K^*=1/\beta^*$ , and the Penrose-shadow curve,  $A=r^*/r_M^*$ , where  $c_{CB}=1$ , and  $A_{CB}=r^*/(c_{M(CB)} \cdot r_{M(CB)})$ , where  $c_{CB} \neq 1$  and  $r_M^* = c_{M(CB)} \cdot r_{M(CB)}$ .

Capital,  $K(0)$ , or per capita capital,  $k(0)$ , is properly estimated if the estimation is done for several years. If each year's  $K(0)$  or  $k(0)$  is estimated during two years, the level of  $K(0)$  or  $k(0)$  may not correctly reflect the transition of the level of technology. I show two cases: (1)  $k(0)$  in equilibrium, where  $B_K/A=c_{CB}=1$ , and (2)  $k(0)$  under the neutrality of financial assets, where  $c_{CB}=1.1$  and  $B_K/A_{CB}=1/c_{CB}$ .

**The case in equilibrium:** Set that  $c_{CB}=1$ ,  $\beta^*=0.4$  (or  $B_K=A=1/\beta^*=2.5$ ),  $\alpha=0.05$ , and labor or population,  $L(0)=10$ .

1. Assume  $r^*=0.1$ ,  $\beta^*=0.4$ , and  $c_{CB}=1$  under equilibrium. Then,  $B_K=A=1/\beta^*=1 \div 0.4=2.5$ , where  $A=r^*/r_M^*$  and  $r_M^*=0.1 \div 2.5=0.04$ . Then,  $r_{CB}=0.1 \div 1=0.1$  using  $r^* = c_{CB} \cdot r_{CB}$ .
2. Assume  $\alpha=0.05$ . Then,  $\Omega^*=\alpha/r^*=0.05/0.1=0.5$ . The value of  $\Omega^*=\Omega(0)$  reflects technology.
3. Calculate  $k(0)=\alpha/x_0$ , using an arbitrary value of  $x_0$ . Suppose  $x_0=0.0025$  after adjustment. Then,  $k(0)=0.05 \div 0.0025=20$ . Now if I raise  $x_0$  up to 0.005,  $k(0)$  will be 10 and if I reduce  $x_0$  down to 0.001,  $k(0)$  will be 50. Finally, if we assume that  $k(0)=20$  and  $L(0)=10$ , then capital is estimated as  $K(0)=20 \times 10=200$ .
4. Confirm  $\alpha = k(0) \cdot x_0 = \Omega(0) \cdot (c_{CB} \cdot r_{CB})$  in Eq. 2:  $0.05=20 \times 0.0025=0.5 \times 1 \times 0.1=0.05$ .
5. Using  $k_e^* = \Omega^{\frac{1}{1-\alpha}} = \Omega(0)^{\frac{1}{1-\alpha}}$  in Eq. 1, the capital-effective labor ratio,  $k_e^*$ , is  $0.5^{1/(1-0.05)}=0.5^{1.05263}=0.482089$ . Then,  $k(0)/k_e^*=20 \div 0.482089=41.48612$  and  $x_e^*=0.05 \div 0.482089=0.103715$ , using  $k_e^* = \alpha/x_e^*$ .

**The case under the neutral financial assets:** setting  $c_{CB}=1.1$  in the above case.

1. First, assuming  $c_{CB}=1.1$ ,  $B_K/A_{CB} = (B_K/A)/c_{CB} = 1 \div 1.1=0.90909$ .
2. Since  $B_K=2.5$ , as above,  $A_{CB} = B_K/(B_K/A_{CB}) = 2.5 \div 0.90909=2.75$ .
3. Calculate  $r_{CB}$ , using  $r^* = c_{CB} \cdot r_{CB}$ :  $r_{CB}=r^*/c_{CB}=0.1 \div 1.1=0.090909$ .

**Instead of setting  $c_{CB}=0.1$ ,** we can directly use the Central Bank interest rate,  $r_{CB}$ .

4. Calculate  $r_{M(CB)}$  using  $A_{CB} = r^*/r_{M(CB)}$ :  $r_{M(CB)} = r^*/A_{CB}=0.1 \div 2.75=0.0363636$ .
5. Calculate  $c_{M(CB)}$ , using  $r_M^* = c_{M(CB)} \cdot r_{M(CB)}$ .

$$c_{M(CB)} = r_M^*/r_{M(CB)} = 0.04 \div 0.0363636 = 1.1, \text{ where } c_{CB} = c_{M(CB)}.$$

For confirmation:

1. Confirm  $A_{CB}$ .  $A_{CB} = r^*/r_{M(CB)} = 0.1 \div 0.0363636 = 2.75$ .
2. Confirm  $B_K/A_{CB}=1/c_{CB}$ .  $B_K/A_{CB} = 2.5 \div 2.75 = 0.90909$ .
3. Confirm  $r^*$ .  $r^* = c_{CB} \cdot r_{CB} = 1.1 \times 0.090909 = 0.1$ , where  $r^* > r_{CB}$ .
4. Confirm  $r_M^*$ .  $r_M^* = c_{M(CB)} \cdot r_{M(CB)} = 1.1 \times 0.0363636 = 0.04$ , where  $r_M^* > r_{M(CB)}$ .

Note that if  $r_M^* > r_{M(CB)}$ , the discount rate of valuation value of capital becomes lower, which results in assets-inflation at  $\beta^*=1$  (under no technology).



## Data 2 Data and ratios necessary for my model (1)

		$s_{\pi}=0.6$						$r^*=r(0)$ and $\Omega^+=\Omega(0)$			$k_e^+=\Omega(0)^{\alpha}/(1-\alpha)$ : using effective labour			
		$g_Y(\text{actual})$	$g_K(\text{actual})$	$n$	$L(0)$	$K(0)$	$S_{\pi}(0)$	$D(0)$	$I(0)$	$W(0)$	$Y(0)$	$S(0)=\Delta K_{net}$	$k(0)/k_e^*$	$ke^*$
1. China	1995	----	----	----	1236.7	2363.1	168.5	112.3	280.9	3229.8	3510.7	1152.4	2.938	0.6503
B. Yuan	1996	0.16781	0.18446	0.00768	1246.2	3268.7	196.8	131.2	328.0	3771.8	4099.8	1308.7	3.355	0.7817
	1997	0.09608	0.06263	(0.00273)	1242.8	3807.2	215.7	143.8	359.5	4134.2	4493.7	1392.0	3.668	0.8351
	840 1998	0.06620	0.70430	0.00893	1253.9	6918.1	230.0	153.3	383.3	4407.9	4791.2	1620.3	3.701	1.4908
\$y by ex.rate	1999	0.02756	0.29397	0.00869	1264.8	9198.5	236.3	157.5	393.9	4529.4	4923.2	1733.8	3.687	1.9728
0.38851	2000	0.08519	0.07569	0.00814	1275.1	10737.7	256.4	171.0	427.4	4915.2	5342.6	1926.7	3.943	2.1356
	2001	0.10752	0.06737	0.00792	1285.2	12693.3	284.0	189.3	473.4	5443.7	5917.1	2202.0	4.308	2.2924
	2002	0.03833	0.09165	0.00755	1294.9	14387.8	294.9	196.6	491.5	5652.4	6143.9	2464.0	4.406	2.5217
								0.0						
2. India	1995	----	----	----	922.0	3688.0	299.4	199.6	499.0	6629.1	7128.1	1714.8	8.124	0.4924
B. Rupees	1996	0.15168	0.01375	0.02885	948.6	4305.8	344.8	229.9	574.6	7634.6	8209.3	1753.5	9.085	0.4996
	1997	0.11280	0.15750	0.01823	965.9	5546.1	383.7	255.8	639.5	8495.8	9135.3	1783.7	9.820	0.5847
	450 1998	0.14344	0.09528	0.01784	983.1	6945.9	438.7	292.5	731.2	9714.4	10445.6	2077.3	10.956	0.6448
\$y by ex.rate	1999	0.10839	0.17282	0.01734	1000.2	9029.2	486.3	324.2	810.4	10767.4	11577.8	2345.1	11.795	0.7654
0.20505	2000	0.09051	0.10052	0.01678	1016.9	10836.2	530.3	353.5	883.8	11742.0	12625.8	2619.3	12.559	0.8484
	2001	0.09112	0.11392	0.01618	1033.4	13170.7	578.6	385.7	964.3	12812.0	13776.3	2802.2	13.376	0.9528
3. Algeria	1995	----	----	----	28.1	382.3	36.1	24.1	60.2	1142.9	1203.0	241.1	45.539	0.2992
B. Dinars	1996	0.28180	0.08615	0.01140	28.4	532.3	46.3	30.8	77.1	1464.9	1542.0	253.9	57.462	0.3264
	1997	0.08179	0.11594	0.01656	28.9	642.6	50.0	33.4	83.4	1584.7	1668.1	221.1	60.798	0.3664
	1580 1998	0.00752	0.24993	0.01560	29.3	809.2	50.4	33.6	84.0	1596.6	1680.7	308.6	59.610	0.4633
\$y by ex.rate	1999	0.14780	0.10113	0.01604	29.8	1022.8	57.9	38.6	96.5	1832.6	1929.1	307.5	66.999	0.5128
0.82439	2000	0.26861	0.16534	0.01579	30.2	1512.0	73.4	48.9	122.4	2324.9	2447.2	257.5	83.004	0.6024
	2001	0.03513	0.15050	0.01687	30.8	1800.7	76.0	50.7	126.7	2406.5	2533.2	296.7	83.874	0.6982
4. Egypt	1995	----	----	----	57.5	39662.1	3672.0	2448.0	6120.0	116280.0	122400.0	2500.0	2258	0.3054
M.Pounds	1996	0.12451	0.07315	0.09233	62.8	47862.9	4129.2	2752.8	6882.0	130758.0	137640.0	2350.0	2316	0.3289
	1997	0.12119	0.08454	0.01910	64.0	58200.0	4629.6	3086.4	7716.0	146604.0	154320.0	9120.0	2537	0.3583
	1490 1998	0.11742	0.01587	0.01906	65.2	66065.8	5173.2	3448.8	8622.0	163818.0	172440.0	18190.0	2780	0.3643
\$y by ex.rate	1999	0.07029	0.01657	0.01916	66.5	71881.1	5536.8	3691.2	9228.0	175332.0	184560.0	17860.0	2917	0.3706
626.21	2000	0.10566	0.00334	0.01940	67.8	79741.2	6121.8	4081.2	10203.0	193857.0	204060.0	13385.0	3163	0.3719
	2001	0.05469	0.04363	0.01977	69.1	87771.4	6456.6	4304.4	10761.0	204459.0	215220.0	9795.0	3264	0.3890
	2002	0.06412	0.04128	0.02011	70.5	97255.2	6870.6	4580.4	11451.0	217569.0	229020.0	10245.0	3398	0.4059
5. Ethiopia	1995	----	----	----	54.7	7190.8	609.9	406.6	1016.6	19314.5	20331.0	486.3	392.94	0.3349
M. Birr	10 1996	0.11961	0.00620	0.07996	59.0	8100.8	682.9	455.3	1138.1	21624.7	22762.8	1555.3	407.23	0.3370
\$y by ex.rate	1997	0.19242	0.04681	0.02796	60.7	10111.7	814.3	542.9	1357.1	25785.7	27142.8	905.3	471.25	0.3537
40.87747	1998	(0.00880)	0.05100	0.02687	62.3	10533.8	807.1	538.1	1345.2	25558.8	26904.0	987.0	453.69	0.3727
	1999	0.08582	0.02978	0.02632	63.9	11778.4	876.4	584.3	1460.64	27752.16	29212.8	486.8	479.25	0.3844
	2000	0.06954	(0.00692)	0.02581	65.6	12510.4	937.3	624.9	1562.22	29682.18	31244.4	183.9	499.87	0.3816
	2001	0.01799	0.03862	0.02561	67.3	13227.2	954.2	636.1	1590.33	30216.27	31806.6	795.35	495.17	0.3971

Data 3 Key ratios calculated using data (1)

		$\alpha$	$\Omega(0)$	$r(0)$	$k(0)$	$y(0)$	$s$	$s_H$	$S_H$	$s_{S//Y}$	$A(0)$	$g_A(\text{actual})$	$c_{CB} = r^*/r_{CB}$	$\Omega_{CB}(0)$
1. China	1995	0.08	0.6731	0.11885	1.91	2.84	0.32827	0.60000	0.29440	0.04800	2.70	----	1.1384	0.6731
B. Yuan	1996	0.08	0.7973	0.10034	2.62	3.29	0.31920	0.60000	0.28487	0.04800	3.05	0.1460	1.1149	0.7973
	1997	0.08	0.8472	0.09443	3.06	3.62	0.30976	0.60000	0.27496	0.04800	3.31	0.0936	1.1044	0.8472
	840 1998	0.08	1.4439	0.05540	5.52	3.82	0.33818	0.60000	0.30482	0.04800	3.33	0.0016	1.2071	1.4439
\$y by ex.rate	1999	0.08	1.8684	0.04282	7.27	3.89	0.35216	0.60000	0.31950	0.04800	3.32	(0.0040)	1.3215	1.8684
<b>0.38851</b>	2000	0.08	2.0098	0.03980	8.42	4.19	0.36063	0.60000	0.32840	0.04800	3.53	0.0716	1.2285	2.0098
	2001	0.08	2.1452	0.03729	9.88	4.60	0.37215	0.60000	0.34049	0.04800	3.83	0.0948	1.1510	2.1452
	<b>2002</b>	0.08	2.3418	0.03416	11.11	4.74	0.40105	0.60000	0.37086	0.04800	3.91	0.0241	1.2652	2.3418
2. India	1995	0.07	0.5174	0.13530	4.00	7.73	0.24057	0.60000	0.20728	0.04200	7.02	----	1.1275	0.5174
B. Rupees	1996	0.07	0.5245	0.13346	4.54	8.65	0.21360	0.60000	0.17912	0.04200	7.78	0.1239	1.1122	0.5245
	1997	0.07	0.6071	0.11530	5.74	9.46	0.19525	0.60000	0.15997	0.04200	8.37	0.0848	1.2811	0.6071
	450 1998	0.07	0.6650	0.10527	7.07	10.63	0.19887	0.60000	0.16374	0.04200	9.27	0.1202	1.1697	0.6650
\$y by ex.rate	1999	0.07	0.7799	0.08976	9.03	11.58	0.20255	0.60000	0.16759	0.04200	9.92	0.0802	1.1220	0.7799
<b>0.20505</b>	2000	0.07	0.8583	0.08156	10.66	12.42	0.20746	0.60000	0.17271	0.04200	10.52	0.0679	1.0195	0.8583
	2001	0.07	0.9560	0.07322	12.75	13.33	0.20341	0.60000	0.16848	0.04200	11.16	0.0681	1.1264	0.9560
3. Algeria	1995	0.05	0.3178	0.15733	13.63	42.87	0.20037	0.60000	0.17564	0.03000	37.62	----	1.1238	0.3178
B. Dinars	1996	0.05	0.3452	0.14485	18.76	54.33	0.16466	0.60000	0.13882	0.03000	46.93	0.2667	1.1142	0.3452
	1997	0.05	0.3852	0.12980	22.27	57.82	0.13253	0.60000	0.10570	0.03000	49.51	0.0603	1.1800	0.3852
	1580 1998	0.05	0.4815	0.10384	27.62	57.36	0.18364	0.60000	0.15839	0.03000	48.59	(0.0198)	1.0931	0.4815
\$y by ex.rate	1999	0.05	0.5302	0.09431	34.36	64.80	0.15942	0.60000	0.13342	0.03000	54.30	0.1275	1.1095	0.5302
<b>0.82439</b>	2000	0.05	0.6178	0.08093	50.00	80.93	0.10522	0.60000	0.07755	0.03000	66.55	0.2453	1.3488	0.6178
	2001	0.05	0.7108	0.07034	58.56	82.38	0.11712	0.60000	0.08982	0.03000	67.21	0.0116	1.1723	0.7108
4. Egypt	1995	0.05	0.3240	0.15430	689.7	2128.3	0.02042	0.60000	(0.00987)	0.03000	1534.99	----	1.1430	0.3240
M.Pounds	1996	0.05	0.3477	0.14379	761.9	2191.0	0.01707	0.60000	(0.01333)	0.03000	1572.36	0.0331	1.1060	0.3477
	1997	0.05	0.3771	0.13258	909.1	2410.5	0.05910	0.60000	0.03000	0.03000	1714.65	0.0988	1.0823	0.3771
	1490 1998	0.05	0.3831	0.13051	1012.7	2643.2	0.10549	0.60000	0.07782	0.03000	1870.04	0.0985	1.0876	0.3831
\$y by ex.rate	1999	0.05	0.3895	0.12838	1081.1	2775.8	0.09677	0.60000	0.06884	0.03000	1957.44	0.0513	1.0698	0.3895
<b>626.208</b>	2000	0.05	0.3908	0.12795	1176.5	3010.6	0.06559	0.60000	0.03669	0.03000	2114.11	0.0871	1.0663	0.3908
	2001	0.05	0.4078	0.12260	1269.8	3113.7	0.04551	0.60000	0.01599	0.03000	2178.17	0.0337	1.1146	0.4078
	<b>2002</b>	0.05	0.4247	0.11774	1379.3	3248.0	0.04473	0.60000	0.01519	0.03000	2262.77	0.0430	1.1774	0.4247
5. Ethiopia	1995	0.05	0.3537	0.14137	131.58	372.02	0.02392	0.60000	(0.00627)	0.03000	291.48	----	1.1781	0.3537
M. Birr	10 1996	0.05	0.3559	0.14050	137.25	385.68	0.06833	0.60000	0.03951	0.03000	301.54	0.0433	1.0093	0.3559
\$y by ex.rate	1997	0.05	0.3725	0.13422	166.67	447.38	0.03335	0.60000	0.00346	0.03000	346.41	0.1635	1.2782	0.3725
<b>40.87747</b>	1998	0.05	0.3915	0.12770	169.08	431.85	0.03669	0.60000	0.00689	0.03000	334.14	(0.0369)	1.2162	0.3915
	1999	0.05	0.4032	0.12401	184.21	456.88	0.0166639	0.6	-0.013749	0.03	351.99	0.0593	1.1721	0.4032
	2000	0.05	0.4004	0.12487	190.74	476.36	0.0058859	0.6	-0.02486	0.03	366.36	0.0454	1.1467	0.4004
	2001	0.05	0.4159	0.12023	196.63	472.82	0.0250058	0.6	-0.005149	0.03	363.09	(0.0083)	1.1061	0.4159

## Data 4 IMF data and

estimated values (1)

		$B_K =$			discount rate		$r_M^* = r^* \beta^*$			$c_{CB} = r^* / r_{CB}$		$(B_K/A) / c_{CB} =$		$c_{M(CB)} = r_M^* / r_{M(CB)}$	
		$\beta^* (\beta=0)$	$i$	$1/\beta^* (\beta=0)$	$F$	$r_{CB}$	$r^*$	$r_M^*$	$A = r^* / r_M^*$	$B_K/A$	$c_{CB}$	$B_K/A_{CB}$	$A_{CB}$	$r_{M(CB)}$	$c_{M(CB)}$
1. China	1995	0.4353	0.2722	2.2972	0.1620	0.1044	0.1188	0.0517	2.2972	1.0000	1.1384	0.8784	2.6151	0.0454	1.1384
B. Yuan	1996	0.4785	0.2650	2.0898	0.1463	0.0900	0.1003	0.0480	2.0898	1.0000	1.1149	0.8969	2.3299	0.0431	1.1149
	1997	0.4740	0.2574	2.1095	0.1325	0.0855	0.0944	0.0448	2.1095	1.0000	1.1044	0.9055	2.3297	0.0405	1.1044
	1998	0.6307	0.2801	1.5854	0.1126	0.0459	0.0554	0.0349	1.5854	1.0000	1.2071	0.8284	1.9137	0.0290	1.2071
	1999	0.6903	0.2913	1.4487	0.0990	0.0324	0.0428	0.0296	1.4487	1.0000	1.3215	0.7567	1.9145	0.0224	1.3215
	2000	0.7049	0.2981	1.4187	0.0962	0.0324	0.0398	0.0281	1.4187	1.0000	1.2285	0.8140	1.7429	0.0228	1.2285
	2001	0.7180	0.3073	1.3927	0.0946	0.0324	0.0373	0.0268	1.3927	1.0000	1.1510	0.8688	1.6030	0.0233	1.1510
	<b>2002</b>	0.7345	0.3304	1.3615	0.0953	0.0270	0.0342	0.0251	1.3615	1.0000	1.2652	0.7904	1.7227	0.0198	1.2652
2. India	1995	0.4113	0.2009	2.4314	0.1485	0.1200	0.1353	0.0556	2.4314	1.0000	1.1275	0.8869	2.7413	0.0494	1.1275
B. Rupees	1996	0.4206	0.1793	2.3776	0.1337	0.1200	0.1335	0.0561	2.3776	1.0000	1.1122	0.8991	2.6442	0.0505	1.1122
	1997	0.4397	0.1646	2.2744	0.1109	0.0900	0.1153	0.0507	2.2744	1.0000	1.2811	0.7806	2.9138	0.0396	1.2811
	1998	0.4622	0.1675	2.1635	0.1083	0.0900	0.1053	0.0487	2.1635	1.0000	1.1697	0.8549	2.5306	0.0416	1.1697
	1999	0.5032	0.1704	1.9873	0.1023	0.0800	0.0898	0.0452	1.9873	1.0000	1.1220	0.8913	2.2297	0.0403	1.1220
	2000	0.5267	0.1744	1.8986	0.0995	0.0800	0.0816	0.0430	1.8986	1.0000	1.0195	0.9809	1.9356	0.0421	1.0195
	2001	0.5551	0.1711	1.8014	0.0924	0.0650	0.0732	0.0406	1.8014	1.0000	1.1264	0.8877	2.0292	0.0361	1.1264
						0.0625									
3. Algeria	1995	0.2691	0.1663	3.7161	0.1338	0.1400	0.1573	0.0423	3.7161	1.0000	1.1238	0.8899	4.1760	0.0377	1.1238
B. Dinars	1996	0.2896	0.1377	3.4525	0.1098	0.1300	0.1448	0.0420	3.4525	1.0000	1.1142	0.8975	3.8468	0.0377	1.1142
	1997	0.3322	0.1120	3.0101	0.0918	0.1100	0.1298	0.0431	3.0101	1.0000	1.1800	0.8475	3.5519	0.0365	1.1800
	1998	0.3722	0.1529	2.6864	0.1123	0.0950	0.1038	0.0387	2.6864	1.0000	1.0931	0.9148	2.9365	0.0354	1.0931
	1999	0.4025	0.1335	2.4845	0.0963	0.0850	0.0943	0.0380	2.4845	1.0000	1.1095	0.9013	2.7565	0.0342	1.1095
	2000	0.4630	0.0902	2.1600	0.0642	0.0600	0.0809	0.0375	2.1600	1.0000	1.3488	0.7414	2.9134	0.0278	1.3488
	2001	0.5004	0.0997	1.9985	0.0667	0.0600	0.0703	0.0352	1.9985	1.0000	1.1723	0.8530	2.3429	0.0300	1.1723
	<b>2002</b>					0.0550									
4. Egypt	1995	1.2472	0.0223	0.8018	0.0817	0.1350	0.1543	0.1924	0.8018	1.0000	1.1430	0.8749	0.9165	0.1684	1.1430
M.Pounds	1996	1.4524	0.0197	0.6885	0.0780	0.1300	0.1438	0.2088	0.6885	1.0000	1.1060	0.9041	0.7615	0.1888	1.1060
	1997	0.3843	0.0533	2.6021	0.0516	0.1225	0.1326	0.0510	2.6021	1.0000	1.0823	0.9240	2.8161	0.0471	1.0823
	1998	0.3485	0.0904	2.8693	0.0781	0.1200	0.1305	0.0455	2.8693	1.0000	1.0876	0.9195	3.1205	0.0418	1.0876
	1999	0.3578	0.0834	2.7949	0.0728	0.1200	0.1284	0.0459	2.7949	1.0000	1.0698	0.9347	2.9901	0.0429	1.0698
	2000	0.3868	0.0585	2.5854	0.0550	0.1200	0.1280	0.0495	2.5854	1.0000	1.0663	0.9379	2.7567	0.0464	1.0663
	2001	0.4367	0.0424	2.2899	0.0431	0.1100	0.1226	0.0535	2.2899	1.0000	1.1146	0.8972	2.5522	0.0480	1.1146
	<b>2002</b>	0.4535	0.0418	2.2048	0.0424	0.1000	0.1177	0.0534	2.2048	1.0000	1.1774	0.8493	2.5960	0.0454	1.1774
5. Ethiopia	1995	1.0894	0.0251	0.9180	0.0735	0.1200	0.1414	0.1540	0.9180	1.0000	1.1781	0.8488	1.0814	0.1307	1.1781
M. Birr	1996	0.6220	0.0607	1.6076	0.1007	0.1392	0.1405	0.0874	1.6076	1.0000	1.0093	0.9908	1.6226	0.0866	1.0093
	1997	0.5144	0.0327	1.9440	0.0429	0.1050	0.1342	0.0690	1.9440	1.0000	1.2782	0.7823	2.4849	0.0540	1.2782
	1998	0.5065	0.0353	1.9745	0.0434	0.1050	0.1277	0.0647	1.9745	1.0000	1.2162	0.8222	2.4014	0.0532	1.2162
	1999	0.6859	0.0193	1.4580	0.0312	0.1058	0.1240	0.0851	1.4580	1.0000	1.1721	0.8532	1.7089	0.0726	1.1721
	2000	0.9755	0.0107	1.0251	0.0248	0.1089	0.1249	0.1218	1.0251	1.0000	1.1467	0.8721	1.1755	0.1062	1.1467
	2001	0.5925	0.0260	1.6876	0.0352	0.1087	0.1202	0.0712	1.6876	1.0000	1.1061	0.9041	1.8667	0.0644	1.1061

Noe: if  $B_K/A < 1$  under  $s < \alpha$ , the situation is assets-inflation oriented (as in African countries), while if  $B_K/A < 1$  under  $s > \alpha$ , assets-inflation oriented (as in China).

Appendix 2-1 The Penrose effect and the valuation ratio using  $I/K$  and  $B_K/A=1.0$ : China 2002, India 2001, and Argeria 2002

1. China 2002															$r_{CB}$ below	
current beta			0.9224	0.08774		$\Omega^*=ke^{*\alpha(1-\alpha)}$	2.28907	$ke^*=\Omega^{*\alpha}(1/(1-\alpha))$		2.4600	$x_e^*/x_0=k(0)/ke^*$		4.51673	$(I/K)/r^*$	$r_{M^*}$ at $\beta^*$	
$n$	$\beta$	$\delta_{\beta=0}$	$s \rightarrow i$	$\Omega(0)$	alpha	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^*=ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_k^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
beta*	$\Omega^*$	$I/K$	$g_k^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_k=r_{M^*}-g_k^*$	$a_y=r^*-I/K$	$v=I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB}=r_{M^*}/r_{CB}$	
0.00755	0.73447	0.33044	2.34181	0.08000	0.0342	11.1111	4.7447	0.00720	2.4600	1.07467	0.0349	0.03252	0.22140	4.1305	0.02509	
0.4	0.5883	0.5617	0.2247	2.5000	0.1360	0.0544	2.5000	1.0000	112.500	(0.1703)	(0.4257)	2.5000	1.0000	2.5000	2.0146	
0.5	0.8765	0.3770	0.1885	2.0000	0.0913	0.0456	2.0000	1.0000	90.000	(0.1429)	(0.2857)	2.0000	1.0000	2.0000	1.6901	
0.6	1.3018	0.2538	0.1523	1.6667	0.0615	0.0369	1.6667	1.0000	75.000	(0.1154)	(0.1924)	1.6667	1.0000	1.6667	1.3656	
0.7	1.9921	0.1659	0.1161	1.4286	0.04016	0.02811	1.4286	1.0000	64.286	(0.0880)	(0.1257)	1.4286	1.0000	1.4286	1.0412	
0.8	3.3075	0.0999	0.0799	1.2500	0.0242	0.0193	1.2500	1.0000	56.250	(0.0606)	(0.0757)	1.2500	1.0000	1.2500	0.7167	
0.9	6.7998	0.0486	0.0437	1.1111	0.0118	0.0106	1.1111	1.0000	50.000	(0.0331)	(0.0368)	1.1111	1.0000	1.1111	0.3922	
0.95	12.2425	0.0270	0.0256	1.0526	0.0065	0.0062	1.0526	1.0000	47.368	(0.0194)	(0.0205)	1.0526	1.0000	1.0526	0.2299	
1	43.7821	0.0075	0.0075	1.0000	0.0018	0.0018	1.0000	1.0000	45.000	(0.0057)	(0.0057)	1.0000	1.0000	1.0000	0.0677	
1.05	-32.8972	-0.0100	-0.0105	0.9524	-0.0024	(0.0026)	0.9524	1.0000	42.857	0.0080	0.0076	0.9524	1.0000	0.9524	-0.0946	
1.1	-12.6910	-0.0260	-0.0286	0.9091	-0.0063	(0.0069)	0.9091	1.0000	40.909	0.0217	0.0197	0.9091	1.0000	0.9091	-0.2568	
1.2	-6.1165	-0.0540	-0.0648	0.8333	-0.0131	(0.0157)	0.8333	1.0000	37.500	0.0491	0.0409	0.8333	1.0000	0.8333	-0.5813	
1.3	-4.2524	-0.0777	-0.1010	0.7692	-0.0188	(0.0245)	0.7692	1.0000	34.615	0.0766	0.0589	0.7692	1.0000	0.7692	-0.9058	

2. India 2001															$r_{CB}$ below	
current beta			0.8267	0.07613		$\Omega^*=ke^{*\alpha(1-\alpha)}$	0.95531	$ke^*=\Omega^{*\alpha}(1/(1-\alpha))$		0.9520	$x_e^*/x_0=k(0)/ke^*$		13.38728	$(I/K)/r^*$	$r_{M^*}$ at $\beta^*$	
$n$	$\beta$	$\delta_{\beta=0}$	$s \rightarrow i$	$\Omega(0)$	alpha	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^*=ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_k^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
beta*	$\Omega^*$	$I/K$	$g_k^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_k=r_{M^*}-g_k^*$	$a_y=r^*-I/K$	$v=I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB}=r_{M^*}/r_{CB}$	
0.01618	0.55511	0.17113	0.95604	0.07000	0.0732	12.7451	13.3312	0.00549	0.9520	0.99656	0.0733	0.07353	0.07470	2.4447	0.04064	
0.4	0.5332	0.3209	0.1284	2.5000	0.1313	0.0525	2.5000	1.0000	112.500	(0.0759)	(0.1896)	2.5000	1.0000	2.5000	0.8078	
0.5	0.7802	0.2193	0.1097	2.0000	0.0897	0.0449	2.0000	1.0000	90.000	(0.0648)	(0.1296)	2.0000	1.0000	2.0000	0.6902	
0.6	1.1287	0.1516	0.0910	1.6667	0.06202	0.03721	1.6667	1.0000	75.000	(0.0538)	(0.0896)	1.6667	1.0000	1.6667	0.5725	
0.7	1.6575	0.1032	0.0723	1.4286	0.0422	0.0296	1.4286	1.0000	64.286	(0.0427)	(0.0610)	1.4286	1.0000	1.4286	0.4548	
0.8	2.5554	0.0670	0.0536	1.2500	0.0274	0.0219	1.2500	1.0000	56.250	(0.0317)	(0.0396)	1.2500	1.0000	1.2500	0.3371	
0.9	4.4162	0.0387	0.0349	1.1111	0.0159	0.0143	1.1111	1.0000	50.000	(0.0206)	(0.0229)	1.1111	1.0000	1.1111	0.2195	
0.95	6.3690	0.0269	0.0255	1.0526	0.0110	0.0104	1.0526	1.0000	47.368	(0.0151)	(0.0159)	1.0526	1.0000	1.0526	0.1606	
1	10.5790	0.0162	0.0162	1.0000	0.0066	0.0066	1.0000	1.0000	45.000	(0.0096)	(0.0096)	1.0000	1.0000	1.0000	0.1018	
1.05	26.3200	0.0065	0.0068	0.9524	0.0027	0.0028	0.9524	1.0000	42.857	(0.0040)	(0.0038)	0.9524	1.0000	0.9524	0.0430	
1.1	-74.6298	-0.0023	-0.0025	0.9091	-0.0009	(0.0010)	0.9091	1.0000	40.909	0.0015	0.0014	0.9091	1.0000	0.9091	-0.0159	
1.2	-9.6770	-0.0177	-0.0212	0.8333	-0.0072	(0.0087)	0.8333	1.0000	37.500	0.0125	0.0105	0.8333	1.0000	0.8333	-0.1335	
1.3	-5.5729	-0.0307	-0.0399	0.7692	-0.0126	(0.0163)	0.7692	1.0000	34.615	0.0236	0.0181	0.7692	1.0000	0.7692	-0.2512	

3. Algeria 2002															$r_{CB}$ below	
current beta			0.5320	0.04981		$\Omega^*=ke^{*\alpha(1-\alpha)}$	0.65505	$ke^*=\Omega^{*\alpha}(1/(1-\alpha))$		0.6406	$x_e^*/x_0=k(0)/ke^*$		91.40769	$(I/K)/r^*$	$r_{M^*}$ at $\beta^*$	
$n$	$\beta$	$\delta_{\beta=0}$	$s \rightarrow i$	$\Omega(0)$	alpha	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^*=ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_k^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
beta*	$\Omega^*$	$I/K$	$g_k^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_k=r_{M^*}-g_k^*$	$a_y=r^*-I/K$	$v=I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB}=r_{M^*}/r_{CB}$	
0.01687	0.50038	0.09970	0.71083	0.05000	0.0703	58.5586	82.3805	0.00085	0.6406	0.97798	0.0763	0.07805	0.01094	1.9940	0.03520	
0.4	0.4930	0.2022	0.0809	2.5000	0.1014	0.0406	2.5000	1.0000	112.500	(0.1008)	(0.1008)	2.5000	1.0000	2.5000	0.7376	
0.5	0.7099	0.1404	0.0702	2.0000	0.0704	0.03522	2.0000	1.0000	90.000	(0.0350)	(0.0700)	2.0000	1.0000	2.0000	0.6403	
0.6	1.0045	0.0993	0.0596	1.6667	0.0498	0.0299	1.6667	1.0000	75.000	(0.0297)	(0.0495)	1.6667	1.0000	1.6667	0.5430	
0.7	1.4278	0.0698	0.0489	1.4286	0.0350	0.0245	1.4286	1.0000	64.286	(0.0244)	(0.0348)	1.4286	1.0000	1.4286	0.4457	
0.8	2.0875	0.0478	0.0382	1.2500	0.0240	0.0192	1.2500	1.0000	56.250	(0.0190)	(0.0238)	1.2500	1.0000	1.2500	0.3484	
0.9	3.2585	0.0306	0.0275	1.1111	0.0153	0.0138	1.1111	1.0000	50.000	(0.0137)	(0.0153)	1.1111	1.0000	1.1111	0.2511	
0.95	4.2662	0.0234	0.0222	1.0526	0.0117	0.0111	1.0526	1.0000	47.368	(0.0111)	(0.0116)	1.0526	1.0000	1.0526	0.2024	
1	5.9116	0.0169	0.0169	1.0000	0.0085	0.0085	1.0000	1.0000	45.000	(0.0084)	(0.0084)	1.0000	1.0000	1.0000	0.1538	
1.05	9.0799	0.0110	0.0115	0.9524	0.0055	0.0058	0.9524	1.0000	42.857	(0.0057)	(0.0055)	0.9524	1.0000	0.9524	0.1051	
1.1	17.7075	0.0056	0.0062	0.9091	0.0028	0.0031	0.9091	1.0000	40.909	(0.0031)	(0.0028)	0.9091	1.0000	0.9091	0.0565	
1.2	-26.7153	-0.0037	-0.0045	0.8333	-0.0019	(0.0022)	0.8333	1.0000	37.500	0.0022	0.0019	0.8333	1.0000	0.8333	-0.0408	
1.3	-8.5551	-0.0117	-0.0152	0.7692	-0.0058	(0.0076)	0.7692	1.0000	34.615	0.0076	0.0058	0.7692	1.0000	0.7692	-0.1381	

Appendix 2-2 The Penrose effect and the valuation ratio using  $I/K$  and  $B_K/A=1.0$ : Egypt 2002, Ethiopia 2001, and Mali 2002

4. Egypt 2002															$r_{CB}$ below	
current beta			0.6088	0.02283		$\Omega^* = ke^{*\alpha}(1-\alpha)$	0.42434	$ke^* = \Omega^{*\alpha}(1/(1-\alpha))$		0.4056	$x_e^*/x_0 = k(0)/ke^*$		3400.479	$(I/K)/r^*$	$r_{CB}$	
$n$	$\beta$	$\delta \rightarrow 0$	$s \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_K^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
0.02011	0.45355		0.04179	0.42466	0.05000	0.1177	1379.3103	3248.0499	0.00004	0.4056	0.95589	0.1178	0.12327	0.00029	0.8357	0.05340
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_K = r_{M^*} - g_K^*$	$a_Y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_{M^*}^*/r_{CB}$	
0.4	0.3554	0.1176	0.0470	2.5000	0.1407	0.0563	2.5000	1.0000	112.500	0.0092	0.0231	2.5000	1.0000	2.5000	0.5628	
0.5	0.4911	0.0851	0.0425	2.0000	0.1018	0.0509	2.0000	1.0000	90.000	0.0084	0.0167	2.0000	1.0000	2.0000	0.5091	
0.6	0.6588	0.0634	0.0381	1.6667	0.0759	0.0455	1.6667	1.0000	75.000	0.0075	0.0125	1.6667	1.0000	1.6667	0.4554	
0.7	0.8713	0.0480	0.0336	1.4286	0.05738	0.04017	1.4286	1.0000	64.286	0.0066	0.0094	1.4286	1.0000	1.4286	0.4017	
0.8	1.1494	0.0364	0.0291	1.2500	0.0435	0.0348	1.2500	1.0000	56.250	0.0057	0.0071	1.2500	1.0000	1.2500	0.3480	
0.9	1.5290	0.0273	0.0246	1.1111	0.0327	0.0294	1.1111	1.0000	50.000	0.0048	0.0054	1.1111	1.0000	1.1111	0.2943	
0.95	1.7759	0.0235	0.0224	1.0526	0.0282	0.0267	1.0526	1.0000	47.368	0.0044	0.0046	1.0526	1.0000	1.0526	0.2675	
1	2.0779	0.0201	0.0201	1.0000	0.0241	0.0241	1.0000	1.0000	45.000	0.0040	0.0040	1.0000	1.0000	1.0000	0.2406	
1.05	2.4558	0.0170	0.0179	0.9524	0.0204	0.0214	0.9524	1.0000	42.857	0.0035	0.0033	0.9524	1.0000	0.9524	0.2138	
1.1	2.9422	0.0142	0.0156	0.9091	0.0170	0.0187	0.9091	1.0000	40.909	0.0031	0.0028	0.9091	1.0000	0.9091	0.1869	
1.2	4.5031	0.0093	0.0111	0.8333	0.0111	0.0133	0.8333	1.0000	37.500	0.0022	0.0018	0.8333	1.0000	0.8333	0.1332	
1.3	8.1707	0.0051	0.0066	0.7692	0.0061	0.0080	0.7692	1.0000	34.615	0.0013	0.0010	0.7692	1.0000	0.7692	0.0796	

5. Ethiopia 2001															$r_{CB}$ below	
current beta			0.9480 (not converg)	0.01060		$\Omega^* = ke^{*\alpha}(1-\alpha)$	0.41760	$ke^* = \Omega^{*\alpha}(1/(1-\alpha))$		0.3988	$x_e^*/x_0 = k(0)/ke^*$		492.99496	$(I/K)/r^*$	$r_{CB}$	
$n$	$\beta$	$\delta \rightarrow 0$	$s \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_K^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
0.02561	0.59255		0.02600	0.41586	0.05000	0.1202	196.6292	472.8200	0.00025	0.3988	0.95508	0.1197	0.12536	0.00203	0.5201	0.07124
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_K = r_{M^*} - g_K^*$	$a_Y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_{M^*}^*/r_{CB}$	
0.4	0.2450	0.1061	0.0425	2.5000	0.2041	0.0816	2.5000	1.0000	112.500	0.0392	0.0979	2.5000	1.0000	2.5000	1.2559	
0.5	0.3279	0.0793	0.0397	2.0000	0.1525	0.0762	2.0000	1.0000	90.000	0.0366	0.0732	2.0000	1.0000	2.0000	1.1729	
0.6	0.4235	0.0614	0.0368	1.6667	0.11807	0.07084	1.6667	1.0000	75.000	0.0340	0.0567	1.6667	1.0000	1.6667	1.0898	
0.7	0.5348	0.0486	0.0340	1.4286	0.0935	0.0654	1.4286	1.0000	64.286	0.0314	0.0449	1.4286	1.0000	1.4286	1.0068	
0.8	0.6662	0.0390	0.0312	1.2500	0.0751	0.0600	1.2500	1.0000	56.250	0.0288	0.0360	1.2500	1.0000	1.2500	0.9238	
0.9	0.8235	0.0316	0.0284	1.1111	0.0607	0.0546	1.1111	1.0000	50.000	0.0262	0.0291	1.1111	1.0000	1.1111	0.8407	
0.95	0.9144	0.0284	0.0270	1.0526	0.0547	0.0519	1.0526	1.0000	47.368	0.0249	0.0262	1.0526	1.0000	1.0526	0.7992	
1	1.0153	0.0256	0.0256	1.0000	0.0492	0.0492	1.0000	1.0000	45.000	0.0236	0.0236	1.0000	1.0000	1.0000	0.7577	
1.05	1.1278	0.0231	0.0242	0.9524	0.0443	0.0465	0.9524	1.0000	42.857	0.0223	0.0213	0.9524	1.0000	0.9524	0.7161	
1.1	1.2543	0.0207	0.0228	0.9091	0.0399	0.0439	0.9091	1.0000	40.909	0.0210	0.0191	0.9091	1.0000	0.9091	0.6746	
1.2	1.5604	0.0167	0.0200	0.8333	0.0320	0.0385	0.8333	1.0000	37.500	0.0185	0.0154	0.8333	1.0000	0.8333	0.5916	
1.3	1.9665	0.0132	0.0172	0.7692	0.0254	0.0331	0.7692	1.0000	34.615	0.0159	0.0122	0.7692	1.0000	0.7692	0.5085	

6. Mali 2002															$r_{CB}$ below	
current beta			0.7230	0.04480		$\Omega^* = ke^{*\alpha}(1-\alpha)$	0.11433	$ke^* = \Omega^{*\alpha}(1/(1-\alpha))$		0.1020	$x_e^*/x_0 = k(0)/ke^*$		125.69313	$(I/K)/r^*$	$r_{CB}$	
$n$	$\beta$	$\delta \rightarrow 0$	$s \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^{*\alpha}$	$r^*$	$x_e = \alpha/ke^*$	$x_0/x_e$	$=g_K^*/r_{M^*}$	$r_{M^*}$ at $\beta^*$
0.02936	0.16581		0.05370	0.11430	0.05000	0.4375	12.8205	112.1696	0.00390	0.1020	0.89213	0.4373	0.49020	0.00796	1.0740	0.07253
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_K = r_{M^*} - g_K^*$	$a_Y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_{M^*}^*/r_{CB}$	
0.4	0.3342	0.1607	0.0643	2.5000	0.1496	0.0598	2.5000	1.0000	112.500	(0.0044)	(0.0111)	2.5000	1.0000	2.5000	0.1496	
0.5	0.4593	0.1169	0.0585	2.0000	0.10886	0.05443	2.0000	1.0000	90.000	(0.0040)	(0.0081)	2.0000	1.0000	2.0000	0.1361	
0.6	0.6121	0.0877	0.0526	1.6667	0.0817	0.0490	1.6667	1.0000	75.000	(0.0036)	(0.0060)	1.6667	1.0000	1.6667	0.1225	
0.7	0.8029	0.0669	0.0468	1.4286	0.0623	0.0436	1.4286	1.0000	64.286	(0.0032)	(0.0046)	1.4286	1.0000	1.4286	0.1090	
0.8	1.0478	0.0513	0.0410	1.2500	0.0477	0.0382	1.2500	1.0000	56.250	(0.0028)	(0.0035)	1.2500	1.0000	1.2500	0.0954	
0.9	1.3737	0.0391	0.0352	1.1111	0.0364	0.0328	1.1111	1.0000	50.000	(0.0024)	(0.0027)	1.1111	1.0000	1.1111	0.0819	
0.95	1.5807	0.0340	0.0323	1.0526	0.0316	0.0300	1.0526	1.0000	47.368	(0.0022)	(0.0023)	1.0526	1.0000	1.0526	0.0751	
1	1.8288	0.0294	0.0294	1.0000	0.0273	0.0273	1.0000	1.0000	45.000	(0.0020)	(0.0020)	1.0000	1.0000	1.0000	0.0684	
1.05	2.1314	0.0252	0.0265	0.9524	0.0235	0.0246	0.9524	1.0000	42.857	(0.0018)	(0.0017)	0.9524	1.0000	0.9524	0.0616	
1.1	2.5088	0.0214	0.0235	0.9091	0.0199	0.0219	0.9091	1.0000	40.909	(0.0016)	(0.0015)	0.9091	1.0000	0.9091	0.0548	
1.2	3.6353	0.0148	0.0177	0.8333	0.0138	0.0165	0.8333	1.0000	37.500	(0.0012)	(0.0010)	0.8333	1.0000	0.8333	0.0413	
1.3	5.8626	0.0092	0.0119	0.7692	0.0085	0.0111	0.7692	1.0000	34.615	(0.0008)	(0.0006)	0.7692	1.0000	0.7692	0.0277	



Appendix 2-3 The Penrose effect and the valuation ratio using  $I/K$  and  $B_K/A=1.0$ : Nigeria 2002, South Africa 2002, and Tanzania 2002

7. Nigeria 2002													$r_{CB}$ below			
$n$	$\beta$	$\Delta \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^* \alpha$	$r^*$	$x_e^*/x_0 = k(0)/ke^*$	$x_0/x_e$	$(I/K)/r^*$	$r_M^*$ at $\beta^*$	$r_{CB}^*$
0.02623	0.32435	0.06112	0.27987	0.05000	0.1787	8.1967	29.2879	0.00610	0.2626	0.93533	0.1781	0.19040	0.03204	1.2225	0.05795	
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_x = r_{M^*} - g_K^*$	$a_y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_M^*/r_{CB}$	
0.4	0.3713	0.1646	0.0658	2.5000	0.1347	0.0539	2.5000	1.0000	112.500	(0.0120)	(0.0300)	2.5000	1.0000	2.5000	0.3264	
0.5	0.5159	0.1185	0.0592	2.0000	0.0969	0.0485	2.0000	1.0000	90.000	(0.0108)	(0.0216)	2.0000	1.0000	2.0000	0.2937	
0.6	0.6967	0.0877	0.0526	1.6667	0.0718	0.0431	1.6667	1.0000	75.000	(0.0096)	(0.0160)	1.6667	1.0000	1.6667	0.2610	
0.7	0.9294	0.0658	0.0460	1.4286	0.05380	0.03766	1.4286	1.0000	64.286	(0.0084)	(0.0120)	1.4286	1.0000	1.4286	0.2282	
0.8	1.2401	0.0493	0.0394	1.2500	0.0403	0.0323	1.2500	1.0000	56.250	(0.0072)	(0.0090)	1.2500	1.0000	1.2500	0.1955	
0.9	1.6757	0.0365	0.0328	1.1111	0.0298	0.0269	1.1111	1.0000	50.000	(0.0060)	(0.0066)	1.1111	1.0000	1.1111	0.1628	
0.95	1.9665	0.0311	0.0295	1.0526	0.0254	0.0242	1.0526	1.0000	47.368	(0.0054)	(0.0057)	1.0526	1.0000	1.0526	0.1464	
1	2.3306	0.0262	0.0262	1.0000	0.0215	0.0215	1.0000	1.0000	45.000	(0.0048)	(0.0048)	1.0000	1.0000	1.0000	0.1300	
1.05	2.7995	0.0218	0.0229	0.9524	0.0179	0.0188	0.9524	1.0000	42.857	(0.0042)	(0.0040)	0.9524	1.0000	0.9524	0.1137	
1.1	3.4262	0.0178	0.0196	0.9091	0.0146	0.0161	0.9091	1.0000	40.909	(0.0036)	(0.0032)	0.9091	1.0000	0.9091	0.0973	
1.2	5.6330	0.0109	0.0130	0.8333	0.0089	0.0107	0.8333	1.0000	37.500	(0.0024)	(0.0020)	0.8333	1.0000	0.8333	0.0646	
1.3	12.3800	0.0049	0.0064	0.7692	0.0040	0.0053	0.7692	1.0000	34.615	(0.0012)	(0.0009)	0.7692	1.0000	0.7692	0.0318	

8. South Africa 2002													$r_{CB}$ below			
$n$	$\beta$	$\Delta \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^* \alpha$	$r^*$	$x_e^*/x_0 = k(0)/ke^*$	$x_0/x_e$	$(I/K)/r^*$	$r_M^*$ at $\beta^*$	$r_{CB}^*$
0.00765	0.34003	0.03800	0.37721	0.05000	0.1326	5555.56	14728	0.00001	0.3566	0.94975	0.1332	0.14022	0.00006	0.7601	0.04507	
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_x = r_{M^*} - g_K^*$	$a_y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_M^*/r_{CB}$	
0.4	0.4774	0.0796	0.0318	2.5000	0.1047	0.0419	2.5000	1.0000	112.500	0.0101	0.0251	2.5000	1.0000	2.5000	0.3103	
0.5	0.6833	0.0556	0.0278	2.0000	0.0732	0.0366	2.0000	1.0000	90.000	0.0088	0.0176	2.0000	1.0000	2.0000	0.2710	
0.6	0.9590	0.0396	0.0238	1.6667	0.05214	0.03128	1.6667	1.0000	75.000	0.0075	0.0125	1.6667	1.0000	1.6667	0.2317	
0.7	1.3472	0.0282	0.0197	1.4286	0.0371	0.0260	1.4286	1.0000	64.286	0.0062	0.0089	1.4286	1.0000	1.4286	0.1924	
0.8	1.9345	0.0196	0.0157	1.2500	0.0258	0.0207	1.2500	1.0000	56.250	0.0050	0.0062	1.2500	1.0000	1.2500	0.1532	
0.9	2.9270	0.0130	0.0117	1.1111	0.0171	0.0154	1.1111	1.0000	50.000	0.0037	0.0041	1.1111	1.0000	1.1111	0.1139	
0.95	3.7337	0.0102	0.0097	1.0526	0.0134	0.0127	1.0526	1.0000	47.368	0.0031	0.0032	1.0526	1.0000	1.0526	0.0942	
1	4.9650	0.0077	0.0077	1.0000	0.0101	0.0101	1.0000	1.0000	45.000	0.0024	0.0024	1.0000	1.0000	1.0000	0.0746	
1.05	7.0767	0.0054	0.0056	0.9524	0.0071	0.0074	0.9524	1.0000	42.857	0.0018	0.0017	0.9524	1.0000	0.9524	0.0550	
1.1	11.5377	0.0033	0.0036	0.9091	0.0043	0.0048	0.9091	1.0000	40.909	0.0011	0.0010	0.9091	1.0000	0.9091	0.0353	
1.2	-111.8404	-0.0003	-0.0004	0.8333	-0.0004	(0.0005)	0.8333	1.0000	37.500	(0.0001)	(0.0001)	0.8333	1.0000	0.8333	-0.0040	
1.3	-11.1303	-0.0034	-0.0044	0.7692	-0.0045	(0.0058)	0.7692	1.0000	34.615	(0.0014)	(0.0011)	0.7692	1.0000	0.7692	-0.0433	

9. Tanzania 2002													$r_{CB}$ below			
$n$	$\beta$	$\Delta \rightarrow i$	$\Omega(0)$	$\alpha$	$r^*$	$k(0)$	$y(0)$	$x_0 = \alpha/k(0)$	calibrat. $ke^*$	$y^* = ke^* \alpha$	$r^*$	$x_e^*/x_0 = k(0)/ke^*$	$x_0/x_e$	$(I/K)/r^*$	$r_M^*$ at $\beta^*$	$r_{CB}^*$
0.02067	0.52157	0.03009	0.43437	0.05000	0.1151	60.0000	138.1296	0.00083	0.4138	0.95684	0.1156	0.12082	0.00690	0.6019	0.06004	
$beta^*$	$\Omega^*$	$I/K$	$g_K^*$	Slope $B_K$	$r^*$	$r_{M^*}$	Slope A	Slope $B_K/A$	Angle $B_K$ (°)	$a_x = r_{M^*} - g_K^*$	$a_y = r^* - I/K$	$v = I/\beta$	$B_K/A$ by Eqs.	$r^*/r_{M^*}$	$c_{CB} = r_M^*/r_{CB}$	
0.4	0.3004	0.1002	0.0401	2.5000	0.1664	0.0666	2.5000	1.0000	112.500	0.0265	0.0663	2.5000	1.0000	2.5000	0.7251	
0.5	0.4085	0.0737	0.0368	2.0000	0.12239	0.0612	2.0000	1.0000	90.000	0.0244	0.0487	2.0000	1.0000	2.0000	0.6666	
0.6	0.5374	0.0560	0.0336	1.6667	0.0930	0.0558	1.6667	1.0000	75.000	0.0222	0.0370	1.6667	1.0000	1.6667	0.6081	
0.7	0.6937	0.0434	0.0304	1.4286	0.0721	0.0505	1.4286	1.0000	64.286	0.0201	0.0287	1.4286	1.0000	1.4286	0.5496	
0.8	0.8873	0.0339	0.0271	1.2500	0.0564	0.0451	1.2500	1.0000	56.250	0.0179	0.0224	1.2500	1.0000	1.2500	0.4911	
0.9	1.1333	0.0266	0.0239	1.1111	0.0441	0.0397	1.1111	1.0000	50.000	0.0158	0.0176	1.1111	1.0000	1.1111	0.4326	
0.95	1.2830	0.0235	0.0223	1.0526	0.0390	0.0370	1.0526	1.0000	47.368	0.0147	0.0155	1.0526	1.0000	1.0526	0.4033	
1	1.4562	0.0207	0.0207	1.0000	0.0343	0.0343	1.0000	1.0000	45.000	0.0137	0.0137	1.0000	1.0000	1.0000	0.3740	
1.05	1.6587	0.0181	0.0190	0.9524	0.0301	0.0317	0.9524	1.0000	42.857	0.0126	0.0120	0.9524	1.0000	0.9524	0.3448	
1.1	1.8989	0.0158	0.0174	0.9091	0.0263	0.0290	0.9091	1.0000	40.909	0.0115	0.0105	0.9091	1.0000	0.9091	0.3155	
1.2	2.5431	0.0118	0.0142	0.8333	0.0197	0.0236	0.8333	1.0000	37.500	0.0094	0.0078	0.8333	1.0000	0.8333	0.2570	
1.3	3.5673	0.0084	0.0110	0.7692	0.0140	0.0182	0.7692	1.0000	34.615	0.0073	0.0056	0.7692	1.0000	0.7692	0.1985	