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A CGE approach to the measurement of PPPs

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1. Introduction

Prasada Rao [1985] gave us a new view of the measurement of Purchasing Power Parities (PPPs). That is, he gave a new interpretation of the GK method (Geary[1958], Khamis[1970]). According to his view, the world prices of the GK method are Walrasian equilibrium prices in a pure exchange framework where each household has a Cobb-Douglas-type utility function.

In line with his thought, many possibilities naturally emerge as alternative aggregation methods for the international comparisons of real GDPs and PPPs. For example, the assumption of Cobb-Douglas utility function may be replaced by that of CES utility function or linear expenditure systems. Or more complex frameworks (or models) could be formulated for the computation of PPPs. For example, production processes could be introduced instead of the pure exchange settings.

This way of finding world prices and hence purchasing power parities may be called “a CGE approach to the measurement of PPPs”, because, typically, some CGE computation techniques might be required for the process.

2. A Simple Model and a Numerical Example

A simple model can be used to give an account of the idea.

Suppose there are two or more groups of households; say two for simplicity, with the same preferences within each group. It is assumed that the preferences can be presented by a Cobb-Douglas utility function with parameters specific to each group. It is assumed that there are two or more kinds of goods; say two for simplicity, in the economy. Each group has some initial endowments.

As in ordinary aggregation procedures in the PPP computation work, we assume the existence of following two matrices.

$$P = (p_{ij}) , Q = (q_{ij}) ,$$

where P is a price matrix the i-j element of which is the price of the i-th commodity (i=a or b) in the j-th country(j=1 or 2), Q is a quantity matrix and its i-j element is the quantity of the i-th commodity in the j-th country. Thus, for example,

$$P = \begin{pmatrix} 2 & 12 \\ 3 & 15 \end{pmatrix} , Q = \begin{pmatrix} 30 & 50 \\ 40 & 60 \end{pmatrix}$$

It is easy to extend this two commodity two country case to more general m- commodity n- country case.

As is well known, the GK method is the aggregation method the world prices or

the international average prices of which are simultaneously determined with *PPPs* by using the data *P* and *Q* above through the following equations;

$$p_i = \sum_j (p_{ij} / PPP_j)(q_{ij} / \sum_j q_{ij})$$

$$PPP_j = \sum_i p_{ij} q_{ij} / \sum_i p_i q_{ij}.$$

In the matrix form, it can be easily shown that the world prices of the GK method is the eigen vector of the following eigen value problem;

$$p' = p'QB'\widehat{Q}i^{-1},$$

where $p' = (p_i)'$, $B = \left(\frac{p_{ij}q_{ij}}{\sum_k p_{kj}q_{kj}} \right)$.

The data for each *country* play a role of each *group of households'* data in the model. *Q* plays a double role. On the one hand, it gives initial endowment vectors. And on the other, *Q* may be considered to be a response to *P*, domestic price matrix. Note that the Cobb-Douglas share parameters can be easily determined by the following expression.

$$\beta_{ij} = \frac{p_{ij}q_{ij}}{\sum_i p_{ij}q_{ij}}.$$

In our numerical example,

$$\beta_1 (= \beta_{a1}) = \frac{2 \times 30}{2 \times 30 + 3 \times 40} = \frac{1}{3}, \quad \beta_{b1} = 1 - \beta_1.$$

$$\beta_2 (= \beta_{a2}) = \frac{12 \times 50}{12 \times 50 + 15 \times 60} = \frac{2}{5}, \quad \beta_{b2} = 1 - \beta_2.$$

3. A General Equilibrium Interpretation of GK method (Prasada Rao[1985])

Given an unknown world price vector, each household group responds to it by the following demand for the *i*-th commodity.

$$q_{ij}^d = \frac{\beta_{ij} \sum_i p_i q_{ij}}{p_i}.$$

It is easy to aggregate the above individual demands and equate it to the aggregate supply of the *i*-th commodity. That is, we obtain the following linear system of equations to determine equilibrium prices;

$$\sum_j \beta_{ij} \sum_i p_i q_{ij} = p_i \sum_j q_{ij}.$$

In our numerical example,

$$\begin{aligned} 30p_a + (560/15)p_b &= 80p_a, \\ 50p_a + (940/15)p_b &= 100p_b. \end{aligned}$$

Either equation leads to the same result. That is, $p_a/p_b = 56/75$. By using this result,

PPP's are obtained as follows;

$$PPP_j = \frac{\sum_i p_i q_{ij}}{\sum_i p_i q_{ij}}.$$

Thus, for country 1 and 2,

$$\begin{aligned} PPP_1 &= \frac{2 \times 30 + 3 \times 40}{56 \times 30 + 75 \times 40} = \frac{180}{4680} = 0.038462, \\ PPP_2 &= \frac{12 \times 50 + 15 \times 60}{56 \times 50 + 75 \times 60} = \frac{1500}{7300} = 0.205479. \end{aligned}$$

Taking 1st country as a base, PPP for the second country can be calculated as 5.342466.

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As was shown in Prasada Rao [1985], this PPP exactly corresponds to the PPP that could be reached by using the GK method. To show his theorem, notice first Cobb-Douglas demand system in TV (transaction value) form can be shown as follows;

$$p' \widehat{q}^d = y' B',$$

where $q^d = \left(\sum_j q_{ij}^d \right)$ is the vector of total demand and B (large beta) is the matrix of Cobb-Douglas share parameters $y' = p' Q$ is the vector of the incomes of household groups. By equating total demand and total supply ($\widehat{q}^d = \widehat{Q}i$), we get the exactly same equation as in the GK case;

$$p' = p' Q B' \widehat{Q}i^{-1}.^2$$

1 Instead of PPPs calculated from expenditures side, it is also possible to have PPPs from the factor side by calculating rK+WL.

2 One implication of his theorem is that the existence and uniqueness of the PPPs of the GK method might be proved as well by using standard assumptions which are typically employed in the proofs of the existence and uniqueness of the Walrasian general equilibrium..

4. An extension to LES (Linear Expenditure System) cases

It is easy to note that the demand system of the model can be modified in various ways. For example, a little generalisation might be one of replacing Cobb-Douglas by LES (Linear Expenditure System). Of course, the linear expenditure system can be derived by assuming the i -th household has the following utility function³;

$$U(q_{i1}, q_{i2}, \dots, q_{im}) = \prod (q_{ij} - \alpha_{ij})^{\beta_{ij}}.$$

In this relatively simple case, the direct calculation may be possible. However, generally, we need to use some CGE procedures.

By using GAMS-HERCULES software⁴ one of the most popular CGE software at least till early 1990's, we obtain the following result, setting committed consumption

part rather arbitrarily as large alpha matrix $A = \begin{pmatrix} 10 & 25 \\ 20 & 30 \end{pmatrix}$;

$$\text{World Price of } A = 55.72$$

$$\text{World Price of } B = 75.225$$

$$PPP_{2/1} = 5.3435.$$

The treatment of government is one of the problematic areas in the ICP. It might be suggested that goods and services government purchases could be treated as if they are committed consumption part of LES demand functions.

5. A CES Utility Case

The procedure for the CES utility case is almost the same as that in the previous section. For example, we can set the sigma parameters (the elasticities of substitution) 3.00 for country 1 and 1.50 for country 2. ⁵ We obtained *exactly* the same result as in the case of GK by the use of typical CGE procedures about CES.

³ See Neary [1997] for some further information about this functional form.

⁴ See Appendix for the calculation procedures.

⁵ In a CES utility function, $U = (\alpha Q_A^\rho + (1-\alpha)Q_B^\rho)^{1/\rho}$, the elasticity of substitution (σ) equals to $-1/(\rho-1)$ and the share parameters are $\alpha^\sigma / (\alpha^\sigma + (1-\alpha)^\sigma)$ and $(1-\alpha)^\sigma / (\alpha^\sigma + (1-\alpha)^\sigma)$. The demand function derived from this form of utility function in value term is share parameter multiplied by (price/price index evaluated at base utility) to the power of 1-sigma by income.

6. Some matrix consistent aggregation methods and their general equilibrium interpretation

As is well known, the GK method is one of the matrix consistent aggregation methods. It is interesting to consider whether other matrix consistent methods like the SRK (Sakuma, Rao, and Kurabayashi [2000]) method can be formulated in the general equilibrium fashion, noting that in such methods, incomes ($y' = p'Q$) and world prices (multiplied by world quantities) can be seen to be linearly or otherwise related. For example, in SRK case, world price vector can be obtained through following eigen value problem;

$$p'QP'\widehat{Q}i^{-1} P' = p'.$$

Or

$$p'QP'\widehat{Q}i^{-1} P'\widehat{Q}i = p'\widehat{Q}i \text{ or } y'\bar{B}' = p'\widehat{Q}i,$$

where $\bar{B}' = P'\widehat{Q}i^{-1} P'\widehat{Q}i$. We can consider the process above as that of finding general equilibrium prices in the case of Cobb-Douglas share parameters given by the matrix \bar{B} .

It is also interesting to note “LES” method above could be formulated as an eigen value problem as follows;

$$p'Q\tilde{B}'(\widehat{Q}i - A)^{-1} = p',$$

where \tilde{B} is the matrix of beta parameters calibrated by giving alpha parameters.

Either in general equilibrium formulation or eigen value problem formulation, it seems easy to introduce redistribution matrix D into the model, remembering $y' = p'Q$.

Thus, $p'QDB'\widehat{Q}i^{-1} = p'$.

7. Other Models

In this section, we will consider several models that include production and other additional elements. Clearly, there are some additional needs of data in order to treat the additional elements that are introduced into the model.

Thus, in order to consider models with production, it is necessary to know something about income distribution for factors, say capital and labour and/or input coefficient matrix about intermediate consumption.

As far as factor distribution for market factors is concerned, it will be assumed that technology is the same between the two countries and the proportion of income distribution by factor is as follows;

$$V = \begin{pmatrix} 1/2 & 1/3 \\ 1/2 & 2/3 \end{pmatrix},$$

where each column represents each activity and each row represents capital and labour respectively.

For intermediate consumption part of models, we assume the following coefficient matrices for country 1 and country 2 respectively;

$$A_1 = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0.1 & 0.4 \\ 0.4 & 0.2 \end{pmatrix}.$$

In the models examined here, the base SAM can be constructed by using GK results almost just as before. As to the model structure, alternative assumptions can be made. The models we examined include the following cases; (1) mobile factors without intermediate consumption case,⁶ (2) mobile capital and immobile labour without intermediate consumption and (3) with intermediate consumption cases, (4) immobile capital and labour without intermediate consumption case, (5) mobile capital and immobile labour without intermediate consumption case in which exogenously given investment is financed through the (proportional) saving of the two groups of households and possibly net import from “rest of the world.” Find below the base numerical SAMs for the second case (Appendix Table 1) and the third case (Appendix Table2)

In the models (1) to (3) and (5), we conducted the same experiment as in section 4 (the LES experiment) and did not find any significant change in the world prices and hence world prices. The following are the results of LES experiment in the case (4);

$$\text{World Price of } A = 56.112$$

$$\text{World Price of } B = 74.925$$

$$PPP_{2/1} = 5.2938.$$

Clearly, unlike the models listed here, there is possibility that the base SAM formulated above does not work well. One of such possibility may be the case of Sraffa-Leontief type models.⁷

8. Concluding Remarks

⁶ Here, “mobile” or “immobile” means “mobile” or “immobile” between countries not between industries.

⁷ See Kurabayashi and Sakuma [1988 and 1990: chap.8].

First of all, it might be worthy of remark that the GK method could be considered to have a large varieties of general equilibrium models from which GK world prices can be generated. Secondly, in a rather large group of models, GK results seem to have a kind of robustness as to the LES experiments. Thirdly, it might be suggested that some of problems frequently encountered, theoretically as well as practically in PPP measurement programmes, might be solved by using CGE approach as described here. Thus, the problem list might include, among others, the treatment of government services, net export, investment goods as well as terms-of-trade effects. Fourthly, the approach suggested here may be useful for constructing models incorporating “real exchange rates.”

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Appendix:

Computation Procedures with the Use of SAM-based CGE Techniques: LES case

The first step is to construct a “national” or “local” SAM for each country. Note that the elements of this SAM are in the value terms. They can be easily obtained by using P and Q matrices above. And then, the first CGE procedure is conducted, in which the beta parameters are calibrated to this numerical SAM data given alpha data.

The second step is to construct a hypothetical “world” or “global” SAM that plays a role of the base SAM for the process. The data for the SAM can be given by the GK result. By remembering Cobb-Douglas is a special case of Linear Expenditure Systems, we can conduct an experiment in which “alpha” and “beta” parameters are changed in accordance with the results of the previous step.

The following are the SAMs for the present calculation. In the tables, “ENDOWA” or “ENDOWB” means endowments for bringing household group(=country) 1 or 2 in goods A or B and H1 and H2 represent household group 1 and 2 ‘s institutional consumption accounts respectively. A and B are the production accounts.

NATIONAL SAM for Country 1

	ENDOWA	ENDOWB	H1	A	B
ENDOWA				60	
ENDOWB					120
H1	60	120			
A			60		
B			120		

NATIONAL SAM for Country 2

	ENDOWA	ENDOWB	H2	A	B
ENDOWA				600	
ENDOWB					900
H2	600	900			
A			600		
B			900		

GLOBAL SAM

	ENDOWA	ENDOWB	H1	H2	A	B
ENDOWA					4480	
ENDOWB						7500
H1	1680	3000				
H2	2800	4500				
A			1560	2920		
B			3120	4380		

Appendix Table 1: Base Global SAM for a Model with Production (Mobile Capital, Immobile Labour Case)

	C	L1	L2	H1	H2	A	B	1A	1B	2A	2B
C								840	1000	1400	1500
L1								840	2000		
L2										1400	3000
H1	1840	2840									
H2	2900		4400								
A				1560	2920						
B				3120	4380						
1A						1680					
1B							3000				
2A						2800					
2B							4500				

Notation:

C; Capital factor accounts,

L1 and L2; Labour factor accounts for country 1 and 2,

H1 and H2; Institutional consumption accounts for household group (=country) 1 and 2,

A and B; Commodity accounts for A and B,

1A, 1B, 2A and 2B; Activity accounts for producing A in the country 1, producing B in the country 1, producing A in the country 2, and producing B in the country 2.

Appendix Table 2: Base Global SAM for a Model (including intermediate consumption)

	C	L1	L2	H1	H2	A	B	1AV	1BV	2AV	2BV	1A	1B	2A	2B
C								481	1239.33	1165.714	1656.19				
L1								481	2478.67						
L2										1165.715	3312.381				
H1	1720.33	2959.67													
H2	2821.904		4478.096												
A				1560	2920							728	1232	640	2960
B				3120	4380							1950	550	3428.571	1982.143
1AV												962			
1BV													3718		
2AV														2331.429	
2BV															4968.571
1A						3640									
1B							5500								
2A						6400									
2B							9910.714								

Additional Notation:

1AV, 1BV, 2AV and 2BV; Value added part of the activity accounts.