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Is Income Mobility Socially Desirable?

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Working Paper

Abstract

In welfare economics the relevance of income mobility has been often highlighted in order to allow for the measurement of equality of opportunity.

In this paper we observe that intra and inter-generational income mobility have very few in common and, in particular, only the latter concerns with equality of opportunity. In contrast with existing literature we focus on the former not the latter, as in social welfare analysis it cannot be transcended that equity and efficiency evolves in a spatio-temporal dimension. If a moral and impartial observer is assumed to be sympathetic to individual interests in a random framework, especially to the loss of well-being due to uncertainty, then income mobility is both source of uncertainty and inequality effects.

In this context, we show that income mobility does not need to be necessarily either socially neutral or socially desirable, instead, it might be well the case that social undesirability occurs ex-ante.

Key words: welfare, mobility, inequality

1 Introduction

In this paper we investigate social desirability of intra-generational exchange income mobility under both certainty and uncertainty conditions. With this purposes in mind we refer to the SDM approach to social welfare whose ethical value judgements are defined over socially relevant attributes in a lifetime

[1] I'm in debt with Prof. M. Fleurbaey, Prof. F. Stroffolini and Prof. D. Van de Gaer for very useful discussions. Errors are mine.

scenario. [2]

As it has been widely observed in the existing literature, income mobility is clearly a multi-faceted concept, and several research interests have been associated to the same economic phenomenon. In such a context, some preliminary notations are required.

Despite of the economic relevance assumed by mobility, it cannot be transcended that origins of this topic trace back to sociology not economics. Indeed, if a socially relevant attribute is assumed to indicate the social status of each individual, then social mobility can be exploited in order to measure the degree of independency between the current status and the origin one. The main idea is that inequality ex-post may not be significant itself. What really matters is inequality ex-ante, that is, the possibility of each individual to exploit the same opportunities of the others. [3]

In economics, instead, the main interest about mobility mostly focuses on the definition of income classes and on the measurement of equality of opportunity between individuals coming from families with different incomes. In this case, income is assumed to be the indicator of social status, and income mobility measures the strength of origin income in determining the current income position.

In comparing the approach developed in economics to the one discussed in sociology, it can be observed that the transition from social mobility to income mobility is not without any implication. Indeed, from one side it is still the case that the mobility transformation which allows to explain how individuals have moved from income class i to income class j can be exploited in order to measure equality of opportunity. From the other side, in economics there might be a stronger interest in investigating the change in income position for the same income unit over shorter periods. For instance, there might be interest in looking for the welfare's effects of yearly changes in income positions for each income unit.

In the existing literature, these two scenarios are usually regarded respectively as inter-generational and intra-generational income mobility. Even if in both cases main analytical tools do not differ strongly, the former (inter) is oriented to the recognition of the the role played by equality of opportunity, while the second is concerned with the analysis of flexibility in the earnings market as

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[2] In this sense, we are concerned with the definition of social welfare which better allows for the recognition of income mobility effects under certainty and uncertainty conditions.

[3] Controversies about inter-generational mobility have been dominated during most of the 19th and 20th century by a violent conflict between what Erikson and Goldthorpe (1992) call the "liberal theory of industrialization on the one hand and the Marxist theory (and various socialist theories) on the other hand". However, it cannot be neglected that theories about goodness of social mobility are clearly overwhelming and a strong improvement in this sense is due to the idea of equality of basic capabilities (Sen, 1970).

well as the definition of a dynamic approach to equity and efficiency within the social welfare function. In the existing literature the inter-generational approach is clearly overwhelming, but this issue is as much relevant as the previous one. "*... a given extent of income inequality in a rigid system in which each family stays in the same position in each period may be more a cause for concern than the same degree of inequality due to great mobility*", (Friedman, 1962). This is particularly evident in poverty analysis, where the real concern is with poverty persistence more than poverty itself.
The difference between intra and inter-generational mobility is substantial. In looking for a definition of maximum inter-generational mobility intended as maximum equality of opportunity, it might be assumed without loss of generality that the maximum is reached when sons' incomes are perfectly independent of fathers' incomes, i.e. whatever the initial income position, each individual has the same opportunity to move in each class at time t+1 (Prais, 1955). Finally, by virtue of the SDM approach to social welfare, almost all theories of redistributive justice would accept the idea that inter-generational income mobility intended as equality of opportunity is socially desirable (then, social desirability of inter-generational income mobility might be imposed *a priori*).
From the other side, it is almost clear that intra-generational income mobility is strictly related to inter-temporal efficiency and equity in the existing allocation of resources, not equality of opportunity. Income mobility intended as changes in income positions has very strong inequality implications, that is, re-rankings positively affects long-run equity. In this sense, the "complete reversal" case might better suit the definition of maximum mobility (Van de gaer et al., 1998).
However, income mobility produces at least another relevant effect. If incomes are assumed to be random (in line with the existing literature), in the sense that they are exogenously given to individuals, mobility involves, ex-ante, uncertainty considerations too. In particular, uncertainty becomes a crucial information whenever individuals are not risk neutral.
The two effects of income mobility discussed above highlight that intra-generational mobility issues cannot be confused with inter-generational ones. In the latter, goodness of income mobility might be imposed without loss of generality as ethical value judgement (Atkinson, 1983). In contrast, when dealing with the former such an ethical value judgement *a priori* would be inopportune: the inequality effect of income mobility is expected to be welfare improving not the uncertainty effect (Gottschalk and Spolaore, 2002). For this reason, in this paper we investigate social desirability of intra-generational income mobility, not inter-generational one.
It's worth observing that the main purpose of this paper - measuring social desirability of intra-generational income mobility - automatically entails the following notation: in order to isolate social effects of intra-generational income mobility we keep all other relevant characteristics constant. In particular, we assume that one-period income distributions are constant over time, so mean

income. This observation automatically leads to the decomposition of income mobility which has been widely investigated in both sociology and economics. In particular, it is opportune distinguishing between structural and exchange mobility. By the former we refer to the change in available income positions in the economy over time. By the latter, instead, we refer to re-rankings within the income distributions. In this paper we transcend from decomposability's issues and mobility is assumed of the exchange kind since the beginning. [4]
The last point to be addressed consists of the timing approach in dealing with social effects of income mobility, whose relevance is implicit in the basic nature of the inequality and uncertainty effects mentioned above. For our purposes we focus on both ex-ante and ex-post social evaluation of intra-generational mobility. In the former social welfare is defined in presence of uncertainty conditions, not in the latter, by which it is clear that a serious trade-off is expected to occur ex-ante, not ex-post, among the inequality and uncertainty effect. [5]
The paper is organized as follows: in section 2 crucial definitions are reported and the main issue is highlighted, i.e. irrelevance of exchange mobility. In section 3 social desirability of income mobility is discussed first in the certain, then in the uncertain scenario. In addition, our approach to the ex-ante social welfare function is compared with the two main references in the existing literature (Hammond, 1983; Dardanoni, 1993). Section 4 concludes.

2 Basic definitions

The basic starting point consists of the definition of social desirability.

Definition 2.1 (Social Desirability of Income Mobility) *Income mobility is socially desirable in a specific society if and only if $W > W^I$, where W and W^I indicate respectively lifetime social welfare in case of actual and null mobility.*

In other words, social desirability concerns with the aggregation of income mobility's effects over the social welfare function. For our purposes we refer

[4] Apart from the opportunity to isolate the mobility effect from structural changes, there are at least two further notations which support our interest in exchange, not structural, mobility. First, income mobility is defined as a movement in terms of income positions (re-rankings), not income. Secondly, in the case of intra-generational mobility it is usually the case that the scenario is strictly defined in the short-run, which would make inconsistent any reference to changes in the income distribution. From now on, by income mobility it is intended only mobility of the exchange kind.
[5] In order to avoid any confusion, we use the expression "*universally ex-post*" when referring to the ex-post approach to uncertainty conditions (Harris, 1978), while by the sole "ex-post" we intent certainty conditions.

to the SDM approach, by which it is assumed that welfare evaluations reflect ethical value judgements unanimously or sufficiently agreed in the society. The two crucial axioms which are generally accepted in the corresponding literature consists of impartiality and the Pigou-Dalton principle. Through the idea of impartiality it is assumed that the SDM evaluates ethically each society assuming that he/she can be associated with equal chance to each income position (uni- or multi-variate). The Pigou-Dalton principle, instead, claims that if a rich-to-poor transfer (absolute or relative) occurs without causing re-rankings then social welfare must necessarily increase. It is well-known that these two principles are equivalent to assuming that the SDM is averse to inequality (Hardy et al., 1934). This allows to handle equity issues.
Given a society of n individuals, $x_t := \{x_{1,t}, ..., x_{n,t}\}$ and $x_{t+1} := \{x_{1,t+1}, ..., x_{n,t+1}\}$ represent the two income vectors at time t and t+1 where $x_{i,t}, x_{i,t+1}$ indicate income of individual i at time t and t+1. Given $x_t, x_{t+1} \in \Re^n_+$, the mobility transformation is usually indicated by $x_t \rightarrow x_{t+1}$. In a discrete scenario the crucial information consists of the contingency table Π that is a $N \times N$ matrix where N is the number of income classes. Elements within the contingency table, π_{ij}'s, indicate the probability of being in class i at time t and class j at time t+1. It must be the case that $\sum_i \sum_j \pi_{ij} = 1$ and the two marginals $\pi_{i,t} = \sum_j \pi_{ij}$ and $\pi_{j,t+1} = \sum_i \pi_{ij}$ correspond respectively to the income distribution at time t and t+1.
Within the existing literature about income mobility it is usually the case that transition matrices are involved. The transition matrix $P := \{\pi_{j|i}\}$ can be directly obtained from the contingency table by conditioning for each row, or better, indicating with $\pi_{j|i}$ the probability of an individual in class i at time t to be in class j at t+1. By construction $\sum_j \pi_{j|i} = 1, \forall i$.[6] Assuming that mobility is just of the exchange kind is equivalent to imposing $\pi_{i,t} = \pi_{i,t+1} \; \forall i$, i.e. the shape of the income distribution in each period is unaffected by the mobility transformation.
In a two period economy the social welfare function can be defined as

$$W = U(X) \tag{1}$$

where X is a $n \times 2$ matrix containing the two income vectors, respectively $x_t, x_{t+1} \in \Re^n_+$. In this sense we use the equivalent notation $W = U(x_t, x_{t+1})$, where bivariate income positions are supposed to be ordered with respect to the sole income vector at time t. If ethical preferences are assumed to be impartially additively separable over income units, then

$$W = \sum_i^N \sum_j^N U(x_i, x_j)\pi_{ij} \tag{2}$$

[6] Obviously, the contingency table and the transition matrix can be replaced by equivalent concepts in the continuous scenario, respectively the joint density function and the conditioned joint density function. For simplicity we stick with the discrete case along the paper, but results can be extended to the continuous case.

where $x_i, x_j, \pi_{i,j}, N$ indicate respectively income associated to class i at time t, income associated to class j at time t+1, joint frequencies ($\sum_i \sum_j \pi_{i,j} = 1$) and the number of income classes. The following result is well-known (Atkinson, 1983): if additive separability over time is assumed, then (2) is equivalent to

$$W = \sum_i U(x_i)\pi_{i,t} + \sum_j U(x_j)\pi_{j,t+1} \tag{3}$$

Unfortunately, (3) automatically entails that joint frequencies (within the contingency table) do not matter for welfare purposes, i.e. (3) is independent of π_{ij}. Instead, only income distributions (at time t and t+1) really do. As a result, given equal marginal income distributions for two societies, perfect immobility ($P = I$), perfect equality of opportunity ($\pi_{j|i} = \frac{1}{N}\ \forall\, i, j$) and complete reversal ($\pi_{j|i} = 0$ everywhere out of the secondary diagonal), must be necessarily welfare equivalent, or equivalently, ex-post social desirability of income mobility is null. However, additive separability over time in presence of ethical preferences turns out to be a serious assumption, because it is implicitly assumed that the SDM cares only about efficiency and equity in each single period and, as a result, he/she is concerned with “welfare trends”, not lifetime welfare (so income mobility).

We use an example to highlight this point. Consider two societies, A and B, which show the following mobility transformations, $(10, 1) \rightarrow_A (1, 10)$ and $(10, 1) \rightarrow_B (10, 1)$. Also, suppose that the two income classes are defined over the two income levels. From (3), as there are no differences in terms of efficiency (mean income), it is possible to allow for welfare evaluations in terms of inequality. Given a generic inequality index $I(x) := \{I : \Re^n_+ \rightarrow \Re\}$, it must be the case that $I_t(A) \sim I_{t+1}(A) \sim I_t(B) \sim I_{t+1}(B)$. When assuming additive separability in (2), only these four indices above really matter and there is no point any further in the recognition of origin and destination incomes across time. In particular, any permutation of the income vector in one period does not affect the result: no matter if the rich at time t+1 was the poor or rich income unit at time t.

Then, the evaluation of income mobility effects cannot transcend from the imposition of inter-temporal ethical value judgement, by which the link between origin and destination incomes is made significant along the (multi) bi-variate income distribution.

Finally, for the construction of social desirability indices we refer to the well-known Sen's abbreviated social welfare function, i.e. $V(\mu, I) = \mu(1-I)$, where μ indicates mean income and, for our purposes, $I := \{\Re^n_+ \rightarrow \Re\}$ is assumed to be Atkinson (1970)'s inequality index. [7]

[7] Notice that this index satisfies all requirements for the abbreviated welfare function as discussed in Kondor (1975).

3 Measuring social desirability

3.1 The certain scenario

There are some observations which are crucial for the measurement of income mobility's effects. First of all, income mobility effects are not concerned with the analysis of welfare or inequality trends. Then, in a two-period economy we need to refer to that definition of lifetime social welfare function such that the link between origin and destination income is involved.
The second crucial observation concerns with the imposition of the P-D principle of transfer in a lifetime and certain framework. In particular, the main focus moves to the definition of "poor income unit". There are at least two approaches which might be considered: i) an income unit is labelled as poor within each static income distribution or ii) an income unit is labelled as poor within the lifetime income distribution, i.e. $(x_t + x_{t+1})$. [8] These two approaches can be immediately translated in terms of two different P-D principles of transfer (i-ii). Unfortunately, the two approaches are not generally consistent. [9]
In this context, a choice between the two specifications of the P-D principles of transfer cannot be avoided, that is, it is not possible to get an unique specification of the social welfare function which satisfies both P-D principles of transfer (i-ii). Because of the nature of income mobility's effects, for our purposes, it seems more appropriate to refer to ii). [10]

Axiom 3.1 (Absolute P-D principle (ii)) *Given two homogenous populations A, B with income vectors resp.* $x_t^A, x_{t+1}^A, x_t^B, x_{t+1}^B \in \Re_+^n$ *such that* $x_{i,t}^A + x_{i,t+1}^A = x_{i,t}^B + x_{i,t+1}^B \; \forall i \neq k, m$ *and* $x_{k,t}^B + x_{k,t+1}^B = x_{k,t}^A + x_{k,t+1}^A + \Delta$, $x_{m,t}^B + x_{m,t+1}^B = x_{m,t}^A + x_{m,t+1}^A - \Delta$, *given that* $x_{m,t}^A + x_{m,t+1}^A > x_{k,t}^A + x_{k,t+1}^A$ *and* $x_{m,t}^B + x_{m,t+1}^B \geq x_{k,t}^B + x_{k,t+1}^B$, *then it must be the case that in a two-period economy* $W(B) > W(A)$.

[8] The feasibility of these two different definitions of poor income units highlights that social welfare in presence of income mobility does not necessarily need to follow the multi-attribute approach to social welfare. Indeed, definition ii) is the immediate counterpart of i) in the case of income mobility, but the same definition would be a non-sense in the multi-attribute literature on social welfare (Tsui, 1999).
[9] For a discussion about this subject see appendix (A).
[10] Notice that this automatically entails the loss of information about the badness of income fluctuations at the individual level. Intuitively, the badness of income fluctuations is concerned with $[u(x_{i,t}) + u(x_{i,t+1})] - 2u\left(\frac{x_{i,t}+x_{i,t+1}}{2}\right)$ where $u''(\cdot) \leq 0$. However, this highlights that the badness of income fluctuations concerns with i), not ii). In addition, it's not straightforward why income fluctuations should be expected to be socially relevant within ethical preferences.

In other words, we assume that any rich-to-poor transfer in terms of lifetime income which is not source of re-rankings must be necessarily welfare improving.
Let's define the $n \times 1$ lifetime income vector y, where y_k indicates lifetime income for income unit k, i.e. $y_k = x_{k,t} + x_{k,t+1}$. Given impartiality, additivity over time and axiom (3.1), the social welfare function can be defined as

$$W = U(y) = \frac{1}{n} \sum_{k=1}^{n} U(y_k) \tag{4}$$

By virtue of Hardy et al. (1934)'s theorem it must be the case that $U''(y) \leq 0$. Notice that this is the same as the social welfare function implicit in Shorrocks (1978b)'s mobility index (see Appendix (B)). Equivalently, let's define the $N \times N$ matrix $Y := \{y_{i,j}\}$, where $y_{i,j}$ indicates lifetime income for an individual starting from position i and ending up in position j. Once that income classes (N) have been defined, (4) can be rewritten as

$$W = \sum_{i=1}^{N} \sum_{j=1}^{N} U(y_{ij}) \pi_{i,j} \tag{5}$$

In line with definition (2.1) and given $V = \mu(1 - I)$, a measure of social desirability of income mobility ex-post can be defined as

$$D^p = V - V^I = \mu_y (I_y^I - I_y) \tag{6}$$

where V^I, μ_y, I_y^I, I_y indicate resp. the abbreviated social welfare function in case of perfect immobility, mean lifetime income, (lifetime) inequality under perfect immobility and (lifetime) inequality under actual mobility. The corresponding relative measure can be defined as

$$D_R^p = I_y^I - I_y \tag{7}$$

With respect to Shorrocks' mobility index ((B.4)) as defined for a two-period economy with mobility of the exchange kind only, (7) is the corresponding relative index obtained within the absolute class of distance measures.

Proposition 3.1 *Exchange income mobility is always socially desirable ex-post.*

Proof 3.1 *For all inequality indices satisfying the P-D principle of transfer the proposition holds by construction.*

In particular, the welfare optimal mobility structure consists of perfect mobility (complete reversal). Finally, if I is intended to be Atkinson's inequality index,

$$D_R^p = \frac{y^{d(\varepsilon)}}{\mu_y} - \frac{[y^I]^{d(\varepsilon)}}{\mu_y} \tag{8}$$

where $y^{d(\varepsilon)}$, $[y^I]^{d(\varepsilon)}$ are respectively the equally distributed equivalent income calculated over (x_t+x_{t+1}) (actual mobility) and (x_t+x_t) (perfect immobility). In addition, D_R^p satisfies symmetry and $D_R^p \in [0,1]$.[11]
At this stage, the conclusive remark in Shorrocks (1978b) turns out to be crucial: "*... to the extent that mobility leads to more pronounced fluctuations and more uncertainty, it is not regarded as socially desirable*".

3.2 The uncertain scenario

3.2.1 Social welfare and uncertainty

The definition of social welfare rises itself several problems which have been attracting the interest of researcher since Aristotele and Plato. However, this is not as much complicated as thinking about a definition of expected lifetime social welfare. Given two societies A and B in a two-period economy with equal income distribution at time t, it can be assumed without loss of generality that the SDM might rank society A as better off than society B just because of better lotteries[12] at time t+1 for each individual. Unfortunately, what happens when both better and worse lotteries occur is not clear at all. Even worst, it is not clear if a "social" attitude to risk can be involved in such a story.
Given the basic framework discussed in the previous section, it might seem straightforward defining social welfare ex-ante as the unweighted sum of expected lifetime utilities, i.e.

$$W = \frac{1}{n}\sum_k E_k U(\tilde{y}_k) \tag{9}$$

where $\tilde{y}_k$ indicates the random lifetime income for income unit k, i.e. $\tilde{y}_k = x_{k,t} + \tilde{x}_{k,t+1}$. Given income classes, $\tilde{y}$ is defined over heterogenous lotteries for each origin income class, i.e. $\mathcal{L}_i := \{\pi_{j|i} \in \Re^N | \pi_{j|i} \geq 0 \,\forall j, \sum_j \pi_{j|i} = 1\}$.[13]
Equivalently, (9) can be rewritten as

$$W = \frac{1}{n}\sum_k U\left[y_k^{d\varepsilon}\right] \tag{10}$$

where the $y_k^{d\varepsilon}$ is the certain equivalent income associated by the SDM to the k-th income position. Then, social welfare is the impartially additively separable aggregation of utilities taken with respect to each individual certain equivalent

[11] $D_R^P = 1$ has not to be taken as the "*complete mobility*" suggested by Shorrocks (1978b).
[12] Stochastic dominance conditions.
[13] In other words, the loss due to uncertainty is handled at the lifetime level for our purposes.

income. In addition, impartiality as well as the P-D principle of transfer are defined at the net value of the uncertainty effect over each income position. [14] Unfortunately, for our purposes, (10) would be not straightforward at all. Indeed, $U(\cdot)$ would reflect at the same time the idea of a social attitude to risk as well as inequality aversion, with an additional implicit assumption: the SDM is equally averse to income uncertainty and inequality. [15]

In contrast, we start from a crucial notation: the degree of aversion to income uncertainty belongs to the set of individual subjective preferences, not ethical ones. So that, the SDM socially evaluates society A with respect to i) the allocation of resources in the two periods and ii) the degree of aversion to uncertainty (λ) which characterizes population A. [16] Then, given impartiality, (10) can be rewritten as

$$W = U(y^{d\lambda}) \tag{11}$$

where $y^{d\lambda}$ indicates the vector of lifetime certain equivalent incomes associated to each individual position as evaluated by the SDM through the "acceptance" of that λ characterizing the corresponding population, i.e.

$$y_k^{d\lambda} = u^{-1}[E_k(u(\tilde{y}_k))] \tag{12}$$

where $u(\cdot)$ involves the degree of aversion to uncertainty (λ) while $U(\cdot)$ aversion to inequality (ε). It's worth observing that (9) can be obtained from the latter (11) assuming that $\varepsilon = \lambda$. Even better, (11) exploits the definition of certain equivalent incomes which allows i) to quantify the loss due to income uncertainty, i.e. Markowitz's risk premium and ii) to define ex-ante inequality

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[14] In the proof for proposition (3.4), we will come back to the relationship between (9) and (4).

[15] Within a different specification of the social welfare function, Gottschalk and Spolaore (2002) assume a non-linear behavior of expected utilities.

[16] Once that impartiality has been imposed, in evaluating socially relevant effects of income mobility ex-ante, the SDM evaluates each income position which consists of a certain income at time t and a heterogenous lottery at time t+1 depending on the origin income class (transition matrix). For instance, let's consider the well-known iso-elastic specification of the social welfare function (Atkinson, 1970), $U(x) = \alpha + \frac{\beta}{1-\varepsilon}x^{1-\varepsilon}$, where $\varepsilon > 0$ (inverse of the elasticity of substitution) indicates the degree of aversion to inequality and x is the vector indicating the allocation of income. Unfortunately, we are not concerned just with an allocation problem, but also with an income lottery. In this context the principle of acceptance might be useful (Harsanyi, 1977): "*Through the principle of acceptance, we acknowledge that everything is as it should be and that this moment is perfect in itself. Through acceptance, we know that it is not necessary to expend negative energy trying to change people or circumstances. We all need to live our own experiences to learn, grow and discover our true self*". In other words, the SDM accepts the specific degree of aversion to income uncertainty $\lambda > 0$ characterizing each society, $u(\tilde{y}_k) = \alpha + \frac{\beta}{1-\lambda}\tilde{y}_k^{1-\lambda}$, where $\tilde{y}_k$ indicates the random lifetime income associated to income unit k (power utility function).

aversion at the net value of the uncertainty effect. [17]
In a discrete approach, given impartial additive separability of $U(\cdot)$, by assuming that individuals starting at the same income position face the same lottery, (11) can be rewritten as

$$W = \sum_i \pi_{i,t} U(y_i^{d\lambda}) \tag{13}$$

where π_t indicates the income distribution at time t.
Then, i) taking Atkinson's iso-elastic welfare function, ii) imposing the power utility function with respect to uncertainty and iii) referring to usual vNM's specification of expected utility, (13) can be rewritten as

$$W = \sum_i \frac{\pi_{i,t}}{1-\varepsilon} \left[\sum_j \pi_{j|i} \left(y_{ij}\right)^{1-\lambda} \right]^{\frac{1-\varepsilon}{1-\lambda}} \tag{14}$$

where $y_{i,j}$ is the state-contingent lifetime income for an individual starting in income class i (time t) and moving to class j (time t+1).
As it will be observed in the next subsection, (14) might be easily regarded as Hammond (1983)'s dual.

3.2.2 Some comparisons

The universally ex-post approach
In the existing literature contributions about ex-ante social welfare are definitely rare. However, there are two different approaches which have been implemented in presence of uncertainty conditions.
Referring to the "*universally ex-post*" approach to uncertainty (Harris, 1978), Hammond (1983) defines first a money-measure of social welfare for each social state (state-contingent income vector), then he suggests to aggregate concavely state-contingent social welfare. [18] Taking Atkinson's iso-elastic specification of

[17] In the existing literature it has been often observed that equity issues might make no sense over uncertain incomes. Instead, the certain equivalent income might be easily interpreted as a certain income whenever a perfect insurance market exists and it is accessible to all individuals. Finally, it seems fair that whenever uncertainty is null, the ex-ante inequality effect is the same as the ex-post one discussed in the previous section.
[18] In order to avoid any misunderstanding, Hammond refers to the definition of the expected social welfare function, by which state-contingent income vectors are defined with respect to the sole income at time t+1. Instead, for our purposes, we assume that the state-contingent income vector concerns with lifetime income over the two periods.

the welfare function,

$$W = \sum_s \pi_s \omega(y_s^{d\varepsilon}) = \sum_s \frac{\pi_s}{1-\lambda} \left[\sum_i \pi_i^s (y_{i,s})^{1-\varepsilon} \right]^{\frac{1-\lambda}{1-\varepsilon}} \tag{15}$$

where π_s is the probability associated to each social state, $\omega(\cdot)$ is the increasing and concave aggregation, $y_s^{d\varepsilon}$ is the equally distributed equivalent lifetime income in social state s, π_i^s is the proportion of individuals in income class i in social state s, $y_{i,s}$ is the lifetime income associated to income unit i in social state s, and ε, λ are usual parameters of aversion. As it is clearly highlighted in Hammond (1983), given a pure socialization of uncertainty [19] , "*On the face of it, this pays no attention to individuals' degrees of risk aversion in mind, but how this should be done is not clear*".

In comparing formally (15) and (14), it's worth observing that what really matters is the procedure: Hammond takes the equally distributed equivalent income in each social state (inequality aversion) then he aggregates with respect to uncertainty. In (14), instead, we first take the certain equivalent income with respect to uncertainty, then the SDM evaluates equity issues.

It's worth observing that, apart from the lack of subjective aversion to income uncertainty in (15), the imposition of a concave aggregation $\omega(\cdot)$ might be not straightforward, especially if implications in terms of inequality aversion are considered. [20]

In order to allow for an immediate comparison between (14) and (15), we assume that π_s can be estimated from the transition matrix by assuming in-

[19] Notice that in (15) we refer to social states and not subjective probabilities associated to each individual.

[20] It might be said that there is no generally or sufficiently agreed ethical value judgement by which the SDM is expected to be averse to uncertainty over state-contingent welfare. In this scenario, it might be useful looking at implications of $\omega(\cdot)$ in terms of inequality aversion. Indeed, concavity of $\omega(\cdot)$ allows to attach larger weight for variations of low $y_s^{d\lambda}$ (P-D principle of transfer as defined over the $y_s^{d\lambda}$). However, if $y_s^{d\lambda}$ is low, then it might be both the case that the cake associated to a particular social state is low, or inequality is high in the same social state. For instance, if two standard rich-to-poor transfers might be alternatively undertaken respectively in social states *s,h* such that the inherited variation of $y_s^{d\lambda}$ and $y_h^{d\lambda}$ is the same (in both cases the state-contingent cake is unchanged), then, given $y_h^{d\lambda} < y_s^{d\lambda}$, the SDM would prefer the rich-to-poor transfer occurring in state h. However, it might be the case that, even if $y_h^{d\lambda} < y_s^{d\lambda}$, indeed, inequality is higher in s than h, that is, the SDM might have chosen both a rich-to-poor transfer occurring in the less unequally distributed social state (h) or a rich-to-poor transfer occurring in the higher unequally distributed social state (h). Summing up, the concave aggregation cannot be justified in terms of inequality aversion, even if this fact does not exclude that "*a sort of*" principle of diminishing transfers defined over states not income positions (Kolm, 1976b) might be invoked.

dependency between each individual lottery, that is,

$$\pi_s = \prod_{i=1}^{N} \pi_{\tilde{q}|i} \quad q := 1, ..., N \tag{16}$$

where $\tilde{q}$ associates a specific j (income position at time t+1) to each i (income position at time t) depending on the social state s.[21] It comes that there are N^N possible social states. Also, we say that combination s belongs to the set of combinations Ω if it is obtained from a transition matrix of the exchange kind ($\pi_t = \pi_{t+1}$).

Once that π_s has been defined in terms of $P := \{\pi_{j|i}\}$, it appears immediately that (15) does not refer to the same definition of aversion to uncertainty that we are concerned with. In other words, in (15), the SDM is averse to uncertainty over different state-contingent income vectors, but he/she is not concerned at all with income uncertainty faced by each individual, i.e. the SDM is not concerned at all with who gets what (income mobility).

However the following result might be of interest.

Proposition 3.2 *If $\varepsilon = \lambda$ and social states probabilities, π_s, are consistent with the transition matrix, $\pi_{j|i}$, then* (14) *and* (15) *are equivalent.*

Proof 3.2 *The proof is given in the appendix* (C).[22]

As it will be shown later on, this result can be explained in terms of mean and inequality effects of uncertainty, in the sense that if $\varepsilon = \lambda$ it is necessarily the case that these two effects are perfectly compensating. In this sense, a serious input comes from the comparison with the utilitarian approach to social welfare under uncertainty conditions.

The utilitarian approach

Even if the universally ex-post approach to social welfare under uncertainty conditions allows to implement the SDM approach to social welfare, as it has observed in the previous section, some inconsistencies would arise. From the other side, whenever the utilitarian approach to welfare is bought, uncertainty issues turn out to be less problematic.

Following the utilitarian tradition, lifetime welfare is defined *sic et simpliciter* as the unweighted sum of individual lifetime expected utilities. This is the usual ex-ante approach to uncertainty conditions which has not to be confused with the universally ex-post framework discussed before.

[21] That is, in (16), $\tilde{q}$ is not required to be the same for each i. For an example see appendix (C).

[22] The main idea is not far from results about consistency between the universally ex-post and ex-ante approach to uncertainty conditions discussed in Diamond (1967) as extended to the case of the expected Bergson's social welfare function. In other words, for $\varepsilon = \lambda$ uncertainty is socially neutral.

Assuming that additive separability applies over time and income units,

$$V_i = u(x_i) + \sum_j \pi_{j|i} u(x_j) \tag{17}$$

where V_i indicates the lifetime expected utility of individual i and $\pi_{j|i}$ is, once again, the probability of an individual starting in class i at time t, to be in class j at t+1.[23] As a result,

$$W = \sum_i \pi_{i,t} V_i \tag{18}$$

where $\pi_{i,t}$ indicates the marginal distribution with respect to time t.[24] This approach is coherent with Arrow-Debreu's general equilibrium theory under uncertainty and $\pi_{j|i}$'s are intended as subjective probabilities.

The specification in (18) allows for an evaluation of income mobility's effects in presence of uncertainty and it relies on the following observation.

If the research interest concerns with the effects of income mobility ex-ante, additive separability is not a strong restriction. Indeed, it is simply assumed that each individual "subjectively" evaluates its expected utility over two periods summing up the corresponding utility level in each period with constant Bernoulli utility function.[25]

In the existing literature it is well-known that income mobility is socially irrelevant in (18) (Dardanoni, 1993). However, given $u''(\cdot) \leq 0$ we claim that irrelevance is not due to the loss income mobility information like in (3), but *sic et simpliciter* it concerns with the perfect compensation between the uncertainty and inequality effect of income mobility, where the loss due to uncertainty refers to the Jensen's effect, while inequality effect is the stochastic counterpart of the ex-post inequality effect discussed in the previous section.

Proposition 3.3 *Exchange mobility is not welfare significant as a perfect compensation arises between the uncertainty and inequality effect.*

Proof 3.3 *The proof is given in the appendix* (D).

Summing up, in comparing our approach to the universally ex-post approach, the latter does not allow to bring back the welfare loss due to uncertainty over each income position. The same problem disappears in the utilitarian

[23] For simplicity we assume that the discount factor is 1.

[24] The same basic framework is implemented by Dardanoni (1993), but an exogenous parameter - decreasing in income - is introduced, by which income mobility is supposed to affect poor income units more than rich ones (losing symmetry). In addition, Dardanoni's approach is defined in a discrete framework and extended to $t \to \infty$.

[25] The main idea is that additive separability is not so restricting when dealing with subjective, not ethical, preferences.

approach but not without a cost. Inequality aversion would be not ethically founded any further.
In the next section we construct ex-ante social desirability indices from (14) by virtue of the same abbreviated social welfare function implemented under certainty conditions.

3.2.3 Indices

The interest in measuring ex-ante social desirability is mostly inherited in the trade-off between the uncertainty and ex-ante inequality effect.
Once again, we define λ as the degree of aversion to income uncertainty for each individual subjective preferences and ε as the degree of aversion to ex-ante inequality as calculated over lifetime certainty equivalent incomes.

Definition 3.1 (Ex-ante social desirability) *Income mobility is socially desirable for a specific society if and only if $W > W^I$ (resp. $V > V^I$), where W and W^I indicate respectively expected lifetime social welfare under actual income mobility and expected lifetime social welfare under the hypothesis of perfect immobility.*

As it has been discussed above, if $\varepsilon = \lambda$, (14) is just equivalent to (9). Then, we first investigate ex-ante social desirability for $\varepsilon = \lambda$.

Proposition 3.4 *Given $x_t, x_{t+1} \in \Re^n_+$ and $\varepsilon = \lambda = z > 0$, exchange income mobility is always ex-ante socially desirable.*

Proof 3.4 *The proof is given in the appendix* (E).

This result may at first be surprising, however the loss of all uncertainty is well-known in the existing literature (Dardanoni, 1993; Gottschalk and Spolaore, 2002).[26] As a result, in (9) the only survived mobility effect would be an inequality effect, which, by construction, is necessarily the same as the ex-post inequality effect discussed in the certain scenario.
Coming back to (14), in a two-period economy the income mobility effect can

[26] It can be shown that the ex-ante inequality effect consists of two components: a) lifetime inequality decreases because of uncertainty (e.g. inequality over certain equivalent incomes is not larger than initial inequality) and b) lifetime inequality decreases because of the recognition of the link between origin and destination income unit across time (e.g. the ex-post inequality effect). In line with Proof (3.3), (9) implicitly assumes that a) is perfectly compensated by the loss of certain equivalent cake due to uncertainty, so that the only inequality effect in b) survives. An example about the two inequality effects might be useful. Given a 3×3 transition matrix such that $\pi_{2|1}, \pi_{1|2}, \pi_{3|3} = 1$, the effect in a) is null (because of null uncertainty), not b).

be defined as

$$V - V^I = \mu_{y^{d\lambda}}(1 - I_{y^{d\lambda}}) - \mu_y(1 - I_y^I) \tag{19}$$

by which

$$D^a = [y^{d\lambda}]^{d\varepsilon} - [y^I]^{d\varepsilon} \tag{20}$$

where $[y^{d\lambda}]^{d\varepsilon}$ indicates the equally distributed equivalent income ($d\varepsilon$) taken over the certain equivalent income ($d\lambda$), i.e.

$$[y^{d\lambda}]^{d\varepsilon} = \left\{ \sum_i \pi_{i,t} \left[\sum_j \pi_{j|i} \left(y_{ij}\right)^{1-\lambda} \right]^{\frac{1-\varepsilon}{1-\lambda}} \right\}^{\frac{1}{1-\varepsilon}} \tag{21}$$

By easy algebraic calculations

$$D^a = (\mu_{y^{d\lambda}} - \mu_y)(1 - I_{y^{d\lambda}}) + \mu_y(I_y^I - I_{y^{d\lambda}}) \tag{22}$$

where the first factor on the RHS is the mean effect (change in mean income given constant inequality), while the latter is the ex-ante time-averaged inequality effect (change in inequality given constant mean income).
Then, the overall ex-ante effect of mobility, D^a, can be decomposed in ex-ante mean and inequality effects,

$$D^u = (\mu_{y^{d\lambda}} - \mu_y)\frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_{y^{d\lambda}}} \tag{23}$$

$$D^i = \left(\frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_{y^{d\lambda}}} - \frac{[y^I]^{d\varepsilon}}{\mu_y}\right)\mu_y \tag{24}$$

It's straightforward that $D^u \leq 0$ and $D^i \geq 0$, i.e. income mobility causes a negative effect over the cake ex-ante and a positive effect over ex-ante time-averaged inequality. In addition, it' worth observing that D^a does not need to be necessarily null.
In line with the previous chapter, in order to answer our initial question (is income mobility socially desirable ex-ante?), we refer to the corresponding relative measures.

$$D_R^a = \frac{[y^{d\lambda}]^{d\varepsilon} - [y^I]^{d\varepsilon}}{\mu_y} \tag{25}$$

where D_R^a does not need to be necessarily positive. In particular, D_R^a indicates the difference among re-scaled time-averaged equality in presence of income mobility and time-averaged equality under the assumption of perfect immobility.
The decomposition of D_R^a in terms of inequality and uncertainty effect leads to the following relative indices,

$$D_R^u = (1 - I_{y^{d\lambda}})\left(\frac{\mu_{y^{d\lambda}} - \mu_y}{\mu_y}\right) = \frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_y} - \frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_{y^{d\lambda}}} \tag{26}$$

$$D_R^i = I_y^I - I_{y^d\lambda} = \frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_{y^{d\lambda}}} - \frac{[y^I]^{d\varepsilon}}{\mu_y} \tag{27}$$

whose interpretations are straightforward once again.
All the indices discussed in this section are relative, replication invariant and symmetric.

Proposition 3.5 $D_R^a \in [-1, 1]$

Proof 3.5 *It's sufficient observing from* (25) *that both* $\frac{[y^{d\lambda}]^{d\varepsilon}}{\mu_y}, \frac{[y^I]^{d\varepsilon}}{\mu_y} \in [0, 1]$.

Proposition 3.6

$$D_R^a \gtreqless 0 \quad if\, and\, only\, if \quad \frac{\mu_y - \mu_{y^{d\lambda}}}{\mu_y} \lesseqgtr \frac{I_y^I - I_{y^{d\lambda}}}{1 - I_{y^{d\lambda}}} \tag{28}$$

Proof 3.6 *Straightforward from* (25).

In other words, mobility is ex-ante socially desirable if and only if the relative welfare loss in terms of mean is lower than the relative welfare gain in terms of expected time-averaged equality.
The following table summarizes the possible behavior of each index at specific transition mechanism

	PI	PE	PM
D_R^p	0	> 0	I_{x_t}
D_R^a	0	$\lesseqgtr 0$	$\min\{I_y\}$
D_R^u	0	< 0	0
D_R^i	0	> 0	$\min\{I_y\}$

Table I: PI, PE and PM indicate resp. perfect immobility, perfect equality of opportunity and perfect mobility. [27]

Finally, it's worth observing that from Table I it can be deduced that the trade-off between mean and inequality effect disappears when the mobility transformation is *stronger* than in PE. [28]

[27] It's worth observing that by construction it must be the case that $\min\{I_y\}$, given the initial income distribution, occurs in the case of perfect mobility.
[28] The real problem in this statement concerns with the definition of strongness (measurement) of income mobility. However, it's worth observing that, starting from PE, any income switching at time t+1 - causing a rich-to-poor transfer in terms of lifetime income - must necessarily reduce the loss due to uncertainty and increase the gain due to equality, i.e. the trade-off disappears.

4 Conclusive remarks

We now go back to our initial question: is income mobility socially desirable? In the existing literature it would not be very easy to find an answer to such a question.

In this paper we have been focusing on intra-generational exchange mobility, that is, the change in income positions over time for the same income unit, excluding the possibility for structural changes in the income distribution. In this context, it has been shown that there are at least two different approaches which deserve particular attention: ex-ante and ex-post.

In the traditional ex-post framework there are no doubts about consistency of the SDM approach in evaluating social desirability of income mobility. In this context, given lifetime ethical preferences, income mobility can be definitely labelled as "good".

In the ex-ante framework, instead, things turn out to be more complicated. Both the SDM and the utilitarian approach do not suit our purposes in defining social welfare under uncertainty conditions (ex-ante). In this sense, we have proposed Hammond's dual, by which both uncertainty and inequality are treated respective to their basic nature. In this sense, inequality aversion belongs to the sphere of SDM's choices, while aversion to uncertainty better suits subjective preferences.

It comes that income mobility is not necessarily socially desirable, that is, the uncertainty effect might be overwhelming with respect to the ex-ante lifetime inequality effect. In line, with the ex-post approach, income mobility is still welfare maximizing at perfect mobility, where minimum uncertainty comes at minimum lifetime inequality (More's society).

The possibility of negative ex-ante social desirability of income mobility is not without any implication for answering our initial question. If the overall social desirability of income mobility might be defined as the aggregation of ex-ante and ex-post effects, the possibility of negative desirability ex-ante highlights that assuming goodness of income mobility *a priori* would be highly misleading. In addition, policy recommendations would strongly differ depending on the sign of D_R^a. Indeed, an increase in exchange income mobility (flexibility in the earnings market) may be recommended for a society with $D_R^a > 0$, while the same recommendation is not generally efficient if $D_R^a < 0$.

APPENDIX

A P-D principles of transfer (i-ii)

The multi-attribute approach to social welfare is not necessarily the unique alternative in dealing with social welfare in presence of income mobility. Indeed, in the latter, the P-D principle of transfer might be imposed over lifetime incomes (ii). As a result, the social welfare function would be defined as $W = U(y)$ where $y = x_t + x_{t+1}$. From the other side, when starting from (i)'s P-D principle of transfer, it might be opportune investigating the implicit inter-temporal P-D principle of transfer.
Let's assume that the utility function of the SDM is additive but not separable over time,

$$W = U(x_t, x_{t+1}) = G[\phi(x_t) + \psi(x_{t+1})] \tag{A.1}$$

where $G'(\cdot), \phi'(\cdot), \psi'(\cdot) > 0$ can be assumed without loss of generality.
Imposing a) impartial additive separability, b) the P-D principle of transfer over each one-period income distribution taken separately and c) non-positive cross-derivative, i.e. $\frac{\partial^2 W}{\partial x_t \partial x_{t+1}} \leq 0$, it must be the case that the following conditions hold

$$\begin{aligned} \frac{\partial^2 W}{\partial x_t^2} &= G''(\cdot)(\phi'(\cdot))^2 + G'(\cdot)\phi''(\cdot) \leq 0 \\ \frac{\partial^2 W}{\partial x_{t+1}^2} &= G''(\cdot)(\psi'(\cdot))^2 + G'(\cdot)\psi''(\cdot) \leq 0 \end{aligned} \tag{A.2}$$

It's worth observing that (A.2) are necessary but not sufficient conditions for concavity of $U(x_t, x_{t+1})$ (that is, concavity of $U(x_t, x_{t+1})$ might be not supported in terms of ethics). Then, the real attention moves to the definition of ethical value judgements by which the non-positive cross-derivative can be imposed (if income positions are assumed to be the same over time, i.e. re-ordered income vectors coincide $x_t^\sigma = x_{t+1}^\sigma$, then the non-positive cross derivative might be translated in terms of the "correlation increasing majorization principle" Boland and Proschan (1988)).
In our context, the non-positive cross-derivative is equivalent to assuming $G''[\cdot] \leq 0$, that is, the implicit lifetime P-D principle of transfer would concern with rich-to-poor transfer in terms of time-averaged utilities, not lifetime incomes. Given (A.1), aversion to inequality over time-averaged utilities and aversion to inequality over lifetime incomes (ii) would be generally consistent if $\phi(\cdot) = \psi(\cdot)$ and $\phi''(\cdot) = \psi''(\cdot) = 0$, that is ordinally equivalent to $W = U(y)$.
It's worth reminding that we are strictly concerned with intra-generational income mobility; the imposition of non-positive cross-derivative might be straightforward when dealing with inter-generational income mobility. Indeed, from one side the lifetime P-D principle of transfer (ii) would lose its interest in

the latter (dynasties). From the other side, if a) $x_t^\sigma = x_{t+1}^\sigma$ and b) equality of opportunity is measured by the correlation coefficient and c) the domain of bivariate income distributions is defined such that the transition matrix is monotone (Conlisk, 1990), then imposing the non-positive cross derivative is equivalent to imposing the following ethical value judgement: equality of opportunity is necessarily welfare improving.

B Shorrocks' mobility index

In the two-period economy Shorrocks' index is defined as

$$S = 1 - \frac{I_y}{\frac{\mu_{x_t}}{\mu_y} I_{x_t} + \frac{\mu_{x_{t+1}}}{\mu_y} I_{x_{t+1}}} \tag{B.1}$$

where $\mu_{x_t}, I_{x_t}, \mu_{x_{t+1}}, I_{x_{t+1}}, \mu_y, I_y$ indicate resp. mean income at time t, inequality at time t, mean income at time t+1, inequality at time t+1, lifetime mean income and lifetime inequality. Given $(V = \mu(1 - I))$, it is easy to show that

$$S = 1 - \frac{V_y^e - V_y}{\sum_{i=0}^{1} (V_{x_{t+i}}^e - V_{x_{t+i}})} \tag{B.2}$$

where V_y is defined over μ_y and I_y, $V_{x_{t+i}}$ is the abbreviated social welfare function defined over $\mu_{x_{t+i}}, I_{x_{t+i}}$ and V^e indicates the abbreviated social welfare function under perfect equality.
If mobility is assumed to be just of the exchange kind, then (B.2) simplifies to

$$S = 1 - \frac{V_y^e - V_y}{V_y^{I,e} - V_y^I} \tag{B.3}$$

where V^I indicates the abbreviated social welfare function under perfect immobility.
Equivalently, because of $\mu_{x_t} = \mu_{x_{t+1}}$, then

$$S = 1 - \frac{I_y}{I_{x_t}} \tag{B.4}$$

where as I is a relative inequality index (h.d.0), then $I_{x_t} = I_y^I$, where I_y^I indicates lifetime inequality in the case of perfect immobility.

C **Proof for proposition** (3.2)

In order to prove that given (16), (15) is equivalent to (14) if $\varepsilon = \lambda = z$, it can be observed that in the former the sum of probabilities among all

feasible social states associated to the event "get x_i at time t, get x_j at time t+1" is by construction $\pi_{j|i}$. We report the simplest example, by which it is assumed that a 2×2 contingency table has been given. We indicate with $\pi_{s=k}, x_{1,t+i}, x_{2,t+i}, \pi_1, \pi_2, \pi_{j|i}$ respectively the probability associated to social state k, income associated to class 1 at time t+i, income associated to class 2 at time t+i, the proportion of people in class 1 at time t (time t+1), the proportion of people in class 2 at time t (time t+1), the probability for an income unit in class i at time t to be in class j at time t+1. In addition, each social state is defined as in (16), that is, we refer to a representative agent for each income class where A and B are respectively the poor and rich representative agent (A is associated to x_1 at time t and B to x_2 at time t).
Given $\varepsilon = \lambda = z$, then (14) can be rewritten as

$$\begin{aligned} W =& \frac{\pi_{1,t}}{1-z}\left[\pi_{1|1}(y_{11})^{1-z}\right] + \frac{\pi_{1,t}}{1-z}\left[\pi_{2|1}(y_{12})^{1-z}\right] + \\ &+ \frac{\pi_{2,t}}{1-z}\left[\pi_{1|2}(y_{21})^{1-z}\right] + \frac{\pi_{2,t}}{1-z}\left[\pi_{2|2}(y_{22})^{1-z}\right] \end{aligned} \tag{C.1}$$

Instead, (15) involves four possible social states at t+1 (state-contingent income distributions): 1) both A and B get x_1, 2) A gets x_1 while B gets x_2, 3) A gets x_2 while B gets x_1 and 4) both A and B get x_2. In line with (16), if A and B's lottery are independent, then $\pi_{s=1} = \pi_{1|1} \cdot \pi_{1|2}$, that is, the probability for both agents to be in class 1 at time t+1. Given these notations, (15) can be rewritten as

$$\begin{aligned} W =& \frac{\pi_{s=1}}{1-z}\left[\pi_{1,t}(y_{11})^{1-z} + \pi_{2,t}(y_{21})^{1-z}\right] + \\ &+ \frac{\pi_{s=2}}{1-z}\left[\pi_{1,t}(y_{11})^{1-z} + \pi_{2,t}(y_{22})^{1-z}\right] + \\ &+ \frac{\pi_{s=3}}{1-z}\left[\pi_{1,t}(y_{12})^{1-z} + \pi_{2,t}(y_{21})^{1-z}\right] + \\ &+ \frac{\pi_{s=4}}{1-z}\left[\pi_{1,t}(y_{12})^{1-z} + \pi_{2,t}(y_{22})^{1-z}\right] \end{aligned} \tag{C.2}$$

by which

$$\begin{aligned} W =& \frac{\pi_{1|1}\pi_{1|2}}{1-z}\pi_{1,t}(y_{11})^{1-z} + \frac{\pi_{1|1}\pi_{1|2}}{1-z}\pi_{2,t}(y_{21})^{1-z} + \\ &+ \frac{\pi_{1|1}\pi_{2|2}}{1-z}\pi_{1,t}(y_{11})^{1-z} + \frac{\pi_{1|1}\pi_{2|2}}{1-z}\pi_{2,t}(y_{22})^{1-z} + \\ &+ \frac{\pi_{2|1}\pi_{1|2}}{1-z}\pi_{1,t}(y_{12})^{1-z} + \frac{\pi_{2|1}\pi_{1|2}}{1-z}\pi_{2,t}(y_{21})^{1-z} + \\ &+ \frac{\pi_{2|1}\pi_{2|2}}{1-z}\pi_{1,t}(y_{12})^{1-z} + \frac{\pi_{2|1}\pi_{2|2}}{1-z}\pi_{2,t}(y_{22})^{1-z} \end{aligned} \tag{C.3}$$

Comparing (C.3) and (C.1) it can be observed that the probability associated to each y_{ij} is the same. For instance, taking the sum of probabilities between the first and third factor on the RHS of (C.3) is just equivalent to the probabilities attached to y_{11} in (C.1).
The same proof can be generalized to $N \times N$ transition matrix.

D Proof for proposition (3.3)

Let's consider (18) in the discrete framework, i.e. $W = \pi_t'[u(\bar{x})+Pu(\bar{x})]$, where $\pi', P, u(\bar{x})$ indicate resp. the $N \times 1$ vector for the income distribution in one period, the $N \times N$ transition matrix and the $N \times 1$ vector of income utilities. Observing that i) the income distribution π and the transition matrix P can be both interpreted in terms of expectations with $Pu(\bar{x})$ the vector of expected utilities, ii) the utility function is increasing and concave, iii) considering the Jensen's decomposition with respect to the vector $Pu(\bar{x})$ and iv) re-arranging, then

$$W = \pi_t' u(\bar{x}) + \pi_t' u(P\bar{x}) + \frac{1}{2}\pi_t'[\overrightarrow{\bar{V} \times u''(P\bar{x})}] \tag{D.1}$$

where $\bar{x}, \bar{V}$ and $(\overrightarrow{\cdot})$ indicate respectively the income vector, the variances' vector and the vectorized product. Let's define with W^I the amount of welfare over two periods when $P = I$ (perfect immobility), i.e. $W^I = 2\pi_t' u(\bar{x})$. It's easy to observe that the mobility effect, $e^m = W^I - W$, concerns only with structural mobility (given vNM utilities), that is $e^m = \pi_t' u(\bar{x}) - \pi_{t+1}' u(\bar{x})$ is null if the two marginals coincide (by construction in the case of exchange mobility). Also, if $W - W^I$ is taken separating the first two factors from the latter in (D.1) and P is assumed to be an exchange mobility matrix, it can be observed that the first difference, $e^i = \pi_t' u(\bar{x}) - \pi_t' u(P\bar{x})$, might be regarded as an utilitarian inequality effect, while $e^u = \frac{1}{2}\pi_t'[\overrightarrow{\bar{V} u''(P\bar{x})}]$, may be intended as uncertainty effect (dividing e^i by $u'(P\bar{x})$, we get Arrow-Pratt risk premium). Recalling the Jensen's decomposition in (D.1)

$$\pi_t' Pu(\bar{x}) = \pi_t' u(P\bar{x}) + \frac{1}{2}\pi_t'[\overrightarrow{\bar{V} \times u''(P\bar{x})} \tag{D.2}$$

and given i) $\pi_t' P = \pi_{t+1}$ (by construction), ii) $\pi_t = \pi_{t+1}$ (exchange mobility), iii) re-arranging, then (D.2) is just $e^i = e^u$.

E Proof for proposition (3.4)

if $\varepsilon = \lambda$ the proof can be established for (9). Given the $N \times N$ matrix $Y := \{y_{i,j}\}$ indicating lifetime income for an individual starting from position i and ending up in position j, (9) can be rewritten as follows

$$W = \sum_i \pi_i \sum_j \pi_{j|i} U(y_{i,j}) = \sum_i \sum_j \pi_{i,j} U(y_{i,j}) \tag{E.1}$$

i.e. it is equivalent to (5). As the ex-ante social welfare function is just the same as the ex-post one, it must be the case that, given (9), income mobility is always ex-ante socially desirable. This is sufficient to establish the proof.

It's worth observing that the same result can be established in terms of the abbreviated social welfare function
Let's consider (13). The abbreviated social welfare function is

$$V = [y^{d\lambda}]^{d\varepsilon} = \left\{ \sum_i \pi_i \left[\sum_j \pi_{j|i} (y_{i,j})^{1-\lambda} \right]^{\frac{1-\varepsilon}{1-\lambda}} \right\}^{\frac{1}{1-\varepsilon}} \tag{E.2}$$

i.e. $[y^{d\lambda}]^{d\varepsilon}$ is the equally distributed equivalent income taken with respect to the distribution of certain equivalent incomes.
In the certain framework, it has been observed that

$$V = y^{d\varepsilon} = \left[\sum_i \sum_j \pi_{i,j} (y_{i,j})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{E.3}$$

Once again it can be observed that (E.2)=(E.3) if $\varepsilon = \lambda$.

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