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Mobility, Inequality, and Horizontal Equity

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Abstract

Mobility implies two distributions and a description of the transition process between these two distributions. A mobility index describes this transition. In most cases, mobility indices have been developed using properties of transition matrices independently of the concepts of inequality and equity. This paper presents a new tool – the Gini index of mobility – that provides an overall consistent framework for the analysis of mobility, inequality, and horizontal equity. The theoretical concepts are illustrated empirically using panel data from rural Mexico.

Key words: mobility, inequality, horizontal equity

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1. Introduction

Mobility implies two distributions (the initial and the final), and a description of the transition process between these two distributions. A mobility index is intended to describe this transition process. Sociologists and economists have proposed a wide range of mobility indices (e.g. Prais, 1955; Bartholomew (1967); Shorrocks, 1978a; Atkinson, 1983; Dardanoni, 1993; see also Atkinson and Bourguignon, 1992, for a review of empirical studies of earnings mobility and Bibby, 1975, for a review of the Sociological literature). In most cases, these indices have been developed using properties of transition matrices, independently of the concepts of inequality and equity that the investigator is intended to use. This paper presents a new tool – the Gini index of mobility – that, together with the Gini index of inequality, provides an overall consistent framework for the analysis of mobility, inequality, and horizontal equity. In so doing, we follow up on a few papers devoted to the relationships between the three concepts. For example, Shorrocks (1978b) shows how income mobility reduces inequality over time. King (1983) develops an index of mobility which can be decomposed into two components, one related to mobility, and the other to horizontal equity.

To motivate the paper and illustrate the relationship between mobility and inequality, consider the system of job rotation in the early days of the Kibbutz. Members of the Kibbutz rotate jobs. Hence, although at each given period of time there is no equality among members, inequality vanishes over time. Inequality is observed only because snap-shots are used to describe an otherwise lengthy process. Another example of the impact of time on inequality is that of the distribution of income over the life cycle. If one is interested in life-time inequality, yearly inequality is inappropriate. Although individuals may have exactly the same pattern of income flow over the life cycle, one will observe inequality simply because the time period used for measurement is too short. A third type of transition over time is associated with uncertainty. If the distribution of income is affected by random shocks, the resulting process can be analyzed formally in the same way as job rotation, except that in the case of job rotation the transition is known in advance to the individuals, while in the case of uncertainty it is not.[2] In all these cases, we will show that a mobility index can help in predicting the appropriate level of inequality over a period of time from a series of snap-shots at any given point in time.

Mobility is also related to horizontal equity. Usually, transition processes take time, but we may also think of instantaneous transition processes with no time dimension attached to them. This applies to changes in incomes due to a reform in taxation. Traditionally, the changes in individual rankings before and after taxes have been analyzed through the concept of horizontal equity. The main principle of horizontal equity in tax reforms is defined by Feldstein (1976: 95) as "*if two individuals would have the same utility level if the tax remained unchanged, they should also have the same utility level if the tax is*

[2] This type of problems may also occur when incomes are registered on a cash flow base rather than on an accrual basis. Different sources of income such as capital gains, farm income, and other types of capital income which are registered on a cash base may have different accumulation and distribution patterns over time. Relying on snap-shots of the distribution may exaggerate the impact of those incomes on inequality in the long run.

changed." The implication of Feldstein's definition of horizontal equality, (and to the best of our knowledge of all measures of horizontal equity), is that rank switching, i.e., the change in the ranking of individuals between the initial (before the reform) and final (after the reform) distributions, is an undesired property. The violation of this norm is the target of horizontal inequity measurement. Measures of horizontal equity are discussed among others in Atkinson (1979) and Plotnick (1981). But clearly, the same rank switching is also the target of mobility measurement, except that mobility is viewed as a desired property to have. The information needed for calculating an index of horizontal equity is identical to that needed for calculating an index of mobility. As will be shown, the Gini mobility index is equivalent to the Atkinson-Plotnick measure of horizontal inequity.

The Gini index of mobility has one more additional property, which is very useful for discussing inequality. Consider the case of having two criteria for ranking the population, such as money income and wealth. Each criterion enables the evaluation of a marginal distribution by a measure of inequality. Changing the criterion changes the level of inequality observed in the population. What can be said if one is interested in a composite measure of inequality taking both criteria into account? One solution is to create a scale to weight the two properties, and to compute the measure of inequality on a weighted average of the two variables. But in many circumstances there is no a-priori agreed upon weighting schemes and one has to search for an appropriate weighting scheme. To evaluate whether this is a serious problem, we state the conditions that enable us to predict the level of inequality when using a weighted average of the two criteria, without assuming specific weights. That is, the Gini mobility index, can also be useful in describing the change in ranking when one moves from using one criterion to the other.

The upshot from the above discussion is that many issues that involve transition from one distribution to another can be represented by the same index. This is shown in this paper using panel data from rural Mexico. The structure of the paper is as follows. In the second section, we introduce the concept of the Gini index of mobility, in both its symmetric and asymmetric forms. We discuss the connection between the Gini mobility index and transition matrices. Then, the use of the index is illustrated using information on rural Mexican households for on income and for wealth as measured by land ownership. Section 3 discusses the use of the Gini mobility index for predicting composite measures of inequality when the analyst is not able, or not willing, to specify the weights to be attributed to each factor in the composite measure of welfare for which an inequality measure is sought. The empirical analysis in that section is devoted to the inequality of a two functions, one depending on income and wealth, and the other depending on income at two points in time. Section 4 shows the equivalence between the Gini index of mobility and the Atkinson-Plotnick measure of horizontal equity, with an application of the impact of a cash transfer program to farmers on inequality and horizontal equity. A conclusion follows with suggestions for further research.

2. The Gini Index of Mobility

2.1. Definitions and properties

The most convenient way to define the Gini index of mobility is by using continuous distributions. However, since we want to analyze the relationship between the index and the literature on mobility, which relies on transition matrices, we will move back and forth from continuous to discrete variables to allow for matrix notation.

Let (Z_1, Z_2) denote a bivariate income distribution in states 1 (initial) and 2 (final). It is assumed that first and second moments exist. Define $Y_j = Z_j / \mu_j$ as the income in terms of mean income. Then (Y_1,Y_2) is a bivariate distribution with $\mu_1 = \mu_2 = 1$. A Mobility index should describe the association between observations in distribution 1 and in 2. We distinguish between symmetric and asymmetric indices of mobility. An index S_{12}, defined over distributions 1 and 2, is symmetric if it satisfies $S_{12} = S_{21}$, for any two distributions. A symmetric index of mobility does not distinguish between the initial and final distribution. The advantage of this property is that the index does not suffer from the index number problem that is typical to directional movement from one state to the other (for example, a price index may be increasing both when moving from state 1 to 2 or from 2 to 1).[3] The Gini symmetric index of mobility is defined as:

$$S_{12} = \frac{COV[(Y_1 - Y_2),(F_1(Y) - F_2(Y))]}{COV(Y_1, F_1(Y)) + COV(Y_2, F_2(Y))} \qquad (1)$$

where $F_j(Y)$ is the (marginal) cumulative distribution j.[4] The discrete version of the definition of the Gini mobility index in (1) is:

$$S_{12} = \frac{\sum_{k=1}^{n}(y_{1k} - y_{2k})(F_{1k} - F_{2k})}{\sum_{k=1}^{n}(y_{1k} - 1)F_{1k} + \sum_{k=1}^{n}(y_{2k} - 1)F_{2k}} \qquad (2)$$

where y_{jk} is the income of household k at time j and F_{jk} is the normalized rank of observation k in the distribution at time j. As shown in Appendix A, the symmetric Gini mobility index in (1) is equal to:

$$S_{12} = \frac{G_1(1-\Gamma_{12}) + G_2(1-\Gamma_{21})}{G_1 + G_2}, \qquad (3)$$

[3] The disadvantage of a symmetric index is that it requires more information than an asymmetric one.
[4] In the sample, the estimator of the marginal cumulative distribution is the rank of the observation divided by the sample size, n.

where G_j = 2 COV(Y_j,F_j(Y)) is the Gini coefficient for period j, while Γ_{js} = COV(Y_j,F_s(Y))/Cov(Y_j,F_j(Y)) is the Gini correlation coefficient (Schechtman and Yitzhaki, 1987 and 1999). The properties of the Gini correlation coefficient that are relevant to this study are:

(a) The Gini correlation coefficient is bounded[5], such that $1 \geq \Gamma_{js} \geq -1$;

(b) If distributions j and s are independent then $\Gamma_{js} = 0$;

(c) Γ_{js} is not sensitive to a monotonic transformations of distribution s;

(d) Γ_{js} is not sensitive to a linear monotonic transformations of distribution j; and

(e) In general, Γ_{js} need not be equal to Γ_{sj} and they may even have negating signs. However, if the random variables Z_j and Z_s are exchangeables then $\Gamma_{js} = \Gamma_{sj}$. Exchangeability essentially means that the shape of the marginal distributions is similar. More formally, as defined by Stuart and Ord (1987: 419), "*a set of random variables is said to be exchangeable if, for all n≥ 1, and for every permutation of n subscripts, the joint distributions of are identical.*" For the two Gini correlations to be equal it is required that the variables be exchangeable up to a linear transformation (Schechtman and Yitzhaki, 1987: 211). Only then can one change the order of the variables without affecting the Gini correlation. [6]

We also define the (directional) asymmetric mobility index $M_{js} = (1-\Gamma_{js})$, where j is the initial state and s is the final state. The symmetric index is a weighted average of the two asymmetric indices, weighted by the inequality in each distribution:

$$S_{12} = w_1 M_{12} + w_2 M_{21}, \tag{4}$$

where $w_i = G_i /(G_1+G_2)$ is the share of inequality of distribution i in the sum of inequality in the two periods. The properties of the various Gini indices of mobility are the following:

(i) Minimum Mobility: consider the case where $y_{2k} = t(y_{1k})$ for each k, where t() is a monotonic non-decreasing transformation. Then $S_{12} = M_{12} = M_{21} = 0$. To see that, note that by property (c) of the Gini correlation coefficient, $\Gamma_{21} = \Gamma_{12} = 1$. If the transition process has not changed the ranking of the units, then the mobility index equals zero. This corresponds to the immobility axiom in Shorrocks (1978). Note, however, that inequality can change between the initial and the final distributions. Examples of such cases are abundant: the application of a pure income tax so that the ranking of before tax income is identical to the ranking of after-tax income[7] or, alternatively, the effect of a tax reform that does not change the ranking of after-tax income. Another example is economic growth that affects all units by a monotonic increase of their incomes, as can be the case where the returns to schooling are changing (that is, distances between adjacent incomes increases or decreases), but the order is not

[5] An important property of the Gini correlation is that the bounds are identical for all marginal distributions. This property does not hold for Pearson correlation coefficient (Schechtman and Yitzhaki, 1999).
[6] See McCall (1991) for the connection between exchangeablity and economic modelling.
[7] Feldstein (1976) . See also measures of progression in the income tax (Lambert , 1993, ch.6)

reversed. Still another example is a macro-economic shock that affects all individuals without causing changes in ranks. Note, however, that this property is asymmetric in the sense that although inequality can change even if there is no change in the ranking, inequality can't change between the two distributions if there is no change in incomes.

(ii) Maximum mobility: assume that $y_{2k} = t(y_{1k})$ for each k, where t() is a monotonic no increasing transformation, then $\Gamma_{12} = \Gamma_{21} = -1$ and $S_{12} = M_{12} = M_{21} = 2$. Maximum mobility occurs if there is a total reverse in the ranks. That is, the richest in distribution 1 is the poorest in distribution 2, the second richest in distribution 1 becomes the second poorest in distribution 2, etc. In this case, the final distribution is derived from the initial distribution by a declining monotonic transformation. Note that in this case, mobility is independent of whether overall inequality increases or decreases between the initial and final distributions.[8]

(iii) Mid-Point: If y_2 and y_1 are statistically independent then $S_{12} = M_{12} = M_{21} = 1$. This is immediate from property (b) of the Gini correlation. Since in most cases of mobility the correlation between the initial and final marginal distributions tends to be positive, some investigators (e. g., Prais (1955) and his followers) defined independence as the extreme case of mobility. Shorrocks (1978a,b) on the other hand prefers to define property (b) as the extreme case. This distinction is not relevant for our purposes.

(iv) Higher mobility: this corresponds to an increase in the Gini mobility indices. That is, the lower the Gini correlations between the initial and the final distributions, the higher the mobility.

2.2. Relationship with transition matrices

Students of mobility have traditionally analyzed transition and turnover matrices.[9] The main interest in this literature is in occupational mobility, while the interest in this paper is in the impact of mobility on inequality. This difference in interest calls for slightly different approach. For convenience and without loss of generality, we divide the initial and the final populations into equi-proportional groups, so that the difference between a transition and a turnover matrix is a multiplication by a constant. In this section, we show, that provided that one is interested on the impact on the Gini index on inequality, then the Gini indices of mobility Γ_{js} and Γ_{sj} are sufficient statistics of the information contained in turnover and transition matrices. This means that transition matrices do not add additional information over the informational content of the mobility indices. To show the relationship between the Gini indices of mobility and transition matrices, it turns out to be convenient to rely on discrete distributions. Let y_{jk} (j=1,2; k=1,..,K) be the normalized income (so that the mean income equals one) and let F_{jk} be the normalized rank (the value of the empirical distribution, a number between zero and 1) of observation k in state j. Let also y_j, F_j be K x 1 vectors of normalized incomes and ranks in state j respectively. Without loss of generality, we will

[8] One can divide the indices by 2 , to keep the index between zero and one.

[9] A turnover matrix is a matrix whose elements sum to one. A turanstion matrix is a matrix whose raws sum to one. Usually, transition matrices represent conditional probabilities while the elements of a turnover matrix represent the joint probability distribution of the two variables.

assume that the vectors F_{jk} are arranged in an increasing order of the ranking of the first period, that is, F_1 is the only vector whose elements which must be arranged in non-decreasing order. Since we are dealing with normalized incomes, the Gini index of distribution j can be written as:

$$G_j = 2\ y'_j F_j - 1. \qquad (5)$$

Using the same procedure, the Gini correlation Γ_{js} is:

$$\Gamma_{js} = (y'_j F_s - 0.5)/(y'_j F_j - 0.5). \qquad (6)$$

Let T_{KK} represent the transition matrix. Since we are interested in an inequality index, aggregation of observations into groups may cause the loss of intra-group inequality. Therefore, the size of the matrix will be required to be the size of the sample. In the sample, the transition matrix will be a permutation matrix of the identity matrix.[10] Note, however, that the transition matrix in the population can take any shape that transition matrices are allowed to have.[11] [12]Since the mobility index is a sufficient statistics, there is no need to really construct the transition matrix and therefore, the size of the transition matrix is irrelevant for any practical purpose.

Let $t_{n,m}$ be an element in transition matrix T. The element $t_{n,m}$ =1 if the observation with rank n in state j moved to rank m in state s. Otherwise, $t_{n,m}$ =1 = 0. Then, it is easy to see that for the vector of ranks, we have

$F'_s = F'_j\, T_{js}$, and $F'_j = F'_s\, T'_{sj,}$

where j and s represent the initial and final distributions, and T_{sj} is the transpose of T_{js}.[13] The Gini correlation coefficient Γ_{js} is then defined as a function of the transition matrix as follows:

$$\Gamma_{js} = (y'_j F_s - 0.5)/(y'_j F_j - 0.5) = (y'_j F_j T_{js} - 0.5)/(y'_j F_j - 0.5)\ . \qquad (7)$$

The Gini correlation Γ_{sj} is obtained in a similar way, and the symmetric mobility index, which includes both Γ_{js} and Γ_{sj}, relies both on the transition matrix and its transpose. Furthermore, assume that a population goes through two consecutive transitional processes, described by the transition matrices T^1 and T^2. Then, the accumulated transition process over the two periods is $A = T^1\ T^2$. To compute the Gini indices of mobility over the two periods, one can proceed as before, using the matrix A as representing the overall transition process. An extension to more than two periods can be done in a similar way. This implies that one can study convergence and ergodic properties by using a series of Gini indices of mobility

[10] In this sense it is a special case of a doubly stochastic matrix, where each column and each raw sum to one. (Marshall and Olkin (1979, ch. 2). Note, however, that each element should be multiplied by a constant.

[11] Note that a similar situation arises whenever the variable is binary variable. Although the probability is a continuous variable, the realization of the variable in the sample is either one or zero.

[12] Traditionally, transition matrices are applied to discrete distributions, due to grouping. The fact that we are not dealing with groups is not due an inability to handle groups. Rather, we define the transition matrix at the household level in order to avoid loss of intra-group inequality, which is relevant for calculating the inequality index

[13] Since T is a permutation of the identity matrix, its inverse is identical to its transpose.

instead of the more complicated series of underlying transition matrices. The convergence of transition matrices to a given matrix will be equivalent to the transition of the Gini mobility index to a given number. Although we will not work with transition matrices in what follows (since we do not need to, thanks to the use of the Gini indices of mobility as sufficient statistics), it is worth to briefly describe the special cases of the Gini mobility indices in terms of the transition matrix:

(a) minimum mobility occurs if the transition matrix is the identity matrix.

(b) maximum mobility occurs if the transition matrix is composed of ones in the diagonal which is opposite to the main diagonal and zeros elsewhere.

(c) mid-point (statistical independence) occurs if the transition matrix is composed of identical lines and columns, and each entry equals 1/k. (Note that this case can only be described for the population, while in the sample, the entries are zero and one.)

(d) higher mobility: a transition process T^1 is defined as more mobile than T^2 if it can be achieved by mean preserving spreads of each raw. Higher mobility implies lower absolute values of Gini correlation, and higher absolute values for the Gini indices of mobility.

Given that one is interested in the Gini coefficient of one marginal distribution, then the Gini correlations represents the only informational content of the mobility matrix that is relevant for predicting the Gini coefficient of the other distribution. Hence, the Gini correlations are sufficient statistics of the sample for predicting the Gini.

2.3. The Relationship with Bartholomew's Index of Mobility

The mobility index that is the closest to the one suggested in this paper is Bartholomew's (1978, p.28, Equation (2.32)) index of mobility. Assuming a discrete distribution of occupations ranked from 1 to k, initial distribution p_i and transition probabilities p_{ij} , the index can be written as:

$$B = \frac{1}{k-1}\sum_{j=1}^{k}\sum_{i=1}^{k} p_i p_{ij} \,|\, i - j \,|. \tag{8}$$

Translating the index to continuous distributions, the index will be written as:

$$B = \int\int | z_1 - z_2 | f(z_1, z_2) dz_1 dz_2 , \tag{9}$$

Where Z_1 and Z_2 are the (non-normalized) variables in each period, respectively. It is difficult to analyze the exact properties of (9). In Appendix B, we show two properties of the index that are relevant to us. Assume that Z_1, Z_2 are independent and identically distributed then Bartholomew's index is identical to Gini's mean difference. This is surprising because as a measure of mobility one should expect it to be equal to the Gini correlation. Moreover, assuming that Z_1, Z_2 are independent but have different distributions, then:

$$B = 2COV_1(Z_1, F_2(Z_1)) + 2COV_2(Z_2, F_1(Z_2)) + 2\mu_2(F_{21} - 0.5) + 2\mu_1(F_{12} - 0.5)\,, \qquad (10)$$

Where $COV_1(Z_1, F_2(Z_1))$ is the covariance over observations of distribution 1, of the variate Z_1

with the ranking of Z_1 had it been ranked according to the properties of distribution 2, and

$$F_{12} = \int_0^\infty F_1(z_2) f_2(z_2) dz_2\,,$$

Is the expected value of the observations of distribution 2, had they been ranked according to the ranking of distribution 1. For a proof of (10), see Yitzhaki, 1994). The implications of Equation (10) are that, in general, Bartholomew's index is sensitive to the initial and final marginal distributions and therefore, may give a misleading picture of the transition process. For example, assume that both the initial and the final distributions are Normal with (μ, σ^2) and ($\mu+\Delta$, σ^2) respectively, and that they are statistically independent. (That is, a constant Δ is added to each observation). Under these assumptions we should have expected maximum mobility. But, Bartholomew's index will be equal to $B = 2\,\Delta\,F_{12}$. That is, the index points out to some degree of immobility even if the society is perfectly mobile. On the other hand, the Gini mobility index, although similar to Bartholomew's index, is not affected by linear transformations of the marginal distributions. [14]

2.4. Empirical illustration

To illustrate the use of the Gini mobility indices (asymmetric and symmetric), we use panel data on income, land owned, land ownership, and cash transfers to rural farmers from a survey conducted by the World Bank in collaboration with the Secretaria de Reforma Agraria of Mexico. The survey was carried in 1994 and 1997 in rural areas, in the so-called ejido sector. Until recently, Mexico’s ejido sector was functioning under a system of communal property whereby land could not be alienated, rented or mortgaged, and usufructuary rights were contingent on occupation and cultivation of the land. A land titling reforms was initiated in 1992 as part of the broader liberalization of Mexico's rural economy, enabling ejidatarios (those living in the ejidos) to own their land on an individual basis. Moreover, in line with the North American Free Trade Agreement requirements, government support programs for agricultural inputs (subsidies) and outputs (guaranteed prices) were terminated. To enable farmers to adjust, the Government created as of 1994 a temporary cash transfer program named Procampo, whereby eligible farmers receive for up to 15 years a fixed sum of money per hectare cultivated (see Cord and Wodon (1999) for details).

The subset of the survey data that will be used, in order to illustrate the properties of the Gini indices of mobility, consists of information on per capita incomes (1994 and 1997), per capita land owned (1994 and 1997), per capita land cultivated (1994 and 1997), and per capita transfers from Procampo (1997) only; (in 1994 the households did not yet receive the program). We also use expansion factors and household sizes in the analysis in order to use the appropriate weights. Summary statistics for all the variables of interest are given in Table 1. The mean quarterly per capita income is slightly higher in 1997 than in 1994 (in constant terms). The lack of growth in income between the two years is in large part due to Mexico's devaluation of December 1994 and subsequent economic downturn in 1995. There are a few households for which per capita incomes are negative, due to the possibility of losses in any given quarter for farmers (the cost of farm inputs may be larger than the revenues from the sale of outputs). These negative values do not represent any problem for the analysis, provided it is recalled that the Gini index of inequality can then be greater than one when the variable of interest has negative values (one such case will appear in the empirical analysis). On average, households own and cultivate two hectares of land per person. The standard deviation for the distribution of land is larger in 1997 than in 1994, as well as the maximum value of the land owned or cultivated. Finally, in 1997, Procampo payments amount on a per capita basis to 332.5 pesos per person on average, which is about 18.9 percent of average per capita income.

Table 2 provides the Gini indices of inequality for per capita income, land owned, and land cultivated in both years, as well as the various Gini indices of mobility. Because of the negative values, the Gini indices of inequality for per capita income are fairly high, at 0.818 in 1994 and 0.830 in 1997. The corresponding measures for land owned and cultivated are somewhat lower, but high as well. They range from 0.567 for land owned in 1994 to 0.628 for land cultivated in 1997. Inequality has increased between both years for both income and land (whether owned or cultivated). There is also substantive mobility between the two years both in terms of income and in terms of land. The highest level of mobility is observed in land cultivated, perhaps in part because of the impact of land reform. Now, that farmers can own their land, it is more easy for them to give it for cultivation to others without loosing their property.

[14] See Boudon, 1973, pp. 51-54 for a discussion of the properties of Bartholomew's index.

There is also a relatively high level of mobility in land owned, indicating that there are sales going on, also as a result of the land titling reform. Mobility is somewhat lower for per capita income, but nevertheless substantial given that only three years separate the two periods. The mobility in per capita income may be due in part to the fact that households having a bad quarter may have negative values in one year, but not in the other. The two asymmetric indices of mobility are fairly close to each other in all cases, which is an indication that there is likely to be exchangeability. The relatively high level of mobility hints that yearly observations suffer from high volatility and extending the time span of measurement can affect the measured inequality significantly.

3. Composite measures of inequality

3.1. Definitions and properties

One useful property of the Gini indices of mobility is that the indices help in estimating composite measures of inequality whereby the analyst is interested in the inequality of a weighted sum of attributes. Let $Y_{(\alpha)} = \alpha Y_1 + (1-\alpha)Y_2$ with $0 \leq \alpha \leq 1$. If α is known, then the Gini for $Y_{(\alpha)}$ can be directly calculated. However, if Y_1 and Y_2 represent two different attributes such as land ownership and income, or if they represent incomes at two different points in time, then one might prefer not to be forced to assume a particular value for α. In this case, it is shown in the Appendix A that the Gini of $Y_{(\alpha)}$ is bounded as follows:

$$\text{Max}\,[0, \alpha G_1 \Gamma_{12} + (1-\alpha)\, G_2 \Gamma_{21}\,] \leq G_{Y(\alpha)} \leq \alpha\, G_1 + (1-\alpha)\, G_2\,. \qquad (11)$$

In a typical case, equation (11) would provide a meaningful range for predicting composite inequality. The upper bound is obtained under perfect Gini correlation whereby the ranks in the two distributions are the same. The lower bound takes into account the Gini correlations between the two variables. The larger the Gini correlations (assuming they are positive, as is the case here), the higher the lower bound, and the smaller the interval of possible values for $G_{Y(\alpha)}$. For example, if $\Gamma_{12} = \Gamma_{21} = 0.9$, then we have $0.9\,[\alpha G_1 + (1-\alpha)\, G_2\,] \leq G_{Y(\alpha)} \leq \alpha G_1 + (1-\alpha)G_2$.

This property enables the evaluation of inequality of a weighted average of variables when one is unwilling to quantify the relative importance of the two variables in the overall distribution (the property is valid for any scalars α and β, i.e. the sum of the scalars need not equal one.) A special case occurs when the two variables entering in the composite index of inequality are exchangeable. If Y_1, Y_2, and $Y_{(\alpha)}$ are exchangeable, as is the case for example for the bi-variate normal distribution, then property (e) of the Gini correlation implies that $\Gamma = \Gamma_{12} = \Gamma_{21}$. Assume also that $\Gamma_{\alpha 1} = \Gamma_{1\alpha;}$ and $\Gamma_{\alpha 2} = \Gamma_{2\alpha}$ (this property holds for normal distributions, but not for log-normal or uniform marginal distributions.) Then, as proven in Appendix A, one can get an explicit solution for estimating $G_{Y(\alpha)}$.

$$G^2_{Y(\alpha)} = \alpha^2 G_1^{\,2} + (1-\alpha)^2\, G_2^{\,2} + 2\alpha(1-\alpha)G_1G_2\Gamma \qquad (12)$$

Equation (12) indicates that, under exchangeability, the Gini behaves in a way similar to the variance. We will not use Equation (12) in the empirical illustration below, which focuses on illustrating the use of equation (11), which is more general.

3.2. Empirical illustration

Assume that both income and wealth determine the well-being of an individual in society, but the relative weights of the two factors can only be approximated. The joint distribution of income and wealth is known. Then one may want to evaluate the inequality of a combined index of these two variables without having to exactly specify the weight attached to each factor. In the case of rural Mexico, we can take the amount of land owned per capita as a proxy for wealth. Using the values in Table 2, and considering different values for α, one gets the results in the first part of Table 3. The lower and upper bounds were computed using equation (11). It can be seen that in the case of income and land ownership, the lower and upper bounds provide a relatively wide interval because the Gini correlation between per capita income and per capita land ownership is not very high. An index of well-being which would take into account both income and land would thus result in a substantial reduction in the measured level of inequality.

Another application relates to inequality over time. If one wants to take into account two or more periods for computing the Gini index of inequality, one can do so without specifying the weights (in this case the discount rate) for the two periods provided one has computed the Gini asymmetric indices of mobility. The results of the simulations obtained for various values of α with the per capita incomes of the two periods are given in the second part of Table 3. The predicted interval is smaller due to the relatively large Gini correlation of income for the two years. Still, given the results in Table 3, it can be seen that income inequality could decrease by a maximum of 25 percentage points if two time periods were taken into account (as an indicator of economic well-being) instead of one.

4. Mobility and Horizontal Equity

4.1. Definitions and properties

The asymmetric mobility index is measuring the change in the ranks between the base period and the final period weighted by the income in the base period. Note that the non-weighted average change in ranks is zero by definition. Hence, the deviation of the index from zero is caused by the correlation between the changes in ranks and in incomes. An increase (decrease) in rank takes place together with an increase (decrease) in income. Since the asymmetric mobility index does not take this simultanous change in incomes into account, it tends to underestimate the impact of changes when ranks increase and to overestimate when ranks decrease. In essence, this is the index number problem. The lower bound in equation (11) is constructed by taking this property into account (see Appendix A for the proof). This property is responsible for the property that the two asymmetric mobility indices of the Gini need not have the same sign, which means that whether mobility has increased or decreased may depend on the choice of the base period.

As pointed out by King (1983) the same measures can be applied to horizontal inequity and mobility. Accordingly, it is easy to show that the Atkinson-Plotnick index of horizontal inequity is actually a special case of the asymmetric Gini mobility index. That is, it can be shown that:

$AP = (1/2)\,(1-\Gamma_{ba})$

where AP indicates the Atkinson-Plotnick (Atkinson (1979) and Plotnick (1981)) index of horizontal inequity, while b and a represent before and after reform distributions. As shown by Lerman and Yitzhaki (1995) the other Gini correlation coefficient (i. e., Γ_{ab}) is a key parameter in Kakwani's (1984) index of horizontal inequity, hence we can conclude that the index number problem can also be demonstrated to appear in indices of horizontal inequality too. If one wants to impose symmetry on the index, then it will be appropriate to use the symmetric version of the Gini mobility index.

4.2. Empirical application

Table 4 provides the results of the impact of Procampo, the program of cash transfers to farmers, on income inequality in rural Mexico, and on horizontal equity. Without Procampo, the Gini index of inequality is 1.02 in 1997 (remember that with negative income values the Gini index can be greater than one). With Procampo, the Gini index of inequality is reduced to 0.830. The Gini correlations between the incomes with and without Procampo are very high, so that the mobility indices are small, at 0.023 and 0.035 depending on which distribution is taken as the base. The index of horizontal equity of Atkinson-Plotnick, which is half the asymmetric mobility index when using the incomes without Procampo as the base, is small at 0.011, implying that Procampo results in fairly limited reranking in the population, in part because so many farmers benefit from the program, in proportion to the land they cultivate (which is itself positively correlated with per capita income).

6. Conclusion

In this paper, we have presented a new index of mobility and its properties. The index provides a summary statistics for transition matrices. It is related to the Gini index of inequality and to measures of horizontal inequity, which is why we have termed it the Gini index of mobility. It is in the eye of the beholder to determine whether the attitude toward an increase in the index should be positive or negative. As we have argued, mobility and horizontal inequity can be viewed as representing the same formal process, except that mobility implies a positive attitude, and horizontal inequity a negative one. Another (well-known) conclusion of our work is that marginal distributions (snap-shots or distributions that are based on one attribute) tend to exaggerate overall inequality. Hence, if the interest is in some kind of convex combination of the marginal distributions, we should expect the inequality of the combination to be lower than the inequality observed from the marginal distributions. In other words, following on the work of Atkinson (1983), Shorrocks (1978), King (1983), and Atkinson and Bourguignon (1992), we have shown that analyzing mobility can be interpreted as adding a dynamic and/or additional dimension to inequality analysis. In our framework, the links between mobility, inequality, and horizontal equity have been made explicit for the special case of the widely used Gini index.

The empirical applications, based on data from Mexico, have shown the wide applicability of the index. We have measured the extent of inequality and income mobility in the ejido sector of rural Mexico between 1994 and 1997; the impact of cash transfers programs on inequality and mobility, with a discussion of horizontal inequality; and how the tools presented can be applied to generate bounds for composite indices of inequality when the weights of the various components of the measure of welfare (such as income and land ownership) are not known.

APPENDIX A:

Proof of Equation (3):

The symmetric mobility index is defined as follows:

$$S_{12} = \{2COV[Y_1-Y_2, F_1(Y)-F_2(Y)]\}/[G_1 + G_2]$$

Using the properties of the covariance, this can be written as:

$$S_{12} = \{2COV[Y_1, F_1(Y)]+2COV[Y_2, F_2(Y)]-2COV[Y_1, F_2(Y)]-2COV[Y_2, F_1(Y)]\}/[G_1+G_2]$$

Using the definitions of the Gini coefficient and Gini correlation completes the proof.

Proof of equation (11):

The proof consists of finding upper and lower bound for $G_{Y(\alpha)}$. The upper bound is

$$\begin{aligned} G_{Y(\alpha)} &= 2\ COV[\alpha Y_1+(1-\alpha)Y_2\ , F(Y(\alpha)] \\ &= 2\ \alpha\ COV[Y_1, F(Y(\alpha)] + 2\ (1-\alpha)\ COV[Y_2, F(Y(\alpha)] \\ &\leq 2\ \alpha\ COV[Y_1, F(Y_1)] + 2\ (1-\alpha)\ COV[Y_2\ , F(Y_2)] = \alpha\ G_1 + (1-\alpha)\ G_2. \end{aligned}$$

The derivation of the upper bound is based on Cauchy-Shwartz inequality, which can be utilized to show that for all Y_j and $Y_{k,}$ $COV[Y_j, F(Y_k)] \leq COV[Y_j, F(Y_j)]$.

The lower bound obtains from the following:

$$\begin{aligned} G_{Y(\alpha)} &= 2\ COV[\alpha Y_1+(1-\alpha)Y_2, F(Y(\alpha))] \\ &= 2\ \alpha\ COV[Y_1, F(Y(\alpha))] + 2\ (1-\alpha)\ COV[Y_2, F(Y(\alpha))] \\ &\geq Max\ [0, 2\ \alpha\ COV[(Y_1, F(Y_2)] + 2\ (1-\alpha)\ COV[Y_2\ , F(Y_1)] \\ &= Max\ [0, \alpha\ G_1\ \Gamma_{12}\ + (1-\alpha)\ G_2\ \Gamma_{21}\ ]. \end{aligned}$$

Proof of Equation (12):

Equation (12) states that when the variables are exchangeable then:

$$G^2_{Y(\alpha)} = \alpha^2 G_1^{\ 2} + (1-\alpha)^2\ G_2^{\ 2} + 2\alpha(1-\alpha)G_1G_2\Gamma^2$$

As before, using the properties of the covariance we can write:

$$\begin{aligned} G_{Y(\alpha)} &= 2\ COV[\alpha Y_1+(1-\alpha)Y_2, F(Y(\alpha))] \\ &= 2\ \alpha\ COV[Y_1, F(Y(\alpha))] + 2\ (1-\alpha)\ COV[Y_2, F(Y(\alpha))] \\ &= \alpha\ \Gamma_{1\alpha}\ G_1 + (1-\alpha)\ \Gamma_{2\alpha}\ G_2 \end{aligned}$$

Under our assumptions (exchangeability and that $\Gamma_{j\alpha} = \Gamma_{\alpha j}$ for j=1,2), we can substitute the above terms in the above equation, use the definition of the Gini correlations in terms of covariances and rearranging terms, to get Equation (12).

Appendix B: The Relationship between Bartholomew's index and the Gini Mobility index.

The aim of this appendix is to show that Bartholomew's index has, on one hand, some similarity to the Gini index but, on the other hand, is also affected by the marginal distributions, in a way, which is not appropriate for an index of mobility.

Let δ be Bartholomew's index. Then, in the general case, with the two variables being continuous, the definition of δ is the absolute difference between first and second period variables. That is:

$$\delta = E\{|Z_1 - Z_2|\}. \tag{B.1}$$

Which can be written explicitly as:

$$\delta = \int_0^\infty \int_0^\infty |z_1 - z_2|\, f(z_1, z_2) dz_1 dz_2 \,, \tag{B.2}$$

Using the definition $|z_1 - z_2| = \mathrm{Max}\,[z_1, z_2] - \mathrm{Min}\,[z_1, z_2]$, then

$$\begin{aligned}\delta &= \int_0^\infty \int_0^{z_2} (z_2 - z_1)\, f(z_1, z_2) dy_1 dy_2 + \int_0^\infty \int_{z_2}^\infty (z_1 - z_2)\, f(z_1, z_2) dz_1 dz_2 = \\ &= \int_0^\infty z_2 \int_0^{z_2} f(z_1, z_2) dz_1 dz_2 - \int_0^\infty \int_0^{z_2} z_1\, f(z_1, z_2) dz_1 dz_2 + \\ &\int_0^\infty \int_{z_2}^\infty z_1\, f(z_1, z_2) dz_1 dz_2 - \int_0^\infty z_2 \int_{z_2}^\infty f(z_1, z_2) dz_1 dz_2 .\end{aligned} \tag{B.3}$$

Equation (B.3) is not easy to handle. To show some of the properties of the index it is assumed that Z_1 and Z_2 are independent with density functions $f_1()$ and $f_2()$ respectively. Then $f(z_1,z_2) = f_1(z_1)\, f_2(z_2)$. Using this property, we can rewrite (B.3) as:

$$\begin{aligned}\delta &= \int_0^\infty z_2 \int_0^{z_2} f_1(z_1) dz_1 f_2(z_2) dz_2 - \int_0^\infty \int_0^{z_2} z_1\, f_1(z_1) dz_1 f_2(z_2) dz_2 + \\ &\int_0^\infty \int_{z_2}^\infty z_1\, f_1(z_1) dz_1 f_2(z_2) dz_2 - \int_0^\infty z_2 \int_{z_2}^\infty f_1(z_1) dz_1, f_2(z_2) dz_2 .\end{aligned} \tag{B.4}$$

Equation (B.2), subject to the independence assumption is investigated in Yitzhaki (1994, Appendix), where it is defined as "the distance between distributions and it is shown to be equal to:

$$\delta = 2COV_1(Z_1, F_2(Z_1)) + 2COV_2(Z_2, F_1(Z_2)) + \mu_1(F_{21} - 0.5) + \mu_2(F_{12} - 0.5)\,, \qquad \text{(B.5)}$$

Where

$$COV_2(Z_2, F_1(Z_2)) = 2\int_0^\infty z_2 F_1(z_2) f_2(z_2) dz_2 - \mu_2 F_{12}\,, \qquad \text{(B.6)}$$

And

$$F_{12} = \int_0^\infty F_1(z_2) f_2(z_2) dz_2\,. \qquad \text{(B.7)}$$

The term $COV_I(Z_i, F_j(Z_i))$ is the covariance between of the observations of distribution i and their rank, had they been ranked according to the ranking of distribution j, while F_{ji} is the expected value of the ranking of observations of distribution i had they been ranked according to the ranking of distribution j.

Equation (B.5) can be evaluated for different marginal distributions so that the properties of Bartholomew's index can be evaluated:

(a) assume that Z_1 and Z_2 are i. i. d. random variables, then

δ=4COV(Z,F(Z)) which, as shown in Lerman and Yitzhaki (1984) is equal to Gini's mean difference.

(b) Assume that that Z_1 and Z_2 are statistically independent with different means. Then, as can be seen from Equation (B.5), the expected value of the distributions affects the index. Boudon (1973) criticizes the Bartholomew's index because of the sensitivity to the properties of marginal distributions.

Table 1: Summary statistics for all variables

Variable (PC = per capita)	*Mean*	*Std. Dev.*	*Min*	*Max*
Year 1994				
PC net income, pesos	1537.32	4089.88	-731.08	71661.95
PC land owned, hectares	1.97	3.25	0.00	65.00
PC land cultivated, hectares	1.85	3.19	0.03	65.00
Year 1997				
PC net income, pesos	1770.30	4202.08	-1257.07	51207.98
PC Procampo transfer, pesos	332.51	522.93	0.00	7878.00
PC land owned, hectares	2.20	5.44	0.00	188.75
PC land cultivated, hectares	2.02	5.74	0.00	201.00

Source: Authors' estimation from World Bank/SRA rural Mexico ejido surveys, 1994 and 1997
Note: the sample is restricted to households for which all variables are available (1027 observations)

Table2: Gini indices of inequality and mobility (symmetric and asymmetric)

	Income	*Land owned*	*Land cultivated*
Gini index of inequality, 1994	0.818	0.567	0.576
Gini index of inequality, 1997	0.830	0.603	0.628
Asymmetric index of mobility, 1994 to 1997	0.261	0.297	0.335
Asymmetric index of mobility, 1997 to 1994	0.267	0.312	0.330
Symmetric index of mobility	0.264	0.305	0.333

Source: Authors' estimation from World Bank/SRA rural Mexico ejido surveys, 1994 and 1997
Note: the sample is restricted to households for which all variables are available (1027 observations)

Table 3: Prediction ranges for composite inequality measures, income and wealth

	Income and land ownership		*Income at two points in time*	
	Gini correlation income and land	Gini correlation Land and income	Gini correlation income 1994 and 97	Gini correlation income 1997 and 94
Γ_{sj}	0.491	0.381	0.739	0.733
Alpha	Lower bound	Upper bound	Lower bound	Upper bound
0.0	0.408	0.830	0.608	0.830
0.1	0.390	0.807	0.608	0.829
0.2	0.372	0.785	0.607	0.828
0.3	0.355	0.762	0.607	0.826
0.4	0.337	0.739	0.607	0.825
0.5	0.319	0.716	0.606	0.824
0.6	0.301	0.694	0.606	0.823
0.7	0.283	0.671	0.606	0.822
0.8	0.266	0.648	0.605	0.820
0.9	0.248	0.625	0.605	0.819
1.0	0.230	0.603	0.605	0.818

Source: Authors' estimation from World Bank/SRA rural Mexico ejido surveys, 1994 and 1997
Note: the sample is restricted to households for which all variables are available (1027 observations)

Table 4: Impact of Procampo cash transfers on income inequality and horizontal equity

Inequality	
Per capita income Gini index with Procampo	0.830
Per capita income Gini index without Procampo	1.002
Mobility	
Asymmetric index between PC income with and without Procampo	0.023
Asymmetric index between PC income without and with Procampo	0.035
Symmetric index of mobility	0.030
Horizontal equity	
Atkinson-Plotnick measure of horizontal inequity	0.011

Source: Authors' estimation from World Bank/SRA rural Mexico ejido surveys, 1994 and 1997
Note: the sample is restricted to households for which all variables are available (1027 observations)

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